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STRUCTURAL CHANGE IN TIME SERIES OF THE EXCHANGE RATES BETWEEN YEN-DOLLAR AND YEN-EURO IN 2001-2004

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Abstract

The economic structural change in the data of the yen exchange rates to US dollar and to euro is investigated. We analyse the distribution of the residuals which are the differences between the smoothed and original data. The probability density function is well approximated by the Gram-Charlier expansion. The existence of the fact that the distribution of the residuals parts from the normal distribution indicates the occurrence of the structural change. Introducing a new index, composed by the skewness and the kurtosis, we can easily find a sign of the structural change. It is shown that both data of the yen-dollar rate and the yen-euro rate from January 2, 2001 to May 18, 2004 undergo the structural changes three times together on May 23, 2001, March 11, 2002, and December 27, 2002.

JEL classification: C51

Key words: structural change, Gram-Charlier expansion, exchange rates, skewness and kurtosis

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1.- Introduction

When we are given a series of data, some of us want to forecast its value at future time, and apply the well-established methods such as ARIMA or regression models. If there are economic structural changes in the data generating mechanism, a naive application of those methods does not give us right answers.

Then the main concern of econometricians was to establish a method by which a sign of the structural change in the data ought to be detected. Akiba (1994), Broemeling and Tsurumi (1987), and Hackl (1989) give surveys about various kinds of methods to find a sign of the structural change. Typical methods that they explained in these books are to examine the stability of the coefficients of the equations that a model such as ARIMA or the regression analysis contains. A subset of the series in which the coefficients of the equations are stable against the recursive estimation constitutes one data regime. The data may have several regimes. A node between the two regimes is the point where the structure changes.

In this paper, we will find a sign of economic structural changes in the time series of the yen exchange rates against US dollar and euro by a different way from the above mentioned methods. In Section 2, we introduce the way of exponential smoothing which does not depend on a type of model such as ARIMA or the regression analysis. We will put stress on residuals associated with the data smoothing. It is shown that the distribution of the residuals departs from the normal distribution.

In Section 3, we show that the Gram-Charlier expansion[1] is useful to approximate the distribution of the residuals. The third and the fourth terms of the Gram-Charlier expansion depend on the skewness and the kurtosis of the data, respectively. Then these higher order moments are important to determine the distribution of the residuals.

In Section 4, we introduce a way to generate some time series from one time series by constituting subsets recursively from the original data set. When the skewness and the kurtosis evaluated in each subset, they are constants. But they are regarded as time series along all the subsets. We define a new index g made from the skewness and the kurtosis. The index g goes to zero, if the distribution of the residuals is close to the normal distribution. A change of the distribution means that of the data structure. Then when the index g goes to zero or departs from zero, we know that the structural change occurs. Taking cumulative sum of this index, we find a sign of the structural change easily as a kink of the cumulated curve. It is shown that the yen exchange rates to US dollar and to euro from January 2, 2001 to May 18, 2004 undergo the structural changes three times together.

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[1] See, for example, Kendall and Stuart (1979).

2.- Residuals for the Data Smoothing

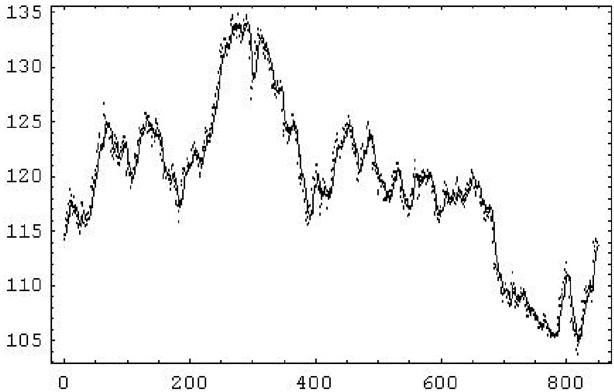
Here are time series of the exchange rates of Japanese yen to US dollar and euro, which are daily series from January 2, 2001 to May 18, 2004.[2] They are illustrated with Graph 1 and 2. Can we find any kinds of structural changes from these data? In order to answer affirmatively, someone will apply ARIMA model to the data. Or other may supplement the given data with other ones and formulate regression equations. Their main concern is to find the break points in the data fitting curves calculated by ARIMA or the regression. Before drawing a conclusion from these methods, we must analyze the property of the residuals associated with those data fittings.

The validity of ARIMA or the regression is based on the stationarity of the residuals. For simplicity we usually postulate that the residuals are normally distributed. The ARIMA or the regression model is a kind of data smoothing. The simplest data smoothing is the moving average. Among others, the exponential smoothing is the most convenient for our present purpose. It is expressed by a formula

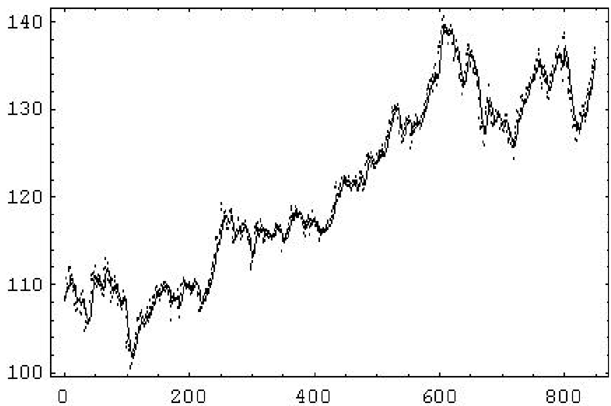
$$\text{(1)} \qquad \overline{x}_t = \overline{x}_{t-1} + a\,(x_t - \overline{x}_{t-1}),\ \ 1 \le t \le 850$$

[2] Source: Pacific Exchange Rate Service, ttp://pacific.commerce.ubc.ca/xr.

Graph 1: Plot of yen-dollar rate and its smoothing



Graph 2: Plot of yen-euro rate and its smoothing



where $\{x_t\}$ is the original series and $\{\overline{x}_t\}$ is the smoothed series.

The characteristic of (1) is that the length of the series $\overline{x}_t$ is the same as x_t. There are two arbitrary constants in (1): one is $\boldsymbol{a}$ with $0<\boldsymbol{a}<1$ and the other is the initial value $\overline{x}_1$. We will put $\boldsymbol{a}=0.3$ and $\overline{x_1}=x_1$ for convenience. The smoothed curves for the yen-dollar rate and the yen-euro rate are given in Graph 1 and 2, respectively.

A residual by the smoothing is defined as

(2) $\Delta x_t = \overline{x_t} - x_t$.

If the residuals are not stationary, the smoothing (ARIMA or the regression) will not give the right results. It is convenient to suppose that the stationary series should be distributed as

(3) $\Delta x_t \sim N(\mu, \sigma^2)$

where μ and σ^2 are the mean value and the variance of Δx_t, respectively. To examine this hypothesis, we take a histogram of $\{x_t\}$. For convenience, we standardize the data as

(4) $$z_t = \frac{\Delta x_t - \mu}{\sigma}$$

(5) $$\mu = \sum_{t=1}^{N} \Delta x_t / N, \quad \sigma^2 = \sum_{t=1}^{N} (\Delta x_t - \mu)^2 / N, \quad N = 850.$$

The standardized density function for the normal distribution is

(6) $$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{z^2}{2} \right].$$

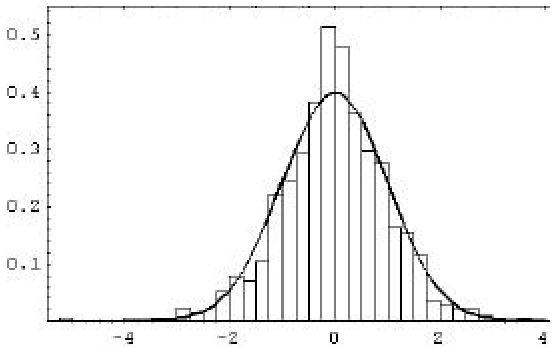
A histogram for the yen-dollar rate is given in Graph 3 together with the plot of (6). The distribution of $\{z_t\}$ seems to be apart from $N(0,1)$. The portmanteau test[3] is as follows:

Q-value=896.7, $\chi^2_{0.95}(44) = 48.6$.

Since $Q > \chi^2_{0.95}$, we reject the hypothesis (3). Then a naive application of the well-known method such as ARIMA to the yen exchange rates may lead to a wrong conclusion.

[3] See, for instance, Box, Jenkins, and Reinsel (1994).

Graph 3: Histogram of yen-dollar rate and normal distribution



3.- The Gram-Charlier Expansion

In this section, we will give a sketch of approximation of the probability density function. The density function $p(x)$ may have the following orthogonal expansion in terms of polynomials $f_n(x)$

(7) $$p(x) = \sum_{n=0}^{\infty} a_n f_n(x) w(x)$$

with the orthogonal property

(8) $$\int f_n(x) f_m(x) w(x) dx = d_{n,m}, \quad n, m = 0,1,2,....$$

It should be noted that the orthogonal relation (8) has the weight function $w(x)$. The coefficients a_n can be obtained by (8) as

(9) $$a_n = \int f_n(x) p(x) dx.$$

When $f_n(x)$ is the Hermite polynomial[4] $H_n(x)$ and $w(x)$ is the standardized density function of the normal distribution $f(z)$, the expression (7) is said the Gram-Charlier expansion. Here are relations

(10) $$f_n(x) = \frac{1}{\sqrt{n!}} H_n\left(\frac{x-m}{s}\right)$$

(11) $$w(x) = f(x) = \frac{1}{\sqrt{2p}} \frac{1}{s} \exp\left[-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right]$$

(12) $$m = \int xp(x)dx, \quad s^2 = \int (x-m)^2 p(x)dx.$$

The Hermite polynomials for some orders are given by

(13) $$H_0(y)=1, \; H_1(y) = y, \; H_2(y) = y^2 - 1$$
$$H_3(y) = y^3 - 3y, \; H_4(y) = y^4 - 6y^2 + 3.$$

From equations (9)-(13), we have

$$a_0 = 1, \; a_1 = 0, \; a_2 = 0$$
$$a_3 = \frac{1}{\sqrt{3!}} \frac{m_3}{s^3}, \quad a_4 = \frac{1}{\sqrt{4!}}\left(\frac{m_4}{s^4} - 3\right)$$

(14) $$m_3 = \int (x-m)^3 p(x)dx, \quad m_4 = \int (x-m)^4 p(x)dx.$$

Using all the results obtained above, we have the Gram-Charlier expansion up to the fourth order:

(15) $$p(x) = \left[1 - \frac{1}{3!} a (3y - y^3) + \frac{1}{4!} b (3 - 6y^2 + y^4)\right] f(x)$$

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4 Kendall and Stuart (1979).

where $y = (x - m)/s$ and

(16) $$a = sk = \frac{m_3}{s^3}$$

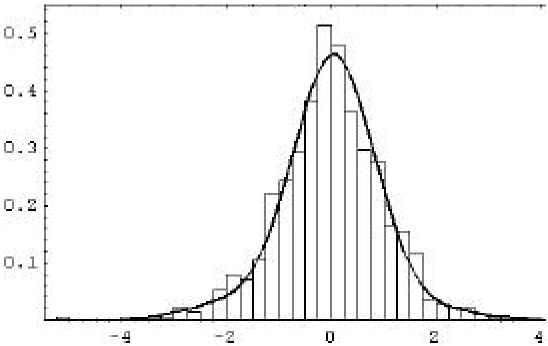
(17) $$b = ku - 3,\ ku = \frac{m_4}{s^4}.$$

The terms sk and ku are called the skewness and the kurtosis, respectively. If the distribution is normal, from (12), (14), (16) and (17), we have sk =0 and ku =3, therefore $a = b = 0$. The significance of the expression (15) is that the magnitude of the deviation of $p(x)$ from $f(x)$ is calculated with the knowledge of a and b.

Now we go back to the series z_t in (4) and replace x in this section with Δx_t in (4) and (5). If we also replace $\int y(x)p(x)\,dx$ for a function $y(x)$ by $\sum_{t=1}^{N} y_t / N$, we can calculate a and b numerically. And we can evaluate the Gram-Charlier expansion (15).

The histogram of z_t and the evaluated Gram-Charlier expansion are illustrated in Graph 4. Comparing Graph 3 with Graph 4, we know that fitting is improved by (15). Granger and Joyeux (1980) pointed out that some economic time series had the distribution with long memory. But this is not the case. The tail of the distribution is as short as that of the normal distribution.

Graph 4: Histogram of yen-dollar rate and Gram-Charlier expansion



4.- A New Index g

In this section, we will closely examine the distribution of the residuals Δx_t. For this purpose, we shall make subsets of $\{\Delta x_t\}$ as follows

$$X_1 = \{\Delta x_1, \Delta x_2, ..., \Delta x_n\}$$
$$X_2 = \{\Delta x_2, \Delta x_3, ..., \Delta x_{n+1}\}$$
$$\cdots \quad \cdots$$
$$X_t = \{\Delta x_t, \Delta x_{t+1}, ..., \Delta x_{t+n-1}\}, \quad 1 \le t \le 650$$

where we set $n = 200$ for convenience. We will calculate the moments in each subset X_t. Now we put

$$m_t = \sum_{k=0}^{n-1} \Delta x_{t+k} / n, \quad s_t^2 = \sum_{k=0}^{n-1} (\Delta x_{t+k} - m_t)^2 / n$$

$$m_t^3 = \sum_{k=0}^{n-1} (\Delta x_{t+k} - m_t)^3 / n, \quad m_t^4 = \sum_{k=0}^{n-1} (\Delta x_{t+k} - m_t)^4 / n.$$

From these, we shall define the t dependence of a and b as

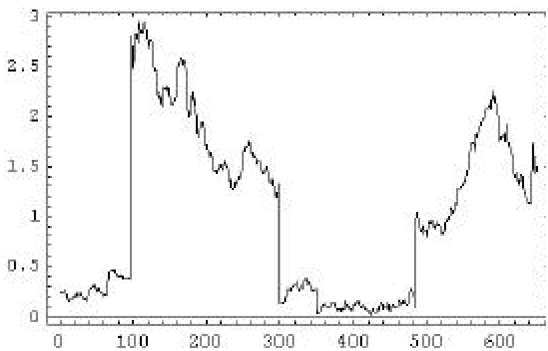
$$a_t = \frac{m_t^3}{s_t^3}, \quad b_t = \frac{m_t^4}{s_t^4}$$

In case the original series $\{\Delta x_t\}$ is stationary, all the moments are independent from t. If X_t distributes normally, then $a_t \approx 0$ and $b_t \approx 0$ for each t. Therefore it is plausible to introduce a new index g_t which indicates the deviation of the distribution from the normal distribution:

$$g_t = \sqrt{a_t^2 + b_t^2}\ .$$

The time dependence of g_t is plotted in Graph 5.

Graph 5: Time dependence of g_t



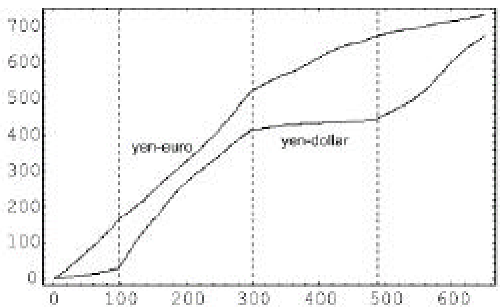
From this graph, we know that the index g_t has sharp jumps to or from the regions of $g_t \approx 0$ around $t = 100$, 300, and 500. To see those facts more clearly, it will be convenient to take the cumulative sum of g_t :

$$G_t = \sum_{t=1}^{t} g_t .$$

The time series G_t for the yen-dollar rate and the yen-euro rate are plotted in Graph 6. In this graph, there are kinks around $t = 100$, 300, and 500. The derivative from the left is different from that from the right at those points. Using the technique of the cumulative sum, we are able to compare patterns of a couple of G_t's in a graph together, in our case, the patterns of the yen-dollar rate and the yen-euro rate. Actually, we know that both the yen rates have the kinks at the same dates. Those dates correspond to the real dates; May 23, 2001, March 11, 2002, and December 27, 2002.

The plot of G_t for the yen-euro rate does not have a sharp kink except for $t = 300$. This is because the value of the parameter a in (1) or the length n of the subset X_i may not be adequate.

Graph 6: Cumulative sum G_t



When we take a difference and the resultant residuals are not stationary, we used to take the second difference. By the same reason, we may need the double smoothing for the yen-euro rate. Anyway we shall name those points in Graph 6 at $t = 100$, 300, and 500 critical points. The data of the yen exchange rates can be divided by the critical points into four subsets which correspond to four phases of the data structure.

5.- Conclusions

In this paper, we discussed how to find structural changes in the time series of the yen exchange rates to US dollar and euro from January 2, 2001 to May 18, 2004. Here are the results:

1) The data are smoothed by the way of the exponential smoothing. The distribution of the residuals associated with the smoothing deviates from the normal distribution. A naïve application of ARIMA or the regression models is not desirable.
2) The density function of the residuals is well approximated by the Gram-Charlier expansion.
3) The data are recursively divided into some subsets. The higher order moments, the skewness and the kurtosis, are calculated in each subset. They form time series along the whole data.
4) The deviation from the normal distribution indicates that the data structure changes from the stationary state. The structural changes are detected from the behavior of these moments as the time series. Introducing a new index composed by those moments, we can find a sign of the

structural change in the yen exchange rates around May 23, 2001, March 11, 2002, and December 27, 2002.

Bibliography

Akiba, H. (1994). *Structural Changes in Foreign Exchange Markets*, Ashish Publishing House, New Delhi.

Box, G..E.P., Jenkins, G,.M., and Reinsel, G..C. (1994). *Time Series Analysis: Forecasting and Control,* third edition, Prentice-Hall International, London.

Broemeling, L. D. and Tsurumi, H. (1987). *Econometrics and Structural Change*, Marcel Dekker, New York.

Granger, C.W. J. and Joyeux, R. (1980). ``An Introduction to Long-Memory Time Series Models and Fractional Differencing," *Journal of Time Series Analysis*, Vol.1, pp.15-19.

Hackl, P. (ed), (1989). *Statistical Analysis and Forecasting of Economic Structural Change*, Springer, Berlin.

Kendall, M. and Stuart, A, (1979). *The Advanced Theory of Statistics*, Vol. 1, fourth edition, Charles Griffin, London.