EXCHANGE RATE MODELS AND INNOVATIONS
A Derivation

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An explicit relationship between innovations in the spot exchange rate and innovations in its driving variables is derived in the context of the 'asset market theory' of the exchange rate. The relationship forms a theoretical basis for regression results in 'news' form that have been reported in the literature.

1. Introduction

In recent years the distinction between *anticipated* and *unanticipated* movements in the exchange rate and its driving variables has been emphasised in the literature [see, e.g., Frenkel and Mussa (1980), Frenkel (1981) and Mussa (1984)]. The essence of this line of thinking is embodied in the 'asset market theory' of the exchange rate that is presented in Frenkel and Mussa (1980). The framework that was developed by Frenkel and Mussa views the exchange rate as a highly sensitive asset price which is immediately affected by an influx of new information. This approach is generally taken to imply that empirical research on the determinants of exchange rates should relate innovations in exchange rates to innovations in a relevant vector of explanatory variables. The idea was first implemented empirically by Frenkel (1981).

While the innovations or 'news' approach is intuitively appealing, no theoretical derivation has been presented in the literature that links the general class of 'news' models that is estimated in Frenkel (1981) with the theoretical model of Frenkel and Mussa (1980). Such a derivation is presented in the next section of this paper.

2. A derivation

Consider the following simple model of exchange rate determination:

$$s(t) = \beta' z(t) + a\mathrm{E}[s(t+1) - s(t) \,|\, t], \qquad H[L]z(t) = F[L]v(t). \qquad (1),(2)$$

Eq. (1) is due to Frenkel and Mussa (1980) and states that the logarithm of the equilibrium spot

exchange rate, $s(t)$, is determined not only by a set of current market fundamentals, but also by the expected rate of change of the exchange rate, $E[s(t+1) - s(t)|t]$, which motivates domestic and foreign residents to move into or out of foreign exchange depending on whether the relative price of foreign exchange is expected to rise or fall. The vector β is a vector of parameters, a ($a > 0$) is a scalar parameter and $E[\cdot|t]$ denotes an expected value conditional on information available at time t. Eq. (1) represents a general relationship which can be derived from a variety of models of exchange rate determination that generally differ in their interpretations of the elements of the vector $z(t)$. In eq. (2) it is assumed that $z(t)$ is generated by a general vector ARMA time series process. $H[L]$ and $F[L]$ are square matrices, assumed of full rank, whose elements are finite polynomials in the lag operator L. The vector $z(t)$ is assumed to be m-dimensional and thus $F[L]$ and $H[L]$ are $m \times m$ matrices. Further, we assume that the $m \times 1$ vector of innovations $v(t)$ has a zero mean, an identity covariance matrix and no serial correlation, that is,

$$E[v(t)] = 0, \qquad E[v(t)v'(r)] = \delta_{rt}I, \tag{3},(4)$$

where I is an $m \times m$ unit matrix and δ_{rt} is the Kronecker delta. Since we assume initially that the process is stationary and invertible (stationarity will be relaxed below), all roots of $\|H[L]\| = 0$ and, $\|F[L]\| = 0$ lie outside the unit circle ($\|\cdot\|$ indicates the determinant of a matrix). Since $H[L]$ is assumed to have full rank, (2) can be solved for $z(t)$,

$$z(t) = \left(H^*[L]/\|H[L]\| \right) F[L]v(t), \tag{5}$$

where $H^*[L]$ is the adjoint matrix (the transpose of the matrix of cofactors) associated with $H[L]$. Eq. (5) expresses $z(t)$ as an infinite, invertible vector moving average process. For simplicity of notation, we define

$$B[L] \equiv \left(H^*[L]/\|H[L]\| \right) F[L]. \tag{6}$$

The matrix $B[L]$ can be written as

$$B[L] = B_0 + B_1L + B_2L^2 + B_3L^3 + \ldots \tag{7}$$

and eq. (5) now reads

$$z(t) = B[L]v(t). \tag{8}$$

Since the vector $z(t)$ is shown to have an invertible moving average representation in eqs. (5) and (8), we can conclude that the sequence of matrices $[B_j; \ j = 0, 1, 2, \ldots]$ is absolutely summable [see, e.g., Fuller (1976, ch. II)]. That is, if we define b_j^{kl} to be the k,lth element of the matrix B_j,

$$\lim_{n \to \infty} \sum_{j=0}^{n} \left| b_j^{kl} \right| < \infty \quad \text{for all } k,l, \tag{9}$$

where $|\cdot|$ denotes an absolute value. This result will be used below.

If we assume that expectations in eq. (1) are formed rationally in the sense that they are consistent with the validity of (1) in all future periods, then forward iteration of (1) and application of an appropriate boundary condition gives

$$E[s(t+i)|t] = \left[\frac{1}{1+a} \right] \sum_{j=0}^{\infty} \left[\frac{a}{1+a} \right]^j \beta'E[z(t+i+j)|t] \tag{10}$$

[see, e.g., Mussa (1984)]. The exchange rate that is currently expected to prevail at time $t + i$ ($i \geq 0$) depends on a weighted average of expected future z's. Setting $i = 0$ in eq. (10) and applying the unexpected change operator $D^u[s(t)] \equiv s(t + 1) - E[s(t + 1) | t]$ gives

$$D^u[s(t)] = \left[\frac{1}{1 + a}\right] \sum_{j=0}^{\infty} \left[\frac{a}{1 + a}\right]^j \beta'(E[z(t + j + 1) | t + 1] - E[z(t + j + 1) | t]).$$ (11)

The unexpected change in the exchange rate is a weighted average of changes in expectations concerning the future z-vectors resulting from new information that is received between times t and $t + 1$. Substituting (8) into (11) gives

$$D^u[s(t)] = \left[\frac{1}{1 + a}\right] \sum_{j=0}^{\infty} \left[\frac{a}{1 + a}\right]^j \beta'(E[B[L]v(t + j + 1) | t + 1] - E[B[L]v(t + j + 1) | t]).$$ (12)

Using the definition of $B[L]$ that is given in (7), we can thus derive

$$D^u[s(t)] = \left[\frac{1}{1 + a}\right] \beta' \left(B_0 + \left[\frac{a}{1 + a}\right] B_1 + \left[\frac{a}{1 + a}\right]^2 B_2 + \ldots\right) v(t + 1).$$ (13)

Making use of the result in (9), we can show that the infinite series,

$$B_0 + \left[\frac{a}{1 + a}\right] B_1 + \left[\frac{a}{1 + a}\right]^2 B_2 + \ldots = \sum_{j=0}^{\infty} \left[\frac{a}{1 + a}\right]^j B_j,$$ (14)

converges to a finite matrix. Again, let b_j^{kl} be the k,lth element of the matrix B_j. Then we have

$$\lim_{n \to \infty} \sum_{j=0}^{n} \left[\frac{a}{1 + a}\right]^j b_j^{kl} \leq \lim_{n \to \infty} \sum_{j=0}^{\infty} \left[\frac{a}{1 + a}\right]^j |b_j^{kl}| < \infty,$$ (15)

where the first inequality follows from the definition of an absolute value and the second from (9). Thus, the infinite series in (14) converges to a finite matrix, say C,

$$\lim_{n \to \infty} \sum_{j=0}^{n} \left[\frac{a}{1 + a}\right]^j B_j = C.$$ (16)

Note that convergence does not critically depend on the stationarity assumption that was made with regard to the process in eq. (2). Convergence is also obtianed for all non-stationary processes that satisfy the right-hand side inequality in (15).

Combining, finally, eq. (13) and (16), we obtain the relationship we set out to derive,

$$D^u[s(t)] = \left[\frac{1}{1 + a}\right] \beta' C v(t + 1).$$ (17)

Eq. (17) is a linear relationship between the innovation in the spot exchange rate between t and $t + 1$ and innovations in the elements of the z-vector in the corresponding period. Given appropriate measures of innovations in the exchange rate and its driving variables, the validity of (17) can be examined empirically. Since innovations are inherently unobservable, any empirical study on the

basis of (17) involves a joint examination of the model and the method that is used to construct innovations. Eq. (17) forms a theoretical basis for the 'news' regressions that are presented in Frenkel (1981).

It should be noted that estimated coefficients in regression equations that are in innovations form do not have the same interpretations as those estimated from standard structural models. That is, when estimating an equation based on (17) one does not recover the parameter vector β, but a complicated vector of coefficients that involves the elements of the vector β, the elements of the matrix C and the scalar a.

References

Frenkel, J.A., 1981, Flexible exchange rates, prices and the role of 'news': Lessons from the 1970s, Journal of Political Economy 89, Aug., 665–705.

Frenkel, J.A. and M.L. Mussa, 1980, The efficiency of foreign exchange markets and measures of turbulence, American Economic Review 70, May, 374–381.

Fuller, W.A., 1976, Introduction to statistical time series (Wiley, New York).

Mussa, M.L., 1984, The theory of exchange rate determination, in: J.F.O. Bilson and R.C. Marston, eds., Exchange rate theory and practice (University of Chicago Press, Chicago, IL) 13–58.

Wolff, C.C.P., 1985, Exchange rate models, parameter variation and innovations: A study on the forecasting performance of empirical models of exchange rate determination, Unpublished Ph.D. dissertation (University of Chicago, Chicago, IL).