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## Does seasonal adjustment induce common cycles?

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### Abstract

In this note we analyze via Monte Carlo simulations how serial correlation common features test statistics behave when X-11 seasonal adjusted data are encountered. We emphasize both size and power distortions. We illustrate the analysis on Japanese consumption/income relationship. © 1998 Elsevier Science S.A.

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### 1. Introduction and motivation

Because official statistical agencies and international institutions often release seasonally corrected data, numerous papers have been studying the effect of seasonal adjustment on econometric inference and hypothesis testing [see inter alia Wallis (1974), Ghysels and Perron (1993), Ghysels et al. (1993) or Maravall (1995)]. One of the most important messages is that estimates are not consistent when lagged dependent variables are present among the regressors; with the exception of cointegrating vectors that are not affected by seasonal filters even if the short run dynamics is. Consequently, in this note we examine, through Monte Carlo experiments, to what extent the common practice of seasonally adjusting data may yield spurious, or misses, serial correlation common feature (SCCF) vectors. Stemming from the work by Engle and Kozicki (1993), Tiao and Tsay (1989), Velu et al. (1986), the goal of SCCF analysis is to find, in a multivariate stationary autoregressive model, linear combinations of economic time series that are white noise processes. This kind of analysis is relevant when searching for short run synchronous comovements in economic time series and yields more efficient estimates due to the reduction in the number of parameters.

We analyze the small sample impact of the well known X-11 linear filter on two different models. In particular we consider:

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$$\Delta y_t = \delta D_t + \sum_{i=1}^p \Phi_i \Delta y_{t-i} + \varepsilon_t \quad t = 1 \dots T \quad (1)$$

$$\Delta_4 y_t = \sum_{i=1}^p \Phi_i \Delta_4 y_{t-i} + \varepsilon_t \quad t = 1 \dots T \quad (2)$$

where we assume, without loss of generality, that  $y_t$  is measured quarterly.  $y_t$  is a  $n$ -dimensional seasonal process with  $\Delta = 1 - L$  and  $\Delta_4 = 1 - L^4$ . As  $(1 - L^4) = (1 - L)(1 + L)(1 + L^2)$ , both models imply only one unit root at the zero frequency. Model (1) displays the usual deterministic pattern structure in which, for the first differenced variables, the seasonal component is summarized by dummy variables. The second model exhibits unit roots at all seasonal frequencies, so the seasonality is said to be stochastic (see Franses (1996) for a survey).

Define  $[\Phi_1 : \Phi_2 : \dots : \Phi_p] = \Phi$ . Notice that common features are present if the rank of  $\Phi$  is smaller than  $n$ . In this paper we analyze the size, i.e. when  $\text{rank}(\Phi) < n$ , as well as the power, i.e. when  $\text{rank}(\Phi) = n$ , of serial correlation common feature tests statistics. As we cannot ignore the presence of seasonal components we first analyze the case in which unadjusted  $y_t$  has been filtered correctly by  $(1 - L)$  and four dummies or by the  $\Delta_4$  operator. These results are then compared to those obtained for time series that have been adjusted by the X-11 filter. In this last situation we transform variables in level by the linear approximation  $y_t^{SA} = \Psi(L)y_t$  where  $\Psi(L) = \sum_{i=-28}^{28} \Psi_i L^i$  with  $\Psi(L) = \Psi(L^{-1})$  and  $\Psi(1) = 1$ . The common feature analysis is then applied to the first or the fourth differences of seasonally adjusted data, i.e. on  $\Delta y_t^{SA}$  or  $\Delta_4 y_t^{SA}$ .

*Proposition 1.1. The common feature space for the unadjusted data is not a common feature space for seasonally adjusted data.*

*Proof.* Following inter alia Ericsson et al. (1994) it is useful to express  $\Psi(L) = \Psi(1) + \Psi^*(L)\Delta$ , where  $\Delta$  is the difference operator and  $\Psi^*(L)$  is a two-sided linear filter with polynomial coefficients  $\Psi_i^*$ . For non cointegrated stationary time series, with for instance  $z_t \equiv \Delta y_t$ , we get  $z_t^{SA} = \Psi(L)z_t = \Psi(1)z_t + \Psi^*(L)\Delta z_t = z_t + \Psi^*(L)\Delta z_t$ . Assume there exists a  $n \times r$  matrix  $\tilde{\beta}$  whose columns span the cofeature space. Premultiplying both side by  $\tilde{\beta}'$ , we get  $\tilde{\beta}' z_t^{SA} = \tilde{\beta}' z_t + \tilde{\beta}' \Psi^*(L)\Delta z_t$ .  $\tilde{\beta}' z_t$  is a white noise by definition of SCCF. Note however that  $\tilde{\beta}' \Psi^*(L)\Delta z_t$  is a weighted sum of an invertible MA(1) process, and hence  $\tilde{\beta}' z_t^{SA}$  will not be a white noise<sup>2</sup>. □

*Proposition 1.2. Inference on common feature space conducted on unadjusted data differs from the one conducted on seasonally adjusted data.*

*Proof.* Let us consider the VAR( $p$ ) defined by (1) where  $\Phi(L) = I - \sum_{i=1}^p \Phi_i L^i$ . Applying the linear approximation of the seasonal filter yields  $\Psi(L)\Phi(L)z_t = \Psi(L)\delta D_t + \Psi(L)\varepsilon_t$ . Substituting  $z_t$  by  $z_t^{SA} + (z_t - z_t^{SA})$  and using the previous filter we obtain<sup>3</sup>

<sup>2</sup>Consequently we end up with a non-synchronous common cycle as defined by Vahid and Engle (1996).

<sup>3</sup>Notice that, if the sum of dummy variables coefficients is zero over a year,  $\Psi(L)\delta D_t = 0$  (Ericsson et al. (1994)).

$$\Psi(L)\Phi(L)z_t^{SA} + \Psi(L)\Phi(L)(z_t - z_t^{SA}) = \Psi(L)\varepsilon_t$$

$$\Phi(L)z_t^{SA} = -\Psi(L)\Phi(L)(z_t - z_t^{SA}) + \Psi(L)\varepsilon_t - \Psi^*(L)\Phi(L)\Delta z_t^{SA}$$

Now, the error process  $\eta_t = -\Psi(L)\Phi(L)(z_t - z_t^{SA}) + \Psi(L)\varepsilon_t - \Psi^*(L)\Phi(L)\Delta z_t^{SA}$  is no longer an innovation and it is not even orthogonal to the regressors, which will induce inconsistency and inefficiency for the reduced rank test statistics.  $\square$

## 2. Test statistics

In the lines of Tiao and Tsay (1989), Gouriéroux and Peaucelle (1993), Ahn and Reinsel (1988), Velu et al. (1986) we test for zero canonical correlations between the  $(n \times T)$  matrix  $W_1 \equiv \Delta Y_t = \{\Delta y_1' \dots \Delta y_T'\}'$  and the  $(n \times p) \times T$  matrix  $W_2 = \{\Delta Y_{t-1}' \dots \Delta Y_{t-p}'\}'$ . Note that for the non X-11 filter case,  $W_1$  and  $W_2$  have to be adjusted for their means and seasonal components, i.e. both terms have been regressed on four centered dummies and the analysis is carried out on residuals. For stochastic seasonality the analysis is conducted between  $\bar{W}_1 \equiv \Delta_4 Y_t = \{\Delta_4 y_1' \dots \Delta_4 y_T'\}'$  and the  $(n \times p) \times T$  matrix  $\bar{W}_2 = \{\Delta_4 Y_{t-1}' \dots \Delta_4 Y_{t-p}'\}'$  where both sets have been adjusted for their means. When the X-11 filter is used, we replace  $W_1$ ,  $W_2$  and  $\bar{W}_1$ ,  $\bar{W}_2$  by  $\tilde{W}_1$  and  $\tilde{W}_2$  where  $\tilde{W}_1 \equiv \Delta Y_t^{SA} = \{\Delta y_1^{SA'} \dots \Delta y_T^{SA'}\}'$  and the  $(n \times p) \times T$  matrix  $\tilde{W}_2 = \{\Delta Y_{t-1}^{SA'} \dots \Delta Y_{t-p}^{SA'}\}'$ . Notice that once the seasonal filter has been applied, it is numerically equivalent to first difference X-11 filtered data when the DGP is model (1) or to take the fourth differences of X-11 filtered data when the DGP is model (2). We consequently do not report both results. In practice, however, the statistical properties of raw data are usually not known. So, we will take most of the time first differences of non stationary seasonally adjusted data.

For a multivariate Gaussian covariance stationary process, the sequence of common feature likelihood ratio test statistics is, for  $H_0: rank[\Phi_1: \dots: \Phi_p] \leq n - r$  against  $H_a: rank[\Phi_1: \dots: \Phi_p] > n - r$  (see Lütkepohl (1991)):

$$\zeta_r = -T \sum_{i=1}^r \log(1 - \lambda_i), \quad r = 1, \dots, n \tag{3}$$

where  $0 \leq \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n \leq 1$  are the ordered eigenvalues of the symmetric matrix

$$A_r = (W_1 W_1')^{-1/2} W_1 W_2' (W_2 W_2')^{-1} W_2 W_1' (W_1 W_1')^{-1/2} \tag{4}$$

The eigenvalues of  $A_r$ , estimated in the usual way, are the squared canonical correlations and the smallest of them measures the relationship between the linear combination of the components of  $W_1$  and the linear combination of the components of  $W_2$  that is the least correlated. If the null hypothesis cannot be rejected, the columns of the matrix  $\alpha \equiv (W_1 W_1')^{-1/2} v_r$ , where  $v_r$  are the eigenvectors of  $A_r$  associated with this smallest correlation, span the cofeature space. This matrix  $\alpha$  whose columns span the left null space of  $\Phi = [\Phi_1: \dots: \Phi_p]$  is such that  $\alpha' \Phi = 0_{(r \times np)}$ . Because of the covariance-stationary hypothesis, the test statistics asymptotically follows, under the null, a  $\chi^2$  with  $rnp - r(n - r)$  degrees of freedom (see Vahid and Engle, 1993).

### 3. The data generating process and simulation results

Under the reduced rank null hypothesis, we generate a bivariate process in which the first as well as the fourth differenced variables follow a VAR of order 2 with a cofeature vector equal to  $[1, -1]$ :

$$\Phi_1 = \begin{bmatrix} 0.2 & -0.5 \\ 0.2 & -0.5 \end{bmatrix}, \Phi_2 = \begin{bmatrix} -0.32 & 0.16 \\ -0.32 & 0.16 \end{bmatrix}, \Omega_\varepsilon = \begin{bmatrix} 9 & 3 \\ 3 & 4 \end{bmatrix},$$

$$\delta = \begin{bmatrix} .1 & 2 & -2 & 1 \\ -6 & 1.5 & -0.5 & 5 \end{bmatrix}$$

Under the alternative, it is easy to choose a DGP that would give a power of 100% in almost all situations. To circumvent this problem, we specify an alternative hypothesis relatively close to the null by fixing  $\Phi_{1,12} = \Phi_{2,21} = 0$ .

In the linear approximation of X-11 filter we do not reconstruct the initial and the final observations in order to avoid the appearance of other problems, such as nonlinearity for instance (Maravall, 1997). Consequently we generate  $52 + 28 + T + 28$  observations. We drop the first 52 to initialize the process and take the  $T$  observation in the middle in each case. The weights of the linear 56th order moving average are often given (in Ghysels and Perron (1993) for instance) up to three digits. In order to minimize rounding errors we recalculate the weights for eight decimals. Also notice that we exclude, in the simulation study, cointegrating relationships both at seasonal and at zero frequencies. We also assume that the deterministic seasonal pattern is constant through time.

Three sample sizes are used, that is  $T=80, 160, 500$ . The first two sample sizes mimic most economic time series for which we often get 20 or 40 years of quarterly data. We also add a larger sample size in order to analyze the behavior of test statistics when the sample increases. All the computations have been done in Gauss 3.14 with the RNDN generator process using 10 000 replications. The estimated model has got successively  $p=1, 2, 4, 8$  lags. Table 1, Table 2 and Table 3 give outcomes of the simulations. Table 1 presents empirical sizes, the nominal one being 5%, for the case in which the correct filters have been applied. Table 2 presents sizes when X-11 filter is used. For these two tables we present the frequencies of rejecting the true null hypothesis that the smallest eigenvalue is zero ( $r=1$ ), the rejection frequency for the sum of the two eigenvalues being 100% in almost all cases. We also present the median of coefficient  $\tilde{\beta}_2$  for the normalized cofeature vector  $[1,$

Table 1  
Empirical sizes and cofeature vector median and interquartile ranges for NSA

DGP	Filter	Lags	T=80			T=160			T=500		
			r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$	r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$	r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$
$\Delta y_t$ +SD	$\Delta y_t$ +SD	p=1	5.10	0.996	0.4253	5.20	0.998	0.2869	5.19	1.001	0.1559
		p=2	5.70	0.993	0.3152	5.21	0.998	0.2172	4.96	0.999	0.1195
		p=4	7.48	0.993	0.3411	5.98	0.998	0.2233	5.36	0.999	0.1211
		p=8	10.76	0.993	0.3809	7.64	0.995	0.2366	5.67	0.999	0.1225
$\Delta_4 y_t$	$\Delta_4 y_t$	p=1	4.33	0.969	0.4085	4.80	0.984	0.2778	5.12	0.996	0.1558
		p=2	4.81	1.005	0.3122	4.77	1.002	0.2147	4.82	1.000	0.1200
		p=4	5.69	1.003	0.3321	5.03	1.002	0.2204	5.00	1.000	0.1204
		p=8	8.03	1.003	0.3742	6.27	0.999	0.2351	5.08	1.000	0.1226

Table 2  
Empirical sizes and cofeature vector median and interquartile ranges for X-11 SA

DGP	Filter	Lags	T=80			T=160			T=500		
			r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$	r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$	r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$
$\Delta y_t$ +SD	$\Delta y_t^{SA}$	p=1	8.62	1.157	0.5151	11.43	1.182	0.3549	16.83	1.192	0.1973
		p=2	19.62	0.976	0.3460	35.80	0.974	0.2321	82.82	0.974	0.1315
		p=4	41.46	0.955	0.4069	72.56	0.953	0.2611	99.95	0.952	0.1448
		p=8	67.29	0.952	0.4782	95.83	0.949	0.2931	100	0.949	0.1539
$\Delta_4 y_t$	$\Delta y_t^{SA}$	p=1	94.03	0.549	0.3014	100	0.740	0.2637	100	0.555	0.1181
		p=2	99.98	0.766	0.3257	100	0.763	0.2280	100	0.760	0.1243
		p=4	100	0.831	0.3103	100	0.831	0.2161	100	0.831	0.1207
		p=8	99.97	0.848	0.3330	100	0.847	0.2300	100	0.846	0.1259

–  $\tilde{\beta}_2$ ], the true one being [1, – 1], as well as the interquartile difference of  $\tilde{\beta}_2$  as a parameter of dispersion. Table 3 deals with the size-adjusted power<sup>4</sup> for T=160 and T=500.

In the first case, the appropriate filter is used. It emerges from Table 1 that there exist no bias nor size distortions in large samples. For small samples, we see a small size distortion if we overspecify the dynamics: empirical sizes go up to 10% in the worst case when T=100 and p=8. A small bias also appears in the fourth difference case if one underspecifies the dynamics, that is taking p=1. The dispersion is smallest when the correct lag length is selected, that is when p=2. The power is also high under the true dynamic structure but decreases for small samples if we overspecify the lag structure.

Once the data have been seasonally adjusted, we however observe huge size distortions yielding

Table 3  
Empirical sizes-adjusted power and cofeature vector median and interquartile ranges

DGP	Filter	Lags	T=160			T=500		
			r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$	r=1	$\beta_{0.5}$	$\beta_{0.75-0.25}$
$\Delta y_t$ +SD	+SD	p=1	49.69	0.145	0.2504	97.62	0.147	0.1344
		p=2	95.02	–0.088	0.3906	100	–0.082	0.2012
		p=4	87.00	–0.089	0.4024	100	–0.081	0.2016
		p=8	70.66	–0.088	0.4214	99.96	–0.081	0.2053
$\Delta_4 y_t$	$\Delta y_t^{SA}$	p=1	0	0.165	0.1623	0	0.166	0.0899
		p=2	7.69	0.059	0.2002	5.46	0.062	0.1086
		p=4	16.14	–0.006	0.2246	24.66	0.000	0.1197
		p=8	14.58	–0.019	0.2433	23.34	–0.014	0.1266
$\Delta y_t$ +SD	$\Delta y_t^{SA}$	p=1	65.08	0.143	0.5346	99.4	0.138	0.2942
		p=2	41.79	–0.154	0.6865	88.70	–0.162	0.3738
		p=4	27.87	–0.127	0.6787	67.90	–0.1339	0.3511
		p=8	19.79	–0.113	0.7578	49.66	–0.1351	0.3808

<sup>4</sup>To get size-adjusted powers, we analyze frequencies of rejecting  $H_a$  when  $H_a$  is true with critical values corresponding to 5% empirical size levels calculated for each specific case.

test statistics too liberal. Moreover, size distortions increase with the number of lags added in the estimated model, a phenomenon which is amplified when the sample size increases. This result can be easily understood with respect to Propositions 1.1 and 1.2. Due to underlying MA(1) structure, the MA parameters are consistently estimated when the sample size increases. Even if the bias is small in this case, it doesn't decrease when  $T$  increases. The power of the test statistic strongly decreases for seasonally adjusted data even when the correct difference operator has been applied. Tests are still consistent in this case because  $\lim_{T \rightarrow \infty} P_{Ha}(\xi_r > c) = 100\%$ . The power is very small if we take the first difference of X-11 filtered data whilst the raw model has also unit roots at seasonal frequencies. Moreover, the consistency of test statistic is questionable with respect to the power evolution from  $T=160$  to  $T=500$ . Unfortunately, taking the first difference of seasonally adjusted data is a popular practice in empirical studies.

#### 4. The Japanese consumption function

We now analyze the impact of seasonal adjustment on the consumption/income relationship. The raw as well as the seasonally adjusted data come from the Japanese national accounts and are recorded quarterly from 1955Q2 up to 1996Q4<sup>5</sup>. Consumption is the total consumption and the income variable we retained is the GDP less government expenditures. The variables are in constant prices. Some seasonal unit root tests and cointegration analyses have previously been done on this data set by Engle et al. (1993) or Hall et al. (1997). As suspected by the last authors we also detect a shift in regime in 1974Q1, so we start our analysis in 1974Q2 in this illustrative example. The log levels of the data are drawn in Fig. 1. As previous studies do not cover the same period, we also use HEGY (see Hylleberg et al., 1990) test statistics in order to analyze if there are some roots at zero or at seasonal frequencies.

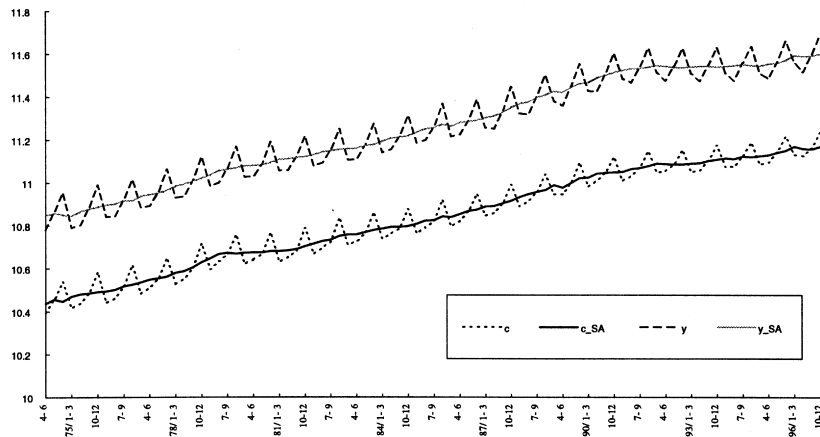


Fig. 1. Japanese consumption and income at constant prices.

<sup>5</sup>The data may be found on the Japanese statistical office Web site <http://www.stat.go.jp/19.htm>.

Table 4  
HEGY unit root test statistics

	$\ln C_t$	$\ln Y_t$	$\ln (C_t/Y_t)$
Lags	1,3,4,5,6,8,10	1,3,4,5,9	1,2,3,6,7,10,11,12
det. part	$c$ , trend, SD	$c$ , trend, SD	$c$ , trend, SD
$\pi_1$	-1.58	-3.24	-1.76
$\pi_2$	-0.19	0.56	-2.60
$\pi_3$	-2.63	-2.16	0.10
$\pi_4$	-1.93	-1.86	-1.57
$\pi_3 \cap \pi_4$	5.26	4.11	1.25

It is shown in Table 4 that the log of both non seasonally and seasonally adjusted data have a unit root at zero frequency, and that raw time series have also unit roots at all seasonal frequencies. Moreover, the vector  $[1, -1]$  is not a cointegrating vector as may be seen in the last column of Table 4.

Table 5 however shows the existence of a long run relationship at the zero frequency but not at seasonal frequencies. The variables are stochastically cointegrated, e.g. are stationary around a deterministic trend, which partly measure the upward trend of the foreign balance. For the raw data the cointegrating relationship is  $c_{1t} = 14.147 + 0.010595trend + 0.637y_{1t}$ , where  $c_{1t} = (1 + L + L^2 + L^3)c_t$  and  $y_{1t} = (1 + L + L^2 + L^3)y_t$ , while  $c_t = 3.439 + 0.002565trend + 0.646y_t$  with the seasonally adjusted data. Notice that both relationships present the same income elasticity as explained by Ericsson et al. (1994).

We may now conduct a common feature analysis in the variables written in their correct ECM form<sup>6</sup>. VAR models which may best characterize the covariance structure of the data are models with 1 or 6 lags for the fourth differences of the seasonally unadjusted model and 2, 6 or 12 lags for the first differences of the seasonally adjusted model. Consequently we choose successively  $p = 2, 6, 12$  in order to examine the stability of results through  $p$ , having in mind that  $p = 6$  seems better for both models.

Table 6 gives the common feature test statistic for the seasonal as well as the non seasonal data set.

Table 5  
HEGY seasonal cointegration tests

$c_t = f(y_t, \text{determ})$		0 frequency	Semi-annual	Annual
Raw data	Lags	1 to 5	1 to 7	1 to 8
	det. part	$c$ , trend	$c$ , SD	$c$ , SD
	Test Stat.	-4.60	-2.64	4.38
Season. adj.	Lags	1 to 2	-	-
	det. part	$c$ , trend	-	-
	Test Stat.	-4.59	-	-

<sup>6</sup>That means with the  $(1 - L^4)$  difference operator and the function of  $c_{1t-1}, y_{1t-1}$  for unadjusted time series and with the  $(1 - L)$  operator and the function of  $c_{t-1}, y_{t-1}$  for the seasonally adjusted data.

Table 6  
Common feature test statistics for Japanese consumption function

Lags	$\Delta_4 y_t$				$\beta$	$\Delta y_t^{SA}$				
	$r=1$	5%cv	$r=2$	5%cv		$r=1$	5%cv	$r=2$	5%cv	$\beta$
$p=2$	33.50	(7.81)	146.52	(15.50)	0.65	7.52	(7.81)	22.11	(15.10)	0.19
$p=6$	60.93	(19.67)	204.63	(36.41)	0.71	16.78	(19.67)	44.68	(36.41)	0.67
$p=12$	84.44	(35.17)	259.76	(65.17)	0.68	30.51	(35.17)	74.03	(65.17)	1.09

We test for a ‘weak form reduced rank structure’ (see Hecq et al. (1997): a situation in which the short run dynamics matrices and the long run coefficients matrix do not have a common left null space as assumed by Vahid and Engle (1993) or Ahn (1997) for seasonal cointegrating vectors. In all cases the model that considers a linear combination of the first (or the fourth differenced data) corrected for long run effects instead of a combination of differenced variables only, has been retained with respect to likelihood ratio test statistics and information criteria (for finite sample properties see Hecq et al. (1997)).

What emerges from Table 6 is that we cannot reject the hypothesis of one serial correlation common feature vector with seasonally adjusted data while we clearly rejected this hypothesis for the unadjusted data. In light of the simulations outcome, this result can be explained by a strong decrease of power with seasonally adjusted data. Also remark that we reject (results not reported) the presence of a common feature vector in the fourth differences of seasonally adjusted data. Due to the presence of unit roots both at zero and at seasonal frequencies, the power of SCCF test statistic is higher when the analysis is applied on fourth differenced adjusted data.

## 5. Conclusion

We have seen, through Monte Carlo simulations, that the practice of using serial common feature test statistics on seasonally adjusted data is a perilous exercise. Due to size distortions, we face the worst situation in which tests are not able to detect common features because of size distortions. Because of lack of power, they spuriously signal discovery of common features when these features do not exist. So, what could be done in empirical studies? It is easy to answer: use seasonally unadjusted data sets. Because the nature of the seasonality present in raw data is crucial, we could at least consider the statistical properties of the data if not from raw data, from previous studies. However it is well known that the power of procedures like HEGY is low in several situations like the presence of a shift in mean or in seasonal pattern, GARCH, nonlinearity...

Consequently, we would advise, when using seasonally adjusted data, to test for non-synchronous common cycle, that is a linear combination of a VARMA( $p,1$ ) for instance which yields a MA(1) process, that is also a scalar component model of order (0,1). The interpretation of the results is however different from Vahid and Engle (1996). In this case, cycles are not really non-synchronous because the vector corresponding to the SCM(0,1) may be a serial correlation common feature for the non-observable raw data. However, this is only valid if we succeed in detecting the correct nature of the seasonality in raw data, otherwise we throw out the dynamics like we would throw out the baby with the bath water.



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