# Common Shocks, Common Dynamics, and the International Business Cycle* 

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#### Abstract

This paper proposes an econometric framework to assess the importance of common shocks and common transmission mechanisms in generating international business cycles. Then we show how to decompose the cyclical effects of permanent-transitory shocks into those due to their domestic and those due to foreign components. Our empirical analysis reveals that the business cycles of the US, Japan, Canada are clearly dominated by their domestic components. The Euro area is more sensitive to foreign shocks compared to the other three countries of our analysis.


Keywords: International business cycles; Permanent-transitory decomposition; Serial correlation common features; Frequency domain analysis.

JEL: C32, E32

[^0]
## 1 Introduction

The expression "international business cycle" refers to the presence of comovements in the cyclical behavior of output data across countries (see e.g. Backus et al., 1995). Although there is convincing empirical evidence in favor of international business cycle linkages (see e.g. Artis et al., 1997; Gregory et al., 1997; Kose et al., 2003), economists and econometricians still dispute on the causes, the consequences, as well as even on the measure of these comovements. For instance, the question concerning the predominant role of common shocks or common propagation mechanisms is far from being resolved (see e.g. Canova and Marrinan, 1998). Other debates concern the influence of foreign shocks in contributing to national business cycles and consequently the degree of openness of economies, as well as the respective role of permanent and transitory (PT hereafter) components. Indeed, if demand shocks are largely responsible for fluctuations, there may be a role for aggregate Keynesian-type policies. There is also no consensus about the type of variables to use in empirical analyses: some researchers favor labor productivity for theoretical reasons (Galì, 1999), while others prefer to work with output series which, for them, measure the movements in the overall economic activity best, as originally advocated by Burns and Mitchell (1946). We use industrial production indexes in this paper because they are more timely available and their statistical properties render them more appropriate for international comparisons. ${ }^{1}$ Finally, we must also mention the discussion concerning the way to transform the data for the extraction of cyclical fluctuations. This paper develops a coherent statistical framework for all these issues.

First, like many others we start by analyzing whether observed fluctuations are due to common shocks. We exploit the low frequency comovements coming from a cointegration analysis to identify groups of shocks according to whether their effects are permanent or transitory. This type of analysis, often performed in the empirical literature, will only be the starting point of our analysis. Once the number of cointegrating vectors is determined, we use the orthogonal decomposition proposed by Centoni and Cubadda (2003) to extract PT components. One of the novelties of our analysis is to include, along with big economies such as Canada, Japan and the US, the Eurozone as a whole. ${ }^{2}$ Indeed, many studies focus on the G7 economies

[^1](e.g. Cheung and Westermann, 2002), and consequently they compare the US with European countries individually. ${ }^{3}$ The difference in the size of countries induces inevitably asymmetries of treatment.

Second, we add a common cyclical feature analysis (Engle and Kozicki, 1993) to the study of common trends. Indeed, the presence of common cycles (Vahid and Engle, 1993) will play a double role. On one hand, this shows whether there exist some common dynamics, namely some common transmission mechanisms of the shocks. One the other hand, imposing the implied restrictions to the estimated model also helps to estimate more accurately the responses to shocks because redundant parameters are excluded. In particular, it is well known that a misleading detection of the number of cointegrating vectors might occur with small sample sizes, and hence can damage the conclusions about the nature and the importance of common shocks. Using an iterative strategy that switches between the cointegrating and cyclical cofeature spaces, we obtain a more precise picture of the number of permanent and transitory components (see Hecq, 2006).

Third, after having carefully detected the number and the type of common shocks, we propose to further decompose the permanent and transitory shocks into a domestic and foreign component. For each country we consequently obtain four types of shocks which contribute to business cycle fluctuations. In particular, we identify the permanent [transitory] countryspecific domestic shock as the component of the common permanent [transitory] shocks that has contemporaneous effect on domestic output. Also the permanent [transitory] country-specific foreign shocks will be the component of the common permanent [transitory] shocks that has no contemporaneous effect on domestic output.

Finally, we asses the importance of such PT domestic and foreign shocks in contributing to national business cycles. So doing we consider fluctuations with a $2-8$ year period using a parametric spectrum decomposition (see Centoni and Cubadda, 2003). In our opinion this approach evaluates the contribution of domestic and foreign shocks to the business cycles more appropriately than the traditional impulse responses or variance decompositions in the time domain.

The proposed approach allows us to answer a series of questions such as: $1^{\circ}$ ) Do international outputs comove because of the existence of common shocks, common dynamics or both? $2^{\circ}$ )

[^2]What is the importance of foreign shocks over national business cycles, and consequently what is the degree of openness of economies? $3^{\circ}$ ) Are business cycles mainly affected by permanent or transitory components?

Previewing the main empirical results, we find that output indices are characterized by both common PT shocks and common propagation mechanisms of these shocks. Similarly to most studies (see e.g. King et al, 1991), our analysis confirms that permanent shocks are the main source of business cycles. However, in contrast to Canova and Marrinan (1998) and Mellander et al. (1992), our results suggest that foreign shocks account for a small portion of the cyclical fluctuations. Only for the Eurozone is the proportion of foreign shock important. Ahmed et al. (1993) and Kwark (1999) reached a similar conclusion for the US economy using a structural VAR approach.

This paper is organized as follows. In Section 2 we briefly review a PT decomposition such that a set of cointegrated time series is separated into independent components (Centoni and Cubadda, 2003), and the notion of serial correlation common feature (Engle and Kozicki, 1993). In Section 3 we propose a statistical measure of the importance of domestic and foreign shocks over the business cycles and we show how to implement it in practice. The contribution of the PT components of domestic and foreign shocks is also investigated. Section 4 presents our empirical analysis of output fluctuations for Japan, Canada, the US and the Eurozone. A final section concludes.

## 2 Common shocks and common propagation mechanisms

### 2.1 Extracting PT shocks

Let $X_{t}$ be a $n$-vector time series such that

$$
\begin{equation*}
A(L) X_{t}=\varepsilon_{t}, \quad t=1, \ldots, T \tag{1}
\end{equation*}
$$

for fixed values of $X_{-p+1}, \ldots, X_{0}$ and where $A(L)=I_{n}-\sum_{i=1}^{p} A_{i} L^{i}, L$ is the lag operator and $\varepsilon_{t}$ are i.i.d. $N_{n}(0,-)$ errors. In the $\operatorname{VAR}(p)$ in (1), deterministic terms have been left out at this level of presentation for notational convenience.

Let us assume that

$$
\begin{equation*}
|A(c)|=0 \text { implies that } c=1 \text { or }|c|>1, \tag{2}
\end{equation*}
$$

then there exist $n \times r$-matrices $\alpha$ and $\beta$ of rank $r$ such that $A(1)=-\alpha \beta^{\prime}$. The matrix $\alpha_{\perp}^{\prime} \Gamma \beta_{\perp}$ has full rank, $\alpha_{\perp}$ and $\beta_{\perp}$ are $n \times(n-r)$-matrices of rank $(n-r)$ such that $\alpha_{\perp}^{\prime} \alpha=\beta_{\perp}^{\prime} \beta=0, \Gamma=$ $I_{n}-\sum_{i=1}^{p-1} \Gamma_{i}$ and $\Gamma_{i}=-\sum_{j=i+1}^{p} A_{j}$ for $i=1,2, \ldots, p-1$. The process $X_{t}$ is cointegrated of order $(1,1)$, the columns of $\beta$ span the cointegrating space, the elements of $\alpha$ are the corresponding adjustment coefficients. We can rewrite Equation (1) in the following Vector Error-Correction Models (henceforth VECM)

$$
\begin{equation*}
\Gamma(L) \Delta X_{t}=\alpha \beta^{\prime} X_{t-1}+\varepsilon_{t} \tag{3}
\end{equation*}
$$

with $\Delta=(1-L)$, and $\Gamma(L)=I_{n}-\sum_{i=1}^{p-1} \Gamma_{i} L^{i}$ (see e.g. Johansen, 1996). Testing for cointegration in such a system is routinely applied by researchers and consequently we refer to the literature for further explanations concerning these tests.

Series $X_{t}$ also admit the following Wold representation

$$
\Delta X_{t}=C(L) \varepsilon_{t},
$$

where $C(L)=I_{n}+\sum_{i=1}^{\infty} C_{i} L^{i}$ is such that $\sum_{i=1}^{\infty} i\left|C_{i}\right|<\infty$.
Under these assumptions, Centoni and Cubadda (2003) derived a PT decomposition where common permanent and transitory shocks are respectively given by

$$
\begin{equation*}
u_{t}^{P}=\alpha_{\perp}^{\prime} \varepsilon_{t} \quad \text { and } \quad u_{t}^{T}=\alpha^{\prime}-{ }^{-1} \varepsilon_{t} . \tag{4}
\end{equation*}
$$

Then the permanent and transitory components of series $X_{t}$ are respectively $P_{t}$ and $T_{t}$, where $X_{t}=P_{t}+T_{t}, \Delta P_{t}=P(L) u_{t}^{P}, \Delta T_{t}=T(L) u_{t}^{T}$, and

$$
\begin{align*}
& P(L)=C(L)-\alpha_{\perp}\left(\alpha_{\perp}^{\prime}-\alpha_{\perp}\right)^{-1}  \tag{5}\\
& T(L)=C(L) \alpha\left(\alpha^{\prime}-{ }^{-1} \alpha\right)^{-1} . \tag{6}
\end{align*}
$$

Since we know from the Granger representation theorem that $C(1)=\beta_{\perp}\left(\alpha_{\perp}^{\prime} \Gamma \beta_{\perp}\right)^{-1} \alpha_{\perp}^{\prime}$ (see e.g. Johansen, 1996), and in view of equations (5) and (6), we obtain $P(1)=C(1)$ and $T(1)=0$. Hence, the shocks $u_{t}^{P}$ only have permanent effects on series $X_{t}$ as required. ${ }^{4}$ Moreover, it is easy to verify that the components $P_{t}$ and $T_{t}$ are uncorrelated at all lags and leads. ${ }^{5}$ Assuming

[^3]hereafter that series $X_{t}$ represent output series of $n$ different countries, our goal in Section 3 will be to further decompose these components into domestic and foreign shocks.

### 2.2 Common transmission mechanisms

Allow us to propose an operational definition of common transmission mechanisms. We rely on the notion of serial correlation common feature (henceforth, SCCF) proposed by Engle and Kozicki (1993) and Vahid and Engle (1993). In this context, series $\Delta X_{t}$ have $s$ SCCF relationships iff there exists an $n \times s$ matrix $\delta$ with full column rank and such that $\delta^{\prime} C(L)=\delta^{\prime}$ for the Wold representation. Hence, SCCF implies that the impulse response functions of series $\Delta X_{t}$ are collinear.

Another way to stress that SCCF involves the presence of common propagation mechanisms among series $\Delta X_{t}$, is done by rewriting the VECM (3) in the following common factor representation

$$
\begin{equation*}
\Delta X_{t}=\delta_{\perp} A^{\prime} W_{t-1}+\varepsilon_{t} \equiv \delta_{\perp} F_{t-1}+\varepsilon_{t}, \tag{7}
\end{equation*}
$$

where $A$ is a $(r+n(p-1)) \times(n-s)$ full-rank matrix, and $W_{t-1}=\left(X_{t-1}^{\prime} \beta, \Delta X_{t-1}^{\prime}, \ldots, \Delta X_{t-p+1}^{\prime}\right)^{\prime}$. Importantly enough, the main characteristic of representation (7) is that all the predictable dynamics of the system are due to the $(n-s)$ common factors $F_{t-1}$. This is not generally the case in the traditional dynamic factor modeling where the idiosyncratic terms may even be more cyclical than the factors themselves. A possible drawback of the SCCF approach is that a matrix such $\delta$ may not exist. However, we can use the less stringent condition that there exists a $n \times s$ polynomial SCCF matrix $\delta(L) \equiv \delta_{0}+\delta_{1} L$ such that $\delta(L)^{\prime} C(L)=\delta_{0}^{\prime}$, see Cubadda and Hecq (2001) for details. Nevertheless, anticipating the results of the empirical analysis in Section 4, we will see that SCCF is quite appropriate for our statistical model of the output series.

Maximum Likelihood (henceforth, ML) inference on SCCF requires to solve the following canonical correlation program,

$$
\begin{equation*}
\operatorname{CanCor}\left\{\Delta X_{t}, W_{t-1} \mid D_{t}\right\}, \tag{8}
\end{equation*}
$$

where CanCor $\{Y, X \mid Z\}$ denotes the partial canonical correlations between the elements of $Y$ and $X$ conditional on $Z$. In this simple case, $Z$ will be $D_{t}$, that is to say a vector of fixed elements such a constant or seasonal dummies. The likelihood ratio test for the null hypothesis that there

[^4]exist at least $s$ SCCF vectors is based on the statistic (see e.g. Anderson, 1984; Velu et al., 1986)
\[

$$
\begin{equation*}
L R=-T \sum_{j=1}^{s} \ln \left(1-\hat{\lambda}_{j}\right), \quad s=1, \ldots, n-r \tag{9}
\end{equation*}
$$

\]

where $\hat{\lambda}_{j}$ is the $j$-th smallest squared canonical correlation coming from the solution of (8). These eigenvalues are obtained from the solution of $\left|\lambda S_{00}-S_{01} S_{11}^{-1} S_{10}\right|=0$, where $S_{h l}, h, l=0,1$ are the second moment matrices of the residuals $R_{0 t}$ and $R_{1 t}$ obtained in multivariate least squares regressions from respectively $\Delta X_{t}$ and $W_{t-1}$ on $D_{t}$.

The test statistic (9) follows asymptotically a $\chi_{(v)}^{2}$ distribution under the null where $v=$ $s \times(n(p-1)+r)-s(n-s)$. Moreover, the canonical variates coefficients of $\Delta X_{t}$ associated with the $s$ smallest eigenvalues $\hat{\lambda}_{1}, \ldots, \hat{\lambda}_{s}$ provide the ML estimate of the SCCF matrix $\delta$ whereas the matrix $A$ in equation (7) is estimated by the canonical variates coefficients of $W_{t-1}$ associated with the $(n-s)$ largest eigenvalues $\hat{\lambda}_{s+1}, \ldots, \hat{\lambda}_{n}$. Finally, the matrix $\delta_{\perp}$ is estimated by a regression of $\Delta X_{t}$ on $F_{t-1}^{\prime}$.

Hecq (2006) also considers a correction for small samples à la Reinsel and Ahn (1992), such that $L R^{\text {cor }}=\frac{T-n(p-1)-r}{T} L R$ as well as the use of information criteria (IC hereafter). For $p$ fixed and $r$ given, they are computed as

$$
I C_{T}(\bar{p}, \bar{r}, s)=-\frac{2}{T} \log \operatorname{lik}+\frac{\vartheta_{T}}{T} \times(\# \text { parameters })
$$

where the penalty $\vartheta_{T}$ is respectively $2, \ln \ln T$, and $\ln T$ for Akaike's Information Criterion (AIC), Hannan-Quinn Criterion (HQ) and Schwarz's Bayesian Criterion (SC). The number of parameters is obtained by comparing the restricted and the unrestricted system. Consequently, the number of parameters is obtained by subtracting the number of restrictions the common dynamics impose (i.e. $v$ ) from $n^{2} \times(p-1)+n r$, that is to say the total number of mean parameters in the VECM (3) for given $r$ and $p$ and $\beta$ superconsistently estimated by $\hat{\beta}$ in a first step (see Vahid and Engle, 1993).

Note that $-\frac{2}{T}$ times the log likelihood is the log of the determinant of the residuals covariance matrix with reduced rank restrictions. As an example, in a VECM with $r$ cointegrating vectors and $s$ cofeature restrictions we have

$$
-\frac{2}{T} \log \operatorname{lik}=\ln \left|S_{00}-S_{01} S_{11}^{-1} S_{10}\right|-\sum_{j=1}^{s} \ln \left(1-\hat{\lambda}_{j}\right)
$$

The limiting distribution of $L R$ in (9) is unaltered when a superconsistent estimate of the cointegrating vectors is used in place of the unknown parameters. However, for small sample sizes it might be interesting to obtain more precise estimates of both cointegrating and cofeature spaces by using the switching algorithm that we present in the next subsection. Indeed the PT decomposition crucially depends on $r$ and consequently it is important to estimate it as accurately as possible.

### 2.3 ML iterative estimation

When both long and short-run comovements are of interest for the researcher, a common practice consists of first estimating superconsistently the cointegrating vectors $\beta$ and next performing a test for common serial correlation by fixing $\beta$ to be equal to their estimates $\hat{\beta}$. The drawback of such a two step procedure is that it does not maximize the likelihood and therefore it might result in a misleading determination of the number of cointegrating and common cyclical feature vectors. Moreover, given the poor small sample performances of the asymptotic Johansen test, ${ }^{6}$ a misspecification on $r$, and hence on the number of the PT shocks, is likely. The determination of $s$ is automatically affected because the constraint $(r+s) \leq n$ applies (see Vahid and Engle 1993).

Let us summarize the main steps of an iterative strategy which maximizes the likelihood function by successively estimating and imposing long and short-run restrictions until convergence is achieved. Derived maxima can then be used both for cointegration and common cyclical feature test statistics, a procedure which performs better for the choice of $r$ and $s$ than the twostep framework (see Hecq, 2006). We first consider the weak form common cycle specification (WF hereafter, see Hecq, Palm and Urbain 2000, 2006) in the searching strategy for SCCF. For the WF we have under the null $\delta^{\prime} \Delta X_{t}=\delta^{* \prime} \beta^{\prime} X_{t-1}+\delta^{\prime} \varepsilon_{t}$ in

$$
\begin{equation*}
\Delta X_{t}=\alpha \beta^{\prime} X_{t-1}+\delta_{\perp} \tilde{A}^{\prime} \tilde{W}_{t-1}+\varepsilon_{t} \tag{10}
\end{equation*}
$$

where $\delta^{* \prime}=\delta^{\prime} \alpha, \tilde{A}$ is a $(n(p-1)) \times(n-s)$, and $\tilde{W}_{t-1}=\left(\Delta X_{t-1}^{\prime}, \ldots, \Delta X_{t-p+1}^{\prime}\right)^{\prime}$. The WF specification involves $\delta^{\prime} \Gamma_{i}=0_{(s \times n)}, i=1, \ldots, p-1$, and allows us to test for cointegration and common feature almost independently. Indeed $\delta$ must not necessarily lie in $\operatorname{sp}\left(\alpha_{\perp}\right)$ and consequently we can have $s>n-r$ weak form cofeature vectors. To estimate (10), we start

[^5]by determining $r$ and estimating $\beta$ with the usual Johansen procedure without any additional restrictions from the short-run. We fix $\hat{\beta}$ and test for the number of WF common feature vectors $s$ using
\[

$$
\begin{equation*}
\text { CanCor }\left\{\Delta X_{t}, \tilde{W}_{t-1} \mid\left(D_{t}^{\prime}, X_{t-1}^{\prime} \hat{\beta}\right)^{\prime}\right\} \tag{11}
\end{equation*}
$$

\]

We next estimate $\tilde{A}^{\prime}$ using the dual problem of the canonical correlation procedure and the vectors associated with the largest eigenvalues. To have trace tests both for cointegration and common cyclical features we must compute a series of quantities. For a given $s=\bar{s}$ and with consequently $\operatorname{rank}(\tilde{A})=n-\bar{s}$, let us compute the determinant of the covariance matrix for $r=n$ cointegrating vectors as

$$
\ln \left|\hat{-}_{r=n, s=\bar{s}}\right|=\ln \left|\tilde{S}_{00}-\tilde{S}_{01} \tilde{S}_{11}^{-1} \tilde{S}_{10}\right|-\sum_{j=1}^{\bar{s}} \ln \left(1-\tilde{\lambda}_{j}^{W F}\right),
$$

where $\tilde{S}_{h l}, h, l=0,1$, refer to cross moment matrices obtained in multivariate least squares regressions from $\Delta X_{t}$ and $\tilde{W}_{t-1}$ on $\left(D_{t}^{\prime}, X_{t-1}^{\prime}\right)^{\prime}$. The derived log-likelihood with $s$ cofeature vectors and $r=n$ will be denoted

$$
L_{\mathrm{max}, r=n, s=\bar{s}}=-\frac{T}{2} \ln \left|\hat{-}_{r=n, s=\bar{s}}\right| .
$$

This estimation does not involve an iterative algorithm because the cointegrating space spans $R^{n}$. For any $r=n-1, n-2, \ldots, 0$ and a given $\bar{s}$, we iterate between $\operatorname{CanCor}\left\{\Delta X_{t}, X_{t-1} \mid\left(D_{t}^{\prime}, \tilde{W}_{t-1}^{\prime} \tilde{A}\right)^{\prime}\right\}$ and CanCor $\left\{\Delta X_{t}, \tilde{W}_{t-1} \mid\left(D_{t}^{\prime}, X_{t-1}^{\prime} \hat{\beta}\right)^{\prime}\right\}$ to obtain more accurate $\hat{\beta}$ and $\tilde{A}$ at each step until convergence is reached. We obtain the maximum for every $r$ such that

$$
\begin{equation*}
L_{\max , r=r_{j}, s=\bar{s}}=-\frac{T}{2}\left\{\ln | |^{\max } \max _{r=r_{j}, s=\bar{s}} \mid-\sum_{j=1}^{\bar{s}} \ln \left(1-\hat{\lambda}_{j, \max }\right)\right\}, \quad r_{j}=n-1, \ldots, 0, \tag{12}
\end{equation*}
$$

where now the eigenvalues and $\hat{-}$ are obtained using the $\hat{\beta}$ computed after convergence is reached and denoted $\hat{\beta}^{\max }$. Trace tests for cointegration are performed by comparing derived maxima of the likelihood function at $r=n$ on the one hand and at $r_{j}=n-1, \ldots, 0$ on the other hand using

$$
\mathrm{H}_{0, r \leq r_{j}, s=\bar{s}}=-2\left(L_{\max , r=r_{j}, s=\bar{s}}-L_{\max , r=n, s=\bar{s}}\right), \quad r_{j}=n-1, \ldots, 0 .
$$

In particular, the previous statistics without additional short-run restrictions (denoted by $\mathrm{H}_{0, r \leq r_{j}, s=0}$ )
is the usual Johansen trace test. The same procedure may be repeated for other $s$ if an analysis of sensitivity is needed.

Similarly, tests for common features can be obtained by evaluating the log-likelihood functions at their maxima for one particular $r=\bar{r}=1, \ldots, n$ for $s=0, \ldots, n$ and by considering twice their differences at derived maxima. Namely for $\bar{r}$ fixed we compute $\mathrm{H}_{0, r=\bar{r}, s=s_{j}=}=$ $2\left(L_{\max , r=\bar{r}, s=s_{j}}-L_{\text {max }, r=\bar{r}, s=0}\right)$, where only $L_{\text {max }, r=\bar{r}, s=s_{j}}$ needs to be found by iteration from (12), while $L_{\text {max }, r=\bar{r}, s=0}$ is the usual $\log$ likelihood obtained in the Johansen test without additional short-run restrictions.

It is shown in Hecq (2006) how to extend this iterative strategy for SCCF based on Hansen and Johansen (1998). As that latter procedure is more tedious to apply we take an approximation in this paper: (i) we take the $r$ and the estimated $\hat{\beta}^{\max }$ obtained with the WF restrictions at the end of the iteration process and (ii) we test for SCCF using that "final" $\hat{\beta}^{\max }$.

## 3 Measuring the business cycle effects of foreign and domestic shocks

In this section we propose a new statistical measure of the importance of foreign and domestic shocks over the business cycle. In particular, we show that the statistics proposed by Centoni and Cubadda (2003) can be modified to decompose the business cycle effects of PT shocks into those due to their domestic and those due to foreign components.

### 3.1 Statistical measures

Having disentangled groups of PT shocks by means of the long-run properties of the data, we resort to short-run identifying restrictions in order to identify the country-specific domestic and foreign components of these shocks. Building on Kwark (1999), we define the permanent [transitory] country-specific domestic shock as the component of the common permanent [transitory] shocks that has a contemporaneous effect on domestic output, and the permanent [transitory] country-specific foreign shocks as the component of the common permanent [transitory] shocks that has no contemporaneous effect on domestic output.

In order to apply the above identifying restrictions, let us decompose innovations $\varepsilon_{t}$ into their PT components as follows

$$
\varepsilon_{t}=\varepsilon_{t}^{P}+\varepsilon_{t}^{T}
$$

where

$$
\begin{equation*}
\varepsilon_{t}^{P}=-\alpha_{\perp}\left(\alpha_{\perp}^{\prime}-\alpha_{\perp}\right)^{-1} u_{t}^{P} \quad \text { and } \quad \varepsilon_{t}^{T}=\alpha\left(\alpha^{\prime}-{ }^{-1} \alpha\right)^{-1} u_{t}^{T} \tag{13}
\end{equation*}
$$

Due to the existence of common PT shocks, we can not orthogonalize $\varepsilon_{t}^{P}$ and $\varepsilon_{t}^{T}$ by a Cholesky decomposition of their variance matrices. However, we can isolate the component of the common permanent [transitory] shocks that is perfectly correlated with the permanent [transitory] innovation $\varepsilon_{j t}^{P}\left[\varepsilon_{j t}^{T}\right]$, for $j=1,2, \ldots, n$, which has a contemporaneous effect on the $j$ th country output. These components are respectively given by

$$
u_{j t}^{P, D}=\mathrm{E}\left(u_{t}^{P} \varepsilon_{j t}^{P}\right)\left[\mathrm{E}\left(\varepsilon_{j t}^{P}\right)^{2}\right]^{-1} \varepsilon_{j t}^{P} \quad \text { and } \quad u_{j t}^{T, D}=\mathrm{E}\left(u_{t}^{T} \varepsilon_{j t}^{T}\right)\left[\mathrm{E}\left(\varepsilon_{j t}^{T}\right)^{2}\right]^{-1} \varepsilon_{j t}^{T}
$$

We define $u_{j t}^{P, D}\left[u_{j t}^{T, D}\right]$ as the permanent [transitory] domestic shocks of the $j$ th country. Consequently, we require that the permanent [transitory] foreign shocks of the $j$ th country are the components of $u_{t}^{P}\left[u_{t}^{T}\right]$ that are uncorrelated with $\varepsilon_{j t}^{P}\left[\varepsilon_{j t}^{T}\right]$. Such PT country-specific foreign shocks respectively read

$$
\begin{equation*}
u_{j t}^{P, F}=u_{t}^{P}-u_{j t}^{P, D} \quad \text { and } \quad u_{j t}^{T, F}=u_{t}^{T}-u_{j t}^{T, D} \tag{14}
\end{equation*}
$$

In view of equations (13) and (14), it is easy to verify that both $u_{j t}^{P, F}$ and $u_{j t}^{T, F}$ have no contemporaneous effects on the $j$ th country output.

The identification of such PT domestic-foreign shocks allows us to decompose the $j$ th country output $X_{j t}$ as follows

$$
\begin{equation*}
X_{j t}=\underbrace{P_{j t}^{D}+P_{j t}^{F}}_{P_{j t}}+\underbrace{T_{j t}^{D}+T_{j t}^{F}}_{T_{j t}}, \tag{15}
\end{equation*}
$$

where $\Delta P_{j t}^{D}=e_{j}^{\prime} P(L) u_{j t}^{P, D}, \Delta P_{j t}^{F}=e_{j}^{\prime} P(L) u_{j t}^{P, F}, \Delta T_{j t}^{D}=e_{j}^{\prime} T(L) u_{j t}^{T, D}, \Delta T_{j t}^{F}=e_{j}^{\prime} T(L) u_{j t}^{T, F}$, and $e_{j}$ is an $n$-vector with unity as its $j$ th element and zeroes elsewhere.

Moreover, since each component in the right hand side of equation (15) is independent from the others, we can write spectrum $F_{j}(\omega)$ of the $j$ th country output as follows

$$
\begin{equation*}
F_{j}(\omega)=F_{P j}^{D}(\omega)+F_{P j}^{F}(\omega)+F_{T j}^{D}(\omega)+F_{T j}^{F}(\omega) \tag{16}
\end{equation*}
$$

where

$$
F_{j}(\omega)=\frac{1}{2 \pi} e_{j}^{\prime} C^{*}(z)-C^{*}\left(z^{-1}\right)^{\prime} e_{j}
$$

$$
\begin{gathered}
F_{P j}^{D}(\omega)=\frac{1}{2 \pi} e_{j}^{\prime} P^{*}(z)-{ }_{j}^{P, D} P^{*}\left(z^{-1}\right)^{\prime} e_{j}, \\
F_{P j}^{F}(\omega)=\frac{1}{2 \pi} e_{j}^{\prime} P^{*}(z)-{ }_{j}^{P, F} P^{*}\left(z^{-1}\right)^{\prime} e_{j}, \\
F_{T j}^{D}(\omega)=\frac{1}{2 \pi} e_{j}^{\prime} T^{*}(z)-{ }_{j}^{T, D} T^{*}\left(z^{-1}\right)^{\prime} e_{j}, \\
F_{T j}^{F}(\omega)=\frac{1}{2 \pi} e_{j}^{\prime} T^{*}(z)-{ }_{j}^{T, F} T^{*}\left(z^{-1}\right)^{\prime} e_{j}, \\
-{ }_{j}^{P, D}=\mathrm{E}\left(u_{j t}^{P, D} u_{j t}^{P, D \prime}\right),-{ }_{j}^{T, D}=\mathrm{E}\left(u_{j t}^{T, D} u_{j t}^{T, D \prime}\right),-{ }_{j}^{P, F}=\mathrm{E}\left(u_{j t}^{P, F} u_{j t}^{P, F \prime}\right),-{ }_{-}^{T, F}=\mathrm{E}\left(u_{j t}^{T, F} u_{j t}^{T, F \prime}\right), \\
\Delta C^{*}(L)=C(L), \Delta P^{*}(L)=P(L), \Delta T^{*}(L)=T(L), z=\exp (-i \omega) \text { for } \omega \in(0, \pi],{ }^{7} \text { and } \\
C^{*}(z)=\left[\Gamma(z)(1-z)-\alpha \beta^{\prime} z\right]^{-1}, \text { for } z \neq 1 .^{8}
\end{gathered}
$$

The spectra in the right hand side of equation (16) can be interpreted as follows. The spectrum $F_{P j}^{F}(\omega)\left[F_{T j}^{F}(\omega)\right]$ measures the variability of the $j$ th country output at frequency $\omega$ that is explained by the $j$ th country permanent [transitory] foreign shocks. Similarly, the spectrum $F_{P j}^{D}(\omega)\left[F_{T j}^{D}(\omega)\right]$ measures the variability of the $j$ th country output at frequency $\omega$ that is explained by the $j$ th country permanent [transitory] domestic shocks. We can finally propose our measures of the contribution of PT foreign [domestic] shocks to the variability of the $j$ th country output at the business cycle frequency band.

Definition 1 (Measures of the business cycle effects of PT foreign shocks). Let $I_{P j}^{F}\left(\omega_{0}, \omega_{1}\right)\left[I_{T j}^{F}\left(\omega_{0}, \omega_{1}\right)\right]$ indicate the relative measure of the spectral mass of the $j$ th country output at the business cycle frequency band $\left[\omega_{0}, \omega_{1}\right]$ that is explained by the $j$ th country permanent [transitory] foreign shocks, where $0<\omega_{0}<\omega_{1} \leq \pi$. Then we have

$$
I_{P j}^{F}\left(\omega_{0}, \omega_{1}\right)=\frac{\int_{\omega_{0}}^{\omega_{1}} F_{P j}^{F}(\omega) d \omega}{\int_{\omega_{0}}^{\omega_{1}} F_{j}(\omega) d \omega} \text { and } I_{T j}^{F}\left(\omega_{0}, \omega_{1}\right)=\frac{\int_{\omega_{0}}^{\omega_{1}} F_{T j}^{F}(\omega) e_{j} d \omega}{\int_{\omega_{0}}^{\omega_{1}} F_{j}(\omega) d \omega},
$$

for $j=1, \ldots, n$.
Definition 2 (Measures of the business cycle effects of PT domestic shocks). Let $I_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)\left[I_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)\right]$ indicate the relative measure of the spectral mass of the $j$ th country

[^6]output at the business cycle frequency band $\left[\omega_{0}, \omega_{1}\right]$ that is explained by the $j$ th country permanent [transitory] domestic shocks. Then we have
$$
I_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)=\frac{\int_{\omega_{0}}^{\omega_{1}} F_{P j}^{D}(\omega) d \omega}{\int_{\omega_{0}}^{\omega_{1}} F_{j}(\omega) d \omega} \text { and } I_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)=\frac{\int_{\omega_{0}}^{\omega_{1}} F_{T j}^{D}(\omega) d \omega}{\int_{\omega_{0}}^{\omega_{1}} F_{j}(\omega) d \omega},
$$
for $j=1, \ldots, n$.
Remark 3 In view of equations (15) and (16), we see that the measures of the business cycle effects of the PT shocks proposed by Centoni and Cubadda (2003) are respectively given by
$$
I_{P j}\left(\omega_{0}, \omega_{1}\right)=I_{P j}^{F}\left(\omega_{0}, \omega_{1}\right)+I_{P j}^{D}\left(\omega_{0}, \omega_{1}\right) \quad \text { and } \quad I_{T j}\left(\omega_{0}, \omega_{1}\right)=I_{T j}^{F}\left(\omega_{0}, \omega_{1}\right)+I_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)
$$
for $j=1, \ldots, n$.

Remark 4 Based on decomposition (15), the relative contributions of foreign and domestic shocks to the variability of the $j$ th country business cycle respectively read

$$
I_{j}^{F}\left(\omega_{0}, \omega_{1}\right)=I_{P j}^{F}\left(\omega_{0}, \omega_{1}\right)+I_{T j}^{F}\left(\omega_{0}, \omega_{1}\right) \quad \text { and } \quad I_{j}^{D}\left(\omega_{0}, \omega_{1}\right)=I_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)+I_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)
$$

for $j=1, \ldots, n$.

### 3.2 Estimation

Estimation of the statistics $I_{P j}^{F}\left(\omega_{0}, \omega_{1}\right), I_{T j}^{F}\left(\omega_{0}, \omega_{1}\right), I_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)$, and $I_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)$ can be summarized by the following six steps:

1. Test for cointegration and SCCF and consequently find and fix $r$ and $s$. Estimate then a VECM, possibly under the SCCF restrictions, and derive consistent estimates of $\alpha, \alpha_{\perp}$, $\beta, \Gamma(L)$, and - respectively denoted by $\widehat{\alpha}, \widehat{\alpha}, \widehat{\beta}, \widehat{\Gamma}(L)$, and $\widehat{-} ;{ }^{9}$
2. Based on the VECM residuals $\widehat{\varepsilon}_{t}$, construct $\widehat{u}_{t}^{P}=\widehat{\alpha}_{\perp}^{\prime} \widehat{\varepsilon}_{t}, \widehat{\varepsilon}_{t}^{P}=\widehat{-} \widehat{\alpha}_{\perp}^{\prime}\left(\widehat{\alpha}_{\perp}^{\prime} \widehat{-} \widehat{\alpha}_{\perp}\right)^{-1} \widehat{u}_{t}^{P}, \widehat{u}_{t}^{T}=$ $\widehat{\alpha}^{\prime}{ }^{-}{ }^{-1} \widehat{\varepsilon}_{t}$, and $\widehat{\varepsilon}_{t}^{T}=\widehat{\alpha}\left(\widehat{\alpha}^{\prime}{ }_{-}{ }^{-1} \widehat{\alpha}\right)^{-1} \widehat{u}_{t}^{T}$;

[^7]3. Compute $\widehat{u}_{j t}^{P, D}\left[\widehat{u}_{j t}^{T, D}\right]$ as the fitted values of a regression of $\widehat{u}_{t}^{P}\left[\widehat{u}_{t}^{T}\right]$ on $\widehat{\varepsilon}_{j t}^{P}\left[\widehat{\varepsilon}_{j t}^{T}\right]$ and construct $\widehat{u}_{j t}^{P, F}=\widehat{u}_{t}^{P}-\widehat{u}_{j t}^{P, D}$ for $j=1,2, \ldots, n$;
4. Obtain $\widehat{-}{ }_{j}{ }_{j}, D, \widehat{\wedge}_{j}^{T, D}$, and $\widehat{-}{ }_{j}, F$ respectively as the sample covariance matrices of the vector series $\widehat{u}_{j t}^{P, D}, \widehat{u}_{j t}^{T, D}$, and $\widehat{u}_{j t}^{P, F}$ for $j=1,2, \ldots, n$;
5. Construct $\widehat{C}^{*}\left(z_{k}\right)=\left[\widehat{\Gamma}\left(z_{k}\right)\left(1-z_{k}\right)-\widehat{\alpha} \widehat{\beta}^{\prime} z_{k}\right]^{-1}, \widehat{P}^{*}\left(z_{k}\right)=\widehat{C}^{*}\left(z_{k}\right)-\widehat{\alpha} \widehat{\alpha}_{\perp}\left(\widehat{\alpha}_{\perp}^{\prime}-\widehat{\alpha} \widehat{\alpha}_{\perp}\right)^{-1}$, and $\widehat{T}^{*}\left(z_{k}\right)=\widehat{C}^{*}\left(z_{k}\right) \widehat{\alpha}\left(\widehat{\alpha}^{\prime}-\widehat{\alpha}\right)^{-1}$, where $z_{k}=\exp \left(-i \omega_{k}\right)$, and $\omega_{k}=\omega_{0}\left(\frac{m-k}{m}\right)+\omega_{1}\left(\frac{k}{m}\right)$ for $k=0,1, \ldots, m$;
6. Obtain
\[

$$
\begin{gathered}
\widehat{I}_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)=\left[\sum_{k=0}^{m} e_{j}^{\prime} \widehat{P}^{*}\left(z_{k}\right) \hat{-}_{j}^{P, D} \widehat{P}^{*}\left(z_{k}^{-1}\right)^{\prime} e_{j}\right]\left[\sum_{k=0}^{m} e_{j}^{\prime} \widehat{C}^{*}\left(z_{k}\right) \widehat{-} \widehat{C}^{*}\left(z_{k}^{-1}\right)^{\prime} e_{j}\right]^{-1}, \\
\widehat{I}_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)=\left[\sum_{k=0}^{m} e_{j}^{\prime} \widehat{T}^{*}\left(z_{k}\right) \widehat{-}_{j}^{T, D} \widehat{T}^{*}\left(z_{k}^{-1}\right)^{\prime} e_{j}\right]\left[\sum_{k=0}^{m} e_{j}^{\prime} \widehat{C}^{*}\left(z_{k}\right) \widehat{-} \widehat{C}^{*}\left(z_{k}^{-1}\right)^{\prime} e_{j}\right]^{-1}, \\
\widehat{I}_{P j}^{F}\left(\omega_{0}, \omega_{1}\right)=\left[\sum_{k=0}^{m} e_{j}^{\prime} \widehat{P}^{*}\left(z_{k}\right) \hat{-}^{P}{ }_{j}, \widehat{P}^{*}\left(z_{k}^{-1}\right)^{\prime} e_{j}\right]\left[\sum_{k=0}^{m} e_{j}^{\prime} \widehat{C}^{*}\left(z_{k}\right) \widehat{-} \widehat{C}^{*}\left(z_{k}^{-1}\right)^{\prime} e_{j}\right]^{-1}, \\
\widehat{I}_{T j}^{F}\left(\omega_{0}, \omega_{1}\right)=1-\widehat{I}_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)-\widehat{I}_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)-\widehat{I}_{P j}^{F}\left(\omega_{0}, \omega_{1}\right),
\end{gathered}
$$
\]

for $j=1, \ldots, n$.

The suggested measures are rather involved functions of the estimated VECM parameters and this complicates the analytical evaluation of their sample variability. Hence, we rely on a bootstrap procedure similar as the one suggested by Gonzalo and Ng (2001). First, we fix both $r$ and $s$ and estimate the VECM by the ML procedure. ${ }^{10}$ Second, we obtain the residuals $\widehat{\varepsilon}_{t}$ by replacing the unknown parameters in equations (3) or (7) with their estimated values. Third, a new sample of data is constructed using a random sample of $\widehat{\varepsilon}_{t}$ with replacement and the initial estimates of the VECM parameters. Fourth, the VECM is re-estimated with the new sample and the associated estimates of the spectral measures are stored. This procedure is repeated 5000 times and the quantiles of the empirical distributions of the bootstrapped $\widehat{I}_{P j}^{F}\left(\omega_{0}, \omega_{1}\right)$, $\widehat{I}_{T j}^{F}\left(\omega_{0}, \omega_{1}\right), \widehat{I}_{P j}^{D}\left(\omega_{0}, \omega_{1}\right)$, and $\widehat{I}_{T j}^{D}\left(\omega_{0}, \omega_{1}\right)$ are then used to construct confidence intervals.

[^8]Table 1: Two-step and iterative Johansen's trace tests

|  | $r=0$ | $r \leq 1$ | $r \leq 2$ | $r \leq 3$ |
| :--- | :---: | :---: | :---: | :---: |
| 2-step | $68.00^{*}$ | 35.86 | 16.74 | 4.81 |
| iterative | $73.42^{*}$ | $41.75^{* *}$ | 20.48 | 7.18 |
| Note: ${ }^{*}\left({ }^{* *}\right)$ reject at $5 \%(10 \%)$ |  |  |  |  |

## 4 Empirical analysis

We apply the previous measures to the industrial production indexes (IPI hereafter) in the manufacturing sector. We consider Canada, the US, Japan as well as the Euro area as a whole. Quarterly seasonally adjusted indexes are taken from OECD databases and the sample spans 1980:Q1 to 2005:Q3, hence $T=103$ observations for $n=4$. A VAR(4) seems to appropriately characterize the dynamic properties of the data according to usual diagnostic test for no autocorrelation, homoskedasticity, and normality of the residuals. Moreover, this model is also checked for parameter constancy by means of the recursive test by Bai et al. (1998). It turns out that 1993:Q4 is the date at which the Chow test statistic takes the largest value in the trimming region $[0.15,0.85]$. However, the sup-Chow test statistic is equal to 26.96 , which is insignificant at the $10 \%$ level according to the critical values tabulated in Andrews (1993).

### 4.1 Common trends - common cycles

In order to test for the presence of common permanent and transitory shocks, we use Johansen's trace statistics for cointegration with a deterministic trend restricted in the long-run in order to capture the differences among the average growth rates of the various national indicators. Table 1 reports the values of both asymptotic and switching trace statistics. In this latter case we have found and consequently imposed one WF cofeature vector, i.e. $s=1$.

With the asymptotic procedure we do not reject the presence of a single cointegrating vector, while two vectors are present at $10 \%$ with the iterative approach ( 42.20 being the critical value at a $5 \%$ level in Johansen, one would probably accept a second cointegrating vector at around $6 \%$ ). This implies that industrial production indexes are driven by two common permanent shocks and two common transitory shocks. The two normalized cointegrating vectors in the iterative approach with $s=1$ are as follows: $\hat{\beta}_{1}^{\max ^{\prime}} X_{t}=\left(\ln E U_{t}-0.65 \ln U S_{t}-0.37 \ln\right.$ Japp+0.0017 trend $)$ and $\hat{\beta}_{2}^{\max ^{\prime}} X_{t}=\left(\ln C a_{t}-0.523 \ln U S_{t}+0.69 \ln E U_{t}-0.0068\right.$ trend $)$. The estimation of standard errors for the long-run components in the iterative approach is still under investigation. However,

Table 2: SCCF test statistics

|  | $s=0$ | $s \geq 1$ | $s \geq 2$ | $s \geq 3$ | $s=4$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| p-value LR | - | 0.14 | $<0.001$ | $<0.001$ | $<0.001$ |
| $p$-value LRcor | - | 0.25 | $<0.001$ | $<0.001$ | $<0.001$ |
| HQ citerion | -35.668 | -35.847 | -35.773 | -35.616 | -35.167 |

we reject at any sensible level ( $p$-values $<0.001$ ) the null hypotheses that a country must be excluded from both cointegrating vectors.

Economic activity growth rates also exhibit a cyclical pattern whose similarity is tested through the SCCF analysis. We fix at $r=2$ the number of cointegrating vectors given the result above and we continue with the SCCF analysis using the cointegrating vectors obtained at the end of the iterative procedure. Table 2 reports the $p$-values associated with both the LR test statistic in (9) and a small-sample corrected version. It emerges that we cannot exclude the presence of one SCCF vector. HQ, the information criteria favoured in Hecq (2006), also indicates $s=1$. The normalized SCCF vector $\Delta \ln U S_{t}-0.820 \Delta \ln C a_{t}-0.045 \Delta \ln J a p_{t}-$ $0.326 \Delta \ln E U_{t}$ mainly links the US and Canada, a result already obtained by Engle and Kozicki (1993) on another period. We conclude that there are three common transmission mechanisms of the national shocks for the four economies.

### 4.2 Contributions of domestic-foreign shocks

In order to asses the relative importance of domestic and foreign PT shocks in contributing to the national business cycle, we apply the measures proposed in the previous section for $p=4$, $s=1$ and $r=2$. Figure 1 shows the log-levels of the various IPIs as well as their permanent components, whereas Figures 2 and 3 provide, respectively, the graphs of the domestic and foreign contributions to the PT components. From Figure 1 it was already clear that the permanent component contributes for the largest part of the observed variations in each IPI. Now, a careful decomposition shows that the variability of the permanent component is due to its domestic contribution in Japan, Canada and to a lesser extent in the US while the amplitude of the foreign component is as large as the domestic part in the Euro area. In Figure 3 we observe large swings for both the domestic and the foreign contributions of the transitory component in the four countries.

Although they are informative on the phases and the amplitudes of important components of the economic activity, it is not trivial to get a simple and concise measure from these pictures.

This is the reason why we now analyze these components in the frequency domain with the view to extracting the information at the business cycle frequencies. Hence, the spectra of each industrial production index and its components are derived from the estimated parameters of the VECM. Figure 4 shows indeed that for the four countries, spectral densities of contributions to shocks have a peak at business cycle frequencies. Moreover, it is noticeable that the permanentdomestic contribution is the dominant component at the business cycle frequency for all countries but the Euro area for which this is the permanent-foreign contribution that has the highest value.

We then focus on the spectra of each IPI and its components at the frequencies corresponding to 8-32 quarter periods. In particular, these spectra are computed for $\omega_{k}=\frac{\pi}{16}\left(\frac{199-k}{199}\right)+\frac{\pi}{4}\left(\frac{k}{199}\right)$ and $k=0,1, \ldots, 199$. Table 3 gives the estimated measures of the cyclical effects of the various shocks along with the $95 \%$ bootstrapped confidence bounds in brackets. Some comments are in order.

First, the results clearly indicate the dominant role of the permanent shocks over the business cycles. However, the contribution of permanent shocks at business cycle frequencies is lower than in several studies. Indeed, from Table 3 we see that permanent shocks account for about $60 \%$ of cyclical variations in the IPIs of the Euro area and the US, and up to $94 \%$ for Japan.

Second, we turn to evaluating the importance of the domestic and foreign shocks on the different economies at the business cycle frequencies. It emerges that for Japan, Canada, and the US the foreign component of the business cycle is small. Due to its higher degree of openness, the Eurozone is more sensitive to foreign shocks with a proportion around $47 \%$.

Third, the larger the degree of exposition to foreign shocks, the larger the contribution of permanent shocks to the foreign component of the business cycle. If one makes the usual assumption that only productivity shocks have permanent effects on the level of output (see e.g. Dufourt, 2005), this result is consistent with the view that international technology diffusion is an important propagation mechanism of permanent shocks across countries. An important force that generates technology spillovers among countries is international trade of input goods, see e.g. Coe and Helpman (1995), and Eaton and Kortum (2001). ${ }^{11}$

Fourth, for all the countries but Japan the domestic component clearly dominates the cyclical effects of transitory shocks. This finding is in line with the interpretation that transitory shocks are mainly connected to country-specific monetary and fiscal policies.

[^9]It is of interest to compare the above results with previous atheoretical analysis based on dynamic factor models, even though most of those studies focused on the distinction between country-specific and international shocks rather than between domestic and foreign shocks. Gregory et al. (1997) and Kose et al. (2003) attributed a more limited role to country-specific shocks over the national business cycles. A possible explanation of these differences is that our specification of the domestic and foreign shocks does not require the overidentifying restrictions that a factor model imposes to the data.

## 5 Conclusions

The empirical example of the previous section shows that the methods proposed in this paper are useful to tackle, in a coherent and integrated setting, issues that were often analyzed independently in previous studies. These issues are precise and testable definitions of common shocks and common propagation mechanisms across countries as well as an assessment of the relative importances of the sources of the business cycles, namely domestic-permanent, foreignpermanent, domestic-transitory and foreign-transitory shocks. With these elements at hand, it is thus possible to provide a detailed statistical picture of the international comovements that theoretical models should account for.

We think that our methodology offers two significative advantages over existing approaches for the analysis of international business cycle. First, differently from most previous studies based on structural VAR's (see e.g. Mellander et al., 1992; Ahmed et al., 1993; Kwark, 1999), our framework allows for unravelling domestic and foreign PT shocks from a set of national outputs without limiting the analysis to two-country models. Second, we mainly propose an atheoritical setting. There are pros and cons concerning the use of an atheoritical framework. On the one hand, we believe that theoretical reasoning can help to disentangle the source of the various shocks within a structural VAR analysis of different variables for a country. This is the case when modelling, for instance, output, money, interest rates and prices. In our setting, in which we want to consider the same variable such as output for a set of countries, not having a priory may be of value added. On the other hand, differently from other atheoretical analyses based on dynamic factor models (see e.g. Gregory et al., 1997; Kose et al., 2003), our study allows for identifying a pair of PT foreign shocks for each country rather than a common worldwide shock that can have both permanent and transitory effects on output data of a set of countries.

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Table 3: Measures of the BC effects of Domestic-Foreign PT shocks ( $\mathrm{r}=2, \mathrm{~s}=1$ )

| Canada |  | Permanent | Transitory | Total |
| :--- | :--- | :---: | :---: | :---: |
|  | Domestic | $0.613[0.315-0.805]$ | $0.223[0.107-0.368]$ | $0.863[0.584-0.942]$ |
|  | Foreign | $0.076[0.005-0.330]$ | $0.088[0.030-0.170]$ | $0.164[0.058-0.416]$ |
|  | Total | $0.689[0.493-0.848]$ | $0.311[0.152-0.506]$ |  |
| US |  | Permanent | Transitory | Total |
|  | Domestic | $0.401[0.083-0.654]$ | $0.311[0.147-0.515]$ | $0.712[0.370-0.881]$ |
|  | Foreign | $0.176[0.027-0.552]$ | $0.112[0.045-0.184]$ | $0.288[0.119-0.629]$ |
|  | Total | $0.577[0.339-0.789]$ | $0.423[0.211-0.661]$ |  |
| Japan | Domestic | $0.925[0.706-0.951]$ | $0.016[0.005-0.092]$ | $0.941[0.738-0.967]$ |
|  | Foreign | $0.013[0.007-0.222]$ | $0.046[0.014-0.085]$ | $0.059[0.033-0.261]$ |
|  | Total | $0.938[0.859-0.971]$ | $0.062[0.029-0.141]$ |  |
|  | Domestic | $0.237[0.034-0.606]$ | $0.289[0.115-0.506]$ | $0.526[0.264-0.808]$ |
|  | Foreign | $0.394[0.115-0.674]$ | $0.080[0.037-0.156]$ | $0.474[0.192-0.736]$ |
|  | Total | $0.631[0.368-0.837]$ | $0.369[0.162-0.632]$ |  |

Figure 1: Industrial production indexes and their permanent components


Figure 2: First differences of the domestic-foreign permanent components


Figure 3: Domestic-foreign transitory components


Figure 4: Spectra of the component growth rates



[^0]:    *Previous versions of this paper were presented at the 58th European Meeting of the Econometric Society in Stockholm, at the Latin American Meeting of the Econometric Society in Panama City, and the 4th Eurostat Colloquium on Modern Tools for Business Cycle Analysis in Luxembourg. We thank the participants for useful comments. Marco Centoni and Gianluca Cubadda gratefully acknowledge financial support from MIUR. Alain Hecq gratefully acknowledges financial support from METEOR through the research project "Macroeconomic Consequences of Financial Instability" as well as the hospitality of the department SEGeS, University of Molise, for the period during which this paper was written.
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[^1]:    ${ }^{1}$ It should be noticed that, since many economic service activities are closely linked with industrial output, the cycles of industrial production and GDP are closely related, see OECD (2003). Moreover, a recent empirical investigation confirms that the industrial production index is the most cyclical coincident indicator of the US business cycle (Cubadda, 2006).
    ${ }^{2}$ Eurozone or Euro area refers to European member states in which the euro has been adopted as the single

[^2]:    currency.
    ${ }^{3}$ We thank referees for having put forward this element. A previous draft of this paper considered the G7 economies.

[^3]:    ${ }^{4}$ The assumption that $\alpha_{\perp}^{\prime} \varepsilon_{t}$ are the permanent shocks is rather common in the literature, see inter alia Warne (1993), Gonzalo and Granger (1995), Johansen (1996), and Gonzalo and Ng (2001).
    ${ }^{5}$ Remarkably, the above decomposition is invariant to rotation of the matrices $\alpha_{\perp}$ and $\alpha$ and non-singular linear transformations of the set of common shocks $u_{t}^{P}$ and $u_{t}^{T}$. Hence, series $X_{t}$ can be separated into independent PT

[^4]:    components without using a priori economic theory.

[^5]:    ${ }^{6}$ See inter alia Ho and Sorensen 1996; Gonzalo and Pitarakis 1999; Cheung and Lai 1993; Söderlind and Vredin 1996; Jacobson, Vredin and Warne 1998.

[^6]:    ${ }^{7}$ We do not consider the case $\omega=0$ since the pseudo-spectral density matrix of series $X_{t}$ is unbounded at frequency zero due to the presence of unit roots at that frequency.
    ${ }^{8}$ As noticed in Cubadda and Centoni (2003), the matrix $A(z) \equiv\left[\Gamma(z)(1-z)-\alpha \beta^{\prime} z\right]$ is invertible for $z \neq 1$ due to Assumption (2).

[^7]:    ${ }^{9}$ See e.g. Gonzalo and Granger (1995) on estimation of $\alpha_{\perp}$.

[^8]:    ${ }^{10}$ The coefficients of the $\beta$ and $\delta$ matrices are estimated even though $r$ and $s$ are fixed.

[^9]:    ${ }^{11}$ See e.g. Keller (2004) for a detailed survey on the importance of various channels of international technology diffusion.

