

The Periodicity of Competitor Pricing

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Abstract

Pricing data reflect multiple decisions (e.g., regular pricing and discounting) often made by multiple decision makers. For example, temporary price reductions (high frequency price changes) can be used to price discriminate in the short run, while regular price adjustments (low frequency price changes) reflect more strategic or long-term goals. It is therefore possible that the “reaction” of one brand’s price to another depends on the frequency of the data analysed. Time disaggregation does not remedy this problem, because frequency aggregation exists even when data are analyzed at the lowest possible level of temporal aggregation. This paper therefore decomposes pricing interactions across multiple frequencies or planning cycles. Using weekly sku-level price data in 37 grocery categories, we shed some light on the nature of pricing interactions across alternative planning horizons. We find that cross-brand correlation in prices occurs at multiple planning horizons, and that the planning horizon of the predominant interaction does typically not coincide with the sampling rate of the data. We next demonstrate that different conclusions about the nature of price competition emerge from different periodicities of pricing data by applying a structural model of competitive price responses to different price periodicities identified by spectral decomposition. Calibrated on short-term price variation, the model indicates that pricing among brands is cooperative, whereas the long-term price variation suggests independent or Nash competitive behavior. We provide alternative interpretations for this finding and we conclude that price periodicity matters for the inference of competitive response.

Keywords: Competition; Competitive Reactions; Price Reactions; Spectral Analysis; Time Series; Vector Autoregression; Empirical Generalizations; Long-term Effects.

1 Introduction

Empirical research pertaining to the measurement and prediction of competitors' price interactions is pervasive in the marketing literature (e.g., Gatignon 1984; Hanssens 1980; Lambin, Naert, and Bultez 1975). More recently, considerable attention has been devoted to the dynamics inherent in competitor price interactions (Dekimpe and Hanssens 1999; Leeflang and Wittink 1992, 1996, 2001; Nijs et al. 2001, Steenkamp et al. 2004). These papers have led to a richer view of competition, engendering the view that response may occur with some delay.

Our aim is to contribute to this literature by considering how inferences regarding pricing interactions vary across different periodicities in pricing data. Weekly pricing data, for example, may embed both regular price decisions (which may change on an infrequent basis) and temporary price reductions (which may occur more frequently). Though these multiple decisions are agglomerated into a single pricing series, the goals of short-term and long-term pricing decisions, as well as consumer response to them, may be different. For example, a manufacturer might use discounts to (1) collude against weaker brands (Lal 1990), (2) exploit asymmetries in price response around a reference price (Greenleaf 1995), (3) price discriminate among brand switchers (Farris and Quelch 1987), (4) increase consumption, (5) induce trial, or (6) meet quarterly goals. In contrast, regular price changes might reflect (1) changes in overall cost structure, (2) a signal of quality, or (3) a drive toward increased profitability. Retailers, likewise, might choose to use discounts for purposes of (1) category management, (2) price discrimination among store switchers, or (3) shifting inventory costs to consumers (Blattberg and Eppen and Lieberman 1981). Moreover, the timing of pricing decisions can differ across agents. Manufacturers, for instance, can react to each other's regular price changes, as the rate of price change for regular prices is sufficiently slow that manufacturers have the opportunity to observe and react to such changes. From discussions with manufacturers, we note that it usually takes 6 months or more to implement a price change into retail (see also Chintagunta, Dube and Singh 2002; Kopalle, Mela and Marsh 1999). In contrast, short-term changes in price often occur too quickly for manufacturers to respond (Leeflang and Wittink 1992).¹ Therefore, it stands to reason that observed pricing interactions may vary across frequencies and that aggregating these across frequencies obscures insights regarding the nature of

¹An exception to this generalization exists when manufacturers can obtain competitive dealing schedules from their retailers well in advance of the deals.

price interactions.

Further substantiating this argument in the context of modeling competition in prices, Kadiyali, Sudhir, and Rao (2001, p. 177) note that “periodicity of decision making and time aggregation/disaggregation are important issues to bear in mind.” They suggest additional research is needed in this area. In this context, we propose an exploratory approach to identify the planning cycles (or periodicities) at which, empirically, prices interact across brands. Using this approach, we next test the conjecture that considering alternative planning cycles in pricing data changes inferences regarding price competition and find that this is indeed the case. In light of this finding, we argue that it is desirable to decompose the data into planning cycles and then let theory inform the modeler which frequency is most germane to the pricing decision of interest. For example, if the aim is to ascertain competitors’ responses to promotion we would select different, and presumably shorter, planning cycles than if we wanted to see whether the data inform us about the competition in regular prices.

The following example illustrates how measures of price interactions depend on planning cycles. For illustrative purposes, we focus on correlation between prices of different brands as a measure of price interaction. This measure is purely descriptive. However, if competitors respond to each other’s price changes, one expects such reactions to manifest themselves as a statistical relation between observed prices.² Consider Figure 1 which depicts more than 4 years of retail price per can for two brands of beer in a Dominick’s Finer Food store in Chicago.

————— insert Figure 1 here —————

The contemporaneous correlation among the price residuals is -0.05 (not significant) for the data in Figure 1. However, as is clear from the graph, there are multiple frequencies represented in the price data. First, there exist high-frequency oscillations in short-term pricing (occurring every five weeks or so). Pricing at this periodicity appears indicative of temporary price reductions. For these frequencies, the correlation in residuals is -0.26 ($t = -3.56$), implying a contemporaneous negative interaction. To the extent short-term price variation is retailer driven, this would suggest that the retailers tend to promote an alternate weeks (Krishna 1994). Second, there exist lower frequency price fluctuations (occurring every 15 weeks or so), perhaps associated with longer-term

²Correlation in the temporal domain is sensitive to whether a competitor interacts with some delay or not. We later propose a frequency based correlation measure that remains unaffected by response delays.

movements in regular price. If one focuses on these long-term variations in prices, the correlation is 0.96 ($t = 45.36$), reflecting changes in regular price (possibly reflecting changes in costs or other factors common across firms).³ Thus, inasmuch as these estimates are taken to be reflective of competitive interaction, different conclusions regarding the nature of competitors' price interactions are drawn by looking at different planning horizons.

In sum, our objectives in this paper are to (1) empirically characterize the degree to which price interactions occur at different periodicities, (2) provide some insight about their dependence on decision horizons and on category and brand characteristics, (3) show that inferred competitor price interactions depend on the periodicity of the pricing data. The spectral approach employed in this paper complements econometric and economic analyses in the time domain (e.g., VAR and NEIO, respectively). Further, the issue of periodicity aggregation is very distinct from time aggregation (Leone 1995). Even when data are disaggregated to the shortest data interval, multiple decision makers and multiple decisions remain combined in the data. Indeed, as data are sampled at higher frequencies (e.g., weekly vs. monthly), more interactions become intermingled in the pricing series. As such, the literature on time aggregation provides limited insights into the periodicity of decision making. In contrast, our analysis seeks to inform the modeler about the frequency or periodicity in which the price interactions occur.

Our paper proceeds as follows. Section 2 outlines the methods used to illustrate our points. Section 3 discusses the data and Section 4 presents the results of the analysis. A key finding is that empirically the periodicity of price interactions generally differs from the sampling rate of the data. In Section 5, we explore further implications of our findings by showing that the frequency of price interactions can affect statistical inferences regarding the nature of price competition. Section 6 concludes the paper.

2 Method

To assess the influence of periodicity on competition, we employ a spectral decomposition of price series. This analysis is purely descriptive and is intended to show how the magnitude and direction of price interactions depend on frequencies, which we denote "planning horizons." In a second

³For this illustration, we isolated the short (long) term variation in the data, by using a high (low) pass filter which eliminates variation below or above quarterly frequencies.

stage analysis, we use regression methods to determine how the magnitude and direction of price interactions depend on a variety of brand and category level variables.

2.1 Spectral decomposition of price covariation

Spectral analysis is a technique that creates a (co)variance decomposition of data (e.g., price) into different planning cycles or frequencies. For this and other reasons, spectral analysis has been widely applied in economics and finance (e.g., Andersen and Bollerslev 1997). However, it has been used only rarely in marketing (Chatfield 1974). We use a *multivariate* spectral decomposition to analyze the competitive price series in many different product categories. The methods for doing this are well-documented (e.g., Hamilton 1994) and therefore we relegate the technical aspects of the analysis to an appendix (Appendix A). To measure whether price covariation is present across frequencies, we compute the coherence at each frequency.

Coherence is equal to the squared correlation coefficient for two or more series of data at a specific frequency (Hassler 1993). Hence, in the context of pricing data, coherence is equal to the squared correlation coefficient for pairs of competing prices at a particular planning horizon ℓ (e.g., weeks, months, quarters). The coherence values range from 0 (no interaction between two competitors at planning horizon ℓ) to 1 (very strong interaction at ℓ). If the coherence is close to 0 at a planning horizon of 3 months, price changes for one brand occurring at 3 month intervals are uncorrelated with those from another brand. In contrast, a coherence close to 1 at the monthly level indicates strong price interactions occurring at roughly 4 week intervals. Importantly, coherence measures the presence of correlation in prices at a particular planning cycle, *regardless of whether competitor price interactions are instantaneous or lagged*. This attractive property ensures that there is no confound between price reactions and the timing of those reactions. For purposes of notation, let $h_{ii'}^c(\omega)$ denote coherence for the prices of brand pair $\{i, i'\}$ in category c , and frequency ω (which implies planning horizon $\ell = 2\pi/\omega$). The coherence measure is symmetric, i.e., $h_{ii'}^c(\omega) = h_{i'i}^c(\omega)$.

As an example, Figure 2 portrays the coherence between Budweiser and Old Milwaukee estimated using the data in Figure 1 and controlling for the other brands' prices (i.e., the pricing analysis is multivariate). The horizontal axis depicts the pricing cycle, or planning horizon (in weeks), and ranges from a low of 200 weeks and a high of 2 weeks (2 weeks implies 26 price cycles

per year – as prices are set weekly, the shortest complete price cycle for weekly data, and thus highest frequency observable in the data is biweekly).⁴ The vertical axis is coherence. The observations in Figure 2 represent the 5%, 25%, 50%, 75% and 95% percentiles in the distribution of the coherence estimates in the beer category at each planning horizon (the methodological section outlines the procedure for estimating these percentiles).

Three areas of high coherence between Budweiser and Old Milwaukee are indicated. First, the highest coherence occurs at cycles of roughly 5 weeks (monthly cycles). The high-frequency price changes evidenced in Figure 1 occur at this periodicity. Second, Figure 2 details strong coherence in pricing for price cycles of approximately 15 weeks (quarterly). A comparison of the top and bottom graphs in Figure 1 suggests that the slower moving regular price shifts are common to both brands, and that longer-term coherence in pricing may be present. Finally, there is a strong biweekly pricing cycle, reflective of weekly price changes indicated in Figure 1. Hence, the interaction of prices between Budweiser and Old Milwaukee contains at least three empirically important planning horizons, and coherence captures them all.

————— insert Figure 2 here —————

In addition to coherence, multivariate spectral analysis provides two other descriptive measures of the statistical interaction of two time series of prices stemming from the polar representation of their spectrum (e.g., Hamilton 1994, p. 275). The first of these measures is called *phase*. This metric captures whether prices tend to move in the same direction or in opposite directions. As with coherence, phase is specific to the planning horizon ω . The value of the phase lies between $-\pi$ and $+\pi$ radians. A phase of 0 implies that prices change concurrently, or “in-phase.” Such series are positively correlated. A phase of $\pm\pi$ indicate that prices are perfectly “out-of-phase.” Such series are negatively correlated. Figure 2’s short-term price interactions (approximately 5 weeks) between Budweiser and Old Milwaukee have a phase close to $+\pi$, indicating that prices are out of phase in the short-term (indicating that the promotions of these brands are negatively correlated). However, the peak in coherence corresponding to approximately 15 weeks evidence a phase very close to 0. As such, it appears that prices move together in the long term, but disparately in

⁴It is noted that cycles (or periods) and frequencies are inversely related. That is letting ℓ denote the length of the price cycle (counted in time units), and ω denote frequency (expressed in radians), the relationship between planning period and frequency can then be expressed as $\ell = (2\pi/\omega)$, $0 < \omega \leq \pi$ (see also Harvey 1975).

the short term. These two facts accord with the price series in Figure 1. The second measure, called gain, is harder to interpret but seeks to express the attenuation of one price by another at a particular frequency. Statistically, its interpretation is similar to the magnitude of a regression coefficient (Brillinger 1981). In the example of Budweiser and Old Milwaukee, Figure 1 suggests the price variation of each of these brands is roughly equal, thus it is not surprising that we find the gain for the short-term and long-term price cycles are both close to 1.

In this paper, we limit our attention to the analysis of coherence for two key reasons. First, our goal is to determine the periodicity at which pricing interactions occur, as this determination is a useful precursor to analyses of pricing competition – and coherence is the only measure related to this objective. Second, coherence is a prerequisite for analyzing other metrics such as phase and gain. Phase and gain have little meaning when coherence is small or insignificant.

2.2 Regression

The next step of our analysis assesses whether competitor interactions, as measured by coherence, systematically vary across planning horizons. To proceed with this analysis, we follow several steps, outlined below:

1. *Categorize the continuous periodicities (ω) into a discrete number of planning horizons ($p = \{0, 1, 2\}$).* Following Leeflang and Wittink (1992), we define short-term reactions as those that occur at intervals of 4 weeks or less (monthly). Leeflang and Wittink (1992; 2001) note that this cutoff corresponds roughly with the period in which manufacturers can not adequately respond to observed changes in retail price activity. We denote medium-term as those reactions that occur between the 4 week period and 13 week (quarterly) period. Such price movements are more likely to include manufacturer reactions to competitors' discounting policies, and may include changes in regular price. We define price changes that occur at greater than a quarterly frequency as longer-term, and these may be more reflective of longer term strategic objectives. Finally, we disregard price changes that occur with a periodicity of more than 26 weeks to ensure a sufficient number of cycles to produce a reliable analysis. In addition, avoiding annual cycles in prices reduces the risk of inadvertently confusing positive covariation in prices due to common seasonality in costs with positive covariation because of strategic long term price matching.

2. *Compute the representative coherence for each planning horizon.* Within each planning

horizon (short, medium, long) there exist multiple coherences, so the problem of selecting the most representative coherence becomes germane. For each planning cycle, we select across all member-frequencies the highest ratio of coherence to its standard deviation.⁵ In other words, the criterion for the presence of price interaction at any frequency range is that coherence should be important for at least *one* frequency that is considered part of that range. For example, the characteristic short term coherence is that for which

$$\max_{\{\omega \text{ s.t. } \omega < 4 \text{ weeks}\}} \frac{\text{mean}(h_{ii'}^c(\omega))}{\text{std}(h_{ii'}^c(\omega))} \quad (1)$$

where $\text{mean}(h_{ii'}^c(\omega))$ and $\text{std}(h_{ii'}^c(\omega))$ are the mean and standard deviation of the sampling draws at of the coherence at planning cycle ω (see Appendix A). We maximize this ratio for each of the three planning horizons (short, medium, and long) to select the representative coherence in each of these planning horizons.

The procedure of selecting the characteristic coherences within a planning horizon can be illustrated by referencing Figure 2. Among all the short run ω 's the ratio of mean coherence to its standard deviation for Budweiser-Old Milwaukee is highest at 2.1 weeks for the short run. Thus the representative coherence for that price pair in the short run is the coherence at 2.1 weeks. Continuing this logic, the medium term coherence corresponds to the mean coherence at 4.9 weeks, and the long-term coherence corresponds to the mean coherence at 15.4 weeks. Although this example contains sizable coherence in all three planning horizons, the subsequent sections show that, empirically, this is not the case in general.

3. *Regress planning horizon, brand and category characteristics on coherence*

Using regression analysis, we investigate whether there are important differences in coherence across pairs of brands, across categories, and across planning horizons. Knowledge of these differences is useful in predicting when periodicity matters (in terms of making inferences about competitive pricing interactions). Appendix B provides details regarding the specification and estimation of the regression model. We note that the regression takes into account unobserved heterogeneity in brand pairs and in categories.

⁵In addition to the highest formulation, we tried a weighted average formulation, wherein the mean coherences were weighted by their variances and summed across the planning cycle. The results were essentially identical.

3 Data

We use the Dominick’s Finer Foods (DFF) data base for this research.⁶ The DFF data are comprised of 7 years of weekly store movement data, and are thus well suited to study longer-term variation in retail prices. The DFF data contain 29 categories, although many of these categories contain multiple subcategories (e.g., grooming products contain razors, shaving cream, and deodorant among other sub-categories). In total, we conducted spectral analyses on 37 subcategories.⁷ From the many UPCs within a category, we select the most important ones for analysis, defined as those that had both high levels of demand within the category and a long duration in the data. When possible, we selected similar UPCs from different brands (but not necessarily different manufacturers) to comprise the different retail price series within a category. For example, in bottled juices, we selected a SKUs related to a particular size (64 oz.) and a particular type of juice (apple juice) for each of the major brands. Selecting similar SKUs from the same store ensures that we are most likely to observe price interactions where they exist. We selected only one retail price series per brand. Though we consider retail prices, it is also possible to use wholesale pricing data to analyze pricing behavior. We refrain from doing so for several reasons. First, many firms only have access to retail prices, so the most useful approach for these firms will focus on retail prices. Second, the wholesale data, by excluding retailer behavior, omits an important player of interest to many firms. Third, in the Dominick’s data, the wholesale prices are not manufacturer prices to the retailer, but rather reflect a weighted average cost of inventory. As such, it is not purged of retailer behavior, because it includes the effect of retail sales data and accounting procedures.

The number of price series per category ranged from 2 to 9. The 37 categories yielded 355 pairs of price series. For the second stage analysis, we therefore have 355 pairs times 3 planning horizons, or 1065 observations of coherence. The unit of analysis consists of UPC prices at the store level. Last, we note that there are a few missing observations scattered about in the data. When these occur, we set the missing prices equal to those of the nearest period. This interpolation approach ensures that prices more closely match the modal prices for regular and sale prices rather than

⁶<http://gsbwww.uchicago.edu/kilts/research/db/dominicks/>

⁷Analgesics, bar soaps, bath soaps, candy bars, cereal, gum, soup, conditioners, cookies, cola, deoderant, floss, fabric softener sheets, fabric softner liquids, frozen dinners, frozen entrees, frozen orange juice, graham crackers, apple juice, liquid dish detergent, liquid laundry detergent, liquid soaps, oatmeal, refrigerated orange juice, paper towels, toilet paper, razors, beer, saltines, shredded cheese, shaving cream, shampoo, sliced cheese, snack crackers, toothbrushes, toothpaste, and tuna fish.

some point in between.⁸ Exceptionally, the series for one UPC are too short, for instance because it is discontinued. In such cases, we resorted to an average of the prices for the most similar UPCs within brand. A sample graph of the UPC level retail price series was presented in Figure 1.

4 Results

4.1 At which planning cycles do prices interact empirically?

Coherence: Figure 3 depicts the retail pricing coherence (averaged over brand pairs) for 4 illustrative categories. The shredded cheese category evidences a sharp reduction in coherence at the four week frequency, and negligible interaction in retail prices beyond 10 weeks. Cereal prices have a very high coherence at about 2 1/2 weeks suggesting that the prices of competitors in the cereal category are strongly correlated. In contrast, prices in the razor category evidence short- and long-term interactions, while prices in the bath-soap category have high degrees of coherence across all planning cycles. Additional variation in the nature of coherence exists across brand pairs within categories. Collectively, the figure suggests that empirical interactions in price occur at very different cycle lengths. These patterns are reflective of patterns in the other categories as well. *In almost all of the 37 categories studied, the most important frequency of price interaction does not coincide with the sample rate of the data (weekly).* This finding is especially important given that all analyses of competition, to our knowledge, focus on variation in price at the sampling rate of the data (which is often weekly). These results further suggest that it is possible to broaden our conception of pricing interactions by focusing on specific and theory-driven frequencies in the data.

————— insert Figure 3 here —————

To underscore the point that coherence manifests across frequencies, we plot, in Figure 4, the 5%, 25%, 50%, 75% and the 95% quantiles of the distribution of the mean coherences across all of the brand pairs in the analysis for the retail pricing series. The 95% quantiles in Figure 4 indicates that substantial coherence exists over all frequencies, and that spectral analysis is a useful approach for divining these planning cycles. Moreover, as we will show, these frequencies play an important

⁸We also tried a c-spline interpolation, and the results were virtually identical. As the number of missing observations is small, alternative common methods of interpolation will probably have inconsequential effects on our results.

role in inferences regarding the nature of competitive interactions. Next, we seek to assess whether there is any systematic variation in the degree of coherence across brands, categories and time.

————— insert Figure 4 here —————

4.2 Do empirical interactions in price depend on the brand, category and planning horizon?

It is apparent from the forgoing analysis that there exists substantial variation in coherence across brands, categories and time. We now explore the existence of patterns in coherence. In particular, we investigate whether our measures of price interaction are affected by brand or category characteristics. Such an analysis could be useful for researchers interested in ascertaining when or whether periodicity is likely to matter. The list of descriptors considered is not exhaustive, but rather reflects the confluence of the measures available in the data, and prior findings in the pricing literature.

4.2.1 Brand level influences on competitor price interactions

We commence by outlining several factors at the level of brand-pairs that are hypothesized to affect the pricing interactions. These brand-pair level variables (defined in Appendix C) are denoted, \mathbf{z}_{ii}^c , and they are as follows:

- *Within Firm Effects.* Reflective of common category planning practices, we expect manufacturers to price their SKUs similarly. This would lead to higher coherence among SKUs.
- *Between Firm Effects.* Price and quality tiers can also affect price interactions (Blattberg and Wisniewski 1989; Bronnenberg and Wathieu 1996).
 - *Private Label.* Given that private label store brands are often in different quality tiers than national brands (Hoch 1996), we expect coherence to be modest for brand pairs that include a store brand.
 - *Price Differential.* Leeflang and Wittink’s (2001) survey of managers indicates that competitor response is more likely when brands are positioned similarly. As such, one might expect coherence to be limited for disparately priced brands.

4.2.2 Category Level Influences on Competitor Price Interactions

In addition to brand descriptors, we delineate a set of category descriptors, denoted by \mathbf{v}_c , that we expect will moderate price interactions. These factors (again defined in the Appendix C) and the directions of our expectations regarding their effects on price interactions, are as follows:

- *can Industry Characteristics.* We consider the following industry factors:
 - *Concentration.* Chen et al. (1992) find that absence of competitive response is more likely with fewer competitors, as more competitors often implies a market of greater strategic importance. This suggests that coherence should be lower in more concentrated markets.
 - *Volatility.* Highly volatile markets with a larger percentage of firms exiting and entering the market can decrease the likelihood that organizations can monitor price activity and this may reduce coherence in pricing.
- *Consumer Characteristics.* Increased consumer price sensitivity is indicative of lower switching costs and thus greater competitive threats. As such, higher price elasticities are associated with a greater likelihood of competitive response (Kuester, Homburg, and Robertson 1999), suggesting increased coherence in price. Leeflang and Wittink (2001) further find that higher cross-brand elasticities lead to greater price reactions.
 - *Penetration.* Increased penetration has been associated with greater price sensitivity (Narasimhan, Neslin and Sen 1996). If this association holds, we expect penetration to contribute to an increase in coherence.
 - *Storability.* Bell, Chiang, and Padmanabhan (1999) find that increased storability leads to increased price sensitivity.⁹ Thus, we expect coherence to increase in storability.

4.2.3 Regression Results

Table 1 presents the results of the coherence regression. Overall, the fit of the model with the data is good and the significant results are consistent with our expectations. Several effects merit spe-

⁹Other factors may influence price sensitivity as well (Bell et al. 1999). However, our attempts to measure them via a survey approach yielded variables that are redundant with the information contained in the variables listed above (for example, stockpiling was highly correlated with interpurchase time).

cial consideration due to the novelty of the findings. In particular, consistent with the cospectrum observed in Figure 4, we find that coherence is more prevalent in the short run than the long run. The result is consistent with Chen, Smith and Grimm (1992), who observe smaller reactions to strategic, as opposed to tactical, actions (as strategic actions involve greater commitment of resources over time). Further, Chen and MacMillan (1992) note that more irreversible decisions (e.g., regular price as opposed to temporary price) can lessen reactions. The effect of same manufacturer is positive and quite substantial, as manufacturers tend to coordinate prices of their brands within a category. In addition, brand pairs with a large price differential show little interaction in pricing. At the category level, we find that more concentrated markets are contain brand pairs that have lower coherence. This indicates that all else equal, the observed prices in concentrated markets tend to be more independent. Storability is positively associated with coherence. This is because storability makes it appealing to consumers to buy when prices are temporarily low. This increased price sensitivity makes price reactions more effective, raising coherence.

————— insert Table 1 here —————

Overall, we conclude that competitor price interactions vary systematically across time, brands, and categories. Next, we argue that these frequency decompositions can be used as a pre-cursor to the specification and interpretation of models of pricing conduct. As correlation in prices does not equate to strategic pricing interactions, we advocate the exploration of coherence in price cycles to screen which planning cycles may be of importance for strategic analysis. In the next section, we seek to demonstrate this point by showing that modeling alternative periodicities in the data leads to different inferences about competitive response.

5 Inferring competitive responses

5.1 Overview

So far we have argued that multiple decisions are represented in weekly pricing data and that some of these decisions are associated with shorter and other with longer planning cycles and reaction speeds. We have further shown that the empirical interaction of shelf prices differs across planning cycles and that for each price pair the dominant interaction does generally not occur at the sample rate of the data. We now take our analysis a step further and show –again using spectral

decomposition— that the impact of periodicity on statistical inferences about price setting can be profound. To accomplish this goal, we first theoretically outline how periodicity can matter and then provide an empirical illustration to show this is the case.

Consider demand q_i given by a linear function in own prices p_i and competitive prices p_j , $j \neq i$,

$$q_i = f(p_i, p_j). \quad (2)$$

Assuming constant marginal cost c_i , the first order condition on profit maximization is that

$$p_i^* - c_i = -\frac{q_i}{dq_i/dp_i} = -\frac{f(p_i, p_j)}{f_{p_i}(p_i, p_j)}. \quad (3)$$

Substituting a linear demand equation for ease of exposition, where b_{ii} indicates an own-price response coefficient and b_{ij} indicates cross-price response coefficient, we obtain

$$f_{p_i}(p_i, p_j) = \frac{dq_i}{dp_i} = b_{ii} + b_{ij} \frac{\partial p_j}{\partial p_i} \quad (4)$$

with $b_{ii} < 0$ and $b_{ij} > 0$.

With Bertrand-Nash competition, all players take each other’s actions as given, i.e., a condition for inference of Bertrand-Nash competition is $\partial p_j / \partial p_i = 0$. On the other hand, if player i thinks that raising its price will make player j do the same, then $\partial p_j / \partial p_i > 0$. Given the signs of b_{ii} and b_{ij} the latter condition is associated with higher margins, and is therefore taken as an indication of “soft” competition or “cooperative” behavior. Conversely, finding that $\partial p_j / \partial p_i < 0$ is associated with lower margins and is taken as evidence of non-cooperative conduct.

Of interest is how pricing decisions at different planning cycles affect these inferences. One particularly interesting case arises when price elasticities vary with the periodicity of prices. For instance, promotional price response (and elasticity) is usually higher than regular price response (Blattberg, Briesch and Fox 1995). Such a result might reflect consumers’ increased tendency to stockpile in response to a temporary price reduction relative to a regular price decrease (Krishna 1994). To assess the impact of this difference on the inferred competitive response, we point out that a firms’ profit margins are invariant to whether data used are quarterly or weekly. This condition is unnecessary but simplifies the argument. From equation (3) and (4) it then follows that the empirical estimate for the sum $b_{ii} + b_{ij} \cdot (\partial p_j / \partial p_i)$ is invariant to whether the weekly or quarterly information in the data is used (i.e., the same margin is estimated). If b_{ii} is more negative with weekly data than with quarterly data, then $b_{ij} \cdot (\partial p_j / \partial p_i)$ needs to be more positive. Then, if

the increase in own price response is larger than the increase in cross-price response, $\partial p_j / \partial p_i$ will be estimated with a higher value from high-frequency data (when demand is more elastic) than from low-frequency data. Thus, estimating the system on weekly data will lead to inferences of more cooperative conduct than on the quarterly data. Though we focus on b_{ii} , it is important to note the argument generalizes to other parameters of interest in the model – as any parameter can change with the periodicity (e.g., retailer, consumer or manufacturer conduct), this increases the likelihood that inferences regarding competitive conduct can change with periodicity, and amplifies our core contention that periodicity matters.

The intuition behind our argument is that under Bertrand-Nash competition margins should shrink when price sensitivity increases. Under the assumptions communicated above the only way in which the system of equations in equation (3) and (4) can deal with the equality of margins across regimes of different frequencies and price responses in weekly or quarterly data is to infer more cooperation when price response is high. Therefore, under such conditions, there appears to be a tendency toward inferring “collusion” or “cooperative conduct” from high frequency high elasticity data.¹⁰

To illustrate these points, we adopt a specification for $f(p_i, p_j)$ similar to Kadiyali et al. (2000). As with Kadiyali et al. (2000, p. 132), we find the log-log, semi-log, and linear demand models to be quite comparable in fit. While the semi-log model enables separate identification of each of the conduct parameters, our goal is to show competitive interactions can differ across planning horizons, and a linear model is sufficient for this purpose. Like Kadiyali et al. (2000), we choose a category with two national brands and a private label; shredded cheese. The shredded cheese category was selected because (1) it evidences a difference in coherence patterns in the long term and short term, (2) the number of brands is low (2 major national brands and 1 private label), thus facilitating estimation of the structural model, and (3) because we have exact knowledge of the variable costs in this category through consultation with a former category manager of one of the major firms in the industry.

¹⁰Rather crucially, this statement builds on the scenario that the only thing that changes across data of different frequencies is own price response b_{ii} . Although this “all else equal” condition substantively holds in our empirical example, we do not wish to suggest it holds in general. Still, if inferences about other parameters change across data of different frequency, it would be a coincidence if the inference about the responses $\partial p_j / \partial p_i$ remained unaffected.

We specify the demand for brand i to be given by

$$q_i = a_i + b_i p_1 + c_i p_2 + d_i p_3 + g_i \cdot \text{ddshift}_i \quad (5)$$

where p_1 is the price of Kraft Shredded Cheese, p_2 is the price of Sargento Shredded Cheese, and p_3 is the price of store brand shredded cheese. The demand shifters (ddshift_i) we use include the total demand of competing brands in the outside stores (i.e., stores other than the one used in estimation) and monthly dummies. It can be shown (see Appendix D) that the estimation equations for shredded cheese are thus given by

$$\begin{aligned} q_1 &= a_1 + b_1 p_1 + c_1 p_2 + d_1 p_3 + g_1 \cdot \text{ddshift}_1 \\ q_2 &= a_2 + b_2 p_1 + c_2 p_2 + d_2 p_3 + g_2 \cdot \text{ddshift}_2 \\ q_3 &= a_3 + b_3 p_1 + c_3 p_2 + d_3 p_3 + g_3 \cdot \text{ddshift}_3 \\ mp_1 &= mc_1 + \gamma_1 q_1 \\ mp_2 &= mc_2 + \gamma_2 q_2 \\ r_1 &= \alpha_1 q_1 + \alpha_2 r_2 + \alpha_3 r_3 \\ r_2 &= \alpha_4 q_2 + \alpha_5 r_2 + \alpha_6 r_3 \\ r_3 &= \alpha_7 q_3 + \alpha_8 r_2 + \alpha_9 r_3 \end{aligned} \quad (6)$$

where mp_i is the manufacturer price of brand $i \in \{1, 2, 3\}$, mc_i is the manufacturer cost, and r_i is the retailer mark-up.

It is also shown in Appendix C that deviations from the Nash condition in the manufacturer pricing equations (mp_i) is given by $k_i = -(1/\gamma_i) - b_i \neq 0$, $i \in \{1, 2\}$. Given estimates for the mean and variance of γ_i and b_i , it is possible to obtain estimates for the variance of k_i , and therefore test for deviations from Nash. The system of equations in 6 are estimated using linear three-stage least-squares (3SLS), with the lagged prices and quantities in outside stores as well as the store under investigation as instruments.

5.2 Inferences at Alternative Planning Cycles

Our first consideration in this example is to ascertain what frequencies to use in our analysis. We used spectral analysis to determine that the best empirical decomposition is obtained around a frequency less than and equal to 4 weeks and a frequency more than 4 weeks (see also Leeflang

and Wittink (1992) for a theoretical rationale as well as the empirical data in Figure 3).¹¹ We then filtered the shredded cheese category to select these frequency components for analysis (see Hamilton 1994). We employ a low pass filter and next a high pass filter to separate the two frequency components.

The key parameter estimates from the structural model are presented in Table 2, and the own- and cross-price coefficients exhibit face validity. We focus our discussion on the inferences about manufacturer pricing rules to illustrate that differences in competitive response can exist over planning horizons. From Table 2 we observe that own price sensitivity is higher in the high frequency than in the low frequency data. Further, we also observe that the cross price sensitivity does not increase with high frequency data as much. A potential explanation for this pattern is that the lion-share of the short term price effect comes from shifting demand around in time rather than across brands.

————— insert Table 2 here —————

A necessary condition for a Bertrand-Nash interaction is that $k_1 = -(1/\gamma_1) - b_1$, $k_2 = -(1/\gamma_2) - b_2 = 0$. To explicitly test for deviations from Nash, we calculate the k_i and its variance using the delta method. Table 3 presents these results. Using the low frequencies in the data, we would infer that the manufacturers compete in a Nash equilibrium, as k_i is not significantly different from 0 for either national brand. On the other hand, using the high frequency data, we would infer that the game deviates significantly from Nash, as $k_i > 0$ for both national brands. Thus, as expected, our inference about competition depends strongly on which frequencies in the data are used for analysis. The differences between long and short run are significant at $t = 9.16$ for k_1 and at $t = 3.19$ for k_2 , with cooperation being greater in the short run.

————— insert Table 3 here —————

The findings are consistent with our speculation that higher price response b_{ii} , all else constant, tends to lead to an inference of more cooperative behavior. As our estimates for own price response for the national brands are 30-90% higher for high frequency data, this implies that the inferred level of cooperation should be higher. This is precisely what we observe. The finding of short-term pricing cooperation is also consistent with Lal (1990) who shows that leading brands collude

¹¹We also used decompositions below and above 13 weeks with the same results as presented here.

through their promotions to lock out lower quality brands. We note that implicit collusion in short-term pricing is possible when manufacturers learn about promotion schedules in advance from the retailer.

In sum, we conclude that our inferences about the nature of the competitive game varies with the periodicity and that more cooperative conduct is inferred with the high frequency data than with the low frequency data. Thus, we demonstrate that it is possible to isolate frequencies of high pricing interactions as inputs for a more structured approach to competitive price analysis. One alternative to our analysis would be to include a discount variable in the supply and demand equations to capture the different goals of regular and discount pricing, and we think this would be a useful extension. However, even with such an approach, it is not clear in which frequency or frequencies pricing variation should predominate.

6 Conclusion

We presented a literature that suggests that price data reflect multiple decisions and multiple decision makers such as retailers and manufacturers, and that these decisions manifest as different pricing interactions across different planning horizons. This literature, therefore, suggests that a single pricing series can exhibit multiple interactions across different frequencies. Using the beer category example, we illustrated that high and low frequency price changes interact differently across competitors. We then formalized this illustration by applying a spectral decomposition to the data in order to uncover the frequencies at which competitor price interactions are most intense. We depicted results for 4 additional categories and found that multiple competitor interactions across pricing frequencies seem prevalent. Next, using data on 37 categories, we generalized these results. We found that competitor interactions, as measured by coherence, do not predominate at the sampling rate of the data. Rather, we find that significant competitor price interactions occur across all planning horizons, be they weekly, monthly, or quarterly. We further demonstrate systematic differences in price interactions across brands and categories, noting that pricing interactions are more prevalent in the short-term, within a given manufacturer's brand portfolio, within price tiers, in less concentrated markets, and in categories that are storable.

We then demonstrated, both theoretically and empirically, that inferences regarding competitive responses require some notion of what constitutes a plausible decision cycle. Using a structural

modeling approach estimated at different planning horizons in the shredded cheese category, we showed that inferences about the nature of competition can deviate significantly across planning horizons. In particular, we find that increased price response, all else equal, tends to favor inferences of cooperation. As such, we envision the spectral approach considered herein to be a useful precursor and complement to more structural approaches for competitive inference. We provide empirical proof for the conjecture that a modeler's choices about periodicity in pricing influence assessments about competitive interaction. We conclude that choices about periodicity ideally combine a spectral exploration of the data with theory about price response, and a modeler's knowledge about managerial constraints in the timing of competitive reactions. Given the influence of periodicity on the inference of primitives such as marginal cost or the nature of competitive interactions, it is important to properly account for this phenomenon when developing managerial insights into these factors.

There is little doubt that studies on price competition will be of continuing importance in the future. Above all, we hope that the research herein is taken as a constructive step in the direction of studying the periodicity of price decision making. Beyond that domain, we hope that spectral analyses may be fruitfully employed to understand the periodicity of decision making in other marketing contexts.

Variable	Parameter^a	t	
<i>Brand Level Variables</i>			
Same Manufacturer	0.70	6.30	**
Private Label	0.03	0.25	
Price Differential	-0.15	-3.14	**
<i>Category Level Variables</i>			
Concentration	-1.88	-3.23	**
Volatility	-0.42	-1.21	
Penetration	-0.00	-0.32	
Storability	0.33	2.83	**
<i>Planning Horizon</i>	-1.15	-14.86	**
<i>Intercept</i>	-0.66	-0.87	
R ²	0.49		

^aNegative effects denote lower coherence

* $p < 0.05$, ** $p < 0.01$

Table 1: Estimation results for coherence

Parameter	low frequency		high frequency	
	Value	t	Value	t
DEMAND EQUATIONS				
<i>Kraft Demand</i>				
Price Kraft (b_1)	-1568.12	-14.00	-2928.24	-27.30
Price Sargento (c_1)	928.11	5.70	1010.44	4.64
Price Dominicks (d_1)	1119.68	6.93	-169.20	-0.95
<i>Sargento Demand</i>				
Price Kraft (b_2)	927.19	8.40	536.20	4.11
Price Sargento (c_2)	-1505.25	-10.35	-2005.80	-16.97
Price Dominicks (d_2)	-19.47	-0.15	-90.98	-0.77
<i>Dominicks Demand</i>				
Price Kraft (b_3)	1496.32	5.40	1706.18	3.75
Price Sargento (c_3)	1012.72	2.71	-485.72	-0.84
Price Dominicks (d_3)	-4113.34	-13.13	-4766.57	-10.78
MANUFACTURER PRICING RULES				
<i>Kraft Rule</i>				
Quantity Kraft (γ_1)	0.000582	30.41	0.000608	36.09
<i>Sargento Rule</i>				
Quantity Sargento (γ_2)	0.000694	32.20	0.000777	50.93
RETAILER PRICING RULES				
<i>Kraft Rule</i>				
Quantity Kraft (a_1)	-0.00017	-15.49	-0.00014	-21.29
Markup Sargento (a_2)	0.728	10.72	0.276	4.60
Markup Dominicks (a_3)	0.245	4.73	0.031	0.69
<i>Sargento Rule</i>				
Quantity Sargento (a_4)	-0.00014	-14.94	-0.00012	-13.00
Markup Kraft (a_5)	0.390	11.23	0.170	3.47
Markup Dominicks (a_6)	0.226	6.31	-0.079	-1.84
<i>Dominick's Rule</i>				
Quantity Dominicks (a_7)	-0.00009	-15.39	-0.00008	-19.52
Markup Kraft (a_8)	0.248	4.88	0.069	1.23
Markup Sargento (a_9)	0.640	9.20	0.003	0.04
SYSTEM-WEIGHTED R^2		0.73		0.70

Table 2: Structural model results

Parameter	low frequency		high frequency	
	Value	t	Value	t
k_1	-150.09	-1.44	1283.50	11.00
k_2	64.33	0.39	718.80	5.95

Table 3: Manufacturing pricing rule parameters

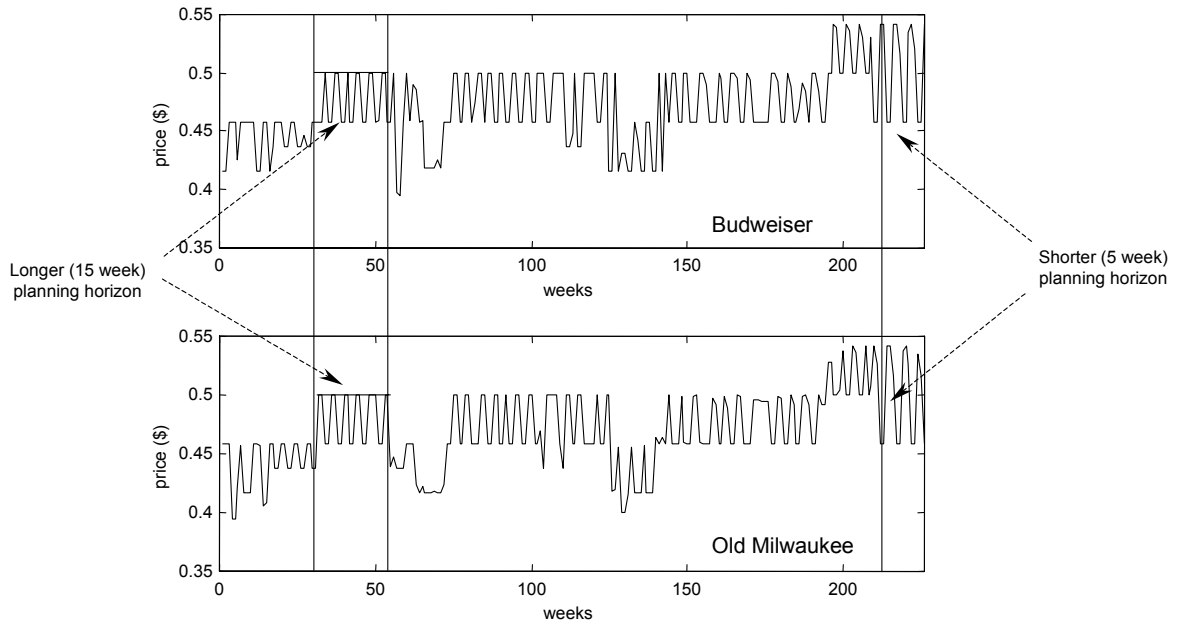


Figure 1: Weekly prices for Budweiser and Old Milwaukee

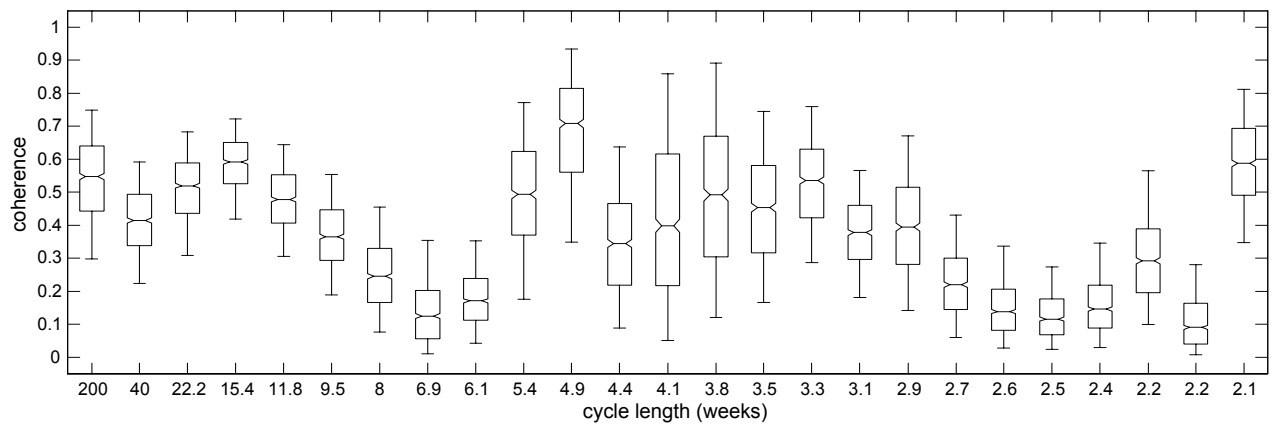


Figure 2: The distribution of coherence between Budweiser and Old Milwaukee

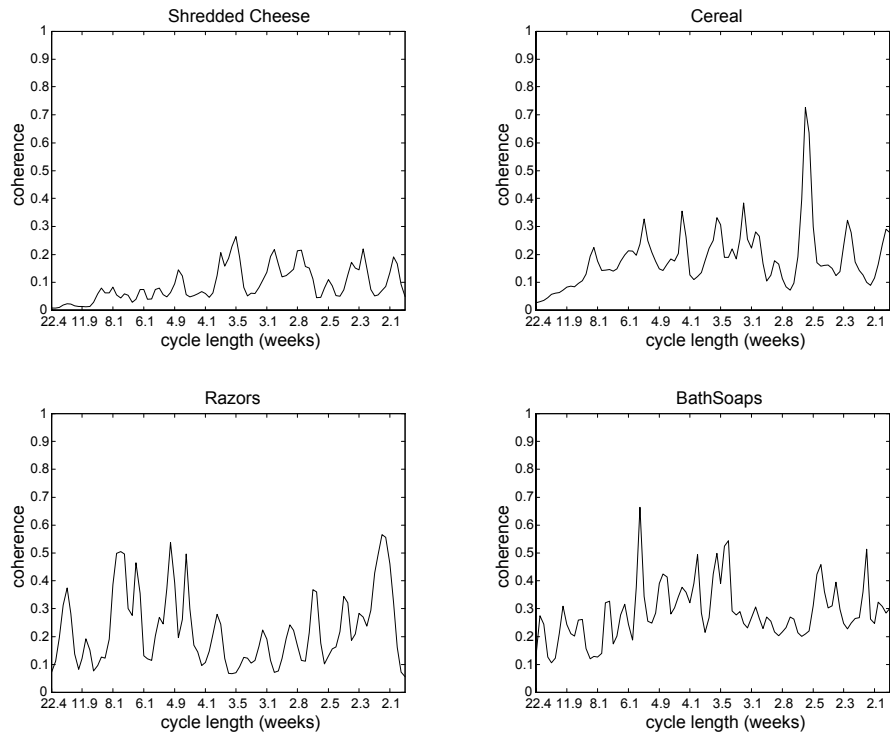


Figure 3: Average coherence for the brand pairs of four different categories

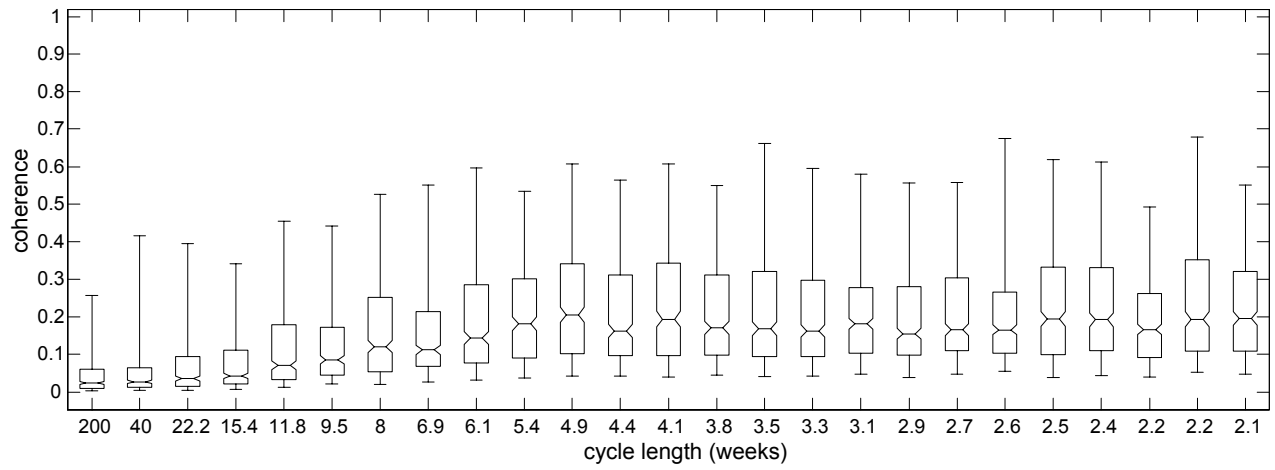


Figure 4: The distribution of coherence across all brand pairs over multiple planning periods.

References

- [1] Andersen, Torben G, and Tim Bollerslev (1997), "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Frequency Returns," *Journal of Finance*, Vol. 52, No. 3 (July), 975-1005.
- [2] Bell, David, R., Jeongwen Chiang, and V. Padmanabhan (1999), "The Decomposition of Promotional Response: An Empirical Generalization," *Marketing Science*, 18, 4, 504-526.
- [3] Blattberg, Robert C., Richard Briesch, and Edward J. Fox (1995), "How Promotions Work," *Marketing Science*, 14, 3, G122-132.
- [4] —————, Gary D. Eppen and Joshua Lieberman (1981), "A Theoretical and Empirical Evaluation of Price Deals for Consumer Non-Durables," *Journal of Marketing*, 45, 1, 116-129.
- [5] —————, and Kenneth J. Wisniewski (1989), "Price-Induced Patterns of Competition," *Marketing Science*, 8, 4, 291-309.
- [6] Brillinger, D. R. (1981), *Time Series, Data Analysis and Theory*, Holden Day, Inc., San Francisco
- [7] Bronnenberg, Bart J. and Luc Wathieu (1996), "Asymmetric Promotion Effects and Brand Positioning," *Marketing Science*, 15, 4, 379-394.
- [8] Chatfield, Christopher (1974), "Some Comments on Spectral Analysis in Marketing," *Journal of Marketing Research*, 11, 1.
- [9] Chan K. Hung, Jack C. Hayya, and J. Kieth Ord (1977), "A Note on Trend Removal Methods: The Case of Polynomial Regression Versus Variate Differencing," *Econometrica*, 45, 3, 737-744.
- [10] Chen, Ming-Jer, and Ian C. MacMillan (1992), "Nonresponse and Delayed Response to Competitive Moves: The Roles of Competitor Dependence and Action Irreversibility," *Academy of Management Journal*, 35 (August), 539-70.
- [11] —————, Ken G. Smith and Curtis M. Grimm (1992), "Action Characteristics as Predictors of Success in Competitive Response," *Management Science*, 38, 3, 439-455.
- [12] Chintagunta, Pradeep, Jean-Pierre Dube, and Vishal Singh (2002), "Balancing Profitability and Customer Welfare in a Supermarket Chain," *Quantitative Marketing and Economics*, forthcoming.
- [13] Dekimpe, Marnik and Dominique M. Hanssens (1999), "Sustained Spending and Persistent Response: A New Look at Long-Term Marketing Profitability," *The Journal of Marketing Research*, 36 (November), 397-412.

- [14] Farris, Paul, and John A. Quelch (1987), "In Defense of Price Promotions," *Sloan Management Review*, 29, 1, 63-69
- [15] Fishman, George S. (1969), "*Spectral Methods in Econometrics*," Harvard University Press, Cambridge, Massachusetts.
- [16] Gatignon, Hubert (1984), "Competition as a Moderator of the Effect of Advertising on Sales," *Journal of Marketing Research*, 21, 4, 387-398.
- [17] Greenleaf, Eric (1995), "The Impact of Reference Price Effects on the Profitability of Promotions," *Marketing Science*, 14, 1, 82-104.
- [18] Hamilton, James (1994), *Time Series Analysis*. Princeton University Press: Princeton, NJ.
- [19] Hanssens, Dominique M. (1980), "Market Response, Competitive Behavior, and Time Series Analysis," *Journal of Marketing Research*, 17, 4, 470-485.
- [20] Harvey, A.C. (1975), "Spectral Analysis in Economics," *Statistician*, 24:1, 1-36
- [21] —————, and A. Jaeger (1993), "Detrending, Stylized Facts, and the Business Cycle," *Journal of Applied Econometrics*, 8, 3, 231-247.
- [22] Hassler, John (1993), "Are They Swinging Together? A Measure of Linear Comovement With an Application to Swedish and Foreign Business Cycles," Working Paper, Institute for International Economic Studies, Stockholm University, Stockholm, Sweden.
- [23] Hoch, Stephen J. (1996), "How Should National Brands Think About Private Labels," *Sloan Management Review*, 37, 2, 89-102.
- [24] Kadiyali, Vrinda, Pradeep Chintagunta, and Naufel Vilcassim (2000), "A Manufacturer-Retailer Channel Interactions and Implications for Channel Power: An Empirical Investigation of Pricing of a Local Market," *Marketing Science*, 19, 2, 127-148.
- [25] —————, K. Sudhir, and Vithala Rao (2001), "Structural Analysis of Competitive Behavior: New Empirical Industrial Organization Methods in Marketing," *International Journal of Research in Marketing*, 18, 1-2(June), 161-186.
- [26] Krishna, Aradhna (1994), "The Impact of Dealing Patterns on Purchase Behavior," *Marketing Science*, 13, 4 (Fall), 351-373
- [27] Kopalle, Praveen, Carl F. Mela, and Lawrence Marsh (1999), "The Dynamic Effect of Discounting on Sales: Empirical Analysis and Normative Pricing Implications," *Marketing Science*, 18, 3 (Summer), 317-332.
- [28] Kuester, Sabine, Christian Homburg, and Thomas S. Robertson (1999), "Retaliatory Behavior to New Product Entry," *Journal of Marketing*, 63, 4, 90-106.
- [29] Lal, Rajiv (1990), "Price Promotions: Limiting Competitive Encroachments," *Marketing Science*, 9, 3, 247-262.

- [30] Lambin, Jean-Jacques, Philippe A. Naert, and Alain V. Bultez (1975), "Optimal Marketing Behavior in an Oligopoly," *European Economic Review*, 6, 105-128.
- [31] Leeflang, Peter S.H., and Dick R. Wittink (1992), "Diagnosing Competitive Reactions Using (Aggregated) Scanner Data," the *International Journal of Research in Marketing*, 9, 39-57.
- [32] ————— and ————— (1996), "Competitive Reaction versus Consumer Response: Do Managers Overreact?," the *International Journal of Research in Marketing*, 13, 2, 103-119.
- [33] ————— and ————— (2001), "Explaining Competitive Reaction Effects," *International Journal of Research in Marketing*, 18, 1-2, 119-137.
- [34] Leone, Robert P., (1995) "Generalizing What is Known About Temporal Aggregation and Advertising Carryover," *Marketing Science*, 14, 3, G141-G150.
- [35] Magnus, Jan-R (1982), "Multivariate Error Components Analysis of Linear and Nonlinear Regression Models of Maximum Likelihood," *Journal of Econometrics*, 19, 2-3 (August), 239-85.
- [36] Narasimhan, C., Scott Neslin and Subrata Sen (1996), "Promotional Elasticities and Category Characteristics," *Journal of Marketing*, 60, 2, 17-30.
- [37] Nelson, Charles R., and Haejoon Kang (1981), "Spurious Periodicity in Inappropriately Detrended Time Series," *Econometrica*, 49, 3, 741-751.
- [38] Nijs, Vincent R., Marnik G. Dekimpe, Jan-Benedict E.M. Steenkamp, and Dominique M. Hanssens (2001), "The Category-Demand Effects of Price Promotions," *Marketing Science*, 20, 1, 1-22.
- [39] Searle, Shayle (1982), "*Matrix Algebra Useful for Statistics*," Wiley, New York, NY.
- [40] Steenkamp, J.B., Vincent R. Nijs, Dominique M. Hanssens and Marnik G. Dekimpe (2004), "Competitive Reactions to Advertising and Promotion Attacks," *Marketing Science*, forthcoming.

A Estimation of Spectral Model

We employ the following procedure to obtain the spectral decomposition of the retail pricing series. The discussion is topical. For more detail, interested readers are referred to Harvey (1975), or for a more advanced treatment, Hamilton (1994).

1. *Difference the data.* Trends are similar to very low frequency cycles (because trends are akin to long-term price movements with an infinite cycle duration). As such, trends increase the apparent power of the lower frequencies in the data, and can lead to spurious low frequency signals in the data (Chan, Hayya, and Ord 1977; Harvey and Jaeger 1993; Nelson and Kang 1981). Moreover, spectral analysis is predicated upon the assumption that the data are stationary. Augmented Dickey-Fuller tests on the price series in our data indicate that this is often not the case. For these reasons, we apply a first differencing filter to the data. The coherence, gain, and phase relationships are invariant to filtering when the same linear filter is applied to all the series (see Fishman 1969, Hassler 1993). Hence, first differencing does not impact the measures of coherence, gain and phase. We note that differencing the data has the advantages of (1) increasing the likelihood that the series are stationary, (2) filtering the zero frequency, (3) preserving the coherence, phase and gain relationships, and (4) controlling for linear trends in price (including linear inflation trends).¹²

2. *Estimate a VAR model.* For all prices y_{ict} , for brands $i = 1, \dots, K_c$, in a category $c = 1, \dots, C$, and time periods $t = 1, \dots, T$, estimate

$$\mathbf{y}_{ct} \underset{(K_c \times 1)}{=} \boldsymbol{\alpha}_{c0} + \sum_{s=1}^P \Psi_{cs} \cdot \mathbf{y}_{ct-s} + \mathbf{e}_{ct} \quad (\text{A.1})$$

where P is the number of lags. The coefficients of this model, Ψ_{cs} , are used to compute the spectral decomposition. Given that regular price changes often show little or no variation for up to six months, we allow for very long lag-shifts of up to 26 weeks (i.e., $P = 26$). We allow for this flexibility because we wish to explore the longer term price interactions for which lower-order VAR models may be less appropriate. On the other hand, the higher order VAR models yields an impractical number of parameters. There is a $K_c \times K_c$ coefficient matrix Ψ to be estimated for each lag. With 5 brands and 26 periods, this would yield more than 600 parameters. This empirical task is impossible, as the number of observations is insufficient to estimate such a large number of parameters. Accordingly, we follow the standard practice of zeroing parameters with t -statistics less than 1.5 (similar to Dekimpe and Hanssens 1999).¹³ We proceed in phases, by first estimating

¹²It is important to note that differencing the data controls for linear trends in inflation in costs and prices. However, ingredient costs in food may be seasonal, and also impact our results. If coherence is driven by costs, we will observe a seasonal peak of 6 or 12 months in the data. We do not observe these peaks.

¹³We also used a cut-off of $t=1.0$ to assess the sensitivity of our results to assess the sensitivity of our results to the inclusion of more parameters. The results remained essentially identical.

a VAR model of order 1, and then retaining the parameters with t -statistics greater than 1.5. We then add a second lag, and repeat the process. This process continues until all P lags are added to the model.

3. *Compute the spectrum and cospectrum.* The spectrum can be interpreted as the proportion of the variance in a price series attributable to a certain frequency. Higher power indicates greater price variation at a given frequency. The cospectrum is then analogous to the covariance, and measures the degree of covariation between two series at a given frequency. We use the coefficient matrices Ψ_{cs} to compute the complete power spectrum (Hamilton 1994). Defining $\Omega_e = E(e_{ct}e'_{ct})$, the spectrum at frequency $\omega = 0, \dots, \pi$ is a square matrix of size K_c that is equal to

$$\mathbf{S}_c(\omega) \underset{(K_c \times K_c)}{=} (2\pi)^{-1} \left[\mathbf{I}_{K_c} - \sum_{s=1}^P \{ \Psi_{cs} e^{-js\omega} \}^{-1} \right] \Omega_e \left[\mathbf{I}_{K_c} - \sum_{s=1}^P \{ \Psi'_{cs} e^{js\omega} \}^{-1} \right], \quad (\text{A.2})$$

where $j = \sqrt{-1}$. In our empirical work, the complex matrix $\mathbf{S}_c(\omega)$ is computed at discrete $\omega = [0, 0.01\pi, 0.02\pi, \dots, \pi]$, and the resulting series $\{\mathbf{S}_c(\omega) \mid \omega = 0, \dots, \pi\}$ defines what is known as the *power spectrum*. Unlike power spectrums estimated from bi-variate VAR models, our approach controls for the observed effects of *all other brands' prices*, because the VAR is estimated on the full set of prices.

4. *Compute the coherence and phase.* Define for each pair of brands i_c and i'_c the following four quantities:

$$\begin{aligned} s_{ii}^c(\omega) &= \mathbf{S}_{ii}^c(\omega) \\ s_{i'i'}^c(\omega) &= \mathbf{S}_{i'i'}^c(\omega) \\ q_{ii'}^c(\omega) &= \text{im} \{ \mathbf{S}_{ii'}^c(\omega) \} \\ c_{ii'}^c(\omega) &= \text{re} \{ \mathbf{S}_{ii'}^c(\omega) \} \end{aligned} \quad (\text{A.3})$$

where $\text{im}\{\arg\}$ is the imaginary part, and $\text{re}\{\arg\}$ is the real part of its arguments. The factor $q_{ii'}^c$ is called the “quadrature” and $c_{ii'}^c$ is called the “cospectrum.” Coherence is computed as follows

$$h_{ii'}^c(\omega) \equiv \frac{[q_{ii'}^c(\omega)]^2 + [c_{ii'}^c(\omega)]^2}{s_{ii}^c(\omega)s_{i'i'}^c(\omega)}. \quad (\text{A.4})$$

Note that the coherence measure is symmetric – it is analogous to an \mathbb{R}^2 measure in regression and measures the strength of association between two series at different frequencies (i.e., planning horizons).

5. *Compute the moments of coherence.* We note that there is no confidence interval around (A.4). To approximate such an interval, we generate 1000 draws from the sampling distribution of the estimated VAR parameters of equation (A.1), and use these draws to obtain an empirical distribution for the power spectrum, and the resulting measure in equation (A.4). For instance,

the distributions underlying the box-plots in Figure 2 are computed from the 1000 replications of the spectral decompositions of the 1000 randomly drawn VAR models. This procedure enables one to ascertain which frequencies are associated with tightly distributed coherences.

B Estimation Of Cross-time, Brand and Category Effects

After collecting the coherences across planning horizons $p = \{0, 1, 2\}$, brand pairs $ii' = 1, \dots, N_c$, and categories $c = 1, \dots, C$, we then estimate the following regressions for coherence,

$$h_{ii'cp} = \delta_{0h} + \mathbf{v}'_c \alpha_h + \mathbf{z}'_{ii'c'} \beta_h + \gamma_h p + \varepsilon_{ii'c} + \eta_{cp} + \xi_{ii'cp}, \quad (\text{B.1})$$

where p is planning horizon and the same manufacturer indicator variable is denoted, $z_{ii'}^c$.¹⁴ The η_{cp} are within category/horizon random effects, the $\varepsilon_{ii'c}$ are within brand-pair effects, and the $\xi_{ii'cp}$ are the observational errors. We specify these random effects to account for potential correlations across observations within the same time horizon and category as well as within the same brand pair across time horizons (as, strictly speaking, these coherence estimates are not independent replicates, and failure to accommodate correlations among these repeated measures might overstate the power of the fixed effects). We specify $\xi_{ii'cp} \sim IIDN(0, \sigma_\xi^2)$, $\eta_{cp} \sim IIDN(0, \sigma_\eta^2)$, and $\varepsilon_{ii'c} \sim IIDN(0, \sigma_\varepsilon^2)$ and assume these three errors to be independent.

Thus, the component of the covariance structure of the within category covariation of coherence is the variance structure of $\eta_{cp} + \xi_{ii'cp}$ across all c , i.e.,

$$\Lambda_c = \begin{bmatrix} \sigma_\xi^2 + \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \vdots & \ddots & \vdots \\ \sigma_\eta^2 & \cdots & \sigma_\xi^2 + \sigma_\eta^2 \end{bmatrix}_{N_c \times N_c} \quad \text{and} \quad \Lambda = \begin{bmatrix} \Lambda_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \Lambda_C \end{bmatrix}_{N \times N}, \quad (\text{B.2})$$

where $N = \sum_{\forall c} N_c$. To obtain the covariance matrix for the system stacked across brand-pairs, categories, and planning horizons we combine Λ with the brand-pair random effects. The resulting covariance matrix is given by

$$\Omega = \mathbf{I}_3 \otimes \Lambda^{-1} + \sigma_\varepsilon^2 \boldsymbol{\iota}_3 \boldsymbol{\iota}'_3 \otimes \mathbf{I}_N, \quad (\text{B.3})$$

where \mathbf{I}_3 is a 3-dimensional identity matrix, \mathbf{I}_N is an N -dimensional identity matrix and $\boldsymbol{\iota}_3 \boldsymbol{\iota}'_3$ is a 3×3 matrix of ones. Ω is thus dimensioned $3N \times 3N$.

We use Maximum Likelihood to estimate the model parameters $\boldsymbol{\theta}_h = [\delta_{0h}, \boldsymbol{\alpha}_h, \boldsymbol{\beta}_h, \sigma_\xi^2, \sigma_\eta^2, \sigma_\varepsilon^2]$. Define $e_{ii'cp} = h_{ii'cp} - \delta_{0h} - \mathbf{v}'_c \alpha_h - \mathbf{z}'_{ii'c'} \beta_h + \gamma_h p$. Array these residuals across brand pairs and categories, so that we obtain an $N \times 1$ vector \mathbf{e}_p , and let the $N \times 3$ matrix $\mathbf{u} \equiv [\mathbf{e}_0 \quad \mathbf{e}_1 \quad \mathbf{e}_2]$. Then the likelihood function is proportional to

$$L(\boldsymbol{\theta}) \propto |\Omega|^{-0.5} \exp(-0.5 \cdot (\text{vec}(\mathbf{u})' \Omega^{-1} \text{vec}(\mathbf{u}))) \quad (\text{B.4})$$

¹⁴Given that coherence is constrained to lie between 0 and 1, we use the logistic transform of coherence in our analysis of the effect of brand and category characteristics on coherence. This transformation is given by $\ln(h_{ii'}(\omega)/(1 - h_{ii'}(\omega)))$. Also note that the transform is taken before any sample moments are computed.

After taking the log of the likelihood and simplifying (Magnus 1982), the log-likelihood function can be written as (ignoring an irrelevant constant),

$$\ln L(\boldsymbol{\theta}) = -0.5 \cdot \ln |\Omega| + 0.5 \cdot \text{trace}(\mathbf{u}' \cdot \Lambda^{-1} \cdot \mathbf{u}) - 0.5 \cdot \text{trace}(\mathbf{u}' \cdot \mathbf{B} \cdot \mathbf{u} \cdot \mathbf{A}) \quad , \quad (\text{B.5})$$

where $\mathbf{A} = \frac{1}{3}\boldsymbol{\nu}_3\boldsymbol{\nu}_3'$, $\mathbf{B} = \Lambda^{-1} - (\Lambda + 3\sigma_\varepsilon^2\mathbf{I}_N)^{-1}$, and the log of the determinant can be shown to equal (using some results on determinants in Magnus (1982) and Searle (1982))

$$\ln |\Omega| = 2 \sum_c [\ln(\sigma_\xi^2 + N_c\sigma_\eta^2) + (N_c - 1) \ln(\sigma_\xi^2)] + \sum_c [\ln(\sigma_\xi^2 + 3\sigma_\varepsilon^2 + N_c\sigma_\eta^2) + (N_c - 1) \ln(\sigma_\xi^2 + 3\sigma_\varepsilon^2)] \quad . \quad (\text{B.6})$$

Note that the log-likelihood when expressed in this form requires only an inversion of a block diagonal $N \times N$ matrix, which is much smaller than Ω . Estimation proceeds by maximizing equation (B.5) over $\boldsymbol{\theta}$.

C Variable Operationalization

The variables in Table 1 are operationalized as follows. *Same manufacturer* is an indicator variable that assumes the value of one if the two brands are produced by the same manufacturer, and 0 otherwise. Note, the same manufacturer is not equivalent to the same brand. *Private label* is an indicator variable that assumes the value of one if one of the brands is a store brand. *Price differential* reflects the mean per unit absolute price difference over time. Price differential was based upon standardized series in order to make the variables comparable across categories (as units of volume differ across categories).

Concentration is measured as the Herfindahl index. *Volatility* is defined as the sum of manufacturer births and deaths in the data expressed as a fraction of the number of manufacturers. A birth is determined by the appearance of a manufacturer sometime over the duration of the data. A death is determined by the disappearance of a manufacturer prior to the end of the data. Each of these is converted to a percent by dividing by the total number of manufacturers. *Penetration* (as the percent of consumers using the category) is obtained from the IRI factbook. *Storability* is rated on a scale of 1-7 via a survey distributed to students in a large MBA program. Table C.1 outlines summary statistics for these variables.

Table C.1 - Variable Means

Variable	Mean	Standard Deviation
<i>Brand Variables</i>		
Same Manufacturer	0.14	0.35
Private Label	0.20	0.40
Price Differential	1.17	0.79
<i>Category Variables</i>		
Concentration	0.26	0.13
Volatility	0.95	0.21
Penetration	71.0	22.3
Storability	4.96	0.56

D Specification and Estimation of Structural Model

We use the following demand equations for three brands (ignoring demand shifters for sake of explication),

$$q_i = a_i + b_i p_1 + c_i p_2 + d_i p_3. \quad (\text{D.1})$$

Then profits for brand 1 are then given by $(mp_1 - mc_1)q$ where mp_1 is the manufacturer price of brand 1, and mc_1 is the manufacturer cost. The derivative with respect to p_1 is given by $(mp_1 - mc_1)'q_1 + (mp_1 - mc_1)q_1'$. This implies $(mp_1 - mc_1)(b_1(\partial p_1/\partial mp_1) + c_1(\partial p_2/\partial mp_1) + d_1(\partial p_3/\partial mp_1)) + q_1 = 0$. Noting that $p_1 = mp_1 + r_1$ where p is retail price and r is markup, then $(mp_1 - mc_1)(b_1(1 + \partial r_1/\partial mp_1) + c_1(\partial p_2/\partial mp_1) + d_1(\partial p_3/\partial mp_1)) + q_1 = 0$. Setting $\partial r_1/\partial mp_1 = t_1$, $\partial p_2/\partial mp_1 = t_2$, and $\partial p_3/\partial mp_1 = t_3$ implies $mp_1 = mc_1 - q_1(b_1 + b_1 t_1 + c_1 t_2 + d_1 t_3)^{-1}$. When t_1 , t_2 , and t_3 are 0, we observe Nash, however these parameters are not separately identified. Setting $k_1 = b_1 t_1 + c_1 t_2 + d_1 t_3$ we obtain $mp_1 = mc_1 - q_1(b_1 + k_1)^{-1}$. Thus, if k_1 differs from 0, the game is not Nash (Kadiyali et al 2000). Note that $k_1 = 0$ is a necessary but not sufficient condition for Nash. Setting $k_1 = -(1/\gamma_1) - b_1$ yields

$$mp_1 = mc_1 + \gamma_1 q_1 \quad (\text{D.2})$$

which is the estimation equation. After estimating γ_1 and b_1 , the variance of k_1 can be inferred using the delta method. A similar equation holds for the other brands. Note that γ is a measure of relative margins inasmuch as the manufacturer margin, $mp_1 - mc_1 = \gamma_1 q_1$.

Retailer profits are given by $r_1 q_1 + r_2 q_2 + r_3 q_3$. Selecting r_1 to maximize profits implies $r_1' q_1 + r_1 q_1' + r_2' q_2 + r_2 q_2' + r_3' q_3 + r_3 q_3' = 0$. Thus, $q_1 + r_1(b_1(\partial mp_1/\partial r_1 + 1) + c_1 \partial mp_2/\partial r_1 + d_1 \partial mp_3/\partial r_1) + r_2(b_2(\partial mp_1/\partial r_1 + 1) + c_2 \partial mp_2/\partial r_1 + d_2 \partial mp_3/\partial r_1) + r_3(b_3(\partial mp_1/\partial r_1 + 1) + c_3 \partial mp_2/\partial r_1 + d_3 \partial mp_3/\partial r_1) = 0$. Again, noting we can not separately identify the conduct parameters, we obtain $r_1 = -(b_1 + k_4)^{-1} q_1 - (b_2 + k_5)(b_1 + k_4)^{-1} r_2 - (b_3 + k_6)(b_1 + k_4)^{-1} r_3$ where $k_4 = b_1 \partial mp_1/\partial r_1 + c_1 \partial mp_2/\partial r_1 + d_1 \partial mp_3/\partial r_1$, $k_5 = b_2 \partial mp_1/\partial r_1 + c_2 \partial mp_2/\partial r_1 + d_2 \partial mp_3/\partial r_1$, and $k_6 = b_3 \partial mp_1/\partial r_1 + c_3 \partial mp_2/\partial r_1 + d_3 \partial mp_3/\partial r_1$. When $k_4, k_5, k_6 = 0$, this is Nash (Kadiyali et al. 2000). Setting $k_4 = (-1 -$

$\alpha_1 b_1)/\alpha_1, k_5 = -b_2 - \alpha_2(b_1 + k_4) = -b_2 + \alpha_2/\alpha_1$, and $k_6 = -b_3 - \alpha_3(b_1 + k_4) = -b_3 - \alpha_3/\alpha_1$ yields the estimation equation,

$$r_1 = \alpha_1 q_1 + \alpha_2 r_2 + \alpha_3 r_3 \tag{D.3}$$

which is linear. Equations (D.1-D.3) together form the system in equation (6)