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Seasonal and long-run fractional integration in the Industrial Production Indexes of some Latin American countries

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Abstract

In this article we propose a new approach that permits us to simultaneously test unit and fractional roots at the long-run and the seasonal frequencies. We examine the Industrial Production Indexes (IPI) in four Latin American countries (Brazil, Argentina, Colombia and Mexico), using new statistical tools based on seasonal and non-seasonal long-memory processes. Results show that the root at the long-run or zero frequency plays a much more important role than the seasonal one. Nevertheless, in the cases of Brazil and Argentina a component of long memory behaviour is also present at the seasonal structure, indicating that shocks modify the seasonal structure for a long period. Policy makers should thus pay attention to this result in choosing the optimal economic policy.

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1. Introduction

The recent economic literature has stressed two major characteristics of macroeconomic series which has been used by academics or economic advisers to better

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understand the consequences of economic policies. First, since the seminal paper of Nelson and Plosser (1982), it appears that almost all series possess a unit root. The consequences of the presence of a stochastic trend are major for the modelling and the decision of economic policies, as a shock affecting one of these series will be persistent and will never die out. The second stylized fact is the presence of seasonal components. Traditionally, seasonal fluctuations have been considered as a nuisance that shadows the most important components of the series (namely the growth and the business cyclical components, e.g., Burns & Mitchell, 1946). They are often removed beforehand via seasonal adjustment procedures. Thus, most of the empirical works considers simply seasonally adjusted data, without paying attention to the role of seasonality. A lot of papers have stressed on one of these two features, but only a few have been concerned with the linkages between the seasonal and the long-run structures.

Hasza and Fuller (1982) have shown that seasonal adjustment biases traditional unit root tests. Such errors are very difficult to measure and so to correct by systematic procedures. Nevertheless, the consequences for political advisors are strong as the persistence of shocks are not correctly determined, leading to misperception in the consequences of economic policies. Moreover, seasonal adjustment procedures do not generally take into account the possible fractional roots at seasonal frequencies, leading thus to erroneous conclusions about its behaviour. For example, a structural economic policy (aiming at modifying the long-run behaviour) may induce a persistent modification of the seasonal structure, ruling out the initial objectives.

In this article we develop a new framework allowing for unit and fractional roots simultaneously at zero (long-run) and the seasonal frequencies. It constitutes an alternative to test for (fractional) unit roots without using any prior seasonal adjustment. It is based on fractionally integrated techniques. To illustrate our theoretical concept, we analyse the monthly structure of several Industrial Production Indexes (IPI) in some Latin American countries. The results suggest that the series can be well characterized in terms of $I(d)$ processes with roots at both at the zero and the seasonal (monthly) frequencies. The structure of the paper is as follows: Section 2 briefly describes a model that incorporates seasonal and non-seasonal roots. Section 3 presents a version of the tests of Robinson (1994) that permits us to test this type of model. In Section 4, the tests are applied to IPI series, while Section 5 contains some concluding comments.

2. A model with seasonal and non-seasonal unit and fractional roots

It is a well-known fact that many time series contain important seasonal components and seasonal first differences is the easiest way to remove the seasonal component. Dickey, Hasza, and Fuller (DHF, 1984); Hylleberg, Engle, Granger, and Yoo (HEGY, 1990), Beaulieu and Miron (1993) and Tam and Reimsel (1997), amongst others proposed test statistics to eliminate the seasonal unit roots in raw

time series. Thus, for example, if x_t is the time series we observe, with a seasonal changing pattern, we can consider the model,

$$(1 - L^s)x_t = u_t, \quad t = 1, 2, \ldots, \tag{1}$$

where L^s is the seasonal lag operator ($L^s x_t = x_{t-s}$); s is the number of time periods in a year, and where u_t is an $I(0)$ process, defined as a covariance stationary process with positive and finite spectral density function at any frequency of the spectrum. Note that the polynomial in (1) can be decomposed into:

$$(1 - L)(1 + L + L^2 + \cdots + L^{s-1}) = (1 - L)S(L). \tag{2}$$

That is, the seasonal difference operator can be broken down into the product of the first difference operator and the moving-average filter $S(L)$, containing further roots of modulus unity. Therefore, the root at the zero frequency appears as a component of the seasonal polynomial in (1). However, there are many cases where this frequency plays a major role, describing not only part of the seasonal structure but also the trending stochastic behaviour of the series. In fact, Hasza and Fuller (1982) consider the model:

$$(1 - L)(1 - L^s)x_t = u_t, \quad t = 1, 2, \ldots, \tag{3}$$

where the first polynomial in (3) represents a stochastic trend, while the second indicates a seasonal structure. In their unit-root test, they suggest to estimate the equations:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-s} + \alpha_3 x_{t-s-1} + u_t,$$

$$x_t = \phi_1 x_{t-1} + \phi_2 (x_{t-s} - \phi_1 x_{t-s-1}) + u_t,$$

and then to test the restrictions $[\alpha_1, \alpha_2, \alpha_3] = [1, 1, -1]$ or $[\phi_1, \phi_2] = [1, 1]$ with a standard F-statistic and using the proper tabulated distributions. Dickey and Pantula (1987) pointed out, however, the inappropriateness of such tests since they are two sided in nature whereas the alternative of stationarity is one-sided.

In this article we generalize the model in the above unit-root test to allow for fractional roots at zero and the seasonal frequencies. Besides, using the LM-based testing procedure developed by Robinson (1994), the tests are the most powerful ones when directed against the appropriate (fractional) alternatives, and due to the use of the latter alternatives, free of the Dickey and Pantula's (1987) criticism. Robinson's (1994) tests have also the advantage of being pivotal, and so finite-sample size distortions can easily be corrected by re-sampling methods such as bootstrap (see, e.g., Candelon & Gil-Alana, 2003).

Hence, model (3) can be viewed as a particular case of a much more general type of long memory processes, allowing fractional roots at zero and the seasonal frequencies of form:

$$(1 - L)^{d_1}(1 - L^s)^{d_2}x_t = u_t, \quad t = 1, 2, \ldots, \tag{4}$$

for given real values d_1 and d_2. Here, the first fractional polynomial can be expressed in terms of its binomial expansion such that for all real d_1,

$$(1-L)^{d_1} = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1(d_1-1)}{2} L^2 - \cdots,$$

and similarly for the seasonal structure,

$$(1-L^s)^{d_2} = \sum_{j=0}^{\infty} \binom{d_2}{j} (-1)^j L^{sj} = 1 - d_2 L^s + \frac{d_2(d_2-1)}{2} L^{2s} - \cdots.$$

Clearly, imposing $d_1 = 0$ and $d_2 = 1$, (4) becomes the seasonal unit root model (1) proposed by Hasza and Fuller (1982), and, imposing $d_1 = d_2 = 1$, we obtain (3). Let us consider first the case of $d_2 = 0$. In this context, if $d_1 > 0$, x_t is said to be a long memory process (at the long-run frequency), so-called because of the strong association between observations widely separated in time. In such a case, the series does not exhibit seasonal movement. The long-run effect of shocks is then crucial for the global dynamic of the process. Similarly, imposing $d_1 = 0$, the model in (4) becomes:

$$(1-L^s)^{d_2} x_t = u_t, \quad t = 1, 2, \ldots, \tag{5}$$

and if $d_2 > 0$, it is called a seasonally fractionally integrated (or seasonal long memory) process. In such a case, the seasonal dynamics are of major importance for the behaviour of the series. Extreme cases appear when $d_2 > 1$. Several studies have explored this topic (see Gil-Alana & Robinson, 2001; Porter Hudak, 1990 inter alia). However, there are not so far empirical works that simultaneously test the degrees of integration at zero and the seasonal frequencies and, in that respect, the present paper considers the general model (4), allowing for simultaneous seasonal and non-seasonal unit and fractional roots, at the same time, trying of being an alternative credible approach to the traditional models based on (1) or (3). This approach constitutes a new framework for empirical analyses with seasonal data. It leads to an unbiased estimation of the degree of the seasonal and the long-run fractional integration. Thus, economics advisers are able to discriminate the optimal economic policy.

3. The testing procedure

Robinson (1994) proposes a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = (d_1, d_2)' = (d_{1o}, d_{2o})' = d_o, \tag{6}$$

in a model given by (4), for any given real numbers d_{1o} and d_{2o}. For the ease of presentation, we consider monthly data, and then assume in the sequel of the paper $s = 12$. Specifically, the test statistic is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4}\hat{a}'\hat{A}^{-1}\hat{a}, \tag{7}$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T}\sum_{j}^{*}\psi(\lambda_j)g(\lambda_j;\hat{\tau})^{-1}I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\hat{\tau})^{-1}I(\lambda_j),$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j}^{*}\psi(\lambda_j)\psi(\lambda_j)' - \sum_{j}^{*}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)'\left(\sum_{j}^{*}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j}^{*}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)'\right)$$

$$\psi(\lambda_j)' = \left[\psi_1(\lambda_j), \psi_2(\lambda_j)\right]; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau});$$

$$\psi_1(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|;$$

$$\psi_2(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right| + \log\left(2\cos\frac{\lambda_j}{2}\right) + \log\left|2\cos\lambda_j\right| + \log\left|2\left(\cos\lambda_j - \cos\frac{\pi}{3}\right)\right| + \log\left|2\left(\cos\lambda_j - \cos\frac{2\pi}{3}\right)\right| + \log\left|2\left(\cos\lambda_j - \cos\frac{\pi}{6}\right)\right| + \log\left|2\left(\cos\lambda_j - \cos\frac{5\pi}{6}\right)\right|,$$

with $\lambda_j = 2\pi j/T$. $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1-L)^{d_{1o}}(1-L^{12})^{d_{2o}}x_t$, and $\hat{\tau} = \arg\min_{\tau\in T^*}\sigma^2(\tau)$ with T^* as a compact subset of the R^q Euclidean space. The function g above is known and comes from the spectral density of u_t:

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi}g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi,$$

and the summation on (*) in the above expressions refers to the discrete bounded frequencies λ_j.

Based on H_o (6), Robinson (1994) established that, under certain regularity conditions:

$$\hat{R} \to_d \quad \chi^2_2, \quad \text{as} \quad T \to \infty. \tag{8}$$

Thus, unlike other procedures, we are in a classical large-sample testing situation by reasons described in Robinson (1994), who also showed that the tests are efficient in the Pitman sense against local departures from the null. A test of (6) will reject H_o against the alternative H_a: $d \neq d_o$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob $(\chi^2_{2,\alpha} > \chi^2_2) = \alpha$.

4. An application to industrial production index in Latin American countries

The time series data used in this section correspond to the log-transformation of the monthly structure of the Industrial Production Indexes series in Brazil, Mexico, Colombia and Argentina, obtained from the IFS database of the International Monetary Fund. These series are seasonally unadjusted. The starting date is January 1980 for Brazil and Mexico, January 1985 for Argentina, and January 1986 for Colombia, and all series end in November-2001.

Fig. 1 contains plots of the original series with their corresponding correlograms and periodograms. The series have a non-stationary appearance and the correlograms clearly show a seasonal pattern, especially for Brazil and Colombia, though a visual inspection at the periodograms also indicates the importance of the root at the long-run frequency. Similar plots for the first differenced data are given in Fig. 2, and the seasonal component, revealed by the 11 peaks in the periodogram, becomes apparent in all cases. Finally, the plots of first and first seasonal (monthly) differences are displayed in Fig. 3, and the correlograms and the periodograms in this figure suggest that the series might now be overdifferenced with respect to both, the zero and the seasonal frequencies.

Denoting the time series by x_t, we employ throughout model (4), testing H_o (6) for values d_{1o}, $d_{2o} = 0$, (0.25), 2, including thus tests for a unit root exclusively at the long-run (zero frequency) ($d_{1o} = 1$, $d_{2o} = 0$); tests for seasonal unit roots ($d_{1o} = 0$, $d_{2o} = 1$); unit and seasonal unit roots ($d_{1o} = d_{2o} = 1$); as well as other fractionally integrated possibilities. We assume that u_t is white noise, and though not reported in the paper, we also consider the case of autocorrelated disturbances, in particular, AR(1) and seasonally monthly AR(1) processes.

The test statistic reported across Table 1 is the one given by $\hat{R}$ in (7). However, instead of presenting the results for all values of d_{1o} and d_{2o}, we just report across the tables those cases where we find at least one non-rejection value for each country. We observe that H_o (6) cannot be rejected for any series when $(d_{1o}, d_{2o}) = (0.00, 1.00)$, implying that a seasonal unit root with white noise u_t may be a plausible way of modelling the series. However, apart from this case, all the remaining non-rejection values take place when $d_{1o} = 1$, 1.25, 1.50 and 1.75 with $d_{2o} = 0$ or 0.25. Thus, we observe higher degrees of integration at the long-run frequency than at the seasonal ones. Some differences appear across countries. More non-rejections are observed for Brazil and Argentina than for Colombia and Mexico. In fact, H_o (6) cannot be rejected when $d_{1o} = 1$, 1.25, 1.50 and 1.75, with $d_{2o} = 0.25$ for Brazil and Argentina, these hypotheses being rejected for the

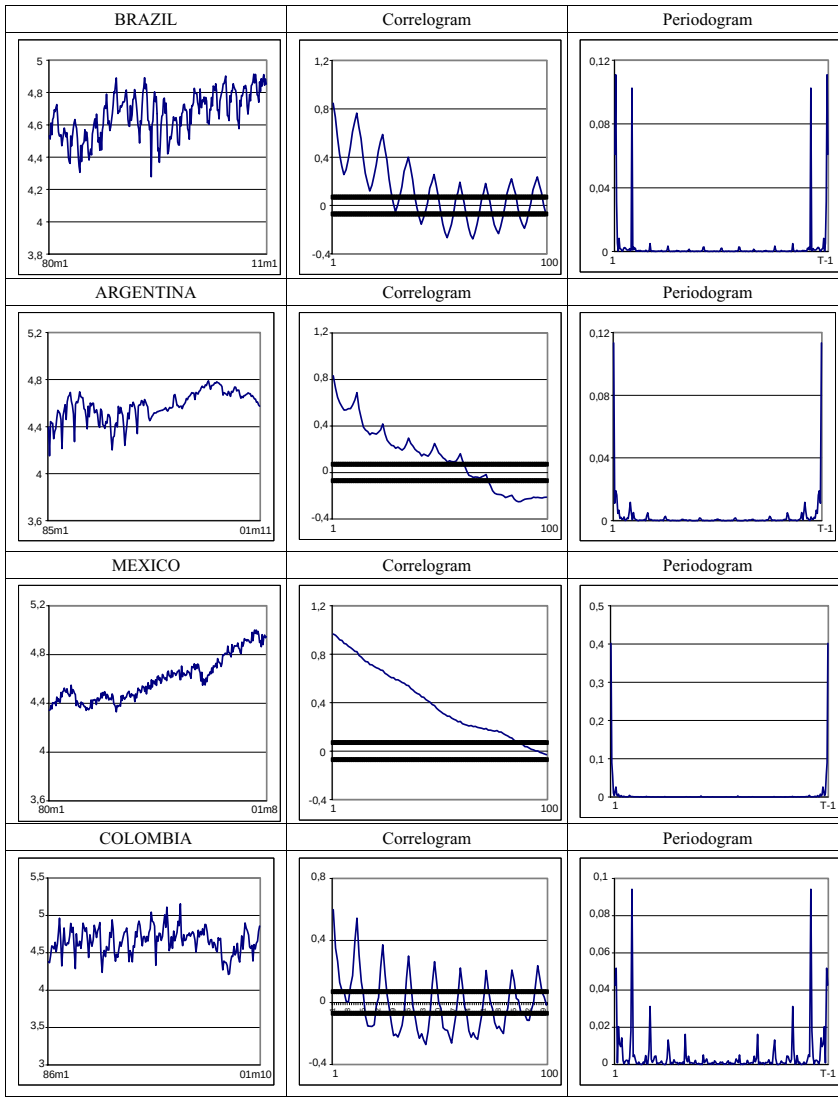

Fig. 1. Log of the Industrial Production Indexes (IPI), with their corresponding correlograms and periodograms. The large-sample standard error under the null hypothesis of no autocorrelation is $1/T^{1/2}$ or roughly 0.07 for series of length considered here.

remaining two countries. Thus, it turns out that models with fractional roots at seasonal frequencies are more rejected for Colombia and Mexico than for Brazil and Argentina, implying that the seasonal structure appears more important for these two later countries. Such a result indicates that seasonal shocks are more persistent in Brazil and Argentina than in Mexico or Colombia.

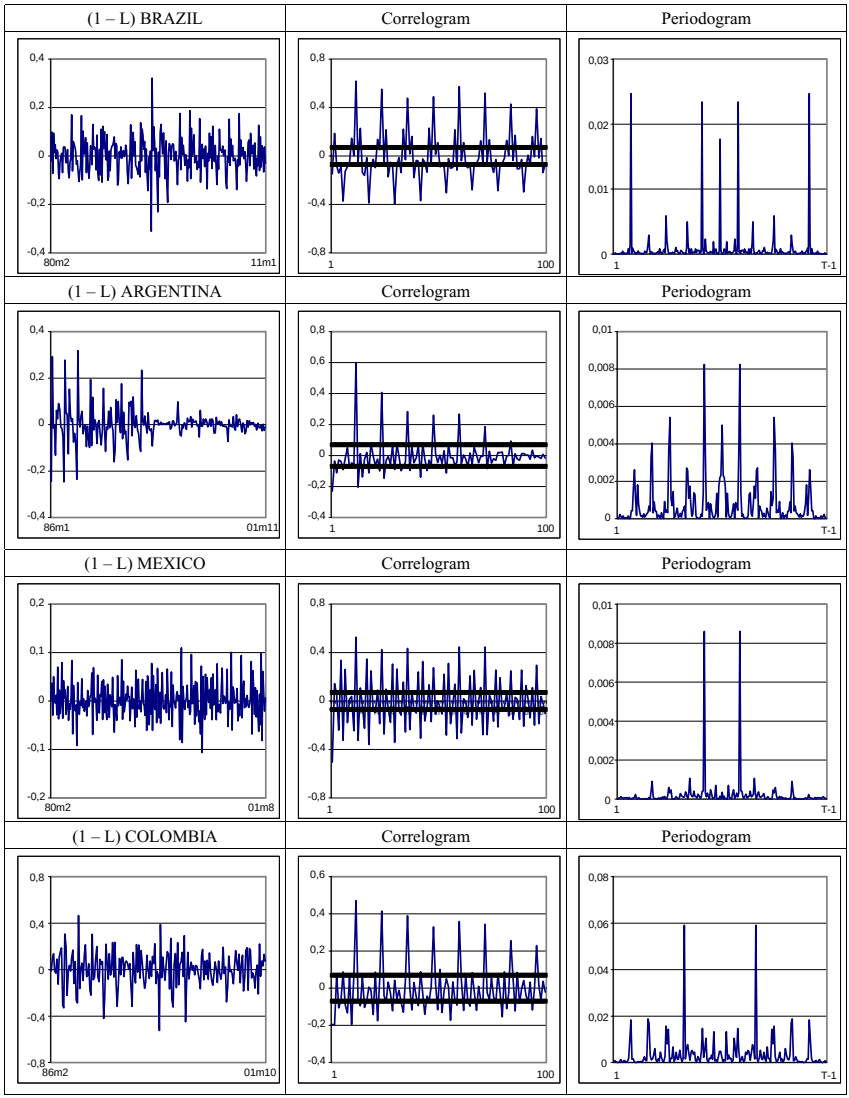

Fig. 2. First differences of the IPI, with their corresponding correlograms and periodograms. The large-sample standard error under the null hypothesis of no autocorrelation is $1/T^{1/2}$ or roughly 0.07 for series of length considered here.

In order to have a more precise view about the message contained in this table, we have performed the same tests, but this time using increments for d_{1o} and d_{2o} of 0.01. Fig. 4 displays the non-rejection regions for each country. We see that d_{1o} is in all cases higher than d_{2o}, stressing once more the importance of the root at zero

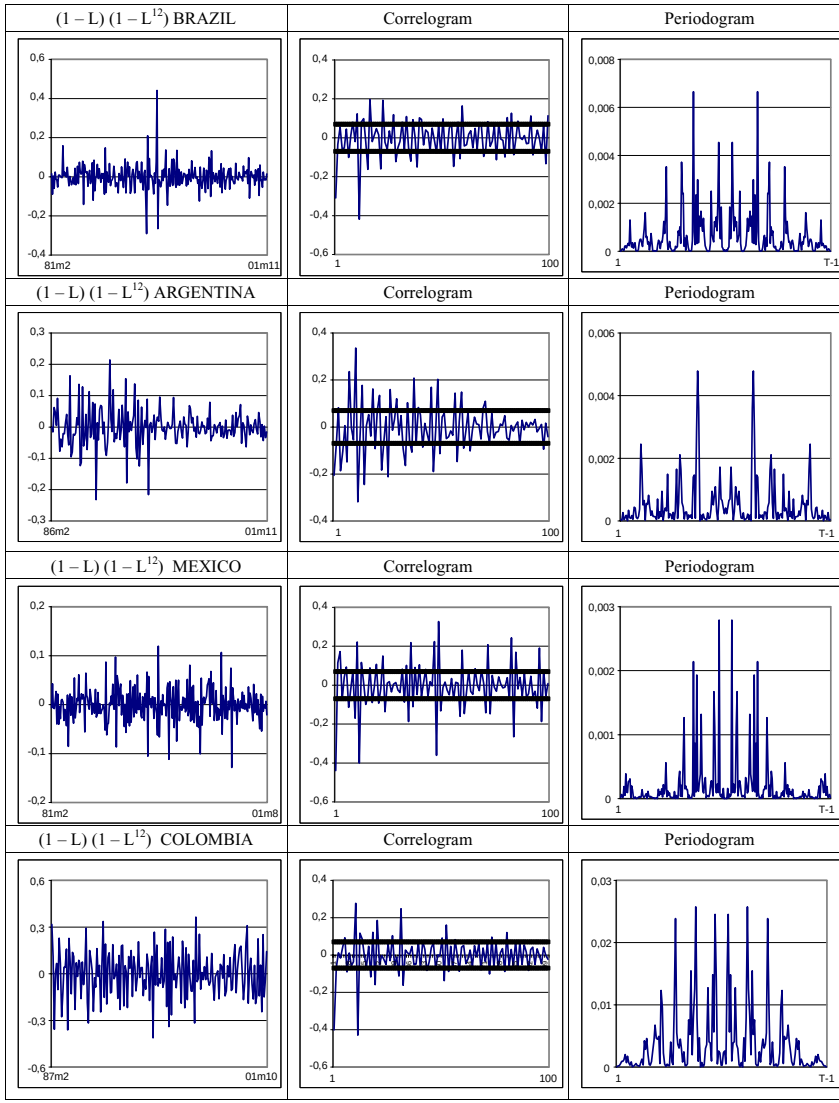

Fig. 3. First and first seasonal differences of the IPI, with their corresponding correlograms and periodograms. The large-sample standard error under the null hypothesis of no autocorrelation is $1/T^{1/2}$ or roughly 0.07 for series of length considered here.

over the seasonal ones. Also, the fact that d_1 is in all cases higher than 0.5 indicates that the component corresponding to the zero frequency is clearly non-stationary. On the other hand, we observe that d_2 is always strictly smaller than 0.5, suggesting that, if the long-run trend is considered, the seasonal structure is stationary though

Table 1
Testing H_o (6) in (4) with the tests of Robinson (1994) and whitenoise disturbances

d_{1o}	d_{2o}	Brasil	Argentina	Mexico	Colombia
0.00	1.00	**1.011**	**0.646**	**1.213**	**3.325**
0.75	0.00	**0.358**	**0.444**	**0.723**	**1.936**
0.75	0.25	**4.105**	**3.330**	**5.751**	**5.528**
1.00	0.00	**0.527**	**0.656**	**1.276**	**2.728**
1.00	0.25	**4.905**	**4.026**	8.089	8.119
1.25	0.00	**0.348**	**0.653**	**1.418**	**2.835**
1.25	0.25	**4.874**	**4.232**	10.943	9.004
1.50	0.00	**0.183**	**0.555**	**1.389**	**2.560**
1.50	0.25	**4.343**	**4.052**	8.885	13.243
1.75	0.00	**0.242**	**0.438**	**1.320**	**2.210**
1.75	0.25	**3.365**	**3.655**	10.045	**5.297**
1.75	0.50	7.654	**5.964**	13.214	11.345

In bold, the non-rejection values at the 95% significance level.

with a component of long memory behaviour. It should also be noted that though the tests are unable to reject the null when $d_{2o} = 0$, i.e., with no seasonal fractional roots, these hypotheses are in all cases "less clearly non-rejected"[1] than when d_{2o} is positive, suggesting that a component of long memory is present in relation with the seasonal part.

These results have thus strong implications in terms of economic policy and planning inference. A particular shock will simultaneously affect the seasonal and the long-run structure of the IPI. Nevertheless, shocks will be mean reverting for the seasonal structure (i.e., their effects disappear in the long-run), whereas the effect will persist on the long-run structure. Thus, a structural economic policy, aiming at modifying the long-run properties of the IPI, can be run out without paying too much attention on the effects on the seasonal structure, as they will quickly die out. Nevertheless, this policy advice has to be modulated for Brazil and Argentina, where the seasonal fractional root is found to lie between 0.3 and 0.6. In these cases, even if shocks are shown to be mean reverting for the seasonal structure, they posses long memory. It means that an economic policy will have a long-run effect and will take time before dying out. Hence, if a structural economic policy is performed in Brazil and Argentina, it will induce a modification of the seasonal structure of the IPI. These effects will not be permanent (the unit-root hypothesis at seasonal frequencies is rejected) but very long lasting. Particular attention should thus be paid to seasonal effects of economic policies in Brazil and Argentina, which is not the case for Colombia and Mexico.

We also performed the tests for the case of autocorrelated disturbances and the results were similar in both cases and also similar to those given in Table 1. Thus,

[1] By 'less clearly non-rejected', we mean that the value of the test statistic is closer to the rejection values.

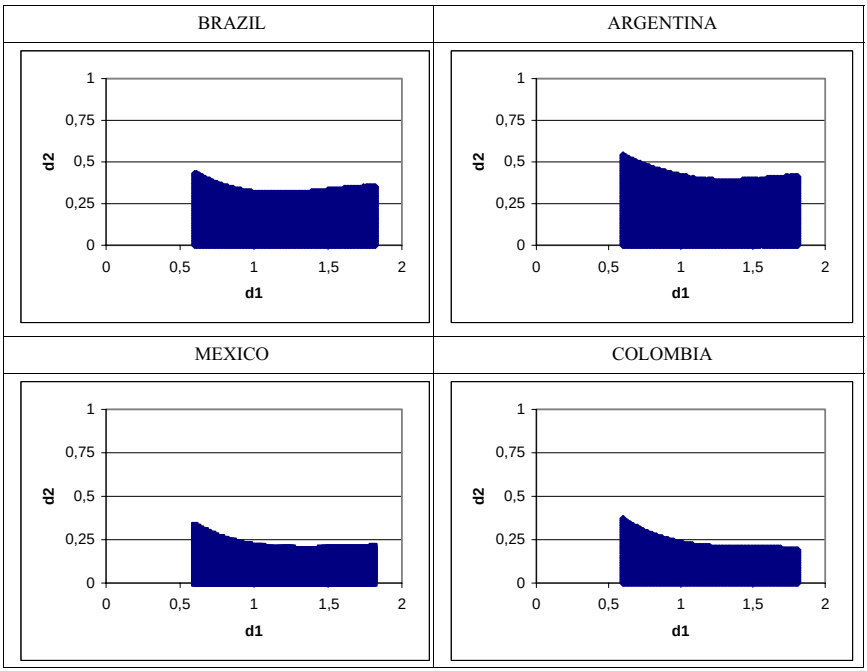

Fig. 4. Regions of values of d_{1o} and d_{2o} where H_o (6) cannot be rejected at the 95% significance level.

apart from the case of $(d_{1o}, d_{2o}) = (0, 1)$, all the other non-rejection values take place when d_{1o} is between 0.75 and 1.75 with $d_{2o} = 0$ or 0.25.

The results presented across this paper present some ambiguous conclusions. Thus, on the one hand, the seasonal unit root model cannot be rejected for any series and any type of disturbances. On the other hand, most of the non-rejections occur when d_{1o} is between 0.75 and 1.75 with $d_{2o} = 0$ or 0.25. This may be explained by the fact that the tests of Robinson (1994), when integer values of d_{1o} and d_{2o} are entertained, have low power to detect fractional models. In fact, we made several Monte Carlo experiments, conducting the tests with $(d_{1o}, d_{2o}) = (0, 1)$ on fractional series with $d_1 = 1$, 1.25, 1.50 and 1.75, and $d_2 = 0$ and 0.25, and the rejection frequencies of the tests never exceeded 0.400. On the other hand, performing the tests with $d_{1o} \in [0.75, 1.75]$ and $d_{2o} \in [0, 0.25]$ on purely seasonal unit-root models, the rejection probabilities were practically 1 in all cases for series of length considered here. In view of this, we can conclude by saying that the non-rejections obtained for the seasonal unit-root model are perhaps spurious and due to the low power of this version of the tests for testing integer differences when fractional models are entertained.

5. Concluding comments

In this article we have examined the monthly structure of the log-transformation of the Industrial Production Indexes in several Latin American countries by means of a version of the tests of Robinson (1994) that permits us to simultaneously test the degrees of integration at zero and the seasonal monthly frequencies. These tests have standard null and local limit distributions and several experiments conducted via Monte Carlo show that the tests perform relatively well in finite samples. The results show that the long-run or zero frequency plays a crucial role when modelling the time series behaviour of the series, with an order of integration ranging between 0.75 and 1.75, though the seasonal root should also be incorporated in the model, with an order of integration ranging between 0 and 0.5 for Argentina and Brazil, and slightly smaller for the other two countries. The presence of the fractional seasonal roots will affect the conclusions obtained by classical unit-root tests.

Policy advices can be drawn from this model as our new framework helps to better characterize the mean reverting properties of the series at seasonal and zero frequencies. Applying it to the IPI in four Latin American countries, it turns out that long-run effects of shocks might not be mean reverting, indicating the importance of long-run effect of economic policies. It is also noticeable that seasonal fluctuations have shorter memory in Colombia and Mexico than in Brazil and Argentina. For these last two countries, policy makers should pay a particular attention at the seasonal effect of the shocks, as they take time before dying out.

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