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Microfoundations and Macro Implications of Indivisible Labor

Casey B. Mulligan*

University of Chicago

ABSTRACT

I show that the "indivisible labor" models of Diamond and Mirrlees (1978, 1986), Hansen (1985), Rogerson (1988), Christiano and Eichenbaum (1992) and many others are, when aggregated across persons with the same marginal utility of income, equivalent to the *divisible* labor model of Lucas and Rapping (1969); any data on aggregate hours and earnings generated by the divisible (indivisible) model can be generated by some parameterization of the indivisible (divisible) model. The same is true when "macro" data are obtained by aggregating over time and across people. This equivalence means that the indivisibility of labor *per se* does not have implications for macroeconomics. Nor does indivisibility have "aggregate" normative implications.

I then build a micro model of the *bunching* of work in continuous time as the consequence of fixed costs and "fatigue effects." Only in a special case does the micro model have as its reduced form the indivisible labor model. In other cases, the bunching of work in time may have unique macro implications. Indivisible and bunching models of labor are shown to have implications for public finance.

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"A small fall in the price of hats or watches will not affect the action of everyone; but it will induce a few persons, who were in doubt whether or not to get a new hat or a new watch, to decide in favor of doing so. ...But the economist has little concern with particular incidents in the lives of individuals ...the variety and fickleness of individual action are merged in the comparatively regular aggregate of the action of many." Alfred Marshall, *Principles of Economics*, III.iii.5.

I. Introduction

Employment is an important "margin" of labor supply (Coleman 1984, Heckman 1993) and many have emphasized the micro-econometric implications of discrete choice in the labor market. But what are the implications of "indivisible labor" for macroeconomic data - measures of economic activity that are aggregated over time and across people? Some (eg., Ashenfelter 1980) suggest that the importance of the employment margin relative to the "hours" margin means that the substitution of work over time must be unimportant. Others such as Hansen (1985), Hansen and Sargent (1988), Rogerson (1988), Plosser (1989), Greenwood and Hercowitz (1991), Kydland and Prescott (1991), Christiano and Eichenbaum (1992), and Cho and Cooley (1994) have argue that the indivisibility of labor means that there is a lot - even infinite - substitution of work over time.

One definition of "indivisibility" is common in the labor and macroeconomics literatures (eg., Diamond and Mirrlees (1978, 1986), Hansen (1985), Rogerson (1988), Hamilton (1988), Christiano and Eichenbaum (1992)): work during some time interval must occur for exactly $\bar{n}$ units of time or there be no work at all. For example, the time interval might be a week with a person working exactly forty hours during a week or not working at all. This indivisible environment can be contrasted with the "divisible" labor environment described by Lucas and Rapping (1969, hereafter LR) where workers may choose to work any fraction of any time interval.

I argue that the hypothesis that all or even some workers in the economy face such an indivisibility constraint has no implications for macroeconomics. In particular, I write down an economy with indivisible labor - of which Hansen's (1985) and Rogerson's (1988) economies are special cases - and show that any macro data generated by the divisible labor economy can be generated by some parameterization of the indivisible economy. Conversely, any macro data generated by the indivisible labor economy can be generated by some parameterization of the divisible economy. Hence, the micro-level indivisibility of labor does not by itself justify the constant marginal disutility of work assumed by Hansen (1985), Hansen and Sargent (1988), Rogerson (1988), Plosser (1989), Greenwood and Hercowitz (1991), Kydland and Prescott (1991), Christiano and Eichenbaum (1992), Cho and Cooley (1994), and others in the literature.

Although commonly used in the literature, Hansen's and Rogerson's definition of indivisibility is a special one. I build a micro model of the optimal bunching of labor in continuous time and show that a special case of the model is equivalent to my indivisible labor model. Thus the optimal bunching of labor in continuous time need not have macro implications. Nevertheless, this leaves open the possibility that some other cases of the optimal bunching model do have macro implications.

The optimal bunching of work in time has naturally led to tax rules that are functions of time aggregated data. I show that the indivisibility or optimal bunching of work together with time aggregated nonlinear tax rules do have implications for both micro and macro data. This is not a point made by Hansen (1985) and those in the literature who have followed, but the interactions between indivisibility and government policy may generate quite interesting implications.

II. Macro Implications of Divisible and Indivisible Labor Supply with Linear Consumption Value Functions

II.A. Consumption Side of the Model

Individuals care about their lifetime consumption and its allocation over time. My analysis uses a "consumption value function" $U(c)$ to summarize this part of a consumer's decision problem:

$$U(c) \equiv \max_{\{c_t\}_{t=1}^{T}} u(c_1, c_2, \ldots, c_T) \quad \text{s.t.} \quad \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t c_t \leq c$$

where $T \geq 1$ and $e^{-\rho(t-1)}Q_t$ is the period t interest rate factor.

Implicit in my use of a consumption value function in the analysis of labor supply is the assumption that consumption and leisure are separable: a different rate of growth of consumption (leisure) does not mean a different intertemporal marginal rate of substitution of leisure (consumption). I assume consumption-leisure separability not for realism, but to derive more transparent results and to facilitate comparisons with previous studies of indivisible labor which typically have assumed separability.

II.B. Lucas and Rapping's Divisible Representative Agent Model

As a contrast to the indivisible labor model, it is useful to begin by introducing a divisible labor environment. During a time interval - whose length I normalize to one - an individual may work from anywhere from 0 to n_{max} units of time. Time interval t work is paid at wage rate w_t. The present value of lifetime consumption C is financed out of the present value of lifetime earnings and an initial asset stock a.

For now I assume that the consumption value function is linear: $U(C) = \lambda C$ where λ is a constant.[1] Hence, the problem (P1) describes the representative agent's labor supply decisions when labor is perfectly divisible:

(P1) ***Optimal Divisible Labor in Discrete Time***

$$\max_{C,\{N_t\}_{t=1}^{T}} \lambda C - \sum_{t=1}^{T} e^{-\rho(t-1)} g_t v(N_t) \quad , \lambda, g_t > 0$$

$$\text{s.t. } N_t \in [0, n_{\max}] \quad , \quad C = a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t N_t$$

where N_t is time worked during time period t, ρ is the rate of time preference, g_t is a preference parameter, a is the consumer's initial asset holdings, w_t is the period t wage rate, and Q_t is the period t interest rate factor (defined net of the rate of time preference).

Because of the assumed separability over time and the assumed separability of consumption and leisure, (P1) is a special case of the LR (1969) model.[2] Like LR, any amount of time worked in the interval $[0,n_{max}]$ is feasible in each time interval for a (P1) consumer.

I make two assumptions (A1)-(A2) and one normalization (A3) about the disutility of work

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[1]I show later that, by introducing employment lotteries and some consumption insurance, identical results can be derived for any increase and concave $U(C)$.

[2]LR's empirical specifications implicitly assume something like separability within and across periods.

$v(N)$ defined on $[0,n_{max}]$:[3]

(A1) $v'(N) > 0$

(A2) $v''(N) > 0$

(A3) $$\int_0^{n_{\max}} \ln v'(N)\, dN = 0$$

For simplicity, I assume that the function $v(N)$ does not vary over time although the marginal disutility of work varies over time to the extent that the parameter g_t varies.

It will be useful to define a "parameterization" of (P1) to be $(a,\rho,\lambda,n_{max},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2\times\mathbb{R}_+^{3T+2}$. Wages and interest rates may be determined as part of a general equilibrium (as they are in Hansen (1985) and Rogerson (1988)), but they are parameters from the point of view of a consumer's decision problem.

The first order condition for the problem (P1) equates the marginal disutility of work to λ times the discounted wage:

$$g_t v'(N_t) = \lambda\, Q_t\, w_t \qquad t = 1, \ldots, T \tag{1}$$

In this special case that λ is a constant, each period's first order condition determines optimal labor supply for that period.

II.C. Modeling the Micro-level Indivisibility of Labor

In the indivisible model, an individual must either work 0 or $\bar{n}$ hours during a time interval. Any amount of work between 0 and $\bar{n}$ is not feasible. Time interval t work is paid at wage rate w_t, so time interval t earnings are either 0 or $w_t\bar{n}$. Hence, problem (P2) describes the labor supply decisions of a consumer whose labor is indivisible:

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[3](A3) is a normalization because, if ln $v'(N)$ integrated to some number $\varepsilon \neq 0$, then same marginal disutility of work could be represented by redefining the parameter g as ge^{ε} and redefining $v(N)$ as $ge^{-\varepsilon}v(N)$.

(P2) ***Indivisible Labor in Discrete Time***

$$\max_{c,\{n_t\}_{t=1}^{T}} \lambda c - \sum_{t=1}^{T} e^{-\rho(t-1)} \gamma_t n_t$$

$$\text{s.t. } n_t \in \{0,\bar{n}\} \quad , \quad c \le a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t n_t$$

where n_t is period t time worked and γ_t is a preference parameter. My notation distinguishes n_t, γ_t and c in problem (P2) from N_t, g_t, and C in problem (P1) because, as shown below, (P1) is assumed to describe aggregate labor supply while (P2) will be assumed to describe micro-level labor supply. As in (P1), w_t is the period t wage rate and $e^{-\rho(t-1)}Q_t$ is the period t interest rate factor.

It will be useful to define a "parameterization" of (P2) to be $(a,\rho,\lambda,\bar{n},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2\times\mathbb{R}_+^{3T+2}$. Wages and interest rates may be determined as part of a general equilibrium (as they are in Hansen (1985) and Rogerson (1988)), but they are parameters from the point of view of a consumer's decision problem.

(P2) describes an environment much like that of Diamond and Mirrlees (1978, 1986) or Hansen (1985) - there are only two feasible choices for time worked during a particular period: $\{0,\bar{n}\}$. Notice that the marginal disutility of work during time interval t $e^{-\rho(t-1)}\gamma_t$ can, in addition to a constant rate of discount, vary over time.

It is relevant that (P2) assumes separability of leisure over time but, because of the assumed indivisibility, not that the disutility of work is linear. To see this, notice that we could start with any disutility of work function $v_t(n_t)$, normalize $v_t(0) = 0$, define $v_t(\bar{n}) = \gamma_t\bar{n}$, and have exactly the problem (P2).

A consumer works or not during time interval t according to:

$$n_t = \begin{matrix}\bar{n}\\0\end{matrix} \text{ as } \lambda Q_t w_t \begin{matrix}\ge\\<\end{matrix} \gamma_t \qquad t = 1, \ldots, T$$

II.D. Aggregation of Problem (P2)

It is useful to aggregate the behavior of heterogeneous individuals solving problems (P2).

(A4) states the assumptions made in the aggregation:

(A4) There are a continuum of individuals solving the problem (P2) (or the problem (P2)' defined in Section III) who differ only according to their life cycle disutility of work profile $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_T\} \in \mathbb{R}_+^T$. The geometric mean preference parameter in period t, g_t, varies over time but the distribution F of preferences around that mean is the same every period:

$$\ln \gamma_t - \ln g_t \sim F \quad \text{all } t$$

$$\int x \, dF(x) = 0$$

F is assumed to be once differentiable and strictly increasing on its support $[x_1, x_2]$. Let N_t denote average labor supply during time interval t.

Notice that (A4) requires consumers to have the same market value of time w_t and same interest rate factor Q_t at each date as well as the same rate of time preference ρ, initial wealth a, intrasession discount rate r, and marginal utility of wealth λ. These might be sources of heterogeneity that are potentially interesting for macroeconomics, but I show that this heterogeneity is not particularly related to the indivisibility of labor.

The fraction Π_t of consumers working at date t is F($\ln(\lambda Q_t w_t / g_t)$). Define N_t to be the date t average labor supply. N_t is computed according to:

$$N_t = \Pi_t \cdot \bar{n} + (1 - \Pi_t) \cdot 0 = F\left(\ln \frac{\lambda Q_t w_t}{g_t} \right) \bar{n} \tag{2}$$

An average budget constraint can easily be computed by averaging the (P2) budget constraint across consumers:

$$C = a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t N_t \tag{3}$$

where C is the average present value of lifetime consumption. Notice the similarity of this average budget constraint with the budget constraint for the divisible problem (P1).

II.E. Divisible and Indivisible Models Generate the Same Macro Data

Proposition 1 When aggregated across individuals according to (A4), macro data on earnings and hours $\{N_t w_t, N_t\}$ generated by any parameterization of the problem (P2) is identical to the macro data on earnings and hours $\{N_t w_t, N_t\}$ generated by some parameterization of the problem (P1).

Proof (i) Choose any set of parameters $(a,\rho,\lambda,\overline{n},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+2}$ for the problem (P2)
(ii) When aggregated according to (A4), average hours $\{N_t\}$ satisfy equation (2).
(iii) Choose the same parameters $(a,\rho,\lambda,n_{max},\{w_t,g_t,Q_t\})$ for the problem (P1) and choose any disutility of wealth function $v(N)$ that satisfies:

$$v'(N) \equiv e^{F^{-1}(N/\overline{n})}$$

with F^{-1} defined at the end points according to $F^{-1}(0) = x_1$ and $F^{-1}(1) = x_2$. Notice that any such $v(N)$ is continuous and satisfies $v'(\mathrm{N})>0$, $v''(N)>0$, and the normalization (A3).
(iv) According to the first order conditions of the problem (P1), average hours $\{N_t\}$ must satisfy (1) which, given the definition of $v'(N)$ above, is the same as (2).

Proposition 2 The macro data $\{N_t w_t, N_t\}$ generated by any parameterization of the problem (P1) is identical to the macro data $\{N_t w_t, N_t\}$ obtained by aggregating some parameterization of the problem (P2) across individuals according to (A4).

Proof (i) Choose any set of parameters $(a,\rho,\lambda,n_{max},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+2}$ for the problem (P1)
(ii) According to the first order conditions of the problem (P1), average hours $\{N_t\}$ must satisfy (1).
(iii) Choose the same parameters $(a,\rho,\lambda,\overline{n},\{w_t,g_t,Q_t\})$ for the problem (P2) and choose the

distribution function F according to:

$$F(x) \equiv \begin{cases} 0 & \text{if} \quad x < \ln v^{/}(0) \\ \dfrac{\phi(e^{x})}{n_{\max}} & \text{if} \quad x \in [\ln v^{/}(0), \ln v^{/}(n_{\max})] \\ 0 & \text{if} \quad x > \ln v^{/}(n_{\max}) \end{cases}$$

where ϕ is the inverse of $v'(N)$, the marginal disutility of work. Notice that any such $F(x)$ satisfies $F \in [0,1]$, $F'(x)>0$, and the normalization of F displayed in (A4).

(iv) When aggregated according to (A4), average hours $\{N_t\}$ from (P2) satisfy equation (2) which, given the definition of $F(x)$ above, is the same as (1).

II.F. Interpretation

It is noteworthy from the proofs of Propositions 1 and 2 that the rate of time preference, the marginal utility of wealth, the level of initial assets, and the sequences of wages, interest rates, and tastes, are the same for (P1) and (P2). Propositions 1 and 2 are the key results of this paper, but it should be noted that a special case of Proposition 1 was stated by Hansen (1985) and Rogerson (1988). They showed that their indivisible labor models were equivalent to a special case of the LR (1969) model - namely the special case when the marginal disutility of leisure is constant. However, their assumptions about preferences, assumptions about the homogeneity of consumers, and definition of equilibrium makes the labor supply side of their models a special case of my problem (P2). When these assumptions are relaxed to allow for heterogeneity, there still exists an equivalent parameterization of the LR model, but that equivalent parameterization need not be the linear parameterization. Proposition 2 shows that there is an equivalent indivisible labor model for *any* parameterization of the problem (P1) - including parameterizations that are not linear and allow for very little substitution over time.

The proofs of Propositions 1-2 obtain because the marginal utility of lifetime consumption does not depend on any particular period's labor supply decision. If the marginal utility of lifetime consumption varied with the level of lifetime consumption, then the smallest possible change in any particular period's labor supply decision in the problem (P2) is a discrete change and would have a "wealth effect" in the sense that it would affect the marginal utility of wealth U' whereas my proofs rely on the constancy of U'. However, to the extent that $e^{-r(t-1)}w_t$ is a small fraction of lifetime wealth - perhaps because period t is sufficiently far into the future or the length of a "period" is short - or that U'' is small in magnitude, we can to a good approximation neglect this wealth effect and enjoy Propositions 1-2 as good approximations.[4] However, rather than proving this claim I introduce trade in lotteries and show that an exact equivalence obtains between the "optimal bunching," "indivisible labor," and "divisible labor" economies even when the consumption value function is nonlinear.

III. Macro Implications of Divisible and Indivisible Labor Supply with Employment Lotteries and Nonlinear Consumption Value Functions

III.A. Representative Agent's Divisible Labor

The problem (P1)' is a separable version of the LR model, including the nonlinear consumption value function:

(P1)' ***Optimal Divisible Labor in Discrete Time***

$$\max_{C,\{N_t\}_{t=1}^{T}} U(C) - \sum_{t=1}^{T} e^{-\rho(t-1)} g_t v(N_t)$$

$$\text{s.t. } N_t \in [0, n_{\max}] \quad , \quad C = a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t N_t$$

(A5) $\quad U'(C) > 0,\ U''(C) \leq 0$

In addition to (A5), I continue to make the two assumptions (A1)-(A2) and the normalization (A3). It will be useful to define a "parameterization" of (P1)' to be $(a,\rho,n_{max},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+1}$.

III.B. Micro-level Indivisible Labor, Employment Lotteries

In the indivisible labor problem (P2), workers choose which periods to work. Consider instead the problem (P2)' where consumers choose a sequence of lotteries $\{L_t\}$ offered by employers.

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[4]The approximation is likely to be poor in Rogerson's (1988) model because he has only one period.

The date t lottery is: (1) an allowance ω_t and (2) a probability of working π_t. Consumers receive their allowance at the beginning of the period regardless of the outcome of the lottery. The firm has property rights over the labor output $w_t\bar{n}$ produced by each consumer chosen by the lottery to work. The market for lotteries is competitive, so the only lotteries that will be traded are those satisfying:

$$\omega_t = \pi_t w_t \bar{n}$$

(P2)' ***Indivisible Labor in Discrete Time***

$$\max_{C,\{\pi_t\}_{t=1}^{T}} U(c) - \sum_{t=1}^{T} e^{-\rho(t-1)} \gamma_t \pi_t \bar{n}$$

$$\text{s.t. } \pi_t \in [0,1] \quad , \quad c = a + P(\Gamma) + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t \pi_t \bar{n}$$

It will be useful to define a "parameterization" of (P2)' to be $(a,\rho,\bar{n},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+1}$.

III.C. Aggregation of Problem (P2)'

It is useful to aggregate the behavior of heterogeneous individuals solving problem (P2)'. In addition to (A4), I make the assumption (A6) in the aggregation:

(A6) Before their lifetime preference profiles $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_T\}$ are revealed at date 0, (P2)' consumers choose an insurance contract $P(\Gamma)$ that has expected value zero and maximizes the expected value of the problem (P2)'. *Ex ante,* each consumer has the same probability of drawing any particular preference profile.

There will be heterogeneity of tastes in an economy generated by aggregating (P2) or (P2)' according to (A4). Some consumers are "unlucky" enough to hate work in those periods when it is most profitable, luck which affects the marginal utility of wealth $U(C)$ in the problem (P2)'. The insurance contracts specified by (A4) compensate unlucky consumers (and penalize lucky consumers) in just the right amounts to guarantee the solutions to (P2)' have the same marginal utility of wealth regardless of Γ.

In this model, consumer i's date t choice of employment lottery is determined according to:

$$\pi_t^i \begin{cases} = 0 & \text{if} \quad \gamma_t^i > \lambda Q_t w_t \\ \in (0,1) & \text{if} \quad \gamma_t^i = \lambda Q_t w_t \\ = 1 & \text{if} \quad \gamma_t^i < \lambda Q_t w_t \end{cases}$$

Because of the date zero trade in insurance contracts, the marginal utility of wealth λ does not vary across consumers and is therefore not indexed by i. In the three cases listed above, two of them are the "trivial" lotteries π=0 and π=1. Nontrivial lotteries occur only when $\gamma_t = \lambda Q_t w_t$ which, given the continuity of F, is a measure zero event.

It is convenient to define the date t "reservation wage" to be the lowest wage at which a consumer is willing to work with positive probability at date t. The date t reservation wage is $\gamma_t/\lambda Q_t$ and work occurs with positive probability at date t whenever w_t exceeds the reservation wage.

The fraction Π_t of consumers working at date t is F(ln($\lambda Q_t w_t/g_t$)). Define N_t to be the date t average labor supply. N_t is computed according to equation (2), which I repeat below for the reader's convenience:

$$N_t = F\left(\ln \frac{\lambda w_t Q_t}{g_t} \right) \bar{n} \tag{2}$$

Since the average insurance premium or award paid at date zero must be zero, an average budget constraint can easily be computed by averaging the (P2)' budget constraint across consumers:

$$C = a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t N_t \tag{3}$$

The average budget constraint (3), the equations (2), and the equation $\lambda = U'(C)$ determine average labor supply at each date.

It is worth noting that only the trivial lotteries π=0 and π=1 are demanded in the competitive equilibrium. This is because of (a) the heterogeneity of consumers and (b) the date zero trade in consumption insurance. That the date zero trade in consumption insurance substitutes eliminates the demand for nontrivial lotteries ($\pi \in (0,1)$) in later periods is shown by Cole and Prescott (1997). But even if consumption insurance were not available, the heterogeneity in tastes for work means there would typically be only some agents who demand nontrivial lotteries. Thus the observation that employment lotteries are rarely (or never) used in the "real world" does not necessarily undermine the empirical relevance of the model.

Despite their lack of equilibrium use, I include the nontrivial lotteries for two reasons. First, the standard analysis can be applied because choice sets are convex. Second and more important, Hansen's and Rogerson's models are literally special cases of my (P2)'.

III.D. Divisible and Indivisible Models Generate the Same Macro Data

Proposition 3 When aggregated across individuals according to (A4) and (A6), macro data on earnings and hours $\{N_t w_t, N_t\}$ for any parameterization of the problem (P2)' is identical to the macro data $\{N_t w_t, N_t\}$ generated by some parameterization of the problem (P1)'.

Proof (i) Choose any set of parameters $(a,\rho,\bar{n},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+1}$ for the problem (P2)'

(ii) When aggregated according to (A4) and (A6), average hours $\{N_t\}$ satisfy (2) and (3), and $\lambda = U'(C)$

(iii) Choose the same parameters $(a,\rho,n_{max},\{w_t,g_t,Q_t\})$ for the problem (P1)' and choose any disutility of wealth function $v(N)$ that satisfies:

$$v'(N) \equiv e^{F^{-1}(N/\bar{n})}$$

with F^{-1} defined at the end points according to $F^{-1}(0) = x_1$ and $F^{-1}(1) = x_2$. Notice that any such $v(N)$ is continuous and satisfies $v'(N)>0$, $v''(N)>0$, and the normalization (A3).

(iv) According to the first order conditions of the problem (P1)', average hours $\{N_t\}$ must satisfy equation (1) with $\lambda = U'(C)$.

(v) By definition of the problem (P1)', average hours $\{N_t\}$ satisfy equation (3).

Proposition 4 The macro data $\{N_t w_t, N_t\}$ generated by any parameterization of the problem (P1)' is identical to the macro data $\{N_t w_t, N_t\}$ obtained by aggregating some parameterization of the problem (P2)' across individuals according to (A4) and (A6).

Proof (i) Choose any set of parameters $(a, \rho, n_{max}, \{w_t, g_t, Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+1}$ for the problem (P1)'

(ii) According to the first order conditions of the problem (P1)', average hours $\{N_t\}$ must satisfy equation (1) with $\lambda = U'(C)$.

(iii) By definition of the problem (P1)', average hours $\{N_t\}$ satisfy equation (3).

(iv) Choose the same parameters $(a, \rho, \overline{n}, \{w_t, g_t, Q_t\})$ for the problem (P2)' and choose the distribution function F according to:

$$F(x) \equiv \begin{cases} 0 & \text{if} \quad x < \ln v'(0) \\ \dfrac{\phi(e^x)}{n_{\max}} & \text{if} \quad x \in [\ln v'(0), \ln v'(n_{\max})] \\ 0 & \text{if} \quad x > \ln v'(n_{\max}) \end{cases}$$

where ϕ is the inverse of $v'(N)$, the marginal disutility of work. Notice that any such $F(x)$ satisfies $F \in [0,1]$, $F'(x) > 0$, and the normalization for F displayed in (A4).

(v) When aggregated according to (A4) and (A6), average hours $\{N_t\}$ from (P2)' satisfy equations (2), (3), and $\lambda = U'(C)$.

III.E. Divisible and Indivisible Models Have the Same Welfare Implications

Because there is some heterogeneity in the indivisible model but not in the divisible model, it is only meaningful to compare aggregate welfare calculations for the two models. Propositions 5 and 6 show there are at least two sensible ways of aggregating welfare in the indivisible model to produce calculations identical to their divisible counterparts:

Proposition 5 For the problem (P2)' with rate of time preference ρ, maximum hours $\bar{n}$, interest rate factors $\{Q_t\}$, and average preference profile $\{g_t\}$ is aggregated across individuals according to (A4) and (A6), the average willingness to pay for a wage increase is the same as the willingness to pay for problem (P1)' with rate of time preference ρ, maximum hours $\bar{n}$, interest rate factors $\{Q_t\}$, and preference profile $\{g_t\}$.

Proof (i) In current value terms, the willingness to pay for a marginal increase in w_t is N_t in the divisible model. This follows from Roy's Identity.

(ii) In current value terms, the willingness to pay for a marginal increase in w_t (taking his insurance premium $P(\Gamma)$ as given) is $\pi_t \bar{n}$ for an individual in the indivisible model who purchases a date t employment lottery offering probability π_t. This follows from Roy's Identity.

(iii) The average willingness to pay in the indivisible model is the average of $\pi_t \bar{n}$, which is N_t.

(iv) Propositions 3 and 4 show that both models generate the same average labor supply N_t, so the average willingness to pay is the same.

Notice that the wage change hypothesized by Proposition 5 is not insured and the willingness to pay varies across individuals in the indivisible model. Proposition 6 considers the willingness to pay for insurable wage changes before each individual's preferences are revealed, defining “indirect utility functions” for each model. $V(a,\{w_t\})$ denotes the maximized value of the representative agent's problem (P1)' and $\tilde{V}(a,\{w_t\})$ the maximized value of (P2)' averaged across individuals.

Proposition 6 When the problem (P2)' with rate of time preference ρ, maximum hours $\bar{n}$, interest rate factors $\{Q_t\}$, and average preference profile $\{g_t\}$ is aggregated across individuals according to (A4) and (A6), each individual's *ex ante* expected indirect utility $\tilde{V}(a,\{w_t\})$ has the same derivatives as the indirect utility $V(a,\{w_t\})$ for problem (P1)' with rate of time preference ρ, maximum hours $\bar{n}$, interest rate factors $\{Q_t\}$, and preference profile $\{g_t\}$.

Proof (i) It is straight-forward to show that, for the problem (P1)', $\partial V/\partial a = U'(C)$ and $\partial V/\partial w_t = U'(C)e^{-\rho(t-1)}Q_t N_t$.

(ii) The expected indirect utility for the problem (P2)' is:

$$\tilde{V}\left(a,\{w_t\}_{t=1}^{T}\right) = U\left(a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t N_t\right) - \sum_{t=1}^{T} e^{-\rho(t-1)} \bar{n} \int_{-\infty}^{\ln \lambda Q_t w_t} e^{x} dF(x)$$

$$\lambda \equiv U'\left(a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t N_t\right)$$

(iii) Using the expression above and the definition of N_t for the problem (P2)', it is straight-forward to show $\partial \tilde{V}/\partial a = U'(C)$ and $\partial \tilde{V}/\partial w_t = U'(C)e^{-\rho(t-1)}Q_t N_t$.
(iv) Since N_t is the same function of $(a,\{w_t\})$ for the two models, higher order derivatives are the same for V and $\tilde{V}$.

Proposition 6 derives an equivalence between the indirect utility function for (P1)' and an aggregate indirect utility function defined for (P2)'. Thus Proposition 6 displays another sense in which the aggregate willingness to pay for an aggregate wage change is the same in the two models.

III.F. Interpretation

If we construct an economy of individuals solving the problem (P2)' with identical preferences at each date, we have the deterministic version of Hansen's (1985) model. To see this, notice that a nontrivial measure of agents may demand nontrivial employment lotteries. Furthermore, because each agent has the same *ex poste* preference profile Γ, the only equilibrium date zero insurance contract is $P(\Gamma) = 0$. In Hansen's homogeneous special case, there exists a sequence of wages and interest rates $\{w_t,Q_t\}$ so that, in Hansen's (1985, p. 318) words, “the elasticity of substitution between leisure in different periods for the 'representative agent' is infinite.” Hansen also shows that indivisibility is not necessary to deliver this result - a linear version of LR's divisible model also implies infinite substitutability over time (see also my Propositions 1 and 3). However, my Propositions 2 and 4 show that indivisible labor is not sufficient to deliver infinite or even substantial substitution over time. This point is important and quite contrary to the spirit of Hansen's and Rogerson's papers, so I demonstrate it in an example.

Choose any set of parameters $(a,\rho,\bar{n},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2\times\mathbb{R}_+^{3T+1}$ for the problem (P2)' and aggregate solutions across individuals according to (A4) and (A6), taking the distribution function $F(x) = e^{x/100}/\bar{n}$ with support $[-\infty, 100 \ln \bar{n}]$. Proposition 4 shows that the same macro data can be generated by the LR model with $v'(N) \equiv N^{1/100}$. This model has practically no scope for substitution over time: the intertemporal elasticity of substitution is 1/100.

In a sense, Hansen's and Rogerson's lotteries are a source of preference heterogeneity because they divide the population into two groups - those who work and those who do not. But the "heterogeneity" is not revealed until *after* agents have made their decisions for the period.[5] Hence Hansen-Rogerson agents must all make the same decisions and, when those decisions are discrete, aggregate behavior must be discrete. Under this interpretation, my departure from Hansen and Rogerson is that heterogeneity is revealed before decisions are made and, as a result, agents make different decisions which are continuous in the aggregate even when discrete at the micro level.

My allowance for heterogeneous reservation wages is also of substantial empirical relevance. Rather than describing aggregate fluctuations as infinitesimal movements along a perfectly elastic labor supply curve (as Hansen does), I allow fluctuations to be discrete movements along an imperfectly elastic one. My agents almost surely strictly prefer their chosen labor force status to the alternative. And it takes a larger wage change to alter the decision of a person whose reservation wage is further from the equilibrium wage. While Hansen (1985) cannot name who will work as a function of the equilibrium wage, I predict that the elderly, children, married mothers of young children, and others with date t reservation wages higher than their "normal" date t market wage will not work unless the date t market wage is substantially higher than what is "normal" for them. Indeed, the elderly, children, and married mothers of young children typically do not work except in extreme circumstances such as wartime (Mulligan 1998b).

As for Propositions 1-2, the proofs of Propositions 3-4 obtain because the marginal utility of lifetime consumption does not depend on any particular period's labor supply decision. The constancy of λ is obtained in the former case by assuming a linear lifetime utility function and in the latter case with some date zero insurance contracts and employment lotteries. These contracts look like "paid sick leave" or perfect disability insurance since they compensate individuals unlucky enough to dislike work in periods when it is most productive. If the date zero insurance were not as perfect as modeled

[5] I owe this analogy to Bob Lucas.

in the problem (P2)', then the smallest possible change in any particular period's labor supply decision in the problems (P2) and (P2)' is a discrete change and would have a "wealth effect" in the sense that it would affect the marginal utility of wealth U' whereas my proofs rely on the constancy of U'. However, to the extent that $e^{-r(t-1)}w_t$ is a small fraction of lifetime wealth (perhaps because period t is sufficiently far into the future or the length of a "period" is short), that available insurance is nearly perfect, or that U'' is small in magnitude, we can to a good approximation neglect this wealth effect and enjoy Propositions 3-4 as good approximations.

Notice that, in my aggregation of Problems (P2) and (P2)', I assume nothing about the serial correlation of an individual's marginal disutility of work γ. It is assumed that the distribution of γ across persons is the same every period (up to an aggregate shifter of the geometric mean g_t), but this could result from each individual's drawing a single deviation from the population mean for his entire lifetime, from many independent draws for each individual, or draws that are imperfectly serially correlated over time for each individual. The single aggregate interpretation suggested by Propositions 1-4 of each of these possibilities means that Mincer's (1962) use of a divisible model to interpret his empirical studies of employment rates is consistent with a more general class of life cycle behaviors than Mincer initially supposed. My single interpretation also seems at odds with Ben-Porath's (1973) claim that the serial correlation of tastes is crucial and with Heckman's (1978) claims that the "interiority" of solutions is crucial. Formally, the difference between my result, Ben-Porath's, and Heckman's, is that Ben-Porath and Heckman do not allow for "tastes insurance," employment lotteries, or a constant marginal utility of wealth λ. And, as discussed above,[6] it is a quantitative question whether or not the model with tastes insurance and employment lotteries or the model with a constant λ closely approximates a model without insurance, without lotteries, and a λ that diminishes with lifetime wealth. For example, more serial correlation of tastes means that more wealth is transferred across agents by my tastes insurance and, with λ diminishing rapidly enough, more heterogeneity of λ across agents in the model without tastes insurance.

The reader may guess that my assumed continuity and monotonicity of F and v' are not

[6]Ben-Porath (1973, p. 700) also points out that it is a quantitative question whether or not the divisible model faithfully describes labor force participation decisions, but he focuses on the serial correlation of tastes and not other relevant quantitative issues such as the quality of insurance markets and the degree to which the marginal utility of wealth diminishes.

necessary to obtain a macro equivalence between the divisible and indivisible economies. v' might be discontinuous if F' were allowed to be zero, and vice versa. v' might be constant over some range if F' were allowed to have mass points, and vice versa. I leave a proof of these conjectures to the reader.

IV. Indivisible Labor as Optimal Bunching in Continuous Time

IV.A. Optimal Bunching as a Dynamic Problem

For one reason or another, people choose to bunch their work and leisure together *in time*. For example, workers typically work for 8 hours or so and separate the 8-hour work sessions with 16-hour intervals of "leisure" or nonwork time. Many workers also work five days in a row and separate these five day sessions with two day "weekends". Several consecutive weeks of work are separated by three or more day "vacations" and several consecutive years of work are separated by months or years of "unemployment," "housework," or "retirement".

Is this bunching of work in time what is meant by "indivisible" labor? If so (and Hansen (1985) suggests this on p. 312), under what conditions do the models (P2) and (P2)' faithfully describe that bunching? I begin with a continuous time labor supply model with a fixed cost of beginning a "work session" and, following Chapman (1909), "fatigue" effects on productivity or utility. In a special case, the model is equivalent to (P2)'. In other cases, (P2)' does not faithfully describe the optimal bunching of work while the model of optimal bunching may have implications for macro data that cannot be derived from a LR model. Hence, a look at the microfoundations of indivisible labor does generate some macro implications which cannot be derived from a divisible model.

Static models of fixed costs and indivisible labor have previously been developed (eg., Barzel 1973, Killingsworth 1983, Cogan 1980, 1981). Whether or not the "fixed cost" is a time or a goods cost matters in the static models and matters much the same way in a dynamic model. However, preferences, wealth, and overall labor productivity are important determinants of the degree to which labor is "indivisible" in the static models. This is not true in a dynamic model with employment lotteries, where the possibilities of substitution over time or trade in lotteries effectively eliminate the nonconvex portions of the "static budget set." As a result, the labor indivisibility depends only on the magnitude of the fixed cost, whether the cost is a time or goods cost, and the way in which productivity and flows of utility depend on the history of labor supply. The dynamic formulation also emphasizes that the equilibrium amount of bunching of labor during one interval of time depends on the preferences and opportunities for work at other points in time.

I derive "indivisible labor" from a tradeoff between fixed costs and fatigue effects, but others (eg., Killingsworth 1983, Weiss 1996, Hamermesh 1997) have suggested that the synchronization of work schedules, rather than fixed costs and fatigue effects, can result in "indivisible labor". I do not explore this possibility and am unaware of a proof in the literature that synchronization is a sufficient condition for work or leisure to be bunched in time.

IV.B. Is One Lifetime Work Session Optimal?

Assume that, in order for work at a point in time to be productive, it must be preceded by either work or the payment of a startup cost. Let a "work session" be an time interval during which only work occurs. In a model with intertemporally separable preferences and constant growth wage and preference profiles, a worker facing such a startup cost would have only one work session in his life - no break-time, no weekends, no vacations, etc. - so that he would avoid multiple payments of the startup cost. The length of that session would depend on tastes, wage rates, etc and the timing of that session would depend on interest rates, time preference, and the shape of the wage profile, but there would be only one work session. With nonmonotonic wage and preference profiles (eg., a child-rearing period), there may be multiple work sessions but there would still be the incentive to bunch work together in long blocks.

IV.C. A Model of Stock Effects

To avoid this rather unrealistic implication, I also suppose that tastes or the marginal product of labor fail to be intertemporally separable at very high frequencies. Suppose, for example, that workers get tired and their productivity falls after several consecutive hours of work. Then it will still be optimal to bunch work in order to economize on the fixed cost, but optimal work plans will involve many relatively short work sessions.

To see the economics of this, consider a continuous time model. As before, t indexes time but now also takes on noninteger values. Each integer time t is the beginning of a potential work session. If the session indexed by an integer t is worked, then time during interval $[t,t+f]$ is spent setting up the work session (but not producing), time in the interval $[t+f, t+T_t]$ is spent working (and producing), and time in the interval $[t+T_t, t+1]$ is spent on leisure. Inclusive of the startup time f, T_t is work time for the time interval indexed by an integer t.

All work sessions must be immediately preceded by some startup time in the amount f.[7] If we interpret this model at a high frequency, f might be thought of as commuting time and a "work session" would be a day at the office or factory. Or f might be "Monday meetings" followed by a four days of work. Or, for seasonal workers, f might be "locating a job" followed by a seasonal work session.

Work productivity $y(t)$ at point in time t depends on the history of time use up to that point:

$$\begin{aligned} y(t) &= A_{\mathrm{int}(t)}\left[1 - G(t \bmod 1)\right] \quad , \quad t \bmod 1 \le T_t \\ G(s) &\in [0,1] \quad , \quad G'(s) \ge 0 \quad , \quad G''(s) \le 0 \end{aligned} \tag{4}$$

where $\mathrm{int}(t)$ denotes the largest integer less than or equal to t and (t mod 1) is (t-int(t)). The productivity parameter $A_{\mathrm{int}(t)}$ is a constant within work sessions, but can vary across work sessions. Total labor product for the work session indexed by the integer t is $\int_f^{T_t} e^{-rs} y(t+s)\, ds$, where r is an intrasession discount rate. Notice that, since $G' \ge 0$, the "margin product" $y(T_t)$ falls (or at least does not increase) as we increase T_t from f to 1 while "average product" (session product per unit time) increases and then falls with T_t.

Since $G' \ge 0$, instantaneous productivity $y(t)$ is relatively high early in the work session and falls with time. Hence, G can be interpreted as the "fatigue" that occurs with extended intervals of work. This fatigue is much like that modeled by Chapman (1909) and, without fixed costs, by Kydland and Prescott (1983), Hotz et al (1988). The only substantial difference is my assumption that fatigue depends on work history only since the last integer t rather than on the entire work

[7]Hansen (1985) describes this setup time as "warm up time," "driving a long distance to work," or "enduring the hassle of putting on a suit and tie" (p. 312). It does not matter whether there is a setup cost of working (so that f is paid at the beginning of a session) or a setup cost of leisure (so that f is paid at the end of a session).

history (including work histories from prior work sessions).[8]

It is relevant that f is a time cost rather than a goods cost. I do not explicitly introduce goods costs into the model, but similar results can be obtained by assuming a negative correlation between f and $A(t)$ across work sessions. An example of a goods setup cost might be the acquisition of a wardrobe or other equipment for the job, although a strict adherence with the model requires that the usefulness of these items terminate when a work session terminates.

I do not explicitly model time-of-day, time-of-year, meteorological, and other naturally recurring effects on labor productivity but the fatigue factor (4) can include these effects. For example, an outdoor construction worker's productivity might be low at the marginal time of day because of poor natural lighting or low at the marginal time of year because of poor weather conditions. However, "naturally cycling" productivity is not enough to explain why five day work sessions are followed by two-day weekends or why factory workers' daily work hours are more sensitive to economic conditions than to, say, annual cycles in the times that the sun rises and sets. Nor is it clear that the economics of labor supply under naturally cycling productivity is so different from other stock effects that they must be analyzed separately.

The fatigue factor represented by the integral in equation (4) can also represent "progressive" labor income taxes which are levied on time-aggregated labor income ("progressive" in the sense that marginal tax rates rise with time-aggregated labor income). In this case, a slightly longer work session increases after-tax earnings by less than session average after-tax earnings because the marginal tax rate is rising.

IV.D. Continuous Time Preferences for Leisure

Workers/consumers get utility from leisure, but not from work or setting up a work session. Continuous time paths for leisure $l(t)$, $t \in [0,T]$ are ranked according to:

$$\begin{gathered}\int_0^T e^{-\rho t}\gamma(t)[1 - l(t)]\,dt \\ l(t) \in \{0,1\} \quad , \quad \gamma(t) > 0\end{gathered} \qquad \textbf{(5)}$$

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[8]Kydland and Prescott (1983) also assume that fatigue enters the utility function rather than affecting labor productivity, an assumption which is of little consequence.

As before, consumption and leisure are separable. What is new in equation (5) is that leisure at any *instant* in time must be either 0 or 1 ((5) does not rule out fractional amounts of leisure during a time *interval*). This rules out variation in the intensity of effort devoted to leisure or work at any instant in time - a possibility which may itself have macro implications.

Assume that $\gamma(t) = \gamma_{\mathrm{int}(t)}$ so that the marginal utility of leisure is constant within work sessions. Henceforth, subscripts are understood to index work sessions, taking only integer values t = 1, 2, 3, ..., T.

IV.E. Lotteries and Consumption Insurance in the Bunching Model

The labor supply decision might be formulated as a choice of which sessions to work and how long each will last. As in the "indivisible" environment of Section II, such a formulation might involve keeping track of the effect of each period's labor supply decision on the marginal utility of wealth. As in Section II, this complication might be avoided by assuming a linear consumption value function. I bypass that rather straightforward analysis and instead introduce lotteries and tastes insurance into the optimal bunching model.

Consumers choose a sequence of lotteries $\{L_t\}$ offered by employers. The date t lottery is: (1) an "allowance" ω_t, (2) a probability of working π_t, and (3) a length of a work session T_t. Consumers receive their allowance at the beginning of the period regardless of the outcome of the lottery. If a consumer is chosen to work, he works for T_t units of time (including the fixed cost f), and the firm has property rights over the output which accrues according to (4). The market for lotteries is competitive, so the only lotteries that will be traded are those satisfying:

$$\omega_t = \pi_t A_t \int_f^{T_t} e^{-rs}\left[1 - G(s)\right] ds$$

where r is an intrasession discount rate.

Problem (P3)'describes optimal labor supply decisions, allowing for a nonlinear consumption value function:

(P3)' ***Optimal Bunching in Continuous-time***

$$\max_{C,\{\omega_t,\pi_t,T_t\}_{t=1}^{T}} U(C) - \sum_{t=1}^{T} e^{-\rho(t-1)} \pi_t \gamma_t \int_0^{T_t} e^{-\rho s} ds$$

$$\text{s.t. } \pi_t \in [0,1] \quad , \quad C = a + P(\Gamma) + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t \omega_t$$

$$T_t \in \{0,[f,1]\} \quad , \quad \omega_t = \pi_t A_t \int_f^{T_t} e^{-rs}\left[1 - G(s)\right] ds$$

There are a continuum of individuals according to their life cycle profile of tastes $\Gamma = \{\gamma_1,\gamma_2,...,\gamma_T\}$ as described by assumption (A4). Differences in preferences across consumers can lead to differences in the marginal utility of wealth $U'(C)$ even if each consumer began life with the same resources because some consumers are unlucky enough to dislike work during those periods when it happens to be most productive. Heterogeneity in the marginal utility of wealth is interesting but not necessarily related to the “indivisibility” of labor so I assume that, at date zero, consumers are *ex ante* identical and trade actuarially fair “tastes” insurance contracts according to (A7):

(A7) Before their lifetime preferences $\Gamma = \{\gamma_1,\gamma_2,...,\gamma_T\}$ are revealed, (P3)' consumers choose the insurance contract $P(\Gamma)$ that has expected value zero and maximizes the expected value of the problem (P3)'. *Ex ante*, each consumer has the same probability of drawing any particular preference profile.

After insurance contracts are settled, consumers choose their demand for employment lotteries according to (P3)'.

It will be useful to define a “parameterization” of (P3)' to be $(a,\rho,r,f,\{A_t,g_t,Q_t\}) \in \mathbb{R}^3\times\mathbb{R}_+^{3T+1}$.

IV.F. Optimal Session Length

When the length of each session is chosen optimally, the marginal product of each session is equated across sessions worked up to the taste parameter:

$$\frac{Q_t A_t [1 - G(T_t)]}{e^{(r-\rho)T_t}\gamma_t} = \frac{Q_s A_s [1 - G(T_s)]}{e^{(r-\rho)T_s}\gamma_s} \quad , \quad all\ \ t, s \mid \pi_t \pi_s > 0 \tag{6}$$

In the case $r = \rho$, it is easy to show that high $Q_t A_t/\gamma_t$ sessions are worked longer, with the relationship between $Q_t A_t/\gamma_t$ and T_t determined by the shape of the intrasession marginal productivity function $G(s)$.

IV.G. Minimum Session Length

Assuming that the utility from leisure is parameterized in a realistic way,[9] it is not optimal to have only one lifetime work session. A very short work session has low average productivity but high marginal productivity. Rather than working such short sessions, it pays to fewer longer sessions in order to incur the fixed costs f less often. It also pays for workers working such short sessions to trade in lotteries so that all enjoy some earnings but only a fraction of them work and incur the fixed cost. Hence, the possibility of substitution over time or trade in lotteries means the "marginal product" of any session worked should be no larger than the "average product" of the next best session to be worked. This implies a minimum length of a work session T_{min}, defined according to the equation of the average and marginal products of a session:[10]

$$\frac{r}{1 - e^{-rT_{\min}}} \int_f^{T_{\min}} e^{-rs}[1 - G(s)]\, ds = 1 - G(T_{\min}) \tag{7}$$

[9]If the preference for leisure is very weak, consumers may work continuously for nearly their entire lifetime. If the preference for leisure is very strong, then consumers may choose to work for only one session.

[10]For simplicity, it is assumed that the T_{min} implicitly defined by (7) is less than one. If not, define $T_{min} \equiv 1$.

No session that is worked will be worked for less than T_{min}. Furthermore, T_{min} depends only on the intrasession interest rate, the fixed cost, and the shape of the intrasession marginal productivity function G(s); T_{min} does not depend on wages, intersession discount rates, or preferences.

IV.H. Equivalence of the Bunching and Indivisible Models

Propositions 7 and 8 show how the optimal bunching of labor in continuous time can be interpreted as a discrete time indivisible labor model. Conversely, the discrete time indivisible labor models of Hansen (1985), Rogerson (1988) and others can be interpreted as the reduced form of a problem describing the optimal bunching of labor in continuous time in the presence of fixed costs and fatigue effects.

Proposition 7 If G(s) takes the form (8), then for every parameterization of the problem (P2)' there exists a parameterization of the Problem (P3)' that yields the same sequence of individual hours and wages, where$\{n_t,w_t\}$ are computed from (P3)' according to (9). The parameters $(a,\rho,\{g_t,Q_t\})$ are indentical for the two problems. As functions of the parameters of (P2)', the other parameters $(r,f,\{A_t\})$ for the problem (P3)' are any element of $\mathbb{R}\times\mathbb{R}_+^{T+1}$ satisfying (10) and $r = \rho$.

$$G(s) \;=\; \begin{cases} 0 & \text{if} \quad s \le \frac{1}{r}\ln\frac{1}{1-r\bar{n}} \in (f,1) \\ 1 & \text{if} \quad s > \frac{1}{r}\ln\frac{1}{1-r\bar{n}} \end{cases} \tag{8}$$

$$n_t \;\equiv\; \frac{1-e^{-rT_t}}{r} \quad , \quad w_t \;\equiv\; \omega_t/\pi_t \tag{9}$$

$$w_t \;\equiv\; A_t\left[1 \;-\; \frac{1-e^{-rf}}{r\bar{n}}\right] \tag{10}$$

Proof (i) Choose any set of parameters $(a,\rho,\bar{n},\{w_t,g_t,Q_t\}) \in \mathbb{R}^2\times\mathbb{R}_+^{3T+1}$ for the problem (P2)'

(ii) For any positive f, let the parameters for the problem (P3)' be $(a,\rho,\rho,f,\{A_t,g_t,Q_t\})$, with $\{A_t\}$ computed from (10).

(iii) When G(s) takes the form (8), we have $T_t = \frac{1}{r} \ln \frac{1}{1 - r\bar{n}}$ for a session t that is worked.

(iv) Rewriting the problem (P3)' using the transformation of variables (9), we have the problem (P2)'.

The Proposition derives Hansen's (1985) environment as a reduced form of the optimal bunching problem when the fatigue function G(s) takes the special form (8).[11] Here there is no fatigue early in the work session and complete fatigue later in the session. The discrete transition from zero to complete fatigue means that all session worked are the same length regardless of wages, tastes, and other variables. If G(s) declined continuously with s, then we see from equation (6) that the length of a work session would vary with wages and other variables.

The proof of Proposition 7 reveals that there are many parameterizations of (P3)' that generate the same data on aggregate hours and earnings as a single parameterization of (P2)'. Indeed, there are equivalent parameterizations of (P3)' not exhibited in Proposition 7. An important reason for multiple equivalent parameterizations of (P3)' is that, without any variation in hours across sessions or across persons, it is impossible to distinguish a session with high productivity A_t and a high fixed cost f from a session with low productivity A_t and a low fixed cost f because both could involve the same hours and output. Nor is it possible to distinguish a high intrasession discount rate from a high marginal disutility of work because each could generate the same reservation wage.

Proposition 8 If G(s) takes the form (8), then for every parameterization of the problem (P3)' there exists a parameterization of the Problem (P2)' that yields the same sequence of individual hours and earnings where $\bar{n}$ and $\{n_k, w_k\}$ are computed from (P3)' according to (9). The parameters $(a,\rho,\{g_t,Q_t\})$ are identical for the two problems. As functions of the parameters of (P3)', the other parameters $\{w_t\}$ for the problem (P2)' are computed according to (10).

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[11]Under one interpretation, Kydland and Prescott (1991) also build a bunching model that has Hansen (1985) as its reduced form. They do not explicitly model "fatigue" and the bunching of labor in continuous time, but their discontinuous utility function defined over "employment" and "hours" can be interpreted as a bunching model where fatigue affects the disutility of work rather than productivity.

Proof (i) Choose any set of parameters $(a,\rho,r,f,\{A_t,g_t,Q_t\}) \in \mathbb{R}^3\times\mathbb{R}_+^{3T+1}$ for the problem (P3)'.
(ii) Let the parameters for the problem (P2)' be $(a,\rho,\bar{n},\{w_t,\hat{g}_t,Q_t\})$, with $\{w_t\}$ computed from (10) and $\{\hat{g}_t\}$ computed from (11).

$$\hat{g}_t = g_t \frac{1 - (1 - r\bar{n})^{\rho/r}}{\rho} \tag{11}$$

(iii) When G(s) takes the form (8), we have $T_t = \frac{1}{r} \ln \frac{1}{1 - r\bar{n}}$ for a session t that is worked.

(iv) Rewriting the problem (P3)' using the transformation of variables (9), we have the parameterization of the problem (P2)' specified in (ii) above.

Propositions 7 and 8 show how the optimal bunching of labor in continuous time can be interpreted as a discrete time indivisible labor model. Conversely, the discrete time indivisible labor models of Hansen (1985), Rogerson (1988) and others can be interpreted as the reduced form of a problem describing the optimal bunching of labor in continuous time in the presence of fixed costs and fatigue effects.

V. "Intensive Margins" in the Indivisible Model

Following Hansen (1985) and Rogerson (1988), my divisible and indivisible macro models have been designed to generate macro data on hours and earnings. But the models might be modified to generate macro data on hours, employment rates, and earnings. Are the two approaches still observationally equivalent? This section shows that the answer depends on the micro model of the "extensive" and "intensive" margins.

V.A. "Intensive" and "Extensive" Margins in the Divisible Economy

Hanoch (1980), Kydland and Prescott (1991), Cho and Cooley (1994), and others have built both "extensive" and "intensive" margins into models of aggregate labor supply. The marginal disutility of work depends separately on the employment rate Π and hours conditional on employment

n:

(P1)'' Divisible Labor with "Intensive" and "Extensive" Margins

$$\max_{C,\{\Pi_t,n_t\}_{t=1}^{T}} U(C) - \sum_{t=1}^{T} e^{-\rho(t-1)} g_t v(\Pi_t, n_t)$$
$$\text{s.t. } \Pi_t \in [0,1] \quad , \quad n_t \in [0, n_{\max}]$$

$$C = a + \sum_{t=1}^{T} e^{-\rho(t-1)} Q_t w_t \Pi_t n_t \quad , \quad w_t = A_t w(\Pi_t n_t)$$

Notice that the wage rate is allowed to depend on the quantity of labor supplied. The only other difference between (P1)' and (P1)" is that (P1)" makes predictions for the decomposition of each date's aggregate hours N_t into an employment rate Π_t and hours conditional on employment n_t. In particular, optimal Π_t and n_t are time-invariant functions of N_t.

As with my other divisible models, I interpret (P1)' as an aggregation across persons. Hanoch (1980) considers (P1)" with $T = 1$ and interprets it as a time-aggregated model of individual-level decisions. In his interpretation, Π_t is the fraction of the 52 weeks in year t during which some work occurs and n_t is average hours worked during those weeks.

V.B. Intensive Margins in the Optimal Bunching Problem

Without assumption (8), the optimal bunching problem (P3)' is a micro model of both extensive and intensive margins of labor supply. Whether or not a session is worked as well as the length of time worked conditional on working vary over time and across people. And, since session length varies over time and across people, so does average hourly earnings.[12] Let n_t and w_t denote the date t session length and hourly earnings *averaged across those who are working at t*.

[12]Because of the lotteries and consumption insurance, optimal lifetime consumption C does not vary across individuals.

$$\Pi_t = F\Big(\ln U'(C)\, Q_t\, A_t [1 - G(T_{\min})] - \ln g_t\Big)$$

$$n_t = \int_{-\infty}^{F^{-1}(\Pi_t)} G^{-1}\Big(1 - e^{x - F^{-1}(\Pi_t)}[1 - G(T_{\min})]\Big)\frac{f(x)}{\Pi_t}\, dx \tag{12}$$

$$N_t = n_t \Pi_t = \int_{-\infty}^{F^{-1}(\Pi_t)} G^{-1}\Big(1 - e^{x - F^{-1}(\Pi_t)}[1 - G(T_{\min})]\Big) f(x)\, dx \equiv E^{-1}(\Pi_t)$$

As before, Π_t denotes the fraction working at date t.

As in the divisible model (P1)", the employment rate Π_t and average hours of those employed n_t are time-invariant functions of aggregate hours N_t. When normalized by the labor productivity parameter A_t, hourly earnings averaged across workers w_t/A_t are also a time invariant function $w(N_t)$ of aggregate hours in both models. That function is computed below for the optimal bunching model:

$$w_t = A_t\, w(N_t)$$

$$w(N) \equiv r \frac{\displaystyle\int_f^{G^{-1}\left(1 - e^{x - F^{-1}(E(N))}[1 - G(T_{\min})]\right)} e^{-rs}[1 - G(s)]\, ds}{1 - e^{rG^{-1}\left(1 - e^{x - F^{-1}(E(N))}[1 - G(T_{\min})]\right)}}\, \frac{f(x)}{E(N)}\, dx \tag{13}$$

Thus the optimal bunching model is a special case of the divisible model (P1)" because the same forces (namely the functions F and G) determining the relationship between employment Π_t and hours n_t also determine the relationship between hours and "wages". In this sense, the indivisibility of labor has refutable macro implications that cannot be derived from the divisible model (P1)'.

The expressions (7), (12) and (13) also show that the optimal bunching model has implications for the relationship between fixed costs f, intrasession discount rates r, employment rates, aggregate hours, and aggregate earnings.

V.C. Intensive Margins as a Consequence of Time Aggregation

Labor supply is almost always measured as a time aggregate: the total time worked during a particular time interval such as a day, week, or year. My Sections II and III follow the previous literature and presume that the time interval over which labor is indivisible coincides with the interval of time aggregation. That is, if labor supply is measured by calendar month, then the calendar month is the time interval during which labor supply must be either 0 or $\bar{n}$. By relaxing that assumption, then the models (P2) and (P2)' can generate time-aggregated macro data that appears to have "intensive" and "extensive" margins.

Let the "measurement period" be an aggregate of K periods (or "potential work sessions") in the model (P2) or (P2)'. Work sessions are indexed $t = 1, ..., TK$ and measurement periods $j = 1, ..., T$. Thus $j = 1$ denotes an aggregate of sessions $t = 1, ..., K$; $j = 2$ denotes an aggregate of sessions $t = K+1, ..., 2K$, etc. As an example, if the potential work session were a month and adult life span equal to 50 years, then K = 12 and T = 50 would model the Census' Bureau's annual measures of labor supply.

A person is said to be "employed" during measurement period j if positive hours are worked during any of the sessions $t = (j\text{-}1)K+1, ..., jK$. Since $n_t \in \{0, \bar{n}/K\}$ for $t = 1, ..., T$, an individual's measured labor supply must be from the set $\{0, \bar{n}/K, 2\bar{n}/K, 3\bar{n}/K, ..., \bar{n}\}$. [13] Assuming that the wage rate $Q_t w_t$, the degree of indivisibility $\bar{n}/K$, and the geometric average marginal disutility of work g_t are constant throughout the measurement period, then the fraction of people "employed" during the measurement period j is:

$$\Pi_j = 1 - \left[1 - F\left(\ln \frac{w_j Q_j \lambda}{g_j}\right)\right]^K$$

As in the time disaggregated model, aggregate hours measured for period s are:

$$N_j = F\left(\ln \frac{w_j Q_j \lambda}{g_j}\right)\bar{n}$$

[13] To facilitate the comparison with the time-disaggregated models, the degree of indivisibility is expressed in units such that the maximum labor supply during the measurement period is $\bar{n}$.

This time aggregation of (P2)' produces a special case of the divisible model (P1)". In particular, it is the special case with $w(N) = 1$ and utility function:

$$v(\Pi, n) \;=\; \hat{v}\left(\ln\left(\frac{n}{\bar{n}} - \frac{1}{K}\right) + \int_0^{\Pi} \frac{1}{x} \frac{1 - (1-x)^{1/K}}{1 - (1-x)^{1/K} - x/K}\, dx \right) \qquad \textbf{(14)}$$

with $\hat{v}(\cdot)$ a monotone increasing function. Mulligan (1998a) uses this model to explain life cycle labor supply, suggesting that, when the measurement period is a year, $K = 2$ fits the life cycle date best. He does not offer a test of the restriction (14).

This analysis shows that Mulligan's (1998a) interpretation of the “hours” margin is substantially different that Ben-Porath's (1973) or Heckman's (1978, 1993). Ben-Porath and Heckman insist that the “hours” and “employment” margins are economically very different while Mulligan suggests that all labor supply decisions are on the extensive margin and that the distinction between employment and hours is only one of time aggregation.

Cho and Cooley motivate their $v(\Pi,n)$ as a reduced form of a time aggregation problem. In fact, their model can be thought of as the time aggregation (as described above) of my optimal bunching model, although they only consider the special case where preferences are homogeneous within the interval of time being aggregated and where $K = \infty$. Of course, their finding of a highly wage-elastic employment rate (as well as Kydland and Prescott's (1991)) is a consequence of their homogeneity assumption.

V.D. Heterogeneous Fatigue Effects

A fourth way of modifying the indivisible model to generate “extensive” and “intensive” margins is to allow agents to differ in the degree of indivisibility $\bar{n}$. Let agent i's date t disutility of leisure be:

$$\bar{g}_t g(\bar{n}^{\,i})\, v(n_t^{\,i})\, e^{\varepsilon_t^i}$$

where $\overline{g}_t$ is a time-specific preference parameter common to all agents, the function $g(\overline{n}^i)$ models the possibility that agents may have systematically different preferences according to their degree of indivisibility, ε_t^i is an agent and time specific preference parameter which is distribution according to the function F, and $v(n)$ reflects variation in the disutility of work with the amount worked (v'>0, v''>0).

Let the date zero "tastes insurance" also insure heterogeneity in the "fatigue effect" parameter $\overline{n}$, so the marginal utility of wealth λ is the same for all agents. Agent i works at date t if and only if:

$$\varepsilon_t^i \leq \ln \frac{\lambda Q_t A_t w(\overline{n}^i)}{g_t} - \ln \frac{g(\overline{n}^i) v(\overline{n}^i)}{\overline{n}^i} \equiv \ln \frac{\lambda Q_t A_t}{g_t} - \ln J(\overline{n}^i)$$

where $A_t w(\overline{n}^i)$ is agent i's date t hourly earnings.

Aggregating across agents, we can compute total hours N_t, the employment rate Π_t, and hours for those employed n_t. As in the divisible model (P1)", Π_t and n_t are time-invariant functions of N_t. In fact, for any parameterization of (P1)", there exists distributions of tastes (F and η) and a wage function $w(\overline{n})$ such that the model of heterogeneous fatigue effects generates the same macro data $\{\Pi_t, n_t, w_t\}$. Similarly, for any distributions of tastes (F and η) and wage function $w(\overline{n})$, there exists a utility function $v(\Pi,n)$ and parameters for the problem (P1)" generating the same macro data $\{\Pi_t, n_t, w_t\}$.[14]

VI. Time-Aggregated Nonlinear Tax Rules

An important reason for the macro equivalence between the divisible and indivisible models of labor supply above is the economic insignificance the interval of time measurement. In fact, the Census Bureau, Survey Research Center, and others ask workers about their work hours aggregated over a week, month, or year because those intervals coincide with some of the bunching of economic activity and, as a consequence, survey respondents can be expected to more accurately remember

[14]Proofs available upon request.

their activity over those intervals than over some arbitrarily chosen interval. For example, one reason that a calendar month or calendar year might be economically significant is because these intervals coincide with federal government time aggregation rules for computing taxes and transfers.

For simplicity, I assume that the government accounting period (a "year") is equal to the interval between work sessions, 1. Period t labor income taxes are a function $\tau_t(e_t)$ of accounting period earnings e_t and, for simplicity, it is assumed that $\tau_t'(e_t)$ exists for all nonnegative e_t. When the indivisible model (P2)' is modified to include these tax rules and aggregated according to (A4) and (A6), aggregate hours for year t are described by equation (14):

$$N_t = F\left(\ln \frac{\lambda w_t (1 - ATR_t) Q_t}{g_t} \right) \bar{n}$$
$$ATR_t \equiv \frac{\tau_t(\bar{n} w_t) - \tau_t(0)}{\bar{n} w_t} \tag{14}$$

where ATR_t is the year t average tax rate.

When the divisible model (P1)' is modified to include these tax rules and the marginal disutility of work function is defined as in Proposition 3, aggregate hours for year t are described by equation (15):[15]

$$N_t = F\left(\ln \frac{\lambda w_t (1 - MTR_t) Q_t}{g_t} \right) \bar{n}$$
$$MTR_t \equiv \tau_t'(w_t N_t) \tag{15}$$

where MTR_t is the year t marginal tax rate.

Thus *average* tax rates determine labor supply in the indivisible model while *marginal* tax rates determine labor supply (or at least its allocation over time) in the divisible model. Equations

[15]See Barro and Sahasakul (1983) for a derivation.

(14) and (15) are interesting not only because they expose a difference between the divisible and indivisible models, but are of significant relevance for public finance. First, it has been extensively argued (eg., Hall and Rabushka 1995) that a revenue neutral flat tax would dramatically increase the efficiency of taxpayers' time allocations. But, of course, this analysis relies heavily on the presumption that - holding constant tax revenue - labor supply depends on the marginal tax rate. My analysis shows that only the average tax rate matters in a model of indivisible labor when the “indivisibility” is at least as long as the tax accounting period. Second, the distinction between average and marginal taxes rates matters for the estimation of labor supply elasticities (which are used for, among other things, tax reform simulations). As shown above for the indivisible model, the observations of MaCurdy (1992) and others that micro or macro labor supply is unresponsive to marginal tax rates is quite consistent with large aggregate labor supply elasticities. Mulligan (1998a) studies two applications of this result, arguing that average tax rates - and thereby the indivisible labor model - fit his data better.

Another difference between the divisible and indivisible labor models is that, in a population of heterogeneous individuals facing the same labor income tax schedule, the former predicts that the distribution of a year *t* sessions earnings has mass points at any “kink” in the tax schedule. This is not true in the indivisible labor model where the distribution of nonzero earnings follows the distribution of $w_t\bar{n}_t$ unless the degree of indivisibility $\bar{n}$ itself responds to tax incentives. This distribution will not typically have mass point unless the underling distributions of tastes or pretax wages have a mass point. Thus the findings of Hausman (1986) and others that the income distribution does not seem to be concentrated at kinks in the individual income tax schedule is consistent with large wage elasticities of labor supply.

Results are more complicated when the tax accounting period is different from the interval between work sessions or the function $\tau_t'(e_t)$ is discontinuous, but it is still true that the indivisible labor model has implications that are distinct from the divisible model.

VII. Conclusions

I build a micro model of the bunching of work and leisure in time. The optimal length of the work session is determined by a tradeoff between session fixed costs and stock effects on productivity or utility. Higher productivity sessions are worked longer and all sessions worked are worked for at least some minimum length of time. The minimum depends on the magnitude of the fixed cost, the form of the stock effects, and the intrasession discount rate, but not worker wealth, wages, or tastes.

For a particular form of the stock effects, all sessions are worked the same length of time and optimal labor supply can be described in reduced form by the "indivisible labor" models Diamond and Mirrlees (1978, 1986), Hansen (1985), Hamilton (1988), Rogerson (1988), Christiano and Eichenbaum (1992) and many others. When aggregated across individuals (and perhaps over time), the indivisible model is equivalent to the divisible model of Lucas and Rapping (1969) defined over aggregated measures.

Thus, I argue that labor indivisibility (as modeled by these authors) *per se* has no implications for macroeconomics. Although such an aggregation result may not be particularly surprising - for example, Marshall suggests this in his *Principles of Economics* (1920/1990) and Hamermesh (1990) derives a smooth labor demand function in a micro model of lumpy adjustment costs - I am able to be precise about the mapping between the heterogeneity and the smoothness of well known models of aggregate behavior. My proofs also show how Hansen's (1985) and Rogerson's (1988) findings of an infinite equilibrium aggregate labor supply elasticity are not a consequence of indivisibility, but of (a) the homogeneity of micro-level decisions, and (b) their definition of equilibrium. My Propositions contradict Rogerson's (1988, pp. 3, 14) claims that indivisibility implies large aggregate labor supply elasticities *even when agents are heterogeneous* in terms of their reservation wages. Nor is it true that substitution along the "extensive" margin must be greater than substitution along the "intensive" margin. Rogerson (1988, p. 14) defends his claims with a parametric example, but my Propositions 2 and 4 show the importance of his assuming a particular distribution for the "reservation wage."[16] Indeed, any nonnegative aggregate labor supply elasticity can be generated by either a divisible or an indivisible model.

I introduce two other models of indivisible labor that do have macro implications. The first is a model of the optimal bunching of work over time in which the length of a work session optimally varies over time and across agents. Because of the fixed costs and fatigue effects that determine the optimal bunching, the model places a restriction on the comovements of employment rates, aggregate hours, and aggregate earnings that are not obtained from a divisible representative agent model of

[16]Defining the "reservation wage" to be the lowest wage at which the constraint $\pi_t \geq 0$ does not bind, Rogerson's example has a degenerate distribution of reservation wages at zero.

employment and hours. In particular, average hourly earnings endogenously vary over time and in a way that is related to the comovements of employment and hours. However, this restriction is rather subtle and, to my knowledge, has not to date been the subject of empirical testing. It cannot be shown that, as a consequence of the endogeneity of average hourly earnings, whether the elasticity of aggregate labor supply with respect to average hourly earnings is necessarily larger (or necessarily smaller) in the indivisible than in the divisible model.

The second model time-aggregates individual-level choices on the "extensive" margin to obtain individual-level measures of "employment" and "hours." The model restricts the comovements of employment rates and aggregate hours, but again this restriction is rather subtle and not the subject of empirical testing to date.

Related, but different, aggregation results can be found in the production theory literature. For example, Hamermesh (1990) studies a model of discrete micro-level labor demand adjustments which look smooth at the aggregate level. Houthakker's (1955) proof - that the input demands of heterogeneous Leontief firms can aggregate to a smooth industry demand - is a closer and more well-known analytical cousin to my result. Houthakker's "fixed proportion" is analogous to my labor indivisibility $\bar{n}$ while each of his firm's profit is analogous to my reservation wage. Houtkakker generates Cobb-Douglas input demand by aggregating across firms with identical profits (namely zero) and fixed proportions distributed Pareto. I generate (in a special case) Cobb-Douglas labor supply by aggregating across consumers with identical "fixed proportions" ($\bar{n}$) and reservation wages distributed exponential.

Smooth aggregate labor supply is generated from a smooth cross-sectional distribution of reservation wages, where the reservation wage is the wage at which a worker demands a lottery with employment probability zero. I obtain from a smooth distribution of the marginal disutility of work, but there are other ways to generate a smooth distribution of reservation wages and hence smooth aggregate labor supply.[17] This might be achieved with heterogeneity in initial assets, heterogeneity in the lifetime history of various shocks, a staggering of time intervals across agents (eg., agent *i*'s "period" begins when agent *j*'s "period" is half completed), or heterogeneity in remaining life expectancy. Mulligan (1998a) also shows that approximately smooth *individual*-level responses can

[17]Heterogeneous marginal disutility need not imply heterogeneous reservation wages (eg., Rogerson 1988, p. 14).

be derived when reservation wages vary over time and "labor supply" is measured as a time aggregate.

I refer to "macro data" in both the divisible and indivisible economies as an aggregation of prices and quantities across individuals with identical initial wealth a, rates of time preference ρ, wages and interest factor profiles $\{w_t, Q_t\}$, and marginal disutility of wealth schedules $U'(C)$. My assumption of homogeneity in these dimensions is only for analytical simplicity. If agents differed, say, in initial wealth a, and the distributions were the same in the divisible and indivisible economies, then both the indivisible and divisible economies would generate the same macro data. This can be proved by first aggregating within groups of individuals with the same a, applying the relevant Propositions 1-4, and then aggregating across groups. What is no longer necessarily true in this case is that (twice) aggregated prices and quantities appear as if they were generated by a representative agent. But this paper does not claim a representative agent always exists, only that indivisible and divisible economies are indistinguishable with macro data. Moreover, while the nonexistence of a representative agent may introduce its own econometric problems (problems which are the basis of criticisms of aggregate studies by Smith (1977, p. 249) and Pencavel (1986, p 34)), aggregation can eliminate some econometric problems.[18]

Labor productivity cannot be directly observed when a person is not working. Moreover, a sample of workers is certainly a sample selected according to labor productivity. This paper does not deny these realities. However, a number of authors in the literature have taken this fact a step further, suggesting that studies of the "hours" margin are immune to (or at least less sensitive to) this form of sample selectivity bias than are studies of the "employment" margin.[19] Their suggestions may be true, but cannot be derived as a matter of logic. Just as we do not know the current labor productivity of a woman who has been out of the labor force for five years, we do not know the summer labor productivity of a school teacher or the late-night labor productivity of a banker who is continuously employed during normal business hours. Since the important econometric problem of inferring labor productivity for those times when a person does not work need not be related to

[18] I emphasize the benefits of aggregation of micro-level discontinuities. See Grunfeld and Griliches (1960) for examples of potential benefits of aggregating micro-level specification errors.

[19] See Heckman (1993) for a clear statement of this claim and a survey of the literature.

the divisibility of labor or to the distinction between "employment" and "hours," I have neglected any discussion of that issue in my analysis of indivisibility.

Time aggregation can be of economic significance, for example, when it is a part of government tax policy. Indivisible labor therefore has important implications for public finance.

Given that measures of economic activity are typically aggregated over periods no longer than a year, I am primarily interested in fairly high frequency bunching of work and leisure. But I suspect that similar economic issues arise at lower frequencies, perhaps with retirement and long-term employment or with life cycle job and occupation changes. In these cases, the "fixed cost" *f* might represent time accumulating firm-specific human capital and the "work session" tenure with a firm. or f might represent time accumulating occupation-specific human capital and the "work session" tenure in that occupation or industry. An interesting extension of my analysis would be to simultaneously introduce fixed costs which are amortized over different horizons and fatigue effects which decay at different rates.

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