IGARCH effect on autoregressive lag length selection and causality tests

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Using Monte Carlo experiments, we show how information criteria determine, in the presence of GARCH errors, an optimal lag length in univariate time series and causality tests. We illustrate the simulations by testing the presence of serial correlation in exchange rates as well as Granger-causality between interest rates.

I. INTRODUCTION AND MOTIVATIONS

In empirical studies, it is usual to determine a parsimonious representation for the dynamics of economic time series by using information criteria.[1] We can use them to test for serial correlation or in the identification phase of ARMA models. Information criteria can also be used to specify a suitable number of lags in the augmented Dickey–Fuller unit root tests and Engle–Granger cointegration tests or in the first step of Johansen maximum likelihood procedure. In Granger causality tests, they can be recommended (Hsiao, 1979) for avoiding arbitary lags and spurious causality (or spurious absence of causality).

However, financial and (some) macroeconomic data, especially high frequency data often have properties which could affect estimation, inference and forecasting: autocorrelation, non-normality, non-stationarity, non-linearity, heteroscedasticity and often an autoregressive form in the conditional variance (ARCH).

ARCH models, introduced by Engle (1982), allow us to take into account time series with varying volatility where both large and low movements are clustered. Generalizing the ARCH process, Bollerslev (1986) introduced a moving average part in the conditional variance (GARCH). Engle and Bollerslev (1986) studied persistence in volatility, i.e. situations in which the sum of volatility and moving average parameters equals one (IGARCH). This last case often occurs with high-frequency data. In this paper we use the term GARCH generically for ARCH, GARCH or IGARCH processes. Many extensions for those models[2] can be found in the literature (EGARCH, ARCH-M, TARCH), but we focus particularly on GARCH (1,1) models as in the following AR(1)-GARCH(1,1):

$$y_t = \mu + \rho y_{t-1} + u_t \tag{1}$$

$$E(U_t|\Phi_{t-1}) = 0 \tag{2}$$

$$V(u_t|\Phi_{t-1}) = h_t = \alpha_0 + \alpha_1 u^2_{t-1} + \beta_1 h_{t-1}, \tag{3}$$

where Φ_{t-1} is the information set available at time t-1, we call α_1 the volatility parameter and β_1 the moving average parameter.

As the conditional mean of the error process u_t equals zero (and thus u_t is a weak white noise), serial correlation should not remain if we estimate an AR(1) model for y_t in Equation 1. Indeed, because $\Phi_{t-k} \subset \Phi_{t-1}$ for $k>0$, we know that:

$$E(u_t|\Phi_{t-k})] = 0$$

and

$$\begin{aligned} E(u_t u_{t-k}) &= E[E(u_t u_{t-k}|\Phi_{t-1})] \\ &= E[u_{t-k}E(u_t|\Phi_{t-1})] \\ &= 0 \end{aligned}$$

However, ARCH errors can induce a kind of spurious serial correlation (Diebold 1986):[3] Box–Pierce or Box–Ljung tests are too liberal and reject too often, at the nominal level, the null of no autocorrelation when they are computed in the usual way.

[1] See for example Judge *et al.* (1985) for a textbook survey.
[2] See for instance Bollerslev *et al.* (1992), Bera and Higgins (1993) for recent survey.
[3] See also Gourieroux (1992).

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Therefore we could be tempted to use information criteria as they are not based on a specific significance level.

Using some small Monte Carlo experiments, we show how the most commonly used information criteria behave when a non-constant conditional variance is introduced. The term *behaviour* should be understood (as in Lütkepohl, 1992) in the sense of maximizing the frequency of finding the lag length of the true model we chose as DGP. The criteria analysed are among the most popular ones in empirical studies: the Schwartz Baysian criterion (SBC), the Hannan Quinn criterion (HQC) and the final prediciton error (FPE). Note that only the first two are strongly consistent (they determine the true model asymptotically), while the FPE (like the AIC) overestimates the true model. We extend the analysis and we test the null hypothesis of non-causality between two stationary and independent processes. We then illustrate the simulation results by analysing the hypothesis of market efficiency in exchange rates as well as asymmetry among EMS-related countries using interest rate data.

II. THE DATA GENERATING PROCESSES (DGP)

In the univariate case, the DGP is a mean stationary autoregressive process of order one with GARCH(1,1) errors, generated using the following equations:

$$y_t = 0.03 + 0.6y_{t-1} + u_t \quad (4)$$

$$u_t = \varepsilon_t(h_t)^{1/2} \quad (5)$$

$$h_t = \alpha_0 + \alpha_1 u^2_{t-1} + \beta_1 h_{t-1} \quad (6)$$

where α_0 depends on the measure unit of y_t, and can be arbitrarily chosen[4] (0.01 for instance) because y_t is generated recursively as a function of the constant. The parameters α_1 (the volatility parameter of the GARCH) and β_1 (the MA one) take the following pairs, including absence of ARCH, a simple ARCH and five IGARCH processes. [0.0], [0.5,0], [0.5,0.50, [0.75,0.25], [0.9,0.1], [0.1,0.9], [0.25,0.75]. Note that the last two pairs give the more realistic empirical models. We have chosen two types of conditional distributions ε_t: a normal one (ε_t~$N(0,1)$ and a more leptokurtic one (ε_t~$t(5)$ i.e. a standardized Student's t with five degrees of freedom). Indeed, an excess of kurtosis is often observed in financial data, a phenomenon which cannot be fully explained by the conditional normal behaviour.[5]

We also test for causality in a system of two stationary and independent variables, y_t and x_t. y_t is generated according to equation 4, and $x_t = 0.5 + 0.3\,x_{t-1} + v_t$. v_t follows a GARCH process. We test for GRANGER non-causality from x_t to y_t (with $\text{cov}(u_t, v_t) = 0$) when both variables follow a GARCH process.[6] We assume that the magnitude of the parameters of the GARCH are the same for x_t and y_t and that there is no common persistence in conditional variances.

III. SIMULATION RESULTS

All the simulations were carried out by using the routine RAND in GAUSS 3.14 with 3000 replications. The first 50 observations were dropped to avoid initial–value problems.

Univariate results

Tables 1 and 2 give for sample sizes of 100 and 500, the frequencies of determining the lag length[7] from 0 to 10 with the three information criteria and the different specifications of GARCH errors.

We would stress the following points:

1. In all situations (GARCH or not), we obtain the well-known finite sample inequality for the frequencies of finding the true model: SBC> HQC> FPE. Even when T increases to 500, the inequality remains.
2. GARCH errors affect the frequency of finding the true model, as they tend to choose a less parsimonious model. The power of the augmented Dickey–Fuller test (as well as Johansen tests for cointegration)[8] and the interpretation of some economic univariate time series results[9] will be strongly affected by adding spurious significant lags. In the same way. Cochrane–Orcutt procedures are not recommended either. These results underline the asymmetric effects of α_1 and β_1: distortions grow, especially with the volatility parameter α_1, but less with the MA part[10] Thus the effect is negligible if empirical results finding a small value for the volatility parameter and a large value for the MA part of the GARCH are confirmed.

[4] Obviously, α_0 has to be strictly greater than zero.
[5] Remember that, even with a normal conditional distribution, GARCH already produces a leptokurtic marginal one.
[6] This case is the less favourable one. Non-causality tests with GARCH erros on one of the two variables only can be found in Hecq (1993).
[7] We are working in terms of non-rejecting the true null hypothesis (1-type 1 error) instead of rejecting the null (type 1 error). Only frequencies from 0 to 5 and more are listred to save place.
[8] See also Boswijk and Franses (1992).
[9] For instance, the hypothesis of market efficiency, which assumes unit root in the price of an asset and no serial correlation in the growth rate (random walk hypothesis).
[10] However, β_1 amplifies a high volatility parameter as can be seen when comparing GARCH(0.5,0) with GARCH(0.5.0.5) results.

Table 1. *Frequencies of finding an AR(1) process: normal conditional distribution*

		T = 100						T = 500					
[α1,β1]	Crit.	0 lag	**1 lag**	2 lags	3 lags	4 lags	≥5	0 lag	**1 lag**	2 lags	3 lags	4 lags	≥5
[0,0]	SBC	0.03	**95.87**	3.3	0.6	0.2	0.03	0	98.83	1.06	0.1	0	0
	HQC	0.03	**86.97**	8.1	2.76	0.76	1.38	0	92.76	5.23	1.43	0.4	0.18
	FPE	0	**69.43**	12.6	6.16	3.33	8.48	0	70.36	11.7	6.56	3.06	8.32
[0.5,0]	SBC	0.33	**88.53**	9.3	1.53	0.2	0.1	0	91.13	7.26	1.4	0.2	0
	HQC	0.26	**77.46**	15.03	4.56	1.43	1.52	0	78.7	14.43	4.86	1.43	0.58
	FPE	0.1	**60.13**	18.4	8.87	4.53	7.97	0	53.26	20.33	10.5	6.03	9.88
[0.5,0.5]	SBC	0.96	**75.26**	13.03	5.06	3.06	2.63	0	58.2	13.53	7.83	5.86	14.58
	HQC	0.46	**58.8**	16.56	8.06	6.43	9.69	0	35.23	13.86	9.5	8.93	32.48
	FPE	0.13	**39.67**	15.20	10.40	9.13	25.47	0	14.16	9.2	7.93	8.4	60.31
[0.25,0.75]	SBC	0.1	**85.16**	8.7	3.26	1.63	1.15	0	**74.36**	10.06	5.1	3.6	6.88
	HQC	0	**70.7**	12.8	6.23	4.06	6.21	0	**50.26**	11.53	8.26	5.93	24.02
	FPE	0	**47.83**	14.13	8.83	7.4	21.81	0	**21.33**	10.16	8.4	6.73	53.38
[0.75,0.25]	SBC	1.56	**71.14**	15.93	5.83	3.3	2.24	0.03	53.33	17.63	10.06	7.16	11.79
	HQC	0.83	**56.66**	19.03	9.36	6.4	7.72	0	34.26	16.33	12.5	10.06	26.85
	FPE	0.23	**38.63**	18.26	12.17	9.63	21.08	0	15.46	11.16	10.96	11	51.42
[0.1,0.9]	SBC	0.03	92.1	5.66	1.56	0.46	0.19	0	92.13	4.93	1.56	0.66	0.72
	HQC	0	80.4	10.73	3.96	2.0	2.91	0	76.13	9.93	5.1	3.16	5.68
	FPE	0	60.63	13.23	7.5	4.86	13.78	0	42.86	10.83	8.53	6.4	31.38
[0.9,0.1]	SBC	1.86	**70.86**	17.3	5.76	2.56	1.66	0	53.43	19.9	11.46	6.26	8.95
	HQC	0.93	**57.93**	20.8	9.43	5.43	6.50	0	35.36	19.83	15.13	10.13	19.55
	FPE	0.46	**40.46**	20.43	12.76	9.0	16.89	0	17.23	14.66	13.7	12.43	41.98

Table 2. *Frequencies of finding an AR(1) process: Student's* t *conditional distribution*

		T = 100						T = 500					
[α1,β1]	Crit.	0 lag	**1 lag**	2 lags	3 lags	4 lags	≥5	0 lag	**1 lag**	2 lags	3 lags	4 lags	≥51
[0,0]	SBC	0	**95.86**	3.4	0.63	0.03	0.06	0	98.56	1.2	0.23	0	0
	HQC	0	**88.86**	7.23	1.96	0.8	1.15	0	92.96	4.93	1.66	0.33	0.12
	FPE	0	**71.36**	11.03	5.63	4.23	7.75	0	72.03	10.43	6.16	3.43	7.95
[0.5,0]	SBC	0.8	**87.53**	10.3	1.06	0.16	0.15	0.03	86.1	11.4	2.06	0.3	0.1
	HQC	0.4	**76.33**	16.53	4.1	1.4	1.24	0.03	73.13	18.83	5.9	1.33	0.11
	FPE	0.26	**59.16**	19.73	7.7	4.76	8.39	0.03	51.1	21.96	11.46	5.46	9.99
[0.5,0.5]	SBC	1.23	**74.73**	12.96	5.13	3.13	2.79	0	58.2	14.86	8.3	5.46	13.18
	HQC	0.53	**58.43**	16.33	8.56	6.7	9.45	0	37	14.93	8	7.06	33.01
	FPE	0.2	**39.73**	16.3	9.96	8.8	25.01	0	16.47	9.83	9.16	8.4	56.14
[0.25,0.75]	SBC	0.13	**83.8**	9.3	3.06	1.56	2.15	0	68.53	12.3	5.93	3.6	9.64
	HQC	0.06	**67.16**	13.16	6.93	4.3	8.39	0	44.85	13.16	7.86	6.43	27.72
	FPE	0	**45.36**	14.26	8.03	7.26	25.09	0	20.2	8.76	7.86	7.03	56.15
[0.75,0.25]	SBC	2.16	**73.4**	14.56	5.3	2.8	1.78	0.06	56.76	18.73	10.1	5.63	8.78
	HQC	1.16	**58.2**	18.96	9.36	6.13	6.19	0	38.26	17.83	13.46	8.6	21.85
	FPE	0.63	**39.76**	19.46	12.2	9.53	18.42	0	18.8	13.1	13.46	10.36	44.28
[0.1,0.9]	SBC	0.03	91.87	5.56	1.36	0.56	0.62	0	87.36	7.5	1.96	1.36	1.82
	HQC	0	79.5	9.63	4.43	2.4	4.04	0	68.16	12.06	6.03	3.96	9.79
	FPE	0	57.73	12.66	7.13	5.8	16.68	0	36.7	10.8	8.33	6.2	37.97
[0.9,0.1]	SBC	2.7	**73.46**	15.73	5.03	1.9	1.18	0	57.36	21.83	10.03	5.23	5.55
	HQC	1.43	**59.5**	21.06	9.2	4.3	4.51	0	40.3	21.43	15.26	8.43	14.58
	FPE	0.83	**42.56**	22.2	12.3	8.7	13.41	0	21.73	16.5	16.3	11.36	34.11

3. Distortions do not vanish when sample size increases. With a value of α_1 greater than 0.5, we note a decrease of the frequency of finding the true model when T is greater than 100. Thus a large volatility parameter looks to affect information criteria, which are thus no longer consistent. This is due to the fact that GARCH processes, particularly because of the moving average parameter, need time to spread out (Hecq and Urbain, 1995). This illustrates why some asymptotic analytical results are difficult to obtain for that type of process, so simulation studies are helpful tools.

Table 3. Y ← X *when both* y_t *and* x_{tt} *follow a GARCH process (SBC criterion)*

T = 500 $\varepsilon_t \sim$ N(0,1)	AR 0	AR 2	AR 2	AR >=3	T = 50d $\varepsilon_t \sim t(5)$	AR 0	AR 1	AR 2	AR >=3
[0,0]	98.5	1.47	0.03	0	[0,0]	98.97	0.93	0.03	0.67
[0.5,0]	98.3	1.40	0.27	0.03	[0.5,0]	98.3	1.23	0.23	0.24
[0.5,0.5]	97.37	1.47	0.33	0.83	[0.5,0.5]	97.20	1.20	0.57	1.03
[0.25,0.5]	97.47	1.8	0.37	0.37	[0.25,0.5]	97.63	1.47	0.40	0.50
[0.75,0.25]	97.13	1.5	0.5	0.87	[0.75,0.25]	97.10	1.2	0.67	1.03
[0.1,0.9]	97.63	2.07	0.13	0.17	[0.1,0.9]	98.10	1.43	0.37	0.10
[0.9,0.1]	96.97	1.6	0.43	1.00	[0.9,0.1]	96.73	1.47	0.57	1.23

Note: We regress by OLS y_t on a constant, y_{t-1} and x_{t-1}, i = 1...10. The regressions based on more than one lag of y_t, as might be expected to avoid spurious augmentations, have given the same results. We are not considering instantaneous Granger causality. However, simulations not reported here have not reported spurious instantaneous relationships.

Table 4. *Optimal lag length in exchange rate differentials (SBC)*

	US$	DM	VPY	CHF	BEF	FRF	GBP	ITL	NLG	ESP	DKK	PTE	IEP	GRD
/USD	–	0	0	0	0	0	0	0	0	0	0	0	0	0
/DM	0	–	0	0	**2**	2	2	1	0	0	3	0	0	0

4. In the presence of ARCH effects, the leptokurtic conditional distribution affects the probability of finding the right lag. In this situation, even a small volatility parameter induces spurious augmentation.
5. In relative terms, the deterioration is less important when using SBC rather than HQC and FPE, as can be seen for instance with a formula of relative loss for choosing the right autoregressive process: (frequency without GARCH – frequency with GARCH)/ (frequency without ARCH). With $\alpha_1 = \beta_1 = 0.5$ and $T = 100$, the reductions of right occurrence frequencies are respectively 21.49%, 32.39% and 42.86% for the SBC, HQC and FPE criteria with the normal conditional distribution, and 22.04%, 34.24% and 44.32% with the Student's t conditional distribution.

Causality tests

We now check whether and how causality tests based on information criteria reject the null of non-causality when two independent variables are affected by GARCH. Table 3 gives the frequency of rejecting the null of non-causality for the SBC. Other criteria reject less often the true null of non-causality in finite sample.[11] In Table 3, AR i means that the lag i minimizes the SBC criterion when searching an optimal causal relationship from X to Y. For instance AR0 means that X is not anterior to Y.

The simulation results do not show evidence of spurious Granger-causality even under the worst circumstances, i.e. with large volatility parameters and T = 500. The Student's t conditional distribution (instead of the normal distribution) does not change the previous results.

We have also tested (results are not reported here) the impact of ARCH under the alternative hypothesis of causality. Only when ARCH residuals are on the regressand (and of course on both variables) can a longer causality relation be observed. Indeed, we observe more causal significant lags from x_t to y_t when at least y_t follows a GARCH process. However, the power against the alternative hypothesis never decreases. Even with volatile data, interpretations of economic relations are never affected, but only the length of links between variables.

IV. EMPIRICAL EVIDENCE

We illustrate the two preceding properties on high frequency data for exchange rates and interest rates.

Are exchange rates 'random walks'

First, note that the expression 'random walk' is not correct in this case because we hypothesize that exchange rates follow GARCH error process, so it would be better to use the term martingale. Assuming a unit root in the log of nominal exchange rates,[12] we test the market efficiency hypothesis by testing for serial correlation in the growth rate of nominal exchange which therefore must follow a difference martingale. Indeed, the efficiency market hypothesis suggests that it should not be possible to predict a change in a bilateral exchange rate on the

[11]For instance, with no-ARCH on both y_t and x_t, and for a sample size of 100 observations. SBC rejects the null with a frequency of 98.5%, HQC with 92.63% and FPE with 69.83%.

[12]See Hecq and Urbain (1993, 1995) for unit root tests in the presence of GARCH.

Table 5. *Estimation of GARCH model under the random walk hypothesis*

Dependent variable Δs_t	constant γ_0	α_0	α_1	β_1	v
DM/US$	2.95e-5 (0.11)	1.85e-6 (1.51)	0.052 (2.82)	0.919 (29.16)	7.04 (3.99)
YPY/US$	–4.05e-4 (–2.17)	8.09e-7 (1.14)	0.055 (3.15)	0.927 (40.24)	4.73 (6.24)
CHF/US$	1.05e-4 (0.37)	9.75e-5 (3.98)	0.074 (1.92)	–0.391 (–1.24)	7.20 (3.88)
BEF/US$	–1.49e-5 (–0.05)	1.92e-6 (1.47)	0.053 (2.82)	0.917 (27.77)	6.90 (3.99)
FRF/US$	3.33e-4 (1.38)	8.69e-7 (0.97)	0.055 (3.40)	0.931 (43.11)	6.88 (3.88)
GBP/US$	2.10e-5 (–0.09)	2.51r-7 (0.81)	0.046 (3.56)	0.951 (66.63)	5.83 (4.22)
ITL/US$	1.59e-4 (0.64)	2.32e-6 (1.90)	0.069 (3.21)	0.897 (28.34)	5.98 (5.35)
NLG/US$	4.67e-5 (0.19)	1.14e-6 (0.95)	0.055 (2.87)	0.934 (38.00)	4.82 (4.31)
ESP/US$	1.40e-4 (0.57)	2.87e-6 (1.99)	0.073 (3.31)	0.886 (25.27)	5.11 (4.94)
DKK/US$	–5.79e-5 (–0.23)	1.47e-6 (1.23)	0.062 (3.54)	0.915 (34.89)	6.95 (4.18)
PTE/US$	1.44e-4 (0.58)	1.43e-6 (1.50)	0.051 (3.06)	0.928 (37.27)	5.48 (4.64)
IEP/US$	–2.39e-5 (–0.09)	3.45e-6 (2.06)	0.082 (3.09)	0.870 (21.43)	4.83 (6.48)
GRD/US$	4.30e-4 (1.87)	1.55e-6 (1.26)	0.060 (3.19)	0.911 (29.60)	7.11 (3.76)
YPY/DM	–2.42e-4 (–1.04)	2.68e-6 (1.71)	0.097 (3.31)	0.853 (19.37)	11.31 (3.95)
CHF/DM	7.74e-6 (0.08)	9.33e-8 (1.65)	0.082 (3.03)	0.821 (15.30)	5.82 (6.83)
BEF/DM	–3.17e-5 (–3.15)	8.00e-9 (0.20)	0.353 (4.16)	0.763 (23.30)	2.95 (8.89)
FRF/DM	–3.31e-5 (–1.22)	1.27e-8 (1.82)	0.13 (4.59)	0.879 (39.89)	4.50 (6.79)
GBP/DM	6.79e-5 (0.75)	1.84e-7 (2.03)	0.126 (4.32)	0.877 (36.56)	4.49 (7.69)
ITL/DM	1.31e-5 (0.29)	6.24e-8 (1.83)	0.249 (3.98)	0.836 (35.08)	3.05 (9.89)
NLG/DM	–5.68e-6 (–1.11)	7.00e-9 (0.22)	0.498 (3.29)	0.746 (19.05)	2.42 (16.14)
ESP/DM	7.77e-7 (0.01)	1.33e-6 (3.08)	0.052 (3.24)	0.610 (12.75)	2.72 (10.02)
DKK/DM	–1.19e-5 (–0.44)	4.50e-9 (0.60)	0.237 (4.69)	0.799 (29.40)	3.39 (9.38)
PTE/DM	1.43e-5 (0.20)	5.78e-7 (1.82)	0.280 (3.97)	0.764 (20.41)	2.98 (11.04)
IEP/DM	1.72e-5 (1.00)	2.49e-8 (0.35)	0.623 (3.11)	0.745 (29.16)	2.44 (13.42)
GRG/DM	3.14e-4 (7.20)	3.53e-7 (2.10)	0.078 (2.23)	0.783 (8.99)	3.77 (7.93)

basis of previous and current known data. This may be tested by running Equation 7 and testing for $\gamma_{1`} = ...\gamma\pi = 0$ or by getting an order of lag equal zero:

$$\Delta S_t = \gamma_0 + \gamma_1 \Delta s_{t-1} + \gamma_2 \Delta s_{t-2} + \ldots + \gamma_p \Delta s_{t-p} + \varepsilon_t \qquad (7)$$

where s_t is the log of the spot nominal exchange rate, Δ is the first difference operator and p is the number of lags we try to determine using information criteria.

Table 4 gives optimal lag length in the growth rates of the bilateral nominal exchange rates either per dollars and per deutschmark of the following currencies: US dollar (US$), deutschmark (DM), Japan yen (YPJ), Swiss franc (CHF), Belgian franc (BEF), pound sterling (GBP), Italian lira (ITL), French franc (FRF), Irish pound (IEP), Greek drachma (GRD), Portugese escudo (ESP), Danish kroner (DKK), Dutch guilder (NLG), Spanish peseta (PTE). We use daily data, which come from the National Bank of Belgium, for the time span 2 January 1991 to 13 April 1994, i.e. 891 observations. We took this period because we wanted to study the speculative pressures affecting the EMS from mid-1992 to mid-1993, pressures which led to the adoption of enlarged margins (15% instead of 2.25%) and the exit of Italy and the United Kingdom from the exchange rate mechanism.

We can reject an absence of serial dependence for all the currencies against the dollar, but five currencies against the deutsch mark present signs of serial correlation.[13] It seems however, surprising that we can reject the efficient market hypothesis for only some EMS related countries. Indeed, arbitrageurs could not leave the market one or two days in these positions without trying to make profit.

We attribute these differences to the GARCH representation of the data. Indeed, if we estimate the GARCH parameters with a Student's t conditional distribution[14] for the growth rate of exchange rates, we see in Table 5 that currencies against the deutschmark present a more leptokurtik conditional distribution and especially a higher parameter of volatility (α_1) and are integrated in variance. The common t statistics are in brackets.

Of course, there are some currencies for which we cannot reject the null hypothesis even if the volatility parameter is relatively high (see NLG/DM, PTE/DM, IEP/DM variables). Notice, however, that our simulations underline a decrease in finding the right lag and not an inability to find it.

[13]Standard errors from OLS regressions also give significant (even using HCSE) lags.
[14]Estimation carried out with a procedure written in RATS 4.10.

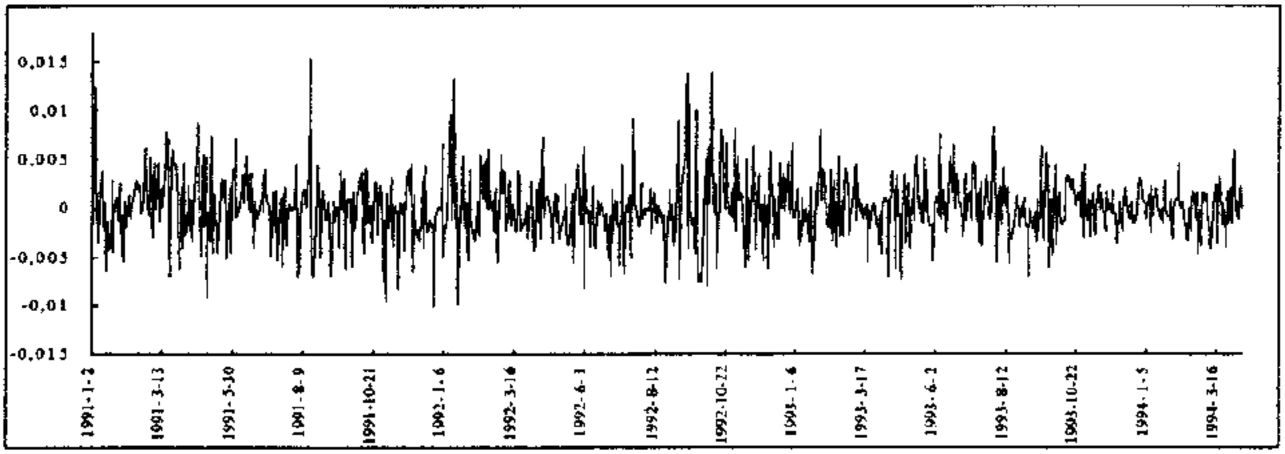

Fig. 1. *Growth rate of French franc per US dollar exchange rate*

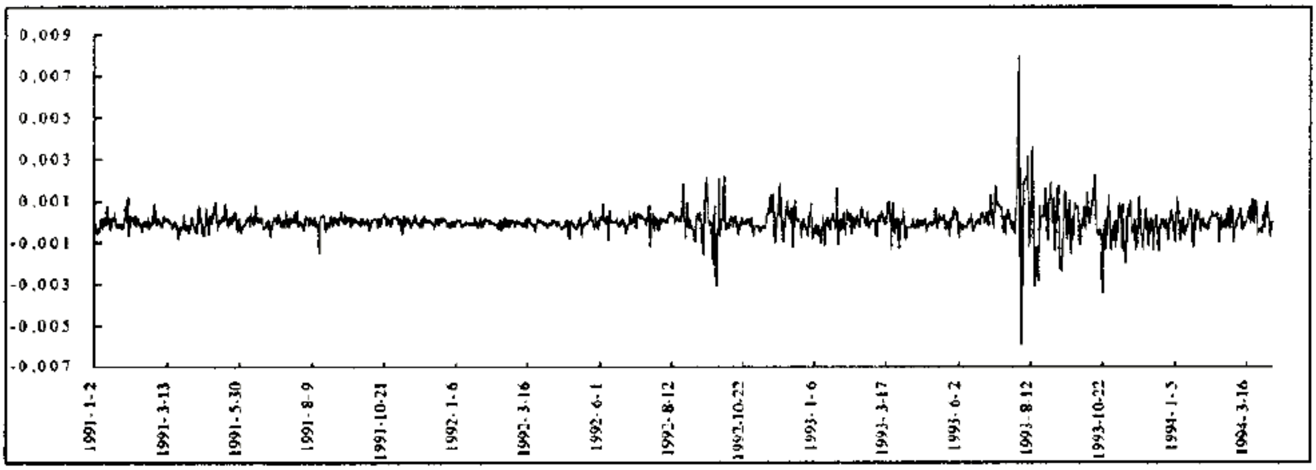

Fig. 2. *Growth rate of French franc per German mark exchange rate*

The difference between the per dollar and per deutschemark bilateral exchange rate can also be seen in Fig. 1 and 2 in which we draw FRF/US$ and FRF/DM exchange rates. It seems that the FRF/US$ can be considered as a strong white noise while FRF/DM clusters at some period and probably follows a GARCH process (weak white noise).

Asymmetry in the EMS

While information criteria are not influenced by GARCH errors they are very useful in testing for Granger-causality. We illustrate this and test for an asymmetric functioning of the EMS, which postulates, following Henry and Weidmann (1994), that the German interest rate influences the rates in the other EMS countries, without there being any influence in the opposite direction. Thus asymmetry uni-directional causality running from the German rate to the other rates.

The data also comes from the National Bank of Belgium database. They consist of daily observations from April 1983 to 13 April 1994 on one month German, Dutch, French, Belgian, British, Italian and Danish Eurorates. The choice of beginning date reflects the date when the exchange rate mechanism entered a phase in which more emphasis was to be laid on policy coordination after the preceding successive realignments. As none of the pairwise data is cointegrated we can proceed to an analysis of causality in first difference without loss of information.

Using information criteria we can easily undertake this analysis. To test for non-causality we begin by choosing an optimal autoregressive process for one variable. We keep this lag fixed and we successively add lags of the second variable and try to minimize the SBC criteria. Then we carry out the same operations in the opposite direction. Table 6 gives the final results.

It can be seen that in neither case is the asymmetry hypothesis accepted. The German eurorate Granger-causes the Dutch interest rate only, but in this case the bidirectional causality exists. The interest rate of Italy and the UK also Granger-causes the German Eurorate. So we conclude that there is non-asymmetric functioning of the EMS over the period studied. However, if we believe the results are not affected by GARCH errors, they are probably sensitive to the usual criticism of causality tests. Indeed, we might suspect that the period is not

Table 6. *Optimal bivariate lags (SBC)*

NLG ← DM	BEF ← DM	FRF ← DM	GBP ← DM	ITL ← DM	DKK ← DM
0 , 1	7 , 0	7 , 0	2 , 0	5 , 0	4 , 0
DM ← NLG	DM ← BEF	DM ← FRF	DM ← GBP	DM ← ITL	DM ← DKK
3 , 1	3 , 0	3 , 0	3 , 4	3 , 3	3 , 0

coherent enough and consequently that coefficients change over time (a recursive analysis confirms our doubts), that we are omitting important variables (such as US Eurorates), that the daily data are too aggregated to observe the causality link which takes place during the day, that Granger-causality means anteriority and not necessarily causation[15] The point, however, is that results are not affected by GARCH errors.

V. CONCLUSION

In order to characterize the dynamic structure of economic data, we can advocate the use of information criteria if we believe that most GARCH time series are better characterized by a small volatility parameter and a large MA one. In all small-sample situations that we considered, the SBC should be preferred to other criteria.

With large volatility parameters, we should be cautious in the use of ADF unit root tests as well as in the augmented Engle–Granger cointegration test (and in the determination of the VAR for the Johansen's ML procedure) and should try to avoid too hasty an interpretation of univariate time series results.

When faced with a large volatility parameter, it may be better, when testing for autocorrelation, to implement the correction factor of the Box–Pierce test, as Diebold does.

We have seen that Granger non-causality tests information criteria are not affected in small samples by GARCH. Spurious causality is not found, even when the two variables follow a GARCH process with a large volatility parameter. Thus information criteria, and especially SBC, are very helpful in empirical studies when testing causality between GARCH time series: money–income causality (with interest rates and prices), consumer–wholesale prices causality, movements of prices in stock exchanges on different markets, or causality between interest rates based on various maturities or in different countries.

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[15]This point is illustrated by the detection of a causal link from the Italian and British to the German Eurorate.