Discussion Paper 111

Institute for Empirical Macroeconomics
Federal Reserve Bank of Minneapolis
250 Marquette Avenue
Minneapolis, Minnesota 55480-0291

June 1996

Limited-Dependent Rational Expectations Models With Jumps

M. Hashem Pesaran*

University of Cambridge

Francisco J. Ruge-Murcia*

Université de Montréal and C.R.D.E.

ABSTRACT

This paper develops a Limited-Dependent Rational Expectations (LD-RE) model where the bounds can be fixed for an extended period, but are subject to occasional jumps. In this case, the behavior of the endogenous variable is affected by the agent's expectations about both the *occurrence* and the *size* of the jump. The RE solution for the one-sided and two-sided band are derived and shown to encompass the cases of perfectly predictable and stochastically varying bounds examined by earlier literature. We demonstrate that the solution for the one-sided band exists and is unique when the coefficient of the expectational variable is less than one. In the case of a two-sided band, the RE solution exists for all the parameter values and is unique if the coefficient of the expectational variable is less than or equal to one. These results hold even when the jump probability is stochastically varying and the error terms are conditionally heteroscedastic. As an illustration, we estimate a model of exchange rate determination in a target zone using data for the Franc/Mark exchange rate. Empirical results provide support for the non-linear model with time-varying realignment probability and indicate that the agents correctly anticipated most of the observed changes in the central parity.

*We have received helpful comments and suggestions from Pierre Perron and seminar participants at the University of Cambridge and the World Congress of the Econometric Society (Tokio, August 1995). Casper de Vries, Philip Stork, and Kees Koedijk graciously provided us with some of the data employed in this project. Financial support from Isaac Newton Trust Fund of Trinity College and the Economic and Social Research Council under grant No. R000233427 is gratefully acknowledged. Correspondence: M. H. Pesaran, Faculty of Economics and Politics, University of Cambridge, Austin Robinson Building, Sidgwick Avenue, Cambridge CB3 9DD, United Kingdom. E-mail: MHP1@econ.cam.ac.uk. An earlier version of this paper was circulated under the title "A Discrete-Time Version of Target Zone Models with Jumps." The views expressed herein are those of the author(s) and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

LIMITED-DEPENDENT RATIONAL EXPECTATIONS MODELS WITH JUMPS

by

M. Hashem Pesaran
University of Cambridge

and

Francisco J. Ruge-Murcia
Université de Montréal and C.R.D.E.

January 1996

I. Introduction

This paper is concerned with the econometric issues that arise in the empirical analysis of markets subject to a government policy aimed at keeping the price movements within publicly announced lower and upper bounds. Important economic examples are the exchange rate target zone mechanism, and the floor and ceiling restrictions on the price movements of some primary commodities [see Ghosh (1987)]. Some of the econometric issues involved in the analysis of such markets are addressed in the recent literature on limited-dependent rational expectations (LD-RE) models. See, for example, Chanda and Maddala (1983, 1984), Shonkwiler and Maddala (1985), Holt and Johnson (1989), Pesaran and Samiei (1992, 1995), Donald and Maddala (1992), and Lee (1994). This literature has focused on the rather restrictive case where the bounds are assumed to be pre-determined and fully *credible*. However, in most applications of interest the publicly announced bounds are not necessarily credible to the agents, and are often subject in practice to sudden and unpredictable movements. In a recent paper, Pesaran and Ruge-Murcia (1995) extend the analysis of the LD-RE models to the case of stochastic thresholds where the bounds vary randomly in *every* period. Using Monte Carlo experiments the authors show that the assumption that the band is perfectly predictable by the agents (when inappropriate) can seriously bias the estimates and the inferences based on them.

In this paper, we consider the more general and relevant case where the bounds can remain fixed over an extended period of time, but are subject to discrete, occasional jumps. The timing and size of these changes in the bounds are assumed to be not fully predictable by the economic agents. This extension is particularly important for modelling (*i*) changes in short-term interest rates targets [see Balduzzi et. al. (1993)], (*ii*) central parity realignments in exchange rates target zones, and (*iii*) unpredictable changes in the floor price of commodity price support schemes. All these examples share the feature that the adjustments to the bounds established by the authorities are implemented infrequently and by finite amounts. Thus, the uncertainty regarding the future value of the bound has two related, but conceptually distinct, components. The first one is the occurrence or timing of the adjustment. The second one is the size or magnitude of the adjustment. We model the occurrence of jumps in the bounds by means of a discrete state variable whose current value is defined by whether there is a change in the bound or not. This state variable is postulated to follow an ergodic Markov-chain with possibly time-varying transition probabilities and its current value is conjectured to be contemporaneously observed by the agents. The later assumption simply follows from the fact that the band is publicly announced and has the implication the state is an observed, rather than a latent variable [as in Hamilton (1989)]. Conditional on there being a jump in the bound, the *size* of the adjustment is then specified as a (continuous) function of the model's forcing variables.

The LD-RE with jumps postulates a more general specification for the bounds while preserving an important feature of the standard LD-RE model. Since the government's intervention prevents the endogenous variable from falling below the band, the observed endogenous variable is censored. The agents' decision is affected by the presence of the band and by the expectations of the government's intervention. Furthermore, expectations affect the behavior of the variable even while it is above the band. In the LD-RE with jumps, the government's intervention is not confined to changes in the fundamentals but it includes the possibility of adjustments to the bounds. Since these adjustments are not perfectly predictable, agents consider both the government's intervention (through the fundamentals) and the stochastic nature of the band when constructing their expectations about the endogenous variable.

The plan of the paper is as follows: Section II provides a general formulation of the LD-RE model with stochastic jumps. Sections II and III derive respectively the exact RE solution for the one-sided and two-sided bounds and show that the solution encompasses the cases of perfectly predictable and continuously and stochastically varying bounds already examined in the literature. (The mathematical proofs are relegated to appendices). It is demonstrated that the solution for the one-sided band exists and is unique when the coefficient of the expectational variable is less than one. In the case of a two-sided band, it is shown that the RE solution exits for all the parameter values and is unique if the coefficient of the expectational variable is less than or equal to one. Section V shows that these results hold even when the jump probability is stochastically varying and the disturbance terms are conditionally heteroscedastic. The log-likelihood function of the model is derived in Section VI. Finally, Section VII applies the model to monthly observations on French Franc/Deutsche Mark bilateral exchange rate between July 1979 and April 1993. During this period, the exchange rate was subject to a target-zone regime, with six (stochastic) jumps taking place in the central parity. In this application, we consider a dynamic, sticky-price exchange rate target-zone model, and show how the exchange rate equation predicted by the theoretical model can be approximated by a simple LD-RE model with jumps. We then estimate four models; the first being a benchmark linear RE model, with the remaining specifications taking account of the effect of the target zone on the agents' expectations, but differing in the way the probability of realignment is modelled. The empirical results support the LD-RE model with a non-zero, time-varying probability of realignment. We also found important asymmetries in the relationship between the realignment probability and deviation of the exchange rate from the central parity. For plausible values of the interest differential, probability of realignment was found to be close to zero when the exchange rate was at the bottom of the band, and became significantly larger than zero (sometimes close to one) when it was in the upper half of the band. Using this framework, we are able to formally address the issue of the credibility of the target zone regime. We conclude that although the target zone has generally been a credible instrument of exchange rate management, in periods preceding parity realignments the announcements by the government about the immutability of the bounds have not been credible to the agents. This is an important shortcoming of the target-zone regime which makes it highly vulnerable to serious bouts of currency speculation.

II. The LD-RE Model and Specification of the Bounds

Consider a variable y_t whose process (in the absence of censoring) is described by the linear rational expectations equation,

$$y_t = \gamma E(y_t | I_{t-1}) + \beta x_t + u_t , \tag{2.1}$$

where γ is a non-zero scalar coefficient, β is a $1 \times k$ vector of parameters, x_t is a $k \times 1$ vector of predetermined variables including an intercept term and (possibly) lagged values of y_t, I_{t-1} is the non-decreasing set of information available to the agents at time t-1, $E(y_t|I_{t-1})$ is the conditional expectation of y_t, and u_t is a disturbance term. For the $\{x_t\}$ process, we adopt the following general linear specification,

$$x_t = \Gamma_1 z_{1,t-1} + \eta_t , \tag{2.2}$$

where Γ_1 is a $k \times m$ matrix of coefficients, $z_{1,t-1}$ is a $m \times 1$ vector of predetermined variables (possibly including lagged values of x_t and y_t), and η_t is a $k \times 1$ vector of random disturbances. Notice that the specification assumed for the x_t process is quite general, in the sense that it encompasses vector autoregressive schemes, does not rule out unit-roots in the process, and accommodates the case when x_t is first-difference stationary.

Consider the situation when an exogenously given lower bound is imposed on the variable y_t. That is, y_t is prevented (for example, through government intervention) from taking values below the lower limit denoted by y_{Lt}. In this case, y_t is a censored variable and the observed dependent variable is given by,

$$y_t = Max\{\gamma E(y_t | I_{t-1}) + \beta x_t + u_t , y_{Lt}\},$$

or equivalently,

$$y_t = \begin{cases} \gamma E(y_t | I_{t-1}) + \beta x_t + u_t , & \text{if } \gamma E(y_t | I_{t-1}) + \beta x_t + u_t > y_{Lt}, \\ y_{Lt} & \text{otherwise}. \end{cases} \tag{2.3}$$

The special case when the bound y_{Lt} is constant and/or perfectly forecastable by the agents has been examined by a number of researchers [*e.g.*, Chanda and Maddala (1983, 1984), Shonkwiler and Maddala (1985), Holt and Johnson (1989), Pesaran and Samiei (1992, 1995), Donald and Maddala (1992), and Lee (1994)]. Pesaran and Ruge-Murcia (1995) consider the more general case when the bound is changing stochastically in *every* period and, consequently, the future value of y_{Lt} is no longer perfectly predictable by the agents.

This paper generalizes the model examined by Pesaran and Ruge-Murcia (1995) by considering a stochastic specification of the bound in which y_{Lt} can be fixed for more than one period but is subject to discrete, occasional jumps. In this case, the behaviour of the endogenous variable is affected by the agents' expectations about both the likely *occurrence* and the *size* of the jump. More specifically, we assume that the stochastic process of the threshold, y_{Lt}, is given by,

$$y_{Lt} = y_{L,t-1} + s_t(\delta_t + v_t), \tag{2.4}$$

where $y_{L,t-1}$ is the level of the bound in effect in the previous period, s_t is a discrete state variable whose value depends on whether an adjustment in the bound takes place at time t ($s_t = 1$) or not ($s_t = 0$). The term $\delta_t + v_t$ decomposes the size of the change in the bound that would occur at time t if $s_t = 1$, into a forecastable component δ_t, and a random, non-predictable part v_t. The predictable part, δ_t, is modelled by,

$$\delta_t = \Gamma_2 z_{2,t-1}, \tag{2.5}$$

where Γ_2 is a $1 \times q$ vector of fixed coefficients and $z_{2,t-1}$ is a $q \times 1$ vector of predetermined variables contained in I_{t-1}.

Under the process in (2.4), the lower bound follows a random walk on random time-steps. Examples of such variables include the targeted Federal Fund Rate used by the monetary authorities in the United States [see Balduzzi et. al. (1993)] and the exchange rate target bands in the EMS. Figure 1 presents monthly observations of the French Franc/Deutsche Mark exchange rate between April 1979 and April 1993. Notice that throughout the sample period six adjustments in the band were implemented by the monetary authorities.[1] In the model developed in this paper, the government affects the endogenous variable, y_t, not only through market intervention but also by means of changes in the bounds. Moreover, in an economy in which agents are rational, agents will consider both the stochastic nature of the band as well as the effect of government intervention when constructing their expectations about the future values of the endogenous variable. Thus, the agents' perception that the bound is not immutable, but that instead is subject to

[1] The realignments on the Ffr/DM parity took place in September 1979, October 1981, June 1982, March 1983, April 1986, and January 1987.

unannounced movements, is allowed to have an effect on the path of the variable the government seeks to control.

It is assumed that at time t-1, the state s_{t-1} is observed by the agents. This conjecture follows directly from the fact that the band is publicly announced and has the implication the state is an observed, rather than a latent variable [as in Hamilton (1989)]. However, notice that the agents still need to construct a forecast of s_t, as part of forming expectations of y_t. We postulate that the current state, s_t, depends *only* on s_{t-1} through a Markov-chain with a (possibly time-variant) matrix of transition probabilities given by,

$$P(t) = \begin{bmatrix} P_{00}(t) & P_{01}(t) \\ P_{10}(t) & P_{11}(t) \end{bmatrix}, \tag{2.6}$$

where $P_{ij}(t) = Pr(s_t = j | s_{t-1} = i)$, for $i, j = 0,1$, and by construction $P_{i0}(t) + P_{i1}(t) = 1$ for $i = 0, 1$. This specification allows additional restrictions to be imposed on the elements of $P(t)$ and/or the parameterisation of the transition probabilities in terms of predetermined variables. Thus, $P_{ij}(t)$ could be written as $P_{ij}(t) = \Psi(z_{3,t-1})$, where $\Psi(\bullet): \Re \rightarrow [0,1]$, and $z_{3,t-1}$ designates a set of predetermined variables contained in the agents' information set at time t-1, possibly including lagged values of y_t and x_t, but *excluding* $E(y_t | I_{t-1})$. Finally, to complete the specification of the model, it is assumed that conditional on x_t and I_{t-1}, the disturbance terms u_t, v_t, and η_t are identically and independently distributed (*i.i.d*) with zero means and constant variance-covariance matrix given by[2]

$$Cov\begin{pmatrix} u_t \\ v_t \\ -- \\ \eta_t \end{pmatrix} = \begin{bmatrix} \sigma_u^2 & 0 & | & 0_{1\times k} \\ 0 & \sigma_v^2 & | & 0_{1\times k} \\ -- & -- & -- & -- \\ 0_{k\times 1} & 0_{k\times 1} & | & \Omega \end{bmatrix}, \tag{2.7}$$

where $0_{1\times q}$ denotes a $1 \times q$ vector of zeros and Ω is the $k \times k$ variance-covariance matrix of η_t.

In what follows, it also proves convenient to define the composite and scaled random variables,

$$\omega_t = u_t + \beta\eta_t, \qquad \text{with} \qquad Var(\omega_t) = \sigma_\omega^2 = \sigma_u^2 + \beta\Omega\beta',$$

[2] The case when the error terms are conditionally heteroscedastic will be examined in Section V.

$$\varepsilon_t = u_t + \beta\eta_t - v_t, \qquad \text{with} \qquad Var(\varepsilon_t) = \sigma_\varepsilon^2 = \sigma_u^2 + \beta\Omega\beta' + \sigma_v^2,$$

$$\upsilon_t = \omega_t/\sigma_\omega, \qquad \text{and} \qquad \xi_t = \varepsilon_t/\sigma_\varepsilon .$$

As shown below, the rational expectations solution depend on the cumulative distributions function of υ_t and ξ_t which are respectively denoted by $F(\bullet)$ and $H(\bullet)$. Other key variables entering the solution are,

$$c_{Lt}^{0} = \left[y_{Lt-1} - \gamma E(y_t \mid I_{t-1}) - \beta x_t^{e}\right]/\sigma_\omega , \tag{2.8}$$

and

$$c_{Lt}^{1} = \left[y_{L,t-1} + \delta_t - \gamma E(y_t \mid I_{t-1}) - \beta x_t^{e}\right]/\sigma_\varepsilon , \tag{2.9}$$

where $x_t^e = E(x_t|I_{t-1})$.

With the above notation, the process for y_t can be written compactly as

$$y_t = \begin{cases} \gamma E(y_t \mid I_{t-1}) + \beta x_t^{e} + \omega_t , & \text{if} \quad \upsilon_t > c_{Lt}^{0}, \\ y_{L,t-1} , & \text{otherwise,} \end{cases} \tag{2.10}$$

when no adjustment in the lower bound takes place at time t (that is, when $s_t = 0$) and,

$$y_t = \begin{cases} \gamma E(y_t \mid I_{t-1}) + \beta x_t^{e} + \omega_t , & \text{if} \quad \xi_t > c_{Lt}^{1}, \\ y_{L,t-1} + \delta_t + v_t , & \text{otherwise ,} \end{cases} \tag{2.11}$$

when a jump in the bound occurs at time t ($s_t = 1$). Thus, the proposed LD-RE model with discrete changes in the bounds encompass the cases when the bound is (*i*) fixed (or perfectly forecastable), and (*ii*) changes stochastically in every period and, consequently, is not perfectly predictable [Pesaran and Ruge-Murcia (1995)]. The present more general specification allows a formal treatment of a wide variety of economic problems where the realignments to the bound are made infrequently and at discrete time intervals, and where the agents' uncertainty about both the occurrence and the size of the adjustment in the band affect the evolution of the endogenous variable.

III. Solution for the One-Sided Case

On the assumption that the state $s_{t-1} = i$, for $i = 0, 1$ is observed by the agent at time t-1, the conditional expectation of y_t can be written as[3]

$$E(y_t \mid I_{t-1}) = E(y_t \mid I_{t-1}, s_t=0) \times P_{i0}(t) + E(y_t \mid I_{t-1}, s_t=1) \times P_{i1}(t), \tag{3.1}$$

for $i = 0, 1$, where the values of $P_{i0}(t)$ and $P_{i1}(t)$ are given by the ith row of the matrix P_t, and satisfy the restriction $P_{i0}(t) + P_{i1}(t) = 1$. From (2.10) and (2.11), it can readily be seen that the conditional expectations on the right hand side of (3.1) can be written as the following weighted averages of the conditional expectations for the case when y_t is above the bound and when it is at the bound,

$$E(y_t \mid I_{t-1}, s_t=0) = E(y_t \mid I_{t-1}, \nu_t > c_{Lt}^0)\, Pr(\nu_t > c_{Lt}^0) + E(y_t \mid I_{t-1}, \nu_t \leq c_{Lt}^0)\, Pr(\nu_t \leq c_{Lt}^0), \tag{3.2}$$

and

$$E(y_t \mid I_{t-1}, s_t=1) = E(y_t \mid I_{t-1}, \xi_t > c_{Lt}^1)\, Pr(\xi_t > c_{Lt}^1) + E(y_t \mid I_{t-1}, \xi_t \leq c_{Lt}^1)\, Pr(\xi_t \leq c_{Lt}^1). \tag{3.3}$$

Using the appropriate expressions for the expectations of y_t conditional on the information set available to agents at time t-1, the state s_t, and the position of y_t with respect to the band [see Lee (1994), and Pesaran and Ruge-Murcia (1995)], and substituting (3.2) and (3.3) into (3.1) we have:

$$\begin{aligned} E(y_t|I_{t-1}) = & \left\{[\gamma E(y_t|I_{t-1}) + \beta x_t^e + \sigma_\omega E(\nu_t|I_{t-1}, \nu_t > c_{Lt}^0)][1 - H(c_{Lt}^0)] + y_{L,t-1} H(c_{Lt}^0)\right\} \\ & \times P_{i0}(t) + \left\{[\gamma E(y_t|I_{t-1}) + \beta x_t^e + \sigma_\varepsilon E(\xi_t|I_{t-1}, \xi_t > c_{Lt}^1)][1 - F(c_{Lt}^1)] \right. \\ & \left. + (y_{L,t-1} + \delta_t) F(c_{Lt}^1)\right\} \times P_{i1}(t), \end{aligned} \tag{3.4}$$

for $i = 0, 1$, where we have made use of the relations $\Pr(\upsilon_t > c_{Lt}^0) = 1 - H(c_{Lt}^0)$, $\Pr(\upsilon_t \leq c_{Lt}^0) = H(c_{Lt}^0)$, $\Pr(\xi_t > c_{Lt}^1) = 1 - F(c_{Lt}^1)$ and $\Pr(\xi_t \leq c_{Lt}^1) = F(c_{Lt}^1)$. The particular form of (3.4) in the case when the disturbance terms are normally distributed is presented in Appendix A. The rational expectations solution for the case of a one-

[3] Strictly speaking, $E(y_t|I_{t-1})$ should be written as $E(y_t|I_{t-1}, s_{t-1} = i)$ to highlight the dependence of the expectations of y_t on whether a realignment has taken place at time t-1 or not. But to simplify the notation we have subsumed this dependence implicitly in the information set I_{t-1}, which contains the value of $s_{t-1} = i$ as a sub-set.

sided band with occasional jumps is given by the value of $E(y_t|I_{t-1})$ that solves the implicit equation (3.4).[4] Below we present the conditions under which this solution exists and is unique.

The relationship of this model with the ones previously considered in the literature is apparent from (3.4). The case in which no adjustment in the band ever takes place or all adjustments are perfectly forecastable corresponds to the situation when $P_{i1}(t) = 0$, and $P_{00}(t) = 1$ for all i and t. Thus, the second term in the right hand side of (3.4) drops out and the solution reduces to one presented by Lee (1994) for a fully predictable, one-sided band. On the other hand, if unpredictable adjustments in the bound take place in *every* period (*i.e.*, $P_{11}(t) = 1$ and $P_{i0}(t) = 0$, for all i and t), then the first term in the right hand side of (2.4) vanishes and the rational expectation solution corresponds to the one established in Pesaran and Ruge-Murcia (1995). The rational expectation solution (3.4) is a weighted average of the solutions obtained in these two polar cases, with the weights given by the transition probabilities in $P(t)$. The weights are time-varying because the particular values of $P_{i0}(t)$ and $P_{i1}(t)$ depend on the state i which is in effect at the time the agents form their expectations and could be specified to be a function of predetermined variables.

The following proposition establishes the sufficient conditions for the existence and uniqueness of the rational expectations solution in the case of a one-sided band with occasional jumps in the bound.

Proposition 1. *If $\gamma < 1$, and $F(\bullet)$ and $H(\bullet)$ are continuous and first-order differentiable distribution functions, then the rational expectations solution for the one-sided band with occasional jumps exists and is unique.*

Proof. See Appendix B. ■

IV. Solution for the Two-Sided Case

The specification presented above can easily be extended to the case where both a lower and an upper bound are imposed on the variable y_t. A simple example of a two-sided band is when y_t is allowed to vary around a central value or "parity" (say y_{Ct}) within a band of fixed width of size θ.[5] The upper and lower values of y_t are then given by $y_{Ut} = y_{Ct} + \theta/2$ and $y_{Lt} = y_{Ct} - \theta/2$, respectively. The process for y_{Ct} is assumed

[4] Note that c_{Lt}^0 and c_{Lt}^1 (defined by (2.8) and (2.9)) are also functions of $E(y_t|I_{t-1})$.

[5] We also considered the case when the width of the band is stochastic, but were unable to find a tractable, closed-form solution.

to be described by

$$y_{Ct} = y_{C,t-1} + s_t(\delta_t + v_t)\,, \tag{4.1}$$

where $y_{C,t-1}$ is the central parity in the preceding period, and all the other variables are defined as before. It the follows that,

$$y_{it} = y_{i,t-1} + s_t(\delta_t + v_t)\,, \qquad \text{for} \quad i = U, L. \tag{4.2}$$

Defining the variables,

$$c_{it}^0 = \left[y_{i,t-1} - \gamma E(y_t|I_{t-1}) \;\; - \beta x_t^e\right] / \sigma_\omega\,, \qquad \text{for} \quad i = U,\ L, \tag{4.3}$$

and

$$c_{it}^1 = \left[y_{i,t-1} + \delta_t - \gamma E(y_t|I_{t-1}) - \beta x_t^e\right] / \sigma_\omega\,, \qquad \text{for} \quad i = U,\ L, \tag{4.4}$$

the observed process for y_t can be written as

$$y_t = \begin{cases} y_{U,t-1}, & \text{if} \quad \upsilon_t \geq c_{Ut}^0, \\ \gamma E(y_t|I_{t-1}) + \beta x_t^e + \omega_t, & \text{if} \quad c_{Lt}^0 < \upsilon_t < c_{Ut}^0,. \\ y_{L,t-1}, & \text{if} \quad \upsilon_t \leq c_{Lt}^0 \end{cases} \tag{4.5}$$

when no adjustment in the central parity take place at time t (that is, when $s_t = 0$) and,

$$y_t = \begin{cases} y_{U,t-1} + \delta_t + v_t, & \text{if} \quad \xi_t \geq c_{Ut}^1, \\ \gamma E(y_t|I_{t-1}) + \beta x_t^e + \omega_t, & \text{if} \quad c_{Lt}^1 < \xi_t < c_{Ut}^1, \\ y_{L,t-1} + \delta_t + v_t, & \text{if} \quad \xi_t \leq c_{Lt}^1. \end{cases} \tag{4.6}$$

when the central parity is displaced by $\delta_t + v_t$ at time t (that is, when $s_t = 1$).

As in the previous section, we assume that s_{t-1} is included in the agents' information set at time t-1 and write the expectation of y_t conditional on I_{t-1} as,

$$E(y_t|I_{t-1}) = E(y_t|I_{t-1}, s_t = 0) \times P_{i0}(t) + E(y_t|I_{t-1}, s_t = 1) \times P_{i1}(t) . \tag{4.7}$$

Once again the conditional expectations in the right hand side of (4.7) can be written as the weighted averages of the expectations conditional on y_t being inside the band or at the upper/lower thresholds:

$$\begin{aligned} E(y_t|I_{t-1}, s_t = 0) &= E(y_t|I_{t-1}, \nu_t \geq c_{Ut}^0) Pr(\nu_t \geq c_{Ut}^0) \\ &+ E(y_t|I_{t-1}, c_{Lt}^0 < \nu_t < c_{Ut}^0) Pr(c_{Lt}^0 < \nu_t < c_{Ut}^0) \\ &+ E(y_t|I_{t-1}, \nu_t \leq c_{Lt}^0) Pr(\nu_t \leq c_{Lt}^0) \end{aligned} \tag{4.8}$$

and

$$\begin{aligned} E(y_t|I_{t-1}, s_t = 1) &= E(y_t|I_{t-1}, \xi_t \geq c_{Ut}^1) Pr(\xi_t \geq c_{Ut}^1) \\ &+ E(y_t|I_{t-1}, c_{Lt}^1 < \xi_t < c_{Ut}^1) Pr(c_{Lt}^1 < \xi_t < c_{Ut}^1) \\ &+ E(y_t|I_{t-1}, \xi_t \leq c_{Lt}^1) Pr(\xi_t \leq c_{Lt}^1) \end{aligned} \tag{4.9}$$

Substituting these results in (4.7) and using the appropriate expressions for the conditional expectations that enter these relations,

$$\begin{aligned} E(y_t|I_{t-1}) &= \Big\{ y_{U,t-1}[1 - H(c_{Ut}^0)] + [\gamma E(y_t|I_{t-1}) + \beta x_t^e \\ &+ \sigma_\omega E(\nu_t|I_{t-1}, c_{Lt}^0 < \nu_t < c_{Ut}^0)][H(c_{Ut}^0) - H(c_{Lt}^0)] + y_{L,t-1} H(c_{Lt}^0) \Big\} \\ &\times P_{i0}(t) + \Big\{ (y_{U,t-1} + \delta_t)[1 - F(c_{Ut}^1)] + [\gamma E(y_t|I_{t-1}) + \beta x_t^e \\ &+ \sigma_\varepsilon E(\xi_t|I_{t-1}, c_{Lt}^1 < \xi_t < c_{Ut}^1)][F(c_{Ut}^1) - F(c_{Lt}^1)] + (y_{L,t-1} + \delta_t) F(c_{Lt}^1) \Big\} \times P_{i1}(t) . \end{aligned} \tag{4.10}$$

As in the one-sided band, the special case when no realignments ever takes place [Lee (1994) and Donald and Maddala (1993)] can be obtained by setting $P_{i1}(t) = 0$ for all i and t, and $P_{00}(t) = 1$ for all t, while the solution for a continuously changing band [Pesaran and Ruge-Murcia (1995)] corresponds to $P_{11}(t) = 1$ for all t, and $P_{i0}(t) = 0$ for all i and t. Therefore, the solution (4.10) can also be interpreted as a time- varying, convex combination of the solutions obtained for fixed and continuously varying bands. Appendix A presents the solution for the case of a two-sided band when the error terms are assumed to be normally distributed.

Proposition 2. *For any $\gamma \in \Re$, and assuming that $H(\bullet)$ and $F(\bullet)$ are continuous and first-order differentiable probability distribution functions, then the rational expectations solution for the two-sided band with occasional jumps in the central parity exits. If $\gamma \leq 1$, then the solution is also unique.*

Proof. See Appendix B. ■

In order to illustrate the extent to which the rational expectations solution of the LD-RE model with jumps varies with the probability of realignments, we computed the RE solution of the following model under three different realignment scenarios:

$$y_t = 0.8E(y_t|I_{t-1}) + x_t + u_t, \qquad u_t \sim i.i.d.\ N(0,0.4^2), \qquad (4.11a)$$

$$x_t = 0.9x_{t-1} + \eta_t, \qquad \eta_t \sim i.i.d.\ N(0,0.2^2), \qquad (4.11b)$$

$$y_{Ct} = y_{C,t-1} + s_t(\delta + v_t), \qquad v_t \sim i.i.d.\ N(0,1.5^2), \qquad (4.11c)$$

where $y_{C,t-1} = 0$, $\theta = 4$, and $s_{t-1} = 0$ (that is, no jump in the central parity has taken place at time t-1). For the realignment probabilities we considered the cases:

(*i*) when the realignment probability is zero and the band is perfectly credible.

(*ii*) when the realignment probability is fixed and set at $P_{01}(t) = 0.25$.

(*iii*) when the probability of realignment is assumed to increase monotonically in x_{t-1} according to a logistic function, namely $P_{01}(t) = exp(5x_{t-1})/[1 + exp(5x_{t-1})]$.

The fundamentals are simulated numerically by iterating successively on (4.11b) using as starting values $x_0 = \eta_0 = 0$. For each simulated value of x_t, the conditional expectations, x_t^e, are calculated as $0.9x_{t-1}$. Taking the additional parameters as given, the rational expectations solution associated with each value of x_t^e is obtained by solving numerically equation (4.10) for $E(y_t|I_{t-1})$.

The RE solutions are displayed in Figures 2 and 3, for $\delta = 0$, and $\delta = 1.5$, respectively. The case $\delta = 0$ is interesting as it represents the situation where the uncertainty about the band is solely characterized by σ_v, which measures the degree of volatility in the band. In contrast, Figure 3 gives the RE solutions when both the volatility of the band and the expectations of a positive realignment are allowed to impact the agent's expectations. As can be seen from these Figures, in all cases the RE solution is a non-linear function of the fundamentals, x_t, and have the distinctive S-shape now familiar from the continuous-time target-zone literature [see, for example, Krugman (1991)]. However, only in cases where the bounds are perfectly forecastable, is the RE solution a symmetric function of the fundamentals. The degree of asymmetry and non-linearity of the solution crucially depends on the probability of realignment and the predictable component of the size of the jump in the central parity. Not surprisingly, the higher is the probability of realignment, the less non-linear is the solution. For example, in the case when the realignment probability is postulated to rise with the fundamentals, the degree of non-linearity of the solution declines steadily as the values of x_t are allowed to increase from -1.0 to 1.5.[6]

V. Extension to the Case of Heteroscedastic Disturbances

In this section we propose a number of time-varying specifications for the variance of the error terms and argue that the results derived above regarding the existence and uniqueness of the RE solution still hold in the more general case when the disturbance terms are heteroscedastic. In particular, we consider (*i*) state-dependent variances, (*ii*) ARCH and GARCH specifications, and (*iii*) other parameterisations of the conditional variance as a function of lagged endogenous or exogenous variables.

A. State-dependent Variance

The variance of the error terms in the model could easily be specified as a function of the state variable s_t. An interesting example would be when the variance of disturbance term to the process of the endogenous variable, namely u_t, is allowed to depend on whether a realignment has taken place at time t or not. Hence, σ_u^2 could take either of two possible values and the variance of the standardized error terms ω_t and ε_t would be further distinguished by the heteroscedasticity of the disturbance term u_t. The association between the state variable, s_t, and the variance of y_t, along with the change in parity itself, would help to econometrically account for the observed jumps in the endogenous variable when a realignment takes place. Another possibility would be to model the variance of the fundamentals (or a subset of them) as a function of the state variable. Notice that the specification with a constant (*i.e.*, non-state-contingent) variance is nested in the more general model where the variance differs across states. Thus, it is possible to employ

[6] In a related paper, Lewis (1995) develops a model of floating exchange rates with occasional interventions and shows that the relationship between the fundamentals and the exchange rate depends directly upon the probability of intervention.

standard tests to verify if the assumption that the variance is the same for all possible values of s_t is in effect supported by the data.

B. Autoregressive Conditionally Heteroscedasticity

Other time-varying configurations for the variance of the disturbance terms are the Autoregressive Conditional Heteroscedasticity (ARCH) proposed by Engle (1982) and the Generalized Autoregressive Conditional Heteroscedasticity or GARCH [Bollerslev (1986)]. These specifications are of special interest when the procedure developed in this paper is applied, for example, to high frequency data on commodity prices, exchange rates, interest rates, or other financial data. The representation for the disturbance term in the $\{x_t\}$ process is easily generalized as $\eta_t | I_{t-1} \sim i.i.d\ (0, \Omega_t)$, where the conditionally time-varying matrix Ω_t could be specified as a function of lagged squared values of the elements of η_t and Ω_t. In particular, consider the following multivariate GARCH (q_1, q_2) specification [see Engle and Kroner (1995)],

$$\Omega_t = \psi + \mu_1 \eta_{t-1} \eta'_{t-1} \mu'_1 + \cdots + \mu_{q_1} \eta_{t-q_1} \eta'_{t-q_1} \mu'_{q_1} + \rho_1 \Omega_{t-1} \rho'_1 + \cdots + \rho_{q_2} \Omega_{t-q_2} \rho'_{q_2}, \tag{5.1}$$

where ψ, μ_i for $i = 1, 2, \ldots, q_1$, and ρ_j for $j = 1, 2, \ldots, q_2$ denote $k \times k$ matrices of parameters. A multivariate ARCH(q_1) process for η_t can be trivially obtained from (5.1) by restricting the elements of ρ_j for $j = 1, 2, \ldots, q_2$ to be zero.

For the error term u_t, notice that the limited-dependent nature of the endogenous variable y_t makes the exact calculation of the residuals for the censored observations infeasible. The difficulty arises because for the case of observations at the bound, the exact values of the residuals are not observed by the econometrician. Thus, it does not seem viable to implement ARCH or GARCH specifications for the variance of the stochastic disturbance u_t. This limitation might not be specially significant in the situations where the heteroscedasticity of the endogenous variable can be modeled as arising directly from the heteroscedasticity of the fundamentals, x_t. In addition, as we argue below, other parameterisations of the conditional variance of u_t in terms of (observable) lagged endogenous or exogenous variables are feasible and straightforward to implement.

C. Other Specifications for the Conditional Variance

In view of the difficulties associated with the use of ARCH type specifications in the context of the limited-dependent variable models, one could employ other parameterizations of the conditional variance of u_t (or of η_t), that do not involve lagged residuals. In the case of the target-zone application discussed below, it is, for example, reasonable to model the conditional variance of u_t as a function of the lagged squared deviation of the exchange rate from the central parity. This specification seeks to account for the observed increase in the volatility of the exchange rate as it approaches the lower/upper bound [see Bertola and Caballero (1992, p. 527)].

D. Existence and Uniqueness of the Solution in the Case of Heteroscedastic Disturbances

Recall that the RE solution of the model was derived by taking expectations conditional on the agent's information for *all* possible states of the system. Also notice that the lagged values of η_t, Ω_t, and all the variables are assumed to be contained in I_{t-1}. Thus, for the more general case when the variance is state-dependent, or the conditional variance is assumed to follow an ARCH/GARCH process or other parameterisation in terms of lagged variables, the RE solution would still be given by (3.4) and (4.10) for the one-sided and two-sided bands, respectively, with the conditional variances of the composite errors ω_t and ε_t varying over time. The existence and uniqueness of the solution in the case of conditionally heteroscedastic disturbances is insured under the conditions set out in Propositions 1 and 2, because σ^2_{ut} and Ω_t are functions of lagged, and not current values of $E(y_t|I_{t-1})$. Thus, results in Appendix B hold for these more general specifications of the error terms.

VI. Derivation of the Likelihood Function

Consider first the case of a one-sided band, and suppose that T observations are available on the various variables that enter the LD-RE model characterized by equations (2.2) to (2.5). Let $w_t = \{x_t, s_t, y_{Lt}, y_t\}$ and notice that since $w_1, w_2, \ldots w_t$ are contained in I_t (the agent's information set at time t), we can write,

$$Pr(w_1, w_2, \cdots, w_t, \cdots, w_T) = Pr(w_1)\, Pr(w_2|I_1) \cdots Pr(w_t|I_{t-1}) \cdots Pr(w_T|I_{T-1})\,, \tag{6.1}$$

where

$$Pr(w_t|I_{t-1}) = Pr(x_t|I_{t-1})\, Pr(s_t|x_t, I_{t-1})\, Pr(y_{Lt}|s_t, x_t, I_{t-1})\, Pr(y_t|y_{Lt}, s_t, x_t, I_{t-1}). \tag{6.2}$$

However, under the assumptions set out in Section II and V, we have:

$$Pr(x_t|I_{t-1}) = (2\pi)^{-k/2}|\Omega_t|^{-\frac{1}{2}} \exp[-\tfrac{1}{2}\, (x_t - \Gamma_1 z_{1,t-1})'\, \Omega_t^{-1} (x_t - \Gamma_1 z_{1,t-1})]\,, \tag{6.3}$$

where Ω_t denotes the (possibly time-varying) conditional variance matrix of the disturbance term η_t. Also since s_t and η_t are assumed to be distributed independently,

$$Pr(s_t|x_t, I_{t-1}) = Pr(s_t|I_{t-1})\,. \tag{6.4}$$

To derive $Pr(y_{Lt}|s_t, x_t, I_{t-1})$ first note that

$$Pr(y_{Lt}|s_t = 0, x_t, I_{t-1}) = 1\,, \tag{6.5}$$

because in this case $y_{Lt} = y_{L,t-1}$ and

$$Pr(y_{Lt}|s_t = 1, x_t, I_{t-1}) = (2\pi\sigma_v^2)^{-\frac{1}{2}} \exp\left[-(\Delta y_{Lt} - \Gamma_2 z_{2,t-1})^2/2\sigma_v^2\right], \tag{6.6}$$

where we have used the assumption that v_t is independently distributed from η_t and s_t. Finally, consider the last term in (6.2) and note that $Pr(y_t|y_{Lt}, s_t, x_t, I_{t-1}) = Pr(y_t|y_{Lt}, x_t, I_{t-1})$, because observing y_{Lt} and $y_{L,t-1}$ (the latter being contained in I_{t-1}) is sufficient to infer s_t (*i.e.*, whether there has been an adjustment in the band at time t or not.) The converse, clearly, does not hold: observations on s_t and $y_{L,t-1}$ do not necessarily deliver the value of y_{Lt}. Having conditioned on x_t and y_{Lt}, the process for y_t is simply given by

$$y_t = \begin{cases} \gamma E(y_t|I_{t-1}) + \beta x_t + u_t, & \text{if } \; u_t > y_{Lt} - \gamma E(y_t|I_{t-1}) - \beta x_t\,, \\ y_{Lt}, & \text{otherwise}. \end{cases} \tag{6.7}$$

The density function of (6.7) is akin to the density that appears in the standard Tobit model. Assuming that the u_t's are normally distributed, for observations lying exactly at the (lower) boundary we have:

$$Pr(y_t|y_{Lt}, x_t, I_{t-1}) = \Phi(c_{Lt})\,, \tag{6.8}$$

where Φ is the cumulative distribution function of the standard normal variable, and

$$c_{Lt} = (y_{Lt} - \gamma E(y_t | I_{t-1}) - \beta x_t)/\sigma_{ut}, \tag{6.9}$$

while for observation above the boundary

$$Pr(y_t | y_{Lt}, x_t, I_{t-1}) = (2\pi\sigma_{ut}^2)^{-½} \exp\left[-(y_t - \gamma E(y_t|I_{t-1}) - \beta x_t)^2/2\sigma_{ut}^2\right]. \tag{6.10}$$

In what follows it will be convenient to represent the unknown parameters in the sets, $\{\Gamma_1, \Omega_t\}$, $\{P_t\}$, $\{\Gamma_2, \sigma_v^2\}$, and $\{\gamma, \beta, \sigma_{ut}^2\}$, by the parameter vectors $\rho_1, \rho_2, \rho_3, \rho_4$, respectively; and let $\rho = \rho_1 \cup \rho_2 \cup \rho_3 \cup \rho_4$. Also denote by Ξ_0 the observations of y_t at the lower band and by Ξ_1 the observations above the band, and define $\Xi = \Xi_0 \cup \Xi_1$. Finally, denote the set of observations at which no realignments has taken place (that is, $s_t = 0$) by τ_0, and the remaining set (*i.e.*, the data points for which $s_t = 1$) by τ_1. Collecting the various expressions given above in (6.1), the log likelihood function in this case is given by

$$L(\rho) = L_x(\rho_1) + L_s(\rho_2) + L_y(\rho_3) + L_y(\rho_4), \tag{6.11}$$

where L_x, L_s, L_{yL}, and L_y denote, respectively, the contributions of x_t, s_t, y_{Lt}, and y_t to the overall log likelihood function. The component log-likelihood functions L_x, L_s, L_{y_L}, and L_y are given by

$$\begin{aligned} L_x = &-(kT/2)\log(2\pi) - (T/2)\log|\Omega_t| \\ &- (½)\sum_{t=1}^{T} (x_t - \Gamma_1 z_{1,t-1})'\Omega_t^{-1}(x_t - \Gamma_1 z_{1,t-1}) \end{aligned} \tag{6.12}$$

$$L_s = \log Pr(s_1) + \log Pr(s_2 | I_1) + \cdots + \log Pr(s_{T-1} | I_{T-2}) + \log Pr(s_t | I_{T-1}), \tag{6.13}$$

$$L_{y_L} = -(1/2)\sum_{t\in\tau_1} \log(2\pi\sigma_v^2) - (1/2\sigma_v^2)\sum_{t\in\tau_1} (\Delta y_{Lt} - \Gamma_2 z_{2,t-1})^2, \tag{6.14}$$

and finally,

$$\begin{aligned} L_y = &\sum_{t\in\Xi_0} \log\Phi(c_{Lt}) - (½)\sum_{t\in\Xi_1} \log(2\pi\sigma_{ut}^2) \\ &- (½\,\sigma_{ut}^2)\sum_{t\in\Xi_1} [y_t - \gamma E(y_t|I_{t-1}) - \beta x_t]^2, \end{aligned} \tag{6.15}$$

where c_{Lt} is already defined by (6.10) and differs from $c^0_{L,t}$ and $c^1_{L,t}$ that enter the RE solution.[7] The conditional expectations, $E(y_t|I_{t-1})$, are given by the solution to the implicit function (3.4). Notice that the above specification of the log-likelihood function includes the cross equation restrictions that are implicit in the dependence of $E(y_t|I_{t-1})$ on the parameters of the process of x_t and the changes in the band.

For the case of a symmetric two-sided band (where $\Delta y_{Ct} = \Delta y_{Lt}$), the log likelihood function can be obtained in a similar fashion and is given by (6.11), except for L_y which is now generalized to

$$L_y = \sum_{t\in\Xi_0} \log\Phi(c_{Lt}) - (\tfrac{1}{2})\sum_{t\in\Xi_1} \log(2\pi\sigma^2_{ut})$$
$$- (\tfrac{1}{2}\sigma^2_{ut}) \sum_{t\in\Xi_1} [y_t - \gamma E(y_t|I_{t-1}) - \beta x_t]^2 + \sum_{t\in\Xi_2} \log[1 - \Phi(c_{Ut})]$$

where $c_{Ut} = (y_{Ut} - \gamma E(y_t|I_{t-1}) - \beta x_t)/\sigma_{ut}$, and Ξ_2 denotes the set of observations of y_t on the upper bound.

VII. Illustration: Exchange Rate Determination in a Target Zone

The LD-RE specification developed in this paper provides a flexible econometric framework for the analysis of the various issues that arise in exchange rate determination within a target zone, and in our view compares favourably with the continuous-time literature on target-zones which have emerged over the recent years following the seminal papers of Krugman (1991) and Flood and Garber (1991):

(*i*) Our proposed model permits a general specification of the fundamentals, and can accommodate both stationary and non-stationary processes. This needs to be contrasted with the continuous-time literature which assumes that the fundamentals follow either Brownian motion [as in Krugman (1991) and Flood and Garber (1991)], or the less tractable Ornstein-Uhlenbeck process [as in Froot and Obstfeld (1989) and Lindberg and Soderling (1991)].

(*ii*) The model's disturbances could be conditionally heteroscedastic and can possess any probability distribution, provided that certain weak restrictions (e.g, continuity and differentiability) are satisfied. To our knowledge none of the continuous-time versions of the target-zone model allow for this important feature

[7] Notice that in (6.14), for the observation in the set with no realignments, namely τ_0, we have log $Pr(y_{Lt}|s_t = 0, x_t, I_{t-1}) = 0$.

of the time series observations.

(*iii*) The specification allows for stochastic jumps in the bounds with a time-varying probability. This is a significant improvement over the continuous-time literature that considers changes in the central parity under a number of special cases, while retaining the assumption that the fundamentals follow a Brownian motion. Svensson (1991), for example, examines the situation when realignments of a constant size take place with fixed probability and independently of the position of the exchange rate in the band; Bertola and Svensson (1993) incorporate an exogenous, time-varying, stochastic devaluation risk that can account for the observed positive correlation between the exchange rate and the interest rate differential; Bertola and Caballero (1992) allow for deterministic changes in the central parity with a constant probability when the exchange rate is at the upper edge of the band [see also Miller and Weller (1988, 1989)]; and Tristani (1994) examines the case of a constant realignment size occurring with a probability which is a linear function of the fundamentals and symmetric with respect to the central parity. Unfortunately, the solution of the continuous-time models loses much of its analytical tractability when the fundamentals do not follow a simple Brownian motion [see for example, Miller and Weller (1989) and Lindberg and Soderling (1991)].

In the discrete-time literature, Koedijk, Stork, and de Vries (1993) examine a version of the Krugman model and show that the S-shape property of the solution is maintained under less restrictive distributional assumptions. Pesaran and Samiei (1992a, 1992b) develop a LD-RE specification with current expectations and perfectly credible bounds whose solution is also characterized by an S-shape relationship between the exchange rate and the fundamentals. Pesaran and Samiei (1995) consider a LD-RE model with future expectations and demonstrate that when the forcing variable is serially independent, the exact analytical solution of the model can be calculated by backward recursion. In addition, the authors show that for the case of a serially correlated forcing variable, it is not feasible to estimate the exact solution of the model and an approximation procedure is required.[8]

The models of exchange rate determination that employ the assumption of UIP usually yield an specification in which the exchange rate is a function of the agents' expectations about its future value [for

[8] In particular, Pesaran and Samiei (1995) adopt as an approximation $E(z_{t+i+s}|I_{t+i}) \approx E(z_{t+i+s}|I_t)$, where (in their notation) z_t is an "intervention" variable which insures that the endogenous variable remains inside the band.

example, see Dornbusch (1976), Frenkel (1976), and Mussa (1976)]. However, as shown in the Appendix C, the process implied by the saddle-path stable solution of a linear future expectations model (with and without a lagged endogenous variable) is mathematically equivalent to the one obtained using the solution of a current expectations model with certain parameter restrictions and a suitable transformation of the forcing variables. For the case of the non-linear RE models in the exchange rate target-zone literature, the appropriately formulated current expectations model might be regarded as an approximation to the standard future expectations model.

A. Model of Exchange Rate Determination

We consider a dynamic, sticky-price exchange rate model [see Dornbusch (1976)] consisting of the following equations:[9]

$$E(y_{t+1} - y_t | I_t) = r_t + \zeta_t \,, \tag{7.1}$$

$$y_t - y_{t-1} = \phi(\bar{y}_t - y_{t-1}), \qquad 0 < \phi < 1, \tag{7.2}$$

$$\bar{y}_t = \alpha_0 + \alpha_1 r_t - \alpha_2 z_t + \alpha_3 m_t + \bar{u}_t \qquad \alpha_1, \alpha_2, \alpha_3 > 0 \tag{7.3}$$

Equation (7.1) is the Uncovered Interest Parity condition where y_t is 100 times the logarithm of the exchange rate, ζ_t is a time-varying risk premium, and r_t denotes the interest rate differential between the home and foreign countries (measured as 100 times the logarithm of the ratio of domestic to foreign interest rates). Equation (7.2) describes the adjustment process of the exchange rate towards its equilibrium level (designated by $\bar{y}_t$). Finally, (7.3) determines the (long-run) equilibrium level of the exchange rate and corresponds to the one obtained in the standard monetarist model with a Cagan money demand equation and the assumption of Purchasing Power Parity [see Frenkel (1976), Kouri (1976), and Mussa (1976)]. Thus, $\bar{y}_t$ is a function of the interest rate differential, r_t, the differences in outputs, z_t, and money supplies, m_t, between the home and foreign country. The variables z_t and m_t are measured as 100 times the logarithms of the ratio of domestic to foreign outputs and money supplies respectively. The parameters α_1, α_2 and α_3 are positive and $\bar{u}_t$ denotes a stochastic disturbance term that is assumed to be serially uncorrelated and normally distributed with mean

[9] Miller and Weller (1990) also examine a continuous-time model of exchange rate target-zones with price inertia.

zero and a time-dependent variance, σ_{ut}^2 [the specification for σ_{ut}^2 will be presented below]. Notice that the above model encompasses the flexible price model as a special case when $\phi = 1$. Eliminating r_t from (7.1) and (7.3), and solving for y_t in terms of the fundamentals we obtain:

$$y_t = \gamma E(y_{t+1} \mid I_t) + \lambda y_{t-1} + w_t, \tag{7.4}$$

where

$$\gamma = \frac{\phi \alpha_1}{1 + \phi \alpha_1}, \quad 0 < \gamma < 1, \qquad \lambda = \frac{1 - \phi}{1 + \phi \alpha_1}, \quad 0 < \lambda < 1,$$

and

$$w_t = \left(\frac{1}{1 + \phi \alpha_1} \right) \left(\alpha_0 \phi - \alpha_2 \phi z_t + \alpha_3 \phi m_t - \alpha_1 \phi \zeta_t + \phi \bar{u}_t \right).$$

As shown in Appendix C, the saddle-path solution of a linear rational expectations model with future expectations is mathematically equivalent to a rational expectations model with current expectations, once the model's forcing variables are appropriately augmented with their lagged changes. In the present application where due to the target zone regime being in effect the model is non-linear, the current expectations version of the model is best regarded as an approximation to the future expectations model, (7.4). Applying Proposition 4 in Appendix C yields

$$y_t \approx \gamma E(y_t \mid I_{t-1}) + c_1 (1 - \gamma) y_{t-1} + \beta x_t + u_t, \tag{7.5}$$

where c_1 is the root of the quadratic equation $\lambda c + \gamma c^{-1} = 1$, that falls inside the unit circle.[10] The augmented set of forcing variables in (7.5) are now m_t, z_t, their lagged changes and the determinants of the risk premium ζ_t, and their lagged changes. In the empirical applications we proxied the determinants of the risk premium by the lagged interest rate differential, r_{t-1}, and the lagged deviations of the exchange rate from its central

[10] Notice that denoting the roots of this equation by c_1 and c_2, we have $(1 - c_1)(c_2 - 1) = \lambda^{-1}(1 - \lambda - \gamma) = \phi/(1 - \phi) > 0$, and hence one of the roots must always lie inside the unit circle.

parity, $y_{t-1} - y_{C,t-1}$, and used Δm_{t-1}, Δm_{t-2}, Δz_{t-1}, Δz_{t-2}, Δr_{t-2}, and $\Delta(y_{t-2} - y_{C,t-2})$, as lagged changes of the forcing variables to control for the error involved in estimating the model with current instead of the future expectations of the exchange rate. Therefore, for $\boldsymbol{x}_t$ we choose the 11×1 vector of explanatory variables,

$$x_t = [1, m_t, z_t, \Delta m_{t-1}, \Delta m_{t-2}, \Delta z_{t-1}, \Delta z_{t-2}, r_{t-1}, (y_{t-1} - y_{C,t-1}), \Delta r_{t-2}, \Delta(y_{t-2} - y_{C,t-2})]' .$$

We estimate (7.5) as a two-sided LD-RE model with jumps using 166 monthly observations on the French Franc/Deutsche Mark bilateral exchange rate between July 1979 and April 1993.[11] During this period, the exchange rate was allowed to fluctuate $\pm 2.25\%$ around an agreed central parity with six changes in the parity taking place on 24 September 1979, 5 October 1981, 14 June 1982, 21 March 1983, 7 April 1986, and 12 January 1987.[12] The data on output was proxied by the index of industrial production for which monthly observations are available. The money supply is measured by seasonally unadjusted M1 and the interest rate corresponds to the end-of-period, nominal interest rate per month. The time series data on all the variables were obtained from the *OECD Main Economic Indicators*, except for the central parities that were kindly provided to us by Casper de Vries. The upper and lower bounds were calculated using the fixed maximum deviation from the central rate (*i.e.*, 2.25 percent).

We now specify the process of the fundamentals of the exchange rate, namely the relative money supplies and output between France and Germany. The money supply differential variable is modeled as an autoregressive, first-difference stationary process,

$$\Delta m_t = \alpha_0 + \alpha_1 \Delta m_{t-1} + \alpha_2 \Delta m_{t-2} + \alpha_3 \Delta m_{t-12} + \eta_{1t}, \tag{7.6}$$

where η_{1t} is a random disturbance term assumed serially uncorrelated and normally distributed with mean zero and variance σ_1^2. The output differential variable is postulated to follow a stationary process in levels,

$$z_t = \varphi_0 + \varphi_1 z_{t-1} + \varphi_2 z_{t-12} + \eta_{2t}, \tag{7.7}$$

where η_{2t} is an error term assumed $N(0,\sigma_2^2)$ and serially uncorrelated. For the estimation of the model, the

[11] In practice our sample period starts in July 1978, but the first twelve observations are employed to construct the lags of the explanatory variables.

[12] After 2 August 1993, the exchange rate was allowed to fluctuate by $\pm 15\%$ around the central parity.

disturbance terms η_{1t} and η_{2t} are allowed to be contemporaneously correlated. The twelfth-order lagged variables have been included in (7.6) and (7.7) in order to capture the seasonal component that may be present in the data.

The realignment process is specified as

$$y_{Ct} = y_{Ct-1} + s_t(\delta + v_t) , \tag{7.8}$$

where y_{Ct} is the central parity at time t and δ is the (constant) forecastable part of the realignment. Finally, the matrix of transition probabilities is given by

$$P(t) = \begin{bmatrix} P_{00}(t) & P_{01}(t) \\ 1 & 0 \end{bmatrix}. \tag{7.9}$$

where $P_{01}(t)$ represents the probability of a realignment at time t and $P_{11}(t)$ has been constrained to be zero since realignments in two successive periods are not encountered in the sample under consideration.

In order to model the increase volatility of the exchange rate as it approaches the upper/lower bounds [see Bertola and Caballero (1992, p. 527)], the variance of the disturbance term to the exchange rate equation (namely u_t) was parameterised to be a function of the lagged square deviation of the central parity. Formally,

$$\sigma^2_{ut} = a + b(y_{t-1} - y_{C,t-1})^2 , \tag{7.10}$$

where a and b are two non-negative scalar coefficients. For the estimation of the models, we assume homoscedastic conditional variances for the random errors in the processes for the fundamentals given by (7.6) and (7.7). We tested for neglected ARCH effects in the disturbances of these processes, but could not reject the hypothesis that disturbances to the money growth and output differential equations are conditionally homoscedastic.[13]

[13] The statistics obtained from the product of the number of observations and the uncentered R^2 of the OLS regression of $\hat{\eta}^2_{it}$ on a constant and four of its lags for $i = 1, 2$ were 6.012 for the money growth differential and 5.513 for the output differential. Under the null hypothesis of no ARCH effects these statistics are asymptotically distributed as chi-squared variates with 4 degrees of freedom.

B. Empirical Results

We estimated four different exchange rate models. The first model (referred to as M_1) is a benchmark linear RE model which does not take into account the effect of the band on expectations. The remaining specifications (models M_2 to M_4) explicitly allow for the effect of the target zone on the agents' expectations of the exchange rate, but differ in the way the probability of realignment is modelled. Specifically, model M_2 assumes that the band is fully credible and, consequently, the probability of adjustments in the band is set equal to zero [as in Pesaran and Samiei (1992b)]; model M_3 allows for a constant, but a non-zero, probability of realignment; and model M_4 postulates a time-varying realignment probability, where $P_{01}(t)$ is specified to be a logistic function of the exchange rate deviations from the central parity, the interest rate, money supply, and output differentials between France and Germany. The parameter estimates for these models, computed by the ML method, are presented in Table 1.[14]

Notice that for the linear model, the point estimates of the coefficients on the money supply and output differential have the *opposite* sign to the one predicted by economic theory. That is, an increase in the French money supply relative to the money supply in Germany would imply an exchange rate appreciation. Similarly, a reduction in output in France *vis a vis* Germany would generate an appreciation of the exchange rate. For this model, none of the estimated coefficients, except for the coefficients of the expectational variable, γ, is different from zero at conventional levels of significance. In contrast, under the non-linear models, all the estimated coefficients have the expected signs, and in the case of models with non-zero realignment probabilities, γ is found to be significantly different from both 0 and 1. Under model M_4, the coefficient of the lagged exchange rate and interest-rate differential variable are also significant at the 5 and 10 per cent level respectively. Notice that none of the other estimated coefficients is statistically

[14] As can be seen in Figure 1, there is a small number of observations of the exchange rate that lie outside the target zone. In particular, the data points for October 1980 and March 1993 are below the lower bound while for October 1988 and December 1990 are above the upper bound. For the estimation of the limited-dependent variable models these observations are treated as censored at their numerical value rather than at the upper/lower limit. Given (*i*) the small number of observations outside the band (4 out of 166) and (*ii*) their numerically small deviation from the bounds, it seems unlikely that the results presented below could be significantly affected by the way we have treated these four observations.

significant; suggesting that even in the case of the non-linear models the deviation from the random walk model might be small.

The RE solution of the model was found under the assumption of saddle path stability. In particular, the root c_1 was postulated to lie inside the unit circle. Using the estimated parameter values it is possible to verify whether this conjecture is indeed satisfied by the data. From (C.9) the coefficient on the lagged endogenous variable was given by $c_1(1 - \gamma)$. In the empirical specification of the model [see Table 1], the coefficient on the lagged endogenous variable is obtained as the sum of coefficients on y_{t-1} and on $y_{t-1} - y_{C,t-1}$.[15] Solving for c_1 in the above relationship and using the estimates in Table 1, yield the estimates 0.997, 1.047, 1.090, and 0.889 for c_1 in the case of the models M_1, M_2, M_3, and M_4, respectively. Thus, except for the model that explicitly allows for a time-varying probability of realignment (i.e. model M_4), the point estimates of c_1 suggest either explosive or near explosive processes for the exchange rate.

The estimates of the time-dependent variance of u_t are presented in Table 2. Notice that in all cases the coefficient on the lagged square deviation of the exchange rate from the central parity are significantly different from zero at conventional levels of significance. This results supports the view that the variability of the exchange rate increases as it approaches the upper/lower limit of the band and provides econometric evidence against one of the implications of the continuous-time, fully credible models of exchange rate target zones that predict a low-exchange rate variability in the neighbourhood of the upper and lower bounds [see Bertola and Caballero (1992, p. 526)].

Since there are 6 realignments of the central parity during the 166 months in the sample, an unconstrained estimate of the probability of realignment (i.e., an estimate *not* subject to the cross equation restrictions of the RE solution) would have been 6/166 = 0.0361. The estimate of the realignment probability in the model with a non-zero, but constant probability is 0.0392 (0.0154). The bracketed figure is the asymptotic standard error of the estimate. The estimates for the time-varying probability of realignment model, M_4, are presented in Table 3. Note that all the explanatory variables, except for the output differentials, are significantly different from zero and have the expected signs. Thus, a rise in the interest rate

[15] Notice that the coefficient on the central parity deviation is also (by definition) a coefficient on the lagged endogenous variable, albeit subject to the linear constraint that its numerical value be the same that on $y_{C,t-1}$.

differential, a higher rate of money growth in France than in Germany, and a larger deviation of the exchange rate from the central parity increase the probability of a realignment.

Figure 4 examines the relationship between the probability of realignment and the interest rate differential and the deviation from the central parity. In constructing this graph, we have fixed the money supply and output differential between France and Germany to their average levels during the sample period. From this figure, it is apparent that for certain values of the interest rate differential, the probability of realignment can increase quite rapidly with the exchange rate deviation from the central parity. For example, for a difference in the rate of interest of 0.5 percent per month between France and Germany, the probability of realignment can increase from almost zero at the lower end, to 0.026 at the central parity, and to 0.754 at the upper end of the band. These results indicate a strong *asymmetry* in the empirical relationship between deviation of the exchange rate from the central parity and the probability of realignment. Thus, for plausible values of the interest rate differential, the probability of realignment is zero or close to zero when the exchange rate is at the bottom of the band and significantly larger than zero (sometimes close to one) when it is in the upper half of the band. This empirical result would seem to undermine the assumption of a symmetry in realignments imposed by some of the researchers in the continuous-time literature [see, e.g., Tristani (1994)].

The graph of the time-varying realignment probability is presented in Figure 5. Notice that the probability rises significantly prior to the realignments in October 1981, June 1982, and January 1987. The probability associated with the adjustments in September 1979 and April 1986 are, however, much smaller. It is interesting to note that the two largest values estimated for the probabilities of the realignment correspond to what turned out to be the largest devaluations in the sample. The realignments on 5 October 1981 and 14 June 1982 were approximately of 8.4 and 10.1 percent respectively compared with 2.0, 7.9, 6.0 and 3.0 percent (approximately) for the devaluations on 24 September 1979, 21 March 1983, 7 April 1986, and 12 January 1987. Similar results are obtained by Koedijk, Stork, and de Vries (1993) that use a linearised, discrete-time, exchange rate model with a GARCH(1,1) disturbance term to calculate the probability of realignment (defined in their analysis as the probability that the exchange rate in the incoming period falls outside the target zone).

The importance of allowing for variations in the probability of realignments can be formally evaluated by testing model M_3 (that assumes a fixed realignment probability) against model M_4 (that postulate a time-varying probability of realignment). Specifically, we test whether the restrictions imposed by M_3 of a fixed, non-zero realignment probability is supported by the data against the alternative that lagged interest rate, money supply, output differentials, and lagged deviations from the central parity have significant explanatory power over the probability of realignment. Using the maximized log-likelihood values in Table 4, the relevant chi-squared statistic for such a test is given by 2(955.973 - 943.921) = 24.104 which is well above the 1 per cent critical value of the chi-squared distribution with 4 degrees of freedom. Thus the restriction of a constant probability of realignment is decisively rejected by the data.

In principle, it is also possible to devise formal statistical tests of the importance of allowing for the bands in the analysis of the exchange rates. There are, however, a number of technical difficulties that need to be resolved, which arise because (*i*) the linear model M_1, and the three non-linear models are non-nested, (*ii*) the parameters of the matrix of transition probabilities would not be identified under the null hypothesis of linearity, and the testing problem will be subject to the so-called Davies' problem [Davies (1977)]. A satisfactory treatment of this problem is beyond the scope of the present paper, but a casual examination of the values of the log-likelihood function and the mean squared forecast errors in Table 4, do seem to support the non-linear specification, M_4, with time-varying realignment probability, as compared to the other three specifications considered in the paper.

C. The Credibility of the Target Zone

We now turn to the question of the credibility of the target zone regime. In what follows we focus on model M_4. Svensson (1993) defines the target zone as credible if the expected future exchange rate is inside the current band. Since under the assumption of UIP and in the absence of a risk premium, the interest rate differential would measure the agents' expectations of devaluation, Svensson uses the spot rate and the interest rate differential in order to construct a series of future expected exchange rate. Subsequent work by Rose and Svensson (1994) and Rose (1993) have focused more precisely on the agents' expectations of changes in the parity. In particular, they distinguish between devaluation within the band and realignment expectations. Total expectations are measured by the interest differential, while devaluation within the band is postulated to be a function of various economic variables. By subtracting the latter from the former, these researchers obtain an empirical estimate of the agents' expectations of a realignment [see also Koedijk, Stork, and de Vries (1993)].

In our preferred specification, M_4, the rational expectations solution provides a close form representation of the agent's expected future value of the exchange rate. Consequently, as in Svensson (1993), it is possible to assess the credibility of the band by examining whether the expected future of the Ffr/DM rate is inside or outside the band. Only for 7 of the 156 observations in the sample, does the expected exchange rate exceeds the upper limit of the target zone. In most cases, this event is associated with the agent's correctly anticipating an incoming realignment of the central parity. Specifically, the expected exchange rate rises above the upper bound in the months of June, August and September 1981 (just before the parity realignment of October 1981), April and May 1982 (prior to the realignment in June 1982), December 1986 (prior to the realignment of January 1987) and finally in October 1988.

In light of the above results, one could conclude that although the target zone has been generally a credible instrument of exchange rate management (as captured by the effect of the band on expectations), in the periods preceding parity realignment the announcements by the government about the stability of the system have not been credible to the agents and that the agents have correctly anticipated most of the changes in the central parity that have taken place over the period between July 1979 and April 1993.

Table 1
Parameter Estimates under Alternative Exchange Rate Models

Explanatory Variables	Linear Model (M_1)	$P_{01}(t) = 0$ (M_2)	$P_{01}(t)$ = constant (M_3)	$P_{01}(t) = f(x_t)$ (M_4)
Intercept	-0.162 (0.399)	-0.139 (0.923)	-0.212 (0.400)	-0.247 (0.291)
$E(y_t\|I_{t-1})$	1.381** (0.640)	0.424 (0.515)	0.424* (0.278)	0.684*** (0.129)
y_{t-1}	-0.378 (0.637)	0.575 (0.515)	0.574 (0.279)	0.317*** (0.128)
m_t	-0.158 (0.386)	0.367 (0.858)	0.283 (0.344)	0.225 (0.242)
z_t	0.435 (0.732)	-0.488 (0.695)	-0.374 (0.413)	-0.280 (0.281)
Δm_{t-1}	-0.644 (1.287)	1.319 (1.969)	1.072 (0.964)	0.380 (0.670)
Δm_{t-2}	-0.637 (1.331)	0.746 (1.347)	0.629 (0.846)	0.668 (0.625)
Δz_{t-1}	-0.131 (0.344)	-0.021 (0.179)	0.012 (0.107)	0.109 (0.254)
Δz_{t-2}	-0.219 (0.431)	0.118 (0.528)	0.089 (0.313)	0.150 (0.263)
r_{t-1}	-0.235 (0.430)	-0.203 (0.485)	-0.174 (0.302)	-0.429* (0.273)
$y_{t-1}-y_{C,t-1}$	-0.002 (0.038)	0.028 (0.073)	0.054 (0.055)	-0.036 (0.045)
Δr_{t-2}	0.147 (0.564)	0.339 (0.914)	0.306 (0.745)	-0.048 (0.309)
$\Delta(y_{t-2}-y_{C,t-2})$	-0.0004 (0.040)	-0.011 (0.125)	-0.005 (0.050)	-0.008 (0.032)

Notes: The dependent variable, y_t, is 100 times the log of the exchange rate (in French Francs per Deutsche Marks), m_t is 100 times log of the relative money supplies, z_t is 100 times log of relative outputs (proxied by indices of industrial production), r_t is 100 times the log of the relative nominal interest rates, y_t-$y_{C,t}$ is the exchange rate deviation from the central parity. Asymptotic standard errors are presented in parenthesis. Specification of $P_{01}(t) = f(x)$ is given in Table 3. The superscripts *, **, and *** respectively indicate statistical significance at the 10, 5 and 1 percent levels.

Table 2
Estimates of the Conditional Variance of u_t Under Alternative Models

Explanatory Variables	Linear Model (M_1)	$P_{01}(t) = 0$ (M_2)	$P_{01}(t)$ = constant (M_3)	$P_{01}(t) = f(x_t)$ (M_4)
Intercept	0.791***	0.385***	0.381***	0.420***
	(0.123)	(0.085)	(0.082)	(0.085)
$(y_{t-1}-y_{C,t-1})^2$	0.147**	0.377***	0.377***	0.271***
	(0.081)	(0.117)	(0.107)	(0.092)

Notes: Asymptotic standard errors are presented in parentheses. For further details also see the notes to Table 1.

Table 3
Estimates of the Probability of Realignment

Explanatory Variable	$P_{01}(t)$ = constant (M_3)	$P_{01}(t) = f(x_t)$ (M_4)
Intercept term	0.039***	-8.709**
	(0.015)	(1.804)
r_{t-1}	–	10.172***
		(2.990)
$y_{t-1} - y_{C,t-1}$	–	2.107***
		(0.563)
$\Delta m_{t-1}-\Delta m_{t-2}$	–	16.135**
		(8.510)
z_{t-1}	–	-3.669
		(3.691)
L_s	-25.593	-19.192

Notes: $P_{01}(t)$ is the probability of realignment (conditional on not having had a realignment in the current period), and is assumed to be a logistic function of $\boldsymbol{x}_t = (1, r_{t-1}, y_{t-1} - y_{C,t-1}, \Delta m_{t-1} - \Delta m_{t-2}, z_{t-1})$. Asymptotic standard errors are presented in parentheses. L_s is the maximized value of the log-likelihood function associated with changes in the central parity. See relation (6.13) in text. For further details see the notes to Table 1.

Table 4
Comparison of the Alternative Exchange Rate Models

Criteria	Linear Model (M_1)	$P_{01}(t) = 0$ (M_2)	$P_{01}(t)$ = constant (M_3)	$P_{01}(t) = f(x_t)$ (M_4)
Mean of Squared Forecast Errors	0.999	1.044	1.044	0.925
L_y	-80.675	-62.505	-62.173	-56.407
$L_x + L_y + L_{y_L} + L_s$	–	-	943.921	955.973
θ	∞	4.50	4.50	4.50

Notes: P_{01} is the probability of realignment (conditional on not having had a realignment in the current period). θ is the assumed value of the band width. L_y, L_x, L_{yL}, and L_s are the maximized values of the log-likelihood functions defined in Section VI.

Appendix A: RE Solution in the Case of Normally Distributed Disturbances

For the econometric estimation of the model, it is often convenient to assume that the disturbance terms are normally distributed. In this case, the standardized variables ξ_t and υ_t are *i.i.d.* $N(0,1)$, with their distribution and density functions, denoted by $\Phi(\bullet)$ and $\phi(\bullet)$, respectively. *The One-Sided Case*

Using the well-known results for censored normal variables [see Maddala (1983, pp. 367)] write,

$$E(\upsilon_t|I_{t-1}, \upsilon_t > c^0_{Lt}) = \phi(c^0_{Lt})/[1-\Phi(c^0_{Lt})], \tag{A.1}$$

and

$$E(\xi_t|I_{t-1}, \xi_t > c^1_{Lt}) = \phi(c^1_{Lt})/[1-\Phi(c^1_{Lt})]. \tag{A.2}$$

Substituting (A.1) and (A.2) into (3.4),

$$E(y_t|I_{t-1}) = \{[\gamma E(y_t|I_{t-1})+\beta x^e_t][1-\Phi(c^0_{Lt})] + y_{L,t-1}\Phi(c^0_{Lt}) + \sigma_\omega\phi(c^0_{Lt})\} \times P_{i0}(t)+$$
$$\{[\gamma E(y_t|I_{t-1})+\beta x^e_t][1-\Phi(c^1_{Lt})] + (y_{L,t-1}+\delta_t)\Phi(c^1_{Lt}) + \sigma_\varepsilon\phi(c^1_{Lt})\} \times P_{i1}(t), \tag{A.3}$$

which implicitly determines the rational expectation solution $E(y_t|I_{t-1})$. Since the conditions stated in Proposition 1 hold, a value for coefficient of the expectational variable $\gamma < 1$, insures that the solution of (A.3) exists and is unique.

The Two-Sided Case

Using the results [Maddala (1983, pp. 366)],

$$E(\upsilon_t|I_{t-1}, c^0_{Lt} < \upsilon_t < c^0_{Ut}) = [\phi(c^0_{Lt})-\phi(c^0_{Ut})]/[\Phi(c^0_{Ut})-\Phi(c^0_{Lt})] \tag{A.4}$$

and

$$E(\xi_t|I_{t-1}, c^1_{Lt} < \xi_t < c^1_{Ut}) = [\phi(c^1_{Lt})-\phi(c^1_{Ut})]/[\Phi(c^1_{Ut})-\Phi(c^1_{Lt})], \tag{A.5}$$

into (4.8), the RE solution can be written as,

$$E(y_t|I_{t-1}) = \{[\gamma E(y_t|I_{t-1})+\beta x^e_t][\Phi(c^0_{Ut})-\Phi(c^0_{Lt})] + y_{U,t-1}[1-\Phi(c^0_{Ut})] \tag{A.6}$$
$$+ y_{L,t-1}\Phi(c^0_{Lt}) + \sigma_\omega[\phi(c^0_{Lt})-\phi(c^0_{Ut})]\} \times P_{i0}(t)$$
$$+ \{[\gamma E(y_t|I_{t-1})+\beta x^e_t][\Phi(c^1_{Ut})-\Phi(c^1_{Lt})]$$
$$+ (y_{U,t-1}+\delta_t)[1-\Phi(c^1_{Ut})] + (y_{L,t-1}+\delta_t)\Phi(c^1_{Lt}) + \sigma_\varepsilon[\phi(c^1_{Lt})-\phi(c^1_{Ut})]\} \times P_{i1}(t).$$

Since the normal distribution function is continuous and differentiable, Proposition 2 insures that the solution of the implicit function (A.6) exists for any value of γ and is unique if $\gamma \le 1$.

Appendix B: Existence and Uniqueness of the Rational Expectations Solution

Below we will establish the conditions for the existence and uniqueness of the rational expectations solution under a general specification for the probability distribution of the error terms for the one-sided and two-sided band.

The One-Sided Case

Using the definitions (2.8) and (2.9), c^{l}_{Lt} can be written in terms of c^{o}_{Lt} as $c^{l}_{Lt} = ac^{o}_{Lt} + b_t$, where $a = \sigma_\omega/\sigma_\varepsilon$, and $b_t = \delta_t/\sigma_\varepsilon$. Note that a is positive and b_t is assumed to be finite. Employing the definition of c^{o}_{Lt}, the rational expectations solution (3.4) is rewritten as

$$\begin{aligned} c^{o}_{Lt} = {} & \gamma c^{o}_{Lt}[1-H(c^{o}_{Lt})P_{i0}(t)-F(ac^{o}_{Lt}+b_t)P_{i1}(t)] \\ & - (\gamma/a)E(\xi_t|I_{t-1}, \xi_t>ac^{o}_{Lt}+b_t)[1-F(ac^{o}_{Lt}+b_t)]P_{i1}(t) \\ & - \gamma E(\upsilon_t|I_{t-1}, \upsilon_t>c^{o}_{Lt})[1-H(c^{o}_{Lt})]P_{i0}(t) - (\gamma/a)b_{1t}F(ac^{o}_{Lt}+b_t)P_{i1}(t) - d_t, \end{aligned} \tag{B.1}$$

where $d_t = [(1-\gamma)/\sigma_\varepsilon][\beta x^{e}_t/(1-\gamma)-y_{Lt-1}]$, and we have used $P_{i0}(t)+P_{i1}(t) = 1$. Now define the function,

$$\begin{aligned} G(c^{o}_{Lt}) = {} & c^{o}_{Lt} - \gamma c^{o}_{Lt}[1-H(c^{o}_{Lt})P_{i0}(t)-F(ac^{o}_{Lt}+b_t)P_{i1}(t)] \\ & + (\gamma/a)E(\xi_t|I_{t-1}, \xi_t>ac^{o}_{Lt}+b_t)[1-F(ac^{o}_{Lt}+b_t)]P_{i1}(t) \\ & + \gamma E(\upsilon_t|I_{t-1}, \upsilon_t > c^{o}_{Lt})[1 - H(c^{o}_{Lt})]P_{i0}(t) + (\gamma/a)b_{1t}F(ac^{o}_{Lt}+b_t)P_{i1} + d_t, \end{aligned}$$

In order to prove that the rational expectations solution exits, we need to show that the function $G(c^{o}_{Lt})$ has a fixed point.[16] Consider first the following result:

Lemma 1. *Assume $E(\upsilon_t|I_{t-1})$ exists, then we have*

$$Lim_{c^{o}_{Lt}\to\infty}E(\upsilon_t|I_{t-1}, \upsilon_t>c^{o}_{Lt})[1-H(c^{o}_{Lt})] = 0, \tag{B.1}$$

and

$$Lim_{c^{o}_{Lt}\to-\infty}E(\upsilon_t|I_{t-1}, \upsilon_t<c^{o}_{Lt})H(c^{o}_{Lt}) = 0. \tag{B.2}$$

Proof. See Pesaran and Ruge-Murcia (1993). ■

Using Lemma 1, the existence and uniqueness of the rational expectations solution can be established.

Proposition 1. *If $\gamma < 1$, and $F(\bullet)$ and $H(\bullet)$ are continuous and first-order differentiable distribution functions,*

[16] Note that c^{o}_{Lt} is a linear function of $E(y_t|I_{t-1})$. Thus, establishing that $G(c^{o}_{Lt})$ has a (unique) fixed point implies that there exists a (unique) value of $E(y_t|I_{t-1})$ which satisfies (3.4).

then the rational expectations solution for the one-sided band with occasional jumps exists and is unique.

Proof. Notice that $G(c^0_{Lt})$ is a continuous on c^0_{Lt} for any continuous distribution functions $F(\bullet)$ and $H(\bullet)$, and consider $Lim_{c^0_{Lt}\to\infty}G(c^0_{Lt})$. Note that d_t is bounded,

$$Lim_{c^0_{Lt}\to\infty}(\gamma/a)b_tF(ac^0_{Lt}+b_t)P_{i1}(t) = (\gamma/a)b_tP_{i1}(t),$$

$$Lim_{c^0_{Lt}\to\infty}E(\upsilon_t|I_{t-1}, \upsilon_t>c^0_{Lt})[1-H(c^0_{Lt})]P_{i0}(t) = 0,$$

by Lemma 1, and

$$Lim_{c^0_{Lt}\to\infty}E(\xi_t|I_{t-1}, \xi_t>ac^0_{Lt}+b_t)[1-F(ac^0_{Lt}+b_t)]P_{i1}(t) = 0,$$

by Lemma 1.

Since the functions $F(\bullet)$ and $H(\bullet)$ are bounded between 0 and 1, and $P_{i0}(t)+P_{i1}(t) = 1$, then

$$Lim_{c^0_{Lt}\to\infty}c^0_{Lt}\{1 - \gamma[1-H(c^0_{Lt})P_{i0}(t)-F(ac^0_{Lt}+b_t)P_{i1}(t)]\} = \infty.$$

Therefore, $Lim_{c_{Lt}\to\infty}G(c_{Lt}) = \infty$. Now consider $Lim_{c_{Lt}\to-\infty}G(c_{Lt})$. Notice that

$Lim_{c_{Lt}\to-\infty}(\gamma/a)b_tF(ac^0_{Lt}+b_t)P_{i1}(t)$ exists and is bounded,

$$Lim_{c^0_{Lt}\to-\infty}E(\upsilon_t|I_{t-1}, \upsilon_t>c^0_{Lt})[1-H(c^0_{Lt})]P_{i0}(t) = E(\upsilon_t|I_{t-1})P_{i0}(t),$$

and

$$Lim_{c^0_{Lt}\to-\infty}E(\xi_t|I_{t-1}, \xi_t>ac^0_{Lt}+b_t)[1-F(ac^0_{Lt}+b_t)]P_{i1}(t) = E(\xi_t|I_{t-1})P_{i1}(t),$$

because $Lim_{c^0_{Lt}\to-\infty}H(c^0_{Lt}) = 0$, $Lim_{c^0_{Lt}\to-\infty}F(ac^0_{Lt}+b_t) = 0$, and as $c^0_{Lt}\to-\infty$, the conditions $\upsilon_t > c^0_{Lt}$ and $\xi_t > ac^0_{Lt}+b_t$ do not impose any restrictions on υ_t and ξ_t. Also

$$Lim_{c_{Lt}\to-\infty}c^0_{Lt}\{1 - \gamma[1-H(c^0_{Lt})P_{i0}(t)-F(ac^0_{Lt}+b_t)P_{i1}(t)]\} = -\infty,$$

as long as $\gamma < 1$, because the functions $F(\bullet)$ and $H(\bullet)$ are bounded between 0 and 1, and $P_{i0}(t)+P_{i1}(t) = 1$. Thus, $Lim_{c^0_{Lt}\to-\infty}G(c^0_{Lt}) = -\infty$. Since $Lim_{c^0_{Lt}\to\infty}G(c^0_{Lt}) = \infty$, $Lim_{c^0_{Lt}\to-\infty}G(c^0_{Lt})=-\infty$, and $G(c^0_{Lt})$ is a continuous function of c^0_{Lt}, then is must be the case that $G(c^0_{Lt})$ crosses the axis $G(c^0_{Lt}) = 0$ at least once. This establishes the existence of the rational expectations solution.

For the second part of the proof, it will suffice to show that the function $G(c^0_{Lt})$ is monotonically increasing. Take the derivative of $G(c^0_{Lt})$ with respect to c^0_{Lt},

$$G'(c^0_{Lt}) = 1 - \gamma[1-H(c^0_{Lt})P_{i0}(t)-F(ac^0_{Lt}+b_t)P_{i1}(t)]$$

Notice that since the functions $F(\bullet)$ and $H(\bullet)$ are bounded between zero and one and $P_{i0}(t)+P_{i1}(t) = 1$, the linear combination of $F(\bullet)$ and $H(\bullet)$ is also bounded between 0 and 1. Therefore the condition $\gamma < 1$ implies that $G'(c^o_{Lt}) > 0$ for all c^o_{Lt}. ■

The Two-Sided Case

We write $c^o_{Ut} = c^o_{Lt}+\psi$, $c^I_{Lt} = ac^o_{Lt}+b_t$, and $c^I_{Ut} = ac^o_{Lt}+a\psi+b_t$, where $\psi = \theta/\sigma_\omega > 0$, and a and b_t are defined as before. Rewrite the RE solution in terms of c^o_{Lt} and define the function,

$$G(c^o_{Lt}) = c^o_{Lt} - \gamma c^o_{Lt}[H(c^o_{Lt}+\psi)-H(c^o_{Lt})]P_{i0}(t) + \gamma c^o_{Lt}[F(ac^o_{Lt}+a\psi+b_t)-F(ac^o_{Lt}+b_t)]P_{i0}(t)$$

$$+ \gamma E(\upsilon_t|I_{t-1}, c^o_{Lt}<\xi_t<c^o_{Lt}+\psi)[H(c^o_{Lt}+\psi)-H(c^o_{Lt})]$$

$$+ \gamma\psi[1-H(c^o_{Lt}+\psi)P_{i0}(t)-F(ac^o_{Lt}+a\psi+b_t)P_{i1}(t)]$$

$$+ (\gamma/a)E(\xi_t|I_{t-1}, ac^o_{Lt}+b_t<\xi_t<ac^o_{Lt}+a\psi+b_t)[F(ac^o_{Lt}+a\psi+b_t)-F(ac^o_{Lt}+b_t)]$$

$$+ (\gamma/a)b_t[1-F(ac^o_{Lt}+a\psi+b_t)+F(ac^o_{Lt}+b_t)]P_{i1}(t) + d_t,$$

where d_t defined as before. In order to prove that the rational expectations solution exits, we need to show that the function $G(c^o_{Lt})$ has a fixed point c^o_{Lt}. Consider first the following result,

Lemma 2. *Assuming $E(\upsilon_t|I_{t-1})$ exists and ψ is finite, then we have*

$$Lim_{c^o_{Lt}\to\infty}E(\upsilon_t|I_{t-1}, c^o_{Lt}<\upsilon_t<c^o_{Lt}+\psi)[H(c^o_{Lt}+\psi)-H(c^o_{Lt})] = 0 \quad \text{(B.3)}$$

and

$$Lim_{c^o_{Lt}\to-\infty}E(\upsilon_t|I_{t-1}, c^o_{Lt}<\xi_t<c^o_{Lt}+\psi)[H(c^o_{Lt}+\psi)-H(c^o_{Lt})] = 0. \quad \text{(B.4)}$$

Proof. See Pesaran and Ruge-Murcia (1993). ■

The conditions for existence and uniqueness of the rational expectations solution is now established in the following proposition:

Proposition 2. *For any $\gamma \in \Re$, and assuming that $H(\bullet)$ and $F(\bullet)$ are continuous and first-order differentiable probability distribution functions, then the rational expectations solution for the two-sided band with occasional jumps in the central parity exits. If $\gamma \leq 1$, then the solution is also unique.*

Proof. Notice that $G(c^o_{Lt})$ is continuous on c^o_{Lt} for any continuous distribution functions $F(\bullet)$ and $H(\bullet)$, and consider $Lim_{c^o_{Lt}\to\infty}G(c^o_{Lt})$. Note that

$$Lim_{c^o_{Lt}\to\infty}\gamma\psi[1-H(c^o_{Lt}+\psi)P_{i0}(t)-F(ac^o_{Lt}+a\psi+b_t)P_{i1}(t)] = 0,$$

$$Lim_{c^o_{Lt}\to\infty}(\gamma/a)b_t[1-F(ac^o_{Lt}+a\psi+b_t)+F(ac^o_{Lt}+b_t)]P_{i1}(t)$$ exists and is bounded by Lemma 2,

$$Lim_{c^o_{Lt}\to\infty}E(\upsilon_t|I_{t-1},\ c^o_{Lt}<\xi_t<c^o_{Lt}+\psi)[H(c^o_{Lt}+\psi)-H(c^o_{Lt})] = 0,$$

and

$$Lim_{c^o_{Lt}\to\infty}E(\xi_t|I_{t-1},\ ac^o_{Lt}+b_t<\xi_t<ac^o_{Lt}+a\psi+b_t)[F(ac^o_{Lt}+a\psi+b_t)-F(ac^o_{Lt}+b_t)] = 0.$$

Finally,

$$Lim_{c^o_{Lt}\to\infty}c_{Lt}\{1 - \gamma[H(c^o_{Lt}+\psi)-H(c^o_{Lt})]P_{i0}(t) - \gamma[F(ac^o_{Lt}+a\psi+b_t) - F(ac^o_{Lt}+b_t)]P_{i1}(t)\} = \infty,$$

because $F(\bullet)$ and $H(\bullet)$ are bounded between 0 and 1 and $P_{i0}(t)+P_{i1}(t) = 1$. Hence, $Lim_{c^o_{Lt}\to\infty}G(c^o_{Lt}) = \infty$. Now consider $Lim_{c^o_{Lt}\to-\infty}G(c^o_{Lt})$. Note that

$$Lim_{c_{Lt}\to-\infty}\gamma\psi[1-H(c^o_{Lt}+\psi)P_{i0}(t)-F(ac^o_{Lt}+a\psi+b_t)P_{i1}(t)] = \gamma\psi,$$

$$Lim_{c^o_{Lt}\to-\infty}(\gamma/a)b_t[1-F(ac^o_{Lt}+a\psi+b_t)+F(ac^o_{Lt}+b_t)]P_{i1}(t) = (\gamma/a)b_t.$$

By Lemma 2 and using the fact that ψ and b_t are finite,

$$Lim_{c^o_{Lt}\to-\infty}E(\upsilon_t|I_{t-1},\ c^o_{Lt}<\xi_t<c^o_{Lt}+\psi)[H(c^o_{Lt}+\psi)-H(c^o_{Lt})] = 0,$$

and

$$Lim_{c^o_{Lt}\to-\infty}E(\xi_t|I_{t-1},\ ac^o_{Lt}+b_t<\xi_t<ac^o_{Lt}+a\psi+b_t)[F(ac^o_{Lt}+a\psi+b_t)-F(ac^o_{Lt}+b_t)] = 0.$$

Finally,

$$Lim_{c^o_{Lt}\to-\infty}c_{Lt}\{1-\gamma[H(c^o_{Lt}+\psi)-H(c^o_{Lt})]P_{i0}(t)-\gamma[F(ac^o_{Lt}+a\psi+b_t)-F(ac^o_{Lt}+b_t)]P_{i1}(t) = -\infty$$

for any value of γ, because $F(\bullet)$ and $H(\bullet)$ are bounded between 0 and 1. Thus $Lim_{c^o_{Lt}\to-\infty}G(c^o_{Lt}) = -\infty$. Therefore, since $Lim_{c^o_{Lt}\to-\infty}G(c^o_{Lt}) = -\infty$, $Lim_{c^o_{Lt}\to\infty}G(c^o_{Lt}) = \infty$, and $G(c^o_{Lt})$ is a continuous function of c^o_{Lt}, then it must be the case that $G(c^o_{Lt})$ crosses the axis $G(c^o_{Lt}) = 0$ at least once, regardless of the value of the parameter γ. This establishes the existence of the rational expectations solution in the case of a two-sided band.

For the second part of the proof, it will suffice to show that the function $G(c^o_{Lt})$ is monotonically increasing in c^o_{Lt}. Take the derivative of $G(c^o_{Lt})$ with respect to c^o_{Lt} and simplify to obtain,

$$G'(c^o_{Lt}) = 1 - \gamma\{[H(c^o_{Lt}+\psi)-H(c^o_{Lt})]P_{i0}(t) + [F(ac^o_{Lt}+a\psi+b_t)-F(ac^o_{Lt}+b_t)]P_{i1}(t)\}.$$

Since $\psi > 0$ and both $F(\bullet)$ and $H(\bullet)$ are a non-decreasing function bounded between zero and one, the condition $\gamma \leq 1$ implies that $G'(c^o_{Lt}) > 0$ for all c^o_{Lt}. ■

Appendix C: On the Equivalence of Future and Current Linear RE Models

The following propositions establish the equivalence of the general future LRE model and a current LRE Model with and without lagged dependent variables.

Proposition 3. *The process of the variable y_t implied by the future LRE Model*

$$y_t = \gamma E(y_{t+1}|I_t) + w_t, \tag{C.1}$$

$$w_t = \alpha(L)\eta_t, \tag{C.2}$$

with $|\gamma| < 1$, is mathematically equivalent to the process generated by the Current LRE Model

$$y_t = \gamma E(y_t \,|\, I_{t-1}) + \left\{\frac{h(L) - \alpha(L)}{\alpha(L)}\right\} \Delta w_t + w_t, \tag{C.3}$$

where $h(L) = [L\alpha(L)\text{-}\gamma\alpha(\gamma)]/(L\text{-}\gamma)$, and w_t is defined as in (C.2).

Proof. In order to prove the Proposition, we will prove that the RE solution of (C.1) and (C.3) are identical. First, note that under the assumption $|\gamma| < 1$, the unique linear stationary solution of (C.1) is given by [see Pesaran (1989), pp. 92],

$$y_t = h(L)\eta_t. \tag{C.4}$$

Second, use (C.2) to rewrite (C.3) as

$$y_t = \gamma E(y_t|I_{t-1}) + h(L)\eta_t - [h(L)\text{-}\alpha(L)]\eta_{t-1}. \tag{C.5}$$

Taking conditional expectations in both sides of (C.5) and solving for $E(y_t|I_{t-1})$ obtain,

$$E(y_t|I_{t-1}) = [h(L)\text{-}h_0]\eta_t/(1\text{-}\gamma) + [h(L)\text{-}\alpha(L)]\eta_{t-1}/(1\text{-}\gamma). \tag{C.6}$$

Plugging (C.6) into (C.5) and using $h(L)\text{-}h_0 = [h(L)\text{-}\alpha(L)]L/\gamma$ yields, $y_t = h(L)\eta_t$, which corresponds exactly to the solution of future LRE in (C.4). ■

Proposition 4. *The process of the variable y_t implied by the future LRE Model*

$$y_t = \gamma E(y_{t+1}|I_t) + \lambda y_{t-1} + w_t, \tag{C.7}$$

$$w_t = \alpha(L)\eta_t, \tag{C.8}$$

where the roots of the equation $1 = \gamma c^{-1} + \lambda c$, are real and satisfy $|c_1| < 1$, $|c_2| > 1$, is mathematically equivalent *to the process generated by the current LRE Model*

$$y_t = \gamma E(y_t | I_{t-1}) + c_1(1-\gamma)y_{t-1} + \left\{\frac{h(L) - \alpha(L)}{\alpha(L)}\right\}\Delta w_t + w_t, \tag{C.9}$$

where $h(L) = [L\alpha(L)\text{-}\gamma\alpha(\gamma)]/(L\text{-}\gamma)$, *and* w_t *is defined as in* (C.8).

Proof. Under the assumptions about c_1 and c_2, the unique linear stationary solution of (C.7) is given by,

$$y_t = c_1 y_{t-1} + h(L)\eta_t. \tag{C.10}$$

Use (C.8) to rewrite (C.9) as

$$y_t = \gamma E(y_t|I_{t-1}) + c_1(1\text{-}\gamma)y_{t-1} + h(L)\eta_t - [h(L)\text{-}\alpha(L)]\eta_{t-1}. \tag{C.11}$$

Take conditional expectations of both sides of (C.11) and solve for $E(y_t|I_{t-1})$ to obtain,

$$E(y_t|I_{t-1}) = c_1 y_{t-1} + [h(L)\text{-}h_0]\eta_t/(1\text{-}\gamma) + [h(L)\text{-}\alpha(L)]\eta_{t-1}/(1\text{-}\gamma). \tag{C.12}$$

Substituting (C.12) into (C.11) and using $h(L)\text{-}h_0 = [h(L)\text{-}\alpha(L)]L/\gamma$ yields,

$$y_t = c_1 y_{t-1} + h(L)\eta_t, \tag{C.13}$$

which corresponds exactly to the solution of the future LRE in (C.7). ■

References

Balduzzi, P, G. Bertola, and S. Foresi. (1993). "A Model of Target Changes and the Term Structure of Interest Rates". *NBER Working Paper No. 4347.*

Bertola, G. and L. E. O. Svensson. (1993). "Stochastic Devaluation Risk and the Empirical Fit of Target-Zone Models". *The Review of Economic Studies* 60, pp 689-712.

Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heteroscedasticity". *Journal of Econometrics* 31, pp. 307-327.

Chanda, A. K. and G. S. Maddala. (1983). "Methods of Estimation for Models of Markets with Bounded Price Variation under Rational Expectations". *Economic Letters* 13, pp. 181-184. Erratum in *Economic Letters* 15, pp. 195-196.

Davies, R. B. (1977). "Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternative". *Biometrika* 74, pp. 247-254.

Donald, S. and G. S. Maddala. (1992). "A Note on the Estimation of Limited Dependent Variable Models under Rational Expectations". *Economic Letters* 38, pp. 17-23.

Dornbusch, R. (1976). "Expectations and Exchange Rate Dynamics". *Journal of Political Economy* 84, pp. 1161-71

Engle, R. F. (1982). "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation". *Econometrica* 50, pp. 987-1007.

Engle, R. F. and K. F. Kroner. (1995). "Multivariate Simultaneous Generalized ARCH". *Econometric Theory*, 11, pp. 122-150.

Flood, R. F. and P. M. Garber. (1991). "The Linkage Between Speculative Attack and Target Zone Models of Exchange Rates". *Quarterly Journal of Economics* CVI, pp. 1367-1372.

Frenkel, J. A. (1976). "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence". *Scandinavian Journal of Economics* 78, pp. 200-224.

Froot, K. A. and M. Obstfeld. (1989). "Exchange-rate Dynamics Under Stochastic Regime Shifts: A Unified Approach". *Journal of International Economics* 31, pp. 203-229.

Ghosh, S. (1987). *Stabilizing Speculative Commodity Markets.* Oxford University Press.

Hamilton, James D. (1989). "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle". *Econometrica* 57, pp. 357-384.

Holt, M. T. and S. R. Johnson. (1989). "Bounded Price Variation and Rational Expectations in an Endogenous Switching Model of the US Corn Market". *Review of Economics and Statistics* 71, pp. 605-613.

Koedijk K. G., P. Stork, and C. de Vries. (1993). "An EMS Target Zone Model in Discrete Time". Tinbergen Institute Discussion Paper TI 93-146.

Kouri, P. J. K. (1976). "The Exchange Rate and the Balance of Payments in the Short Run and the Long Run: A Monetary Approach". *Scandinavian Journal of Economics* 78, pp. 280-304.

Krugman, P. (1991). "Target Zones and Exchange Rate Dynamics". *Quarterly Journal of Economics* 106, pp. 669-682.

Lee, L-F. (1994). "Rational Expectations in Limited Dependent Variable Models", *Economic Letters*, 46, pp. 97-104.

Lewis, Karen K. (1995). "Occasional Interventions to Target Rates". *American Economic Review* 85, pp. 691-715

Maddala G. S. (1983). *Limited-Dependent and Qualitative Variables in Econometrics*. Econometric Society Monographs No. 3. Cambridge University Press.

Maddala G. S. and F. D. Nelson. (1975). "Switching Regression Models with Exogenous and Endogenous Switching." *Proceedings of the American Statistical Association*, pp, 423-426.

Miller, M. and P. Weller. (1989). "Exchange Rate Bands and Realignments in a Stationary Stochastic Setting" In *Blueprints for Exchange Rate Management*, edited by M. Miller et al. Academic Press: New York.

Miller, M. and P. Weller. (1990). "Exchange Rate Bands with Price Inertia". University of Warwick, *Working Paper No. 337.*

Mussa, M. L. (1976). "The Exchange Rate, the Balance of Payments, and Monetary and Fiscal Policy Under a Regime of Controlled Floating" *Scandinavian Journal of Economics* 78, pp. 229-248.

Pesaran, M. H. (1989). *The Limits to Rational Expectations*. Basil Blackwell, Oxford.

Pesaran, M. H. and F. J. Ruge-Murcia. (1993). "Limited-Dependent Rational Expectations Models with Stochastic Thresholds". DAE Working Paper No. 9318. University of Cambridge.

Pesaran, M. H. and F. J. Ruge-Murcia. (1995). "Limited-Dependent Rational Expectations Models with Stochastic Thresholds". *Economic Letters*. Forthcoming.

Pesaran, M. H. and S. H. Samiei. (1992a). "Estimating Limited-Dependent Rational Expectations Models: With an Application to Exchange Rate Determination in a Target Zone". *Journal of Econometrics* 53, pp. 141-163.

Pesaran, M.H. and S.H. Samiei. (1992b). An Analysis of the Determination of Deutsche Mark/French Franc Exchange Rate in a Discrete-Time Target-Zone Model. *Economic Journal*, 102, pp. 388-401.

Pesaran, M. H. and S. H. Samiei. (1995). "Limited-Dependent Rational Expectations Models with Future Expectations", *Journal of Economic Dynamics and Control* 19, pp. 1325-1354.

Rose, A. K. (1993). "Sterling's ERM Credibility: Did the Dog Bark in the Night". *Economic Letters* 41, pp. 419-427.

Rose, A. K. and L. E. O. Svensson. (1994). "European Exchange Rate Creditability Before the Fall", *European Economic Review*, 38, pp. 1185-1216.

Shonkwiler, J. S. and G. S. Maddala. (1985). "Modelling Expectations of Bounded Prices: An Application to the Market for Corn". *Review of Economics and Statistics* 67, pp. 634-41.

Svensson, L. E. O. (1991). "The Term Structure of Interest Rate Differentials in a Target Zone: Theory and Swedish Data". *Journal of Monetary Economics* 28, pp. 87-116.

Svensson, L. E. O. (1993). "Assessing Target Zone Credibility". *European Economic Review*. 37, 763-802.

Tristani, O. (1994). "Variable Probability of Realignment in a Target Zone". *Scandinavian Journal of Economics*, 96, pp.1-14.

Figure 1. Ffr/DM Exchange Rate

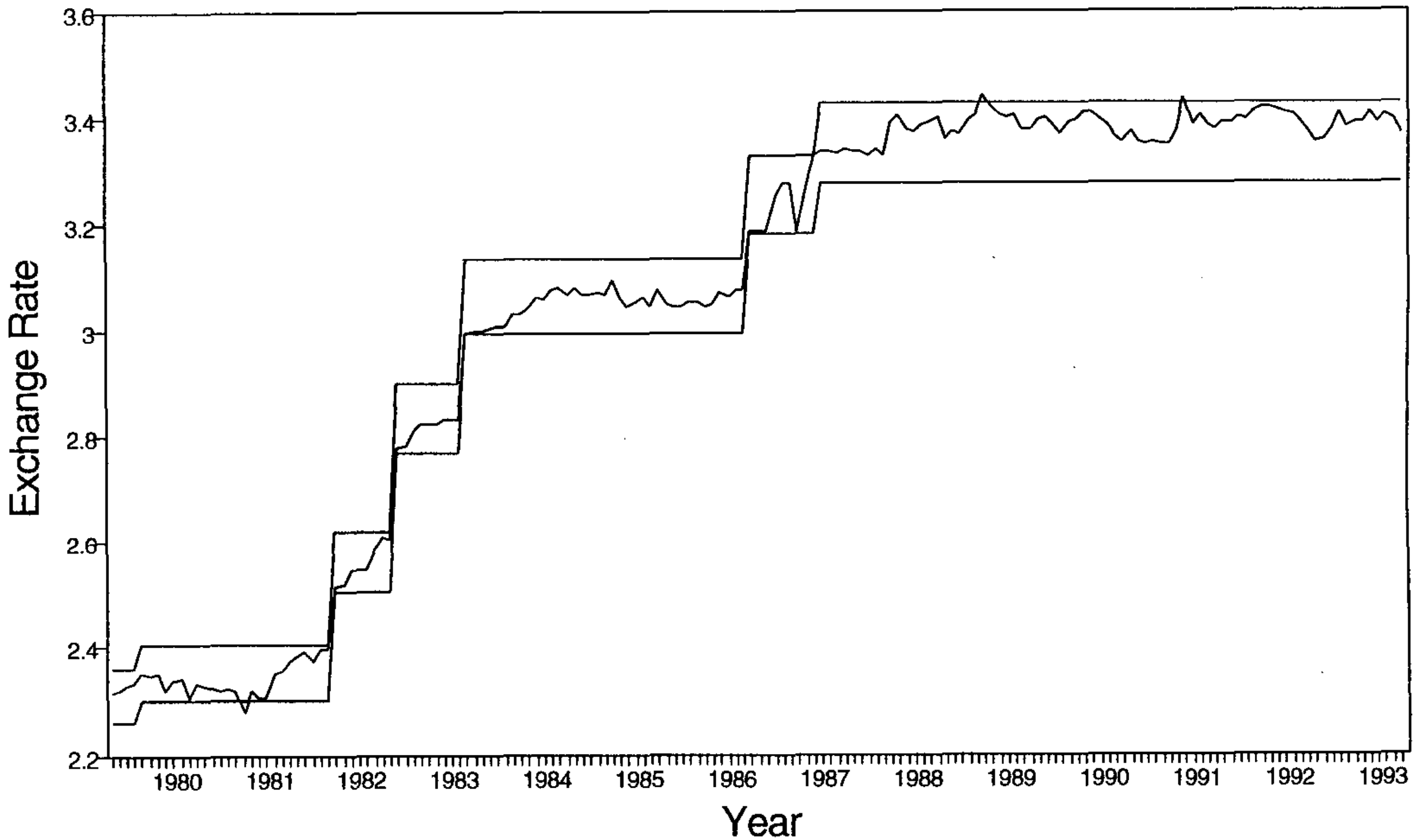


Figure 2. RE Solutions under Different Realignment Probabilities (case $\delta = 0$)

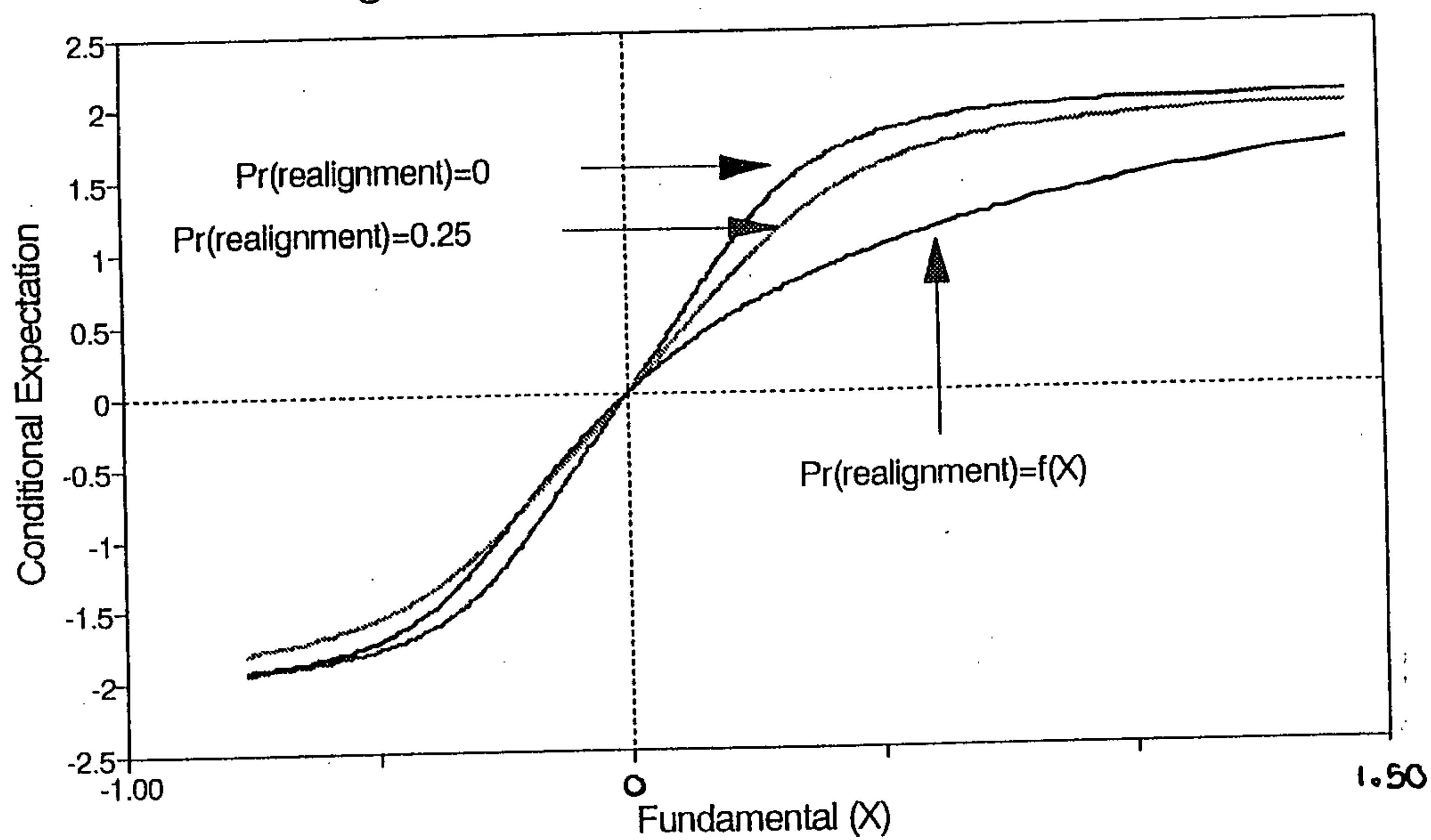
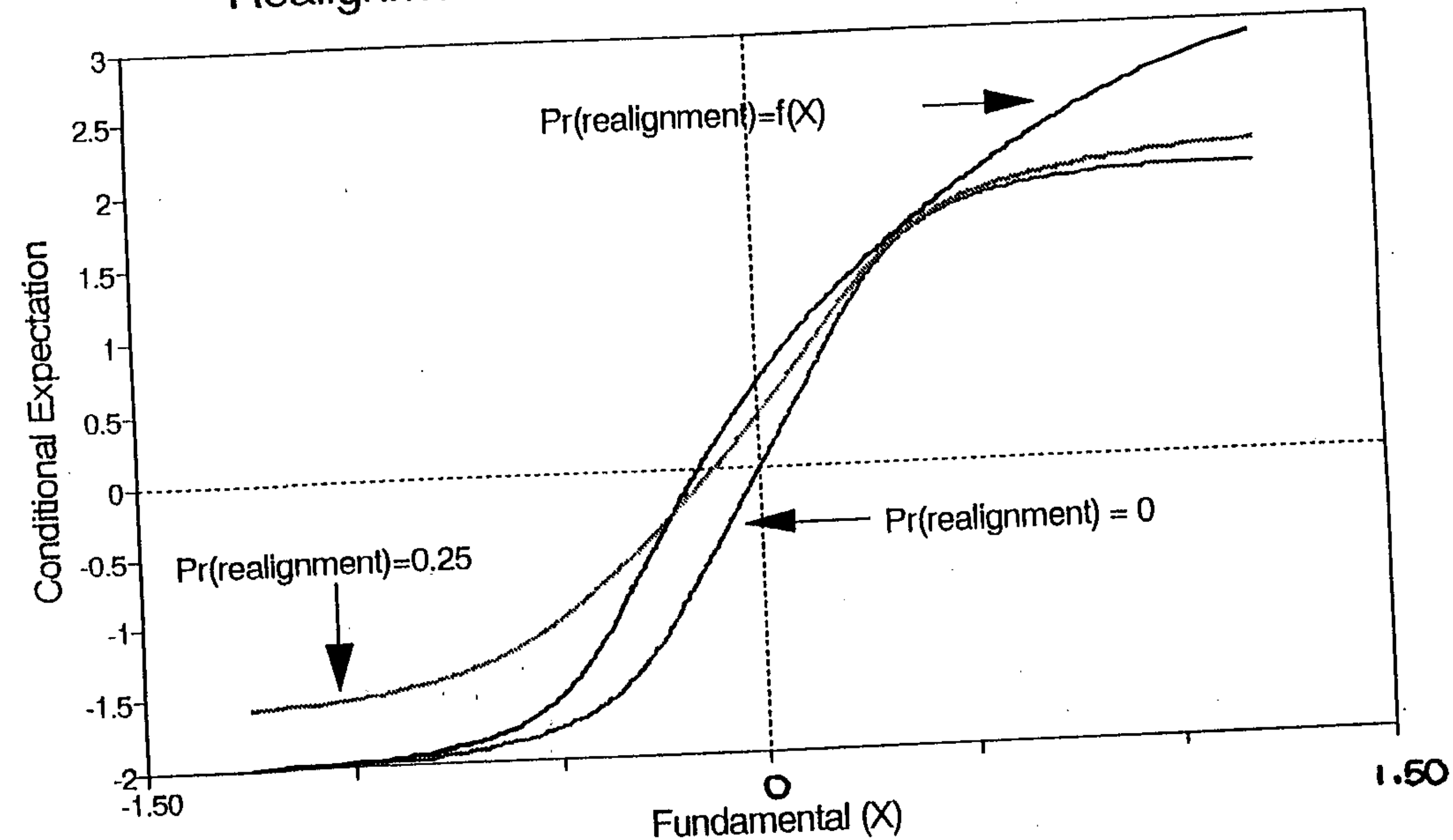


Figure 3. RE Solutions under Different Realignment Probabilities (case δ=1.5)



GAUSS   Mon Mar 6 16:52:51 1995

Figure 4. Probability of Realignment

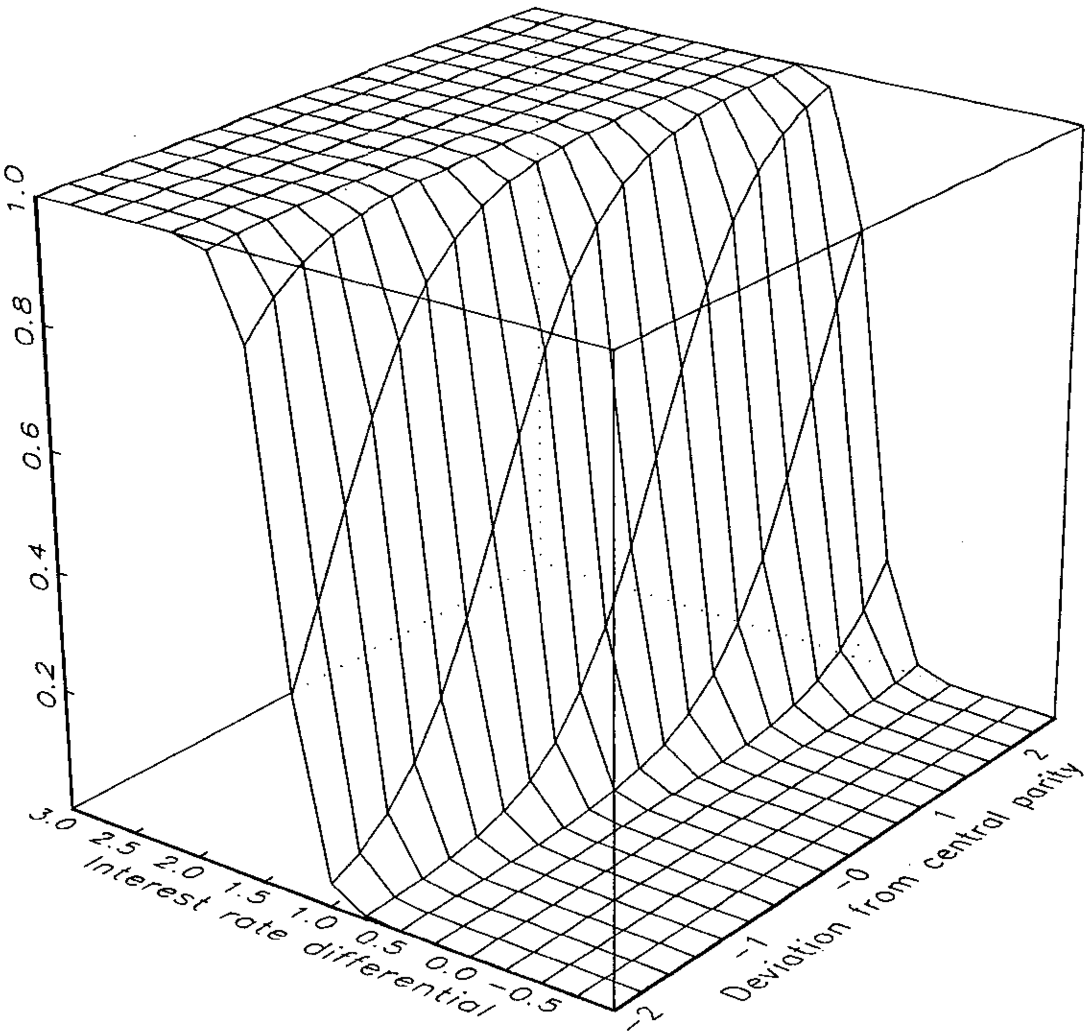


Figure 5. Time-varying Probability of Realignment