

# Adjustment Costs and Time-To-Build in Factor Demand in the U.S. Manufacturing Industry

FRANZ C. PALM, H.M.M. PEETERS and G.A. PFANN<sup>1</sup>

Faculty of Economics and Business Administration, Department of Quantitative Economics, Tongersestraat 53, 6211 LM Maastricht, The Netherlands

*Abstract:* In order to explain cyclical behavior of factor demand, the static neoclassical model of the firm has been extended to include either adjustment costs (e.g. Lucas (1967)) or time-to-build considerations as in Kydland and Prescott (1982). This paper presents an intertemporal factor demand model which accounts for adjustment costs and gestation lags. The closed form solution of the model is a highly restricted vector ARMA-process that is estimated using quarterly data for the manufacturing industry in the U.S., 1960–1988.

The main conclusion is that both sources of dynamics of factor demand are identifiable and found to be empirically of importance.

*Key Words:* factor demand models, adjustment costs, gestation lags.

*JEL Classification System-Numbers:* C32, C5, E22

## 1 Introduction

In order to explain cyclical behavior of factor demand, the static neoclassical model of the firm has been extended along two alternative lines. Adjustment costs proposed by Lucas (1967) and time-to-build considerations as in Kydland and Prescott (1982) lead to factor demand models which have different dynamic properties. A large number of theoretical and empirical analyses of factor demand uses the adjustment costs approach. A survey is presented in Palm and Pfann (1990). Since the pioneering contribution of Kydland and Prescott (1982), some authors have included time-to-build considerations in their models, usually in a general equilibrium real business cycle framework.

The two approaches have generally been presented as alternative explanations of the dynamics of factor demand. For instance, Rossi (1988) develops Bayesian posterior odds to compare an adjustment-cost model of factor demand with a gestation-lag model as non-nested hypotheses.

This paper presents an intertemporal optimization model for labor, structures and equipment demand which accounts for adjustment and gestation lags in factor demand. A similar approach in a general equilibrium model has been

<sup>1</sup> This research was sponsored by the Economics Research Foundation, which is part of the Netherlands Organization for Scientific Research (NWO) and by the Royal Netherlands Academy of Arts and Sciences (K.N.A.W.).

adopted by Park (1984). The aim of this study is to investigate to what extent gestation lags and adjustment costs are needed to explain the dynamics of factor demand in a partial equilibrium model. While adopting a partial equilibrium framework we test if the main determinants of factor demand are indeed parametric for the decision makers in the model in which case a partial equilibrium analysis becomes legitimate. Investment flows are modeled instead of capital stock series because the latter series have been constructed from investment series without taking into account time-to-build considerations. The model is estimated using quarterly data for the manufacturing industry in the U.S., 1960–1988. The analyses have been carried out along the lines of a modeling approach which integrates time series and econometric analyses of a dynamic model derived from intertemporal optimizing behavior.

The main features of the approach are as follows. Economic theory is used to specify a dynamic model for the series to be analyzed. Adding the process generating the exogenous variables, the model is completed. In line with the Simplicity Principle, we start with a linear-quadratic optimization model for which the first order necessary conditions are linear and derive a forward-looking closed-form solution for the demand for labor, structures and equipment. Univariate time series methods are used to investigate the properties of the initial theoretical model. In particular, the order of integration, the serial correlation properties and the distribution of the series are analyzed.

As mentioned above when adopting a partial equilibrium approach, Granger-causality of the conditioning variables plays a crucial role in the intertemporal optimization models. The directions of causality between factor input and factor prices are examined. Non-stationarity is modeled jointly with other dynamic features in the series instead of being eliminated prior to the econometric analysis.

The implications of the optimization model for the presence of cointegrating regressions are also investigated. The reduced form of the joint model is estimated and model specification tests are performed. The structural parameters are computed from the reduced form parameter estimates using a minimum distance estimator. This methodology iterates into a model that is theoretically sensible and in agreement with the information in the data. Finally, plots of the impulse response functions for the endogenous variables give additional insights in the dynamics of the model.

The paper is organized as follows. In section 2, an econometric specification for the dynamic factor demand model is derived using linear-quadratic approximations. Section 3 describes the results of the empirical analysis. Finally, in section 4 some concluding remarks are made and further extensions are discussed. Some technical derivations, data definitions and results for additional tests are given in appendices.

## 2 An Intertemporal Model of Factor Demand

A firm is assumed to maximize its real present value of profits over an infinite horizon. It employs labor  $N_t$ , uses capital in the form of structures  $K_t^s$  and equipment  $K_t^e$ . Decisions to change inputs are costly.

The technology assumes that time is required to build structures. The firm operates under uncertainty. In order to determine the optimal level of the variables  $N_t$ ,  $K_t^s$  and  $K_t^e$ , the firm uses all the relevant information available up to time  $t$ . Current factor prices are part of the information set. Real factor costs, that is factor prices deflated by the price of output, are assumed to be given to the firm in the sense that they are not Granger-caused by factor demand. The production technology obeys the usual regularity conditions ( $\partial Q_t / \partial X_t > 0$ ,  $X_t' = [N_t, K_t^s, K_t^e]$  and  $Q_t$  denotes real output per time period; the Hessian matrix is negative definite) and is locally approximated by a quadratic function

$$Q_t = (\alpha + \lambda_t)' X_t - \frac{1}{2} X_t' A X_t, \tag{2.1}$$

where  $\alpha$  is a  $(3 \times 1)$ -vector with constant coefficients  $\alpha_i$  ( $i = 1, 2, 3$ ),  $\lambda_t$  denotes a  $(3 \times 1)$ -vector of exogenous stochastic shocks to the marginal productivity of the inputs and  $A$  is a positive definite matrix. Adjustment costs (AC) are assumed to be quadratic in net changes in labor and capital stocks

$$AC_t = \frac{1}{2} \Delta X_t' \Gamma \Delta X_t, \tag{2.2}$$

where  $\Delta$  denotes the first differences and  $\Gamma$  a positive definite matrix. Structures and equipment depreciate at the rates  $\kappa^s$  and  $\kappa^e$  respectively. The law of motion for equipment yields

$$K_t^e = (1 - \kappa^e) K_{t-1}^e + I_{t-1}^e, \quad 0 < \kappa^e < 1, \tag{2.3}$$

where  $I_t^e$  denotes gross investment in equipment.

Contrary to equipment, structures need to be built. Following Kydland and Prescott (1982), structures with fixed plans are specified by

$$K_t^s = (1 - \kappa^s) K_{t-1}^s + S_{1,t-1}, \quad 0 < \kappa^s < 1 \tag{2.4a}$$

$$S_{j,t} = S_{j+1,t-1}, \quad j = 1, 2, \dots, J - 1 \tag{2.4b}$$

$$I_t^s = \sum_{j=1}^J \delta_j S_{j,t} \tag{2.4c}$$

and

$$\sum_{j=1}^J \delta_j = 1, \quad 0 \leq \delta_j < 1, \tag{2.4d}$$

where  $S_{j,t}$  are the total expenditures of structure projects  $j$  periods from completion at period  $t$  and  $J$  denotes the total time to build structures.

New investment projects initiated in period  $t$  are denoted by  $S_{j,t}$  and equation (2.4b) implies that the expenditures necessary to complete these projects do

not change during the gestation. From equation (2.4a) it follows that finished time-to-build capital  $S_{1,t-1}$  is added to linearly depreciating structures, so that at time  $t$  productive structures are  $K_t^s$ . During the gestation period,  $J$ , expenditures to current structure projects are given by  $\delta_j, \delta_{j-1} \dots \delta_1$ , where the production stage  $j$  ( $j = J, J-1, \dots, 1$ ) indicates the number of periods a project is away from completion. Equation (2.4d) implies that for all time-to-build structure projects the distribution of investments as well as the gestation  $J$  are assumed to be the same. Gross investments at time  $t$ ,  $I_t^s$ , consist of investments in all current structure projects ( $S_{1,t}, S_{2,t} \dots S_{J,t}$ ) with  $\delta_j S_{j,t}$  ( $j = 1, 2 \dots J$ ) being the amount of investment in each project (equation (2.4c)). Notice that if  $J = 1$  the structure accumulation equation (2.4a) is similar to equation (2.3).

Variable costs are defined as

$$VC_t = Y_t' P_t, \quad (2.5)$$

with  $Y_t' = [N_t, I_t^s, I_t^e]$  and  $P_t' = [W_t, C_t^s, C_t^e]$  being the vector of factor inputs and real factor prices respectively.

The firm's objective is to maximize its real value of current and future profits (PV) at time  $t$ ,

$$PV_t = \lim_{H \rightarrow \infty} E_t \sum_{h=0}^H \beta^h [Q_{t+h} - VC_{t+h} - AC_{t+h}], \quad 0 < \beta < 1, \quad (2.6)$$

where  $\beta$  is the constant real discount factor and  $E_t(\cdot) = E(\cdot | \Omega_t)$  with  $\Omega_t$  being the available information set at time  $t$ .

Substitution of (2.4b) into (2.4a) and then into (2.4c) yields

$$S_{j,t} = S_{1,t+j-1} = K_{t+j}^s - (1 - \kappa^s) K_{t+j-1}^s, \quad j = 1, 2, \dots, J, \quad (2.7)$$

and

$$I_t^s = \sum_{j=1}^J \delta_j (K_{t+j}^s - (1 - \kappa^s) K_{t+j-1}^s) = \sum_{j=0}^J \phi_j K_{t+j}^s, \quad (2.8)$$

with

$$\phi_0 = (\kappa^s - 1) \delta_1$$

$$\phi_j = \delta_j + (\kappa^s - 1) \delta_{j+1}, \quad j = 1, 2, \dots, J-1$$

$$\phi_J = \delta_J.$$

Notice that gross structures investments are the sum of current and future net structures investments and depreciation ( $\Delta K_{t+j}^s + \kappa^s K_{t+j-1}^s, j = 1, 2 \dots J$ ) weighted by the time-to-build parameters ( $\delta_j, j = 1 \dots J$ ).

Variable factor costs can then be expressed as

$$VC_t = W_t N_t + C_t^s \left[ \sum_{j=0}^J \phi_j K_{t+j}^s \right] + C_t^e [K_{t+1}^e + (\kappa^e - 1) K_t^e]. \quad (2.9)$$

At period  $t$  decisions are made concerning labor  $N_t$ , and the new capital projects  $S_{j,t}$  and investment  $I_t^s$  that determine  $K_{t+J}^s$  and  $K_{t+1}^e$  respectively. The criterion

function (2.6) is maximized with respect to the decision variables  $N_{t+h}$ ,  $K_{t+J+h}^s$  and  $K_{t+1+h}^e$  ( $h = 0, 1 \dots$ ). First order necessary conditions for maximization of (2.6) consist of the set of Euler equations and a pair of transversality conditions assuring the finiteness of the process. We further assume absence of interrelations in the production- and adjustment cost-functions, that is the matrices  $A$  and  $\Gamma$  are diagonal with  $i$ -th ( $i = 1, 2, 3$ ) diagonal elements denoted by  $a_{ii}$  and  $\gamma_{ii}$  respectively. This assumption substantially simplifies the derivation of the solution.

The Euler equations for  $N_t$ ,  $K_{t+J}^s$  and  $K_{t+1}^e$  can be written as

$$\alpha_1 + \lambda_{1t} - \alpha_{11}N_t - W_t - \gamma_{11}\Delta N_t + \beta\gamma_{11}E_t\Delta N_{t+1} = 0 \tag{2.10a}$$

$$\beta^J[\alpha_2 + E_t\lambda_{2,t+J} - a_{22}K_{t+J}^s] - \sum_{j=0}^J \beta^j\phi_{J-j}E_tC_{t+j}^s - \beta^J\gamma_{22}\Delta K_{t+J}^s + \beta^{J+1}\gamma_{22}E_t\Delta K_{t+J+1}^s = 0 \tag{2.10b}$$

$$\beta[\alpha_3 + E_t\lambda_{3,t+1} - a_{33}K_{t+1}^e] - C_t^e - \beta(\kappa^e - 1)E_tC_{t+1}^e - \beta\gamma_{33}\Delta K_{t+1}^e + \beta^2\gamma_{33}E_t\Delta K_{t+2}^e = 0 \tag{2.10c}$$

The transversality conditions are satisfied if each variable has an exponential order less than  $1/\sqrt{\beta}$ . Defining  $X_t^d = [N_t, K_{t+J}^s, K_{t+1}^e]'$  and using the method of Blanchard and Kahn (1980) the solution of the Euler equations can be expressed as follows

$$X_t^d = F_1 X_{t-1}^d - \sum_{i=0}^{\infty} (\beta F_1)^{i+1} D E_t(Z_{t+i}) \tag{2.11}$$

$Z_t$  contains the observed variables  $W_t$ ,  $C_t^s$ ,  $C_t^e$  and the unobservable variables  $\lambda_{1t}$ ,  $E_t\lambda_{2,t+J}$ ,  $E_t\lambda_{3,t+1}$ ,  $E_tC_{t+k}^s$  ( $k = 1, 2, \dots, J$ ) and  $E_tC_{t+1}^e$ . A derivation and description of the matrices  $F_1$  and  $D$  can be found in appendix I.

Since we use investment data for the empirical analysis we rewrite the second and third equation in (2.11) in terms of gross investment. The first equation remains unaltered. From (2.8) it follows that:

$$I_t^s = \sum_{j=0}^J \phi_j K_{t+j}^s \quad \text{and} \quad \sum_{j=0}^J \phi_j = \kappa^s \sum_{j=1}^J \delta_j = \kappa^s \tag{2.12}$$

Defining  $[\cdot]_2$  as the second row of the matrix in square brackets, the second equation in (2.11) together with (2.12) then gives

$$K_{t+J}^s = f_2 K_{t+J-1}^s - \sum_{i=0}^{\infty} (\beta f_2)^{i+1} [D]_2 E_t(Z_{t+i}) \quad \Leftrightarrow$$

$$\sum_{j=0}^J \phi_j K_{t+j}^s = f_2 \sum_{j=0}^J \phi_j K_{t+j-1}^s - \sum_{j=0}^J \phi_j \sum_{i=0}^{\infty} (\beta f_2)^{i+1} [D]_2 E_{t+j-J}(Z_{t+i+j-J}) \quad \Leftrightarrow$$

$$I_t^s = f_2 I_{t-1}^s - \sum_{j=0}^J \phi_j \sum_{i=0}^{\infty} (\beta f_2)^{i+1} [D]_2 E_{t+j-J}(Z_{t+i+j-J}) \tag{2.13a}$$

Defining  $\phi_0^e = \kappa^e - 1$  and  $\phi_1^e = 1$ , the third equation is obtained in the same way

$$I_t^e = f_3 I_{t-1}^e - \sum_{j=0}^1 \phi_j^e \sum_{i=0}^{\infty} (\beta f_3)^{i+1} [D]_3 E_{t+j-1}(Z_{t+i+j-1}) . \tag{2.13b}$$

Reminding that  $Y_t' = [N_t, I_t^s, I_t^e]$ , the system of equations (2.11) becomes

$$Y_t = F_1 Y_{t-1} + \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^{i+1} D E_{t-j}(Z_{t+i-j}) , \tag{2.14}$$

where

$$D_0 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\phi_J & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\phi_{J-1} & 0 \\ 0 & 0 & (1 - \kappa^e) \end{bmatrix},$$

$$D_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\phi_{J-k} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad k = 2, 3, \dots, J .$$

Contrary to productive capital ( $K_t^s$  and  $K_t^e$ ) the gross investments of capital ( $I_t^s$  and  $I_t^e$ ) depend on realized real prices for the total gestation period due to the fact that gross investments are a weighted sum of investment projects that were initiated during the period  $t - J$  and  $t$ . For example  $K_t^s$  in (2.13a) depends on  $C_{t-J}^s$  but not on  $C_{t-J+1}^s, \dots, C_{t-1}^s, C_t^s$ , whereas  $I_t^s$  depends on all these prices because  $K_{t+1}^s, K_{t+2}^s \dots K_{t+J}^s$  are included.

In order to obtain a model which relates the decision variables  $Y_t$  to observed exogenous variables, we have to specify the processes that generate the exogenous variables. We assume that the productivity shocks and the first differences of factor prices are generated by first order vector-autoregressions

$$\lambda_t = R \lambda_{t-1} + \varepsilon_t^\lambda, \quad \text{with } E_t \varepsilon_t^\lambda = 0, \quad E_t \varepsilon_t^\lambda \varepsilon_t^{\lambda'} = \Sigma^\lambda, \quad E_t \varepsilon_t^\lambda \varepsilon_t^{s'} = 0, \quad t \neq s \tag{2.15}$$

$R$  being a diagonal matrix with typical diagonal element  $\rho_{ii}$  ( $i = 1, 2, 3$ ), and

$$\Delta P_t = \bar{M} \Delta P_{t-1} + \varepsilon_t^p \quad \text{with } E_t \varepsilon_t^p = 0, \quad E_t \varepsilon_t^p \varepsilon_t^{p'} = \Sigma^p, \quad E_t \varepsilon_t^p \varepsilon_t^{s'} = 0, \quad t \neq s \tag{2.16}$$

and a typical element of  $\bar{M}$  being denoted by  $\mu_{ij}$ . These assumptions are permissive and will be tested in the empirical analysis.

As described in appendix I, along the lines of Pfann (1990) from (2.15) and (2.16) one can obtain an expression for the expected future values of the exogenous variables and substitute it into (2.14). After applying a Koyck transformation to eliminate the serial correlation in the unobservable technology components  $\lambda_t$ , the forward-looking linear decision rules for the production factors yield

$$Y_t = C + (F_1 + R) Y_{t-1} - R F_1 Y_{t-2} + \sum_{j=0}^{J+2} M_j P_{t-j} + \varepsilon_t^y, \tag{2.17}$$

where  $C, M_j$  ( $j = 0, 1, \dots, J + 2$ ) and  $\varepsilon_t^y$  are defined in appendix I. The process of prices (in (2.16)) which can be written as

$$P_t = (I + \bar{M})P_{t-1} - \bar{M}P_{t-2} + \varepsilon_t^p, \tag{2.18}$$

and the system (2.17) form the model for factor demand and factor prices which we complete by assuming that

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^p \end{bmatrix} \sim N(0, \Sigma), \quad \text{where } \varepsilon_t^y = \begin{bmatrix} \varepsilon_{1t}^y \\ \varepsilon_{2t}^y \\ \varepsilon_{3t}^y \end{bmatrix}, \quad \varepsilon_t^p = \begin{bmatrix} \varepsilon_{1t}^p \\ \varepsilon_{2t}^p \\ \varepsilon_{3t}^p \end{bmatrix},$$

$$\text{Cov}(\varepsilon_t^y, \varepsilon_{t-k}^p) = 0 \quad \text{for all } k$$

and

$$\text{Cov}(\varepsilon_{1t}^y, \varepsilon_{1s}^y) = 0 \quad t \neq s$$

$$\begin{aligned} \text{Cov}(\varepsilon_{2t}^y, \varepsilon_{2,t-k}^y) &= \sum_{j=0}^{J-k} \phi_j \phi_{j+k} [\Sigma^{\lambda^*}]_{22} & k = 1, 2, \dots, J \\ &= 0 & k > J \end{aligned}$$

$$\begin{aligned} \text{Cov}(\varepsilon_{3t}^y, \varepsilon_{3,t-k}^y) &= (\kappa^e - 1) [\Sigma^{\lambda^*}]_{33} & k = 1 \\ &= 0 & k > 1 \end{aligned}$$

$$\text{Cov}(\varepsilon_{it}^p, \varepsilon_{is}^p) = 0 \quad t \neq s \quad i = 1, 2, 3,$$

where  $[\Sigma^{\lambda^*}]_{ij}$  is the  $(i, j)$ -th element of covariance matrix  $\Sigma^{\lambda^*}$  multiplied by a constant. Notice that these assumptions imply that technology shocks can not instantaneously influence production prices (and vice versa).

The subsystem (2.17) is a multivariate linear dynamic regression model with diagonal autoregressive matrices because of the diagonality of both  $R$  and  $F_1$ . The matrices  $M_i$  ( $i = 4 \dots J + 2$ ) only have non-zero elements in the second row.  $M_3$  has zero restrictions in the first row, whereas  $M_0, M_1$  and  $M_2$  are unrestricted.

System (2.17) is a trivariate ARMAX-model. The equations for labor, structures and equipment are ARX(2, 2), ARMAX(2,  $J + 2, J$ ) and ARMAX(2, 3, 1) processes with the figures between parentheses indicating the order of the AR part, the number of lagged factor prices and the degree of the MA part respectively. The contemporaneous correlations among the three disturbances  $\varepsilon_{1t}^y, \varepsilon_{2t}^y, \varepsilon_{3t}^y$  are unrestricted.

In system (2.17) the first lagged dependent variable appears because of the adjustment cost assumption. The lagged prices  $P_{t-1}$  and  $P_{t-2}$  are included in all equations due to the first order autoregressive process of price differences. The second equation in system (2.17) contains the largest number of lags. Gross investment of structures is a weighted sum of current capital projects. Current projects were started during periods  $t - J$  up to  $t$ . The information set in this model includes the technology shocks and prices of all production factors. Prices realized during the periods  $t - J - 2$  to  $t$  appear as explanatory variables. Technology shocks realized during the periods  $t - J$  to  $t$  are unobserved. Their influence is found in the model as the persistent part ( $R$ ) and the transitory part ( $\varepsilon_t^{\lambda}$ ). The persistent part is represented by the last lagged dependent variable

and the last lagged vector of prices, present in all equations in (2.17). The transitory part of technology shocks is represented by the moving average disturbance with weights  $\phi_j$  ( $j = J, J - 1, \dots, 0$ ). The interpretation is as follows. At time  $t - J$ , structure project  $S_{J,t-J}$  is initiated, a decision that is influenced by the technology disturbance  $\varepsilon_{2,t-J}^{\lambda}$  but only by the part  $\phi_j$  which is invested in that period. During the gestation time  $t - J, t - J + 1, \dots, t$  this decision has an impact that can not be changed due to the fixed investment plan assumption. The impact of the disturbance  $\varepsilon_{2,t-J}^{\lambda}$  on gross investments is  $\phi_J \varepsilon_{2,t-J}^{\lambda}, \phi_{J-1} \varepsilon_{2,t-J}^{\lambda}, \dots, \phi_0 \varepsilon_{2,t-J}^{\lambda}$ . Gross investments at time  $t$ , building up  $K_t^s, K_{t+1}^s, \dots, K_{t+J}^s$  are thus influenced by the random part of the technology shocks by the amounts  $\phi_0 \varepsilon_{2,t-J}^{\lambda}, \phi_1 \varepsilon_{2,t-J+1}^{\lambda}, \dots, \phi_J \varepsilon_{2,t}^{\lambda}$ . In a similar way the matrices  $M_i$  ( $i = 0, 1, \dots, J + 2$ ) bear the time-to-build parameters.

The structural parameters are the parameters of the production technology, of the adjustment cost functions, the time-to-build coefficients, the depreciation rates, the real discount factor and the parameters in the processes for  $P_t$  and  $\lambda_t$ . The structural parameters are subject to nonlinear restrictions. There are also cross-equations restrictions between the parameters of the subsystems (2.17) and (2.18). In appendix I, it is shown that the structural parameters are all identified from the reduced form estimates of (2.17) and (2.18) and the additional restrictions (see appendix I (A.6) and (2.4d)).

Given that the parameters of the decision rules (2.17) and of the process for  $P_t$  in (2.18) are functionally related, one cannot expect  $P_t$  to be weakly exogenous with respect to the parameters in (2.17) (see Engle et al. (1983)). Therefore,  $P_t$  cannot be superexogenous either which is a different way of stating the Lucas' critique; structural changes which affect the conditional mean of  $P_t$  and/or  $\lambda_t$  will also affect the parameters in the factor demand system (2.17) in a way that can be predicted. Although the economic model implies absence of weak and super-exogeneity, it relies on the assumption of no Granger-causality from  $Y_t$  to  $P_t$  and  $\lambda_t$ .

With respect to the non-stationarity of  $P_t$ ,  $\lambda_t$  and  $Y_t$ , a number of interesting features can be mentioned. For instance,  $\lambda_t$  can be interpreted as the impact of technological change on production or as total factor productivity (the so called Solow residual). If  $\lambda_t$  is assumed to be generated by a random walk, i.e. if  $R \equiv I$ , conditional on the past of  $P_t$ , the autoregressive part of the model for  $Y_t$  has also roots on the unit circle. A random walk for  $\lambda_t$  is consistent with the idea that technical change is a cumulative process of generating knowledge, a point which has been forcefully argued by Lippi and Reichlin (1990) among others. As  $\lambda_t$  is not directly observed, the random walk hypothesis has to be tested indirectly. For instance, an estimate of  $R$  which is close to  $I$  is evidence in favor of the random walk hypothesis on  $\lambda_t$ . Alternatively, if both  $P_t$  and  $Y_t$  are found to be integrated of order one, cointegration tests can be used to check whether the non-stationarity of  $Y_t$  can be fully accounted for by the non-stationarity of  $P_t$ . This discussion also illustrates that non-stationarities in economic series can be and in our view should be investigated and interpreted within the framework of a theoretical model.



One can estimate the complete model as a reduced form model without imposing the nonlinear restrictions and analyze its dynamic specification. The adjustment cost model and the pure time-to-build model are nested in the model (2.17). Without time-to-build considerations (i.e.  $J = 1$ ), the second equation reduces to a ARMA(2, 1) with three lagged prices as explanatory variables. Alternatively, if adjustment costs are absent (i.e. if  $\Gamma = 0$ ), the second autoregressive term disappears in all three equation. The general set-up of the model (2.17) allows us to investigate the appropriateness of adjustment costs and gestation lags for explaining the dynamics of factor demand.

### 3 Empirical Analysis

In this section we apply the model for dynamic factor demand to quarterly U.S. manufacturing industry data for the period 1960–1988. A description of the data and the sources are given in appendix II.

Section 3 is divided into three parts. The first part reports the results of the time series properties of the data and relates them to assumptions underlying the theoretical model. In the second part a comparison of the adjustment cost specification and the gestation lag dynamics with the reduced form model given in (2.17) and (2.18) is carried out. Finally, the third part presents the parameter estimates of the reduced form and the structural model. It should be noted that quarterly dummies are included in the model to take account of the presence of seasonality in the data.

#### 3.1 *Integration, Order Selection and Cointegration*

An important issue in time series analysis is the determination of the order of integration of the individual series. In model (2.18), it is assumed that the variables in the vector  $P_t$  are integrated of order 1. Under the assumption that the innovations  $\lambda_t$  are stationary, the elements of vector  $Y_t$  in (2.17) are integrated of order 1 as well. If technology shocks  $\lambda_t$  are  $I(1)$  as one might expect, factor demand and factor prices in (2.17) will not be cointegrated.

In table I, we report the values of Fuller's  $\hat{\tau}_\mu$  and  $\hat{\tau}_t$  statistics for the six variables in the system. The results in the upper part of table I indicate that with the exception of  $N_t$  and  $I_t^s$  the null hypothesis of a unit root cannot be rejected at the conventional 5% significance level. The inclusion of additional lags (see the last rows of table I) does not affect this conclusion for labor and structures either. The hypothesis of second order integration is strongly rejected for all

Table I. Tests for integration

	$N_t$	$I_t^s$	$I_t^e$	$W_t$	$C_t^s$	$C_t^e$
Fuller's $\hat{t}_\mu$	-3.37*	-3.74*	-2.11	-2.14	-1.31	-1.63
Adjusted $R^2$	0.62	0.67	0.37	0.11	0.21	0.23
Fuller's $\hat{t}_\tau$	-3.38	-3.83*	-3.03	-2.00	-1.78	-3.06*
$t$ -value trend	0.85	-1.10	2.35*	0.14	1.20	-2.71*
Adjusted $R^2$	0.62	0.67	0.39	0.11	0.21	0.27
Fuller's $\hat{t}_\mu$	23.17*	51.20*	23.82*	16.46*	23.71*	26.81*
Fuller's $\hat{t}_\mu$	-3.07	-2.00	-2.11	-2.23	-1.16	-1.11
Fuller's $\hat{t}_\tau$	-3.11	-2.18	-3.2	-2.14	-1.56	-2.21

Model:  $\Delta V_t = u_0 + u_1 t + u_2 V_{t-1} + u_3 \Delta V_{t-1} + u_4 \Delta V_{t-2} + \varepsilon_t^v$  with  $V_t \in \{N_t, I_t^s, I_t^e, W_t, C_t^s, C_t^e\}$

The first row gives the statistics for the hypothesis  $H_0: u_2 = 0$  while  $u_1 = u_4 = 0$ . The third row gives the statistics for the hypothesis  $H_0: u_2 = 0$  while  $u_4 = 0$ . The fourth row gives the  $t$ -value of  $u_1$ . The sixth row gives the same statistics as the first row where the variables  $V_t$  in the model are differenced once more. The seventh and eighth rows give the statistics for the hypothesis  $H_0: u_2 = 0$  while  $u_1 = 0$  and  $H_0: u_2 = 0$  respectively.

\* Significant at the 5% level.

series. Thus according to these results all variables are assumed to be integrated of order 1. Labor and structures seem to be on the verge of stationarity and non-stationarity possibly due to the fact that they have risen until the mid sixties and did not rise significantly during the seventies and eighties (see also the graphs in appendix II). Also from the differences between the  $\hat{t}_\mu$  and  $\hat{t}_\tau$  statistics and from the  $t$ -values of the linear trend, it appears that including a linear trend significantly improves the fit of the investment and price process of equipment. The signs of the trends suggest that equipment investments have risen whereas prices have fallen during the time period under analysis.

Next, we identify the order of the dynamics of  $\Delta P_t$  from information in the data. Estimated autocorrelations and partial autocorrelations for each time series suggest a first order univariate autoregressive model for  $\Delta W_t$ ,  $\Delta C_t^s$  and  $\Delta C_t^e$ . The results of order selections for the trivariate system are presented table II.

The cross-correlation matrices and the Schwarz (1978) criterion suggest a first order trivariate autoregression for  $\Delta P_t$ . This finding and the outcomes of the unit roots tests in table I corroborate the first order VAR specification for the process of  $\Delta P_t$  in (2.18) and the implied order of integration of  $Y_t$  in (2.17). The  $\chi^2$  and the Akaike criteria however suggest a higher order autoregressive process. Notice that the Schwarz criterion is also at the high (conservative) end of the acceptable range of penalty terms in certain model selection settings (see Potscher (1989)). Assuming a higher order autoregressive process for prices would entail including longer lags for prices in (2.18). We shall come back to this point in section 3.2.

Finally, we investigate whether the variables  $Y_t$  and  $P_t$  in (2.17) and (2.18) are cointegrated. Under the assumption that the productivity shocks are stationary, the theoretical model implies that there exist three cointegration rela-

**Table II.** Order selection for the trivariate process of factor prices

Lag <i>j</i>	1	2	3	4	5
Schwarz( <i>j</i> )	-26.22	-26.12	-25.92	-25.63	-25.35
AIC	-26.44	-26.56	-26.58	-26.51	-26.45
CHI <sup>2</sup> ( <i>j</i> )	54.27	31.43	20.23	10.75	11.56
Schwarz( <i>j</i> ) = ln  $\mathcal{L}$   + (9 <i>j</i> * ln( <i>T</i> ))/ <i>T</i>				<i>j</i> = 1, 2, ..., 5	<i>T</i> = 110
AIC( <i>j</i> ) = ln  $\mathcal{L}$   + (18 <i>j</i> )/ <i>T</i>				<i>j</i> = 1, 2, ..., 5	
CHI <sup>2</sup> ( <i>j</i> ) = <i>T</i> (ln  $\mathcal{L}_{j-1}$   - ln  $\mathcal{L}_j$  ) ~ $\chi^2_{0.05}(9)$ = 16.92				<i>j</i> = 1, 2, ..., 5	

Cross-correlation matrices of  $\Delta P_t^i = [\Delta W_t, \Delta C_t^s, C_t^e]$

	$\Delta W_{t-1}$	$\Delta C_{t-1}^s$	$\Delta C_{t-1}^e$	$\Delta W_{t-2}$	$\Delta C_{t-2}^s$	$\Delta C_{t-2}^e$	$\Delta W_{t-3}$	$\Delta C_{t-3}^s$	$\Delta C_{t-3}^e$
$\Delta W_t$	0.22*	0.37*	0.34*	0.16	0.15	0.18	0.18	0.09	0.21*
$\Delta C_t^s$	0.34*	0.45*	0.33*	0.25*	0.09	0.04	0.16	0.07	0.00
$\Delta C_t^e$	0.29*	0.27*	0.49*	-0.08	0.01	0.06	-0.10	0.22*	-0.06

\* Significant cross-correlations, i.e. larger than  $2/\sqrt{T} \approx 0.189$  (*T* = 112)

tionships between factor demand and factor prices as then the disturbance vector of (2.17) is stationary [see Nickell (1985) and Palm and Pfann (1991) for a discussion of the relationship between intertemporal optimization models and cointegration]. Rejecting cointegration points towards the nonstationarity of technological shocks and/or other explanatory variables that have not been included in the model.

In order to test for the number of cointegrating relationships, we adopt the approach put forward by Johansen and Juselius (1990) and estimate the following unrestricted VAR model by ordinary least squares

$$\Delta Z_t = \mu_0 + \tau t + \Gamma_1 \Delta Z_{t-1} + \Gamma_2 \Delta Z_{t-2} + \Gamma_3 \Delta Z_{t-3} + \Pi Z_{t-4} + \varepsilon_t^z, \tag{3.1}$$

where  $Z_t^i = (Y_t^i, P_t^i)'$  and  $\mu_0$  and  $\tau$  are two  $6 \times 1$  vectors and  $\Gamma_i$  (*i* = 1, 2, 3) and  $\Pi$  are  $6 \times 6$  matrices with constant coefficients. The residuals of system (3.1) are analyzed. The results are presented in table III. Both the skewness and the excess kurtosis of the residuals do not deviate significantly from those of a normal distribution. The LM-statistics (see Jarque and Bera (1980)) do not reject normality either. Using the Box-Pierce test, the hypothesis of zero residual autocorrelation in (3.1) is not rejected at the conventional significance levels either. The results for Hosking's (1980) multivariate portmanteau test show that serial cross-correlation is not significant when more than ten lags are taken. This result is not unsatisfactory in view of the fact that the number of lags is less than 10 percent of the total sample and therefore the approximation of the asymptotic  $\chi^2$ -distribution still holds.

We use the trace based likelihood ratio (LR) statistic  $-2 \ln LR(H_1|H_0) = -T \sum_{j=r+1}^n \ln(1 - Eig(j))$  to test  $H_0$ : the rank of  $\Pi$  is  $r < n$  against  $H_1$ :  $\Pi$  has full rank  $n$  with  $Eig(j)$  denoting the maximum likelihood estimate of the *j*-th eigenvalue of  $\Pi$ . No cointegrating relations were found when the linear trend was not included in (3.1), which is possibly due to the fact that labor and struc-

**Table III.** Test statistics for the normality and zero disturbance serial correlation and for cointegration in the model (3.1)

	<i>N</i>	<i>I<sup>s</sup></i>	<i>I<sup>e</sup></i>	<i>W</i>	<i>C<sup>s</sup></i>	<i>C<sup>e</sup></i>
Skewness (SK)	-0.16	0.42	-0.32	0.39	0.25	-0.21
Excess Kurtosis (EK)	1.01	0.36	0.42	0.54	-0.22	0.08
Normality (NORM)	5.24	3.88	2.75	4.28	1.40	0.83
Autocorrelation <i>Q</i> (10)	7.64	6.22	5.85	10.50	7.09	9.46

<i>s</i>	HQ	$\chi^2_{0.05}(36s-144)$
5	99.15*	53.94
10	263.90*	258.11
11	290.93	297.38
12	316.59	336.42
13	347.93	375.28
14	374.86	413.98
15	416.02	452.54

<i>H</i> <sub>0</sub>	<i>i</i>	Eig( <i>i</i> )	Lambda <sub>max</sub> -test <i>U</i> ( <i>i</i> )	Trace-test <i>CU</i> ( <i>i</i> )
<i>r</i> ≤ 5	6	0.053	6.09	6.09
<i>r</i> ≤ 4	5	0.072	8.38	14.47
<i>r</i> ≤ 3	4	0.132	15.86	30.33
<i>r</i> ≤ 2	3	0.175	21.60	51.93
<i>r</i> ≤ 1	2	0.232	29.63	81.56
<i>r</i> = 0	1	0.342	46.82	128.38*

\* Significant at the 5% level

$$\text{NORM} = (\text{SK}^2 + \text{EK}^2/4)/6 \sim \chi(2)$$

$$Q(10) = T \sum_{r=1}^s r_i^2 (i = 1, 2, \dots, 10) \sim \chi(10)$$

$$\text{HQ} = T \sum_{r=1}^s \text{trace}(C_r C_0^{-1} C_r C_0^{-1}), \text{ with } C_r = (1/T) \sum_{i=1}^T e_i^r e_i^{r'} \sim \chi^2(36 \cdot (s-4))$$

$$U(i) = -T \ln(1 - \text{Eig}(i))$$

$$CU(i) = \sum_{j=i}^6 U(j)$$

tures are close to stationarity, whereas the other series are not. Comparing the outcomes of the trace test statistics with the critical values given by Johansen (1991) table (V) suggests that the null hypothesis of  $r \leq 1$  is not rejected whereas the null hypothesis of  $r = 0$  is rejected at the 5 percent level, implying that the number of cointegrating regressions is one, a finding that suggests that the technological shocks are non-stationary. However, the same analysis for model (3.1) with five lagged dependent variables (the results are not given here) suggests that there exist three cointegration relationships.

Therefore we also test for cointegration between the variables in subsets which include one series on factor demand and the three price series. Because the matrices *R* and *F* in (2.17) are diagonal, if  $\lambda_i$  is stationary, at least one cointegrating relationship per subsystem should be found. If *R* is nondiagonal, because of interrelation more than one longrun relationship can hold for each subsystem. The results are presented in table IV. In almost all cases, normality

of the residuals of the cointegration relations is not rejected. Only for labor, the residuals seem to be leptokurtic. This result can possibly be explained by the near stationarity of labor and the nonstationarity of the three prices and the linear trend. The results for autocorrelation, both univariate and multivariate, corroborate the model due to the fact that in each subsystem up to four lags of the dependent variable were included. The conclusion from the cointegration tests is that for each subsystem of a production factor and factor prices one equilibrium relationship is found. We also investigated model (3.1) for the three factor prices only. The results are given in the bottom part of table IV and suggest that no cointegration relationship among prices exists.

In summary, the existence of no cointegration relationship among prices and of one cointegration relationship per subsystem is predicted by the theoretical model presented in section 2. The empirical results do not deny that this is consistent with the information in the data.

### 3.2 *Reduced Form Tests*

In order to estimate the restricted VAR model (2.17) and (2.18) the length of the time to build structures has to be determined. For the mid fifties Mayer (1960) investigated the length of time required to complete investment projects by individual U.S. firms, and found that the average construction time to complete plants equals 11 months. In line with this finding, we assume  $J$  to be equal to 4 quarters. Imposing only exclusion restrictions, model (2.17)–(2.18) can be estimated as a system of seemingly unrelated ARMAX equations. In table V we give the log-likelihood values of several variants model (2.17)–(2.18). Three quarterly seasonal dummies and a linear trend (as a consequence of the cointegration test results) were included in each equation. The number of variables given in table V does not include these variables.

When the joint model is compared with the model without adjustment costs (see table V, part A), which is a model with only one lagged dependent variable in (2.17), the likelihood ratio (LR) statistic is 49.77. As it is much larger than the  $\chi^2_{0.01}(3)$ -value of 11.34, the null hypothesis  $\Gamma = 0$  is rejected. The same conclusion holds for the exclusion of the time-to-build aspect, that is the exclusion of the lagged factor prices  $P_{t-i}$  ( $i = 4, 5, 6$ ) and the moving average part  $\varepsilon_{2,t-j}$  ( $j = 2, 3, 4$ ) for which we have a likelihood ratio statistic of  $36.85 > \chi^2_{0.01}(12) = 26.217$ .

The technology process in the model was assumed to be a first order autoregressive process with a diagonal autoregressive matrix  $R$  in (2.15).

If interrelations in the form of a non-diagonal matrix  $R$  are allowed for, the matrices  $R + F$  and  $-RF$  in the autoregressive part of (2.17) are no longer diagonal. The likelihood ratio statistic for the test of the model with diagonal matrix  $R$  against that with an unrestricted matrix  $R$  is 27.66 (see table V, part

**Table IV.** Test statistics for the normality and zero disturbance serial correlation and for cointegration in subsystems

Labor						
	$N$	$W$	$C^s$	$C^e$		
Skewness (SK)	0.15	0.14	0.10	-0.13		
Excess Kurtosis (EK)	2.45*	0.41	0.36	-0.00		
Normality (NORM)	28.32*	1.18	0.78	0.33		
Autocorrelation Q(10)	5.70	3.70	7.43	8.71		
Structures						
	$I^s$	$W$	$C^s$	$C^e$		
Skewness (SK)	0.37	0.06	0.21	-0.12		
Excess Kurtosis (EK)	0.57	-0.24	-0.29	-0.33		
Normality (NORM)	4.00	0.33	1.22	0.74		
Autocorrelation Q(10)	11.10	6.20	7.51	12.10		
Equipment						
	$I^e$	$W$	$C^s$	$C^e$		
Skewness (SK)	-0.26	0.23	0.24	-0.02		
Excess Kurtosis (EK)	-0.03	-0.13	-0.06	-0.19		
Normality (NORM)	1.26	1.06	1.05	0.17		
Autocorrelation Q(10)	7.56	5.02	6.90	8.34		
Hosking test						
$s$	Labor	Structures	Equipment	$\chi^2_{0.05}(16.s-64)$		
5	33.37*	36.75*	35.71*	28.33		
6	53.59*	46.72	48.17	48.98		
7	71.09*	58.24	55.99	68.52		
8	85.81	78.56	77.57	87.51		
9	100.55	97.37	107.20	106.14		
10	121.63	111.47	118.03	124.51		
15	189.90	174.61	195.21	214.14		
$H_0$	$i$	Labor CU( $i$ )	Structures CU( $i$ )	Equipment CU( $i$ )	Prices CU( $i$ )	
$r \leq 3$	4	4.80	7.23	3.64		
$r \leq 2$	3	15.24	19.63	14.43	7.44	
$r \leq 1$	2	34.84	35.95	36.09	17.52	
$r = 0$	1	66.15*	67.30*	68.63*	38.34	
Prices	$W$	$C^s$	$C^e$	$s$	Hosk.	$\chi^2_{0.05}(9.s-36)$
SK	0.03	-0.00	-0.01	5	13.84	18.50
EK	0.20	0.14	0.21	6	27.58	31.02
NORM	0.21	0.09	0.22	7	32.06	42.69
Q(10)	5.25	7.26	12.4			

\* Significant at the 5% level.

Table V. Reduced form tests<sup>†</sup>

A) Adjustment costs, time-to-build and technology (1961.IV–1988.IV)			
Model (2.17)	$-2 \cdot \loglik.$	Number of variables	LR-statistic
with $J = 4$	1188.42	59	—
without adjustment costs	1238.19	56	49.77**
without time-to-build ( $J = 1$ )	1225.27	47	36.85**
with $R$ unrestricted	1160.76	71	27.66**
B) Granger-causality (1962.III–1988.IV)			
Process of prices	AR(2)	AR(4)	AR(5)
Wald	24.57**	21.37*	20.72**
LR	23.06**	22.49**	21.76*
C) Dynamics of (2.17) implied by the order of the process for $P_i$ (1962.III–1988.IV)			
	LR-statistic	Degrees of freedom	
Process of prices is ARI(3, 1)	52.73**	18	
Process of prices is ARI(4, 1)	64.65**	27	
D) The length of the time-to-build ( $J$ ) in model (2.17) (1962.I–1988.IV).			
$J$	2	3	4
LR-statistic			
$H_0: J$ against			
$H_1: J + 1$	11.96*	14.39**	0.01
Degrees of freedom	4	4	4

<sup>†</sup> The models are estimated using the computer package SCA from Liu et al. (1986). The likelihood values are all "exact".

\* Significant at the 5%-level (using the table for the  $\chi^2$ -distribution).

\*\* Significant at the 1%-level.

A). The number of degrees of freedom involved is 12 and the test is significant at the conventional level of 5%. The assumption of a non-diagonal  $R$  has also implications for the moving average part of (2.17) ( $R^*$  and  $D_j$  in (A.14) in appendix I are no longer interchangeable and as a consequence a vector Koyck-transformation is required). Although a more general technology process seems reasonable according to the above test, the exact implications of this extension are not obvious. Notice however that when  $R$  is nondiagonal, capital stock is included in each Euler equation. The implied model for gross investment has a moving average part in each equation.

A crucial assumption in the specification of the theoretical model is the unidirectional causality relationship between real production prices and production factors. If an individual entrepreneur could affect factor prices by varying factor demand, the assumption (2.18) should also include lagged production factors ( $Y_{t-i}$ ,  $i > 0$ ) and the closed-form solution (2.17) should be extended with price decision equations. In table V, part B we estimate a system of factor price equations

$$P_t = \sum_{j=1}^p \bar{M}_j P_{t-j} + D Y_{t-1} + \varepsilon_t^p$$

including an intercept, seasonal dummies and a linear trend and test the hypothesis  $H_0: D = 0$  against the hypothesis  $H_1: D \neq 0$ . The number of degrees of freedom involved is 9.

The number of lagged prices (prices are expressed in levels for reason of comparison with the factors in levels) is taken to be 2, 4, 5 respectively as suggested by the order selection criteria reported in table II. However, as shown by Toda and Phillips (1991), the conventional causality tests are valid asymptotically as  $\chi^2$ -criteria only when there is sufficient cointegration with respect to the variables whose causal effects are being tested. When this cointegration condition fails to hold, the limit distribution involves a mixture of a  $\chi^2$ -distribution and a non-standard distribution, which generally involves nuisance parameters. Simulation results obtained by Toda and Phillips (1991) indicate that the rejection frequency associated with the conventional Wald test is usually much larger than the nominal size of 5%. Therefore, our finding of Wald and LR-values in the range between 20.72 and 24.57 for the complete system are too low to be taken as evidence in favor of the existence of Granger causality from demand to prices. A natural extension would be concerned with analyzing the causality structure between the subvectors  $P_t$  and  $Y_t$  in the framework of the vector ARMA process (2.17)–(2.18) along the lines of Boudjellaba et al. (1992), a point left for future research.

The test results in table II, concerning the order of the autoregressive process of prices is reconsidered in table V, part C. The order of 3 and 4 for the autoregressive part, suggested by the Akaike and the  $\chi^2$ -criteria implies longer lags in prices in the system of factor demand equations (2.17). For instance, if the process of prices is ARI(3,1) then up to eight lagged prices should be included in (2.17) with 18 additional non-zero elements. When compared with model (2.17) with  $P_t$  being ARI(1, 1), this extension of the model (2.17) is significant according to the LR-test. A LR-test of the implications for (2.17) of an ARI(4, 1) process for prices is also significant when it is compared with the ARI(1, 1) model. However, the ARI(4, 1) model is not significant when it is compared with the ARI(3, 1) model. Testing the time-to-build assumption within the extension of model (2.17) when the process for prices is ARI(3, 1) and ARI(4, 1) give the LR-statistic 23.36 and 22.69 that are still significant ( $\chi_{0.05}^2(12) = 21.03$ ), implying that even in this extended model time-to-build is found to be relevant. Moreover, the Schwarz criterion leads us to select an ARI(1, 1) process for  $P_t$  whereas the AIC tends to overestimate the order of the process, we prefer to stick to the low order process for  $P_t$ .

To test whether the findings of Mayer (1960) about the length of the time-to-build are consistent with the findings for our model, we vary the length of the time-to-build up to five quarters. The results are given in the lower panel of table V. The specifications with four quarters gestation lags ( $J = 4$ ) are significantly different from those with a gestation of two or three quarters. According



to these results, the extension to a gestation period from four to five quarters is not significant. Therefore, the length of the gestation period in model (2.17) is consistent with the survey findings of Mayer.

### 3.3 *Reduced and Structural Form Estimates*

In table VI the reduced form estimates of model (2.17)–(2.18) subject to exclusion restrictions implied by the theoretical specification are given.

The following conclusions emerge from the estimation results for the reduced form in table VI. The parameters of the AR matrices  $F_1 + R$  and  $F_1 R$  and of the MA matrices  $\Phi_i$ ,  $i = 1, \dots, 4$  are highly significant. The deterministic trend appears to be relevant only in the demand equation for labor.

Table VII contains diagnostic tests for the labor, structures and equipment equations in (2.17). The first statistics show that the assumption of normality can only be rejected for the labor-residuals. The LM-statistic is significant because of the presence of leptokurtosis. A similar result was also found when testing for cointegration (see table III) with a VAR(4) model. The ARCH-statistics in table VII are not significant. No univariate autocorrelation is found as the Box-Pierce statistic  $Q$  is not significant. Hosking's multivariate autocorrelation indicates that there is significant cross-correlation in the residuals of the model (2.17)–(2.18).

In the lower panel of table VII, the same test statistics are reported for the model in (2.17)–(2.18) estimated without restricting  $R$  to be diagonal as suggested by the empirical findings in table V, part A. Even in this extended model a very high order multivariate autocorrelation is found. Notice that in this case the autocorrelation is also caused by autocorrelation of the residuals of the equipment equation.

The theoretical model (2.17)–(2.18) contains 68 reduced form parameters, whose estimates are given in table VI. These parameters are functions of the structural parameters, that can be estimated using the method of Asymptotic Least Squares (see for example Kodde et al. (1990)). Estimates of the structural parameters are given in table VIII. The estimates for the matrix of the process of prices are identical with the reduced form estimates.

The discount rate ( $\beta$ ) and both depreciation rates ( $\kappa^s$  and  $\kappa^e$ ) are not identified. The discount rate is assumed to be 0.96. According to depreciation rates used by the OECD when calculating capital stock ("Flows and Stocks of fixed capital", see appendix II) it is assumed that  $\kappa^s = 0.0125$  and  $\kappa^e = 0.025$ . The estimates of the structural coefficients of the production function, the adjustment cost function, the technology process and the time-to-build parameters are conditional on these values. As the data have been seasonally adjusted using dummies, the parameter vector  $\alpha$  in the production function cannot be identified from the intercept. The time-to-build parameters  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$  are identified from

Table VI. Reduced form estimates (Standard errors within parentheses)

$$C = \begin{bmatrix} -0.91 & (0.35) \\ -0.50 & (0.69) \\ -0.12 & (0.03) \end{bmatrix}$$

$$\tau = \begin{bmatrix} 0.02 & (0.01) \\ 0.01 & (0.01) \\ 0.00 & (0.00) \end{bmatrix}$$

$$F_1 + R = \begin{bmatrix} 1.20 & (0.08) & 0 & 0 \\ 0 & 1.80 & (0.05) & 0 \\ 0 & 0 & 0 & 1.70 & (0.10) \end{bmatrix}$$

$$-F_1 \cdot R = \begin{bmatrix} -0.37 & (0.08) & 0 \\ 0 & -0.84 & (0.05) & 0 \\ 0 & 0 & 0 & -0.78 & (0.10) \end{bmatrix}$$

$$M_0 = \begin{bmatrix} 0.15 & (0.17) & -0.26 & (0.26) & 0.19 & (0.84) \\ -0.16 & (0.57) & 1.09 & (0.88) & 6.43 & (3.01) \\ -0.01 & (0.02) & -0.01 & (0.03) & 0.13 & (0.10) \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 0.32 & (0.22) & -0.16 & (0.30) & 1.07 & (1.32) \\ 0.05 & (0.64) & -1.43 & (0.89) & -7.87 & (4.14) \\ 0.01 & (0.03) & -0.00 & (0.04) & -0.28 & (0.19) \end{bmatrix}$$

$$M_2 = \begin{bmatrix} -0.20 & (0.19) & 0.04 & (0.24) & -1.76 & (0.84) \\ 1.39 & (0.69) & 0.26 & (0.95) & 2.15 & (3.85) \\ -0.00 & (0.03) & -0.00 & (0.04) & 0.16 & (0.18) \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1.29 & (0.64) & -0.68 & (0.90) & 1.32 & (3.56) \\ 0.01 & (0.02) & -0.00 & (0.02) & -0.04 & (0.08) \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 0 & 0 & 0 \\ -0.42 & (0.61) & -2.34 & (0.89) & -5.38 & (3.50) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 0 & 0 & 0 \\ 1.79 & (0.59) & -0.17 & (0.76) & 4.39 & (3.37) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_6 = \begin{bmatrix} 0 & 0 & 0 \\ -0.01 & (0.57) & 1.23 & (0.69) & -0.28 & (2.31) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\phi_0 = I_3 \quad \phi_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.18 & (0.10) & 0 \\ 0 & 0 & 0 & 0.53 & (0.15) \end{bmatrix}$$

$$\phi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.41 & (0.09) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \phi_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.38 & (0.08) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \phi_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.39 & (0.10) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{M} = \begin{bmatrix} -0.05 & (0.15) & 0.36 & (0.20) & 0.23 & (0.14) \\ -0.07 & (0.11) & 0.34 & (0.12) & 0.10 & (0.14) \\ 0.05 & (0.11) & 0.13 & (0.15) & 0.48 & (0.11) \end{bmatrix}$$

$$\chi^2 = \begin{bmatrix} 0.41 \\ 0.44 & 3.42 \\ 0.02 & 0.06 & 0.01 \end{bmatrix} \quad \chi^p = \begin{bmatrix} 0.0002 \\ 0.00008 & 0.00013 \\ 0.00008 & 0.00006 & 0.00018 \end{bmatrix}$$

The following model was estimated after seasonal adjustment:

$$Y_t = C + \tau t + (F_1 + R)Y_{t-1} - RF_1 Y_{t-2} + \sum_{j=0}^{J+2} M_j P_{t-j} - \sum_{i=0}^J \phi_i \varepsilon_{t-i}^A \tag{2.17}$$

$$AP_t = \bar{M} \Delta P_{t-1} + \varepsilon_t^P \tag{2.18}$$

Assumption:  $J = 4$   
 Number of observations = 109 (1961:IV, 1988:IV)  
 2.loglik = 1188.42

Table VII. Tests on the residuals (see table V)

A. R diagonal (table VI)	Labor	Structures	Equipment
SKEW	0.34	0.05	0.03
EX.KURT.	1.19*	-0.22	-0.06
LM	12.22*	0.27	0.04
ARCH			
1	0.85	1.35	0.99
2	3.50	2.15	1.92
3	5.29	2.33	2.39
4	7.28	2.45	5.22
5	7.52	2.77	7.26
Q(10)	13.5	10.6	13.0
s	Hosking	$\chi^2_{0.05}(9.s-53)$	
10	99.84*	55.17	
15	139.18*	108.44	
20	169.87*	159.60	
25	204.55	209.72	
30	234.89	259.21	
40	309.02	356.95	
B. R unrestricted	Labor	Structures	Equipment
SKEW	0.38	0.02	0.11
EX.KURT.	1.42*	-0.23	0.50
LM	11.75*	0.25	1.32
Q(10)	13.6	10.5	21.3*
s	Hosking	$\chi^2_{0.05}(9.s-65)$	
8	67.15*	15.49	
9	79.53*	28.33	
10	83.74*	40.14	
15	118.82*	94.53	
20	149.73*	146.08	
25	183.68	196.43	
30	215.75	246.06	
40	287.71	343.99	

\* Significant at the 5% level.

the moving average parameters of the structures equation. The value of  $\delta_4$  can then be determined from (2.4d), that is from the assumption that the  $\delta$ 's add up to one.

Given these time-to-build parameters, the coefficients  $f_1, f_2, f_3, \rho_1, \rho_2, \rho_3$  and  $\gamma_{11}, \gamma_{22}, \gamma_{33}$  can be determined from the six parameters of the autoregressive part and all the parameters of  $M_0$ , thereby imposing all restrictions on  $M_0$  (see (A.11) and (A.12)). The value of  $f_4, f_5$  and  $f_6$  then follows from (A.6a). Finally, the production coefficients  $a_{11}, a_{22}$  and  $a_{33}$  can be obtained from (A.6b) and the restrictions on  $b_i$  (see the expression below (A.1)). Notice that by only taking

**Table VIII.** Structural form estimates (ALS) (Standard errors within parentheses)

Assumptions:			
$\beta = 0.96$			
$\kappa^s = 0.0125$			
$\kappa^e = 0.025$			
	Labor	Structures	Equipment
Production function	$a_{11} : 2.64$ (2.51)	$a_{22} : 0.006$ (0.017)	$a_{33} : 0.58$ (1.83)
Adjustment costs	$\gamma_{11} : 12.19$ (11.56)	$\gamma_{22} : 0.35$ (0.93)	$\gamma_{33} : 27.60$ (86.60)
Technology	$\rho_1 : 0.47$ (0.35)	$\rho_2 : 0.86$ (1.10)	$\rho_3 : 0.96$ (0.67)
Eigenvalues	$f_1 : 0.64$ (0.29)	$f_2 : 0.89$ (1.06)	$f_3 : 0.88$ (0.57)
	$f_4 : 1.63$ (0.73)	$f_5 : 1.17$ (1.39)	$f_6 : 1.18$ (0.77)
Process of prices	$\bar{M} = \begin{bmatrix} -0.05 & (0.15) & 0.36 & (0.20) & 0.23 & (0.14) \\ -0.07 & (0.11) & 0.34 & (0.12) & 0.10 & (0.14) \\ 0.05 & (0.11) & 0.13 & (0.15) & 0.48 & (0.11) \end{bmatrix}$		
Time-to-build			
$\phi_3/\phi_4 : 0.18$ (0.10)	$\phi_2/\phi_4 : -0.41$ (0.09)	$\phi_1/\phi_4 : -0.38$ (0.08)	$\phi_0/\phi_4 : -0.39$ (0.10)
$\delta_1 : 0.114$ (0.063)	$\delta_2 : 0.237$ (0.083)	$\delta_3 : 0.360$ (0.074)	$\delta_4 : 0.289$ (0.054)
Number of reduced form parameters = 68			
Number of structural parameters = 27			

into account  $M_0$ , the restrictions of  $M_1$  to  $M_6$  are not considered. Imposing all restrictions on this highly overidentified model is complicated. As a result of ignoring some restrictions, the estimates of the structural parameters in table VIII are not fully efficient.

As can be seen from table VIII, almost all time-to-build parameters are significant at the 5% level. These estimates imply a hump-shaped distribution of investments during the gestation period. Most other coefficients in table VIII are not significant, the reason being, the insignificance of many coefficients of  $M_0$  in table VI. The value of the Wald statistic associated with the restriction imposed when estimating the structural parameters from the reduced form parameters using asymptotic least squares equals 1.09 ( $< \chi^2_{0.05}(1) = 3.84$ ). As a consequence, the restriction imposed between  $\phi_i/\phi_4$ ,  $i = 0, 1, 2, 3$  and  $\delta_i$ ,  $i = 1, 2, 3$  is not rejected.

A high persistence of technological shocks in the equipment and structures equations with estimates of  $\rho_1$  and  $\rho_2$  of 0.86 and 0.96 respectively is found. Other factors of persistence ( $f_i$ ) in the reduced form depend on the adjustment

costs coefficients  $\gamma_{it}$  and the production function coefficients  $a_{it}$  (see the equation for  $b_i$  below (A.1) and (A.6b)). There is obviously a trade-off between the eigenvalues and the adjustment costs provided the marginal productivity is kept constant.

From the estimation results for the structures and equipment equations, it can be concluded that  $f_2$  and  $f_3$  are almost equal, whereas adjustment costs and marginal productivity for these production factors are quite different. A possible explanation of this finding is the time-to-build for structures.

Finally, again we want to stress the fact that care has to be taken when interpreting these results. More precise estimates could be obtained when all restrictions implied by the theoretical model were imposed on the structural coefficients.

### 3.4 *The Dynamics of the Model*

In order to study the dynamic properties of the different models we computed the moving average representation (MAR) for the first differences of the variables  $I_t^s$ ,  $I_t^e$  and  $N_t$ . The results are given in the figures 1–15. The figures presenting the impulse response of  $\Delta I^s$  to a shock in the productivity  $\varepsilon_{3t}^A$  and that of  $\Delta I^e$  and  $\Delta I^e$  to a shock in  $\varepsilon_{1t}^A$  are not reported, as these responses are zero (see also expression (A.9) in the appendix I with  $\Gamma$  being diagonal). The MAR has been obtained from the reduced form estimates of the models with respectively time-to-build and adjustment costs (t.t.b. and a.c.), time-to-build only (t.t.b.) and adjustment costs only (a.c.) and  $J = 4$ . The estimates for the model with time-to-build and adjustment costs are given in table VI. The contemporaneous covariance matrix of the technology and factor price shocks  $\mathcal{Z}$  has been decomposed such that in the recursive form the variables are ordered as follows  $C_t^s$ ,  $C_t^e$ ,  $W_t$ ,  $I_t^s$ ,  $I_t^e$ ,  $N_t$  with  $C_t^s$  appearing in all six equations,  $C_t^e$  appearing in five equations (with the exception of the equation for  $C_t^s$ ) and so on. The size of the impulse is equal to one standard deviation of the associated shock.

Several features emerge from the impulse responses in the figures 1–15. First there is no evidence of persistence in the impact of the shocks on the first differences of factor demand. Second, the shortrun dynamics of the different models differ in several respects. The model allowing for time-to-build dynamics only exhibits in general a fairly smooth monotonic response pattern. Adjustment costs instead lead to cyclical patterns in the impulse response function with peaks and troughs generally appearing between quarters 4 and 12. The model with time-to-build and adjustment costs exhibits the largest fluctuations in the impulse response. Third, the response of  $N_t$  to the different shocks is hardly sensitive to the choice of the specification for the dynamics. Fourth, the reaction of the model with time-to-build only has immediately the expected sign. A positive technology shock leads to an increase in  $\Delta I^s$  and  $\Delta I^e$ , whereas

price shocks imply a decrease in the change of  $I^s$  and  $I^e$ . Fifth, the response of  $\Delta I^s$  and  $\Delta I^e$  to respectively  $\varepsilon_{2t}^\lambda$  and  $\varepsilon_{3t}^\lambda$  is hump-shaped for the models with adjustment costs included. Notice that Park (1984) also finds a hump-shaped response in the change of gross private domestic fixed investment using U.S. quarterly data for the period 1948–1981. Sixth, the positive reaction of  $\Delta I^s$  and  $\Delta I^e$  quickly after the shocks  $\varepsilon_{2t}^p$  and  $\varepsilon_{3t}^p$ , respectively for the model with time-to-build only, resembles that found by Park (1984) for the change in gross private domestic investment in the U.S. While, because of sampling errors some care has to be taken when interpreting these findings, one can conclude that the specification of the dynamics strongly affects the shape of the estimated response function.

Finally, notice that our findings cannot be directly compared with the simulation results in Rouwenhorst (1991) where the impulse response of the capital stock is reported instead of that of investment. The figures 1–15 represent impulse responses of  $\varepsilon$  shocks in a partial equilibrium model of factor inputs. Real business cycle models describe economies instead of demand or supply functions only.

#### 4 Conclusions

In this paper, we formulated a dynamic factor demand model which incorporates adjustment costs and time-to-build considerations and derived the closed form solution for the demand for labor, structures and equipment. The inclusion of time-to-build features for structures adds – beside the adjustment costs of all production factors – another dynamic dimension to the model. Both sources of dynamics can be identified from the information in the data. Given the processes for the exogenous variables the factor prices and technology, the dynamics and the nonstationarity of factor demand are completely determined by the theoretical model.

The model was analyzed using quarterly data U.S. manufacturing industry data for the period 1960.I–1988.IV. The findings of the univariate time series and the econometric analyses were interpreted in the framework of the initial optimization model. The joint model performs reasonably well when it is applied to quarterly U.S. manufacturing data. In particular, assumptions on the order of integration, on the number of cointegration relationships were found to be supported by the information in the data. Tests on assumptions concerning the process of technology indicate that the theoretical model could be extended by a more general specification where interrelations between production factors arise.

When accounting for the fact that in the presence of integrated regressors, the Wald statistic has a non-standard distribution, the results from LR-tests

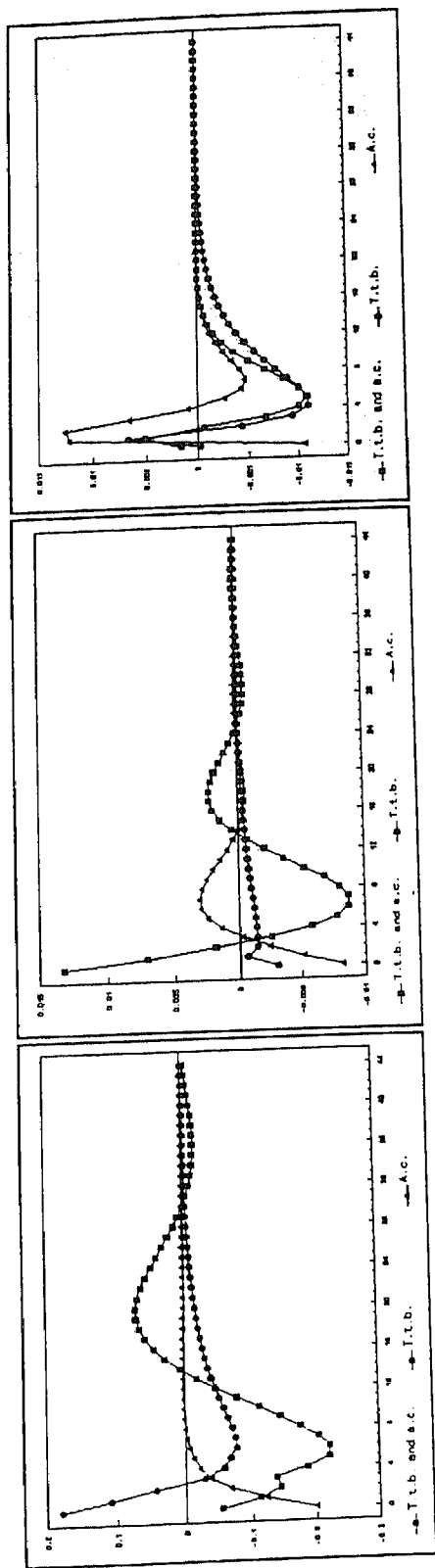


Fig. 1. Impulse from  $\varepsilon_{2t}^s$  to  $I^s$

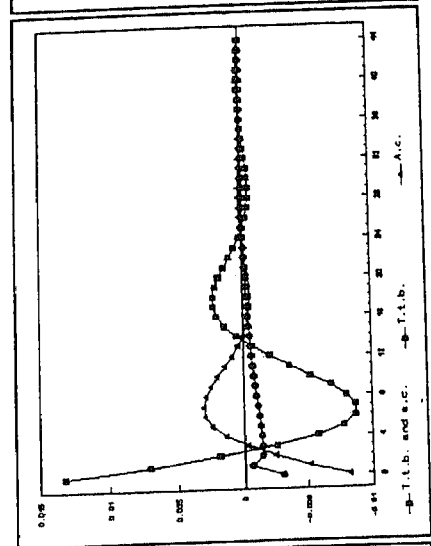


Fig. 2. Impulse from  $\varepsilon_{2t}^e$  to  $I^e$

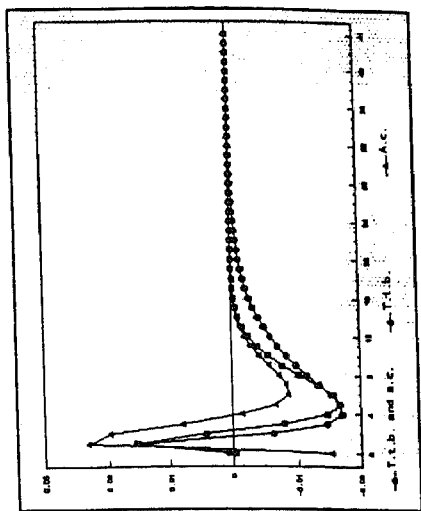


Fig. 3. Impulse from  $\varepsilon_{2t}^i$  to  $N$

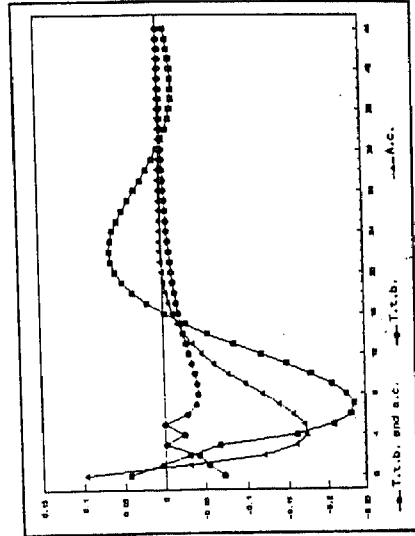


Fig. 4. Impulse from  $\varepsilon_{2t}^c$  to  $I^c$

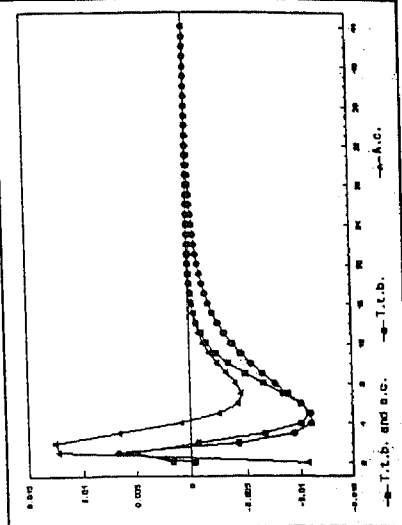


Fig. 5. Impulse from  $\varepsilon_{2t}^f$  to  $I^f$

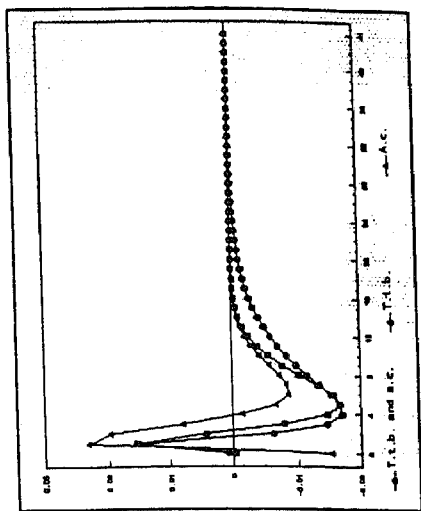


Fig. 6. Impulse from  $\varepsilon_{2t}^g$  to  $N$



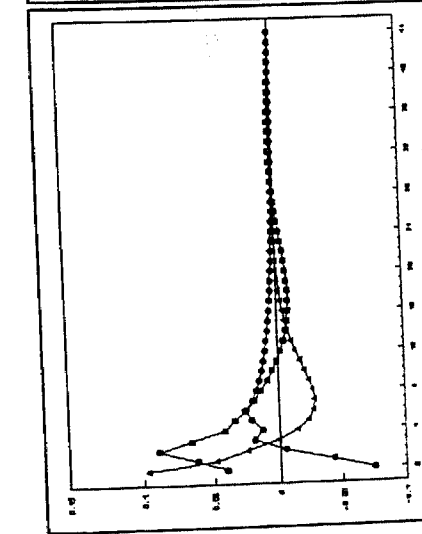
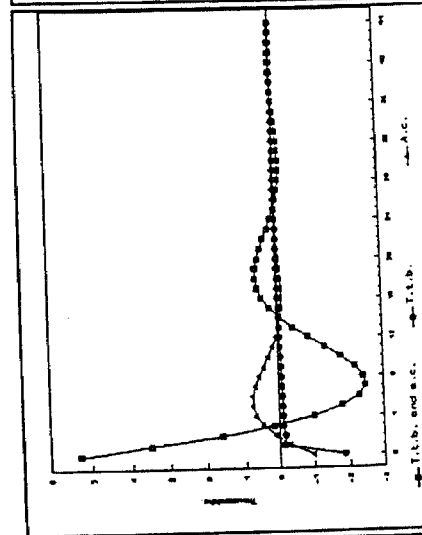
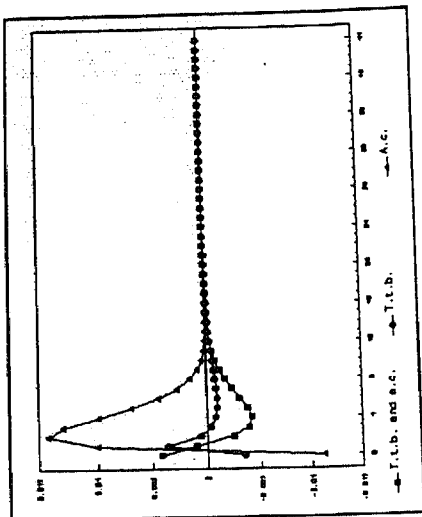


Fig. 9. Impulse from  $\epsilon_1^T$  to  $N$

Fig. 8. Impulse from  $\epsilon_1^T$  to  $I^e$

Fig. 7. Impulse from  $\epsilon_1^T$  to  $I^c$

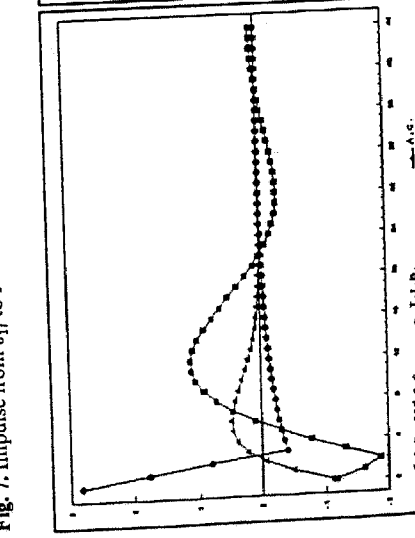
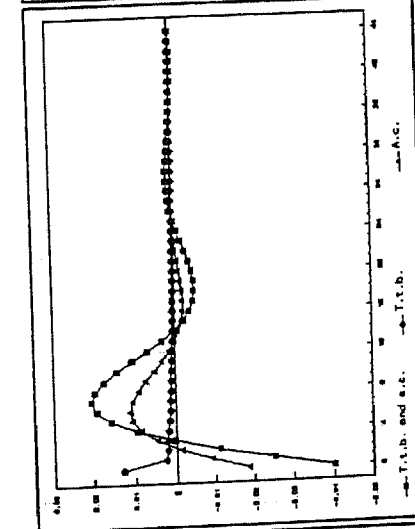
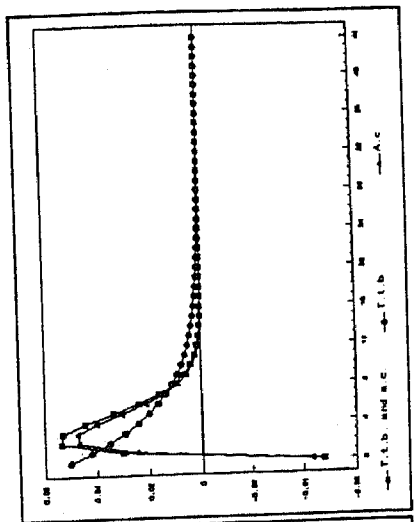
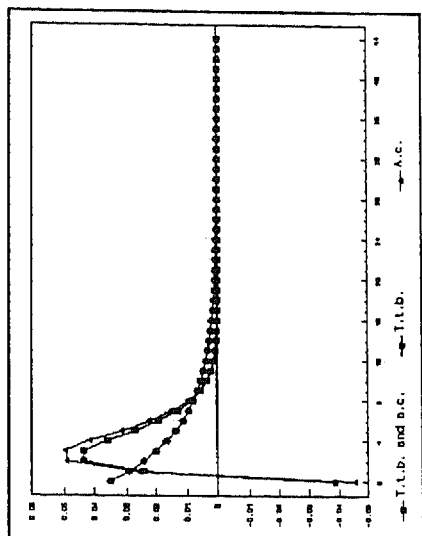
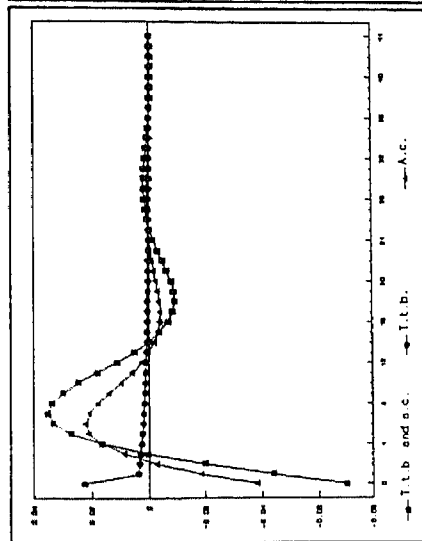
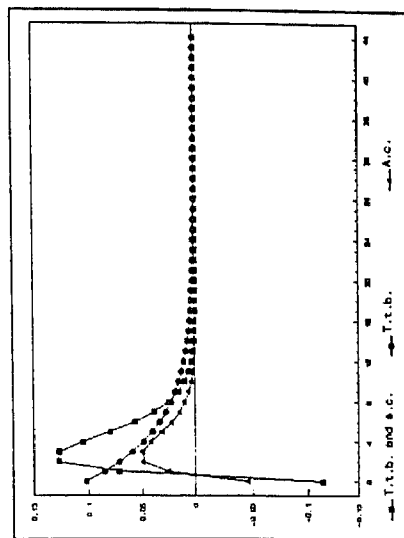


Fig. 12. Impulse from  $\epsilon_2^T$  to  $N$

Fig. 11. Impulse from  $\epsilon_2^T$  to  $I^e$

Fig. 10. Impulse from  $\epsilon_2^T$  to  $I^c$

Fig. 13. Impulse from  $\epsilon_{3r}^i$  to  $I^c$ Fig. 14. Impulse from  $\epsilon_{3r}^i$  to  $N$ Fig. 15. Impulse from  $\epsilon_{3r}^i$  to  $N$

on uni-directional Granger causality from production factors to factor prices do not reject the assumptions made in the theoretical model.

Taking up these remarks the consequences of assuming interrelation of production factors within both the production function and the adjustment cost function are important. More information can be gathered with respect to the degree of complementarity or substitutability among labor, structures and equipment. The difficulty in analyzing these features in a model with time-to-build arises from the differences in lead times between production factors.

Most importantly, it was found that given all simplifying assumptions made, time-to-build considerations are found to be at least as important as adjustment costs in our model. The impulse response function associated with the model with time-to-build dynamics only has a much smoother shape than the response function of the models based on adjustment costs. Comparisons of the results presented in this paper with the stylized facts from the real business cycle (RBC) literature, however, should be made with great caution. This paper investigates a partial equilibrium model of factor inputs, whereas RBC models describe economies instead of just demand or supply functions.

### Appendix I Solving the Euler Equations for the Rational Expectations

We define  $X_t^d = [N_t, K_{t+J}^s, K_{t+1}^e]'$ , normalize the system (2.10a)–(2.10c) in the expectations variables and extend it by adding identities to yield in matrix form

$$\begin{bmatrix} X_t^d \\ E_t X_{t+1}^d \end{bmatrix} = \begin{bmatrix} 0 & I_3 \\ -\beta^{-1}I_3 & B_{22} \end{bmatrix} \begin{bmatrix} X_{t-1}^d \\ X_t^d \end{bmatrix} + \begin{bmatrix} 0 \\ D \end{bmatrix} Z_t, \tag{A.1}$$

where  $B_{22}$  is a  $3 \times 3$  diagonal matrix with typical diagonal element  $b_i = a_{ii}(\beta\gamma_{ii})^{-1} + \beta^{-1} + 1$ , and  $D$  and  $Z_t$  given by

$$D = (\beta\Gamma)^{-1} \begin{bmatrix} -\alpha_1 & -1 & 0 & 0 & 1 & 0 \\ -\alpha_2 & 0 & -1 & 0 & 0 & \phi_J\beta^{-J} \\ -\alpha_3 & 0 & 0 & -1 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & \dots & 0 \\ & & 0 & \phi_{J-1}\beta^{-J+1} & 0 & \phi_{J-2}\beta^{-J+2} & \dots & \phi_0 \\ & & \beta^{-1} & 0 & (\kappa^e - 1) & 0 & \dots & 0 \end{bmatrix}$$

$$Z_t = [1 \quad \lambda_{1t} \quad E_t \lambda_{2,t+J} \quad E_t \lambda_{3,t+1} \quad W_t \quad C_t^s \quad C_t^e \quad E_t C_{t+1}^s \quad E_t C_{t+1}^e \quad E_t C_{t+2}^s \quad \dots \quad E_t C_{t+J}^s] \tag{A.2}$$

Define

$$B = \begin{bmatrix} 0 & I_3 \\ -\beta^{-1}I_3 & B_{22} \end{bmatrix}.$$

If we can decompose  $B$  as

$$B = G^{-1}FG \quad (\text{A.3})$$

and partition  $F$  and  $G$  in blocks of  $3 \times 3$  matrices as

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \quad F = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix} \quad \text{with} \quad F_1 = \begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} f_4 & 0 & 0 \\ 0 & f_5 & 0 \\ 0 & 0 & f_6 \end{bmatrix},$$

the application of Blanchard and Kahn (1980) expresses factor demand  $X_t^d$  as

$$X_t^d = -G_{22}^{-1}G_{21}X_{t-1}^d - G_{22}^{-1} \sum_{i=0}^{\infty} F_2^{-i-1}G_{22}DE_t(Z_{t+i}) \quad (\text{A.4})$$

From (A.3) it follows that

$$GB = FG \Leftrightarrow \begin{bmatrix} -\beta^{-1}G_{12} & G_{11} + G_{12}B_{22} \\ -\beta^{-1}G_{22} & G_{21} + G_{22}B_{22} \end{bmatrix} = \begin{bmatrix} F_1G_{11} & F_1G_{12} \\ F_2G_{21} & F_2G_{22} \end{bmatrix}$$

such that under the condition that the submatrices are invertible, the equality of the (2, 1) blocks gives

$$-\beta^{-1}G_{22} = F_2G_{21} \Leftrightarrow -G_{22}^{-1}G_{21} = G_{21}^{-1}(\beta F_2)^{-1}G_{21}. \quad (\text{A.5})$$

Because  $F_2$  is diagonal, the autoregressive part in the solution (A.4) is diagonal if  $G_{21}$  is diagonal. The characteristic polynomial of the matrix  $B$  can be written as (see also Palm and Pfann (1990))

$$(-f^2 + b_1f - \beta^{-1})(-f^2 + b_2f - \beta^{-1})(-f^2 + b_3f - \beta^{-1}),$$

where  $b_1, b_2, b_3$  are the eigenvalues of  $B_{22}$ . From this follows that the six eigenvalues  $f_i$  ( $i = 1, 2, \dots, 6$ ) of  $B$  satisfy

$$f_1f_4 = f_2f_5 = f_3f_6 = \beta^{-1} \quad (\text{A.6a})$$

and

$$f_1 + f_4 = b_1, \quad f_2 + f_5 = b_2, \quad f_3 + f_6 = b_3, \quad (\text{A.6b})$$

where it is assumed that

$$|f_i| \leq 1, \quad i = 1, 2, 3 \quad \text{and} \quad |f_i| > 1, \quad i = 4, 5, 6.$$

The equality

$$-G_{22}^{-1}G_{21} = G_{21}^{-1}(\beta F_2)^{-1}G_{21} = G_{21}^{-1}F_1G_{21} \quad (\text{A.7})$$

(see (A.5) and (A.6a)) then also holds.

Because  $B_{22}$  is diagonal, the decomposition (A.3) is easy to obtain

$$G^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ f_1 & 0 & 0 & f_4 & 0 & 0 \\ 0 & f_2 & 0 & 0 & f_5 & 0 \\ 0 & 0 & f_3 & 0 & 0 & f_6 \end{bmatrix}, \quad \begin{aligned} f_i &= \frac{1}{2}b_i - \frac{1}{2}[b_i^2 - 4\beta^{-1}]^{1/2}, & i = 1, 2, 3, \\ f_i &= \frac{1}{2}b_{i-3} + \frac{1}{2}[b_{i-3}^2 - 4\beta^{-1}]^{1/2} & i = 4, 5, 6. \end{aligned}$$

Thanks to (A.7) and the diagonality of  $F_1$  and  $G_{22}$ , formula (A.4) reduces to

$$X_t^d = F_1 X_{t-1}^d - \sum_{i=0}^{\infty} (\beta F_1)^{i+1} D E_t(Z_{t+i}). \tag{A.8}$$

Notice that the necessary condition  $|f_i| \leq 1, i = 1, 2, 3$  and  $|f_i| > 1, i = 4, 5, 6$  follows from the assumptions  $0 < \beta < 1, \gamma_{ii} > 0 (i = 1, 2, 3)$  and  $a_{ii} > 0 (i = 1, 2, 3)$ .

The rewriting of (A.8) in terms of  $Y_t$  gives (2.14). Separating the components of  $Z_t$  into a constant term, a technology component and a price component, we then rewrite (2.14) as

$$\begin{aligned} Y_t &= C^* + F_1 Y_{t-1} - F_1 \Gamma^{-1} \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^i E_{t-j} [\lambda_{1,t+i-j}, \lambda_{2,t+i+J-j}, \lambda_{3,t+i+1-j}] \\ &\quad - F_1 \Gamma^{-1} \sum_{k=0}^J D_k^* \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^i E_{t-j} P_{t+i+k-j}, \end{aligned} \tag{A.9}$$

where (see also (2.12))

$$C^* = - \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^{i+1} \begin{bmatrix} \alpha_1 (\beta \gamma_{11})^{-1} \\ \alpha_2 (\beta \gamma_{22})^{-1} \\ \alpha_3 (\beta \gamma_{33})^{-1} \end{bmatrix} = \begin{bmatrix} \alpha_1 f_1 (\gamma_{11} (1 - \beta f_1))^{-1} \\ \kappa^s \alpha_2 f_2 (\gamma_{22} (1 - \beta f_2))^{-1} \\ \kappa^e \alpha_3 f_3 (\gamma_{33} (1 - \beta f_3))^{-1} \end{bmatrix}$$

and (see (2.14))

$$D_k^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta^{-J+k} & 0 \\ 0 & 0 & \beta^{-1+k} \end{bmatrix} D_k, \quad k = 0, 1, \dots, J.$$

Using the assumption that  $\lambda_t$  is generated by a first order VAR (2.15), the part in (A.9) which is explained by technology shocks can be expressed as

$$\begin{aligned} &F_1 \Gamma^{-1} \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^i E_{t-j} \begin{bmatrix} \lambda_{1,t+i-j} \\ \lambda_{2,t+i+J-j} \\ \lambda_{3,t+i+1-j} \end{bmatrix} \\ &= F_1 \Gamma^{-1} \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^i \begin{bmatrix} \rho_1^i & 0 & 0 \\ 0 & \rho_2^{i+J} & 0 \\ 0 & 0 & \rho_3^{i+1} \end{bmatrix} \lambda_{t-j} = R^* \sum_{j=0}^J D_j \lambda_{t-j}, \end{aligned} \tag{A.10}$$

where

$$R^* = F_1 \Gamma^{-1} (I - \beta F_1 R)^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_2^J & 0 \\ 0 & 0 & \rho_3 \end{bmatrix}.$$

Similarly but in a far less obvious way, from the assumption of a first order VAR for  $\Delta P_t$  (2.16), it follows that

$$\sum_{i=0}^{\infty} (\beta F_1)^i E_{t-j}(P_{t+i+k-j}) = V_k P_{t-j} - \left( \sum_{i=1}^k \bar{M}^i + \beta F_1 V_k \bar{M} \right) P_{t-1-j},$$

with

$$V_k - \beta F_1 V_k (I + \bar{M}) + (\beta F_1)^2 V_k \bar{M} = (I - \beta F_1) \sum_{i=1}^k \bar{M}^i + I_3 \quad k \geq 0. \quad (\text{A.11})$$

such that

$$-F_1 \Gamma^{-1} \sum_{k=0}^J D_k^* \sum_{j=0}^J D_j \sum_{i=0}^{\infty} (\beta F_1)^i E_{t-j}(P_{t+i+k-j}) = \sum_{i=0}^{J+1} M_i^* P_{t-i}, \quad (\text{A.12})$$

where

$$M_0^* = -F_1 \Gamma^{-1} \sum_{k=0}^J D_k^* D_0 V_k$$

$$M_i^* = -F_1 \Gamma^{-1} \sum_{k=0}^J D_k^* \left[ D_i V_k - D_{i-1} \left( \sum_{j=1}^k \bar{M}^j + \beta F_1 V_k \bar{M} \right) \right] \quad i=1, 2, \dots, J$$

$$M_{J+1}^* = F_1 \Gamma^{-1} \sum_{k=0}^J D_k^* D_J \left( \sum_{j=1}^k \bar{M}^j + \beta F_1 V_k \bar{M} \right).$$

Only the matrices  $M_0^*$  and  $M_1^*$  have no zero elements.  $M_2^*$  has zero elements on the first row whereas  $M_k^*$  ( $k = 3, 4, \dots, J+1$ ) has zero elements on the first and third row.

After substituting (A.10) and (A.12) into (A.9) and applying a Koyck transformation to eliminate the unobservable technology components  $\lambda_t$ , we obtain the model

$$Y_t = C + (R + F_1) Y_{t-1} - R F_1 Y_{t-2} + \sum_{j=0}^{J+2} M_j P_{t-j} + \varepsilon_t^y \quad (\text{A.13})$$

with

$$C = (I - R) C^*$$

$$M_0 = M_0^*$$

$$M_i = M_i^* - R M_{i-1}^*, \quad i = 1, 2, \dots, J+1$$

$$M_{J+2} = -R M_{J+2}^*$$

$$\varepsilon_t^y = -R^* \sum_{j=0}^J D_j \varepsilon_{t-j}^\lambda = R^* \begin{bmatrix} \varepsilon_{1,t}^\lambda \\ \sum_{j=0}^J \Phi_{J-j} \varepsilon_{2,t-j}^\lambda \\ \varepsilon_{3,t}^\lambda + (\kappa^e - 1) \varepsilon_{3,t-1}^\lambda \end{bmatrix}.$$

## Appendix II Quarterly Data for the Manufacturing Industry, 1960–1988

The variables are:

- $N$  Average weekly hours, that is  $L * H$  where:  
 $L$  = number of all employees  
 $H$  = weekly hours of work.
- $I^s, I^e$  Gross fixed capital formation, structures and equipment respectively, in constant prices.
- $W$  Real hourly earnings, that is hourly earnings deflated by the producer price index of industrial goods  $P_y$ .
- $C^s, C^e$  Real costs of gross investments, structures and equipment respectively, that is  $(C_c^i/I^i)/P_y$  ( $i = s, e$ ) where  $C_c^i$  = gross fixed capital formation of  $i$  in current prices, and  $I^i$  = gross fixed capital formation of  $i$  in constant prices.

Sources:

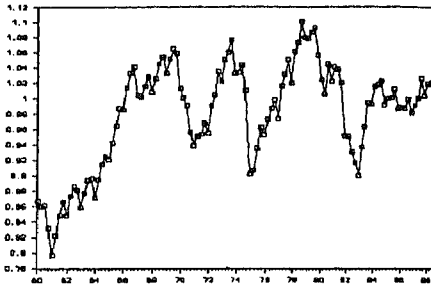
- $MEI$  Main Economic Indicators, Organisation for Economic Cooperation and Development (OECD), various issues.
- $FS$  Flows and Stocks of Fixed Capital, OECD, various issues.
- $QNA$  Databank of Quarterly National Accounts, OECD.

The time series  $I^s, I^e, C_c^s, C_c^e$ , are taken from FS. All other time series are taken from MEI and are quarterly.

The annual (end of the year) series  $I^s, I^e, C_c^s, C_c^e$  are interpolated using the Ginsburgh method. National (non-residential) investment series from QNA for structures and equipment in both current and constant prices are used to describe quarterly fluctuations.

All series are seasonally unadjusted and are indexed at 1985.II. The graphs of the time series  $N, W, I^s, C^s, I^e, C^e$  are given in the next figures.

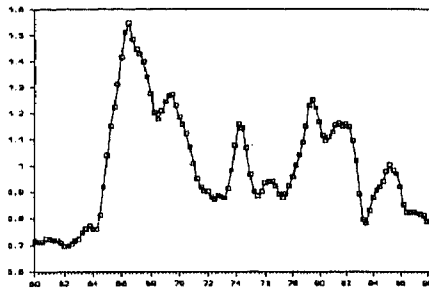
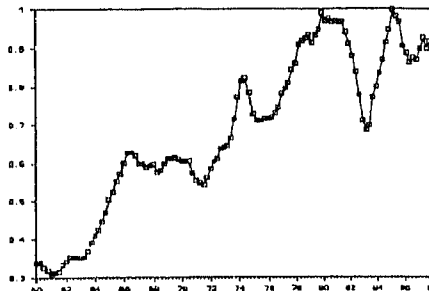
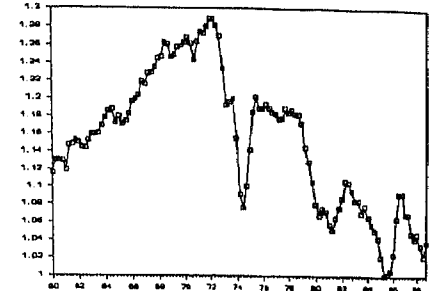
## United States Manufacturing Industry 1960.I-1988.IV



Labor (N)



Wages (W)

Structures Investment ( $I^S$ )Price Structures Investment ( $C^S$ )Equipment Investment ( $I^E$ )Price Equipment Investment ( $C^E$ )



## References

- Blanchard OJ, Kahn CM (1980) The solution of linear difference models under rational expectations. *Econometrica* 48:1305–11
- Boudjellaba H, Dufour J-M, Roy R (1992) Testing causality between two vectors in multivariate ARMA models. *Journal of the American Statistical Association* 87:1082–1090
- Engle RF, Hendry DF, J-F Richard (1983) Exogeneity. *Econometrica* 51:277–304
- Hosking JRM (1980) The multivariate portmanteau statistic. *Journal of the American Statistical Association* 75:602–608
- Jarque CM, Bera AK (1980) Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics Letters* 6:255–259
- Johansen S (1991) The role of the constant term in cointegration analysis of nonstationary variables. Mimeo, Copenhagen
- Johansen S, Juselius K (1990) Maximum likelihood estimation and inference on cointegration – with applications to the demand for money. *Oxford Bulletin of Economics and Statistics* 52:169–210
- Kodde DA, Palm FC, Pfann GA (1990) Asymptotic least-squares estimation – efficiency considerations and applications. *Journal of Applied Econometrics* 5:229–243
- Kydland FE, Prescott EC (1982) Time to build and aggregate fluctuations. *Econometrica* 50:1345–70
- Lippi M, Reichlin L (1990) Diffusion of technical change and the identification of the trend component in real GNP. *Observatoire Français des Conjonctures Economiques, Paris, Document de travail no. 90–94*
- Liu LM, Hudak GB, Box GEP, Muller ME, Tiao GC (1986) *The SCA Statistical System – Reference Manual for Forecasting and Time Series Analysis*. Illinois: SCA-Press
- Lucas RE (1967) Adjustment costs and the theory of supply. *Journal of Political Economy* 75:321–34
- Mayer Th (1960) Plant and equipment lead times. *The Journal of Business* 33:127–132
- Nickell SJ (1985) Error correction, partial adjustment and all that: an expository note. *Oxford Bulletin of Economics and Statistics* 47:119–129.
- Palm FC, Pfann GA (1990) Interrelated demand rational expectations models for two types of labour. *Oxford Bulletin of Economics and Statistics* 52:45–68
- Palm FC, Pfann GA (1991) Interrelation, structural changes, and cointegration in a model for manufacturing factor demand in the Netherlands. *Recherches Economiques de Louvain* 51:221–243
- Park JA (1984) Gestation lags with variable plans: An empirical study of aggregate investment. Ph.D. dissertation, Carnegie-Mellon University
- Pfann GA (1990) *Stochastic Adjustment Models of Labour Demand*, Berlin Springer-Verlag
- Pötscher BM (1989) Model selection under nonstationarity; autoregressive models and stochastic linear regressions models. *The Annals of Statistics* 17:1257–1274
- Rossi PE (1988) Comparison of dynamic factor demand models. In: Barnett WA, Berndt ER, White H (eds) *Dynamic econometric modeling, proceedings of the third international symposium in economic theory & econometrics*, Cambridge, Cambridge University Press 357–376
- Rouwenhorst KG (1991) Time to build and aggregate fluctuations. *Journal of Monetary Economics* 27:241–254
- Schwarz G (1978) Estimating the dimension of a model. *Annals of Statistics* 6:461–464
- Toda HY, Phillips PCB (1991) Vector autoregression and causality: A theoretical overview and simulation study. Cowles Foundation Discussion Paper no 1001, Yale University