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The Effect of Tax-Favored Retirement Accounts on Capital Accumulation and Welfare*

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1. INTRODUCTION

Since 1962, self-employed individuals have been permitted to contribute to tax-favored retirement accounts, and in 1974 this privilege was extended to workers whose employers did not provide pension coverage. A primary motivation for this legislation was to extend to such workers the tax benefits enjoyed by those participating in employer-provided pension plans. The Economic Recovery Tax Act (ERTA) of 1981 extended eligibility to contribute to Individual Retirement Accounts (IRAs) to all employees and raised the annual contribution limit, and the Tax Reform Act of 1986 partially reversed this extension of eligibility.

A major purpose of expanding the scope of tax-favored retirement accounts, including the 1981 extension of IRA eligibility, was to encourage private saving. Encouragement of saving is also the primary motivation behind current proposals to expand individual tax-favored retirement plans. In 1984, total IRA contributions were more than $35 billion, or 40 percent of personal saving. After the curtailment of eligibility in 1986, annual IRA contributions fell to less than a third of their previous level, while contributions to 401(k) plans rose to about $50 billion by 1990. Although contributions to these various accounts are directly observable, their effect on saving is not. Consequently, the effectiveness of tax-favored retirement plans in stimulating saving is a matter of debate.

Most of the discussion of individual tax-favored retirement plans to date has concentrated on IRAs. Proponents of IRAs argue that the favorable tax treatment of such accounts stimulates substantial incremental saving that would not occur otherwise. A second view holds that most contributions are shifted from previously accumulated assets or from saving that would occur in the absence of IRAs. According to this view, savers avail themselves of the tax benefits IRAs offer but change their overall saving little in response to these tax benefits. One reason suggested for the small effect on saving is the limit on annual IRA contributions. For anyone already saving more than the maximum permissible contribution, IRAs do not affect the rate of return on incremental saving.

Empirical evidence on the saving effect of IRAs is mixed. For example, Venti and Wise (1990) conclude that 96 percent of IRA contributions constitutes incremental private saving (of which 31 percent takes the form of reduced tax liability and 65 percent is incremental national saving) and only 4 percent is shifted from non-IRA assets. On the other hand, Gale and Scholz (1990) conclude that only between −2 percent and 25 percent of IRA contributions constitutes incremental national saving. All of these conclusions are based on data from cross sections of households during the 1980s.

A major difficulty arises in attempting to draw such inferences from cross-sectional data, as both saving and IRA contributions are endogenous variables that households choose subject to legal and budgetary restrictions. Thus, a positive correlation between IRA contributions and saving does not prove that IRAs "cause" saving. Because the government does not compel IRA contributions, contributions are not an exogenous instrument of government policy. Instead, the policy instrument is the regulations governing eligibility to make contributions.

Most if not all cross-sectional studies of the saving effect of IRAs have examined data with no meaningful variation in IRA eligibility. Such data do not permit direct estimation of the response of saving to exogenous changes in eligibility. Instead, inferences must be based on the correlation of saving and contributions with each other and with other variables such as income and the demographic characteristics of households. In addition, inferences require some assumptions about the structure of the model that describes IRA and non-IRA saving. Given these structural assumptions, one can derive and estimate reduced forms relating IRA contributions and non-IRA saving to various exogenous variables. From the reduced-form estimates, one can then calculate the saving effect of IRAs. However, these estimated reduced forms do not themselves distinguish between alternative economic structures, and thus do not provide unambiguous estimates of the saving effects. For example, Joines and Manegold (1991) show that the reduced form estimated by Venti and Wise (1986, 1987, 1989, 1990) is consistent with a structure in which IRA eligibility has no effect on saving. Because of these problems, existing empirical studies do not provide sharp or reliable estimates of the saving effect of IRAs.

Given the inconclusiveness of empirical evidence, substantial insight can be gained from a carefully designed simulation exercise. For such an exercise to be convincing, the model used must be substantially more explicit than those that have appeared to date in the IRA literature. In particular, such a model must adequately represent intertemporal consumption choice and must include the major institutional features of IRAs. The institutional features (in place up through 1986 and present in some current proposals) are that (1) contributions to IRAs are deductible from taxable income in the year of the contribution, (2) funds in the IRA accumulate tax-free until withdrawal, at which time they are subjected to the income tax, (3) annual contributions are subject to a maximum that depends on the demographic characteristics of the household, and (4) early withdrawals from IRAs are subject to an additional tax penalty.

Without an early-withdrawal penalty, IRAs are a perfect substitute for ordinary saving, apart from differences in the tax treatment of interest payments. In such a case, individuals saving more than the IRA contribution limit would trivially contribute the maximum to their IRAs before accumulating any other assets. To capture the possibility of pre-retirement withdrawals from IRAs, a model must have more than two periods. In addition, it should incorporate uncertainty about pre-retirement income. Finally, any interesting model of IRAs must include some borrowing constraint, at least in the form of a divergence between the rates of interest households face on borrowing and lending. If the two rates were equal and interest payments were tax deductible (as was the case until 1986), then households would trivially make the maximum allowable contribution to their IRAs even in a world of uncertainty and early-withdrawal penalties.

The institutional features described above, together with some form of restriction on individual borrowing, induce "kinks" in the intertemporal budget constraint and make solution of a multiperiod model with uncertainty a difficult task. The model developed in İmrohoroğlu, İmrohoroğlu and Joines (1993,1994) involved similar kinks due to borrowing constraints. Here, we adapt that model to accommodate the institutional features of IRAs and to model those features much more realistically than has been done to date.

The models used to derive predictions about saving behavior in most previous studies of IRAs have been inadequate in one or more respects. For example, the various papers of Venti and Wise contain no explicit model of intertemporal choice or uncertainty, and the early-withdrawal penalty does not appear explicitly in the agent's choice problem. Instead, IRAs and non-IRA assets appear directly in the individual's objective function. They are imperfect substitutes by assumption rather than because of institutional features. Kotlikoff (1990), Gale and Scholz (1990), and Gravelle (1991) have criticized the Venti-Wise model for not being well-grounded in utility theory.

Other papers present very simple two- or three-period models, generally assuming a world of perfect certainty, and do not capture the relevant institutional features of IRAs. Important exceptions are the three-period uncertainty model of Gale and Scholz and the multiperiod uncertainty model of Engen and Gale (1993). These models share with ours the realistic feature that IRAs and other assets are imperfect substitutes only because of the early-withdrawal penalty and some restriction on individual borrowing. All of these models incorporate a traditional representation of intertemporal choice in that individuals derive utility only from consumption and get no direct satisfaction from holding various assets and liabilities. Instead, they acquire assets and liabilities solely for the consumption these assets and liabilities finance.

In this paper we develop a computable general equilibrium model that consists of overlapping generations of 65-period lived individuals facing mortality risk and individual income risk. Individuals are not permitted to have negative net asset positions. During their working years, individuals face stochastic employment opportunities. They supply labor inelastically whenever they are given the opportunity to work. If they are not given the opportunity to work, they receive unemployment insurance. Because they face liquidity constraints, individuals in our economy save through private asset holdings in order to self-insure against future income fluctuations and provide for old-age consumption. There are two types of private assets through which they can self-insure, ordinary assets and tax-favored retirement accounts which, for simplicity, we refer to as IRAs.

Our model corresponds closely to the main institutional features of IRAs as they existed during the early 1980s.[1] Contributions are tax-deductible and cannot exceed an upper limit. IRA balances compound at a before-tax rate of return. Withdrawals are taxable and may be subject to an early-withdrawal penalty. After the mandatory retirement age, individuals rely on social security benefits and private savings for their consumption. Social security payments are financed with a payroll tax on the employed young. The interest rate and the relative wage are determined in part by the profit maximizing behavior of a firm with a constant returns to scale technology.[2]

After the model is calibrated to match certain features of the U.S. economy, we specify the optimization problem of the individual as a finite-state, finite horizon, dynamic programming problem and use numerical methods to compute equilibria under alternative contri-

[1] Other tax-favored retirement plans, such as Keogh accounts and 401(k) plans, differ in some respects from IRAs, although these differences are small relative to those separating all such plans from ordinary, taxable saving. Thus, the closest empirical analogue to our model's IRAs is probably all individual tax-favored retirement plans rather than narrowly defined IRAs. However, see Poterba, Venti, and Wise (1994) for a discussion of the institutional differences between IRAs and 401(k) plans and for evidence that the two plans may have different effects on saving behavior.

[2] In a recent paper, Engen and Gale (1993) independently examine the effects of IRAs in a framework similar to ours. One important difference between the two models is that Engen and Gale assume that the before-tax interest rate and before-tax age-earnings profiles are fixed exogenously and are unaffected by equilibrium adjustments of the capital stock. Another difference is that our equilibrium framework permits welfare comparisons across different IRA arrangements, whereas Engen and Gale are primarily concerned with the effects of IRAs on asset accumulation.

bution limits for IRAs. Our results indicate that an IRA system with a contribution limit similar to that in effect during the early 1980s raises the net national saving rate by 2.74 percent and the steady-state capital stock by 3.40 percent. This increase in the capital stock raises steady-state welfare by at most an amount equivalent to a lump-sum consumption supplement of 0.5 percent of GNP. Approximately six percent of IRA contributions constitute incremental saving, a finding consistent with the empirical estimates of Gale and Scholz and much lower than those of Venti and Wise. Most of these conclusions are robust to a variety of alternative model specifications. In particular, the fraction of IRA contributions constituting incremental saving invariably lies within the range estimated by Gale and Scholz and is much lower than the Venti-Wise estimates. Thus, our results suggest that the brief experiment with universal IRA eligibility during the 1980s probably had a minimal effect on the U.S. capital stock.

Our results also indicate that the contribution limit is an important factor in explaining the effects of the IRA programs we examine. Middle-aged workers in our model economy generally save substantially more than they are permitted to invest in IRAs when the contribution limit is similar to that in effect during the early 1980s. IRAs do not raise the marginal rate of return for agents who would save substantially more than the contribution limit by accumulating ordinary assets. The fact that a modest contribution limit appears to be binding for a substantial fraction of savers may explain why IRAs have a relatively small effect on aggregate saving. Thus, our results lend support to recent suggestions [e.g., Bernheim and Scholz (1992)] that retirement accounts with favorable tax treatment only for contributions above some base amount would provide more stimulus to saving than conventional IRAs.

The paper is organized as follows. Section 2 describes the model economy. Section 3 contains the computational details including calibration and a description of the numerical methods. Section 4 discusses our findings. Concluding remarks are given in section 5.

2. THE MODEL ECONOMY

The model economy is populated with overlapping generations of ex ante identical individuals. In each time period, a new generation is born with probability $\psi_1 = 1$. These individuals survive from age j to age $j+1$ with probability $\psi_{j+1} > 0$ until age J, beyond which death is certain. The share of age-j individuals in the economy is denotedby $\mu_j \geq 0$. These shares are determined from $\mu_j = \psi_j \frac{\mu_{j-1}}{(1+\rho)}$ for $j \geq 2$ and $\sum_{j=1}^{J} \mu_j = 1$, where ρ denotes the rate of growth of population.

A. *Preferences*

Individuals rank lifetime consumption paths according to

$$E\sum_{j=1}^{J}\beta^{j}\left[\prod_{k=1}^{j}\psi_k\right]\frac{c_j^{1-\gamma}}{1-\gamma}\,, \tag{1}$$

where β is the subjective discount factor and γ is the coefficient of relative risk aversion.

B. *Earnings*

At the beginning of each age until the mandatory retirement age of j^*, surviving individuals in this economy receive a draw from nature determining their employment status.[3] If they are employed, they earn the age-dependent employed wage $w_j^e = w\hat{h}\epsilon_j$ where w is the marginal product of labor, $\hat{h}$ is the number of hours spent working, and ϵ_j is the efficiency index of an age-j individual.[4] Otherwise, they receive unemployment insurance benefits equal to a fraction ϕ of the employed wage: $w_j^u = \phi w_j^e$. After retirement, individuals draw on their social security benefits, which are a fraction θ of the average lifetime employed wage of all members of their cohort. The disposable income q_j of the individual is given by

$$q_j = \begin{cases} (1-\tau-\tau_s-\tau_u)w_j^e & \text{for } j=1,2,\ldots,j^*-1, \text{ if } s=e, \\ w_j^u & \text{for } j=1,2,\ldots,j^*-1, \text{ if } s=u, \\ b & \text{for } j=j^*,j^*+1,\ldots,J, \end{cases} \tag{2}$$

where the social security and unemployment insurance tax rates are denoted by τ_s and τ_u, respectively, and the income tax rate is denoted by τ. Social security benefits are given by

$$b = \theta\frac{\sum_{j=1}^{j^*-1} w_j^e}{j^*-1}\quad. \tag{3}$$

C. *Individual Budget Constraints with IRAs*

We can now describe the institutional structure of Individual Retirement Accounts and the age-dependent individual budget constraints that arise from the assumed setup. Let

[3] Due to computational difficulties, we will restrict our immediate attention to computing steady-state equilibria.

[4] Our use of indivisible labor follows Hansen (1985) and Rogerson (1988).

y_j and a_j denote, respectively, ordinary asset holdings and the IRA balance at the end of period j. Consumption at age j is denoted by c_j. We assume that $y_j \geq 0$, that is individuals are liquidity constrained.[5] Also, we restrict $a_j \geq 0$. Because the time of death is uncertain, individuals who die before age J will leave positive asset balances. These accidental bequests are assumed to be distributed equally to all survivors in a lump-sum fashion.

In the last period of life, the individual's budget constraint is given by

$$\begin{aligned} c_J = {} & [1 + (1-\tau)r](y_{J-1} + \alpha_1) + b + \alpha_2 \\ & + (1-\tau)\left[(1+r)a_{J-1}\right] , \end{aligned} \tag{4}$$

where α_1 is a lump-sum distribution of accidental bequests, α_2 is a lump-sum government transfer, and r is the marginal product of capital (net of depreciation). This constraint incorporates the assumptions that IRA assets compound at a before-tax rate and that withdrawals are subject to the income tax but face no additional penalty. Absent a bequest motive, individuals choose not to hold positive asset balances at the end of age J, so that $y_J = 0$ and $a_J = 0$.

During retirement years, i.e. for $j \in [j^*, J)$, the individual's budget equation is written as

$$\begin{aligned} c_j = {} & [1 + (1-\tau)r](y_{j-1} + \alpha_1) + q_j + \alpha_2 - y_j \\ & + (1-\tau)\left[(1+r)a_{j-1} - a_j\right] \quad \text{and} \end{aligned} \tag{5}$$

$$a_j \leq (1+r)a_{j-1} . \tag{6}$$

The last equation reflects the restriction that retirees cannot contribute to their IRA accounts.

In line with the provisions of ERTA, individuals who are older than 60 but younger than 65 can contribute to IRAs if they have labor income. They can also withdraw without penalty. Hence, for $j \in [j^* - 5, j^* - 1]$ and $s = e$, consumption is still given by equation (5), but the constraint (6) is replaced by

$$a_j - (1+r)a_{j-1} \leq \iota , \tag{7}$$

[5] This assumption is sometimes interpreted as permitting collateralized loans, but prohibiting individuals from having negative net worth. With IRAs in the model, we also assume that individuals are prohibited from borrowing against their IRA balances.

where ι denotes the ceiling on IRA contributions. If an individual is in the same age group as above but unemployed, then the individual cannot contribute to an IRA but can still withdraw without penalty. So, for $j \in [j^* - 5, j^* - 1]$ and $s = u$, consumption is given by equation (5) subject to restriction (6).

For $j \in [1, j^* - 6]$ and $s = e$ (employed), a contributor's consumption is given by equation (5) subject to

$$(1 + r)a_{j-1} < a_j \leq (1 + r)a_{j-1} + \iota \; . \tag{8}$$

Note once more that an unemployed individual cannot contribute to his IRA. For $j \in [1, j^* - 6]$, regardless of employment status, withdrawals are subject to a penalty τ_w. Thus, consumption for the unemployed and employed noncontributors is

$$\begin{aligned} c_j = {} & [1 + (1 - \tau)r](y_{j-1} + \alpha_1) + q_j + \alpha_2 - y_j \\ & + (1 - \tau - \tau_w)\left[(1 + r)a_{j-1} - a_j\right] \; , \text{ where} \end{aligned} \tag{9}$$

$$a_j \leq (1 + r)a_{j-1} \; . \tag{10}$$

D. *Technology*

The aggregate production technology is given by

$$Q = f(K, N) \equiv BK^{1-\alpha}N^{\alpha} \quad , \tag{11}$$

where $B > 0$, $\alpha \in (0, 1)$ is labor's share of output, and K and N are aggregate capital and labor inputs, respectively. These are computed from individual behavior as follows:

$$K = \alpha_1 + \sum_{j=1}^{J} \sum_a \sum_y \sum_s \mu_j \lambda_j(a, y, s) \left[A_{j-1}(a, y, s) + Y_{j-1}(a, y, s)\right],$$

$$N = \sum_{j=1}^{j^*-1} \sum_a \sum_y \sum_s \mu_j \lambda_j(a, y, s) \hat{h} \epsilon_j \quad ,$$

where $A(\cdot, \cdot, \cdot)$ and $Y(\cdot, \cdot, \cdot)$ are individuals' decision rules, μ_j is the share of age-j individuals in the population, and $\lambda_j(\cdot, \cdot, \cdot)$ is the age-dependent distribution of agents. The aggregate capital stock is assumed to depreciate at the rate δ. The profit-maximizing behavior of the firm gives rise to first-order conditions which determine the net real return to capital and the real wage

$$(12)\qquad r = (1-\alpha)B\Big[\frac{K}{N}\Big]^{-\alpha} - \delta \quad , \quad w = \alpha B\Big[\frac{K}{N}\Big]^{(1-\alpha)} \ .$$

E. *Government's Budget*

The government agencies that oversee the social security and unemployment insurance programs are assumed to balance their budgets every period. Each period, the government purchases G of goods and services that provide no utility to the private sector. In addition, the government makes a lump-sum transfer of α_2 to each individual. To finance these expenditures, the government imposes taxes on income from labor and capital. Tax receipts from interest income on non-IRA assets plus tax collections on net withdrawals from IRAs (which may be negative) constitute revenue from capital income. Thus, the government's budget is given by:

$$(13)\qquad G + \alpha_2 = R_K + \tau w N \ ,$$

where R_K denotes the government's revenue from capital income. The tax rate is adjusted so that the budget is always in balance.

3. STATIONARY EQUILIBRIA

Let $D^R = \{d_1^R, d_2^R, \ldots, d_m^R\}$ and $D^I = \{d_1^I, d_2^I, \ldots, d_m^I\}$ denote the discrete grids of points on which regular and IRA asset holdings are required to fall. For any beginning-of-period asset holding and employment status $(a, y, s) \in D^I \times D^R \times S$, define the constraint set of an age-j agent $\Omega_j(a, y, s) \in R_+^3$ as all triplets (c_j, a_j, y_j) such that budget constraints (2)–(10) are satisfied, in addition to the following nonnegativity constraints:

$$c_j \geq 0 \ , \ a_j \geq 0 \ , \ y_j \geq 0 \ .$$

Let $V_j(a, y, s)$ be the value of the objective function of an age-j agent with beginning-of-period asset holdings and employment status (a, y, s). $V_j(a, y, s)$ is defined as the solution to the dynamic program

$$(14)\qquad V_j(a, y, s) = \max_{(c, a', y') \in \Omega_j(a, y, s)} \Big\{ U(c) + \beta \psi_{j+1} E_{s'} V_{j+1}(a', y', s') \Big\} \ .$$

Given a set of constant fiscal policy arrangements $\{G, \alpha_2, \theta, \phi, \tau, \tau_s, \tau_u, \tau_w, \iota\}$, a steady-state equilibrium is a collection of value functions $V_j(a, y, s)$, individual policy rules $C_j : D^R \times D^I \times S \to R_+, A_j : D^R \times D^I \times S \to D^I$, and $Y_j : D^R \times D^I \times S \to D^R$, age-dependent (but time-invariant) measures of agent types $\lambda_j(a, y, s)$ for each age $j = 1, 2, \ldots, J$, relative prices of labor and capital $\{w, r\}$, and lump-sum transfers α_1 and α_2 such that individuals maximize utility subject to budget constraints (2)–(10), the government budget constraint is satisfied, and the goods market clears.[6] Formally, the following conditions must hold in equilibrium:

i. Aggregate variables are computed from individual behavior:

$$K = \alpha_1 + \sum_{j=1}^{J} \sum_{a} \sum_{y} \sum_{s} \mu_j \lambda_j(a, y, s) \left[A_{j-1}(a, y, s) + Y_{j-1}(a, y, s)\right],$$

$$K' = \sum_{j=1}^{J} \sum_{a} \sum_{y} \sum_{s} \mu_j \lambda_j(a, y, s) \left[A_j(a, y, s) + Y_j(a, y, s)\right],$$

$$N = \sum_{j=1}^{J} \sum_{a} \sum_{y} \sum_{s} \sum_{j} \mu_j \lambda_j(a, y, s) \hat{h} \epsilon_j,$$

ii. The relative prices $\{w, r\}$ solve the firm's profit maximization problem by satisfying equation (12),

iii. Given relative prices $\{w, r\}$, government fiscal policy $\{G, \alpha_2, \theta, \phi, \tau, \tau_s, \tau_u, \tau_w, \iota\}$, and accidental bequests α_1, the individual policy rules $C_j(a, y, s), A_j(a, y, s)$ and $Y_j(a, y, s)$ solve the individuals' dynamic program (14),

iv. The commodity market clears,

$$\begin{aligned} &\sum_{j} \sum_{a} \sum_{y} \sum_{s} \mu_j \lambda_j(a, y, s) C_j(a, y, s) + \left[K' - (1 - \delta) K\right] + G \\ &\qquad = f(K, N) \ , \end{aligned} \tag{17}$$

where the initial wealth distribution of agents, A_0, is taken as given,

v. The collection of age-dependent, time-invariant measures $\lambda_j(a, y, s)$ for $j = 1, 2, \ldots, J$, satisfies

$$\lambda_j(a', y', s') = \sum_{s} \sum_{a: a' = A_j(a, y, s)} \sum_{y: y' = Y_j(a, y, s)} \Pi(s', s) \lambda_{j-1}(a, y, s) \quad , \tag{18}$$

[6] For a detailed definition of equilibrium in a similar environment, see Diaz-Gimenez and Prescott (1992), Rios-Rull (1991) and İmrohoroğlu, İmrohoroğlu and Joines (1993,1994).

where the initial measure of agents at birth, λ_1, is taken as given,

vi. The social security system is self-financing:

$$\tau_s = \frac{\sum_{j=j^*}^{J} \sum_a \sum_y \mu_j \lambda_j(a,y,s) b}{\sum_{j=1}^{j^*-1} \sum_a \sum_y \mu_j \lambda_j(a,y,s=e) w \epsilon_j \hat{h}}$$
$$(19) \qquad = \frac{b \sum_{j=j^*}^{J} \mu_j}{w\hat{h} \sum_{j=1}^{j^*-1} \mu_j \lambda_j(a,y,s) \epsilon_j} ,$$

vii. The unemployment insurance benefits program is self-financing:

$$\tau_u = \frac{\sum_{j=1}^{j^*-1} \sum_a \sum_y \mu_j \lambda_j(a,y,s=u) \xi w \hat{h}}{\sum_{j=1}^{j^*-1} \sum_a \sum_y \mu_j \lambda_j(a,y,s=e) w \epsilon_j \hat{h}}$$
$$(20) \qquad = \frac{\xi \sum_{j=1}^{j^*-1} \mu_j}{\sum_{j=1}^{j^*-1} \mu_j \epsilon_j} ,$$

viii. The lump-sum distribution of accidental bequests is determined by

$$(21) \qquad \alpha_1 = \sum_j \sum_a \sum_y \sum_s \mu_j \lambda_j(a,y,s)(1-\psi_{j+1})[A_j(a,y,s) + Y_j(a,y,s)] \ .$$

ix. The government's budget satisfies:

$$(15) \qquad G + \alpha_2 = R_K + \tau w N \ .$$

A. *Measure of Utility*

In order to compare the quantitative impact of alternative IRA limits, we need a measure of "average utility." Given a policy arrangement $\Omega = \{G, \alpha_2, \theta, \phi, \tau, \tau_s, \tau_u, \tau_w, \iota\}$, we calculate

$$(15) \qquad W(\Omega) = \sum_{j=1}^{J} \sum_a \sum_y \sum_s \beta^{j-1} \Big[\prod_{k=1}^{j} \psi_k \Big] \lambda_j(a,y,s) U\Big(C_j(a,y,s)\Big) \ ,$$

as our measure of utility. $W(\Omega)$ is the expected discounted utility a newly born individual derives from the lifetime consumption contingency plan $\{C_j(a,y,s)\}_{j=1}^{J}$ under a given IRA limit ι.

B. *Calibration*

In order to obtain numerical solutions to the model and examine the effects of tax-favored retirement accounts, we need to choose particular values for the parameters of the model. We calibrate our model under the assumption that the model period is one year.[7]

Individuals are assumed to be born at the real-time age of 21 and they can live a maximum of $J = 65$ years, to the real-time age of 85. After age 85, death is certain.[8] The sequence of conditional survival probabilities $\{\psi_j\}_{j=1}^{J}$ is taken from Faber (1982). The share of age groups in the population, μ_j, is calculated from the relations $\mu_{j+1} = \frac{\psi_{j+1}}{(1+\rho)}\mu_j$ and $\sum_{j=1}^{J}\mu_j = 1$, where ρ is the growth rate of the population, which has averaged 1.2 percent per year in the United States over the last fifty years. The mandatory retirement age is taken to be $j^* = 45$, which corresponds to the real-time age of 65. The efficiency index $\{\epsilon_j\}$ is intended to provide a realistic cross-sectional age distribution of earnings at a point in time. This index is taken from Hansen (1991), interpolated to in-between years, and normalized to average one between $j = 1$ and $j = j^* - 1$; after $j = j^* - 1$ we assume that $\epsilon_j = 0$. Raw hours of work, $\hat{h}$, are taken as 0.45, which assumes that individuals devote 45 hours a week (out of a possible 98 hours) to work. Given an employment rate of 94%, the aggregate labor input is computed as $N = 0.94\hat{h}\sum_{j=1}^{j^*-1}\mu_j\epsilon_j$. Note that μ_j and therefore N depend on the population growth rate ρ and the probability of survival ψ_j.

Following Prescott (1986), the exponent of labor in the production function, α, is taken to be 0.64, which is labor's share of GNP. Auerbach and Kotlikoff (1987) use 0.75, and Hubbard and Judd (1987) use 0.70. The parameter B in the production function is fixed at 1.3193 so that output is normalized at one for a capital-output ratio of 3 given an aggregate labor input of 0.3496. The rate of depreciation of capital, δ, is taken as 0.06. We also compute results for a depreciation rate of 0.08.

There seems to be a wide range of empirical estimates for the intertemporal elasticity of substitution, $1/\gamma$. Hurd's (1989) estimates are 1.4 and .89, which are larger than those usually found in the literature. For example, Hall (1988) uses annual data on consumption and estimates negative values for the intertemporal elasticity of substitution. Mehra and Prescott (1985) cite various empirical studies that suggest that the coefficient of relative risk aversion, γ, is between one and two. We take $1/\gamma$=0.67 as our base case and also report results for the cases where $1/\gamma$=0.33. As one would expect, the results are sensitive to this parameter.

[7] Taking the period as six months would double the computational burden of the model.

[8] This assumption does not appear to be crucial; according to Faber (1982), we are leaving out less than three percent of the U.S. population.

In an overlapping generations setting, economic theory does not impose any restriction on the size of the rate of time preference.[9] The subjective time discount factor β has traditionally been taken to be less than unity. For example, Auerbach and Kotlikoff (1987) use $\beta = 0.9852$ in a representative agent, life cycle model with certain lifetimes. Hubbard and Judd (1987) use the same value for β in a representative agent, life cycle model with lifetime uncertainty.

Recent empirical evidence on the value of β suggests that a subjective discount factor greater than unity is plausible. Hansen and Singleton (1983) fit several different models to aggregate U.S. time series data and estimate β to be greater than unity in about half the cases. Using the Panel Study of Income Dynamics, Hotz, Kydland and Sedlacek (1988) obtain estimates of β ranging from 1.0123 to 1.2041 that are all statistically larger than unity.[10] The empirical study most relevant for our purposes is that of Hurd (1989), who explicitly incorporates mortality risk into a life cycle model. Using the Retirement History Survey, he estimates the coefficient of risk aversion and the subjective time discount factor. His nonlinear 2SLS estimates imply a β of 1.011, and no statistically significant role for planned bequests in his sample.

Figure 1 shows the sequence of effective discount factors $\left\{\beta^{j-1}\prod_{k=1}^{j}\psi_k\right\}_{j=1}^{J}$ under alternative values of β. For $\beta = 0.98$, the effective discount factor shows an increasing disregard for future consumption under lifetime certainty. With mortality risk, the utility of future consumption is even more heavily discounted. For $\beta = 1.011$ under certain lifetimes, there is increasing preference for consumption in old age. For $\beta = 1.011$ under lifetime uncertainty, the effective time discount factor shows a slight increase in the weight attached to consumption up to about the fortieth period of life (real-time age 60), followed by a decline as unconditional mortality risk becomes large.

We take $\beta = 1.011$ for our benchmark model economy. We choose this value of β not only because it has empirical support but also because it results in a wealth-income ratio close to that observed in the United States, while $\beta = 0.98$ does not. Auerbach and Kotlikoff also based their choice of β in part on its ability to reproduce a plausible wealth-income

[9] See Benninga and Protopapadakis (1990), Kocherlakota (1990) and Deaton (1991) for a discussion of restrictions on the subjective discount factor in economies with infinitely lived agents.

[10] They argue that the negative estimates for the rate of time preference may reflect a systematic variation of preferences over the life cycle, for example due to the need to alter expenditures as the family size changes over the life cycle. Also, Davies (1981) and Rios-Rull (1991) use a β that exceeds unity in their simulation models.

ratio. They chose a value less than unity, which gave a reasonable wealth-income ratio in an economy without social security but failed to do so once social security was introduced. We choose a value of β that delivers reasonable wealth-income ratios when the social security replacement rate is in the neighborhood of 50 to 60 percent. Cooley and Prescott (1994) also argue that β should be chosen to reproduce plausible wealth-income ratios.

The transition probabilities are chosen to make the probability of employment equal to 0.94, independent of employment in the previous period. The transition probabilities matrix is then given by

$$\Pi(s', s) = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix} .$$

The average duration of unemployment is therefore $1/(1-0.94) = 1.0638$ model periods.[11]

Unemployment benefits are set equal to 40 percent of the employed wage. The social security replacement rate, θ, is taken to be 50 percent. General revenues, which are used to fund government purchases and transfer programs other than social security and unemployment insurance $(G + \alpha_2)$, are fixed at 0.2085.

C. Computing the Stationary Equilibria

Our numerical solution method begins by discretizing the state space $D^R \times D^I \times S$ where $S = \{e, u\}, D^R = \{d_1^R, d_2^R, \ldots, d_m^R\}$ and $D^I = \{d_1^I, d_2^I, \ldots, d_m^I\}$. To implement our assumption of liquidity constraints, we chose the lower bound on asset holdings as zero: $d_1^R = d_1^I = 0$. In order to examine the sensitivity of the results to the size of the grid, we have experimented with two sets of upper bounds on asset holdings, $d_m^R = d_m^I = 10$, and $d_m^R = 4$ and $d_m^I = 10$. In all cases, the upper limit was large enough to be non-binding. An upper limit of 10 is about 10 times the annual earnings of an employed individual. The spacing was even between zero and 10 with a total of m grid points. Hence, the state space has $m \times m \times 2$ points for individuals below retirement age, and $m \times m \times 1$ points for retired individuals. The control space is $m \times m$ for all individuals. [12]

[11] Although the unemployment rate of 0.06 does match the postwar U.S. average, the duration clearly exceeds that in the U.S. economy. A possible remedy for this is to shorten the model period from one year to one quarter, at the expense of quadrupling the computational burden. Incorporating persistence in unemployment would further increase its average duration.

[12] The choice of m was dictated to be 129 due to memory limitations. However, we were able to examine the sensitivity of the results to the grid size by changing the upper limit on asset holdings.

The definition of stationary competitive equilibrium described above instructs us to seek the fixed point of an operator implicitly defined over the aggregate capital stock K. Hence, the numerical algorithm we use focuses on obtaining convergence in the aggregate capital stock. Given a guess for K, the decision rules for each cohort are found by computing a backward recursion starting from the last period of life at $j = J$ until the first age $j = 1$ using the dynamic program in equation (14). Using the definition of aggregate K,we calculate a new K and iterate on this procedure until convergence.

In the present case, 65 age-specific decision rule matrices, 44 matrices of dimension $m \times m \times 2$ for individuals below the retirement age and 21 matrices of dimension $m \times m$ for retirees, must be computed and kept in core memory for subsequent use in calculating the distribution of agent types and aggregate variables.[13] Also, a similar number of matrices with the same dimensions representing the distribution of agent types is computed by a forward recursion and kept in memory. Because of the large amount of core memory required by the addition of a second asset, we have thus far been unable to use as fine a grid as in our earlier papers.[14] We initially took $m = 129$, giving us a total of $44 \times 129 \times 129 \times 2$ plus $21 \times 129 \times 129 \times 1$, or $1,797,228$ agent types.

Performing a grid search over each of the $129 \times 129 = 16,641$ combinations of IRA holdings and ordinary assets for each of the 1,797,228 possible agent types proved very time-consuming. To speed the computation of the agents' decision rules, we took advantage of features specific to the maximization problem at hand. Because IRAs and ordinary assets are close substitutes, the expected utility of a given individual (viewed as a function of the end-of-period levels of these two assets chosen by the agent) has a sharply defined ridge. An example (using a coarse 61×61 grid) is shown in Figure 2, which displays the expected utility for a 64-year old agent. To find a maximum of this expected utility, we found the maximizing value of IRA holdings assuming no end-of-period ordinary assets. We then searched south-eastward along the entire ridge of expected utility and recorded the maximizing combination of ordinary assets and IRA holdings.

4. FINDINGS

[13] The curse of dimensionality poses a serious problem in this case since there are two distinct assets. We employed various time and memory saving devices which rely on the global concavity of the value functions. For details on these procedures, see İmrohoroğlu, İmrohoroğlu and Joines (1993).

[14] With one asset, the state space has $m \times 2$ points for individuals below retirement age and $m \times 1$ points for retirees, and the control space has only $m \times 1$ points.

In this section we describe the findings from our benchmark model economy and then examine the sensitivity of those results to certain characteristics of the economy. The sensitivity analysis is conducted along several dimensions: first, using alternative values for the intertemporal elasticity of substitution and the rate of time preference; second, assuming that retirees face a lower tax rate than workers; and finally, considering ex-ante differences in the human capital endowments of the agents.

For each variant of our model economy, we compare steady-state equilibria under different IRA arrangements. We examine economies with IRA contribution limits of zero (no IRAs), 0.08, and 0.16. A contribution limit of 0.08 corresponds to about eleven percent of labor income per employed worker, which is similar to the actual contribution limit as a percent of average wages and salaries per worker during the period of universal IRA eligibility. For some variants of the model, we also examine an IRA program with no limit on contributions and no penalty for early withdrawals. In the steady state, this IRA arrangement is equivalent to a tax system having only a consumption or a labor income tax.

Because we require the government budget to be balanced under each IRA arrangement, our experiments involve revenue-neutral changes in the tax structure. Government purchases and transfer payments are fixed, and the income tax rate is adjusted to ensure a balanced budget. Engen and Gale examine IRA programs in which the resulting steady-state revenue losses are not recouped through other tax changes or reductions in government spending. Because governments cannot run primary deficits in the steady state (McCallum, 1984), the policies examined by Engen and Gale are feasible only under the assumption that the government would run sufficiently large primary surpluses in the absence of IRAs. Given that the federal budget has shown a primary surplus in only four years since 1970, the plausibility of such an assumption is doubtful.[15]

A. *Benchmark Model*

Table 1 shows results for an economy with no IRAs, economies with contribution limits of 0.08 and 0.16, and an economy with a de facto consumption tax.

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[15] Kotlikoff (1990) simulates the effects of a temporary deficit-financed reduction in capital income tax rates. Because the budget is returned to balance at the expiration of the temporary tax reduction, his experiment does not violate the government's intertemporal budget constraint.

Table 1. Steady-State Equilibria in the Benchmark Economy

	IRA Contribution Limit			
	$\iota = 0.00$	$\iota = 0.08$	$\iota = 0.16$	$\iota = \infty^*$
$\tau_n = \tau_k$	0.2429	0.2611	0.2682	0.2680
IRA Assets	0.0000	2.1006	3.4942	4.6858
Ord. Assets	3.7475	1.7699	0.6422	0.0000
α_1	0.0472	0.0531	0.0562	0.0566
K	3.7947	3.9236	4.1926	4.7424
r	0.0431	0.0410	0.0368	0.0293
w	1.9903	2.0145	2.0630	2.1591
C	0.6453	0.6487	0.6555	0.6674
Q	1.0872	1.1004	1.1269	1.1794
Utility	−177.5786	−176.7434	−175.6242	−174.2595

* No withdrawal penalty.

Moving from a regime without IRAs to a contribution limit of 0.08 raises the net national saving rate from 0.0530 to 0.0544, an increase of 2.74 percent. The capital stock increases by 3.40 percent.[16] Approximately 94 percent of IRA contributions are shifted from non-IRA saving, while about six percent constitute incremental saving.[17] These percentages are

[16] In the steady state of the Solow growth model, the gross national saving rate is equal to $(\delta + \rho)K/Q$ and the net national saving rate is equal to $\rho/(Q/K - \delta)$. With a Cobb-Douglas production technology, the proportionate change in the gross saving rate is equal to α times the proportionate change in the capital stock, where α is labor's share of output.

[17] Period-t per capita ordinary assets are computed as weighted averages of balances held at the end of period $t-1$, where the weights are period-t population shares. Per capita IRA assets are defined in a similar manner. These aggregates thus include only those assets held at the end of period $t-1$ by individuals who survive to period t. Assets of those who do not survive are

similar to those estimated empirically by Gale and Scholz and are much lower than the Venti-Wise estimates. Total IRA balances are about 54 percent of the capital stock, an outcome never observed historically. Note, however, that the table reports only steady-state results. IRA balances would obviously constitute a smaller fraction of the capital stock during the early stages of the transition from a no-IRA regime than in a steady state. In addition, the fraction of these early IRA contributions constituting incremental saving would likely be smaller than in the steady state, as initial contributions could be financed largely out of previously accumulated assets. These results suggest that the brief experiment with universal IRA eligibility during the 1980s probably had a minimal effect on the U.S. capital stock.

An increase in the contribution limit from 0.08 to 0.16 further increases the net national saving rate by 5.59 percent and the capital stock by 6.86 percent, with about 19 percent of the additional IRA contributions constituting incremental saving. IRA balances account for 83 percent of the capital stock.

Figure 3 shows age-asset profiles for IRA contribution limits of 0.08 and 0.16. As expected, an increase in the IRA contribution limit causes agents to increase their IRA holdings and reduce their ordinary assets. Regardless of the contribution limit, retirees exhaust their regular assets before they start to withdraw from their IRAs. This behavior is due to the fact that the early withdrawal penalty ceases to apply at model age 40, making IRAs and ordinary assets equally liquid for retirees. Because IRA balances continue to compound at a before-tax rate, retirees prefer not to reduce these balances as long as they can draw upon ordinary assets to finance their consumption.[18] Figure 3 also indicates that the contribution limit is an important feature of the IRA programs we examine. IRAs do not raise the marginal rate of return for agents who would save substantially more than the contribution limit in the absence of IRAs. Middle-aged workers in our model economy do generally save substantially more than they are permitted to invest in IRAs when the contribution limit is 0.08. Thus, their ordinary assets exceed their IRA balances throughout much of their working lives, and IRA balances constitute only 36 percent of total financial assets

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distributed in a lump-sum fashion to the survivors. The size of this lump-sum distribution is α_1. The aggregate capital stock is thus the sum of IRA assets, ordinary assets, and the lump-sum distribution α_1.

[18] Agents in our model are not required to begin withdrawing from their IRAs at any particular age. Imposing a mandatory withdrawal schedule similar to that contained in ERTA would have little effect, because agents generally begin drawing down their IRA balances by model age 51 if the contribution limit is 0.08 and even earlier if the limit is 0.16.

for workers of model age 25. With a contribution limit of 0.16, middle-aged workers make larger contributions to their IRAs, and IRA balances account for 72 percent of total financial assets at model age 25. The fact that a contribution limit of 0.08 appears to be binding for a substantial fraction of savers may explain why IRAs with such a limit have a relatively modest effect on aggregate saving.

Removing the IRA contribution limit and the early withdrawal penalty results in a further increase in the capital stock of about 13 percent, with about 46 percent of incremental IRA contributions constituting new saving. Comparing this economy with that described in the first column indicates that removing the tax on income from capital results in an increase in the capital stock of about 25 percent above that in an economy with a pure income tax. This increase in the capital stock is seven times as large as that resulting from IRAs with a contribution limit of 0.08.

In addition to providing an analysis of the effects of IRAs on the capital stock, our model permits us to make welfare comparisons of different tax structures. Table 1 indicates that an increase in the IRA contribution limit raises expected lifetime utility.[19] A more informative way of quantifying these welfare effects is in terms of a lump-sum consumption supplement given to each agent in the no-IRA economy that would result in an expected lifetime utility equal to that attained in each of the other economies. According to these calculations, a consumption supplement equal to 0.53 percent of GNP would give agents living in the no-IRA economy the same expected utility as they would attain in an economy with an IRA contribution limit of 0.08. A consumption supplement equal to 1.24 percent of GNP would give agents in the no-IRA economy the same expected utility as with a contribution limit of 0.16, and a consumption supplement equal to 2.11 percent of GNP would give them the same expected utility as under a consumption tax. While not negligible, these welfare effects are smaller than those reported by İmrohoroğlu, İmrohoroğlu and Joines (1994) for an unfunded social security system.

Two caveats should be kept in mind in interpreting these welfare effects. First, the reported effects obtain only in the steady state and ignore any transition gains or losses. Second, because labor is supplied inelastically in our model, these figures are upper limits

[19] The increased capital stock resulting from a higher contribution limit does not necessarily imply an increase in expected utility. A change in fiscal policy may alter the typical age-consumption profile in such a way as to reduce utility. In addition, a higher capital stock results in lower aggregate consumption if the economy starts from a dynamically inefficient steady state. All of the steady states that we examine are dynamically efficient. See İmrohoroğlu, İmrohoroğlu and Joines (1994).

on the welfare gains that could actually be achieved. Favorable tax treatment of income from capital requires a higher tax rate on labor income. With an inelastic supply of labor, a labor income tax is effectively a lump-sum tax, and it entails a lower welfare cost than would be the case with a variable labor supply. Most of the increase in the labor income tax, and thus a large portion of any resulting distortion of the labor supply decision, occurs in moving from a no-IRA economy to one with a contribution limit of 0.08. Holding labor input constant, this policy change causes a reduction in the after-tax wage rate of about 1.2 percent. The two remaining economies actually have higher after-tax wage rates than the no-IRA economy, and the after- tax wage rate under a de facto consumption tax is five percent above that in the no-IRA economy.

In summary, the results from our benchmark model indicate that a modest IRA contribution limit similar to that contained in ERTA leaves the marginal rate of return unchanged for many agents and causes small increases in the steady-state capital stock and welfare. Only a small fraction of IRA contributions constitutes incremental saving. A larger contribution limit raises the marginal rate of return for more agents, with the result that a larger fraction of contributions constitutes incremental saving. Thus, our results lend support to recent suggestions that retirement accounts with favorable tax treatment only for contributions above some base amount would provide more stimulus to saving than conventional IRAs.[20]

B. *Alternative Preference Parameters*

We now examine the sensitivity of the results from our benchmark model economy to alternative values of the intertemporal elasticity of substitution in consumption and the rate of time preference.

In our benchmark economy, we assumed an intertemporal elasticity of substitution of 0.67, implied by our choice of ($\gamma = 1.5$). Table 2 reports findings for an elasticity of 0.33 ($\gamma = 3.0$). Columns two and three of the table repeat some of the information from Table 1 for the economy with an intertemporal elasticity of substitution of 0.67. The next two columns show the characteristics of the economy with a lower intertemporal elasticity of substitution. In this economy, an increase in the contribution limit from zero to 0.08 raises

[20] Bernheim and Scholz (1992) have proposed such a saving incentive. They also argue that the interest elasticity of saving rises with income, with the result that low-income households, the group for which conventional IRAs are most likely to raise the after-tax return, are also the group least likely to increase their saving in response to a higher rate of return.

the capital stock by about 25 percent. Roughly 24 percent of IRA contributions constitutes incremental saving, while 76 percent is shifted from non-IRA saving.

Table 2. Alternative Preference Parameters

	$\beta = 1.011$, $\gamma = 1.5$		$\beta = 1.011$, $\gamma = 3.0$		$\beta = 0.98$, $\gamma = 1.5$	
	$\iota = 0.0$	$\iota = 0.08$	$\iota = 0.0$	$\iota = 0.08$	$\iota = 0.0$	$\iota = 0.08$
K	3.7904	3.9236	2.3055	2.8932	2.2492	2.7342
IRA Assets	0.0000	2.1006	0.0000	2.4105	0.0000	2.4803
Ord. Assets	3.7424	1.7699	2.3055	0.4827	2.2492	0.2113
C	0.6448	0.6487	0.5734	0.6078	0.5691	0.5999
Q	1.0868	1.1004	0.9087	0.9861	0.9006	0.9662
r	0.0432	0.0410	0.0819	0.0627	0.0842	0.0672

We also conducted simulations using a discount factor of $\beta = 0.98$ in place of our benchmark value of 1.011. Although there is empirical support for a discount factor greater than unity, previous analyses using models similar to ours, such as Auerbach and Kotlikoff (1987) and Hubbard and Judd (1987), have used a value smaller than unity. The results in this case, presented in columns six and seven of Table 2, indicate that an increase in the contribution limit from zero to 0.08 increases the capital stock by about 22 percent, with about 18 percent of IRA contributions constituting incremental saving.[21]

The preference parameters used in Table 2 seem to imply that IRAs are more effective in stimulating saving than was the case in our benchmark economy. The reliability of inferences drawn from these alternative specifications is questionable, however. Both the model economy with $\beta = 0.98$ and that with $\gamma = 3.0$ fail to produce wealth-output ratios

[21] We have examined the sensitivity of these results to the depreciation rate and to the size of the grid as well. For example, in this economy with a depreciation rate of 0.08 instead of 0.06, an increase in the contribution limit from zero to 0.08 increases the capital stock by 17.25 percent. With a finer grid of 0.031 points, IRAs with a contributiom limit of 0.08 cause a 22.49 percent increase in the capital stock.

consistent with those observed in the United States, whereas the wealth-output ratios in the benchmark economy are empirically plausible. Because the alternative specifications fail to explain observed saving behavior, their predictions concerning the effects of IRAs on saving are suspect.[22] Nevertheless, even with these alternative preference specifications, the fraction of IRA contributions constituting incremental saving falls within the range estimated empirically by Gale and Scholz (1990) and is much lower than the estimates reported by Venti and Wise (1990).

C. *Lower Tax Rate During Retirement*

With a progressive income tax, individuals typically face a lower tax rate during retirement than during their working years. This feature of the tax code increases the value of the tax deferral on IRA contributions and potentially makes IRAs more attractive. As part of our sensitivity analysis, we modified the tax system in our benchmark model economy so that retirees face a tax rate only half as large as that applicable to workers. Table 3 shows the results of this analysis. Adopting IRAs with a contribution limit of 0.08 raises the capital stock by only 0.25 percent as compared with the no-IRA economy. Less than one half of one percent of IRA contributions constitutes new saving, and IRA balances equal 57 percent of total assets in the steady state. Moving from a contribution limit of 0.08 to a limit of 0.16 increases the capital stock by 10.11 percent. About 27 percent of incremental IRA contributions constitutes new saving, and IRA balances account for 85 percent of total assets. Note that, because less revenue is collected from retirees, workers face a higher tax rate than was the case in the corresponding economy from Table 1. This tax rate applies to the returns on ordinary assets of workers as well as to their to labor income, and it would seem to reinforce the tax advantage of IRAs. However, if withdrawals from IRAs are taxed at a reduced rate during retirement, smaller IRA balances must be accumulated to attain any given retirement consumption path than was the case in Table 1. Thus, taxing retirees at a lower rate than workers actually seems to reduce the effectiveness of IRAs in stimulating saving.

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[22] Auerbach and Kotlikoff (1987) report reasonable wealth- output ratios in their benchmark model with $\beta = 0.9852$ but, like us, find that incorporating social security into the model results in implausibly low wealth-output ratios.

Table 3. Lower Tax Rate During Retirement

	IRA Contribution Limit		
	$\iota = 0.00$	$\iota = 0.08$	$\iota = 0.16$
$\tau_n = \tau_k$	0.2481	0.2736	0.2854
IRA Assets	0.0000	2.1415	3.5216
Ord. Assets	3.7424	1.6072	0.6053
α_1	0.0480	0.0510	0.0571
K	3.7904	3.7997	4.1840
r	0.0432	0.0431	0.0369
C	0.6448	0.6456	0.6564
Q	1.0868	1.10877	1.1261
Utility	−177.6407	−177.2596	−175.6445

D. *Ex-ante Differences in Human Capital*

In the economies considered above, all agents are identical at birth. At any point in time, these agents differ in their age and in their employment histories, and these differences result in heterogeneity in asset holdings. However, the degree of heterogeneity in income and wealth is less than that observed in the U.S. economy. The asset profiles in our benchmark economy indicate that most middle- aged workers are constrained by an IRA contribution limit of 0.08, with the result that IRAs subject to such a limit do not increase the marginal after- tax rate of return for most workers during the years when they accumulate much of their wealth. The IRA contribution limit is less likely to be binding for agents with low income and assets than for wealthier agents. Thus, it is possible that the effect of IRAs could be different in a model with greater heterogeneity in income and wealth than in our benchmark economy.

In this section we introduce three types of agents who differ ex ante in terms of their endowments of non-reproducible human capital. These agents face different age-earnings profiles throughout their lives. We calibrate the age-earnings profiles faced by these agents using money earnings for full-time male and female workers who differ in terms of their education. The three age-earnings profiles correspond to agents with no or some high school education, agents with high school and some college education, and agents with college and post college education.[23] The respective shares for these groups are 14%, 63%, and 23%. Figure 4 shows the corresponding age-earnings profiles.

Table 4. Differences in Human Capital Endowments

	IRA Contribution Limit		
	$\iota = 0.00$	$\iota = 0.08$	$\iota = 0.16$
$\tau_n = \tau_k$	0.2502	0.2674	0.2725
IRA Assets	0.0000	2.0234	3.3378
Ord. Assets	3.4381	1.6425	0.6362
α_1	0.0469	0.0538	0.0572
K	3.4850	3.7197	4.0312
r	0.0479	0.0435	0.0383
C	0.6252	0.6331	0.6423
Q	1.0448	1.0696	1.1010

Table 4 shows the results for the economy in which agents differ in terms of their human capital. In general, total asset holdings are lower in this economy than in the benchmark economy with ex ante identical agents. Moving from a regime without IRAs to a regime with a 0.08 contribution limit increases the capital stock by 6.73 percent, with about 11.26 percent of IRA contributions constituting new saving. The implied increase in the saving rate is 5.38 percent. Although these percentages are almost twice as large as those from

[23] The data are from U.S. Bureau of the Census, Money Income of Households, Families, and Persons in the United States, 1990, Current Population Reports, Series P-60, no. 174 (August 1991), Table 30.

the benchmark economy, the fraction of contributions constituting new saving is still much closer to the empirical estimates obtained by Gale and Scholz than to the various Venti-Wise estimates. Increasing the contribution limit from 0.08 to 0.16 raises the capital stock by 8.37 percent, with 23.44 percent of incremental IRA contributions constituting new saving. These percentages are slightly larger than those from the benchmark economy.

5. CONCLUSIONS

In this paper we develop a computable general equilibrium model to examine the effects of tax-favored retirement accounts on the capital stock and welfare. The model consists of overlapping generations of 65-period lived individuals facing mortality risk, individual income risk, and liquidity constraints. There are two types of private assets, ordinary assets and tax-favored retirement accounts that we refer to as IRAs.

The results from our benchmark model indicate that an IRA system with a contribution limit similar to that in effect during the early 1980s raises the net national saving rate by 2.74 percent and the steady-state capital stock by 3.40 percent. This increase in the capital stock raises steady-state welfare by an amount equivalent to a lump-sum consumption supplement of 0.5 percent of GNP. Because of our assumption that labor is supplied inelastically, this estimate is an upper bound on the welfare gain. In addition, this estimate ignores any welfare losses during the transition to the new steady state. Approximately six percent of IRA contributions constitutes incremental saving. Note, however, that our results apply only to the steady state. The fraction of IRA contributions constituting incremental saving would likely be smaller during the early stages of the transition to a universal IRA regime than in the steady state, as initial contributions could be financed largely out of previously accumulated assets. Thus, our results suggest that the brief experiment with universal IRA eligibility during the 1980s probably had a minimal effect on the U.S. capital stock.

These conclusions are robust to most of the alternative model specifications that we have tried. Allowing for ex ante differences in human capital endowments increases the effectiveness of IRAs in stimulating saving. Assuming the elderly face an income tax rate half as large as that applicable to workers slightly reduces the effectiveness of IRAs. Using a finer grid on asset holdings has a negligible effect, while assuming a higher depreciation rate reduces the effectiveness of IRAs. The only specifications suggesting that IRAs have a substantially larger effect on saving are those using alternative preference parameters. These conclusions are suspect, however, because the alternative preference specifications imply unrealistically low wealth-output ratios in the absence of IRAs. In all specifications, the fraction of IRA contributions constituting incremental saving lies within the range estimated by Gale and Scholz and is much lower than the Venti-Wise estimates obtained from U.S. data for the early 1980s.

Our results indicate that the contribution limit is an important factor in explaining the effects of the IRA programs we examine. IRAs do not raise the marginal rate of return for agents who would save substantially more than the contribution limit in the absence of IRAs. Middle-aged workers in our model economy do generally save substantially more than they are permitted to invest in IRAs when the contribution limit is similar to that in effect during the early 1980s. Our findings indicate that adopting a contribution limit twice this large would triple the effect of IRAs on the capital stock. In addition, a larger fraction of contributions would constitute incremental saving. This result is due to the fact that a less binding contribution limit causes more agents to experience an increase in the after-tax return on saving. Completely removing the contribution limit and the early withdrawal penalty results in a capital stock about 25 percent above that in an economy with a pure income tax. This increase in the capital stock is seven times as large as that resulting from IRAs with a contribution limit of 0.08.

The fact that a modest contribution limit appears to be binding for a substantial fraction of savers may explain why IRAs with such a limit have a relatively small effect on aggregate saving. A modest contribution limit leaves the marginal rate of return unchanged for many agents and causes only a small increase in the steady-state capital stock. Thus, our results lend support to recent suggestions that retirement accounts with favorable tax treatment only for contributions above some base amount would provide more stimulus to saving than conventional IRAs.

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Figure 1. Sequence of Effective Discount Factors

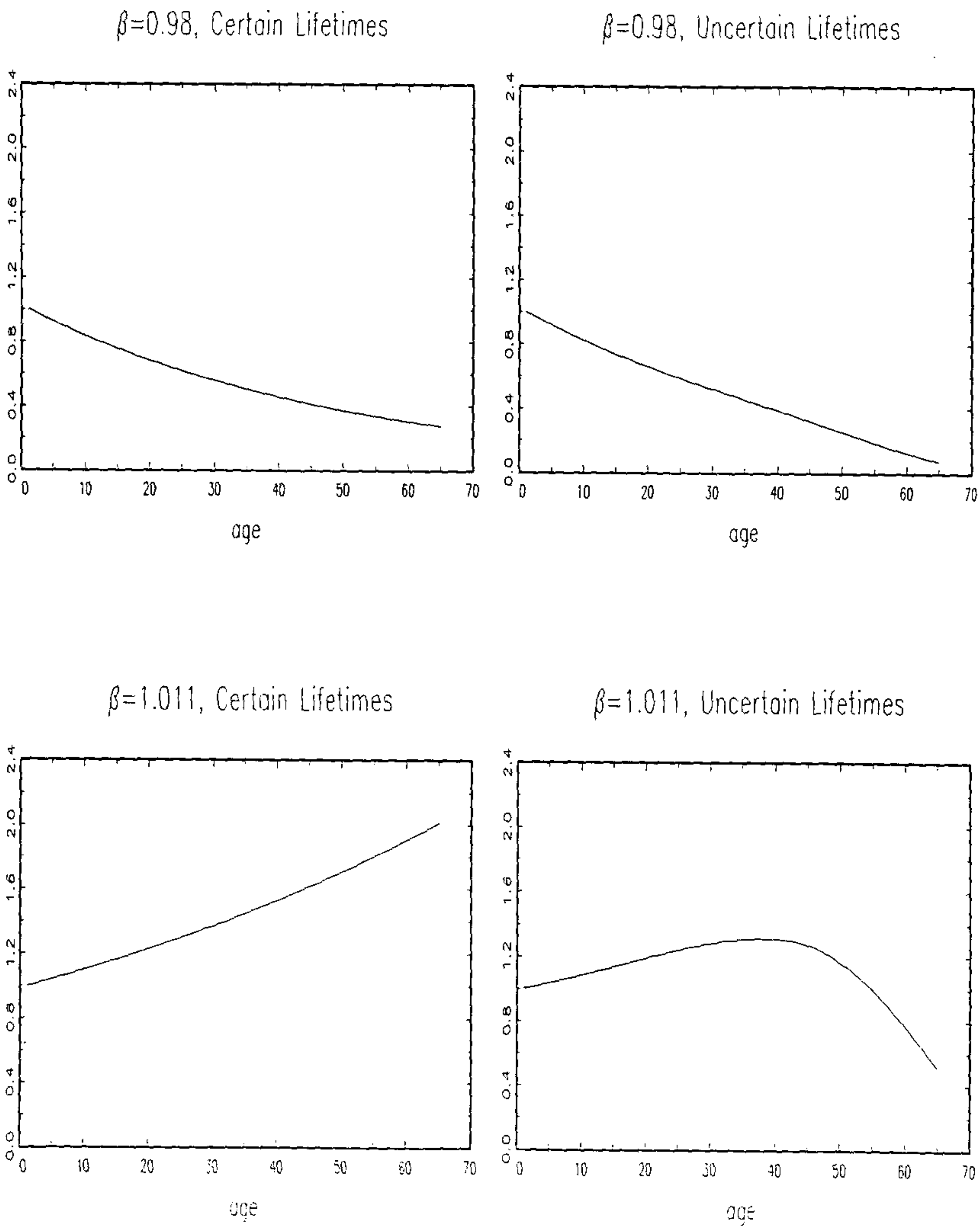

Figure 2. Expected Utility as a Function of the Two Assets

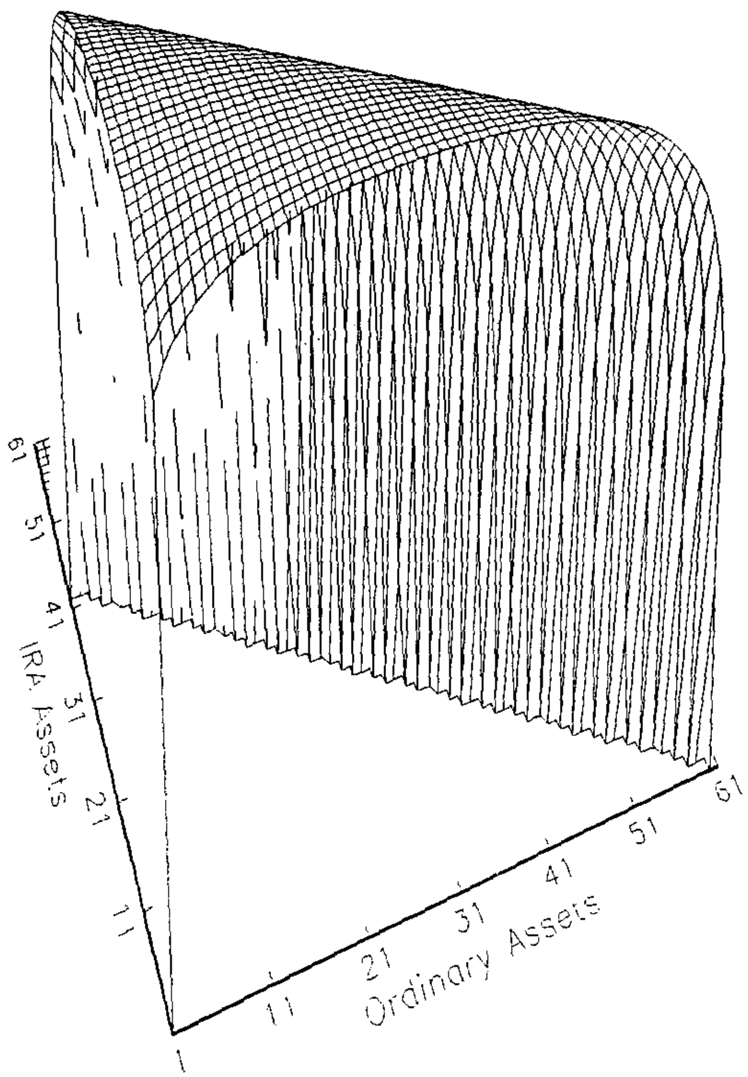

Figure 3. Ordinary and IRA Asset Profiles

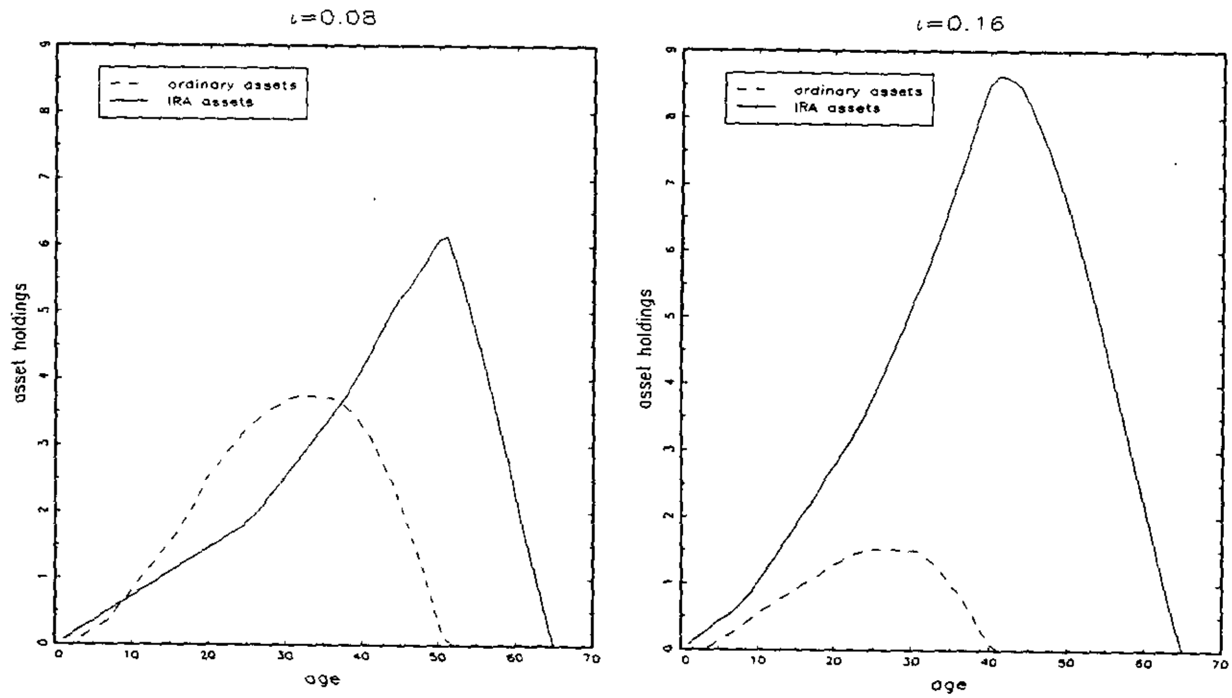

Figure 4. Differences in Human Capital