

# Common Cycles and Common Trends in Latin America

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**Abstract:** *This paper applies three common cyclical feature models for analyzing business cycle co-movements among the real gross domestic product of five Latin American countries. It emerges that these economies are strongly related and share long and short-run propagation mechanisms.*

## 1. Introduction

Economic series display a vast array of similarities, which can be removed by linearly combining them, and are labeled **Common Features**. Such common features arise when the series exhibit co-movements, i.e. when they are generated by common factors. Examples are: common stochastic trends (cointegration), common serial correlation (common cycles), common autoregressive conditional heteroscedasticity (common ARCH), common structural breaks (co-breaking), common seasonality, etc. These similarities have several implications for economic time series modeling as well as for estimation, testing and forecasting. Indeed, common features can be exploited to reduce the number of parameters to estimate and consequently can increase efficiency and improve forecast accuracy since redundant factors are removed. By determining the number and the nature of the common factors, the analysis not only leads to a more parsimonious parametrized model, it also yields information that is crucial from an economic point of view because economic theory often predicts and explains such co-movements.

It is then clear that a common feature analysis could be applied to anything that is present in individual variables and that disappears by some appropriate combinations because variables share this feature. However, due to the importance of the spurious regression issue (Granger and Newbold, 1974), the bulk of the literature has mainly focused on long-run co-movements, namely the search for common stochastic trends through cointegration analyses (see Stock and Watson, 1988; Johansen, 1995). More recently, some authors have also analyzed the existence of short-run co-movements between stationary time series or between first differences of cointegrated I(1) series, namely the presence of common cyclical features (see Engle and Kozicki, 1993). These will be associated with common business cycles and sometimes interpreted as a condition for economic convergence and sustainable monetary unions (see Beine, Candelon and Hecq, 2000).

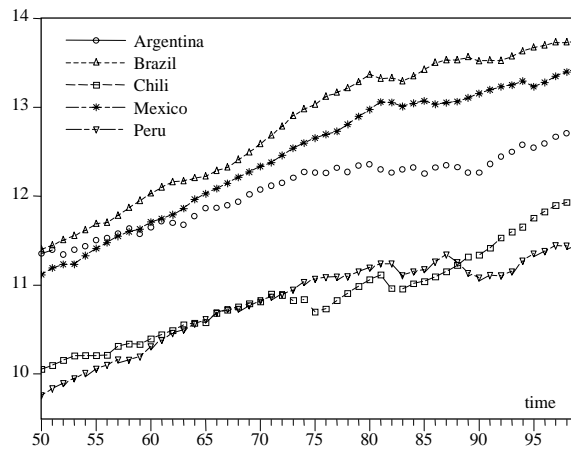
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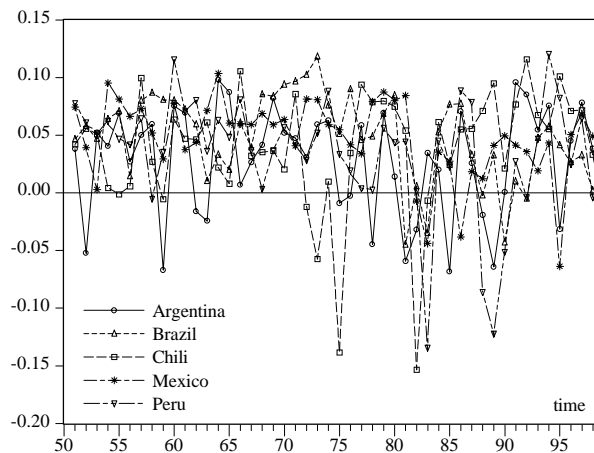
This paper investigates the degree of long-run and short-run dependence among five major Latin American economies. Figures 1 and 2 present both the log-levels and the growth rates of the annual real gross domestic product for Argentina, Brazil, Mexico, Peru and Chile. From Figure 1, it is obvious that these series share some positive tendencies. Figure 2 also shows some evidence of short-run co-movements. Multivariate cointegration and common feature tests are performed to formally test the hypothesis that these countries share a reduced number of trends and cycles.

The novelty of this paper is to confront three types of common cyclical feature models. These are the Serial Correlation Common Feature (SCCF) and two alternative models that have been proposed to relax the strong assumptions underlying SCCF: the Weak Form reduced rank structure (WF) and the Polynomial Serial Correlation Common Features (PSCCF). Section 2 summarizes these representations while Section 3 reports the empirical findings. We also compare results based on formal likelihood ratio tests with the use of information criteria. Section 4 concludes.

**Figure 1** – Log-levels of real gross domestic products (1950-1999)



**Figure 2** – Growth rates of real gross domestic products (1950-1999)



## 2. Three Reduced Rank Models

This section presents three common cyclical feature models. These three models impose additional restrictions to the vector error-correction model (VECM) and consequently should be considered as complementary tools to cointegration. Indeed, whilst cointegration analyzes long-run co-movements, a common cyclical feature study focuses on short-run co-movements; whilst cointegration stresses the common trends from a set of economic time series, common cyclical features reveal the common cycles.

Let us start with the vector autoregressive model of order  $p$ , i.e. a VAR( $p$ ), for a  $n$ -vector of I(1) time series  $y_t$  over the period  $t = 1 \dots T$ :

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \dots + \Phi_p y_{t-p} \equiv \Phi(L) y_t = \varepsilon_t, \quad (1)$$

for fixed initial values  $y_{-p+1}, \dots, y_0$  and with  $\Phi(L) = I - \sum_{i=1}^p \Phi_i L^i$ .  $\varepsilon_t$  is a  $n$ -dimensional homoscedastic Gaussian mean innovation process with nonsingular covariance matrix  $\Omega$ . For notational convenience, deterministic terms are omitted at this level of presentation. Let us further assume that  $y_t$  is cointegrated of order (1,1). Hence, the rank of  $-\Phi(1) = \sum_{i=1}^p \Phi_i - I$  is  $r$ ,  $0 < r < n$ , and  $-\Phi(1)$  can be expressed as the product  $\alpha\beta'$  with  $\alpha$  and  $\beta$  both  $(n \times r)$  matrices of rank  $r$ . Then the VAR can be written as a vector error-correction model (VECM):

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t, \quad (2)$$

with  $\Gamma_i = -\sum_{j=i+1}^p \Phi_j$  for  $i = 1 \dots p-1$ . The columns of  $\beta$  span the space of cointegrating vectors, and the elements of  $\alpha$  are the corresponding adjustment coefficients.

Each variable in  $\Delta y_t$  is serially correlated and we consider three types of common feature restrictions on (2). We index by  $a$ ,  $b$  or  $c$  the co-feature matrices associated with these models. The first model, labeled Serial Correlation Common Features or SCCF (Engle and Kozicki, 1993; Vahid and Engle, 1993), arises if the serial correlation in the series  $\Delta y_t$  is such that there exist some linear combinations of  $\Delta y_t$  which do not exhibit autocorrelation. More formally, this implies that there exist a matrix  $\delta_a$  such that  $\delta_a' \Delta y_t = \delta_a' \varepsilon_t$  is a  $s$ -dimensional white noise process and that consequently the following restrictions to the VECM in (2) jointly hold: 1)  $\delta_a' \alpha = 0$  and 2)  $\delta_a' \Gamma_i = 0$ ,  $i = 1 \dots p-1$ . A second model, called the Weak Form reduced rank structure or WF (Hecq, Palm and Urbain, 2000, 2001, 2002), arises when  $s$  linear combinations  $\delta_b$  of  $\Delta y_t$  in deviation from the error-correction terms  $\alpha\beta' y_{t-1}$  are white noises. It corresponds to the restrictions  $\delta_b' \Gamma_i = 0$ ,  $i = 1 \dots p-1$ . Finally, the Polynomial Serial Correlation Common Features or PSCCF (Cubadda and Hecq, 2001, 2002) arises when there exists a polynomial matrix

$\delta_c(L) = \delta_{0,c} - \delta_{1,c}L$  such that  $\delta_c(L)' \Delta y_t = \delta_{0,c}' \varepsilon_t$ . PSCCF implies the following restrictions on the VECM in (2): 1)  $\delta_{0,c}' \alpha = 0$  and 2)  $\delta_{0,c}' \Gamma_i = 0$  if  $i > 1$  and  $\delta_{0,c}' \Gamma_i = \delta_{1,c}'$  if  $i = 1$ . The SCCF is the more restrictive form among the three specifications. Both WF and PSCCF aim at relaxing the assumptions underlying SCCF and allow for adjustment delays in the synchronization of the cycles.<sup>2</sup>

These sets of common feature restrictions give rise to a full description of the trend and the cyclical components of  $y_t$ . Indeed, since the stationary process  $\Delta y_t$  admits the Wold representation  $\Delta y_t = C(L)\varepsilon_t$ , with  $\sum_{j=1}^{\infty} j |C_j| < \infty$ ,  $C_0 = I_n$ , and using the associated polynomial factorization  $C(L) = C(1) + \Delta C^*(L)$  where  $C_i^* = -\sum_{j=i+1}^{\infty} C_j$  for  $i \geq 0$ , we obtain the Beveridge-Nelson permanent transitory decomposition of  $y_t$  such that:

$$y_t = \tau_t + \xi_t \equiv \text{trend} + \text{cycle}, \quad (3)$$

where  $\Delta \tau_t = C(1)\varepsilon_t$  and  $\xi_t = C^*(L)\varepsilon_t$  are respectively the first difference of the trend component and the level of cyclical component. Under cointegration  $C(1)$  is of reduced rank  $n - r$  and we know that these common stochastic trends are annihilated by the cointegrating vectors such that  $\beta' \tau_t = 0$ . Similarly, under common feature  $C^*(L)$  is of reduced rank  $n - s$  and we have  $\delta_a' \xi_t = 0$  or  $\delta_c(L)' \xi_t = \delta_{0,c}' \varepsilon_t$  under respectively SCFF and PSCCF. The decomposition for the WF is more complicated and can be found in Hecq et al. (2000).

We use the well-known Johansen's maximum likelihood estimator to test for cointegration. We also use a canonical correlation approach to determine the rank of the common feature space, namely the number of zero squared canonical correlations that will be associated at the number of linearly independent co-feature vectors.<sup>3</sup> Let the expression  $\text{Cancor}(X_t, Z_t | W_t)$  denotes the partial canonical correlations between  $X_t$  and  $Z_t$  conditional on  $W_t$ . For instance, the "Johansen's test" with a constant in the short-run involves  $X_t = \Delta y_t$ ,  $Z_t = y_{t-1}$  and  $W_t = (1, \Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$ . Statistical inference for the various common feature specifications can be obtained by solving similar canonical correlation problems for particular choices of  $X_t$ ,  $Z_t$  and  $W_t$ . Table 1 reports these variables as well as the number of restrictions these models imply. Notice that for the WF and the PSCCF, there are two ways to proceed depending upon whether we want to net out the effect of the vector that is not annihilated by the co-feature vector. The number of restrictions is used to compute the degrees of freedom. Under stationarity,

<sup>2</sup> We have not considered in this paper the codependent cycle approach of Vahid and Engle (1997) where a linear combination of a VAR is a MA( $q$ ), with  $q$  small.

<sup>3</sup> Notice that a GMM (or IV) approach is also feasible but less convenient in the case of multiple co-feature vectors.

these tests follow an asymptotic  $\chi^2_{(v)}$  distribution under the null of common features. Likelihood ratio tests are given by:

$$Test_h = -T \sum_{i=1}^s \ln(1 - \hat{\lambda}_i^h), \quad (4)$$

where  $s = 1 \dots n - r$  for the SCCF and the PSCCF and  $s = 1 \dots n$  for the WF;  $h = a, b, c$  denotes the model which is considered.  $\hat{\lambda}_i$  with  $0 \leq \hat{\lambda}_1 < \dots < \hat{\lambda}_i < \dots < 1$  are the estimated eigenvalues (i.e. smallest squared canonical correlations) given by the solution of  $|\lambda - S_{xx}^{-1/2} S_{xz} S_{zz}^{-1} S_{zx} S_{xx}^{-1/2}| = 0$  where the product moment matrices such as  $S_{xx} = T^{-1} \sum_{i=1}^T \tilde{X}_i \tilde{X}_i'$  or  $S_{xz} = T^{-1} \sum_{i=1}^T \tilde{X}_i \tilde{Z}_i'$  imply elements of  $X_t$  and  $Z_t$  concentrated out  $W_t$ .  $D_t$  stands for any deterministic terms such as a constant, a deterministic trend, seasonal dummies, etc.

**Table 1** - Testing for common features using reduced rank regressions

Models	$X_t$	$Z_t$	$W_t$	Number of restrictions
SCCF	$\Delta y_t$	$(y'_{t-1} \beta, \Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$	$D_t$	$s(n(p-2) + r + s)$
WF	a) $(\Delta y'_t, y'_{t-1} \beta)'$	$(y'_{t-1} \beta, \Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$	$D_t$	$s(n(p-2) + s)$
	b) $\Delta y_t$	$(\Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$	$(y'_{t-1} \beta, D_t)'$	$s(n(p-2) + s)$
PSCCF	a) $(\Delta y'_t, \Delta y'_{t-1})'$	$(y'_{t-1} \beta, \Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$	$D_t$	$s(n(p-3) + r + s)$
	b) $\Delta y_t$	$(y'_{t-1} \beta, \Delta y'_{t-2}, \dots, \Delta y'_{t-p+1})'$	$(\Delta y'_{t-1}, D_t)'$	$s(n(p-3) + r + s)$

The last column itself reveals one of the interest in common features, namely the strong reduction in the number of parameters that need to be estimated. Indeed, without deterministic terms, an unrestricted VAR( $p$ ) has  $n^2 p$  mean parameters. The restricted model is simply given by the set of  $s$  pseudo structural equations to which one stacks  $n - s$  unrestricted equations as in the following example for the SCCF,<sup>4</sup>

<sup>4</sup> For the WF and the PSCCF we replace respectively the first and the second zero of the right-hand side matrix by parameter matrices.

$$\begin{pmatrix} I_s & \bar{\delta}'_a \\ 0 & I_{n-s} \end{pmatrix} \Delta y_t = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \tilde{\alpha} & \tilde{\Gamma}_1 & \dots & \tilde{\Gamma}_{p-1} \end{pmatrix} \begin{pmatrix} \beta' y_{t-1} \\ \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} + \omega_t. \quad (5)$$

The co-feature matrix  $\delta'_a = (I_s : \bar{\delta}'_a)$  has been normalized on the first  $s$  variables using the identity matrix;  $\tilde{\alpha}, \tilde{\Gamma}_i$  are the parameter matrices for the  $n - s$  remaining equations of the VECM. For the case we illustrate in Section 3, i.e. a system with five variables and three lags, we have in the unrestricted VAR, 75 mean parameters. Imposing two cointegrating vectors reduces this number to 66. If we add three common feature vectors, there only remain 36, 42 and 51 parameters that need to be estimated under respectively SCCF, WF and PSCCF. Contrary to cointegration this gain in terms of parameter reduction increases with the number of lags. Note finally that alternatively to (5), common feature restrictions can be expressed as a dynamic factor model with, as for the SCCF for instance,  $n - s$  factors  $F_{a,t} = C'_a (y'_{t-1} \beta, \Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$  which are linear combinations of the right-hand side variables in (2):

$$\Delta y_t = \delta_{a\perp} F_{a,t} + \varepsilon_t, \quad (6)$$

where  $\delta_{a\perp}$  is a  $n \times (n - s)$  matrix of loadings and consequently the orthogonal complement to  $\delta_a$  such that  $\delta'_a \delta_{a\perp} = 0$ ;  $C'_a$  being a  $(n - s) \times (n(p - 1))$  matrix of coefficients.

### 3. Empirical Analysis

This section investigates the short and the long-run interactions between the real gross domestic product of five major Latin American economies: Brazil, Argentina, Mexico, Peru and Chili. All the annual series are derived from the Total Economy Database<sup>5</sup> and span the period 1950-1999. We cannot use too many lags with such a small sample but a VAR with three lags seems to capture the dynamics of this multivariate process. Johansen's ML tests detect the presence of two cointegrating vectors for the model with a constant only and three cointegrating vectors for the model with a deterministic trend constrained in the long-run.<sup>6</sup> This latter is used to capture the possible presence of a deterministic convergence process or the existence of trend stationary variables. The modules of largest roots of the companion matrix are respectively (0.9904, 0.9843, 0.9843, 0.9280, 0.9280, 0.8528) and (0.9826, 0.9826, 0.8970, 0.8970, 0.7733, 0.7733) for

<sup>5</sup> University of Groningen and The Conference Board, GGDC Total Economy Database, 2002, [www.eco.rug.nl/ggdc](http://www.eco.rug.nl/ggdc). The variables are expressed in 1990 US dollars and converted at "Geary-Khamis" purchasing power parities.

<sup>6</sup> Because most people are familiar with cointegration techniques we do not reproduce the results to save place but complete results such as outputs, graphs and data are available upon request.

the model with a constant and the model with a deterministic trend, confirming the presence of respectively three and two common trends. More interesting for this paper is the number of common feature vectors. Table 2 and 3 report the eigenvalues (i.e. squared canonical correlations), the value of the log-likelihood as well as the  $p$ -value associated with the null hypothesis that there exist at least  $s$  co-feature vectors. An entry  $<0.001$  means that the probability of not rejecting an additional vector is very small and is less than 0.001.

**Table 2** – Common features test statistics,  $r=2$  (constant only)

	SCCF			WF			PSCCF		
	$\lambda_i$	$p$ -val	loglik	$\lambda_i$	$p$ -val	Loglik	$\lambda_i$	$p$ -val	loglik
$s \geq 1$	0.08	0.83	828.727	0.08	0.65	828.748	0.01	0.94	830.645
$s \geq 2$	0.21	0.64	823.219	0.17	0.49	824.113	0.10	0.71	828.116
$s \geq 3$	0.24	0.54	816.579	0.21	0.41	818.407	0.14	0.65	824.683
$s \geq 4$	0.72	<0.001	786.177	0.59	0.001	797.239	0.65	<0.001	799.791
$s = 5$	0.82	<0.001	746.136	0.69	<0.001	769.381	0.76	<0.001	766.554

**Table 3** – Common features test statistics,  $r=3$  (deterministic trend)

	SCCF			WF			PSCCF		
	$\lambda_i$	$p$ -val	loglik	$\lambda_i$	$p$ -val	loglik	$\lambda_i$	$p$ -val	loglik
$s \geq 1$	0.09	0.86	843.73	0.08	0.68	844.069	0.01	0.97	845.78
$s \geq 2$	0.21	0.71	837.971	0.17	0.53	839.632	0.14	0.67	842.284
$s \geq 3$	0.55	0.01	819.051	0.27	0.27	832.141	0.47	0.01	827.52
$s \geq 4$	0.75	<0.001	786.267	0.62	<0.001	809.077	0.69	<0.001	800.063
$s = 5$	0.82	<0.001	746.136	0.70	<0.001	780.489	0.76	<0.001	766.554

From the tables, it turns out that with  $r=2$ , we cannot reject the presence of three co-feature vectors of each kind. But because SCCF imposes more restrictions, it will be our favorite parsimonious model.<sup>7</sup> Once we introduce a linear trend in the long-run, we do not reject the existence of three cointegrating vectors. Consequently, due to the linear independence between the cointegrating and the common feature spaces under SCCF and PSCCF, i.e.  $r + s \leq n$ , we are only able to detect two co-feature vectors. On the other hand, the WF is robust to an overidentification of the number of long-run relationships and still reports three co-feature vectors (see Hecq et al., 2001).

Instead of using the previous two step formal approach in which we first determine  $p$  and then  $s$ , we could also rely on a joint determination procedure and let  $s$  and  $p$  vary inside each class of model (see Vahid and Issler, 2002). To do so we can use one of the well know information criteria, namely:

<sup>7</sup> See Hecq et al. (2001) for formal testing of SCCF against WF.

$$AIC(p, s, \hat{r}) = -\frac{2}{T} \loglik + \frac{2}{T} (\# \text{ parameters})$$

$$HQ(p, s, \hat{r}) = -\frac{2}{T} \loglik + \frac{2 \ln \ln T}{T} (\# \text{ parameters})$$

$$SC(p, s, \hat{r}) = -\frac{2}{T} \loglik + \frac{\ln T}{T} (\# \text{ parameters})$$

where  $r$  is assumed fixed and is denoted by  $\hat{r}$  in the formulas although the parameters of cointegrating vectors are estimated for each sample and each lag. The number of parameters is given by those needed in the VECM once the normalized cointegrating vectors are known, namely  $nr + n^2(p - 1)$ , less the number of restrictions given in Table 1 for each model. To let  $s$  and  $p$  vary, we choose to take  $p=2\dots pmax$  with  $pmax=5$ . Consequently all the models are estimated on the same subsample with  $T=45$ . Table 4 and 5 report the results for the Hannan Quinn criterion for both the constant and the constrained deterministic trend models.

**Table 4** – Common features restrictions with HQ,  $\hat{r} = 2$  (constant only)

	<b>SCCF</b>			
	$p=5$	$p=4$	$p=3$	$p=2$
$s = 0$	-32.0544	-31.8236	-31.793	-31.8749
$s = 1$	-32.6714	-32.3139	-32.1787	-32.0139
$s = 2$	-33.1381	-32.5499	-32.5273	-32.1868
$s = 3$	<b>-33.4326</b>	-32.7165	-32.9186	-32.3793
$s = 4$	-32.6712	-32.4603	-32.4513	-32.1896
$s = 5$	-31.6432	-31.6432	-31.6432	-31.6432
	<b>WF</b>			
	$p=5$	$p=4$	$p=3$	$p=2$
$s = 0$	-32.0544	-31.8236	-31.7930	-30.2822
$s = 1$	-32.5766	-32.2030	-32.0606	-30.7005
$s = 2$	-33.0066	-32.4458	-32.3369	-31.1786
$s = 3$	<b>-33.1853</b>	-32.5151	-32.6388	-31.7322
$s = 4$	-32.7124	-32.3692	-32.4572	-31.8528
$s = 5$	-32.0393	-32.0306	-32.0750	-31.8232
	<b>PSCCF</b>			
	$p=5$	$p=4$	$p=3$	$p=2$
$s = 0$	-32.0544	-31.8236	-31.7930	-31.8749
$s = 1$	-32.5310	-32.1332	-31.9610	-31.7561
$s = 2$	-32.8322	-32.2690	-32.1819	-31.7561
$s = 3$	<b>-32.9604</b>	-32.2638	-32.4343	-31.8749
$s = 4$	-32.1437	-31.9102	-31.9125	-31.6522
$s = 5$	-31.1137	-31.1137	-31.1137	-31.1137



**Table 5** – Common features restrictions with HQ,  $\hat{r} = 3$  (deterministic trend)

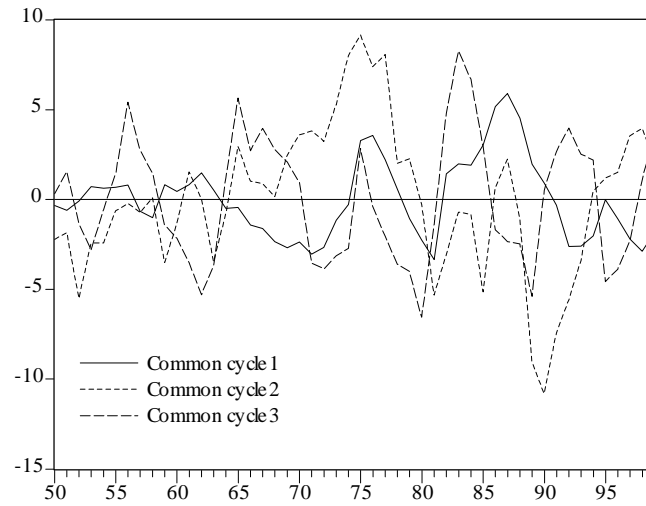
<b>SCCF</b>				
	<i>p=5</i>	<i>p=4</i>	<i>p=3</i>	<i>p=2</i>
<i>s</i> = 0	-33.1588	-32.0586	-32.1607	-32.0299
<i>s</i> = 1	-33.7090	-32.5859	-32.5952	-32.2216
<i>s</i> = 2	<b>-34.1386</b>	-32.7460	-32.9970	-32.4318
<i>s</i> = 3	-33.6641	-32.7377	-32.9508	-32.3471
<i>s</i> = 4	-32.6559	32.3918	-32.4161	-32.1738
<i>s</i> = 5	-31.6432	31.6432	-31.6432	-31.6432
<b>WF</b>				
	<i>p=5</i>	<i>p=4</i>	<i>p=3</i>	<i>p=2</i>
<i>s</i> = 0	-33.1588	-32.0586	-32.1607	-32.0299
<i>s</i> = 1	-33.5938	-32.5493	-32.4324	-32.0885
<i>s</i> = 2	<b>-34.0416</b>	-32.7483	-32.7149	-32.2136
<i>s</i> = 3	-33.7408	-32.8057	-32.9904	-32.4226
<i>s</i> = 4	-33.0481	-32.6117	-32.7268	-32.5107
<i>s</i> = 5	-32.2687	-32.2856	-32.2971	-32.3589
<b>PSCCF</b>				
	<i>p=5</i>	<i>p=4</i>	<i>p=3</i>	<i>p=2</i>
<i>s</i> = 0	-33.1588	-32.0586	-32.1607	-32.0299
<i>s</i> = 1	-33.4565	-32.3627	-32.3835	-31.9704
<i>s</i> = 2	<b>-33.7949</b>	-32.4044	-32.5885	-32.0299
<i>s</i> = 3	-33.2795	-32.2936	-32.4652	-31.8555
<i>s</i> = 4	-32.2305	-31.8376	-31.8818	-31.6295
<i>s</i> = 5	-31.1137	-31.1137	-31.1137	-31.1137

One can see from Table 4 that information criteria also favor  $s = 3$  and the SCCF model but now  $p = 5$  minimizes these criteria. Consequently, with a reduced number of propagation mechanisms, the dynamics is more important than we believed. Table 5 reports, for the model with a constrained deterministic trend, that a SCCF with  $s = 2$  and  $p = 5$  is the best overall model. This latter will be the final specification we will retain for the decomposition. Notice that both Tables 4 and 5 deliver a very particular situation because we have  $r + s = n$ . In that case, it can be shown that there exists a unique common trend common cycle decomposition. Moreover the Gonzalo-Granger (1995) permanent transitory decomposition is equivalent to the Beveridge-Nelson one which is also equivalent to the decomposition proposed by Vahid and Engle (1993), namely (see Proietti, 1997; Hecq et al. 2000):

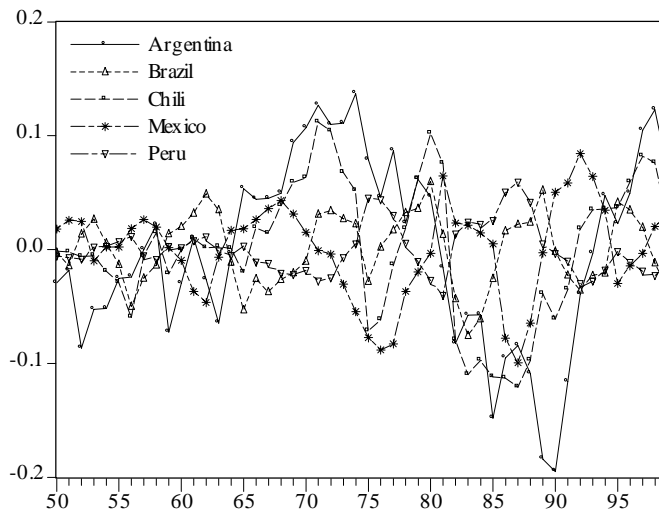
$$y_t = \tau_t + \xi_t \equiv \delta_a (\delta_a' \delta_a)^{-1} \delta_a' y_t + \beta (\beta' \beta)^{-1} \beta' y_t \quad (7)$$

where the  $n - r = 2$  common trends are given by  $\delta_a' y_t$ . The  $n - s = 3$  common cycles  $\beta' y_t$  are plotted in Figures 3 and the individual cyclical components  $\beta (\beta' \beta)^{-1} \beta' y_t$  in Figure 4.

**Figure 3 – Common cycles**



**Figure 4 – Cyclical components**



We could now analyze in more details these cyclical components using cross correlograms in order to see how series lead and lag. Multivariate regressions or impulse responses would also be interesting tools to use. To simplify things here we only reproduce the simple correlation coefficients between the five cyclical components and the three common cycles. From the high correlations, it appears that the Brazilian and Peruvian cyclical components determine respectively the third and the first common cycles. To a lesser extent the second common cycle is strongly related to Argentina. We also observe that Argentina and Brazil have independent business cycles; some countries like Argentina and Chile are procyclical while Peru is countercyclical with respect to Mexico and Chile.

**Table 6** – Correlation matrix

	Arg	Bra	Chili	Mex	Peru	CoCy1	CoCy2	CoCy3
Arg	1	0.05	0.71	-0.12	-0.36	-0.55	0.83	-0.08
Bra		1	0.37	-0.49	-0.03	0.054	0.06	-0.99
Chili			1	0.27	-0.83	-0.87	0.25	-0.39
Mex				1	-0.74	-0.68	-0.59	0.50
Peru					1	0.97	0.21	0.04

## 4. Conclusion

In this paper we have examined the dynamics of five Latin American economies. Our results suggest that these countries share long and short-run co-movements. We observe a special and, say, rare situation in which the number of cointegrating and co-feature vectors add up to  $n$ . Quite often other models are needed to relax the strong assumptions of the SCCF. Their own decomposition as well as the decomposition of SCCF when  $r + s < n$  can be found in the referenced papers. The specification we have obtained can be used for dating business cycle turning points, to interpret impulse responses, to compute forecasts or to test for contagion of financial crises (Candelon, Hecq and Verschoor, 2002).

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