

## INTERVENTION OF JAPANESE MONETARY AUTHORITY IN THE FOREIGN EXCHANGE MARKET

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### Abstract

The Japanese monetary authority intervenes into the foreign exchange market on and off. The influence of the operations on the fluctuation of the yen exchange rate to U.S. dollar is examined in the period of September 17 through 28, 2001. A cyclic behavior is found in correlation functions of the series. It is shown that the cycle of the fluctuation is elongated after the intervention. A model that represents these properties of correlation functions is proposed. The Fokker-Planck equation of this model is solved. The expectation value derived from the solution is cyclical and the variance is constant. Those results are different from what the Ornstein-Uhlenbeck process has.

JEL classification: C51

Key words: Intervention, Fokker-Planck equation, Langevin equation, exchange rate fluctuation

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### 1.- Introduction

The present author, Obara (2004), investigated the structural changes in the foreign exchange market of Japanese yen against U. S. dollar and euro in the data during 2001 and 2004. It was shown that the time when the structural change happened could be determined by the magnitude of the deviation of the probability density functions from the normal distribution.

There may be several reasons why the data structure changes. One of them, that must be most important, is the intervention of the monetary authority. If we analyse such data as the intervention, we can refine the theory of the structural change in the foreign exchange market.

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The Ministry of Finance, Japan, intervened in the foreign exchange market five times in the period of 2001 through 2004. They are classified as:

September 17 through 28, 2001; May 22 through June 28, 2002  
January 15 through 29, 2003; February 24 through March 10, 2003  
May 8 through December 31, 2003.

These data are given in the web site of the Ministry of Finance, Japan.<sup>1</sup> The results of the interventions have their own characteristic features and it seems difficult to summarize all of them in a unified way. If the purpose of the authority is to stop the appreciation of yen, the first and third interventions were successful judging from the results that the trend turned into the phase of depreciation of yen. The second intervention was operated in the midst of the trend of the yen appreciation, but the operation did not stop the trend. The characteristic of the fourth intervention was buying of euro. The attempt was unsuccessful. In the fifth term, the authority intervened on and off for a long period without any results. The intention of them was not clear. Then it will be interesting to investigate the first intervention and to know what happened in the data structure during the intervention.

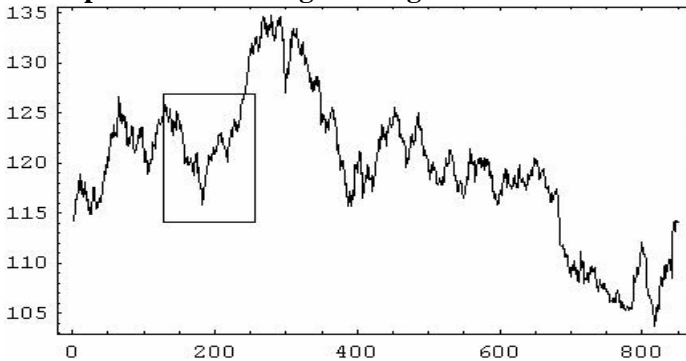
Graph 1 shows a pattern of the daily series of yen exchange rate against U. S. dollar from January 2, 2001 through May 18, 2004.<sup>2</sup> It is composed of 850 samples. Graph 2 is zooming in the rectangular section of Graph 1. The series is from August 1, 2001 through October 31, 2001. The rectangular section of Graph 2 is the first ten-day's intervention. As mentioned above, the purpose of the previous paper was to find the structural change of the yen exchange rate. The method was a modified moving average. In this paper, we will also focus on the structural change of the data. The characteristics of the time series appear in the behaviours of the auto-correlation functions of the series.

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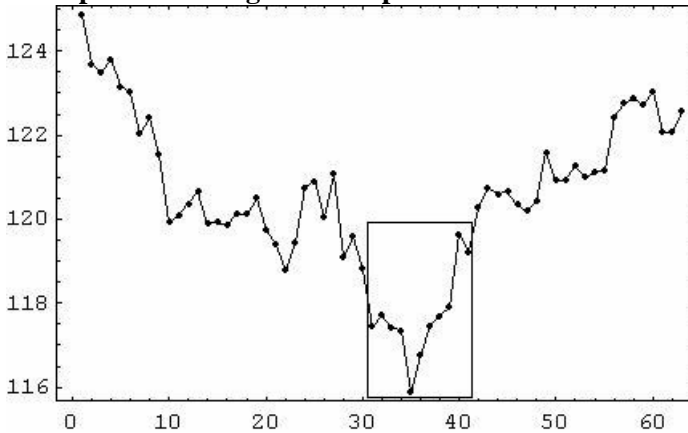
<sup>1</sup> <http://www.mof.go.jp/feio/monthly>.

<sup>2</sup> Source: Pacific Exchange Rate Service, <http://pacific.commerce.ubc.ca/xr>.

**Graph 1. Yen exchange rate against U. S. dollar**



**Graph 2. Zooming in of Graph 1**



The operation at September 21, 2001 was selling yen to buy  $1.08 \times 10^{10}$  U. S. dollar. This amount is huge: 10% over the average turn-over of September,  $0.93 \times 10^{10}$  U. S. dollar.<sup>3</sup>

A new method will be introduced to analyze the auto-correlation functions. Usually the correlation function is considered to be zero, if the fluctuation of it is within a certain confidence level. But we think that there should be meaningful information in the fluctuation thrown away statistically.

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<sup>3</sup> Source: The Bank of Japan, <http://www.boj.or.jp/stat/bis/deri0404.html>

It is shown in Section 2 that the correlation functions have cyclic behavior that was discarded in many cases. The cycle of the fluctuation is different before and after the intervention.

In Section 3, we postulate that the data generating system is controlled by a Langevin equation. In Section 4, coefficients of the Langevin equation are determined by nonlinear regressions. We know that the cycle of the oscillation is elongated by the intervention. The process generated by the Langevin equation is a Markov process. Then it satisfies a Chapman-Kolmogorov equation. This equation is turned into a Fokker-Planck equation, if an equation of motion (in our case the Langevin equation) is given.<sup>4</sup> In Section 5, the Fokker-Planck equation is solved in the periodic boundary condition. The solution, that is the probability density function, has different features from the ordinary random walk and the equation of Ornstein-Uhlenbeck (1954). The distribution has harmonically oscillating expectation value and a constant variance. The final section is the conclusions.

## 2. Sample correlation functions

Let the exchange rate be  $z(t)$ ,  $t=1,2,\dots,N$ . A random process usually has a trend, so it is convenient to take a difference not to be bothered with the trend. We put  $x(t)=z(t+1) - z(t)$

The behavior of a stochastic process will be examined by a correlation function. Our sample correlation function is

$$\rho(k) = \frac{\sum_{t=1}^{N-k} x(t+k)x(t)}{\sum_{t=1}^N x(t)^2} \quad (1)$$

where the average of  $x(t)$  should be zero. The parameter  $k$  is called a lag and takes a positive integer up to 20.

We will divide the 63 whole data from August 1 through October 31, 2001 into 3 periods:

- $1 \leq t \leq 31$  before intervention;
- $32 \leq t \leq 42$  intervention
- $43 \leq t \leq 63$  after intervention.

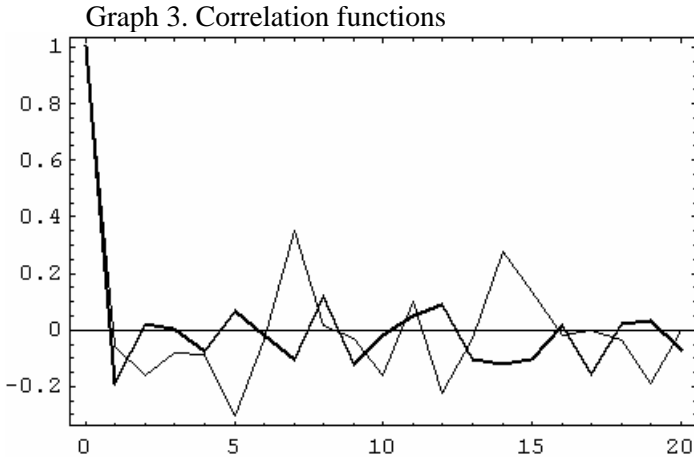
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<sup>4</sup> See for instance Gardiner (2003).

Corresponding to each period, we have correlation functions:

$$\rho(k) = \rho_1(k) \quad \text{for } 1 \leq t \leq 31, \quad \rho(k) = \rho_2(k) \quad \text{for } 43 \leq t \leq 63.$$

Since the number of data during the intervention is small, we do not consider the correlation function for this period. The plots of those correlation functions are given in Graph 3.



The thick line is  $\rho_1$ , and the thin line is  $\rho_2$ . Both correlation functions are usually said “statistically zero,” because the 95% confidence level is given by  $\pm 2/\sqrt{N} \approx 0.25$ . If we take a close look at them, they are oscillating and their cycles look different. In the next section, we propose a model which represents a cyclical oscillation of the correlation function.

### 3. Langevin equation and correlation function

Suppose that  $x(t)$  should satisfy the following Langevin equation

$$\frac{d}{dt}x(t) = -\lambda x(t) + f(t) + \varepsilon(t) \quad (2)$$

where  $\lambda$  is a positive constant and it means a damping factor. The function  $f(t)$  represents an external force imposed from the outside of this system. We suppose

$$f(t) = a \sin \varphi t \tag{3}$$

where  $a$  and  $\varphi$  are positive constants. The function  $\varepsilon(t)$  is a random force, in other words, white noise whose properties are

$$\langle \varepsilon(t) \rangle = 0 \tag{4}$$

$$\langle \varepsilon(t) \varepsilon(t') \rangle = c \delta(t - t')$$

where  $\langle \dots \rangle$  means an expectation value and  $c$  is a positive constant. When we want to solve the equation (2), it is convenient to use the Fourier transform

$$x(t) = \int_{-\infty}^{\infty} d\omega e^{i\omega t} x[\omega]. \tag{5}$$

The inverse transform of this is

$$x[\omega] = \int_{-\infty}^{\infty} dt e^{-i\omega t} x(t). \tag{6}$$

We introduce a truncated Fourier integral whose meaning will be revealed later

$$x_T[\omega] = \int_{-T}^T dt e^{-i\omega t} x(t). \tag{7}$$

Next we define a power spectrum of  $x_T[\omega]$

$$I_x[\omega] = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle |x_T[\omega]|^2 \rangle. \tag{8}$$

It should be noted that the limit should be taken after calculating the expectation value in (8). The expression (6) which is the infinite integral cannot be substituted into the right hand side of (8).

We introduce a new correlation function

$$\phi(\tau) = \langle x(t + \tau)x(t) \rangle \tag{9}$$

which is different from the sample correlation function (1) by the normalization. The relationship between (8) and (9) is expressed by a formula

$$\phi(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{i\omega\tau} I_x[\omega]. \tag{10}$$

This relation is known as the Wiener-Khinchin theorem.<sup>5</sup>

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<sup>5</sup> See also Gardiner (2003).

In order to get the correlation function (9), we will calculate  $x_T[\omega]$  and then  $I_x[\omega]$ . Multiplying (2) by  $e^{-i\omega t}$  and integrating it from  $-T$  to  $T$ , we have the left hand side of (2)

$$lhs = \int_{-T}^T dt e^{-i\omega t} \frac{d}{dt} x(t) = i\omega x_T[\omega] + K[\omega, T] \quad (11)$$

where

$$K[\omega, T] = e^{-i\omega t} x(t) \Big|_{-T}^T.$$

We postulate that there is a positive constant  $k$  such that

$$\lim_{T \rightarrow \infty} |K[\omega, T]| < k. \quad (12)$$

On the other hand, the right hand side of (2) is

$$rhs = -\lambda x_T[\omega] + f_T[\omega] + \varepsilon_T[\omega] \quad (13)$$

$$\text{where } f_T[\omega] = \int_{-T}^T dt e^{-i\omega t} a \cos \theta, \quad \varepsilon_T[\omega] = \int_{-T}^T dt e^{-i\omega t} \varepsilon(t). \quad (14)$$

From (11) and (13), we have

$$x_T[\omega] = \frac{1}{i\omega + \lambda} \{f_T[\omega] + \varepsilon_T[\omega] - K[\omega, T]\}. \quad (15)$$

From the first expression of (4), we have

$$\langle |x_T[\omega]|^2 \rangle = \left| \frac{1}{i\omega + \lambda} \right|^2 \left\{ |f_T[\omega]|^2 + \langle |\varepsilon_T[\omega]|^2 \rangle + R[\omega, t] \right\} \quad (16)$$

$$R[\omega, T] = K^2[\omega, T] - 2K[\omega, T]f_T[\omega].$$

The first term of (15) is easily calculated from (14)

$$f_T[\omega] = a \left\{ \frac{\sin(\omega + \varphi)T}{\omega + \varphi} + \frac{\sin(\omega - \varphi)T}{\omega - \varphi} \right\}. \quad (17)$$

The power spectrum of  $f(t)$  is given by, corresponding to (8),

$$I_f[\omega] = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle f_T^2[\omega] \rangle$$

$$= \lim_{T \rightarrow \infty} \frac{a^2}{2} \left\{ T \left( \frac{\sin(\omega + \varphi)T}{\omega + \varphi} \right)^2 + \frac{1}{T} \left( \frac{\sin(\omega + \varphi)T}{\omega + \varphi} \right) \left( \frac{\sin(\omega - \varphi)T}{\omega - \varphi} \right) + T \left( \frac{\sin(\omega - \varphi)T}{\omega - \varphi} \right)^2 \right\}. \quad (18)$$

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There are two useful expressions of the Dirac  $\delta$  function

$$\lim_{T \rightarrow \infty} \frac{1}{\pi} \frac{\sin \alpha T}{\alpha} = \delta(\alpha), \quad \lim_{T \rightarrow \infty} \frac{T}{\pi} \left( \frac{\sin \alpha T}{\alpha T} \right)^2 = \delta(\alpha). \quad (19)$$

It is easy to show that these two expressions satisfy the definition of the  $\delta$  function:

1.  $\delta(\alpha) = 0$  for  $\alpha \neq 0$ ,  $\delta(\alpha) = \infty$  for  $\alpha = 0$
2.  $\int_{-\infty}^{\infty} \delta(\alpha) d\alpha = 1$ .

Using the second expression of (19), we know that the first and third terms in the curl brackets of (18) become  $\pi\delta(\omega + \varphi)$  and  $\pi\delta(\omega - \varphi)$ , respectively. Let the second term be

$$A = \lim_{T \rightarrow \infty} \frac{1}{T} \left( \frac{\sin(\omega + \varphi)T}{\omega + \varphi} \right) \left( \frac{\sin(\omega - \varphi)T}{\omega - \varphi} \right) \quad (20)$$

If neither  $\omega + \varphi$  nor  $\omega - \varphi$  is zero,  $A = 0$ . We consider the case that  $\omega + \varphi$  and  $\omega - \varphi$  are zero. Since the terms  $\omega + \varphi$  and  $\omega - \varphi$  do not go to zero simultaneously, we put  $\omega + \varphi = 0$ .

Since each term has finite limit when  $T \rightarrow \infty$ , we have, using the first expression of (19),

$$A = \lim_{T \rightarrow \infty} \frac{1}{T} \left( \frac{\sin(\omega - \varphi)T}{\omega - \varphi} \right) \pi\delta(\omega + \varphi) = 0.$$

We know  $A = 0$  for any choices of  $\omega + \varphi$  and  $\omega - \varphi$ . This leads us to

$$I_f[\omega] = \frac{\pi a^2}{2} \{ \delta(\omega + \varphi) + \delta(\omega - \varphi) \}. \quad (21)$$

Next we calculate  $\langle |\varepsilon_T[\omega]|^2 \rangle$  and  $I_\varepsilon[\omega]$ . From (14), we have

$$\langle |\varepsilon_T[\omega]|^2 \rangle = \int_{-T}^T dt \int_{-T}^T dt' e^{-i\omega(t-t')} \langle \varepsilon(t)\varepsilon(t') \rangle. \quad (22)$$

To perform the double integral, the following transformation is useful:  $\tau = t - t'$  and  $\eta = t'$ . Therefore the integral is transformed to



$$\int_{-T}^T dt \int_{-T}^T dt' \rightarrow \int_{-T}^T d\eta \int_{-T-\eta}^{T-\eta} d\tau.$$

From (4), (22) becomes

$$\langle |\varepsilon_T[\omega]|^2 \rangle = c \int_{-T}^T d\eta \int_{-T-\eta}^{T-\eta} d\tau e^{-i\omega\tau} \delta(\tau) = 2cT. \quad (23)$$

Then we have

$$I_\varepsilon[\omega] = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle |\varepsilon_T[\omega]|^2 \rangle = c. \quad (24)$$

The third term  $R[\omega, T]$  in (16) does not contribute to (8) because of (12). Then the right hand side of (8) can be calculated by (16), (18), (21), (23), and (24) as

$$I_x[\omega] = \left| \frac{1}{i\omega + \lambda} \right|^2 \left[ \frac{\pi a^2}{2} \{ \delta(\omega + \varphi) + \delta(\omega - \varphi) \} + c \right]. \quad (25)$$

Substitute (25) into (10), we have two kinds of integration. The first one is the integrals of delta functions, and the result is

$$\phi_1 = \frac{a^2}{2} \frac{\cos \varphi \tau}{\lambda^2 + \varphi^2}. \quad (26)$$

The second integral concerning  $|1/(i\omega + \lambda)|^2 c$  is

$$\phi_2 = \frac{c}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega\tau} \frac{1}{2i\lambda} \left( \frac{1}{\omega - i\lambda} - \frac{1}{\omega + i\lambda} \right). \quad (27)$$

Since we postulate  $\tau > 0$ , the contour in the complex  $\omega$  plane should be taken in the region  $\text{Im } \omega > 0$ . Then the second pole in (27) does not contribute, and (27) becomes simply

$$\phi_2 = \frac{c}{2\lambda} e^{-\lambda\tau}.$$

Finally we have the correlation function (10) as

$$\phi(\tau) = \phi_1 + \phi_2 = \frac{a^2}{2} \frac{\cos \varphi \tau}{\lambda^2 + \varphi^2} + \frac{c}{2\lambda} e^{-\lambda\tau}. \quad (28)$$

In the next section, we will compare the formal correlation function (28) with the sample correlation function (1).

#### 4. Regression analysis

In the expression (28), there are parameters;  $a, c, \lambda$ , and  $\varphi$ . We will estimate their value by the regression analysis. The parameter  $a$  determines the magnitude of the external force. Then it should be considered as an exogenous parameter. Another parameter  $c$  is the magnitude of the white noise that cannot be determined by this model, then it is also considered as a given constant in this paper. The sample correlation function (1) is normalized by  $\sum_{t=1}^N x(t)^2$ .

We will normalize the expression (28) as

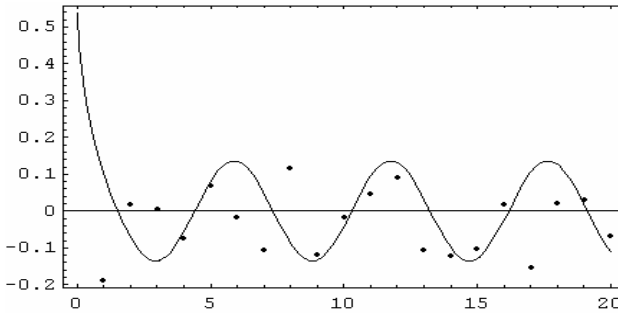
$$\phi(\tau) = \frac{\cos \varphi \tau}{\lambda^2 + \varphi^2} + \frac{e^{-\lambda \tau}}{\lambda}. \tag{29}$$

This function is equated with the sample correlation function (1) and regarded as the regressand. The parameters  $\lambda$  and  $\varphi$  are the regressors. The results of the nonlinear regressions are given in Table 1. In this table,  $T$  is the cycle of the oscillation:  $T = 2\pi / \varphi$ . If we put the results of Table 1 into (29), we can depict fitting curves of  $\rho_1$  and  $\rho_2$ , which are given in Graph 4 and 5. The numbers of sample for  $\rho_1$  and  $\rho_2$  are just twenty. This is not enough to get an accurate estimation and fitting.

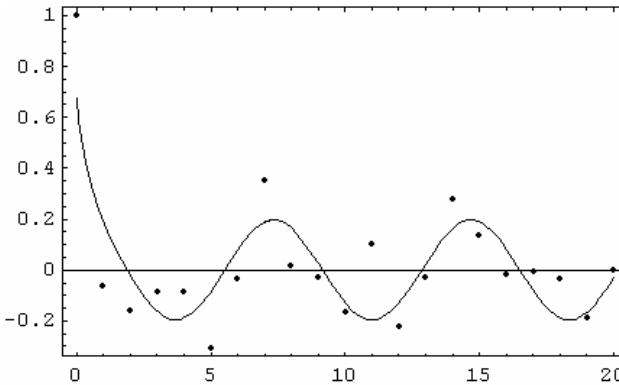
Table 1. Nonlinear regressions

	$\lambda$	$\varphi$	$T$
$\rho_1(k)$	2.493	1.068	5.88
$\rho_2(k)$	2.085	0.856	7.34

Graph 4: Fitting of  $\rho_1(k)$



Graph 5: Fitting of  $\rho_2(k)$



From the table and Graph 4 and 5, we can conclude that after the intervention the cycles of the oscillation of the correlation functions become longer. And the damping factors do not change much before and after the intervention.

## 5. Fokker-Planck equation

The Langevin equation (2) determines the behavior of  $x(t)$ . Above all the random force  $\varepsilon(t)$  is an essential origin of the stochastic behavior of  $x(t)$ . This process is said the Markov process. When a process is the Markov process, it has the Chapman-Kolmogorov equation for the transition probability function  $p(t, x)$ .

If the process satisfies a specific equation, in our case the Langevin equation (2), the Chapman-Kolmogorov equation is turned into the Fokker-Planck equation. The Fokker-Planck equation derived from (2) is

$$\frac{\partial}{\partial t} p(t, x) = c \frac{\partial^2}{\partial x^2} p(t, x) + (\lambda x - a \cos \varphi t) \frac{\partial}{\partial x} p(t, x) + \lambda p(t, x). \quad (30)$$

This equation is different from that of the ordinary Brownian motion or Ornstein-Uhlenbeck process.

For the O-U process, in which  $a = 0$ , we can get a solution under an initial condition:  $p(0, x) = \delta(x)$ . (31)

It is well-known that the solution is the normal distribution whose expectation value and the variance are proportional to  $e^{-\lambda t}$  and  $1 - e^{-2\lambda t}$ .<sup>6</sup>

In our case, the initial condition (31) is not suitable, and we will see later that a periodic boundary condition will be reasonable. A common technique to solve a differential equation is to use the Fourier transformation

$$p(t, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{i\omega x} p[t, \omega]. \quad (32)$$

Substituting this expression into (30), we have

$$\frac{\partial}{\partial t} p[t, \omega] = -c\omega^2 p[t, \omega] - \lambda\omega \frac{\partial}{\partial \omega} p[t, \omega] - i\omega a \cos \varphi t p[t, \omega]. \quad (33)$$

This is the first order partial differential equation, then this is equivalent to the following subsidiary equation

$$\frac{dt}{1} = \frac{d\omega}{\lambda\omega} = - \frac{dp}{(c\omega + i\omega a \cos \varphi t)p}.$$

(34)

This equation can be solved as

$$p[t, \omega] = \exp[-\alpha\omega^2 - i\beta(t)\omega] \gamma(\omega e^{\lambda t}) \quad (35)$$

$$\alpha = \frac{c}{2\lambda}, \quad \beta(t) = \frac{a}{\sqrt{\lambda^2 + \varphi^2}} \sin(\varphi t + \theta), \quad \theta = \tan^{-1} \frac{\lambda}{\varphi}$$

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<sup>6</sup> See for example Bellac (2001).

where  $\gamma$  is an arbitrary function of  $\omega e^{-\lambda t}$ . In order to decide  $\gamma$ , it is convenient to put a boundary condition

$$p[t, \omega] = p[t + nT, \omega], \quad n = \pm 1, \pm 2, \dots, \quad T = 2\pi / \varphi. \quad (36)$$

From this, we have

$$\gamma(\omega e^{-n\lambda T}) = \gamma(\omega). \quad (37)$$

If  $\gamma$  is differentiable, the only solution which satisfies (37) is  $\gamma = \text{const}$ . Next we will decide this constant. Substituting (35) with  $\gamma = \text{const}$  into (38), we have

$$p(t, x) = \frac{1}{2\sqrt{\pi\alpha}} \exp\left[-\frac{(x - \beta(t))^2}{4\alpha}\right] \gamma. \quad (38)$$

Since probability function should satisfy

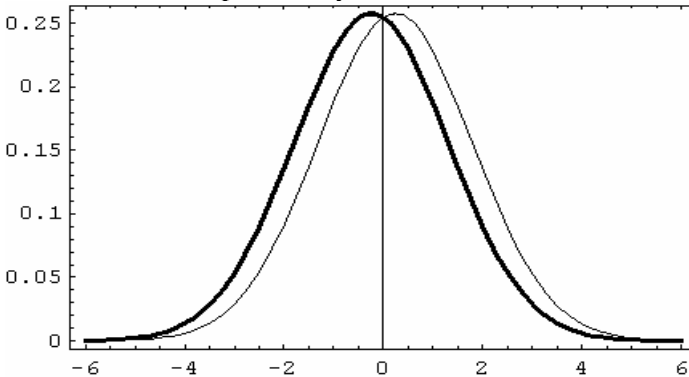
$$\int_{-\infty}^{\infty} p(t, x) dx = 1 \quad \text{we know that } \gamma = 1.$$

It is straightforward to calculate the expectation value and the variance:

$$E(x) = \beta(t), \quad V(x) = 2\alpha.$$

These results are different from what is derived from the O-U equation as mentioned above. The plot of  $p(t, x)$  is given in Graph 6.

Graph 6: Oscillation of probability function



The thin curve depicts  $p(0, x)$  and the thick curve  $p(T/2, x)$ . The shape of the probability function does not alter but it swings periodically along the horizontal axis in  $1 \leq t \leq T$ . The problem of the uniqueness of the solution (38) will be discussed somewhere.

## **6. Conclusions**

The Japanese monetary authority intervenes in the foreign exchange market by selling yen and buying U. S. dollar. In the period of September 17 through 28, 2001, the operation was successful: the appreciation of yen stopped and the trend turned into depreciation of yen against U. S. dollar. We investigated the effect of the intervention on the correlation function. It was found that after the intervention, the cycle of the oscillation of the correlation function became longer than before.

We proposed a Langevin equation that represented the cyclic oscillation of the correlation function.

The Fokker-Planck equation associated with our Langevin equation was solved. The property of the solution was different from that of the ordinary Ornstein-Uhlenbeck equation; the expectation value of the data oscillated harmonically and the variance was constant.

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