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Chain of Production as a Monetary Propagation Mechanism

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ABSTRACT

This paper studies a general equilibrium model with multiple stages of production and asynchronized price setting that provides a new explanation for the observed persistent real effects of monetary shocks. The key feature of the model is a vertical chain-of-production structure. In this model, the effects of monetary shocks on price adjustment are gradually dampened via the interactions of firms through their input-output relations and the timing of their price decisions. The model predicts that prices adjust by a smaller amount and less rapidly at later stages than at earlier stages, which is supported by empirical evidence. More importantly, an increase in the total number of stages in the model leads to not only uniformly larger and longer-lasting real effects but also flatter paths of aggregate output response. With sufficiently many stages, the price level adjustment becomes arbitrarily close to zero and the aggregate output tends to carry the full burden of adjustment. Thus, the chain-of-production mechanism goes a long way in propagating the shocks.

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1 Introduction

That production of final consumption goods typically requires multiple stages of processing has long been recognized by economists (e.g., Smith (1776, 1937 ed., p.11)). A thesis of this paper is that the assumption of multiple-stage production is crucial for explaining long-lasting real effects of monetary policy shocks.[1] A dynamic stochastic general equilibrium model is constructed which incorporates a chain of production and staggered price contracts. The model embodies a powerful monetary propagation mechanism.

The importance of chain of production in helping explain the observed relationships between money and other economic variables has not received as much attention as it deserves. Although empirical studies reveal that prices at different stages of production behave differently and input-output relations across stages may potentially be an important source of friction in price adjustment (e.g., Gordon (1981), Blanchard (1987), and Clark (1996)), theoretical research on monetary propagation mechanisms usually abstracts from the input-output relations and confines in a single-stage paradigm. A departure from the paradigm in our view may be a key to resolve some of the ongoing puzzles in the literature. For instance, while most literature considers staggered price contracts as capable of generating persistent real effects of monetary shocks (e.g., Taylor (1980, 1998)), the recent general equilibrium work by Chari, Kehoe, and McGrattan (CKM) (1998a) suggests an anomaly because, in their model, factor prices change too quickly following the shocks and so do goods prices. Since slow price level adjustment is likely to be essential for generating output persistence, a mechanism that can generate sluggish price level adjustment seems to be an important ingredient of a model that can generate real persistence.[2] The aforementioned empirical studies are a manifestation that the chain of production is such a mechanism, as is confirmed by Blanchard (1983).

The purpose of this paper is to show that, by simply incorporating a chain of production into the baseline model of CKM (1998a), monetary shocks can generate not only sluggish price level adjustment but also persistent output response. Our model differs from Blanchard (1983) in that it incorporates optimizing individuals and frictionless factor markets, and in that it features horizontal interactions within each stage via staggered price setting and vertical interactions

[1] For empirical evidence on the persistent real effects of monetary shocks, see, for example, Christiano, Eichenbaum, and Evans (1998).

[2] The "sluggish price level adjustment" refers to the price level inertia of Blanchard (1983)) in the sense that the response of the price level to shocks is small on a period-by-period basis and the price level does not fully adjust for a long period of time.

across stages via input-output relations, rather than vertical interactions via both input-output relations and staggered price setting across different stages.

In our model economy, there are multiple stages of production and a continuum of firms at each stage producing differentiated goods. The outputs of firms at the first stage are used as inputs by firms at the second stage, whose outputs are then used as inputs by firms at the third stage, and so on. There is a representative household who consumes and invests a composite of goods produced at the final stage and supplies labor and capital to firms at the first stage. Factor markets are perfectly competitive, while goods markets are monopolistically competitive (e.g., Blanchard and Kiyotaki (1987)). In the spirit of Taylor (1980) and CKM (1998a), it is assumed that goods prices are set in a staggered fashion. More specifically, in each period and at each stage, half of the firms sets new prices while the other half does not; once a price is set, it has to be fixed for two periods. The household is infinitely lived and has preferences over consumption, leisure, and real money balances. There is a government that conducts monetary policy by injecting or extracting money via lump-sum transfers or taxes.

Our contributions in this paper can be summarized in four results. We first show that, in the special case with a single production stage as in CKM (1998a), the model cannot generate price level inertia or output persistence beyond the initial contract period following a monetary shock. The intuition can be illustrated in the case with an expansionary shock. In the impact period, half of the firms cannot set new prices because they have already set prices in the previous period. Since these firms' prices are relatively lower, the demand for their outputs becomes relatively higher. In meeting the output demand, they increase their demand for labor and capital, causing factor prices to rise. Thus the marginal cost facing all firms fully rises as soon as the shock occurs. In consequence, all firms choose to fully raise their prices whenever they have the chance to set new prices. At the end of the second period when all firms have had the chance to adjust prices, the price index is entirely composed of fully raised prices and thus fully rises as well. Therefore there is no price level inertia. Since each contract lasts for two periods, the output returns to the steady state as soon as the initial contract period is over. Hence there is no output persistence.

One approach commonly taken in the literature to amplify the persistence is to introduce factor market frictions to prevent factor prices from moving too quickly in response to a shock (e.g., Jeanne (1998), Huang and Liu (1998), and Gust (1997)). We take here an alternative approach by examining the ability of a production chain in generating price level inertia and output persistence. To focus on the role of the chain in dampening the effects of the shock on prices, we maintain the assumption that factor markets are perfectly competitive and frictionless.

The dampening mechanism of the chain can be illustrated in the case with two stages. Following an expansionary shock, for the reasons discussed above, firms at the first stage immediately face fully raised marginal costs and choose to fully raise their prices whenever they have the chance to set new prices. What is different here is that firms at the second stage do not face a fully raised marginal cost until the second period arrives. This is because their marginal cost is equal to the first-stage price index, which does not fully rise in the impact period as it then must record both the newly adjusted prices and the prices that were set before the shock occurs. In the impact period, therefore, firms at the second stage that can set new prices would choose *not* to fully raise their prices. In the second period, facing fully raised marginal costs, firms at the second stage that can set new prices do choose to fully raise their prices. Nevertheless, the second-stage price index does not fully rise as it then must record the newly adjusted prices and the prices partially adjusted in the impact period. We have thus seen that, the first-stage prices that were set before the shock occurs serve to dampen the effects of the shock on the second-stage price decisions in the impact period and on the second-stage price index in the entire initial contract period. Compared to the first-stage prices, therefore, the second-stage prices adjust by a smaller amount (vertical dampening) and less rapidly (horizontal dampening)—it takes an extra period for them to fully adjust. Consequently, the aggregate output continues to stay above the steady state even when the initial contract period is over.

When there are more stages, the impact of the shock on prices diminishes from earlier to later stages as the dampened fractions of the impact via earlier-stage prices that were set before the shock occurs accumulate across stages. In general, if the shock occurs in period 0, the prices at any stage $n \geq 1$ that were set before the shock occurs serve to dampen the effects of the shock on the price decisions in periods 0 through $m-n-1$ and on the price index in periods 0 through $m-n$ at any later stage $m > n$. Hence, as is shown in our second result, there is an equilibrium "snake effect" as emphasized by Blanchard (1983) in the sense that prices adjust by a smaller amount and less rapidly at later stages than at earlier stages in response to the shock.[3]

The snake effect directly leads to an equilibrium price level inertia. When there are N total production stages, the final-stage price index does not fully rise until period N arrives. Since the monetary shock is divided between movements in the price level and movements in the aggregate output, it follows that the aggregate output stays above the steady state in periods 0 through $N-1$. As the number of stages increases, the response of the price level decreases and that of

[3]For empirical evidence of the snake effect, see, for example, Blanchard (1987), Clark (1997), and Christiano, et. al. (1998).

the output increases on a period-by-period basis and it takes longer periods for the output to return to the steady state. Nonetheless, care must be taken to note that a uniformly larger and longer-lasting output response does not always lead to a more persistent response. To have more persistence, it is also needed that the output response dies out more gradually, i.e., it calls for a flatter impulse response function of the output. In short, even though the chain is able to generate the snake effect and thus the price level inertia, whether it can indeed help magnify output persistence remains a non-trivial question.

Our third result establishes the strict monotonicity of output persistence in the total number of production stages in terms of a general measure of persistence. To be specific, the measure of persistence employed in this paper is the collection of the ratios of output response in period t to that in period $t-1$, for all t such that $1 \leq t \leq N-1$, where N is the total number of stages. These ratios together provide a fairly accurate measurement of the flatness of the impulse response function of the output. It is shown that these ratios are strictly increasing in N. Thus, the larger is the total number of stages, the flatter the output impulse response function is. In our baseline model, for instance, when the number of stages is increased from one to five and then to ten, the ratio of output response at the end of the initial contract period to that in the impact period (the "contract multiplier") increases from 0 to 0.46 and then to 0.62. Since our persistence measure also nests the "half-life" of output response, i.e., the number of periods it takes for the output to return to half of the level of its initial response, the half-life also rises as the number of stages increases.

The remaining question is then: How long a way can the chain-of-production mechanism go in helping amplify the persistence? Our final result provides an encouraging answer. It is shown that, when the number of stages is sufficiently large, the price level response becomes sufficiently close to zero, and the aggregate output tends to carry the full burden of adjustment.

The paper is organized as follows. Section 2 describes the baseline model where, for analytical convenience, we abstract away from capital accumulation. Section 3 presents the main results based on analytical solutions to the model. Section 4 shows that neither incorporating capital accumulation nor replacing the model's input-output structure with a sparse input-output matrix will affect the results. Section 5 concludes. All proofs are contained in Appendix A. A model with capital accumulation is described in Appendix B.

2 The Model Economy

This section sets up a baseline model. There are multiple stages of production and a continuum of firms at each stage producing differentiated goods and setting prices in a staggered fashion. To focus on the role of this chain-of-production mechanism in generating price level inertia and output persistence following monetary shocks, we consider simple two-period staggered price contracts. To help exposition, we abstract away here from capital accumulation, as well as the sparse nature of the input-output matrix in the data noted by Basu (1995). It will be shown in Section 4 that these model simplifications are without loss of generality for the results obtained in the next section.

In our model economy, production of a consumption good requires N stages of processing, from crude material to intermediate goods, then to more advanced intermediate goods, and so on. At each stage, there is a continuum of firms indexed in the interval $[0, 1]$, each producing an intermediate good differentiated from other intermediate goods produced at the same stage. Production at a stage $n \in \{2, \ldots, N\}$ requires all intermediate goods produced at the previous stage $n-1$, while production at the first stage ($n = 1$) requires homogeneous labor services provided by a representative household (see Figure 1 for an illustration of this chain-of-production structure). In each period t, the economy experiences a realization of shocks s_t, while the history of events up to date t is $s^t \equiv (s_0, \cdots, s_t)$ with probability $\pi(s^t)$. The initial realization s_0 is given.

The household has a utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[\ln C(s^t) + \Phi \ln \left(\frac{M(s^t)}{\bar{P}_N(s^t)} \right) - \Psi L(s^t) \right],$$

where $\beta \in (0, 1)$ is a subjective discount factor, $M(s^t)$ and $L(s^t)$ denote money balances and labor hours, respectively, and $\bar{P}_N(s^t)$ is the final-stage price index. The household's consumption $C(s^t)$ is a Dixit-Stiglitz (1977) composite of goods produced at the final stage

$$(1) \quad C(s^t) = \left[\int_0^1 Y_N(i, s^t)^{\frac{\theta_N - 1}{\theta_N}} di \right]^{\frac{\theta_N}{\theta_N - 1}} \equiv Y(s^t),$$

where $Y_N(i, s^t)$ is a type $i \in [0, 1]$ good and $\theta_N > 1$ is the elasticity of substitution among all types of goods produced at the final stage. Note that $Y(s^t)$ can be interpreted as an aggregate output and $\bar{P}_N(s^t)$ as an aggregate price level.

The household is endowed with one unit of time in each period, thus $0 \leq L(s^t) \leq 1$. Upon the realization of s^t, it solves the utility maximization problem by choosing $\{Y_N(i, s^t)\}_{i \in [0,1]}$, $M(s^t)$, $L(s^t)$, and one-period nominal bonds $B(s^{t+1})$, taking nominal wage rate $W(s^t)$, bond price $D(s^{t+1}|s^t)$, final-stage prices $\{P_N(i, s^t)\}_{i\in[0,1]}$ and the price level $\bar{P}_N(s^t)$ as given. The utility maximization is subject to (1), a sequence of budget constraints

$$\int_0^1 P_N(i, s^t)Y_N(i, s^t)di + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(s^{t+1}) + M(s^t) \\ \leq W(s^t)L(s^t) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t),$$

and a borrowing constraint $B(s^t) \geq -\bar{B}$ for some large positive number $\bar{B}$, for each s^t and each $t \geq 0$, with initial conditions $M(s^{-1})$ and $B(s^0)$ given. Here, $B(s^{t+1})$ is a one-period nominal bond that costs $D(s^{t+1}|s^t)$ dollars at s^t and pays off one dollar in the next period contingent upon the realization of s^{t+1}, $\Pi(s^t)$ is the household's claim to all firms' profits, and $T(s^t)$ is a nominal lump-sum transfer from the government. The demand $Y_N^d(i, s^t)$ for a type $i \in [0, 1]$ good produced at the final stage is derived from the first order conditions and is given by

$$(2) \quad Y_N^d(i, s^t) = \left[\frac{P_N(i, s^t)}{\bar{P}_N(s^t)}\right]^{-\theta_N} Y(s^t),$$

where $\bar{P}_N(s^t) = \left[\int_0^1 P_N(i, s^t)^{1-\theta_N} di\right]^{\frac{1}{1-\theta_N}}$.

Production technology of a firm $i \in [0, 1]$ at a stage $n \in \{2, \ldots, N\}$ is a standard Dixit-Stiglitz (1977) type of production function

$$(3) \quad Y_n(i, s^t) = \left[\int_0^1 Y_{n-1}(i, j, s^t)^{\frac{\theta_{n-1}-1}{\theta_{n-1}}} dj\right]^{\frac{\theta_{n-1}}{\theta_{n-1}-1}},$$

where $Y_n(i, s^t)$ is i's output, $Y_{n-1}(i, j, s^t)$ is the output produced by a firm $j \in [0, 1]$ at the previous stage $n-1$ that is used by i as input, and $\theta_{n-1} > 1$ is the elasticity of substitution among all goods produced at stage $n-1$. Production technology of a firm $i \in [0, 1]$ at the first stage is a standard constant returns to scale production function $Y_1(i, s^t) = L(i, s^t)$, where $Y_1(i, s^t)$ and $L(i, s^t)$ are i's output and labor input, respectively.

Firms are monopolistic competitors in their outputs' markets and price-takers in their inputs' markets. They set prices in a staggered fashion to maximize profits, taking their outputs' demand schedules as given. At each stage and in each period t, half of the firms can set new prices upon the realization of s^t. Once a price is set, it remains fixed for two periods. We sort the index of firms at each stage so that those indexed $i \in [0, 1/2]$ set new prices in periods $0, 2, 4, \ldots$, while those indexed $i \in (1/2, 1]$ set new prices in periods $1, 3, 5, \ldots$, and so on.

Upon the realization of s^t, a firm $i \in [0, 1]$ at a stage $n \in \{1, \ldots, N\}$ that can set a new price chooses its output price $P_n(i, s^t)$ to solve a two-period profit-maximization problem

$$\text{Max} \quad \sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau|s^t)[P_n(i, s^t) - V_n(i, s^\tau)]Y_n^d(i, s^\tau),$$

taking its unit cost function $V_n(i, s^\tau)$ and its output demand schedule $Y_n^d(i, s^\tau)$ as given. If $n = 1$, the unit cost is simply $V_1(i, s^\tau) = W(s^\tau)$, since labor is the only input used in the first-stage production. If $n \in \{2, \ldots, N\}$, the unit cost is derived by choosing $Y_{n-1}(i, j)$ to minimize $\int_0^1 P_{n-1}(j) Y_{n-1}(i, j) dj$ subject to (3). Solving this cost minimization problem yields the demand function by firm i for a good $j \in [0, 1]$ produced at stage $n - 1$

$$Y_{n-1}^d(i, j, s^\tau) = \left[\frac{P_{n-1}(j, s^\tau)}{\bar{P}_{n-1}(s^\tau)}\right]^{-\theta_{n-1}} Y_n(i, s^\tau),$$

where $n \in \{2, \ldots, N\}$, and $\bar{P}_{n-1}(s^\tau) \equiv \left[\int_0^1 P_{n-1}(j, s^\tau)^{1-\theta_{n-1}} dj\right]^{1/(1-\theta_{n-1})}$ is the price index at stage $n - 1$. Therefore, the demand schedule for good j can be obtained by summing up its demand by all firms at stage n, that is

$$(4) \quad Y_{n-1}^d(j, s^\tau) \equiv \int_0^1 Y_{n-1}^d(i, j, s^\tau) di = \left[\frac{P_{n-1}(j, s^\tau)}{\bar{P}_{n-1}(s^\tau)}\right]^{-\theta_{n-1}} Y_n(s^\tau),$$

where $n \in \{2, \ldots, N\}$ and $Y_n(s^\tau) \equiv \int_0^1 Y_n(i, s^\tau) di$. The unit production cost, which, due to constant returns to scale, is also the marginal cost, of firm i derived from the cost-minimization problem is then given by

$$(5) \quad V_n(s^\tau) \equiv V_n(i, s^\tau) = \bar{P}_{n-1}(s^\tau),$$

where $n \in \{1, \ldots, N\}$, with the convention that $\bar{P}_0(s^\tau) \equiv W(s^\tau)$. Note that the unit cost (5) is firm independent. Taking (4) and (5) as given, firm i's profit maximization problem yields its optimal price setting rule

$$(6) \quad P_n(i, s^t) = \frac{\theta_n}{\theta_n - 1} \frac{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) \bar{P}_n(s^\tau)^{\theta_n} Y_{n+1}(s^\tau) V_n(s^\tau)}{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) \bar{P}_n(s^\tau)^{\theta_n} Y_{n+1}(s^\tau)},$$

where $n \in \{1, \ldots, N\}$, with the convention that $Y_{N+1}(s^\tau) \equiv Y(s^\tau)$. To understand (6), notice that the firm sets its price equal to a constant markup over a weighted average of its marginal costs in the subsequent two periods, while inspecting (2) and (4) reveals that the weights are (normalized) discounted total demand for its output in the corresponding periods.

We close the descriptions of the model economy by specifying a monetary policy. The nominal money supply process is given by $M^s(s^t) = \mu(s^t) M^s(s^{t-1})$, where $\ln \mu(s^t)$ follows a stationary AR(1) process. Newly created money is injected into the economy via a lump-sum transfer by the government to the household, that is, $T(s^t) = M^s(s^t) - M^s(s^{t-1})$.

Definition 1 *An equilibrium for this economy consists of allocations* $\{Y_N(i, s^t)\}_{i \in [0,1]}$, $L(s^t)$, $M(s^t)$, *and* $B(s^{t+1})$ *for the household, allocations* $\{L(i, s^t)\}_{i \in [0,1]}$ *and prices* $\{P_1(i, s^t)\}_{i \in [0,1]}$ *for firms at the first stage, allocations* $\{Y_{n-1}(i, j, s^t)\}_{i,j \in [0,1]}$ *and prices* $\{P_n(i, s^t)\}_{i \in [0,1]}$ *for firms at* *each stage $n \in \{2, \ldots, N\}$, wage rate $W(s^t)$, bond prices $D(s^{t+1}|s^t)$, and price indices $\bar{P}_n(s^t)$ for each stage $n \in \{1, \ldots, N\}$, that satisfy the following conditions: (i) taking wage and prices as given, the household's allocations solve its utility maximization problem; (ii) taking wage and all prices but its own as given, each firm's allocation and price solve its profit maximization problem; (iii) markets for labor, money, and bonds clear; (iv) monetary policy is as specified above.*

In what follows, we focus on a symmetric equilibrium in which firms in the same cohort at each stage make identical decisions. As a consequence, each firm is completely identified by the stage at which it produces and the time at which it can set a new price. Thus from now on we can drop the indices i and j for individual firms, and let $P_n(t)$ denote the prices set at a time t for goods produced at a stage $n \in \{1, \ldots, N\}$.

3 Main Results

This section presents the main results of this paper. We begin by reducing the equilibrium conditions to $2N+2$ equations, including N price decision equations, a labor supply decision equation, a money demand equation, and N equations defining price indices. We then log-linearize the equilibrium conditions around the deterministic steady state. In the following equilibrium conditions, the variables are logarithmic deviations of the corresponding level variables from their steady state values.

The linearized price decision equation (6) for firms at a stage $n \in \{1, \ldots, N\}$ is given by

(7) $$p_n(t) = \frac{1}{1+\beta}\bar{p}_{n-1}(t) + \frac{\beta}{1+\beta}E_t[\bar{p}_{n-1}(t+1)],$$

where the notation $\bar{p}_0(t)$ denotes $w(t)$ and E_t is a conditional expectation operator, for each $t \geq 0$.

The labor supply decision equation derived from the household's problem is given by $\Psi Y(s^t) = W(s^t)/\bar{P}_N(s^t)$, and its linearized version is

(8) $$w(t) = \bar{p}_N(t) + y(t).$$

Next, by log-linearizing the money demand equation obtained from the household's problem around the steady state, we get

(9) $$\bar{p}_N(t) + y(t) = (1-\beta)m(t) + \beta E_t[\bar{p}_N(t+1) + y(t+1)].$$

Finally, the linearized price index at a stage $n \in \{1, \ldots, N\}$ is simply a weighted average of the ongoing prices at the same stage and is given by

(10) $$\bar{p}_n(t) = \frac{1}{2}p_n(t-1) + \frac{1}{2}p_n(t).$$

To gain insights into the chain's ability in generating price level inertia and output persistence, we derive analytical solutions to the linearized system of equilibrium conditions in the case where the logarithm of money supply follows a random walk process given by $m(t) = m(t-1) + \epsilon(t)$, where $\epsilon(t)$ is a white noise. Suppose that at time 0 there is a one percent shock to the disturbance term so that $\epsilon(0) = 1$ while $\epsilon(t) = 0$ for all $t \geq 1$. Our objective here is to compute the impulse response functions to determine how the money shock is divided into movements in prices and movements in the aggregate output. For this purpose, we focus on a perfect foresight equilibrium and thus drop the expectation operator E_t in (7) and (9). The following proposition partially characterizes the equilibrium.

Proposition 1 *There is a unique perfect foresight equilibrium in which*

$$(11) \quad w(t) = 1, \quad t \geq 0,$$

$$(12) \quad p_n(t) = 1, \quad t \geq n-1, \quad n \in \{1, \ldots, N\},$$

$$(13) \quad \bar{p}_n(t) = 1, \quad t \geq n, \quad n \in \{1, \ldots, N\},$$

$$(14) \quad y(t) = 0, \quad t \geq N.$$

Proposition 1, among other things, establishes the no-persistence result of CKM (1998a) corresponding to the case with a single production stage. When $N = 1$, (14) implies that the output returns to the steady state as soon as the initial contract period is over ($y(t) = 0$ for $t \geq 1$). This is because every firm faces a fully raised marginal cost in each period ($w(t) = 1$ for $t \geq 0$), thus chooses to fully raise its price whenever it can set a new price ($p_1(t) = 1$ for $t \geq 0$). At the end of the initial contract period ($t = 1$) when all firms have had the chance to adjust their prices, the price index is entirely composed of fully raised prices and thus fully rises as well ($\bar{p}_1(1) = 1$). Thus there is no price level inertia. Here, staggered price contracts make the shock non-neutral only in the impact period ($t = 0$) when half of the ongoing prices was set before the shock occurs.

Having more than one stage is thus necessary if the model is to generate price level inertia or output persistence beyond the initial contract period. In what follows we show that this is also sufficient.

The key to understand the chain's ability in generating price level inertia is to understand how the effects of the shock on prices can be gradually dampened through the chain from earlier to later stages. This can be easily seen in the case with two stages, as illustrated in Figure 2. To make the illustration as simple as possible, the value of β is set to 1. The arrows in the figure describe the equilibrium relations between price decisions and price indices within and across stages according to (7) and (10). (The bold-faced letters in the figure denote price indices. For example, $\mathbf{P}_2(0)$ denotes $\bar{p}_2(0)$.) Following the shock, for the reasons discussed above, firms at the first stage immediately face fully raised marginal costs and choose to fully raise their prices whenever they have the chance to set new prices. What is different here is that firms at the second stage do not face a fully raised marginal cost until the second period arrives. This is because their marginal cost is equal to the first-stage price index, which does not fully rise in the impact period as it ($\bar{p}_1(0)$) then by (10) must record both the newly adjusted prices ($p_1(0) = 1$) and the prices that were set before the shock occurs ($p_1(-1) = 0$). That is, $\bar{p}_1(0) = (1/2)p_1(-1) + (1/2)p_1(0) = 1/2$. Thus, it is not until the second period and on that the first-stage price index fully rises ($\bar{p}_1(t) = (1/2)p_1(t-1) + (1/2)p_1(t) = 1$ for $t \geq 1$). In the impact period, therefore, firms at the second stage that can set new prices ($p_2(0)$) would choose not to fully raise their prices, since by (7), their optimizing prices are an average of their marginal costs in the impact period ($\bar{p}_1(0)$) and in the second period ($\bar{p}_1(1)$). That is, $p_2(0) = (1/2)\bar{p}_1(0) + (1/2)\bar{p}_1(1) = 3/4$. At the end of the initial contract period, firms at the second stage face fully raised marginal costs and thus those of them that can set new prices choose to fully raise their prices ($p_2(1) = 1$). Yet, the second-stage price index $\bar{p}_2(1)$ does not fully rise as it then by (10) must record the newly adjusted prices ($p_2(1) = 1$) and the prices partially adjusted in the impact period ($p_2(0) = 3/4$). That is, $\bar{p}_2(1) = (1/2)p_2(0) + (1/2)p_2(1) = 7/8$.

In summary, the first-stage prices that were set before the shock occurs ($p_1(-1)$) serve to dampen the effects of the shock on the second-stage price decisions in the impact period ($p_2(0)$) and on the second-stage price index in the entire initial contract period ($\bar{p}_2(0)$ and $\bar{p}_2(1)$). For instance, $p_2(0) = 3/4$, thus a fraction $1/4$ of the effect of the shock on $p_2(0)$ is dampened. This fraction is equal to the multiplication of the weight $1/2$ on $p_1(-1)$ in the price index equation (10) for $\bar{p}_1(0)$ and the weight $1/2$ on $\bar{p}_1(0)$ in the price decision equation (7) for $p_2(0)$. The dampened fractions of the effects of the shock on $\bar{p}_2(0)$ and $\bar{p}_2(1)$ can be similarly interpreted. As a consequence, compared to the first-stage prices, the second-stage prices adjust by a smaller amount and less rapidly ($p_2(0) = 3/4 < 1 = p_1(0)$, $\bar{p}_2(0) = 3/8 < 1/2 = \bar{p}_1(0)$, and $\bar{p}_2(1) = 7/8 < 1 = \bar{p}_1(1)$). This pattern of price adjustment from earlier to later stages mimics a snake-like movement and is thus termed "snake effect" as in Blanchard (1983). The snake effect directly results in an equilibrium price level inertia. In particular, the second-stage price index—which is also the price level in this case—does not fully rise even when the initial contract period is over.

When there are more stages, the impact of the shock on prices diminishes from earlier to later stages as the dampened fractions of the impact via earlier-stage prices that were set before the shock occurs accumulate across stages. In general, the prices at any stage $n \geq 1$ that were set before the shock occurs serve to dampen the effects of the shock on the price decisions in periods 0 through $m-n-1$ and on the price index in periods 0 through $m-n$ at any later stage $m > n$. Such accumulation intensifies the snake effect and the price level inertia, as is illustrated in Table I for the case with $N = 20$. In computing the equilibrium prices displayed in the table, we choose $\beta = 0.96^{1/2}$ so that the period length corresponds to half a year. According to the table, prices adjust by a smaller amount and less rapidly at later stages than at earlier stages. Implicitly, the more the stages, the larger the magnitude of the price level inertia. Our next proposition formally establishes the result.

Proposition 2 *Suppose that $N \geq 2$. In the perfect foresight equilibrium, the following holds for each $n \in \{1, \ldots, N-1\}$:*

(15) $p_{n+1}(t) \quad < \quad p_n(t), \quad 0 \leq t \leq n-1,$

(16) $\bar{p}_{n+1}(t) \quad < \quad \bar{p}_n(t), \quad 0 \leq t \leq n.$

According to Proposition 2, the effects of the shock on prices are extenuated through the chain from earlier to later stages. It is thus inferred that the price level inertia increases with the number of stages in the sense that the response of the final-stage price index decreases on a period-by-period basis and it takes longer periods for it to fully adjust. With N stages, the price level does not fully rise until period N arrives.

It follows immediately from Propositions 1 and 2 that, as the number of stages increases, the response of the aggregate output increases on a period-by-period basis and it takes longer periods for the output to return to the steady state. With N stages, the aggregate output stays above the steady state in periods 0 through $N-1$.

It is important to emphasize that a uniformly larger and longer-lasting output response due to a further increase in the number of stages does not always correspond to a more persistent response. To have more persistence, we need to show that the output response in later periods relative to that in earlier periods also becomes larger as the number of stages increases, so that the initial output response dies out more gradually. That is, we need a flatter impulse response function of the output. In short, although by Propositions 1 and 2 the chain is able to generate the snake effect and thus the price level inertia, whether it can indeed help magnify the output persistence remains a non-trivial question.

To gain some intuitions and confidence, we first examine the implications of our model in terms of two special measures of persistence proposed in the literature. One is the ratio of output response at the end of the initial contract period to that in the impact period (the "contract multiplier"), that is, $y(1)/y(0)$. The other is the number of periods it takes for the output to return to half of the level of its initial response (the "half-life"), that is, a time $t^{\star}$ such that $y(t^{\star})/y(0) = 0.5$. As Table II exhibits, both the contract multiplier and the half-life of output response are strictly increasing functions of N. Since the two measures describe to some extent how flat the output impulse response function is, the outcomes reported in Table II are a manifestation that an increase in the number of stages may help magnify output persistence in terms of these measures.

In fact, the monotonicity of output persistence in the number of stages holds true not only in terms of the contract multiplier and the half-life of output response, but also for a more general measure of persistence. This is established in the next proposition. To be specific, the measure of persistence adopted here is the collection of the ratios of output response in period t to that in period $t-1$, for all t such that $1 \leq t \leq N-1$, where N is the total number of stages. This measure, while nesting the contract multiplier and the half-life of output response as special cases, provides a fairly accurate measurement of the flatness of the impulse response function of the output. The following proposition shows that these ratios are strictly increasing in N. To help exposition, we make explicit here the dependence of the aggregate output on N. More specifically, we use the equilibrium relation that $y(t) = 1 - \bar{p}_N(t)$ if there are N stages, and $y(t) = 1 - \bar{p}_{N+1}(t)$ if there are $N+1$ stages.

Proposition 3 *(Monotonicity of Output Persistence) The following holds in the perfect foresight equilibrium:*

$$(17) \quad \frac{1-\bar{p}_{N+1}(t)}{1-\bar{p}_{N+1}(t-1)} > \frac{1-\bar{p}_N(t)}{1-\bar{p}_N(t-1)}, \quad 1 \leq t \leq N,$$

for $N \geq 1$.

Propositions 2 and 3 imply that, with more stages, the chain generates not only uniformly larger and longer-lasting real effects but also flatter paths of dynamic output response. Since the monotonicity displayed in (17) is strict, the chain has a promising potential in generating substantial output persistence. For instance, as Table II reveals, when the number of stages is increased from one to five and then to ten, the ratio of the output response at the end of the initial contract period to that in the impact period (the contract multiplier) increases from 0 to 0.46 and then to 0.62, a substantial increase.

The remaining question is then, how long a way the chain mechanism can go in helping increase output persistence in terms of the general persistence measure. We provide here an encouraging answer. Our next proposition shows that, with sufficiently many stages, the response of the price level at any time is sufficiently close to zero, and that of the aggregate output at any time is sufficiently close to one percent.

Proposition 4 *As the total number of production stages N approaches ∞, the price level $\bar{p}_N(t)$ approaches 0 and thus the aggregate output $y(t) = 1 - \bar{p}_N(t)$ approaches 1, for all $t \geq 0$.*

Our findings in this section can be summarized as follows: (i) with a single stage, the price level fully adjusts and the output returns to the steady state as soon as the initial contract period is over; (ii) with multiple stages, the effects of the shock on prices are gradually dampened through the chain, inducing an equilibrium snake effect in the sense that prices adjust by a smaller amount and less rapidly at later stages than at earlier stages, so that the magnitude of the price level inertia increases with the number of stages; (iii) with more stages, the chain generates not only uniformly larger and longer-lasting real effects but also flatter paths of dynamic output response; and (iv) with sufficiently many stages, the price level response at any time is sufficiently close to zero, and the aggregate output tends to carry the full burden of adjustment.

4 Robustness

To help exposition, we have made the baseline model as simple as possible. In particular, there is no capital accumulation, and production of a firm at a later stage requires the outputs of all firms at the previous stage. This section is devoted to showing that the results obtained so far are robust and general, and do not hinge on these model simplifications.

4.1 Input-Output Structure

It is important to emphasize that, even though the dense input-output structure in the baseline model seems to be very special, it is employed there just for mathematical elegance, and is not essential for the results obtained in the previous section. What matters is only that, for firms of the second stage and beyond, their input-supplying firms set prices in a staggered fashion, while it does not matter whether the input-supplying firms constitute all or just part of the firms of the previous stage. As long as this essential feature of the model is retained, it is still true that, following a shock, firms at later stages do not face fully adjusted marginal costs and thus they have no incentives to fully adjust their prices for extended periods of time. In consequence, it remains the case that the effects of the shock on prices are gradually dampened through the chain from earlier to later stages, and the results obtained from the baseline model continue to hold.

To make this point clear, we now show that the results in the previous section remain valid without modifications when we incorporate into the model the sparse nature of the input-output matrix in the data. Consider the following input-output structure. There are N production stages and a continuum of firms at each stage producing differentiated goods. Firms at each stage are divided into H cohorts: $[0, \frac{1}{H}], (\frac{1}{H}, \frac{2}{H}], \ldots, (\frac{H-1}{H}, 1]$. Without loss of generality, H is assumed to be an even natural number. Production technology at the first stage is the same as in the baseline model. Production of a firm in a cohort $h \in \{1, 2, \ldots, H\}$ at a stage n requires using the outputs of firms in cohorts h and $h+1$ at the previous stage $n-1$, modulo H, for each $n \in \{2, \ldots, , N\}$, according to a Dixit-Stiglitz (1977) type of production function. Therefore, the firm uses in its production only the outputs of $2/H$ fraction of the firms of the previous stage. Since H can be arbitrarily large, this fraction can be arbitrarily small and the corresponding input-output structure can be arbitrarily sparse.[4]

The staggered price contracts are specified as follows. In periods $0, 2, 4, \ldots$, firms in cohorts $2, 4, \ldots, H$ at each stage can set new prices while the rest cannot. In periods $1, 3, 5, \ldots$, firms in cohorts $1, 3, \ldots, H-1$ at each stage get the chance to set new prices while the others do not. Once a price is set, it has to remain fixed for two periods. Therefore, in each period and at each stage, half of the firms can set new prices while the other half cannot. It can be similarly shown as in Section 2 that the optimizing price of a firm in a cohort h at a stage $n \in \{2, \ldots, N\}$ is a markup over a weighted average of its marginal costs in the current period and in the next period, where the marginal costs are an average of the prices of firms in cohorts h and $h+1$ at the previous stage $n-1$ in the corresponding periods. Since each firm at the first stage simply faces the wage rate as its marginal cost, its optimizing price is a markup over a weighted average of the wage rates in the current period and in the next period. In a symmetric equilibrium, firms at the same stage face the same production costs and those of them that can set new prices at the same time make identical price decisions. Hence, the price decisions of each firm only depend on the stage at which it produces and the time at which it can set a new price, but neither on the index of the firm nor on the index of the cohort that the firm belongs to. It follows that, the price index at each stage, which is an average of all firms' prices at the stage, is also an average

[4]We are grateful to Narayana Kocherlakota for suggesting that we use this input-output structure to capture the sparse nature of the input-output matrix in the data.

of the prices of firms in any two cohorts at the same stage that set prices in an asynchronized fashion.

It is then straightforward to verify that the current model is equivalent to the baseline model in Section 2 in terms of the equilibrium price deicisions, price indices, and aggregate output, and thus the log-linearized equilibrium conditions are identical to those in the baseline model given by (7)-(10). In consequence, the effects of the shock on prices are gradually dampened through the chain in an identical manner, and the results in Section 3 continue to hold.

4.2 Capital Accumulation

It is equally important to point out that, for the validity of the results in Section 3, the abstraction of the baseline model from capital accumulation is not essential either. Incorporating capital into the baseline model directly affects the production costs and thus the price decisions of firms of the first stage; and, through input-output relations, indirectly affects the price decisions of firms of later stages. However, the dampening mechanism of the chain works in the same way as in the baseline model in attenuating the effects of the shock on goods prices. The impact of factor price movements on goods prices diminishes from earlier to later stages, as the dampened fractions of the impact via the earlier-stage prices that were set before the shock occurs accumulate across stages and eventually become dominant in determining the division of the shock into movements in the price level and movements in the aggregate output.

To make this point clear, we formally construct a model with capital accumulation. Since in this case it is difficult to obtain analytical solutions, we resort to numerical methods. We log-linearize the equilibrium conditions of the model around the deterministic steady state, and solve the linearized system numerically to obtain the impulse response functions of the price level and of the aggregate output following a monetary shock. The stochastic process for the money growth rate is given by

(18) $\ln \mu(t) = \rho \ln \mu(t-1) + \varepsilon(t),$

where $0 < \rho < 1$, and $\varepsilon(t)$ has an i.i.d. normal distribution with zero mean and finite variance. In computing the equilibrium impulse response functions, we choose the magnitude of $\varepsilon(0)$ so that money stock rises by 1% one year after the shock occurs (that is, at the end of the initial contract period). To help exposition, the response of the output is pointwise normalized by its initial response. Detailed model specifications, computation methods, and calibration strategies are contained in Appendix B.

Figure 3 plots the impulse response function of the aggregate output for $N = 1, 5, 10, 20$, respectively. From the figure, in terms of our general measure of persistence, the response of the output becomes more persistent when there are more stages. With $N = 1$, following an initial rise in the impact period, the output returns to the steady state before the initial contract period is over. With more and more stages, the output stays above the steady state for longer and longer periods of time, and returns to the steady state along flatter and flatter paths. Figure 4 plots the impulse response function of the price level for $N = 1, 5, 10, 20$, respectively. According to the figure, as the number of stages increases, the response of the price level diminishes on a period-by-period basis and converges to a new steady state level along a flatter path.

In summary, the basic conclusions lying with our analytical results in Section 3 continue to hold when capital accumulation and the sparse nature of the real-world input-output matrix are taken into account.

5 Conclusions

We have shown in the current paper that a model with multiple production stages and asynchronized price setting at each stage can generate sluggish price level adjustment and persistent aggregate output response following monetary shocks. The intuition behind our results is simple: the effects of the shock on prices diminish from earlier to later stages as the dampened fractions of the effects via the earlier-stage prices that were set before the shock occurs accumulate across stages. In consequence, a chain with more stages generates not only uniformly larger and longer-lasting output response but also flatter paths of the response. The results stand firm in the presence of capital accumulation and sparse input-output structures.

Our results can be extended to generalizations of our model's physical environment. For example, all but the "limiting" result carry over directly to a model in which production at later stages requires labor and capital in addition to outputs produced at earlier stages. Such a chain-of-production structure works in the same way as our baseline model in dampening the effects of the shock on prices. The only difference here is that the dampened fractions are now relatively smaller, which however does not affect the conclusions lying with the first three propositions, although which does imply that the conclusion drawn in Proposition 4 for the limiting case has to be amended to respect the shares of factor inputs at later stages in determining the division of the shock into movements in the price level and movements in the aggregate output. Also, versions of our results can be obtained for environments in which prices at some but not necessarily at all stages are set in an asynchronized fashion.

Our model is the first to incorporate the chain-of-production feature in the real-world economy into a general equilibrium framework. Although the model is tailored to address the persistence issue of monetary shocks, it can be extended to study other important questions as well. For instance, despite the overwhelming empirical evidence that asset prices are much more volatile than goods prices, there are few satisfactory theoretical explanations. Our model may help interpret this observation based on the following intuitions. As we have shown in the text, from earlier to later stages, the response of goods prices to a shock becomes smaller while that of outputs becomes larger and more persistent, and thus the response of dividends tends to be more persistent. Since the current prices of assets are equal to the discounted sums of future dividend streams, assuming no price bubbles, there tend to be larger gaps at later stages between the volatility of asset prices and that of goods prices. For a detailed analysis of this issue, see Huang and Liu (1999a).

Our model can also be a good starting point to study issues regarding international business cycle comovements and real exchange rate persistence. Most international business cycle models with technology shocks as a driving force of aggregate fluctuations predict that cross-country correlations of aggregate output, investment, and labor hours are small or even negative, while the correlations in the data are large and positive. This is known as the international comovement puzzle (e.g., Baxter (1995)). A related puzzle is the large and persistent deviations of the real exchange rates in the data from the purchasing power parity. The recent work by Chari, Kehoe, and McGrattan (1998b) resorts to long-period of exogenous price stickiness to amplify the persistence of real exchange rate. An extension of our current model to a two-country economy is likely to resolve both the international comovement puzzle and the real exchange rate persistence puzzle. With multiple stages of production and staggered price contracts in each country, a domestic shock tends to increase demand for goods produced in both countries through cross-country input-output relations. The response of real exchange rate to a country-specific shock is also likely to be persistent because the real exchange rate is related to the relative consumption in the two countries, as suggested by most exchange-rate theories (e.g., Stockman (1998), Chari, et al. (1998b)). In Huang and Liu (1999b), we provide a more detailed analysis of these issues.

Appendix A

PROOF OF PROPOSITION 1: Using (8), (9) and $m(t) = 1$ for $t \geq 0$, we obtain

$$(19) \quad w(t) = 1 + [w(0) - 1]/\beta^t$$

for each $t \geq 0$. Subsituting (19) into (7) for the case with $n = 1$ yields

$$(20) \quad p_1(t) = 1 + 2[w(0) - 1]/[\beta^t(1 + \beta)]$$

for each $t \geq 0$. Substituting (10) into (7) leads to

$$(21) \quad p_n(t) = \frac{1}{2(1+\beta)} p_{n-1}(t-1) + \frac{1}{2} p_{n-1}(t) + \frac{\beta}{2(1+\beta)} p_{n-1}(t+1)$$

for each $t \geq 0$ and each $n \in \{2, \ldots, N\}$. Using (20) and (21), we can prove by induction on n that

$$(22) \quad p_n(t) = 1 + 2[w(0) - 1]/[\beta^t(1 + \beta)]$$

for each $t \geq n - 1$ and each $n \in \{2, \ldots, N\}$. It then follows from (10), (20) and (22) that

$$(23) \quad \bar{p}_n(t) = w(t)$$

for each $t \geq n$ and each $n \in \{1, \ldots, N\}$.

We claim that the only value of $w(0)$ that is consistent with an equilibrium is $w(0) = 1$. If otherwise, $w(0) > 1$ or $w(0) < 1$, then by (19), as t goes to infinity, $w(t)$ diverges to plus or minus infinity at a rate of $1/\beta$, so does the price level $\bar{p}_N(t)$ as implied by (23). These possibilities, however, can be ruled out as in Obstfeld and Rogoff (1983, 1986). The hyper-inflationary path with $w(t) \to \infty$ cannot be an equilibrium, because with the log-utility in real balances the household would suffer an infinite utility loss as real balances approach zero along such a path. The hyper-deflationary path with $w(t) \to -\infty$ cannot be an equilibrium either, because it would violate the appropriate transversality condition with respect to real balances. Therefore, $w(0) = 1$, and there is a unique equilibrium in which $w(t) = 1$ for all $t \geq 0$ according to (19). That is, equation (11) holds. Equations (12) and (13) then follow from (20), (22) and (23). Finally, equation (14) follows from (8), (11) and (13). This completes the proof. $\square$

PROOF OF PROPOSITION 2: We prove (15) by induction on n. We first verify (15) for $n = 1$. Equation (12) implies that $p_1(0) = 1$ and thus $\bar{p}_1(0) = 1/2$ according to (10). This together with $\bar{p}_1(1) = 1$ by (13) results in $p_2(0) = 1 - 1/[2(1 + \beta)] < 1$ according to (7). Therefore, (15) holds for $n = 1$. This would be the end of the proof of (15) if $N = 2$. Without loss of generality, we assume $N > 2$. Suppose that (15) holds for n with $1 \leq n \leq N-2$. We need to show that (15) holds for $n+1$, that is,

$$(24) \quad p_{n+2}(t) < p_{n+1}(t), \quad 0 \leq t \leq n.$$

Fix an arbitrary t with $0 \leq t \leq n$. It follows that $-1 \leq t-1 \leq n-1$ and $1 \leq t+1 \leq n+1$. By the induction hypothesis and (12), we have

$$p_{n+1}(t-1) \leq p_n(t-1), \qquad p_{n+1}(t) \leq p_n(t), \qquad p_{n+1}(t+1) \leq p_n(t+1),$$

with at least one strict inequality. Noticing that relation (21) holds for each $t \geq 0$ and each $n \in \{2, \ldots, N\}$, we have

$$p_{n+2}(t) - p_{n+1}(t) = \frac{1}{2(1+\beta)}[p_{n+1}(t-1) - p_n(t-1)] + \frac{1}{2}[p_{n+1}(t) - p_n(t)] + \frac{\beta}{2(1+\beta)}[p_{n+1}(t+1) - p_n(t+1)] < 0,$$

which establishes (24). This completes the proof of (15).

To prove (16), fix an arbitrary $n \in \{1, \ldots, N-1\}$ and an arbitrary t with $0 \leq t \leq n$. It follows that $-1 \leq t-1 \leq n-1$. Then (12) and (15) imply that

$$p_{n+1}(t-1) \leq p_n(t-1), \quad p_{n+1}(t) \leq p_n(t),$$

with at least one strict inequality, which together with (10) leads to

$$\bar{p}_{n+1}(t) - \bar{p}_n(t) = \frac{1}{2}[p_{n+1}(t-1) - p_n(t-1)] + \frac{1}{2}[p_{n+1}(t) - p_n(t)] < 0.$$

This establishes (16), and thus completes the proof of the proposition. □

PROOF OF PROPOSITION 3: We prove the proposition by induction on N. To simplify expressions, we denote $1 - \bar{p}_N(t)$ by $y_N(t)$ so that (17) can be expressed as

$$(25) \quad \frac{y_{N+1}(t)}{y_{N+1}(t-1)} > \frac{y_N(t)}{y_N(t-1)}, \quad 1 \leq t \leq N.$$

We shall verify in Lemma 1 that (25) holds for $N = 1, 2, 3$. Suppose that (25) holds for $N \geq 3$. We need to show that (25) holds for $N+1$, that is,

$$(26) \quad \frac{y_{N+2}(t)}{y_{N+2}(t-1)} > \frac{y_{N+1}(t)}{y_{N+1}(t-1)}, \quad 1 \leq t \leq N+1.$$

We proceed by first noting that, when adding an additional stage to a chain with N stages, (7)-(10) remain to be equilibrium conditions for the modified economy with $N+1$ stages where

$N+1$ replaces N everywhere, since none of these conditions hinge on the total number of stages. Manipulating (7) and (10) for index $N+1$ leads to

$$(27)\quad \bar{p}_{N+1}(t) = \begin{cases} \frac{1}{2(1+\beta)}\bar{p}_N(t-1) + \frac{1}{2}\bar{p}_N(t) + \frac{\beta}{2(1+\beta)}\bar{p}_N(t+1), & \text{if } t \geq 1, \\ \frac{1}{2(1+\beta)}\bar{p}_N(0) + \frac{\beta}{2(1+\beta)}\bar{p}_N(1), & \text{if } t = 0, \end{cases}$$

which, along with the notations $y_N(t) \equiv 1 - \bar{p}_N(t)$ and $y_{N+1}(t) \equiv 1 - \bar{p}_{N+1}(t)$, implies that

$$(28)\quad y_{N+1}(t) = \begin{cases} \frac{1}{2(1+\beta)}y_N(t-1) + \frac{1}{2}y_N(t) + \frac{\beta}{2(1+\beta)}y_N(t+1), & \text{if } t \geq 1, \\ \frac{1}{2(1+\beta)}y_N(0) + \frac{1}{2} + \frac{\beta}{2(1+\beta)}y_N(1), & \text{if } t = 0. \end{cases}$$

When adding one more stage to a chain with $N+1$ stages, we can derive a similar expression by replacing in (28) the index N with $N+1$ on the right-hand side and the index $N+1$ with $N+2$ on the left-hand side. We write it down here for future reference:

$$(29)\quad y_{N+2}(t) = \begin{cases} \frac{1}{2(1+\beta)}y_{N+1}(t-1) + \frac{1}{2}y_{N+1}(t) + \frac{\beta}{2(1+\beta)}y_{N+1}(t+1), & \text{if } t \geq 1, \\ \frac{1}{2(1+\beta)}y_{N+1}(0) + \frac{1}{2} + \frac{\beta}{2(1+\beta)}y_{N+1}(1), & \text{if } t = 0. \end{cases}$$

We then note that a version of Proposition 1 holds for the modified economy with $N+1$ stages where $N+1$ replaces N everywhere. Using (13) for indices N and $N+1$, along with the definition $y_N(t) \equiv 1 - \bar{p}_N(t)$ and the induction hypothesis, we have:

$$(30)\quad \frac{y_{N+1}(t+1)}{y_{N+1}(t)} > \frac{y_N(t+1)}{y_N(t)}, \quad \frac{y_{N+1}(t)}{y_{N+1}(t-1)} > \frac{y_N(t)}{y_N(t-1)}, \\ \frac{y_{N+1}(t-1)}{y_{N+1}(t-2)} > \frac{y_N(t-1)}{y_N(t-2)}, \quad \text{if} \quad 2 \leq t \leq N-1;$$

$$(31)\quad y_{N+1}(t+1) = y_N(t+1) = y_N(t) = 0, \quad \frac{y_{N+1}(t)}{y_{N+1}(t-1)} > \frac{y_N(t)}{y_N(t-1)}, \\ \frac{y_{N+1}(t-1)}{y_{N+1}(t-2)} > \frac{y_N(t-1)}{y_N(t-2)}, \quad \text{if} \quad t = N;$$

$$(32)\quad y_{N+1}(t+1) = y_{N+1}(t) = y_N(t+1) = y_N(t) = y_N(t-1) = 0, \\ \frac{y_{N+1}(t-1)}{y_{N+1}(t-2)} > \frac{y_N(t-1)}{y_N(t-2)}, \quad \text{if} \quad t = N+1.$$

Finally, (28)-(32) along with Lemma 2 establish that

$$\frac{y_{N+2}(t)}{y_{N+2}(t-1)} > \frac{y_{N+1}(t)}{y_{N+1}(t-1)}, \quad 2 \leq t \leq N+1.$$

To establish (26), it thus remains to show that

$$\frac{y_{N+2}(1)}{y_{N+2}(0)} > \frac{y_{N+1}(1)}{y_{N+1}(0)},$$

which, given (28) and (29), is equivalent to showing that

$$\text{(33)}\quad \frac{\frac{1}{2(1+\beta)}y_{N+1}(0)+\frac{1}{2}y_{N+1}(1)+\frac{\beta}{2(1+\beta)}y_{N+1}(2)}{\frac{1}{2(1+\beta)}y_{N+1}(0)+\frac{1}{2}+\frac{\beta}{2(1+\beta)}y_{N+1}(1)} > \frac{\frac{1}{2(1+\beta)}y_{N}(0)+\frac{1}{2}y_{N}(1)+\frac{\beta}{2(1+\beta)}y_{N}(2)}{\frac{1}{2(1+\beta)}y_{N}(0)+\frac{1}{2}+\frac{\beta}{2(1+\beta)}y_{N}(1)}.$$

To establish (33), by Lemma 2, it suffices to show that

$$\frac{y_{N+1}(2)}{y_{N+1}(1)} > \frac{y_N(2)}{y_N(1)}, \quad \frac{y_{N+1}(1)}{1} > \frac{y_N(1)}{1}, \quad \frac{y_{N+1}(0)}{y_{N+1}(0)} = \frac{y_N(0)}{y_N(0)}.$$

The first inequality follows from the induction hypothesis, the second follows from (16) for index N in an economy with $N+1$ stages, and the last equality is trivial. This establishes (26), and thus completes the proof of the proposition. □

Proof of Proposition 4: In light of (8), (10) and (11), it suffices to show that, for each $t \geq 0$,

$$\text{(34)}\quad \lim_{N\to\infty} p_N(t) = 0.$$

We proceed by first showing that the limit exists. Similarly as in the proofs of Propositions 1 and 2, it can be shown that $p_N(t)$ is monotonically decreasing in N. The recursive relations in (21) imply that, for all $N \geq 2$, $p_N(t)$ is a weighted average of the first-stage prices $p_1(-1)$, $p_1(0), \ldots,$ and $p_1(t+N-1)$. This together with (12) and the fact that $p_1(-1) = 0$ implies that $p_N(t)$ is uniformly bounded from below by 0 and from above by 1. Therefore, for each $t \geq -1$, the limit of $p_N(t)$ as $N \to \infty$ exists. Denote this limit by $p(t)$. Then, trivially $p(-1) = 0$, and

$$\text{(35)}\quad 0 \leq p(t) \leq 1,$$

for each $t \geq 0$. It remains to show that $p(t) = 0$ for each $t \geq 0$. For convenience, we rewrite here (21) for index N and for each $t \geq 0$:

$$p_N(t) = \frac{1}{2(1+\beta)}p_{N-1}(t-1) + \frac{1}{2}p_{N-1}(t) + \frac{\beta}{2(1+\beta)}p_{N-1}(t+1).$$

Since each of the four terms in the above equation converges to a finite limit, taking $N \to \infty$ on both sides of the equation leads to

$$p(t) = \frac{1}{2(1+\beta)}p(t-1) + \frac{1}{2}p(t) + \frac{\beta}{2(1+\beta)}p(t+1),$$

which can be rewritten as $p(t+1) - p(t) = [p(t) - p(t-1)]/\beta$. By iterating on t, we get

$$\text{(36)}\quad p(t+1) - p(t) = \left(\frac{1}{\beta}\right)^{t+1}[p(0) - p(-1)].$$

Summing up both sides of (36) through periods $0, \ldots, t$, and using $p(-1) = 0$ and $0 < \beta < 1$, we have $p(t+1) = p(0)[(1/\beta)^{t+2} - 1]/[(1/\beta) - 1]$.[5] It follows that, for each $t \geq 0$,

$$(37) \quad p(t) = \left[\frac{(1/\beta)^{t+1} - 1}{(1/\beta) - 1} \right] p(0).$$

Equation (37) implies that $p(0) = 0$. If otherwise $p(0) > 0$, then there exists some $\tau \geq 0$ such that $p(t) > 1$ for $t \geq \tau$, a contradiction to (35). It follows immediately that $p(t) = 0$ for $t \geq 0$. This completes the proof. □

Lemma 1 *In the perfect foresight equilibrium,*

$$\frac{1 - \bar{p}_{N+1}(t)}{1 - \bar{p}_{N+1}(t-1)} > \frac{1 - \bar{p}_N(t)}{1 - \bar{p}_N(t-1)}, \quad 1 \leq t \leq N$$

for $N = 1, 2, 3$.

PROOF: Equations (10), (12), and (13) together with $p_1(-1) = 0$ imply that

$$\bar{p}_1(0) = \frac{1}{2}, \quad \bar{p}_1(t) = 1, \quad t \geq 1.$$

Using the above solutions and repeatedly applying (27) result in the following solutions:

$$\bar{p}_2(0) = \frac{1 + 2\beta}{4(1+\beta)}, \quad \bar{p}_2(1) = \frac{3 + 4\beta}{4(1+\beta)}, \quad \bar{p}_2(t) = 1, \quad t \geq 2,$$

$$\bar{p}_3(0) = \frac{1 + 4\beta}{8(1+\beta)}, \quad \bar{p}_3(1) = \frac{4 + 13\beta + 8\beta^2}{8(1+\beta)^2},$$

$$\bar{p}_3(2) = \frac{7 + 16\beta + 8\beta^2}{8(1+\beta)^2}, \quad \bar{p}_3(t) = 1, \quad t \geq 3,$$

$$\bar{p}_4(0) = \frac{1 + 9\beta + 17\beta^2 + 8\beta^3}{16(1+\beta)^3}, \quad \bar{p}_4(1) = \frac{5 + 29\beta + 41\beta^2 + 16\beta^3}{16(1+\beta)^3},$$

$$\bar{p}_4(2) = \frac{11 + 44\beta + 48\beta^2 + 16\beta^3}{16(1+\beta)^3}, \quad \bar{p}_4(3) = \frac{15 + 48\beta + 48\beta^2 + 16\beta^3}{16(1+\beta)^3},$$

$$\bar{p}_4(t) = 1, \quad t \geq 4.$$

It is then straightforward to verify the claimed inequality by direct substitutions. □

[5] Proposition 4 in fact holds even in the case without discounting, i.e., with $\beta = 1$. To see this, note that in this case (36) implies that $p(t) = tp(0)$ for all $t \geq 1$. Therefore, the only value that $p(0)$ can take is 0. If otherwise $p(0) > 0$, then $p(t) > 1$ for all $t \geq 1/p(0)$, a contradiction to (35). It then follows immediately that $p(t) = 0$ for all $t \geq 0$.

Lemma 2 *Let A, B, C, D and a, b, c, d be arbitrary nonnegative real numebrs. Then,*

$$(38)\quad \frac{\frac{1}{2(1+\beta)}B + \frac{1}{2}C + \frac{\beta}{2(1+\beta)}D}{\frac{1}{2(1+\beta)}A + \frac{1}{2}B + \frac{\beta}{2(1+\beta)}C} > \frac{\frac{1}{2(1+\beta)}b + \frac{1}{2}c + \frac{\beta}{2(1+\beta)}d}{\frac{1}{2(1+\beta)}a + \frac{1}{2}b + \frac{\beta}{2(1+\beta)}c}$$

if one of the following three conditions holds:

(i) $\frac{D}{C} \geq \frac{d}{c}$, $\frac{C}{B} \geq \frac{c}{b}$, $\frac{B}{A} \geq \frac{b}{a}$, *with at least one strict inequality,*

(ii) $D = d = c = 0$, $\frac{C}{B} \geq \frac{c}{b}$, $\frac{B}{A} \geq \frac{b}{a}$, *with at least one strict inequality,*

(iii) $C = D = b = c = d = 0$, $\frac{B}{A} > \frac{b}{a}$,

where all variables are strictly positive unless specified otherwise.

PROOF: We first prove (38) under (i). Cross multiplying the terms on both sides of (38) and expanding the resulting expressions show that (38) is equivalent to the following inequality:

$$(39)\quad \begin{aligned} &\frac{1}{4(1+\beta)^2}Ba + \frac{1}{4(1+\beta)}Bb + \frac{\beta}{4(1+\beta)^2}Bc + \frac{1}{4(1+\beta)}Ca + \frac{1}{4}Cb + \frac{\beta}{4(1+\beta)}Cc \\ &+ \frac{\beta}{4(1+\beta)^2}Da + \frac{\beta}{4(1+\beta)}Db + \frac{\beta^2}{4(1+\beta)^2}Dc \\ &> \frac{1}{4(1+\beta)^2}Ab + \frac{1}{4(1+\beta)}Ac + \frac{\beta}{4(1+\beta)^2}Ad + \frac{1}{4(1+\beta)}Bb + \frac{1}{4}Bc + \frac{\beta}{4(1+\beta)}Bd \\ &+ \frac{\beta}{4(1+\beta)^2}Cb + \frac{\beta}{4(1+\beta)}Cc + \frac{\beta^2}{4(1+\beta)^2}Cd \end{aligned}$$

Using (i) to compare the two sides of (39) term by term leads to a conclusion that the terms on the left-hand side are always larger than or equal to the corresponding terms on the right-hand side, except for those terms involving Bc and Cb. We thus need to show that

$$\frac{\beta}{4(1+\beta)^2}Bc + \frac{1}{4}Cb \geq \frac{1}{4}Bc + \frac{\beta}{4(1+\beta)^2}Cb,$$

or, by collecting terms, that

$$\frac{1}{4}\left[1 - \frac{\beta}{(1+\beta)^2}\right](Bc - Cb) \leq 0.$$

The above inequality holds since $0 < \beta < 1$ and $Bc \leq Cb$ by (i). Since there is at least one strict inequality in (i), (39) holds, and so does (38). The proof of (38) under (ii) or (iii) is similar, with the specified zero terms imposed in (39). This completes the proof. □

Appendix B

This appendix presents a model of chain-of-production with capital accumulation. The model is identical to the baseline model in Section 2 with two exceptions. First, firms at the first stage now use both labor and capital as inputs. Second, the household's problem now involves a decision on capital accumulation.

B.1. The Model

We first describe the household's problem. The utility function is the same as in the baseline model. The budget constraint is now given by

$$\begin{aligned}(40)\qquad & \bar{P}_N(s^t)C(s^t) + \bar{P}_N(s^t)I(s^t)\left[1+\phi\left(\frac{I(s^t)}{K(s^{t-1})}\right)\right] + \sum_{s^{t+1}} D(s^{t+1}|s^t)B(s^{t+1}) + M(s^t) \\ \leq\ & W(s^t)L(s^t) + R^k(s^t)K(s^{t-1}) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t),\end{aligned}$$

where $I(s^t)$, $K(s^t)$, and $R^k(s^t)$ denote investment, capital stock, and the nominal rental rate on capital, respectively, and $\phi(I(s^t)/K(s^{t-1}))$ is the capital adjustment cost, for each s^t and each $t \geq 0$. The other notations are the same as in the baseline model. The consumption/investment good is a Dixit-Stiglitz (1977) composite of goods produced at the final stage, that is,

$$(41)\quad C(s^t) + I(s^t)\left[1+\phi\left(\frac{I(s^t)}{K(s^{t-1})}\right)\right] = \left[\int_0^1 Y_N(i,s^t)^{\frac{\theta_N-1}{\theta_N}}\, di\right]^{\frac{\theta_N}{\theta_N-1}} \equiv Y(s^t).$$

Additionally, the capital accumulation rule is given by

$$(42)\quad I(s^t) = K(s^t) - (1-\delta)K(s^{t-1}),$$

where $0 < \delta < 1$ is a capital depreciation rate.

The representative household chooses $C(s^t)$, $I(s^t)$, $\{Y_N(i,s^t)\}_{i\in[0,1]}$, $M(s^t)$, $L(s^t)$, and $B(s^{t+1})$ to maximize utility subject to (40)-(42) and a borrowing constraint $B(s^t) \geq -\bar{B}$ for some large positive number $\bar{B}$, taking prices $W(s^t)$, $R^k(s^t)$, $D(s^{t+1}|s^t)$, $\{P_N(i,s^t)\}_{i\in[0,1]}$, and $\bar{P}_N(s^t)$ and initial conditions $K(s^{-1})$, $M(s^{-1})$, and $B(s^0)$ as given. To simplify notations, we denote by $Q(s^t)$ the investment-capital ratio $I(s^t)/K(s^{t-1})$, and by $F(Q)$ the effective cost of capital $1 + \phi(Q) + Q\phi'(Q)$. The first order conditions are then given by

$$(43)\quad -\frac{U_l(s^t)}{U_c(s^t)} = \frac{W(s^t)}{\bar{P}_N(s^t)},$$

$$(44)\quad \frac{U_m(s^t)}{U_c(s^t)} = 1 - \beta\sum_{s^{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})\bar{P}_N(s^t)}{U_c(s^t)\bar{P}_N(s^{t+1})},$$

$$(45)\quad D(s^\tau|s^t) = \beta^{\tau-t}\pi(s^\tau|s^t)\frac{U_c(s^\tau)\bar{P}_N(s^t)}{U_c(s^t)\bar{P}_N(s^\tau)},\ \ \tau \geq t,$$

$$\begin{aligned}(46)\quad U_c(s^t)F(Q(s^t)) = \beta\sum_{s^{t+1}} & \pi(s^{t+1}|s^t)U_c(s^{t+1})[R^k(s^{t+1})/\bar{P}_N(s^{t+1}) \\ & +(1-\delta)F(Q(s^{t+1})) + Q(s^{t+1})^2\phi'(Q(s^{t+1}))],\end{aligned}$$

where $U_c(s^t)$, $U_l(s^t)$, and $U_m(s^t)$ denote the marginal utility of consumption, leisure, and real money balances, respectively, and $\pi(s^\tau|s^t) = \pi(s^\tau)/\pi(s^t)$ is the conditional probability of s^τ given s^t. Equations (43)-(46) are standard first order conditions with respect to the household's choice of labor, money, bond, and capital, respectively.

We next specify the problem of firms at the first stage. Production technology of a firm $i \in [0,1]$ at the first stage is a standard Cobb-Douglas production function

$$(47) \quad Y_1(i,s^t) = K(i,s^t)^\alpha L(i,s^t)^{1-\alpha},$$

where $0 < \alpha < 1$. Minimizing $R(s^t)K + W(s^t)L$ subject to (47) yields firm i's factor demand functions

$$(48) \quad L^d(i,s^t) = \left[\frac{1-\alpha}{\alpha}\frac{R^k(s^t)}{W(s^t)}\right]^\alpha Y_1(i,s^t), \quad K^d(i,s^t) = \left[\frac{\alpha}{1-\alpha}\frac{W(s^t)}{R^k(s^t)}\right]^{1-\alpha} Y_1(i,s^t).$$

It follows that the marginal cost function (which is also the unit cost function due to constant returns) at the first stage is firm-independent and is given by

$$(49) \quad V_1(s^t) = \tilde{\alpha} R^k(s^t)^\alpha W(s^t)^{1-\alpha},$$

where $\tilde{\alpha} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. The other optimization conditions are the same as in Section 2, and an equilibrium can be defined analogously.

B.2. The Computation

We now describe how to compute equilibrium decision rules. With appropriate substitutions, the equilibrium conditions can be reduced to $2N+3$ equations, including an aggregate resource constraint, a capital Euler equation, a money demand equation, N price decision equations, and N equations defining price indices. The decision variables are N current prices, aggregate consumption, aggregate labor, and aggregate capital stock. We focus on a symmetric equilibrium in which firms in the same cohort at each stage make identical decisions so that the price decision of a firm only depends on the stage at which it produces and the time at which it can set a new price.

We begin with the aggregate resource constraint. By integrating the goods demand functions (2) and (4), we get, for $n \in \{1, \ldots, N\}$,

$$(50) \quad Y_n(s^t) = \left[\prod_{k=n}^{N} G_k(s^t)\right] Y(s^t),$$

where $G_k \equiv \int_0^1 [P_k(i)/\bar{P}_k]^{-\theta_k} di$ for $k \in \{1, 2, \ldots, N\}$. Integrating (48) over i and using (50) lead to

$$(51)\quad Y(s^t) = \left[\prod_{n=1}^{N} G_n(s^t)\right]^{-1} K(s^{t-1})^{\alpha} L(s^t)^{1-\alpha},$$

where we have used the factor market clearing conditions that $\int_0^1 K^d(i, s^t) di = K(s^{t-1})$ and $\int_0^1 L^d(i, s^t) di = L(s^t)$. Note that the capital stock available for rent in period t is $K(s^{t-1})$ and it is chosen by the household in period $t-1$, while each firm i at the first stage chooses its factor demand after the realization of s^t and its demand for capital is $K^d(i, s^t)$. Substituting (51) into (41) gives the aggregate resource constraint.

Next, we express all variables in the N price decision equations in terms of the aggregate variables. This involves the N unit cost functions and price indices, in addition to the the stage-specific demand functions Y_n which are related to the aggregate output by (50). In light of (43), (48), and (49), the unit cost at the first stage is given by

$$V_1(s^t) = \frac{1}{1-\alpha}\left(\frac{L(s^t)}{K(s^{t-1})}\right)^{\alpha}\left(\frac{-U_l(s^t)}{U_c(s^t)}\right)\bar{P}_N(s^t),$$

where we have used the relation $R^k(s^t) = (\alpha/(1-\alpha))(L(s^t)/K(s^{t-1}))W(s^t)$ derived from (48) and the implication of constant returns to scale that the labor-capital ratio is firm-independent. The unit cost at a later stage ($n \geq 2$) is simply the price index at the previous stage as indicated by (5). In a symmetric equilibrium, firms in the same cohort at each stage make identical price decisions, and thus the price index at a stage $n \in \{1, \ldots, N\}$ is given by

$$(52)\quad \bar{P}_n(s^t) = \left[\frac{1}{2}P_n(s^{t-1})^{1-\theta_n} + \frac{1}{2}P_n(s^t)^{1-\theta_n}\right]^{\frac{1}{1-\theta_n}},$$

and the term G_n in (50) is given by $G_n(s^t) = \bar{P}_n(s^t)^{\theta_n}[P_n(s^{t-1})^{-\theta_n} + P_n(s^t)^{-\theta_n}]/2$.

Finally, we use (43) and (48) to substitute for $R^k(s^t)$ in the capital Euler equation, and (52) to substitute for $\bar{P}_N(s^t)$ in the money demand equation.

Given the Markov money supply process (18), a stationary equilibrium in this economy consists of stationary decision rules which are functions of the state of the economy. In each period, half of the ongoing prices was set in the previous period due to staggered price contracts. Thus, in period t, the state records the prices set in period $t-1$ in addition to the beginning-of-period capital stock and the exogenous money growth rate. We normalize all prices by dividing them by the money stock to induce stationarity. Thus, the state in this economy in event s^t is given by $[P_1(s^{t-1})/M(s^t), \cdots, P_N(s^{t-1})/M(s^t), k(s^{t-1}), \mu(s^t)]$.

B.3. The Calibration

The parameters to be calibrated include the subjective discount factor β, the parameters Φ and Ψ that determine the relative weights of real money balances and leisure time in the utility function, the capital share α, the capital depreciation rate δ, the goods demand elasticity parameters θ_n, the monetary policy parameter ρ, and the parameters in the capital adjustment cost function. The calibrated values are summarized in Table III.

Following the standard business cycle literature, we choose $\beta = 0.96^{1/2}$. To assign a value for Φ, we use the implied money demand equation

$$\Phi = \frac{M(s^t)}{\bar{P}_N(s^t)\, C(s^t)} \left(\frac{R(s^t) - 1}{R(s^t)} \right),$$

where $R(s^t) = \left(\sum_{s^{t+1}} D(s^{t+1}|s^t)\right)^{-1}$ is the gross nominal interest rate. We choose $\Phi = 0.028$ so that the model is consistent with a steady state consumption velocity of 3.3 and a nominal interest of 1.05, both at annualized levels. The serial correlation parameter ρ for money growth is set to 0.57^2, based on quarterly U.S. data on M1 from 1959:3 through 1995:2 obtained from Citibase (see also CKM (1998a)).

We next choose $\delta = 1 - 0.92^{1/2}$ and $\Psi = 1.56$ so that the model predicts an investment-output ratio of 0.23 and an average share of time allocated to market activity of 1/3, as in most business cycle studies.

We set $\theta_n = \theta$, and assign a value for θ so that the model implies a steady state price-cost margin of 0.26, which is the value found by Domowitz, Hubbard and Petersen (1986). The price-cost margin (PCM) in our model is the overall *net* markup of the final stage price over the ultimate unit cost $v \equiv V_1/\bar{P}_N$, that is,

$$PCM = \frac{\bar{P}_N - v\bar{P}_N}{\bar{P}_N} = 1 - v,$$

where we have used the steady state relation that $\bar{P}_N = P_N$. The implied steady-state unit cost is $v = 0.74$. We then determine the value of θ using the following steady state relationship:[6]

$$v = \prod_{n=1}^{N} \left[\frac{\theta_n - 1}{\theta_n} \right] = \left(\frac{\theta - 1}{\theta} \right)^N.$$

Given v, the value of α is assigned so that the model predicts an annualized capital-output ratio of 2.65. Specifically, we use the steady state condition $K/Y = (\alpha v)/r$, where $r \equiv R^k/\bar{P}_N = 0.06$ is the steady state real rental rate on capital. If follows that $\alpha = 0.45$. This value is larger

[6] In an unreported sensitivity test, we find that our results are not sensitive to the value of θ. Indeed, θ does not play any important role as can be seen from the analytical solutions in Section 3.

than what is usually used in the standard business cycle literature, because it is here the share of capital income in total cost, rather than that in total revenue.[7]

Finally, we assume that the capital adjustment cost function takes the form

$$\phi\left(\frac{I}{K}\right)=\frac{\psi}{2}\left(\frac{I}{K}\right)^2,$$

where ψ is adjusted as we vary the number of production stages so that the model predicts a standard deviation of aggregate investment being 3.23 times as large as that of output, in accordance with the U.S. data.

[7]Hall (1988) shows that the measure of α based on total cost instead of total revenue is a more accurate measure of the elasticity of output with respect to capital input, especially with imperfect competition.

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TABLE I

SNAKE EFFECT IN THE BASELINE MODEL

$\bar{p}_n(t)$	$n=1$	$n=2$	$n=5$	$n=10$	$n=20$
$\bar{p}_n(0)$	0.50	0.37	0.24	0.17	0.12
$\bar{p}_n(1)$	1.00	0.87	0.65	0.49	0.36
$\bar{p}_n(2)$	1.00	1.00	0.89	0.73	0.56
$\bar{p}_n(3)$	1.00	1.00	0.98	0.88	0.72

TABLE II

OUTPUT PERSISTENCE IN THE BASELINE MODEL

	$N=1$	$N=2$	$N=5$	$N=10$	$N=20$
Contract Multiplier	0.00	0.20	0.46	0.62	0.73
Half Life	0.50	0.63	0.93	1.41	2.01

TABLE III

CALIBRATED PARAMETERS

Preferences: $U(C, M/\bar{P}_N, L) = \log C + \Phi \log(M/\bar{P}) - \Psi L$,	$\Phi = 0.028,\ \Psi = 1.56$
Production Technology at the First Stage: $Y = K^\alpha L^{1-\alpha}$,	$\alpha = 0.45$
Production Technology at Stage $n \geq 1$: $Y_{n+1} = \left[\int Y_n(j)^{\frac{\theta_n - 1}{\theta_n}} dj\right]^{\frac{\theta_n}{\theta_n - 1}}$,	$\theta_n = \theta$ adjusted
Capital Accumulation: $K_t = I_t + (1-\delta)K_{t-1}$,	$\delta = 1 - 0.92^{1/2}$
Adjustment Cost Function: $\phi(I_t/K_{t-1}) = \frac{\psi}{2}(I_t/K_{t-1})^2$	ψ adjusted
Money Growth: $\log \mu_t = \rho \log(\mu_{t-1}) + \varepsilon_t$	$\rho = 0.57^2$
Subjective Discount Factor	$\beta = 0.96^{1/2}$

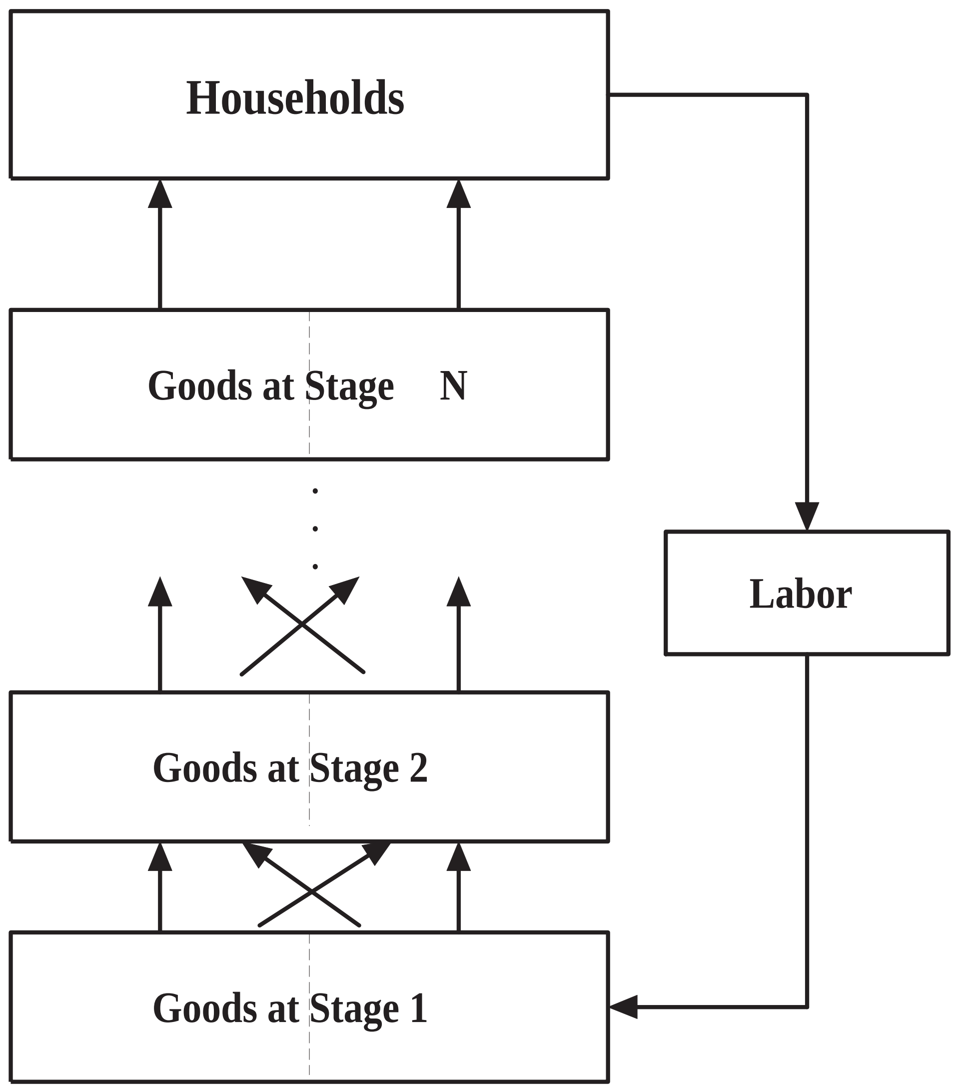

Figure 1: —Chain structure of the economy

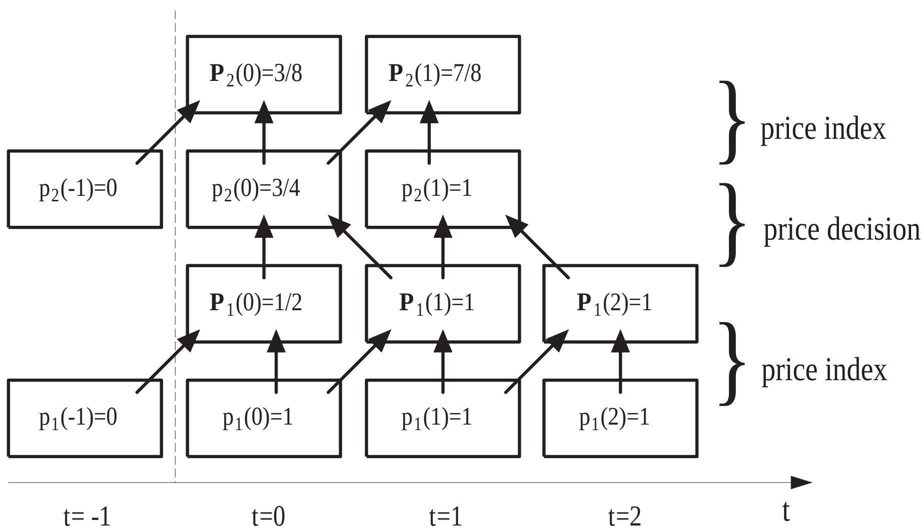

Figure 2: —Snake effect illustrated ($N = 2$, $\beta = 1$)

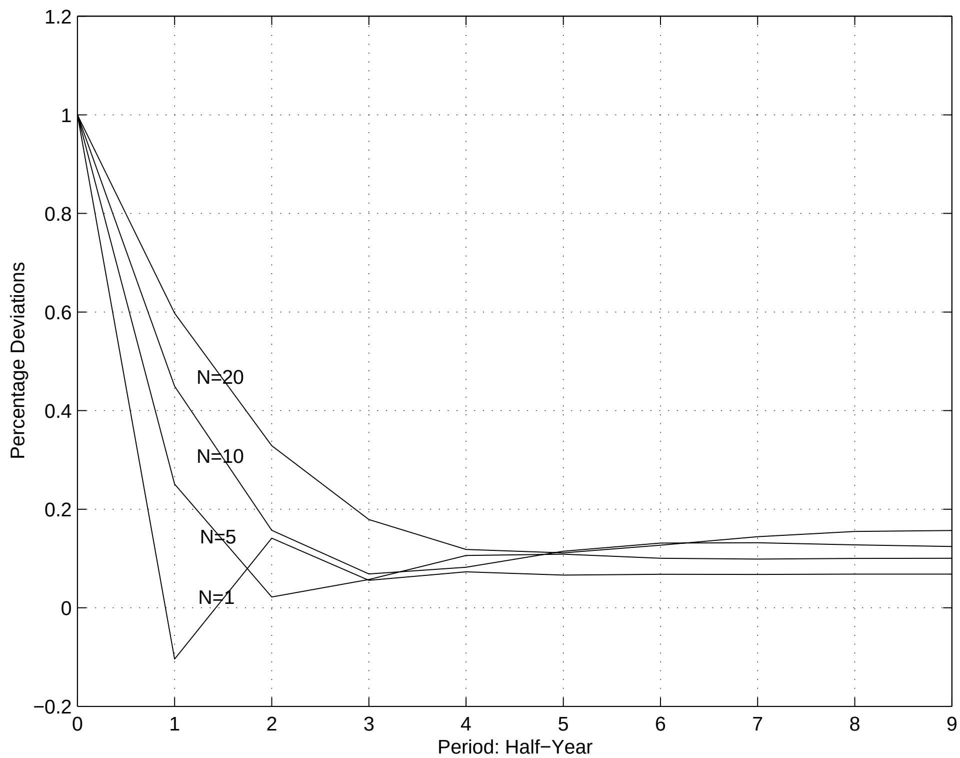

Figure 3: —Relative response of aggregate output to monetary shocks

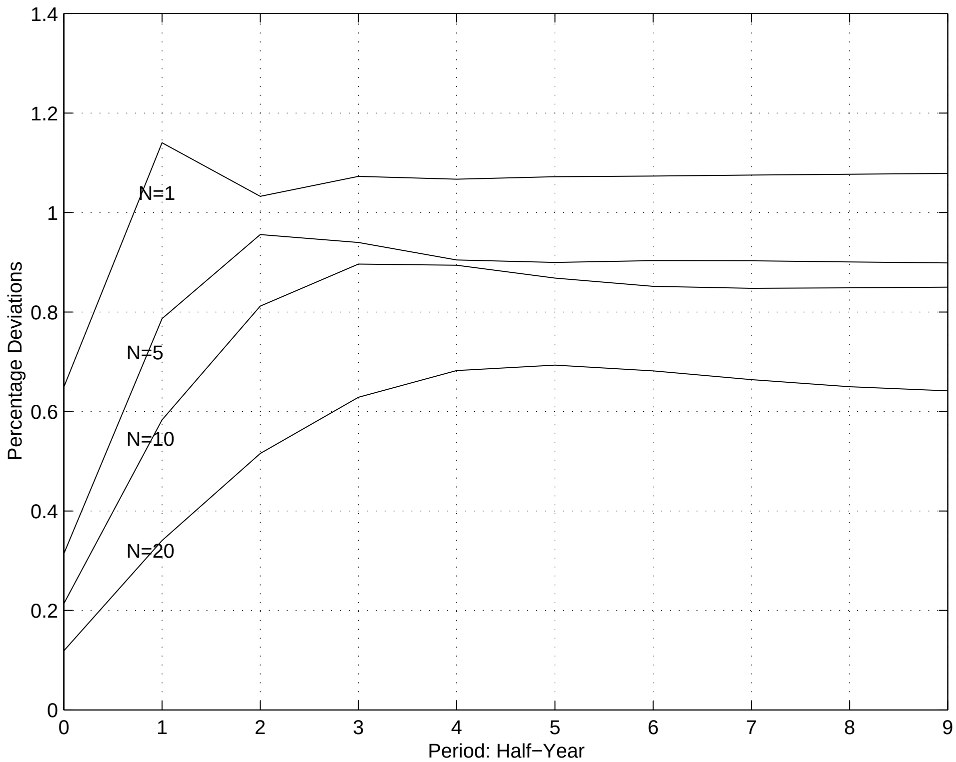

Figure 4: —Response of price level to monetary shocks