THE OPTIMAL DEMAND FOR FOREIGN EXCHANGE RESERVES IN PAKISTAN

JALIL, Abdul*
BOKHARI, Sheharyar

Abstract:
Using monthly data on foreign exchange reserves from June 1995 through June 2005, we find that in line with other country-specific studies the opportunity cost of holding reserves played a greater role than reserve volatility in determining the level of reserves in Pakistan. Our finding is in contrast with the hypothesis of increased capital mobility that is commonly set forth in explaining the precautionary motive for reserve holdings. As also pointed by Ramachandran (2004), this result could perhaps be attributed to the fact that capital outflow in Pakistan (as also in India) is not as free as capital inflow and a large part of the recent reserve accumulation is due to non-debt reserve inflows.
JEL classification: E58, F32. F37 Keywords: Foreign Exchange Reserves, Optimal Demand, GARCH, Pakistan

1. Introduction

The arrival of the new millennium witnessed a historic accumulation of foreign exchange reserves in Pakistan, from an average of US $1.5bn in late 1990’s to US $10bn in 2003. Recent debates on the sustainability of these newly acquired reserves raised the more relevant question of identifying the factors that affect the central bank’s decision to hold these reserves. In fact, some economists have argued that a proper supply policy cannot be

* Mr. Jalil is Research Analyst (State Bank of Pakistan) and a PhD student in Wuhan University, under the supervision of Dr. Ma Ying, and can be reached at jalil.hanif@gmail.com. Mr. Bokhari is a recent M.A economics graduate from Tufts University, Massachusetts, U.S.A. The authors are grateful to Mr. Riaz Riazuddin for his encouragement and support of this project. Finally, the views expressed in this paper are solely of the authors. Any errors in the paper are solely ours.

formulated unless something is known about the central bank's demand for reserves.[1]

This paper is an attempt to determine the optimal demand for foreign exchange reserves in Pakistan through a buffer stock model of reserves. The buffer stock, or inventory model has been remarkably successful in explaining international reserve holdings in emerging markets (see for instance Flood and Marion (2002)). The model postulates that a central bank will choose an initial level of reserve holdings that minimizes its total expected costs. It identifies two costs incurred by the central bank; the opportunity cost of reserves and the adjustment cost that is incurred whenever reserves reach some lower bound. The opportunity cost of holding reserves is largely determined by the level of the reserves in the country and would be the difference between the returns from an alternative investment and the yield of reserves. The adjustment cost is generally interpreted as the output or welfare lost due to costly policy measures that are needed to generate sufficiently large external payments surplus necessary for reserve accumulation in times of actual reserves reaching some lower limit. For the purposes of this paper and without any loss of generality, we will assume a lower bound of zero. Furthermore, the two costs are interrelated since a higher stock of reserves reduces the probability of having to adjust and thus reduces the expected cost of adjustment, but this benefit comes at the cost of higher foregone earnings. In this paper, we use Frenkel and Jovanovic (1981) (henceforth FJ's) stochastic buffer stock model that obtains the optimal reserves by minimizing the sum of the expected value of both components of cost. The advantage of using their approach is that they provide an explicit formula for the optimal level of reserves. While most other studies tell us only the relative and qualitative nature of the optimal level of international reserves, FJ's formula, in principle, enables us to tell whether the actual level of reserves is absolutely larger or smaller than the optimal level. A recent cross-sectional time series analysis by Flood and Marion (2002) shows that the FJ style reserve demand holds true when applied to the new emerging markets data.

[1] See for example Kelly (1970) and Grubel (1971).

In our study, we find that both adjustment and opportunity costs play a significant role in the accumulation of reserves in Pakistan, with the later being a more predominant determinant. We also find a structural shift in the reserves data from June 2000 that is likely a result of reforms by the SBP, the increase in remittances and other exogenous capital inflows that marked the turn of the century. These structural changes have substantially increased the holdings of international reserves.

The rest of this paper is organized into four sections. The first section highlights the historical reserve build up in the Pakistan from the mid 1990s to the year 2005. A brief review of the literature on the topic is given in section two. The third section outlines the mechanics of the FJ model and section four takes the model to the data on reserves in Pakistan during the period June 1995 to June 2005. Finally, the sixth section concludes with the implications of the study.

2. Historical Reserve Position in Pakistan

Since the start of the new millennium, there has been a gradual built up of foreign exchange reserves that stood at $13.3 bn as of June 2005. However, these reserves were substantially lower ten years earlier when they stood at $3.7 bn in June 1995. Figure 1 illustrates the foreign exchange reserves in Pakistan over the period 1995 to 2005. As shown in the figure, relatively low levels of reserves that were sensitive to occasional lumpy inflows characterized the latter half of the 1990s. For instance, a temporary surge in the inflows took place in 1994 when Pakistan experienced foreign direct investment of $1.5bn that primarily constituted of proceeds from the PTC vouchers and the advent of Hubco. These, however, were one-off inflows that quickly dried up in the following year.

During the same period, there was a substantial loss of remittances to the informal channel in Pakistan that was more efficient than the official banking system and also provided a better exchange rate. It was only in 1999 that the State Bank of Pakistan (SBP) began purchasing foreign currency from the kerb market in the wake of the nuclear test and the subsequent imposition of multi-layer

sanctions. The SBP also took various measures to divert worker's remittances towards the formal banking channel and its efforts were expedited after September 11th, when fear of possible association with terrorists prompted many expatriates to use the official system when sending their funds.
In the next section, we turn to give a brief overview of the literature on the demand for foreign exchange reserves.

3. Brief Review of the Literature

The literature exploring the determinants of the demand for international reserves can be characterized broadly into two groups. One of these groups determines the demand for reserves for a cross section of countries over a period of time (see Table 1). The second group determines the demand for international reserves for individual countries (see Table 2). As the purpose of this paper is to determine the demand for reserves in Pakistan, we will focus our discussion on the methodology used in country-specific studies. In this regard, a good number of studies were primarily motivated by the monetary approach to balance of payments that asserted that in the presence of a fixed or pegged exchange rate regime, the domestic money market disequilibria affected reserve holdings in the short-run (e.g. Elbadawi (1990), Badinger (2004)).

Figur:1 Foreign Exchange Reserves of Pakistan

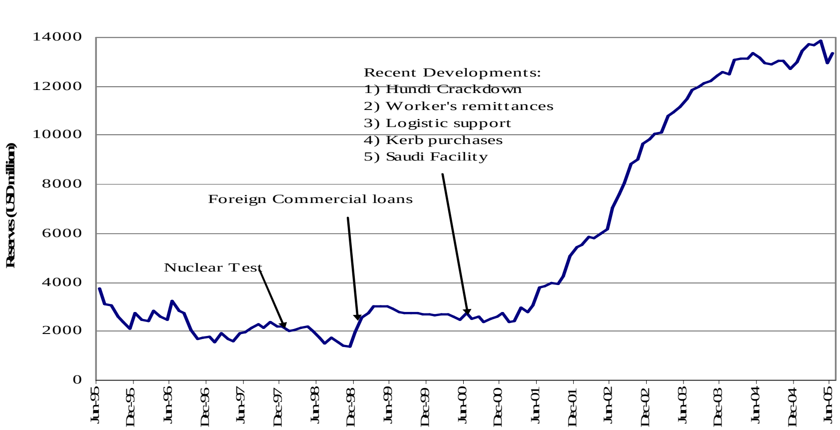


Table 1: Representative Panel Studies on the Demand for International Reserve

Author	Exchange Rate Regime	Dependent Variable	Explanatory Variables	*Methodology*
Kelly ('70)	Pegged	R = level of reserves	GDP/capita (proxy for Opp. cost), Std. Dev. Of X, M/GDP.	Pooled OLS on 46 countries from 1953-65
Aizenman and Marion ('02) IMF Working paper	Various	R/P (deflated by U.S GDP deflator)	GDP/capita, Population, X volatility, Nominal Eff. Exch. Rate volatility, M/GDP, Corruption Index	Panel Regression of 122 developing countries from 1980-2002
Flood and Marion ('02) IMF Working paper	Various	R, R/M, R/P, R/GNP or R/M2	Volatility of Reserves, Opp. Cost Nominal Eff. Exch Rate volatility, (X+M)/GDP, Gross Capital flows)/GDP	GMM with fixed effects on 35 countries from 1988-1999
Edison ('03) IMF Economic Outlook	Various	R/P (Deflated by the U.S consumer price index)	GDP/capita, Population, M/GDP, Export volatility, Exch. Rate volatility, opportunity cost, Capital Flows/GDP, Broad Money/GDP	Panel Regression with Fixed Effects

Table 2: Country Specific Studies on the Demand for International Reserves

Author	Exchange Rate Regime	Dependent Variable	Explanatory Variables	Methodology
Elbadawi (1990) Sudan	Fixed	First Difference of R (the level of Reserves)	M/GDP, Uncertainty of Reserves, Money mkt. disequilibrium, Remittances	ECM/OLS on Sudanese data from 1971-82
Ford and Huang (1993) China	Pegged	First Difference of R (the level of Reserves)	Ind. + Agri Ouput, Uncertainty of Reserves, M/GDP, Mny mkt. deq.	ECM on Chinese data from 1956-89
Badinger (2004) Austria	Fixed	First Difference of R	M (size of intl transact ions), Uncertainty of Reserves, Opp. cost (i-rate differential), Mny mkt. deq.	VEC on Austrian data from 1985-97
Silva and da Silva (2004) Brazil	Fixed and Floating	Level of Reserves	Volatility (own measure), Dummy for regime change, Opportunity. Cost	GARCH/ EGARCH on Brazilian data from 1995-2004
Ramacha ndran (2004) India	Floating	Level of Reserves	Volatility of Reserves, Opportunity cost (call money rate)	GARCH/ARCH on weekly Indian data from 1999-2003
Khan and Ahmed (2005) Pakistan	Floating	Level of Reserves (for long run) and First Difference of the Level of Reserves (for short-run)	M/GDP, Uncertainty of Reserves, Mny mkt deq, opportunity cost, Remittances	VEC on quarterly data from 1982-2003

Additionally, they note that the short run reserve movements are most driven by the domestic monetary disequilibrium. The coefficient on the money disequilibrium was 0.5, which means that

50% of the domestic monetary market disequilibrium is eliminated by changing domestic credit while the remaining 50% is eliminated by changes in reserves. Finally, they also find that reserves respond positively to variations in the balance of payments in the short-run, thus confirming the important role of this variable in both the long- and the short-run. The FJ framework is employed in Ramachandran (2004) and Silva et al (2004) in determining the optimal demand for reserves in India and Brazil, respectively. Using a generalized autoregressive conditional hetroscedasticity (GARCH) model for the volatility of reserves, Ramachandran (2004) finds that the reserve demand in India is predominantly determined by opportunity cost than reserve volatility, a result that is in stark contrast to the overall evidence of the emerging market economies.

Similarly, Silva et al (2004) use an Exponential-GARCH model for reserve volatility. Upon controlling for a structural shift from a fixed to a floating exchange rate in Brazil, they find that reserve volatility has a significantly positive affect on reserve demand, whereas opportunity cost has the expected negative affect. Arguing that the buffer stock model is a good forecast of the optimal reserve holdings, they recommend that the central bank of Brazil use the difference between optimal and real reserve holdings in accessing the need for new money from the IMF. We find this an interesting observation for the FJ model, for in the past it has been the case that Pakistan has borrowed money from the IMF that was often accompanied by strict conditionalities that incurred substantial fiscal and political costs.

We now turn to the next section to discuss the FJ framework in greater detail and identify the equation that will subsequently be used for the empirical analysis.

3. Model of Reserves

FJ hypothesized that changes in reserve holdings $dR(t)$ can be characterized by the following stochastic equation:

$$dR(t) = -\mu dt + \sigma dW(t) \quad \text{, where } R(0) = R_0, \mu \geq 0$$

where W(t) is the standard Wiener process[2], with mean zero and with variance *t*. For the Weiner process, the change in reserves in a small time *dt* is a normal variable with mean $-\mu dt$ and variance $\sigma^2 dt$ and is temporarily independent. At any point in time, the distribution of reserve holdings R (t) is characterized by:

$$R(t) = R_0 - \mu t + \sigma W(t)$$

and

$$R(t) \sim N(R_0 - \mu t, \sigma^2 t)$$

In the above equation, R_0 denotes the initial stock of reserves (assumed to be optimal level) and μ denotes the deterministic part of the instantaneous change in reserves (assumed to have a negative drift). In the special case for which on average the balance of payment is balanced, μ is zero and the stochastic process that governs the changes in reserves is without a drift. Under this special case, FJ show that a second order Taylor approximation of optimal reserves holdings yields the following reserve demand equation:

$$\ln R_0 = \beta + 0.5 * \ln \sigma_t - 0.25 * \ln r_t$$

where R_0 is again is the optimal starting level for reserves after restocking, and β, σ and r are the fixed (or country-specific), adjustment and opportunity costs, respectively. Moreover, under the further assumption that the observed reserves R_t are proportional to optimal reserves R_0 up to an error term u_t, we get the following estimable equation

$$\ln R_t = b_0 + b_1 \ln \sigma_t + b_2 \ln r_t + u_t \quad \ldots\ldots\ldots\ldots (1)$$

The theoretical predictions of the model for the parameters b_1 and b_2 are .5 and .25 respectively. Essentially, equation (1) determines

[2] The Weiner Process (a white noise Brownian Motion) is a continuous time analog of a simple random walk with independent increments. This process is rather general and all diffusion processes (i.e. processes whose paths are continuous with probability 1) can be represented as a functional transformation of the Weiner Process. In the section on empirics, we will discuss if it is appropriate to model the changes in reserves as being stochastic.

the optimal stock of reserves by considering two components of cost; the adjustment cost (σ_t), which is incurred once reserves reach an undesirable lower bound, and the foregone earning on reserve holdings (r_t). The cost of adjustment, which is assumed to depend on the frequency of adjustment, stems from the need to reduce expenditures relative to income so as to yield the desired balance of payments surplus that is necessary for the accumulation of reserves[3]. The adjustment and opportunity costs are interrelated: a higher stock of reserves reduces the probability of having to adjust and thereby reduces the expected cost of adjustment, but this cushion is acquired at the cost of higher foregone earnings. Finally, due to the postulated stochastic process, reserve holdings are random and therefore, it is assumed that the optimal stock is obtained by minimizing the sum of the expected value of both components of cost.

Equipped with the theoretical model, we now turn to carrying an empirical analysis of the optimal demand for reserves in Pakistan during the period June 1995 to June 2005.

4. Empirical Analysis

We estimate equation (1) using monthly data for the period from June 1995 to June 2005, a total of 121 observations. The data on reserves and the money market rate are obtained from the State Bank of Pakistan's Research Department and the *Handbook of Statistics on the Pakistan Economy*, respectively. Following Ramachandran

[3] The exact mechanism through which real expenditures are reduced and the adjustment is effected depends on the circumstances. FJ state that it may take the form of a decline in real income due to a deterioration in the terms of trade, or take the form of a decline in the real value of nominal assets due to a rise in the price level that accompanies the deterioration in the terms of trade. Furthermore, FJ's specification of the cost of adjustment in terms of a reduction of expenditures relative to income implicitly assumes that international borrowing opportunities for reserve built up are limited and costly. Thus, by and large, adjustment requires a current account surplus. It is important to note that in the case of Pakistan that borrowed internationally in the 1990s, our estimate of the above-defined adjustment cost will necessarily be less than .5 or suboptimal.

(2004) and Silva et al (2004), we measure the adjustment cost (σ_t) by the volatility of reserves, the sign on the coefficient of which is expected to be positive. Similarly, we are consistent with the literature on reserves by using the call money rate as a proxy for the opportunity cost of reserves.[4]

In FJ's model, it is assumed that changes in reserves are random. Therefore, as a diagnostic test of the appropriateness of this approach, we check if the monthly changes in the log of reserves displayed any significant correlation. We fail to find any significant correlation, which means that we can model Pakistan's balance of payments with a stochastic model. Furthermore, the Engle (1982) LM test for ARCH effects rejects the null hypothesis of no ARCH effects at the 5% significance level. This makes it appropriate to model the variance of reserves changes through GARCH specifications, as in Ramachandran (2004) and de Silva et al (2004).

To measure the volatility of the change in reserves, one needs to estimate the following regression:

$\Delta R_t = \omega + \upsilon_t \sqrt{\sigma_t^2}$ where υ_t is normally distributed and σ_t^2 is the conditional variance. For modeling the conditional variance, Engle et al (2001) suggest using the Schwartz Information Criterion (SIC) to find the best model in the GARCH (*p*,*q*) class for p ε [1,5] and q ε [1,2]. Following this method, we find that the best model with the lowest SIC in this class is a GARCH (1,2) where the conditional variance is defined as

$$\sigma_t^2 = \delta + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2 + c\sigma_{t-2}^2$$

where $\varepsilon_{t-1} = (R_{t-1} - \omega)$ is the ARCH term and σ_{t-1} and σ_{t-2} are the GARCH terms. If $a=b=c$, then there is no GARCH effect and the

[4] Since, in practice, international reserves are typically held in interest-earning liquid assets, *r* should be viewed as the difference between what could have been earned and what is actually earned. FJ have called this difference the penalty rate that is incurred by investing reserves in liquid short-term assets. As the data on reserve investment is unavailable, the use of the call money rate as a proxy, assumes that variations in the call rate reflect variations in the penalty rate.

variance is a constant, δ. The results for the GARCH (1,2) model are as follows:

$$\sigma_t^2 = .0001_{(.00008)} + .228_{(.0567)}\varepsilon_{t-1}^2 + 1.211_{(.0731)}\sigma_{t-1}^2 - .419_{(.0448)}\sigma_{t-2}^2$$

The parameters of the conditional variance were obtained via maximum likelihood and Bollerslev and Wooldridge (1992) robust standard errors are reported in the parenthesis.[5] All parameters are significant at the 1% level, except the constant term, which is significant at the 10% level. Engle (2001) suggests that examining the correlation in the squared residuals can do a check of whether the volatility model captures all the persistence in the variance of the changes in reserves. On observation, we find that the standardized squared residuals are correlated, i.e. there are still ARCH effects remaining. However, no higher order GARCH specification could be fit on the given data. A check on the stationary properties of our variables reveals that the log of reserves ($\ln R_t$) and the log of the conditional standard deviation of reserves ($ln\sigma_{gt}$) as derived from the GARCH (1,2) model are non-stationary. However, the log of the money market rate ($\ln r_t$) is stationary. A Dickey Fuller test on the first difference of $\ln R_t$ and first difference of $\ln\sigma_{gt}$ rejects the null hypothesis of a unit root with a p-value of 0. Since these two variables are integrated of the same order I(1), we estimate equation 1 and check for the stationary of the residuals

$$\ln R_t = 8.447_{(.685)} + .373_{(.186)}\ln\sigma_{gt} - .586_{(.129)}\ln r_t$$
$$R^2 = 0.65\ ,\ \ F = 106.4$$

The estimates obtained for all variables are significant at the 5% level and Newey-West standard errors (autocorrelation consistent) are reported in parenthesis. Further, the Dickey Fuller test rejects the null hypothesis of a unit root in the residuals at the 5% significance level. Hence, this specification in levels is spurious-free.

---

[5] Bollerslev and Wooldridge (1992) showed that the maximum likelihood estimates of the parameters of the GARCH model assuming normal errors are consistent even if the true distribution of the innovations is not normal. The usual standard errors of the estimates are not consistent when the assumption of normality of errors are violated, so Bollerslev and Wooldridge supply a method for obtaining consistent estimates of these.

As compared with the theoretical predictions of the elasticities of 0.5 on $\ln\sigma_{gt}$ and -0.25 on $\ln r_t$, we obtain 0.373 and -.586, respectively. The signs on the coefficients are as expected, and our results corroborate the findings on reserves in Pakistan by Khan and Ahmad (2005). However, we have strong reservations about the magnitude of these estimates for the reason that there may be a structural break in the reserves data from some point in the year 2000. The banking reforms by SBP, the substantial increase in remittances and other capital inflows since 2000 provide a good reason to believe that there has been a shift in the structural parameters being estimated. To illustrate this point, we estimate the volatility in reserves for the periods before and after June 2000. The choice of this date is arbitrary as it is in the middle of the year and it also divides the dataset into 60 observations for pre-June 2000 period and 61 observations for the post-June 2000 period. As shown in Figure 2, we can see that as compared to the period after June 2000, the previous period exhibited a higher volatility in reserves. GARCH (1,1) was the only model in the GARCH class that could be fit on these two different groups.

The estimated conditional variance for the period before June 2000 is

$$\sigma_t^2 = .00003_{(.00008)} + .734_{(.249)}\varepsilon_{t-1}^2 + .518_{(.0731)}\sigma_{t-1}^2$$

and that for the period after 2000 is

$$\sigma_t^2 = .00009_{(.00007)} + .188_{(.096)}\varepsilon_{t-1}^2 + .774_{(.0458)}\sigma_{t-1}^2$$

Clearly, the parameter estimates are quite different in the two periods, which suggests a structural break in the data[6]. This makes sense in the historical context of Pakistan, as there were more frequent fluctuations before 2000.

To incorporate the effects of this structural break in the data, we re-estimate the long-run relationship of reserves by including a dummy variable that accounts for the structural change after June 2000. The new estimates are as follows:

$$\ln R_t = 7.952_{(.432)} + .270_{(.107)}\ln\sigma_{gt} - .363_{(.109)}\ln r_t + .687_{(.188)}dummy$$

(2) $R^2 = 0.78$, F = 112.1

---

[6] Robust formal tests for structural breaks in GARCH models are still part of active research. Smith (2006) provides a survey of such tests and discusses robustness issues.

All variables are significant at the 5% level and Newey-West standard errors are presented in the parenthesis. By including the dummy variable, the R^2 of the model increases by more than 10%.

The sign on the dummy is positive, implying that the reserve holdings substantially increased after June 2000 due to structural changes in the banking system and exogenous supply factors.

Further, we also examine the statistical properties of the residuals obtained from the estimation in (2). The Jarque-Bera normality statistic is 4.07, which is insignificant. This means that the residuals are normally distributed. An inspection of the squared residuals reveals that the squared errors are correlated, which implies that there are still un-captured ARCH effects.

Figure 2: Volatility of Reserves (Conditional Variance as modeled by GARCH(1,1))

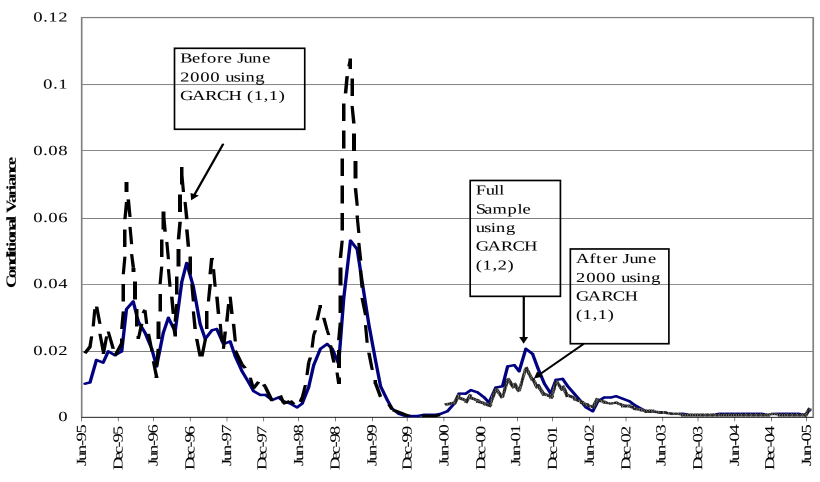
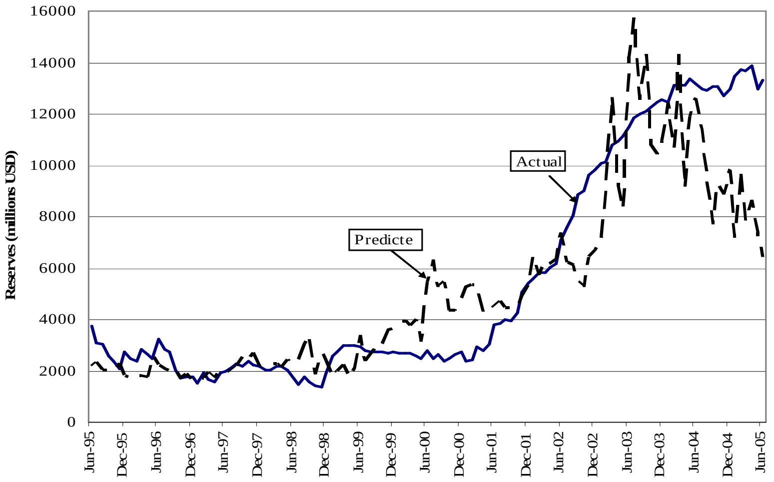


Figure 3: Optimal Reserves from Eq. (2) vs. Actual Reserves during 1995 - 2005



Ramachandran (2004) found a similar problem with the Indian data. Finally, the Dickey Fuller test rejects the null hypothesis of a unit root at the 5% significance level. In comparison with Ramachandran (2004) and Silva et al, our estimate of the adjustment cost ($\ln\sigma_{gt}$) of .27 is very similar to the case of India (.28) and Brazil (.14). The estimate of -.36 on the opportunity cost is smaller than found for India (-.70) and bigger than that for Brazil (-.19). This suggests that, in the case of Pakistan, after accounting for exogenous structural changes, the reserve built-up as determined by reserve volatility showed similar pattern as in India and Brazil. This provides evidence to the extent to which precautionary demand for reserves have been in importance in these countries.

Figure 3 shows the behavior of predicted values from equation (2) and the actual reserves. During June 1995 to June 2000, SBP held reserves that were on average lower than optimal by about $16 million whereas it has been in excess of optimal by about $780 in the period since July 2000. However, if we make a comparison for the period June 2004 through to June 2005, we find that SBP has been holding about $ 4.3 billion in excess of optimal reserves. This suggests that recent reserve accumulation does not reflect the optimal behavior as predicted by the buffer stock model.

5. Conclusion

In line with the findings of country-studies on India and Brazil, we find that opportunity cost played a greater role than reserve volatility in determining the level of reserves in Pakistan, but not to the predominant effect as in India. This result is in contrast with the hypothesis of increased capital mobility that is commonly set forth in explaining the precautionary motive for reserve holdings. As also pointed by Ramachandran (2004), this could be perhaps attributed to the fact that capital outflow in Pakistan as in India, is not as free as capital inflow and a large part of the recent reserve accumulation is due to non-debt reserve inflows.

References

Aizenman, Joshua and Nancy Marion (2002), “The High Demand for International Reserves in the Far East: What’s going on?” NBER Working Paper No. 9266, *National Bureau of Economic Research.*

Badinger, Harald (2004), “Austria’s Demand for International Reserves and Monetary Disequilibrium: The Case of a Small Open Economy with a Fixed Exchange Rate Regime”, *Economica*, 71: pp. 39-55

Bollerslev, Tim and Jeffrey M. Wooldridge (1992), “Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances.” *Econometric Reviews*, 11(2): pp. 143-172.

Edison, Hali (2003), “Are Foreign Exchange Reserves in Asia Too High?” *World Economic Outlook*, September: pp. 78-92

Elbadawi, I.A. (1990), “The Sudan Demand for International Reserve: A Case of a Labor Exporting Country”, *Economica*, 57: pp 73-89

Engle, Robert (1982), “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation.” *Econometrica*, 50(4): pp. 987-1007

Engle, Robert (2001), “GARCH 101: The Use of ARCH/GARCH models in Applied Econometrics”, *The Journal of Economic Perspectives*, 15(4): pp. 157-168

Engle, Robert and Andrew J. Patton (2001), "What Good is a Volatility Model?", *Quantitative Finance*, 1(2): pp 237-245

Flood, Robert and Nancy Marion (2002), "Holding Reserves in an Era of High Capital Mobility", IMF Working Paper No. 62, *International Monetary Fund.*

Ford, J.L and Guobo Huang (1993), "The Demand for International Reserves in China: An ECM Model with Domestic Monetary Disequilibrium", *Economica*, 67: pp. 379-397

Frenkel, J. A. and B. Jovanovic (1981), "Optimal International Reserves: A Stochastic Framework", *The Economic Journal*, 91: pp. 507-514.

Grubel, Herbert (1971), "The Demand for International Reserves: A Critical Review of Literature", *Journal of Economic Literature*, 9(4): pp. 1148-1166

Heller, H. Robert (1966), "Optimal International Reserves", *The Economic Journal*, 76: pp. 296-311.

Kelly, Michael (1970), "The Demand for International Reserves", *American Economic Review*, 55(4): pp. 655-667.

Khan, Karim and Eatzaz Ahmad (2005), "The Demand for International Reserves: A Case Study of Pakistan", *Proceedings of the 21st Annual General Meeting & Conference*, Pakistan Institute of Development Economics.

Ramachandran, M. (2004), "The Optimal Level of International Reserves: Evidence for India", *Economics Letters*, 83: pp. 365-370.

Silva, Antonio F. and Erica D. da Silva (2004), "Optimal International Reserves Holdings in Emerging Markets Economies: The Brazilian Case", Proceedings of the 32th Brazilian Economics Meeting, No 078, *ANPEC-Brazilian Association of Graduate Programs in Economics*.

State Bank of Pakistan (2005), *Handbook of Statistics on the Pakistan Economy*.