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Measuring the forward foreign exchange risk premium: multi-country evidence from unobserved components models☆

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Abstract

We investigate the nature of the foreign exchange risk premium for a wide range of currencies, using unobserved components models with exactly matched spot and forward exchange rate data. Significant time-variation of the risk premium is documented for most currencies. Our estimates indicate considerable persistence in the risk premium, and suggest that the variability of the risk premium is quite low relative to the variability of the forward forecast error. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Forward discount bias is a phenomenon that was studied extensively in the literature. In addition to the forward exchange unbiasedness being rejected, it is generally found that the change in the future exchange rate is negatively related to the forward discount. A prominent explanation for the rejection of forward rate unbiasedness is the existence of a time-varying risk premium. Other explanations

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involve peso problems, irrationality of expectations, learning behavior, and market inefficiency. Useful surveys of the empirical findings in this area are provided by Hodrick (1987), Lewis (1995), and Engel (1996). In this article we attempt to obtain more information about the nature of the risk premium. One approach to model the risk premium is the unobserved components (or signal extraction) methodology that was introduced in this literature by Wolff (1987) and Nijman et al. (1993). They showed how the risk premium can be interpreted as an unobserved component, and how models of this type can be identified and estimated. The available empirical evidence related to this approach is quite limited as only one relatively small sample, containing three currencies relative to the US Dollar, was studied. In addition to being limited, this sample is by now fairly dated. The primary objective of this article is to further assess the relevance of the unobserved components approach by studying a large, up-to-date dataset, covering 20 years of exchange rate data and 15 different countries. Care is taken, contrary to the earlier studies, to follow the sampling procedure of Bekaert and Hodrick (1993) to match spot and forward data, in order to avoid the introduction of measurement error.

2. The risk premium as an unobserved component

The logarithm of the forward exchange rate can be divided into an expected future spot rate component and a risk premium component:

$$F_t^{t+1} = E_t[S_{t+1}] + P_t \tag{1}$$

where F_t^{t+1} is the natural logarithm of the forward rate at time t for a contract maturing at $t+1$, $E[S_{t+1}]$ is the rational expectation, based on information available at time t, of the log of the spot exchange rate at time $t+1$, and P_t is the risk premium. Subtracting S_{t+1} from both sides of Eq. (1) and defining $v_{t+1} \equiv E[S_{t+1}] - S_{t+1}$, we obtain

$$y_t \equiv F_t^{t+1} - S_{t+1} = P_t + v_{t+1} \tag{2}$$

Eq. (2) states that y_t, the forecast error resulting from the forward rate as a predictor of the subsequent spot rate, consists of a risk premium and a white noise error term: 'signal' and 'noise'. We will attempt to model the signal as an unobserved component in order to extract it from its noisy environment. An important advantage of this methodology is that the expectation of the future spot rate need not be modeled explicitly.

Our modeling strategy focuses on the premium itself. Following Wolff (1987), Nijman et al. (1993) and Huisman et al. (1998), the signal is assumed to be generated by a model from the autoregressive integrated moving average (or ARIMA) class of models. This assumption is consistent with theoretical models that have been studied in the literature, see Nijman et al., 1993. In order to explain our modeling strategy, let us assume that P_t is generated by an ARMA(1,1) model:

$$(1 - \phi L)P_t = (1 - \omega L)a_t \tag{3}$$

where L is the lag operator, $|\phi| < 1$, $|\omega| \leqslant 1$, $\phi \neq \omega$, and a_t is a white noise sequence with mean zero and constance variance σ_a^2 which is uncorrelated with $v_{t'}$ for all t and t'. We assume that $E[a_t v_{t+i}] = 0(\forall i)$[1]. Substituting Eq. (3) into Eq. (2) we obtain.

$$(1 - \phi L)y_t = (1 - \omega L)a_t + (1 - \phi L)v_{t+1} \tag{4}$$

The right-hand side of Eq. (4) consists of two uncorrelated MA(1) sequences. The summation theorem for moving averages in Ansley et al. (1977) states that the summation of two moving average processes of orders q_1 and q_2 has a MA(q^*) representation with $q^* \leqslant \max[q_1,q_2]$. Making use of this theorem, we can conclude that y_t in equation Eq. (4) has an ARMA(1,1) representation, so that Eq. (4) can be rewritten as

$$(1 - \phi L)y_t = (1 - \theta L)\varepsilon_t \tag{5}$$

where $|\theta| \leqslant 1$ and εt has mean zero and constant variance σ_ε^2. If P_t is generated by a general ARMA(p,q) process, it can be shown that y_t follows an ARMA ($\leqslant p$, $\leqslant \max[p,q]$) process, the parameters of which are identified as long as $p \geq q + 1$, see Hotta (1989).

3. Data

In this study we employ end-of-the-month spot and forward exchange rates which cover the period January 1976 through March 1996.[2] The maturity of the forward rates is one month. All raw data are London closing mid-prices against the Pound Sterling, obtained from Datastream, for 15 countries: Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Spain, Sweden, Switzerland and United States.[3] In our empirical analysis below all exchange rates were crossed in order to report our results in US Dollar terms. The data are sampled following the procedure described in Bekaert and Hodrick (1993), using exact delivery dates of the forward exchange contracts. To find the delivery date on a forward contract made today, one first finds today's spot value date, which is two business days in the future. Delivery takes place on the calendar day in the next month that corresponds to the current spot value date, under the condition that the delivery day is a business day. If not, delivery takes place on the next business day if it falls within the same calendar month. If the latter condition is not met, delivery takes place on the first previous business day. This rule is followed except when the spot value day is the last business day of the

[1] This assumption could be relaxed in favor of allowing for (intertemporal) correlation between forecast errors and risk premiums.

[2] We thank Ronald Huisman for kindly providing the data employed in this study.

[3] Note that the data for the case of Japan are only available from June 1978.

Table 1
Least squares estimates of ARMA model (5) for y_t

Currency	ϕ[a]	θ[a]	σ_ε^2 ($\times 10^4$)	σ_a^2 ($\times 10^4$)	σ_v^2 ($\times 10^4$)	Lower bound[b] σ_a^2 ($\times 10^4$)	Upper bound[c] σ_v^2 ($\times 10^4$)
Austria	0.74***	0.59***	12.14	1.39	9.67	0.34	10.13
	(0.16)	(0.20)					
Belgium	0.80***	0.68***	12.67	0.89	10.72	0.22	11.00
	(0.15)	(0.18)					
Canada	–	−0.09	1.98	–	–	0.18	1.63
		(0.06)					
Denmark	0.77***	0.63***	11.81	1.09	9.67	0.27	10.02
	(0.15)	(0.18)					
France	0.81***	0.69***	11.65	0.72	9.99	0.18	10.21
	(0.14)	(0.18)					
Germany	0.77***	0.64***	12.49	1.03	10.46	0.25	10.79
	(0.16)	(0.20)					
Ireland	0.74***	0.60***	11.92	1.31	9.56	0.32	10.00
	(0.17)	(0.20)					
Italy	–	−0.17***	11.23	–	–	1.96	7.64
		(0.06)					
Japan	–	−0.20***	14.26	–	–	2.82	9.18
		(0.07)					
Netherlands	0.72***	0.56***	12.33	1.61	9.64	0.39	10.18
	(0.18)	(0.20)					
Norway	0.64***	0.50*	9.10	1.34	7.12	0.32	7.61
	(0.24)	(0.27)					
Spain	–	−0.13**	11.90	–	–	1.58	8.95
		(0.06)					
Sweden	–	−0.15**	10.45	–	–	1.54	7.60
		(0.06)					
Switzerland	–	−0.18***	16.15	–	–	2.92	10.84
		(0.06)					
UK	–	−0.13**	13.42	–	–	1.79	10.09
		(0.06)					

[a] Large-sample standard errors are provided in parentheses.
[b] The corresponding upper bound is σ_ε^2.
[c] The corresponding lower bound is zero.
* Denotes statistical significance at the 10% level.
** Denotes statistical significance at the 5% level.
*** Denotes statistical significance at the 1% level.

current month, in which case the forward value date is the last business day of the next month. Unless one follows these rules precisely, measurement error is introduced into the analysis.

4. The empirical results

As we aim to estimate time-series models along the line of Eq. (5) for all available currencies relative to the US Dollar, models need to be identified (in the Box-Jenkins sense) for all of these currencies. This was accomplished using the procedures described in Box and Jenkins (1976). Additional guidance was obtained from the Schwarz information criterion for laglength selection. In all cases an ARMA(1,1) or a MA(1) model for y_t was found to be parsimonious and adequate. Note that an ARMA(1,1) model for y_t is consistent with an AR(1) or ARMA(1,1) model for the risk premium, P_t, itself. Similarly a MA(1) model for y_t is consistent with a MA(1) model for P_t. Our estimated models for y_t, therefore, at the same provide information about the time-series behavior of the risk premium, P_t. Least squares estimates for the models are presented in Table 1. In all cases a very insignificant constant term was removed prior to estimation.

In the second and third columns of Table 1 the parameter estimates for the ARMA(1,1) and MA(1) models, respectively, for y_t are presented. Large-sample standard errors are provided in parentheses. For Austria, Belgium, Denmark, France, Germany, Ireland, the Netherlands and Norway we fit ARMA(1,1) models for y_t, which are consistent with either an AR(1) or an ARMA(1,1) model for the risk premium. All parameter estimates in these cases are highly statistically significant. If one is willing to restrict ω a priori to be equal to zero, i.e. to choose the AR(1) model for the risk premium — both σ_a^2 (the variance of the random disturbance term in Eq. (3) for the risk premium) and σ_v^2 (the variance of the 'noise'

Table 2
Tests of H_0: $\phi = \theta$ for estimated ARMA(1,1) models

Currency	F-statistic[a]	P-value
Austria	6.628**	0.0106
Belgium	5.091**	0.0250
Denmark	6.182**	0.0136
France	4.659**	0.0319
Germany	4.974**	0.0267
Ireland	6.447**	0.0117
Netherlands	7.007***	0.0087
Norway	5.206**	0.0234

[a] Under the null hypothesis the test statistic is distributed as F(1,239).
** Indicates statistical significance at the 5% level.
*** Indicates statistical significance at the 1% level.

term in Eq. (2) above) can be identified from ϕ, θ, and σ_ε^2 (the variance of the disturbance term in Eq. (5) for y_t). The estimates of σ_ε^2, σ_a^2, and σ_v^2 (the latter two variances under the assumption that $\omega = 0$) are reported in columns four through six. If one is not willing to restrict the risk premium to being generated by an AR(1) model, but allows it to be governed by an ARMA(1,1) model, σ_a^2 and σ_v^2 are not identified. Nijman et al. (1993), however, derived bounds on these two variances: $\sigma_\varepsilon^2[(1+\theta^2)\phi-(1+\phi^2)\theta]/(1+\phi)^2 \leqslant \sigma_a^2 \leqslant \sigma_\varepsilon^2$ and $0 \leqslant \sigma_v^2 \leqslant \sigma_\varepsilon^2(1+\theta)^2/(1+\phi)^2$. These bounds are presented in the seventh and eighth columns in the table.

For Canada, Italy, Japan, Spain, Sweden, Switzerland and the United Kingdom we fitted MA(1) models. These are consistent with MA(1) models for the underlying risk premium processes, too. The estimates for the MA(1) coefficient θ are reported in the third column. For these models the underlying parameters ω, σ_a^2, and σ_v^2 are not identified, but bounds on the variances are again available: $|\theta|\sigma_\varepsilon^2 \leqslant \sigma_a^2 \leqslant \sigma_\varepsilon^2$ and $0 \leqslant \sigma_v^2 \leqslant (1+\theta)^2\sigma_\varepsilon^2$. For the MA(1) models these bounds are reported in the seventh and eighth columns. Note that all estimated θ-coefficients are highly statistically significant, with the exception of the Canadian Dollar.

For all of the models (except in the Canadian case) there is evidence of nonnegligible time variation in the risk premium. It is interesting to note that the models' point estimates for the parameters ϕ and θ imply that y_t displays positive first-order serial correlation. For instance, the point estimate $\theta = -0.20$ for the Japanese Yen implies a first-order autocorrelation for y_t of $-\theta/(1+\theta^2) = 0.19$, see Box and Jenkins (1976). This positive serial correlation was of course also apparent at the identification stage of the modeling procedure. The case of the Canadian Dollar is quite different from the other currencies in terms of statistical significance. This is likely to be related to the fact that the variance of the forward forecast error, the dependent variable, for the Canadian Dollar is only about fifteen percent of the average variance level for the other currencies. For the ARMA(1,1) cases, if again one is willing to assume that $\omega = 0$, the positive ϕ-estimates would correspond directly to estimated AR(1) coefficients for the underlying risk premium models. These estimates would thus indicate considerable persistence in the risk premium. For the MA(1) models it can be shown that the negative θ-estimates must also imply negative ω-values, which in turn implies, again, that the risk premia show persistence (in the sense of positive serial correlation) over time.

When considering the point estimates of ϕ and θ for the ARMA(1,1) models, one notices that they are of similar orders of magnitude in all cases. This issue merits investigation because if indeed they are identical to each other, $(1\text{-}\phi L)$ will cancel $(1\text{-}\theta L)$, so that the forward forecast error would be a white noise process. Formal (large-sample) tests of H_0: $\phi = \theta$ are provided in Table 2.

Table 2 provides F-statistics and associated P-values. The results are quite uniform: in all cases equality of ϕ and θ is rejected at the ninety-five percent confidence level and sometimes (the case of the Netherlands) even at the ninety-nine percent level. Hence, we can conclude that indeed the persistence described above is present also in the ARMA(1,1) cases.

It is also interesting to study the various variance estimates in detail. First, we consider the ARMA(1,1) models. Here, σ_ε^2, the variance of the innovation in the y_t

process is a useful benchmark. If we are willing to assume that $\omega = 0$, the relatively low σ_a^2 values indicate that the variability of the risk premium is quite low relative to the variability of the forward forecast error, as well as relative to the 'noise' variance σ_v^2, which is a conclusion that differs from the earlier result in Wolff (1987). If we are not willing to assume that $\omega = 0$, the variance bounds in the last two columns are appropriate. They are quite wide and, therefore, not very informative. The variance bounds for the MA(1) models are somewhat more informative, but still do not allow for precise comparisons of variance magnitudes.

5. Conclusions

On the basis of unobserved components models we have studied the nature of the foreign exchange risk premium in this article for fifteen currencies relative to the US Dollar. For all of the currencies but one, we find significant evidence of a time-varying risk premium. The stochastic structure of the forward forecast error can be captured adequately with low-order models of the (AR)MA class. Our model estimates suggest persistence in the risk premium, in the sense of positive serial correlation, thus corroborating the results of Nijman et al. (1993) and Wolff (1987). For a number of models our results indicate (assuming an AR(1) model for the risk premium) that the variance of the risk premium is quite low relative to the variance of the forward forecast error. It is interesting to note that we have identified two different stochastic structures for our samples: an ARMA(1,1) model applies to a subset of continental European countries (including 'core EMS' countries). The model estimates are quite similar for all these cases. An MA(1) model applies to almost all of the remaining countries (Canada being the exception). Within this group the model estimates are also very similar for different currencies.

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