

An Empirical Comparison of Methods for Temporal Distribution and Interpolation at the National Accounts

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Abstract

This study evaluates five mathematical and five statistical methods for temporal disaggregation in an attempt to select the most suitable method(s) for routine compilation of sub-annual estimates through temporal distribution and interpolation in the national accounts at BEA. The evaluation is conducted using 60 series of annual data from the National Economic Accounts, and the final sub-annual estimates are evaluated according to specific criteria to ensure high quality final estimates that are in compliance with operational policy at the national accounts. The study covers the cases of temporal disaggregation when 1) both annual and sub-annual information is available; 2) only annual data are available; 3) sub-annual estimates have both temporal and contemporaneous constraints; and 4) annual data contain negative values. The estimation results show that the modified Denton proportional first difference method outperforms the other methods, though the Casey-Trager growth preservation model is a close competitor in certain cases. Lagrange polynomial interpolation procedure is inferior to all other methods.

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1. Introduction

National accounts face the task of deriving a large number of monthly or quarterly estimates using mostly annual data and some less reliable or less comprehensive quarterly or monthly information from proxy variables known as indicators. The annual data are usually detailed and of high precision but not very timely; they provide the most reliable information on the overall level and long-term movements in the series. The quarterly or monthly indicators are less detailed and of lower precision but they are timely; they provide the only explicit information about the short-term movements in the series. Statistically speaking, the process of deriving high frequency data from low frequency data and, if it is available, related high frequency information can be described as temporal disaggregation. To simplify the exposition, we follow the terminology used in the literature to refer to the low frequency data as annual data and the high frequency data as sub-annual data (Cholette and Dagum, 2006).

Typically the annual sums of the sub-annual indicator series are not consistent with the annual values, which are regarded as temporal aggregation constraints or benchmarks, and quite often the annual and sub-annual series exhibit inconsistent annual movements. The primary objective of temporal disaggregation is to obtain sub-annual estimates that preserve as much as possible the short-term movements in the indicator series under the restriction provided by the annual data which exhibit long term movements of the series. Temporal disaggregation combines the relative strength of the less frequent annual data and the more frequent sub-annual information.

Temporal disaggregation also covers the cases where sub-annual information is not available. In addition, it covers the cases where a set of sub-annual series are linked by some accounting relationship regarded as a contemporaneous aggregation constraint. In such cases, temporal disaggregation should

produce sub-annual estimates that are consistent with both temporal and contemporaneous aggregation constraints.

Temporal disaggregation is closely related to but different from benchmarking. They are related in that benchmarking or temporal disaggregation arises because of the need to remove discrepancies between annual benchmarks and corresponding sums of the sub-annual values. They are different because the benchmarking problem arises when time series data for the same target variable are measured at different frequencies with different level of accuracy, whereas temporal disaggregation deals with the problem where the sub-annual data are not for the same target variable as the annual data.

There are two facets of the temporal disaggregation problem: *temporal distribution* and *interpolation*. Temporal distribution is needed when annual data are either the sum or averages of sub-annual data. In general distribution deals with all *flow* and all *index* variables. Interpolation, on the other hand, deals with estimation of missing values of a *stock* variable, which at points in time have been systematically skipped by the observation process. In the estimation of quarterly variables, interpolation is used for all stock variables whose annual values equal those of the fourth (or the first) quarter of the same year. An example is population at the end of a year which equals the population at the end of the fourth quarter of the same year. Sometimes sub-annual estimates need to be made before the relevant annual and/or quarterly information is available. In such cases temporal distribution and interpolation are combined with extrapolation to produce sub-annual estimates.

There is a wide variety of methods for temporal distribution, interpolation and extrapolation. Because benchmarking and temporal disaggregation deal with similar statistical problems, the methods used for temporal disaggregation are very similar to those used for benchmarking. To a large extent, the choice of methods depends on two factors:

basic information available for estimation, and preference or operational criteria suitable for either a mathematical approach. There are three general cases regarding the availability of information: 1) information is available on both an annual and sub-annual basis; 2) information is only available on an annual basis; and 3) annual and/or sub-annual information is not yet available at the time of estimation.

If both annual and sub-annual information is available, the choices are pure numerical procedures, mathematical benchmarking methods, or statistical methods. Examples of pure numerical procedures are linear interpolation or distribution procedures, and Lagrange interpolation procedure. The most widely used mathematical benchmarking methods are the Denton adjustment method (Denton, 1971) and its variants (Helfand et al, 1977; Cholette, 1979) and the Causey-Trager method (Causey and Trager, 1981; Bozik and Otto, 1988). The most commonly used statistical methods are the Chow-Lin regression methods (1971) and their extensions (Bournay and Laroque, 1979; Fernandez, 1981; Litterman, 1983); time-series ARIMA models (Hillmer and Trabelsi, 1987); and generalized regression-based methods (Cholette and Dagum, 2006). More recently, time series models such as benchmarking based on signal extraction (Durbin and Quenneville, 1997; Chen, Cholette and Dagum, 1997) and the structural time series models in state space representation for interpolation and temporal distribution (Harvey and Pierse, 1984; Gudmundson, 1999; Proietti, 1999; and Aadland, 2000) have also been developed.

If, on the other hand, only annual data are available, choices are mathematical, numerical smoothing methods, and time series ARIMA models. Commonly used smoothing methods are those developed by Boot, Feibes, and Lisman (1970), and by Denton (1971). A commonly used numerical smoothing procedure is the cubic spline. The ARIMA models developed to generate sub-annual estimates in the absence of sub-annual information are those by Stram and Wei (1986) and Wei and Stram (1990).

When annual and/or sub-annual information for the current year is not yet available at the time of estimation, extrapolation is necessary. If only sub-annual indicator values are available, they can be used to extrapolate the annual aggregate, which can then be used to distribute or interpolate the sub-annual estimates. The idea is that the indicator and the annual series have the same time profile and, thus, they have the same growth rate. However, if no information on the indicator is yet available, the sub-annual estimates for the current periods can only be derived from the extrapolation of previous sub-annual estimates or from interpolation of the extrapolated annual data. If the available information is not sufficiently reliable and/or not complete, it is then necessary to consider some classical extrapolation methods based on previously available sub-annual information or based on the methods that use related series.

Like most government statistical agencies, deriving sub-annual estimates using only annual data and incomplete sub-annual information is a routine practice in the national accounts at BEA. The national accounts at BEA have experimented with a variety of techniques over the years. Up to the 1970s, the Bassie adjustment procedure (Bassie, 1958) was the major method used at BEA. Bassie was the first to develop a method for constructing sub-annual series whose short-term movements would closely reflect those of a related (indicator) series while maintaining consistency with annual aggregates. The Bassie procedure tends to smooth the series, and hence, it can seriously disturb the period-to-period rates of change in sub-annual series that exhibit strong short-term variation. Because the Bassie method operates on only two consecutive years of data, using the Bassie method often results in a step problem if data of several years are adjusted simultaneously. Moreover, the Bassie method does not support extrapolation.

Because of the unsatisfactory results from the Bassie procedure, the Minimum Constrained Variance Internal Moving Average (MCVIM) procedure was introduced to the national accounts

during the 1980s, and both Bassie and MCVIM were used for interpolation and temporal distribution. MCVIM method is based on the idea of deriving sub-annual estimates by minimizing the variance in the period-to-period changes in the estimated sub-annual series. However, in the 1990s, the Bassie and MCVIM procedures were largely replaced by a purely numerical procedure known as the Lagrange polynomial interpolation procedure when BEA transferred to the AREMOS time-series database.

Polynomial interpolation is a method that takes a collection of n points of annual data and constructs a polynomial function of degree $n-1$ that passes these annual values. Using the Lagrange polynomial interpolation procedure assumes that the polynomial function that passes the annual values is a Lagrange polynomial; the n points of annual values are called Lagrange data. In general, Lagrange polynomial interpolation can be considered if only the approximation of level is needed. Lagrange polynomial interpolation procedure has some serious known drawbacks. First of all, Lagrange polynomial interpolation is a global method, and hence, it requires some information about the function globally. In practice, the approximated values based on a certain assumed degree of polynomial could sharply disagree with the actual values of the function due to lack of information about the function globally. Secondly, the approximation error may be unbounded as the degree of polynomial increases. Thirdly, computation of the Lagrange is costly, because it requires a large number of evaluations. The Lagrange polynomial interpolation formula uses $3n$ additions, $2n$ divisions and $2n$ multiplications even when done most efficiently.

Lagrange polynomial interpolation has not proven satisfactory in the national accounts. The major problems encountered are inconsistency in the short-term movements shown in the indicator and final estimated sub-annual series, the sharp increase or decrease at the end of the interpolated series, and the jumpy links between the newly interpolated span of a series and the previously benchmarked span of the series. An added

difficulty arises in practice because Lagrange polynomial interpolation in AREMOS uses five annual data points, but under the current revision policy at the national accounts, only data of the most recent three years can be used in estimation. Thus, it requires a forward extrapolation of two annual data points before Lagrange polynomial interpolation can be performed. Although there is a recommended extrapolation procedure, analysts often have to make judgmental decisions on the extrapolation procedure to be used when encountering various data problems. Sub-annual estimates generated from the Lagrange polynomial interpolation procedure in AREMOS are directly affected by the choices of extrapolation procedures.

Because of the difficulties experienced using Lagrange polynomial interpolation, alternative methods such as linear interpolation, Bassie and MCVIM methods continue to be used in various parts of the national accounts. To improve the quality of sub-annual estimates in the national accounts, there is strong interest in finding an appropriate standardized method for temporal disaggregation at the national accounts. Various attempts have been made in recent years by researchers at BEA in a search for a better method for temporal disaggregation (Klick, 2000; Loebach 2002).

The objective of this study is to evaluate a number of existing mathematical and statistical methods for temporal disaggregation based on certain specific criteria and to recommend the most suitable method(s) for practice in the national accounts. The evaluation is conducted using 60 annual data series from the National Economic Accounts. The study will cover the cases where 1) both annual and indicator series are available; 2) only annual data are available; 3) annual series contain both positive and negative values; and 4) annual series have both temporal and contemporaneous constraints. To evaluate different methods for temporal disaggregation, three software packages were used: ECOTRIM from Eurostats, BENCH from the Statistics of Canada, and BMARK from the U.S. Census Bureau.

Each software package offers some unique features and has advantages over the others in certain aspects of computation.

The plan for the rest of the paper is as follows. Section 2 presents various mathematical and statistical methods for temporal disaggregation. Section 3 discusses the five criteria for evaluation. Section 4 presents and discusses the estimation results. Section 5 evaluates the methods and software package used in the experiment and suggests method(s) and software for temporal disaggregation at the national accounts. Section 6 concludes the paper.

2. Methods for Temporal Disaggregation

There is a variety of mathematical and statistical methods developed for temporal disaggregation. The distinction between a mathematical and a statistical method is that a mathematical model treats the process of an unknown sub-annual series as deterministic and treats the annual constraints as binding, whereas a statistical model treats the process of an unknown sub-annual series as stochastic and allows the annual constraints to be either binding or not binding. The choice of a particular method depends to a large extent on the information available for estimation and subjective preference or operational criteria.

Because the objective of this study is to search for the most suitable method(s) for interpolation and temporal distribution for routine compilation of sub-annual estimates in the national accounts, we focus on three mathematical methods: the Denton adjustment method (1971) and its variants; the Causey-Trager growth preservation method (Causey and Trager, 1981); and the smoothing method (Boot, Feibes and Lisman, 1970). We shall also review five extensions of the Chow-Lin regression method (Chow and Lin, 1971), which include an AR(1) model estimated by applying maximum likelihood (ML) and generalized least squares (GLS), the Fernandez random walk model (1981), and the Litterman random walk-Markov model (1998) estimated by applying ML and GLS.

In this section we shall briefly describe the mathematical and statistical methods being reviewed in this study and discuss the advantages and disadvantages of these methods.

2.1 Mathematical Methods for Temporal Disaggregation

We shall start with the original Denton adjustment method (Denton, 1971) and its variants, and then describe the Causey-Trager method (Causey and Trager, 1981). These methods are suitable for interpolation or distribution when both annual and sub-annual indicator values are available. We shall then discuss the Boot, Feibes and Lisman smoothing method for distribution and interpolation when only annual values are available.

To formalize our discussion, we first define some notation. Let z_t and x_t denote the sub-annual indicator series and sub-annual estimates from distribution or interpolation for sub-annual period $t = 1, 2 \dots, T$, where T is the total number of sub-annual periods in the sample. Let y_m denote the annual value of year m for $m = 1, 2 \dots, M$, where M is the total number of annual values in the sample. Let k_{mt} denote the coverage fraction of sub-annual estimate of period t , e.g. for quarterly or monthly series of index variables, $k_{mt} = 1/4$ or $k_{mt} = 1/12$; for flow variables, $k_{mt} = 1$. Also let t_{1m} and t_{2m} denote the first and last periods covered by the m -th annual value, e.g. if $m = 2$, for a quarterly series, $t = t_{1m} = 5$ and $t = t_{2m} = 8$ respectively. Let Δ denote backward first difference operator, e.g., $\Delta x_t = x_t - x_{t-1}$, $\Delta z_t = z_t - z_{t-1}$, $\Delta x_t - \Delta z_t = \Delta(x_t - z_t) = (x_t - z_t) - (x_{t-1} - z_{t-1})$. Let Δ^2 denote the backward second difference operator, e.g., $\Delta^2 x_t - \Delta^2 z_t = \Delta^2(x_t - z_t) = \Delta(x_t - z_t) - \Delta(x_{t-1} - z_{t-1})$.

2.1.1 The original Denton adjustment method and its variants

The original Denton adjustment method is based on the principle of movement preservation. According to this principle,

the sub-annual series x_t should preserve the movement in the indicator series, because the movement in the indicator series is the only information available. Formally, we specify a penalty function, $P(x, z)$, where $x = (x_1 \dots, x_T)'$ and $z = (z_1 \dots, z_T)'$ are $T \times 1$ column vectors of final sub-annual and indicator values. The mathematical problem of the original Denton adjustment model is to choose final sub-annual estimates, x , so as to minimize the penalty function $P(x, z)$, subject to the temporal aggregation constraints,

$$Y_m = \sum_{t=t_{1m}}^{t_{2m}} k_{mt} x_t ,$$

for $m = 1, 2 \dots, M$, in the cases of distribution of index or flow variables. In the cases of interpolation of stock variables, the constraint becomes

$$Y_m = x_{t_{1m}} , \quad \text{or} \quad Y_m = x_{t_{2m}} ,$$

for $m = 1, 2 \dots, M$, with either the first or the last sub-annual estimate equals the annual value for year m .

Let $y = (y_1 \dots, y_M)'$ be a column vector of M annual values; let B be a $T \times M$ matrix that maps sub-annual estimates to annual constraints; and let A be a $T \times T$ weighting matrix. Then, the original Denton model can be specified in the matrix form as

$$(1) \quad \text{Min}_x P(x, z) = (x - z)' A(x - z),$$

Subject to

$$(2) \quad y = B'x.$$

The solution to the final sub-annual estimate is

$$(3) \quad x = z + C(y - B'z),$$

where $y - B'z$ measures the annual discrepancy and C represents the distribution rule determining how the annual discrepancy is distributed to each sub-annual period. If A is $I_{T \times T}$, then C will be the inverse of the number of sub-annual periods in a year. The annual discrepancy would then be distributed evenly to each sub-annual period. Apparently, that is not a good choice of distribution rule.

Denton proposed several variants of the original movement preservation model based on the first or higher order difference of the final sub-annual estimates and the indicator series. The most widely used are the additive and proportional first and second difference variants.

1) *The additive first difference variant* preserves the sample period-to-period change in the level of the final sub-annual estimates and the indicator values, $(x_t - z_t)$, under the annual constraint. As a result, x_t tends to be parallel to z_t . The objective in this case is achieved by minimizing the sum of squares of $\Delta(x_t - z_t)$, so the penalty function is $P(x, z) =$

$$\sum_{t=1}^T [\Delta(x_t - z_t)]^2.$$

2) *The proportional first difference variant* preserves the proportional period-to-period change in the final sub-annual estimates and the indicator series, x_t/z_t . As a result, x_t tends to have the same period-to-period growth rate as z_t . The objective here is achieved by minimizing the sum of squares of $\Delta(x_t/z_t) = (x_t/z_t) - (x_{t-1}/z_{t-1})$, and the penalty function in this

$$\text{case is } P(x, z) = \sum_{t=1}^T \left(\frac{x_t}{z_t} - \frac{x_{t-1}}{z_{t-1}} \right)^2.$$

3) *The additive second difference variant* preserves the sample period-to-period change in $\Delta(x_t - z_t)$ as linear as possible. In this case, the objective is to minimize the sum of squares of $\Delta^2(x_t - z_t)$, and the penalty function is $P(x, z) = \sum_{t=1}^T [\Delta^2(x_t - z_t)]^2$.

4) *The proportional second difference variant* preserves the sample period-to-period change in $\Delta(x_t/z_t)$ as linear as possible. The objective in this case is to minimize the sum of squares of $\Delta^2(\frac{x_t}{z_t})$, and the penalty function is $P(x, z) = \sum_{t=1}^T [\Delta(\frac{x_t}{z_t} - \frac{x_{t-1}}{z_{t-1}})]^2$.

Denton imposed the initial conditions that no adjustments are to be made in the indicator values outside of the sample. Thus, in the first difference variants, the initial condition boils down to $x_0 = z_0$, and in the second difference variants, the initial conditions boil down to $x_0 = z_0$ and $x_{-1} = z_{-1}$. Such initial conditions result in a major short-coming of the original Denton method, because it induces a transient movement at the beginning of the series. It forces the final sub-annual estimates to equal the original series at time zero and results in the minimization of the first change ($x_1 - z_1$). Such transient movement defeats the principle of movement preservation. Helfand et al. (1977) solved this problem by modifying the penalty

function to $P(x, z) = \sum_{t=2}^T [\Delta(x_t - z_t)]^2$ for the additive first

difference variant, and $P(x, z) = \sum_{t=2}^T (\Delta \frac{x_t}{z_t})^2$ for the proportional

first difference variant. The modified first difference variants precisely omit this first term to solve this short-coming of the original Denton method. The penalty functions for the second difference variants are modified accordingly.

2.1.2 The Causey-Trager growth preservation model

The growth preservation model is first developed by Causey and Trager (1981) and later revised by Trager (1982). They propose that instead of preserving the proportional period-to-period change in final sub-annual estimates and indicator series x_t/z_t , the objective should be to preserve the period-to-period percentage change in the indicator series. As a result, the period-to-period percentage change in x_t tends to be very close to that in z_t . This objective is achieved by minimizing the sum of squares of $(x_t/x_{t-1} - z_t/z_{t-1})$. Formally, the mathematical problem of the Causey-Trager model is

$$(4) \quad \text{Min}_x P(x, z) = \sum_{t=2}^T \left[\frac{x_t}{x_{t-1}} - \frac{z_t}{z_{t-1}} \right]^2,$$

subject to (2), the same temporal constraints as those specified in the original Denton model.

The Causey-Trager model is non-linear in the final sub-annual estimates x_t , and thus, it does not have a closed form solution. The Causey-Trager model is solved iteratively using the solution to the Denton proportional first difference model as the starting values. Causey (1981) provided a numerical algorithm using steepest descent to obtain the final sub-annual estimates x_t , $t = 1, 2 \dots, T$. This numerical algorithm was later revised by Trager (1982). Because the Causey-Trager model is non-linear and is solved iteratively, the computational cost is higher compared to the Denton proportional first difference model, which has a closed form solution.

2.1.3. The Boot, Feibes and Lisman smoothing method (BFL)

Smoothing methods are suitable for univariate temporal disaggregation when only annual data are available. The basic

assumption of smoothing methods is that the unknown sub-annual trend can be conveniently described by a mathematical function of time. Because no sub-annual information is available, the sub-annual path is given a priori or chosen within a larger class, such as that the necessary condition of satisfying temporal aggregation constraints and the desirable condition of smoothness are both met. One such smoothing method is the BFL method. There are two versions of the BFL method, the first difference model and the second difference model.

The objective of the first difference model is to minimize period-to-period change in the level of final sub-annual estimates Δx_t subject to the annual constraints. Formally, the problem is to

$$(5) \quad \text{Min}_x P(x) = \sum_{t=2}^T (x_t - x_{t-1})^2 ,$$

subject to (2). The constraints are the same as those specified in the original Denton model.

The objective of the second difference model is to keep the period-to-period change in Δx_t as linear as possible. This objective is achieved by minimizing the sum of squares of $\Delta^2 x_t = (\Delta x_t - \Delta x_{t-1})$ subject to annual constraints. Formally, the second difference model is

$$(6) \quad \text{Min}_x P(x) = \sum_{t=2}^T [\Delta(x_t - x_{t-1})]^2 ,$$

subject to (2).

Like other smoothing methods, the BFL method does not allow extrapolation of sub-annual estimates, because it is designed only to give a sub-annual breakdown of the available annual data. Thus, to produce the current year estimates, a forecast value of

the current year annual value is needed. All sub-annual periods of the current year are then estimated at the same time.

2.2 Regression Methods

There have been quite a number of statistical methods developed for temporal disaggregation. These methods are designed to improve upon the mathematical methods discussed above, which do not take into consideration certain behaviors of economic time series data. Some examples of the statistical benchmarking methods are optimal regression models (Chow and Lin, 1971; Bournay and Laroque; 1979; Fernandez, 1983; Litterman, 1983), dynamic regression models (De Fonzo, 2002), unobserved component models or structural time series models using Kalman filter to solve for optimal estimates of the missing sub-annual observations (Gudmundson, 1999; Hotta and Vasconcellos, 1999; Proietti, 1999; Gomez, 2000), and Cholette-Dagum regression-based benchmarking methods (Cholette and Dagum, 1994, 2006).

For our purpose, we will limit our discussion on the more widely used Chow-Lin regression model and its variants, which are used in our experiment. We shall also briefly discuss the Cholette-Dagum regression-based method, because, in some respect, it can be considered a generalization of the Denton benchmarking approach. The structural time series models formulated in state space representation for interpolation and temporal distribution are not yet fully corroborated by empirical applications, and, therefore, will not be discussed here.

2.2.1 The Chow-Lin regression method

Chow-Lin (1971) developed a static multivariate regression based method for temporal disaggregation. They argue that sub-annual series to be estimated could be related to multiple series, and the relationship between the sub-annual series and the observed related sub-annual series is

$$(7) \quad x_t = z_t\beta + u_t,$$

where x is a $T \times 1$ vector, z is a $T \times p$ matrix of p related series, β is $p \times 1$ vector of coefficients, u_t is a $T \times 1$ vector of random variables with mean zero and $T \times T$ covariance matrix V . They further assume that there is no serial correlation in the residuals of the sub-annual estimates.

In matrix form, the relationship between annual and sub-annual series can be expressed as

$$(8) \quad y = B'x = B'z\beta + B'u.$$

Chow-Lin derives the solutions by means of the minimum variance linear unbiased estimator approach. The estimated coefficient $\hat{\beta}$ is the GLS estimator with y being the dependent variable and annual sums of the related sub-annual series as the independent variables. The estimated coefficients are

$$(9) \quad \hat{\beta} = [z'B(B'VB)^{-1}B'z]^{-1}z'B(B'VB)^{-1}y,$$

and the linear unbiased estimator of x is

$$(10) \quad \hat{x} = z\hat{\beta} + VB(B'VB)^{-1}[y - B'z\hat{\beta}].$$

The first term in (10) applies $\hat{\beta}$ to the observed related sub-annual series of the explanatory variables. The second term is an estimate of the $T \times 1$ vector u of residuals obtained by distributing the annual residuals $y - B'z\hat{\beta}$ with the $T \times M$ matrix $VB(B'VB)^{-1}$. This implies that if the sub-annual residuals are serially uncorrelated, each with variance σ^2 , then $V = \sigma^2 I_{T \times T}$, and

then annual discrepancies are distributed in exactly the same fashion as Denton's basic model with $A = I_{T \times T}$.

The assumption of no serial correlation in the residuals of sub-annual estimates is generally not supported by empirical evidence. Chow-Lin proposes a method to estimate the covariance matrix V under the assumption that the errors follow a first-order autoregressive AR(1) process. There are two static variants of the Chow-Lin approach intended to correct the serial correlation in the sub-annual estimates. One is the random walk model developed by Fernandez (1981), and the other is the random walk-Markov model developed by Litterman (1983).

2.2.2 The Random walk model

Fernandez argues that economic time series data are often composed of a trend and a cyclical component, and he proposes to transform the series to eliminate the trend before estimation. He sets up the relationship between the annual and the sub-annual series as that in equation (8) and derives $\hat{\beta}$ and the linear unbiased estimator of x as those in equations (9) and (10). However, Fernandez argues that the sub-annual residuals follow the process

$$(11) \quad u_t = u_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim N(0, V_{T \times T})$, is a vector of random variables with mean zero and covariance matrix $V_{T \times T}$. Based on this specification, the relationship between x and z is

$$(12) \quad x_t - x_{t-1} = z_t \beta - z_{t-1} \beta + u_t - u_{t-1},$$

which can be expressed as

$$(13) \quad Dx_t = Dz_t \beta + Du_t,$$

where D is the first difference operator.

The relationship between the annual and sub-annual series also needs to be transformed accordingly, because the sum of Dx_t is not equal to y . Fernandez shows such relationship in the matrix form as

$$(14) \quad \Delta y = QDx = QDz\beta + QDu,$$

where Δ is similar to D but the dimension of Δy is $M \times M$, and Q is $M \times T$ matrix. This specification holds if the final sub-annual estimates x in year 0 are constant, an assumption considered reasonable for large sample size.

Given the sub-annual residual process specified by Fernandez and setting $QD = B'$, the solutions are

$$(15) \quad \hat{x} = z\hat{\beta} + (D'D)^{-1}B(B'(D'D)^{-1}B)^{-1}[y - B'z\hat{\beta}],$$

$$(16) \quad \hat{\beta} = [z'B(B'(D'D)^{-1}B)^{-1}B'z]^{-1}z'B(B'(D'D)^{-1}B)^{-1}y.$$

If $A = D'D$, these solutions are identical to those derived from the first difference regression model. Fernandez concludes that 1) before estimating the sub-annual series through interpolation or distribution, the behavior of the series should be studied. If the series is non-stationary and serially correlated, then the first difference data should be used to transform the data in order to obtain stationary and uncorrelated series; 2) if the first difference is not enough, other transformation is needed to convert residuals to serially uncorrelated and stationary variables; and 3) given proper transformation, the degree of serial correlation can be tested by generalized least square estimation.

2.2.3 The Random walk-Markov model

Litterman (1983) argues that the relationship between short-run movements in x and $z\beta$ is fairly stable in most cases, but the levels of x and $z\beta$ may vary over time. He points out that Chow-Lin's specification of the covariance matrix, $V = I_{n \times n} \sigma^2$, is not adequate if the sub-annual residuals exhibit serial correlation, because this specification would lead to step discontinuity of the sub-annual estimates between the annual periods as it allocates each annual residual among all sub-annual estimates. He also argues that Chow-Lin's treatment of serial correlation is only adequate if the error process is stationary.

Litterman argues that Fernandez' random walk assumption for the sub-annual residual term could remove all serial correlation in the annual residuals when the model is correct. However, in some cases, Fernandez' specification does not remove all serial correlation. Litterman proposes the following generalization of the Fernandez method,

$$(17) \quad x_t = z_t \beta + u_t,$$

$$(18) \quad u_t = u_{t-1} + \varepsilon_t,$$

$$(19) \quad \varepsilon_t = \alpha \varepsilon_{t-1} + e_t,$$

where $e_t \sim N(0, V)$, is a vector of random variables with mean zero and covariance matrix V , and the implicit initial condition is that $u_0 = 0$. In fact, given the specification of his model, Litterman's model is considered an ARIMA (1, 1, 0) model.

Under this assumption of the sub-annual residual process, Litterman transforms the annual residual vector into $E = HDu$ and derived the covariance matrix V as

$$(20) \quad V = (D'H'HD)^{-1}\sigma^2,$$

where H is an $T \times T$ matrix with 1 in the diagonal elements and $-\alpha$ in the entries below the diagonal elements. The solutions of x and β are respectively

$$(21) \quad \hat{x} = z\hat{\beta} + (D'H'HD)^{-1}B'[B(D'H'HD)^{-1}B']^{-1}[Y - B'z\hat{\beta}],$$

$$(22) \quad \hat{\beta} = [z'B(B'(D'H'HD)^{-1}B')^{-1}B'z]^{-1}z'B(B'(D'H'HD)^{-1}B')^{-1}Y.$$

Litterman suggests two steps to estimate β and derives the linear unbiased estimator of x . The first step is to follow the estimator derived by Fernandez and to generate the annual residuals, $\hat{U} = B\hat{U}$. The second step is to estimate α by forming the first-order autocorrelation coefficient of the first difference of the annual residuals and solving for α . Therefore, Litterman's method also uses first difference data rather than the level data.

2.2.4 AR(1) model

Apart from the random walk models, there are also attempts to model the errors as an AR(1) process. Bournay and Laroque (1979) propose that the sub-annual errors follow an AR(1) process

$$(11) \quad u_t = \rho u_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim N(0, \sigma^2)$ is white noise, and $|\rho| < 1$. The value of the coefficient ρ represents the strength of movement preservation of the distribution or interpolation model. Bournay and Laroque suggested that $\rho = .999$, which represents very strong movement preservation. The AR(1) model can be estimated by applying ML or

GLS. Cholette and Dagum (2006) suggest that ρ be set to .9 for monthly series and to .9³ for quarterly series.

Cholette and Dagum developed Regression-based benchmarking method (1994), which consists of two basic models, the additive model and the multiplicative model.

The Cholette-Dagum additive benchmarking model is formulated as follows,

$$(12) \quad z_t = \sum_{h=1}^H r_{th} \beta_h + x_t + e_t, \quad t = 1, \dots, T,$$

subject to

$$(13) \quad Y_m = \sum_{t=t_{1m}}^{t_{2m}} j_{mt} x_t + \varepsilon_t, \quad m = 1, \dots, M,$$

where $E(e_t) = 0$, $E(e_t e_{t-1}) = \sigma_t^\lambda \sigma_{t-1}^\lambda \omega_t$, λ measures degree of heteroscedasticity, $E(\varepsilon_m) = 0$ and $E(\varepsilon_m e_t) = 0$.

The first term in (12) specifies deterministic time effects. If $H = 1$ and $r_{th} = -1$, this term captures the average level discrepancy between the annual and the sub-annual data. In some cases, a second regressor is used to capture a deterministic trend in the discrepancy. In some cases, r_{th} may be absent, which implies $H = 0$. The Cholette-Dagum model allows the annual constraint to be not binding. This is the case if ε_t is nonzero. The above additive model can be modified into proportional model. One can see from (12) and (13) that by setting the parameters to certain default values, the Cholette-Dagum additive model can be modified to approximate the Denton additive and proportional models.

The Cholette-Dagum multiplicative benchmarking model is formulated as follows,

$$(14) \quad \ln z_t = \sum_{h=1}^H r_{th} \ln \beta_h + \ln x_t + \ln e_t, \quad t = 1, \dots, T,$$

subject to (13), where $E(\varepsilon_m) = 0$ and $E(\varepsilon_m \varepsilon_m) = \sigma_{\varepsilon_m}^2$.

The multiplicative model requires that both annual and sub-annual indicator observations to be positive in order to avoid negative final estimates of sub-annual values. Typically, the deterministic regressor is a constant, i.e. $H = 1$ and $r_{Ht} = -1$, which captures the average proportional level difference between the annual and the sub-annual indicator data. If this is the case, the first term in (13) is a weighted average of the proportional annual discrepancies. Similar as in the additive model, in some cases, a second regressor is used to capture a deterministic trend in the proportional discrepancy, or the regressor may be absent, in which case $H = 0$. By setting the parameters in the multiplicative model to certain default values, the multiplicative model can be modified to approximate the Causey-Trager growth preservation model.

3. Test Criteria and Estimation Strategy

Our objective is to select a method or methods most suitable for routine temporal distribution and interpolation in the national accounts. The most suitable method(s) should generate final sub-annual estimates that best satisfy certain pre-specified criteria and should be easy to implement given the operational criteria set by the national accounts.

3.1 Test Criteria

Five basic criteria should be used to evaluate the final sub-annual estimates generated using different methods.

1) Temporal aggregation constraint must be satisfied. This means that for each annual period, the sub-annual estimates must aggregate or average to the annual benchmarks. The temporal discrepancy can be measured with respect to the indicator series or to the estimated final sub-annual series. Temporal

discrepancies with respect to indicator series show how much the indicator series need to be adjusted so that the temporal constraints can be satisfied, and could be large because the indicator and the annual benchmark in general do not directly measure the same target variable. Temporal discrepancies with respect to the final sub-annual estimates measure how well the temporal aggregation constraints are satisfied, and should be null if the annual benchmarks are binding.

Temporal discrepancies can be measured algebraically (in level) or proportionally. For year $m = 1, 2 \dots, M$, algebraic temporal discrepancy with respect to final sub-annual estimates is computed as

$$(3.1) \quad D_x^A = y_m - \sum_{t=t_{1m}}^{t_{2m}} k_{mt} x_t, \quad \text{for indicator and flow variables,}$$

or

$$D_x^A = y_m - x_{t_{1m}} \text{ (or } y_m - x_{t_{2m}}), \text{ for stock variables.}$$

Correspondingly, proportional temporal discrepancy with respect to final sub-annual estimates is computed as

$$(3.2) \quad D_x^P = y_m / \sum_{t=t_{1m}}^{t_{2m}} k_{mt} x_t, \text{ for indicator and flow variables,}$$

or

$$D_x^P = y_m / x_{t_{1m}} \text{ (or } y_m / x_{t_{2m}}), \text{ for stock variables.}$$

Algebraic and proportional discrepancies with respect to the indicator series D_z^A and D_z^P can be written out simply by replacing x_t with z_t in (3.1) and (3.2).

Two statistics of temporal discrepancies are empirically useful. The means of discrepancies measure the level or proportional difference between the annual benchmarks and the indicator series, or between the annual benchmarks and the estimated sub-annual series, of all annual periods in the sample.

The standard deviation of discrepancies measures the dispersion of discrepancies of all sample periods. A large standard deviation may imply erratic discrepancies over sample periods, suggesting a contradiction between the annual and indicator variables, and it may also imply that in the process of satisfying the annual benchmarks, temporal distribution or interpolation distorts the movements of the indicator series. Erratic discrepancies may suggest low reliability of the indicator series.

2) Short-term movements in the indicator series should be preserved as much as possible. Short-term movement preservation can be measured in terms of level, proportion, and growth rates. Different methods are designed to achieve different objectives of short-term movement preservation. For example, the Denton additive first difference method is designed to preserve period-to-period movements in the indicator series. Thus, the objective is to minimize period-to-period change between the sub-annual and indicator series. The resulting sub-annual estimates tend to be parallel to the indicator series. The Denton proportional first difference method is designed to preserve proportional period-to-period movements in the final sub-annual estimates and the indicator series. Therefore, the objective is to minimize period-to-period change in the ratio of the final sub-annual estimates to the indicator series. Final sub-annual series estimated using this method tends to have the same period-to-period percentage changes as the indicator series. The Causey-Trager method is designed to preserve period-to-period growth rate in the indicator series. The resulting sub-annual estimates and the indicator series tend to have the same growth rates.

Two statistics can be used to measure short-term movement preservation: 1) *the average absolute change in period-to-period differences* between the final sub-annual estimates and the indicator series of all sub-annual periods in the sample, c^L ; and 2) *the average absolute change in period-to-period growth rates*

between the final sub-annual estimates and the indicator series of all periods in the sample, c^P . These two statistics are computed as follows:

$$(3.3) \quad c^L = \frac{\sum_{t=2}^T |(x_t - x_{t-1}) - (z_t - z_{t-1})|}{(T - 1)},$$

$$(3.4) \quad c^P = \frac{\sum_{t=2}^T |[(x_t / x_{t-1}) / (z_t / z_{t-1})] - 1.0|}{(T - 1)}.$$

The first statistic c^L measures changes in the period-to-period differences, and thus, it is more relevant to additive benchmarking. The second statistic c^P measures changes in the period-to-period growth rates, and thus, it is more relevant to proportional or growth rate preserving benchmarking.

3) Final sub-annual estimates should not exhibit drastic distortions at the breaks between years. By distortion we mean that the movements in the sub-annual estimates are inconsistent with the movements in the indicator series, unless such inconsistent movements are observed in the annual values. Some benchmarking methods may generate large percentage changes in the sub-annual periods at the breaks between years. In a comparative study of benchmarking methods Hood (2002) shows that the average *absolute percentage change* between the estimated sub-annual series and the indicator is larger during the months from November to February than that during the months from March to October. For some benchmarking methods, the distortion can be quite large.

To evaluate estimates obtained using different methods, for monthly series we compute the *average absolute change in period-to-period growth rate* from November to February, and from March to October, of all years in the sample. We denote the first grouped average as C_B and the second grouped average as C_M , where

B stands for periods at breaks and M stands for periods in the middle of a year. For quarterly series we compute C_B as the average absolute period-to-period change in growth rate from the fourth quarter to the following first quarter, and C_M as the average from the second to the third quarter, of all sample years. We compare the two grouped averages computed using final sub-annual series estimated by each method. A good benchmarking method should generate the least distortions at the breaks between years when compared with other methods.

4. Final sub-annual estimates should not exhibit step discontinuity or drastic distortions at the beginning and ending periods of the sample. To evaluate final estimates from different methods, we compare the absolute change in period-to-period growth rate between the final estimates and the indicator values for the second and the last periods of the sample. Note that the second period is when the first period-to-period growth can be computed.

A related issue is how smoothly the final estimates interpolated using revised annual and indicator data link to the previously benchmarked series. The national accounts revise the annual and the subannual values of the previous three calendar years during annual revision in each July. Under the current revision policy only the final sub-annual estimates of the most recent three calendar years are updated during annual revision. Final estimates for the periods prior to the three years being revised are considered previously benchmarked and not to be revised.

There are two alternative ways to comply with the current revision policy. The first alternative is to incorporate linking as an initial condition in the optimization problem. That is to require the optimal solution to satisfy the condition that the newly revised final estimates be linked to the unrevised estimate of the last sub-annual period prior to the three years being revised. The second alternative is to simply replace the

previous three years of annual and indicator data with the revised data in a sample of many years, and re-interpolate the whole sample. Then use the newly interpolated final estimates of the most recent three years to replace the estimates obtained prior to the annual revision. The rationale for the second alternative is that re-interpolating the whole sample rather than just the sample of the previous three years may lead to more gradual transition from the span of the sample that is previously benchmarked to the span of the newly revised sample. To find out which alternative provides smoother linking, we compute the absolute percentage change in the linking period using the final estimates generated using each method and denote it as C_L .

5. Contemporaneous constraint, if present, should be satisfied.

Some series have both temporal and contemporaneous constraints.

Two examples are the 16 quarterly series of government taxes on production and imports and the 15 quarterly series of industry transfer payments to government. Each tax and transfer payment series has temporal constraints to be satisfied. For each quarter, the quarterly total of the taxes serves as contemporaneous constraint for the 16 series on taxes, and the quarterly total of the transfer payments serve as contemporaneous constraint for the 15 series on transfer payments.

Satisfying contemporaneous constraint is a reconciliation issue rather than benchmarking issue. Ideally, the software for benchmarking should also be able to handle reconciliation. Unfortunately, most benchmarking programs are designed for benchmarking only and do not provide the option for reconciliation. To evaluate different methods, we compute the *contemporaneous discrepancy* as the percentage difference between the sum of the final sub-annual estimates and the contemporaneous aggregate for each sub-annual period, and compare the contemporaneous discrepancies of the final sub-annual estimates interpolated using different methods.

3.2 Methods for Evaluation and Software Used for Estimation

The methods selected for evaluation are five mathematical methods and five regression methods discussed in Section 2. The five mathematical methods are: the modified Denton additive first difference and proportional first difference methods; the Causey-Trager growth preservation method, and the first and second difference Boot-Feibes-Lisman smoothing methods. The five regression methods are: an AR(1) model by Bournay and Laroque (1975) estimated by applying ML and GLS; the random walk model by Fernandez (1981); and the Random walk-Markov model by Litterman (1983) estimated by applying ML and GLS.

We use three software programs for estimation: 1) a FORTRAN program BMARK developed by the Statistical Methodology Research Division at the Census Bureau; 2) a FORTRAN program BENCH developed by the Statistical Research Division at Statistics Canada; and 3) ECOTRIM program for Windows based on Visual Basic and C++ languages developed by Eurostat. The BMARK program is designed for univariate benchmarking and it supplies procedures based on four mathematical benchmarking methods, two for benchmarking seasonally unadjusted series and two for benchmarking seasonally adjusted series. The two options relevant for interpolation and distribution for the national accounts are the modified Denton proportional first difference method and the iterative, non-linear Causey-Trager growth preservation method. In the BMARK program these methods are referred to as RATIO and TREND models. We shall refer to these methods as RATIO and TREND in the following discussion.

The BENCH program is designed for univariate benchmarking and for temporal disaggregation. It is developed for a generalization of the Denton methods based on GLS regression techniques. It provides options for specifying binding or non-binding benchmarks, benchmarks for particular years only, and sub-annual benchmarks. It allows for incorporating particular information about the error generating process. For instance,

the autocorrelation of the errors may be modeled by assuming that the errors follow a stationary ARMA process, and the reliability of each annual and sub-annual observation may be characterized by their variances. Although the program is designed for benchmarking using regression based methods, the program can be used to approximate the modified Denton additive and proportional first difference methods by assigning a set of parameters to the default values.

The ECOTRIM program is developed for Windows by Eurostat. It supplies procedures based on temporal disaggregation of low frequency series using mathematical and statistical methods. It allows for univariate and multivariate temporal disaggregation of time series. For univariate series with indicator or related series, ECOTRIM provides the options of five regression methods listed above. For univariate series with no indicators, ECOTRIM provides the options of first and second difference smoothing methods by Denton and by Boot-Feibes-Lisman. For temporal disaggregation of multivariate series with respect to both temporal and contemporaneous constraints, ECOTRIM provides procedures using the Fernandez random walk model, the Chow-Lin white noise model, the Denton adjustment methods and the Rossi regression model (1982). Moreover, ECOTRIM provides both interactive and batch mode for temporal disaggregation.

We use all three software programs to generate final sub-annual estimates using the selected methods and evaluate the final estimates according to the five criteria discussed above.

4. Estimation Results

When compiling sub-annual estimates by temporal distribution and interpolation, the national accounts encounter the following cases: 1) both annual and sub-annual indicator data are available; 2) only annual data are available; 3) both temporal and contemporaneous constraints are presents; and 4)

annual data contain negative values. In order to have a proper understanding of how each method works, we choose series so that all these cases are included in our experiments.

We have selected 60 series for temporal distribution and interpolation, 15 from the National Income and Wealth division (NIWD) and 45 from the Government division (GOVD). Table A1 in the Appendix lists the annual series and their indicator series, if available, included in the estimation experiment. Data used in estimation were obtained after the 2005 annual revision. For the 15 series from NIWD, some are quarterly variables and some are monthly variables. Indicator series are available for 14 out of the 15 series. Of the 45 series from the GOVD, 16 are government taxes on production and imports, 15 are transfer payments to the federal government, and 14 are series from the Federal and the State and Local Government branches. The series on taxes and transfer payments are quarterly variables, and they have both temporal and contemporaneous constraints. The contemporaneous constraints for taxes and transfer payments are, respectively, quarterly total of the taxes and quarterly total of the transfer payments. No indicator series are available for the 14 series from the Federal and State and Local Governments branches, and some of these series have multiple negative annual values in the sample.

Because choices of methods for temporal distribution or interpolation largely depend on the basic information available for estimation, we separate the 60 series into two categories: 1) annual series with sub-annual indicators; and 2) annual series without sub-annual indicators. We shall discuss the results in each category according to the criteria discussed in Section 3.

4.1 Temporal disaggregation of annual series with indicators

Of the 60 series included in the experiment, 45 series fall into this category, 14 of which are from NIWD, and the remaining 31 are from the GOVD, which have both temporal and

contemporaneous constraints. The sample sizes are between 8 to 12 years. For some series we have pre- and post-revised annual and indicator values from 2002 to 2004.

The indicator series selected for distribution and interpolation are not the same target variables measured by the annual data. They are intended to provide information on the short-term movements in the target variables. Thus, indicators selected should be closely correlated with the target variables. To see if the indicators are closely correlated with the target variables, we computed the correlation coefficient ρ between each pair of annual series and annual aggregates of sub-annual indicator series. Table 1 shows that all but one ρ values are in the range of .8130 and .9999, and 39 of the 45 pairs have a ρ value greater than .9, an indication of strong correlation between the annual and the corresponding indicator series. (Most tables are included at the end of the report.)

Table 1 is here

We estimated these series using the following methods: the modified Denton additive first difference (DAFD) and proportional first difference (DPFD) methods; the Casey-Trager growth rate preservation method (TREND); the Fernandez random-walk model (RAWK); the Litterman random-walk Markov model estimated by applying ML (RAWKM MAX) and GLS (RAWKM MIN) and the Bournay and Laroque AR(1) model also estimated by applying maximum likelihood (AR(1) MAX) and GLS (AR(1) MIN). To compare these methods with Lagrange polynomial interpolation, we also included the final estimates from Lagrange polynomial interpolation procedure (LPI).

We used the options of the modified Denton additive and proportional first difference methods from BENCH program; the options of the modified Denton proportional first difference method and the Casey-Trager method from BMARK program; and the options for the five regression based methods from ECOTRIM

program. For the regression models, the parameter to be estimated is the one in the autocorrelation process of the errors. ECOTRIM program allows the options of having the user choose the parameter's value or having the parameter be optimally estimated. By choosing the parameter's value, the user decides how strong the short-term movement preservation should be. In this experiment, we choose the option of having the parameter be optimally determined in the estimation. Both BENCH and BMARK programs provide the option of the modified Denton proportional first difference method, and we used both to compare the results from the two software programs. We continue to refer to the Denton proportional first difference option from BMARK as RATIO. To simplify the exposition, we shall use the abbreviations of the methods in the following discussion of the results.

The final sub-annual estimates from the mathematical and regression methods are quite close in level. See Figures 1-1 to 1-10 for details. (All figures are included at the end of the report.) However, the final estimates from LPI procedure sometimes display a significant jump or dip at the beginning and/or ending periods of the sample. Four such examples are provided in Figures 1-1 to 1-4. In Figure 1-1, the big dip at the end of the final estimates from LPI exhibits contradictory movement as seen in the indicator series. In Figure 1-2 and 1-3, such contradiction can be seen both at the beginning and at the end of the final estimates from LPI. In Figure 1-4, the final estimates from LPI are flat for all periods in 2004, while the indicator values increase mildly and the annual value for 2004 increases sharply. In this case, the balance between the short-term movements in the indicator series and the long-term movements in the annual series is lost in the final estimates from LPI. In some cases, the final estimates from LPI display movements inconsistent with those seen in the annual and indicator series for some periods in the sample. For example, in Figure 1-5 the movements in the LPI estimates from the beginning of 2001 to the end of 2002 do not match the movements seen in the

indicator series. In Figure 1-6 the LPI estimates display a zigzagged pattern that is in sharp contrast to the smooth movements in the indicator series. Reasons for such behaviors in the LPI final estimates are discussed in the introduction.

Final estimates from the modified Denton additive first difference method (DAFD) may also display a pattern that is not observed in the indicator series. Two such examples are shown in Figure 1-7 and 1-8. In Figure 1-7a and 1-7b, the indicator series is quite volatile, especially in the later periods in the sample. However, these movements are quite moderate compared with the progressively sharper zigzagged pattern seen in the final estimates from DAFD. This zigzagged pattern may be caused by the mechanism of the DAFD model to minimize period-to-period difference between the indicator series and estimated final sub-annual series. Figure 1-8a and 1-8b show a similar example. These examples suggest that the DAFD method may not be the proper choice for distribution and interpolation if there are frequent rises and falls in the indicator series, because by keeping the indicator values and final estimates parallel, some volatile pattern is generated. Next we shall compare the final estimates according to the 5 test criteria. Final estimates generated by the 5 regression methods are quite close in level. Figure 1-9 and 1-10 are two examples.

Temporal aggregation constraint

As discussed in Section 3 proportional annual discrepancy with respect to indicator series D_z^P measures the annual values relative to the annual aggregates of the indicator values. The more different is D_z^P from one, the more adjustments in the indicator series are needed to satisfy annual constraints. In most cases, the computed D_z^P is very different from one, indicating that the annual values and the annual aggregates of the indicator values of these series are quite different in level

(See Table 2-1 and 2-2). This is because the indicator and the annual data do not directly measure the same target variables. The computed proportional annual discrepancy with respect to final sub-annual estimates D_x^P is equal to one for all final estimates, except for 6 final sub-annual series from LPI. In these 6 cases, D_x^P is significantly different from one, indicating that the temporal constraints are not satisfied. For all other methods evaluated in this study, temporal aggregation constraints are satisfied.

Table 2-1 and 2-2 are here

Short-term movement preservation

Recall that the two statistics which measure the short-term preservation are the average absolute change in period-to-period difference C_x^L and the average absolute change in period-to-period growth rate between the final sub-annual estimates and the indicator series C_x^P . These two statistics can be interpreted as correction in level and in percentage or as adjustment to the indicator series in order to satisfy temporal constraints. Because in the national accounts the emphasis is placed on achieving smooth period-to-period percentage changes, we focus on the comparison of C_x^P value from final estimates interpolated using each method.

Table 3-1 contains the C_x^P values of the 14 NIWD series from each method. The final estimates from LPI often have larger C_x^P values when compared with the final estimates from other methods. One can also see from Figure 2-1 to 2-4 that LPI final estimates have larger dispersions in the period-to-period difference in growth rates. In some cases, C_x^P of the final estimates from LPI

is greater by a factor of at least 10 compared with C_x^P values of the final estimates from the other methods.

Table 3-1 is here

For each of the 14 series, the C_x^P values of the final estimates from DAFD, DPDFD, RATIO and TREND are quite close, except for the two cases where C_x^P values of the final estimates from DAFD are unreasonably large. These large C_x^P values correspond to the cases where the DAFD final estimates exhibit a sharp zigzagged pattern. For some series, the C_x^P values of the final estimates from the five regression based methods are more varied. See Figure 3-1 to 3-4 for details.

No single method produces the minimum C_x^P in all 14 cases. To have some idea about which method on average better preserves the short-term movements, we computed the mean of the 14 C_x^P values for each method. From Table 3-1 we can see that the smallest mean of the 14 C_x^P values is from the final estimates of DPDFD. Thus, for the 14 series from NIWD, the modified Denton proportional first difference (DPDFD) option from BENCH program is on average the best in preserving the short-term movements in the indicator series. A comparison of the C_x^P values from the mathematical methods with the C_x^P values from the regression methods shows that the means of C_x^P values from the regression based methods are in general significantly larger.

For the 16 series of taxes on production and imports and the 15 series of transfer payments to the federal government, we only compare the final sub-annual estimates from the DPDFD, RATIO, TREND and LPI methods¹. From Table 3-2 one can see that for each

¹ Because of the erratic behaviors of the final estimates from DAFD, we excluded DAFD from experiment after estimating the 14 series from NIWD. We did not include the estimates

tax or transfer payment series, the C_x^P values from DPFD, RATIO and TREND are only slightly different. The final estimates of taxes from DPFD has the smallest mean of the 16 C_x^P values, and the final estimates of transfer payments from TREND has the smallest mean of the 15 C_x^P values. We should point out that the differences between the means of C_x^P values are only marginal. Thus, we could say that for the series on taxes and transfer payments, the DPFD option from BENCH and the TREND option from BMAK preserve short-term movements equally well.

Table 3-2 is here

Minimum distortion at breaks between years

Final estimates from all methods included in the experiment exhibit some degree of distortion at breaks between years. Table 4-1a and 4-1b contain the C_B and C_M values for the final sub-annual estimates of the 14 series from NIWD estimated using each method. It is clear that for each series of final estimates, C_B is greater than C_M . The pattern of distortion at breaks between years can also be observed easily from Figures 2-1 to 2-10. In almost all cases, the final estimates from LPI generate larger distortions at breaks between years. See Figures 2-1 to 2-8 for examples. In a few cases, the final estimates from DAFD also display much larger distortions than the final estimates from the other methods. Figures 2-9 and 2-10 give two such examples. The unusually large distortions occur when the final estimates from DAFD exhibit zigzag patterns as shown in Figure 1-9 and 1-10.

Table 4-1a and 4-1b are here

A comparison of the C_B values in Table 4-1a shows that DPF_D method has out-performed the other methods for the 14 series from NIWD, and DPF_D has also produced the smallest mean of all 14 C_B values. A comparison of the C_B values from the regression methods show that the AR(1) methods generate smaller distortions than the other regression methods. However, when compared with the C_B values from the mathematical methods, the C_B values from the regression methods are in general larger, indicating that the final estimates from these regression methods generate larger distortions at breaks between years.

Table 4-2a and 4-2b contain the C_B and C_M values for the final estimates of taxes and transfer payments. From Table 4-2a we can see that for each series, the C_B values from DPF_D, RATIO and TREND are very close. The minimum mean of the 16 C_B values for tax series and the minimum of the 15 C_B values for the transfer payment series are both from DPF_D, though the means of C_B values from different methods are only marginally different. Thus, we can say that the modified Denton proportional first difference method (DPF_D from BENCH and RATIO from BMAK) and the Casey-Trager method (TREND) generate similar distortions at breaks between years.

Table 4-2a and 4-2b are here

Minimum distortion in the beginning and ending periods

A good method for temporal disaggregation should not generate final estimates that impose large distortion at the beginning and ending periods of the sample. Such distortion is measured by inconsistent period-to-period growth rate between the final estimates and the indicator series. There are two aspects we need to consider when examining distortions at the beginning and ending periods of final estimates. First, we look at the absolute change in period-to-period growth rates between the

final estimates and the indicator values in the second and the last sub-annual periods, and we denote them as C_2 and C_T .

Table 5-1a and 5-1b contain the values of C_2 and C_T of the 14 NIWD series from the final sub-annual estimates of all methods under evaluation. One can see that for almost all series, C_2 and C_T values from LPI estimate are much larger than those from the final estimates of the other methods. This clearly shows that Lagrange polynomial interpolation method has a tendency to generate distortions at the beginning and ending periods of the sample. Such distortions can also be observed in Figures 2-1 to 2-8. In two cases, C_2 and C_T from DAFD have unreasonably large values, which correspond to the cases where the final estimates exhibit sharp zigzagged patterns.

Table 5-1a and 5-1b are here

A careful comparison of the C_2 and C_T values show that there is not a single method that produces the minimum C_2 and C_T for all 14 series. We computed the means of the 14 C_2 and C_T values and found that the minimum means of C_2 and C_T are both computed from DPFD final estimates, though the differences between the means of C_2 and C_T of DPFD, RATIO and TREND methods are fairly small. However, for most series, the C_2 and C_T values from the regression methods are much larger, and so are the means of the C_2 and C_T values. These results suggest that on average, DPFD method generates the least distortion at the beginning and ending sub-annual periods of the sample.

Table 5-2a and 5-2b are here

Table 5-2a and 5-2b show the C_2 and C_T values of the final sub-annual estimates of the 16 tax and 15 transfer payment series from LPI, DPFD, RATIO and TREND methods. Again, for almost all series, the C_2 and C_T values from LPI are much larger than those

from the other methods. However, the results for the other methods are mixed. For the final estimates of both taxes and transfer payments the minimum mean of C_2 is from the DPFDF method, whereas the minimum C_T is from the TREND method for both taxes and transfer payments. One should note that the means for C_2 and C_T from different methods differ very marginally.

Next we shall look at how smoothly the revised estimates link to the previously benchmarked series. We have both pre-revised and revised data from 2002 to 2004 for 9 of the 14 series from NIWD, and we experimented with both alternatives for linking using DPFDF, RATIO and TREND methods. We decided not to use the regression methods for the linking test, because the sample size of 3 years is too small for any reliable statistical results.

Tables 6-1 and 6-2 compare the absolute change in period-to-period growth rate between the newly revised final sub-annual estimate and the indicator value in the first period of the revised sample. Table 6-1 shows the results if the revised final estimates are linked to the previously benchmarked estimates obtained using the same method. Table 6-2 shows the results if the revised final estimates are linked to the previously benchmarked estimates from LPI. One would expect less smooth transition if the revised estimates are linked to the previously benchmarked estimates obtained using a different method, especially a method with serious known problems.

Table 6-1 and 6-2 are here

The left panels in both Table 6-1 and 6-2 show the results using the first alternative for linking; and the right panels show the results using the second alternative for linking. We observe the following from these tables: 1) the absolute difference in growth rates shown in the left panels are smaller in most cases than those shown in the corresponding columns in the right panels, indicating that setting linking as an initial

condition in the optimization problem tend to lead to smoother transition; 2) a comparison of the means of the 9 series in Table 6-1 and 6-2 show that DPDFD method combined with the first alternative for linking generates the smoothest transition; and 3) transition is less smooth if the revised final estimates are linked to the previously benchmarked estimates obtained using LPI.

Contemporaneous constraint

The 16 series on taxes have a contemporaneous constraint, and so do the 15 series on transfer payments. Ideally, a benchmarking program should provide an option for temporal disaggregation when contemporaneous constraint is present. Unfortunately, BENCH and BMARK programs do not provide such an option. ECOTRIM provides an option of a two-stage method for disaggregation of multivariate series. In the first stage, temporal constraint of each series is satisfied, and the estimates from the first stage are used as preliminary estimates in the second stage when the contemporaneous constraint is included in estimation. The options for the second stage temporal distribution and interpolation are the modified Denton proportional first and second difference methods and a regression method developed by Rossi (1982).

We chose to use the option of the modified Denton proportional first difference for multivariate series in ECOTRIM for the second stage estimation, using the estimates obtained from LPI, DPDFD, RATIO, and TREND as the preliminary estimates in the second stage. Table 7-1 and 7-2 show how well the first stage estimates from DPDFD, RATIO, TREND, and LPI satisfy the contemporaneous constraint, and how well the second stage estimates satisfy the contemporaneous constraint. We computed the contemporaneous discrepancy measured by the level difference and by the percentage difference between the sum of the final

estimates of the 16 taxes (15 transfer payments) and the total taxes (total transfer payments) for each quarter in the sample.

Table 7-1 and 7-2 are here

From Table 7-1 and 7-2 one can see that for both taxes and transfer payments the contemporaneous discrepancies from the first stage estimation are very small for estimates from DPDFD, RATIO and TREND, whereas the contemporaneous discrepancies for the estimates obtained using LPI is much larger. One can also see that the contemporaneous constraints for both taxes and transfer payments are perfectly satisfied in the second stage estimation. For each component series of taxes and transfer payments, the estimates obtained using DPDFD, RATIO and TREND in the first stage estimation are only adjusted mildly in the second stage estimation, whereas some component series estimated using LPI in the first stage are adjusted by much bigger margins. The results suggest that the modified Denton proportional first difference for multivariate series in ECOTRIM is a good candidate for the second stage temporal disaggregation when contemporaneous constraint is present.

4.2 Temporal Disaggregation of Annual Series without Indicators

Of the 60 series included in the experiment, 15 series have no sub-annual information available, 14 of which are from GOVD and 1 is from NIWD. The annual data are from 1993 to 2004. 5 out of the 15 series have negative values in the annual series. The methods available for disaggregation without sub-annual information are the smoothing methods developed by Boot, Feibes and Lisman (BFL) (1970), ARIMA models by Stram and Wei (1986, 1990), and numerical procedures such as LPI and cubic spline interpolation. We focus on the BFL and LPI methods. We did not include the ARIMA models in our experiment, because the annual

series are not sufficiently long and because such methods require estimation of unobserved covariance matrix. ECOTRIM program provides the options of the first difference and second difference BFL models (BFLFD, BFLSD); BENCH program provides an equivalent version of the first difference BFL model by setting the indicator value to 1 for all sub-annual periods in the DAFD method; BMARK program also provides a similar option in the RATIO method. ECOTRIM and BENCH programs are able to disaggregate the annual series that have negative values, but BMARK program is not able to handle such cases. This is probably due to programming details. Nevertheless, this is a limitation of BMARK program.

When sub-annual information is not available, some test criteria need to be modified. Instead of examining short-term movement preservation measured by the average absolute change in period-to-period growth rate between the final and the indicator series, we compare the smoothness of final sub-annual estimates. Instead of examining distortions of sub-annual movements at the beginning and/or ending periods, we compare the smoothness of the final estimates at the beginning and ending periods.

Temporal aggregation constraint

Temporal aggregation constraints are satisfied by the final sub-annual estimates obtained using LPI, DAFD, BFLFD, BFLSD, and RATIO methods. The final estimates from FAFD, BFLFD and RATIO are fairly close in level. From Figures 4-1 to 4-5 one can see that the final estimates obtained from LPI may display sharp increase or decrease at the beginning or the ending period of the series, and they may also exhibit sharp turns in the middle portion of the series. Similarly, as shown in Figure 4-5 and 4-6, the final estimates from BFLSD method may also show large increase or decrease at the beginning or ending periods of the final sub-annual series.

Smoothness of the final estimates

We compare smoothness of the final estimates from different methods using the average absolute period-to-period growth rates of the final sub-annual estimates. For the 5 series which have negative values in the annual series, the estimated final sub-annual series also has negative values. Consequently, period-to-period growth rate cannot be computed. Thus, for these series, we compare period-to-period change in level as a proxy for smoothness. When comparing the final estimates obtained using different methods, the smoothest series should have the least volatile changes between periods.

Table 8-1a shows the average absolute period-to-period growth rates of the final sub-annual estimates of the 10 non-negative series estimated using LPI, DAFD, BFLFD, BFLSD and RATIO methods. DAFD and RATIO are BFLFD equivalent from BENCH and BMARK. One can see that no single method has generated the minimum average absolute period-to-period growth rate in all 10 cases. A comparison of the means of the average absolute period-to-period growth rates of the 10 series shows that DAFD (BENCH version of BFLFD) generates the smoothest final estimates among the five methods, though the means from DAFD, BFLFD, BFLSD and RATIO are quite close. Figure 4-7 to 4-10 show that period-to-period growth rate of final estimates from LPI and BFLSD can be quite volatile.

Table 8-1a is here

Table 8-1b shows the average absolute period-to-period change in level of the 5 series which have negative values in the annual data. No results from RATIO because BMARK program cannot disaggregate series that have negative values. A comparison of the means of the average absolute period-to-period level change of the 5 series shows that DAFD generates smoothest final estimates. Figure 4-11 to 4-12 are examples to show that the

final estimates from LPI or BFLSD may have much larger period-to-period change than the final estimates from BFLFD.

Table 8-1b is here

Smoothness in the beginning and ending periods

A good smoothing method should not produce sharp jump or dip in the beginning and/or ending periods of the final sub-annual series, if such movement is not observed in the annual data. We evaluate the smoothness of all final sub-annual estimates in the beginning and ending periods by comparing the period-to-period growth rate in the second and the last period of the series. The left panel of Table 8-2a shows the absolute percentage change of the 10 non-negative series in the second period. It is obvious that the final estimates from LPI and BFLSD may have much larger percentage change in the second period. In fact, this is the case for 9 out of 10 series of LPI estimates and 6 out of 10 series of BFLSD estimates.

Table 8-2a is here

The right panel of Table 8-2a shows the absolute period-to-period growth rates of the 10 series in the last period of the sample. Similarly, the final estimates from LPI and BFLSD exhibit much larger percentage change compared with other estimates. Four examples are shown in Figure 4-7 to 4-10. A comparison of the growth rates from all series and from all methods shows that on average the final estimates from DAFD have the smallest period-to-period growth rate in the second and the last periods of the sample.

Table 8-2b shows the absolute period-to-period level change in the beginning and ending periods of the 5 series that have negative values. Again, the final estimates from LPI and BFLSD

exhibit much larger period-to-period change in the second and last periods. Figure 4-11 and 4-12 show two such examples. Moreover, on average, the final estimates from DAFD method exhibit the smallest period-to-period level change in the beginning and ending periods of the series.

Table 8-2b is here

In sum, for diagggregation of annual series that have no sub-annual information available, BFLFD method seems to perform the best. Out of three versions of BFLFD model, the one provided in BENCH program performs slightly better than the other versions. BMARK program fail to handle disaggregation of annual series that have negative values.

4.3 Comparative Summary of the Estimation Results

In this section we shall provide a comparative summary of the results from the estimation of 60 series using a variety of methods. Table 9-1 compares the 4 mathematical methods and 5 regression methods used to estimate the 14 series from NIWD. The 6 statistics used for the comparative evaluation are: 1) \bar{D}_x^p , the mean of proportional temporal discrepancy with respect to the final estimates; 2) \bar{C}^p , the mean of the average absolute change in period-to-period growth rate between the final estimates and the indicator series; 3) \bar{C}_b , the mean of the average absolute change in period-to-period growth rate between the final estimates and the indicator series during all periods at breaks between years in the sample; 4) \bar{C}_2 , the mean of the absolute change in growth rate between the final estimates and the indicator values in the second period of the sample; 5) \bar{C}_T , the mean of the absolute change in growth rate between the final estimates and the indicator values in the last period of the sample; and 6) \bar{C}_L , the mean of the absolute change in growth rate

between the final estimates and the indicator values in the linking period. These statistics are taken from the tables used in earlier discussion.

Recall that C_x^P measures short-term movement preservation, C_B is the statistic to measure the distortion at the breaks between years, C_2 and C_T measure the distortions at the beginning and ending periods of the sample, and C_L measures how well the newly revised final estimates link to the previously benchmarked series. These 5 statistics measure the difference in various aspects of short-term movements between the final estimates and the indicator series. Thus, the smaller the number the better the results.

Table 9-1 is here

It is clear from Table 9-1 that the modified Denton proportional first difference method (DPFD) from BENCH program outperforms the other methods, though the differences between DPFD, RATIO, and TREND are fairly small. The results from DAFD are contaminated by the two cases in which the final estimates exhibit very volatile patterns. The results also show that the mean statistics are in general larger for regression methods, and LPI is inferior to all the other methods.

Table 9-2 contains the comparative results for the 4 mathematical methods used to estimate the 30 series from GOVD. Because these series are not used to test linking of the newly revised data due to lack of pre-revision data, and because they have contemporaneous constraints, \bar{C}_L is replaced with \bar{D}_x^C the mean of the contemporaneous discrepancy. The left panel shows the comparative statistics for the taxes and the right panel shows the comparative statistics for the transfer payments.

Table 9-2 is here

There is no clear winner in all 5 test criteria. However, the modified Denton proportional first difference model (DPFD) from BENCH and the Causey-Trager model (TREND) from BMARK are close competitors. Again, differences between DPFD, RATIO and TREND are only marginal. LPI is inferior to the others methods.

Table 9-3 compares the mathematical methods used to estimate the 15 annual series that have no sub-annual information. For the 10 series that have no negative values, the statistics used to evaluate the smoothness of the final estimates are: 1) \bar{g}_t , the mean of the average period-to-period growth rate of the sample; 2) \bar{g}_2 and \bar{g}_T , the mean of the period-to-period percentage change in the second and the last period. For the 5 series with negative values, means of the corresponding period-to-period level changes, $\Delta\bar{x}_t$, $\Delta\bar{x}_2$ and $\Delta\bar{x}_T$, are used instead.

Table 9-3 is here

The comparative results show that for series with or without negative values in the annual data, the first difference BFL approximation provided by BENCH (DAFD) outperforms the other methods. The BMARK approximation of the first difference BFL model cannot handle the series with negative annual values. The LPI and BFLSD methods often generate large percentage change in the beginning and ending periods of the final estimates.

5. Evaluation of the Methods for Temporal Disaggregation in NEA

We have experimented with 6 mathematical methods, 5 regression methods and 3 software programs for temporal disaggregation in our search for the most suitable method(s) for routine compilation of sub-annual estimates through temporal distribution and interpolation in the national accounts. We now discuss how each method has performed.

It is very clear from our results that Lagrange polynomial interpolation procedure is inferior to all other methods when evaluated according to the 5 test criteria, and thus, is not an appropriate choice for temporal distribution and interpolation in the national accounts.

Among the mathematical methods for temporal disaggregation of annual series that have sub-annual indicators, the modified Denton proportional first difference method (DPFD) from BENCH program outperforms the other methods, though the Causey-Trager growth preservation method (TREND) is a close competitor in the temporal distribution of the tax and transfer payment series. BMARK version of the modified Denton proportional first difference model (RATIO) slightly underperforms DPFD. Cautions should be taken when considering the modified Denton additive first difference method (DAFD), because it may generate final estimates that exhibit erratic patterns.

For temporal disaggregation of annual series with no sub-annual indicator, the BENCH version of the first difference FBL smoothing method (DAFD) outperforms the ECOTRIM version of the same method (BFLFD) and the second difference smoothing method (BFLSD). TREND cannot be used for temporal disaggregation of annual series without sub-annual indicators. BMARK version of the first difference BFL smoothing model (RATIO) cannot handle annual series with negative values.

We should evaluate the regression methods used in the experiment with caution, because these methods are not extensively tested. We chose the option of having parameter ρ in the autoregressive error process be optimally estimated, and the outcome of this choice may be more smoothed final estimates but somewhat weaker preservation of the short-term movement in the indicator series. There is a trade-off between achieving smoothness of the final estimates and strong short-term movement preservation. Some studies suggest that setting ρ to a certain value helps achieve proper short-term movement preservation.

More tests with different parameter values should be conducted to reach more conclusive evaluation.

We should also point out that the current revision policy in the national accounts limit the advantages that regression methods can offer. Most of the series used in the experiment are clearly serially correlated. Using regression methods to correct serial correlation should improve the final estimates. However, because only the most recent 3 years of data can be used in estimation, it would be very difficult for any regression method to produce reliable results.

6. Conclusion

We have experimented with 5 mathematical and 5 regression methods for temporal distribution and interpolation using 3 software programs designed for benchmarking and temporal disaggregation. Because we have used a large number of series that allow us to study a variety of cases routinely encountered in temporal distribution and interpolation at the national accounts, we have acquired a good understanding of how each method works in each case. Such knowledge provides us with a solid basis for selecting the most suitable method(s) for routine compilation at the national accounts.

This study has also helped familiarize us with the frontier research in this area. New research on temporal disaggregation and new software programs developed for that purpose continue to emerge, which should further benefit our research in the future. One can see from the estimation results that the mathematical methods are easier to use, but they do not correct any serial correlation in the time-series data. Thus, in our future research, we would like to further examine regression-based methods which would help correct serial correlation in the time-series data. The best method for temporal disaggregation at the national accounts should produce sub-annual estimates that

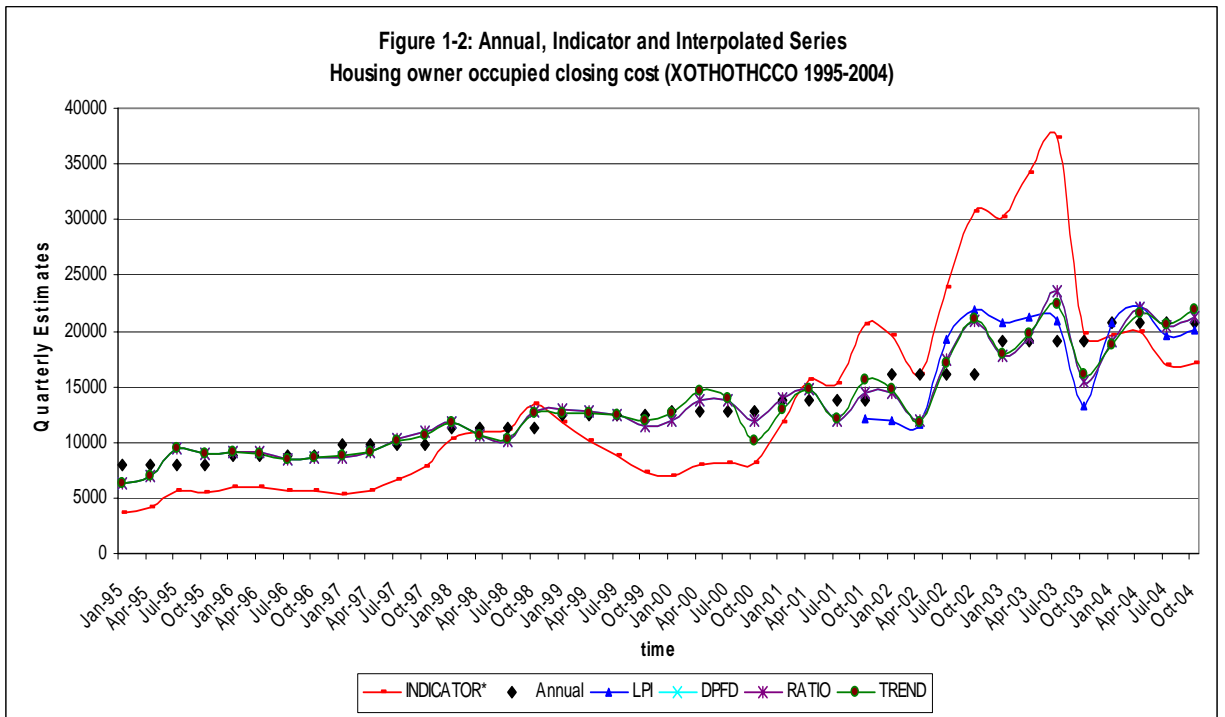
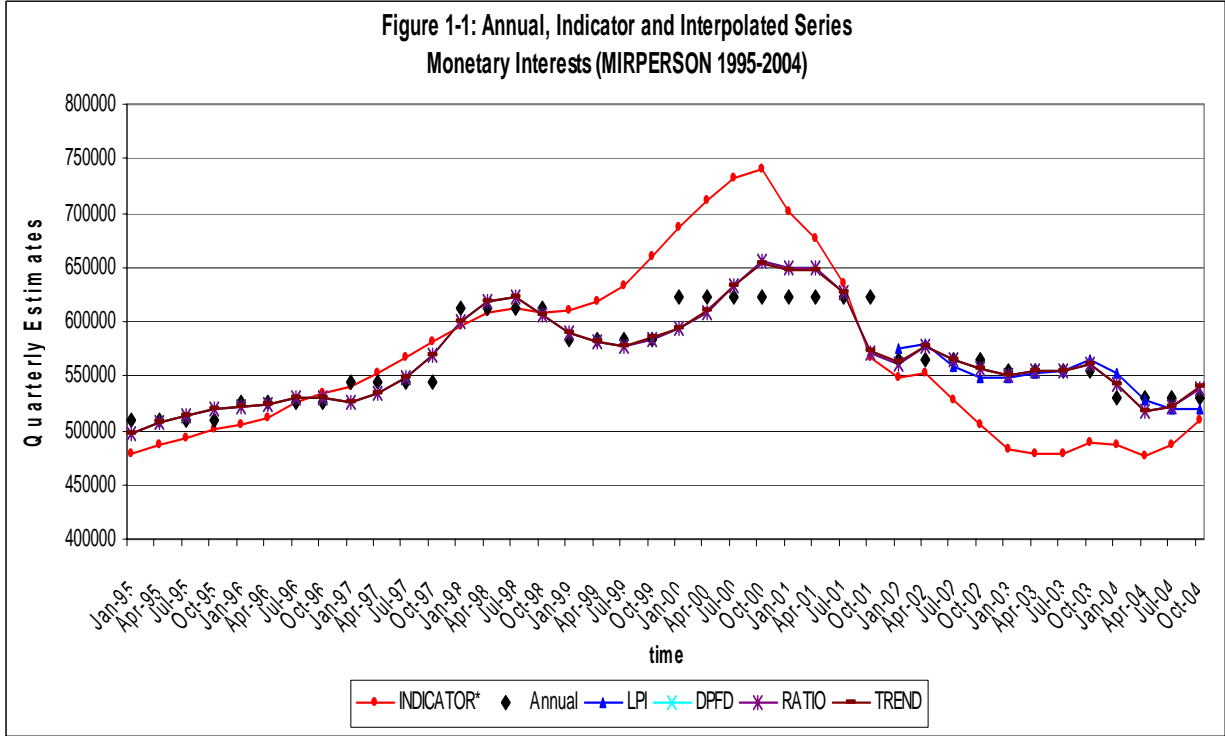
satisfy the criteria used in our evaluation and are also free of serial correlation. Toward that goal we shall continue our research.

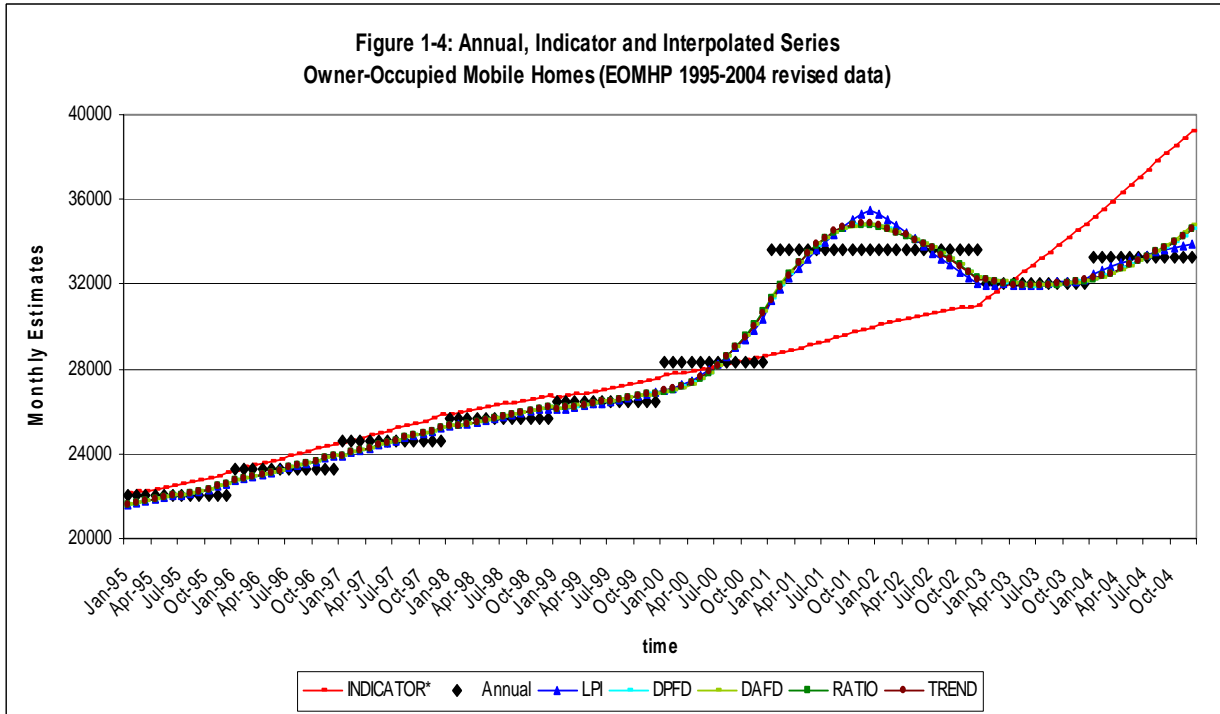
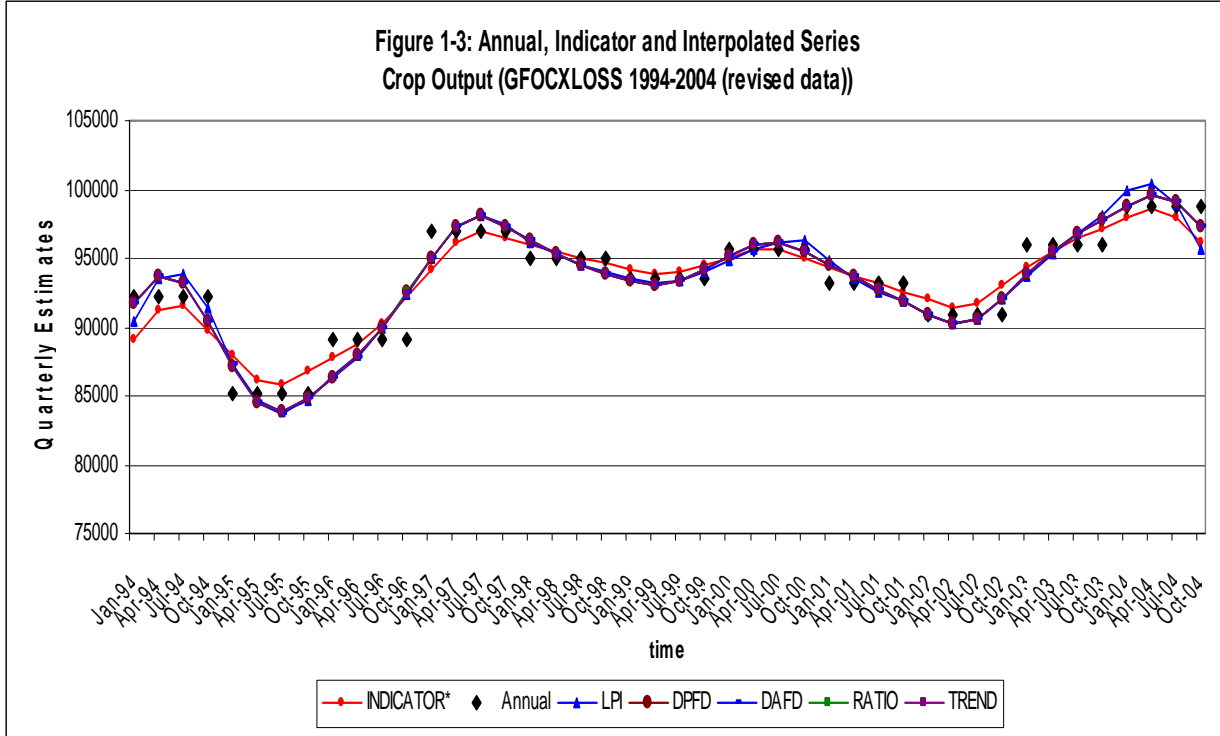
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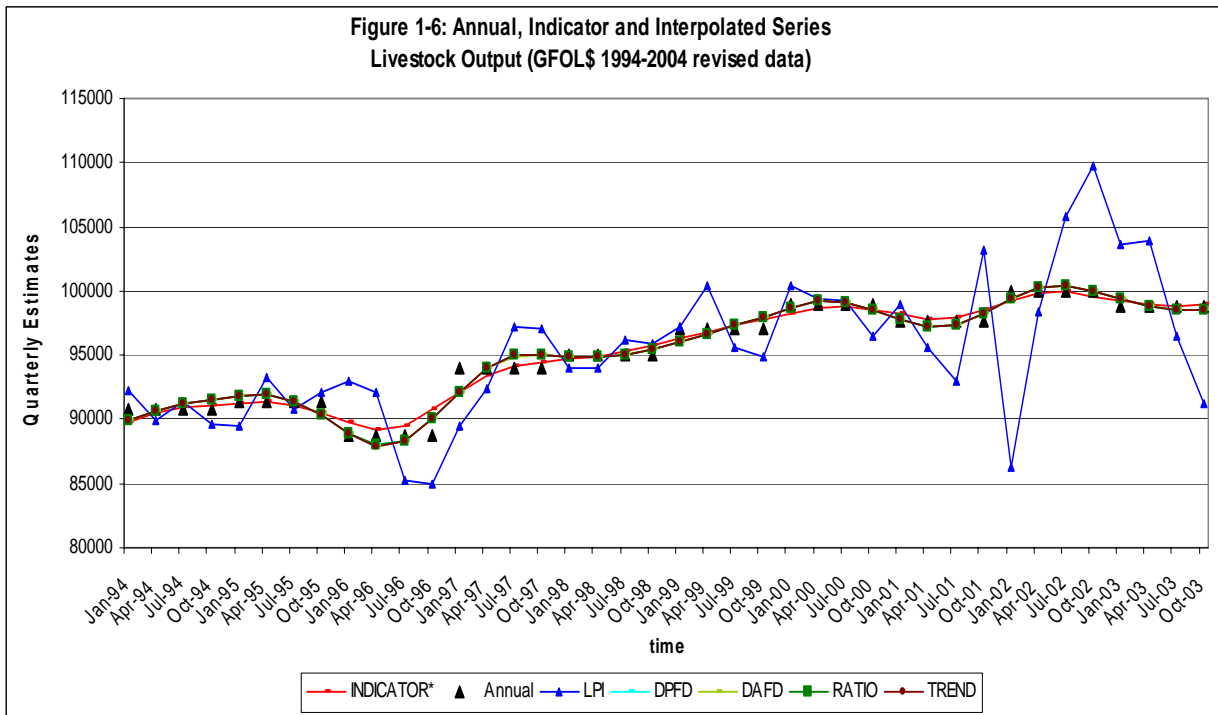
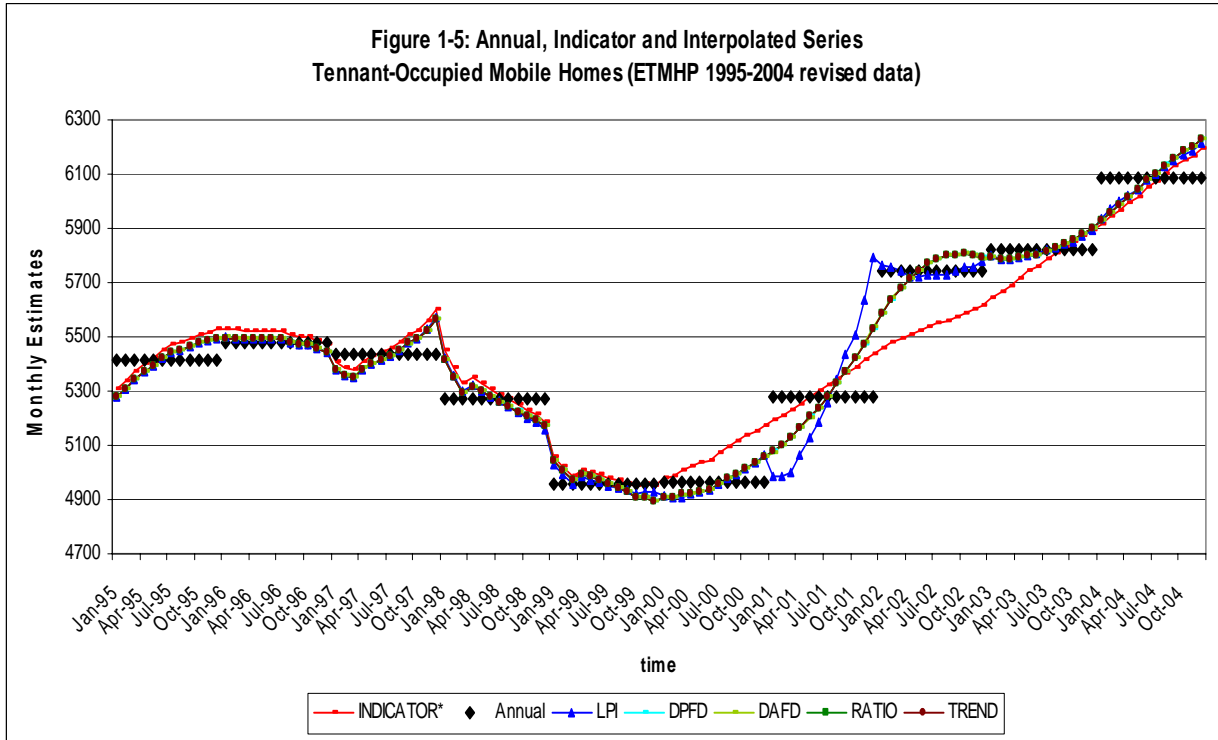
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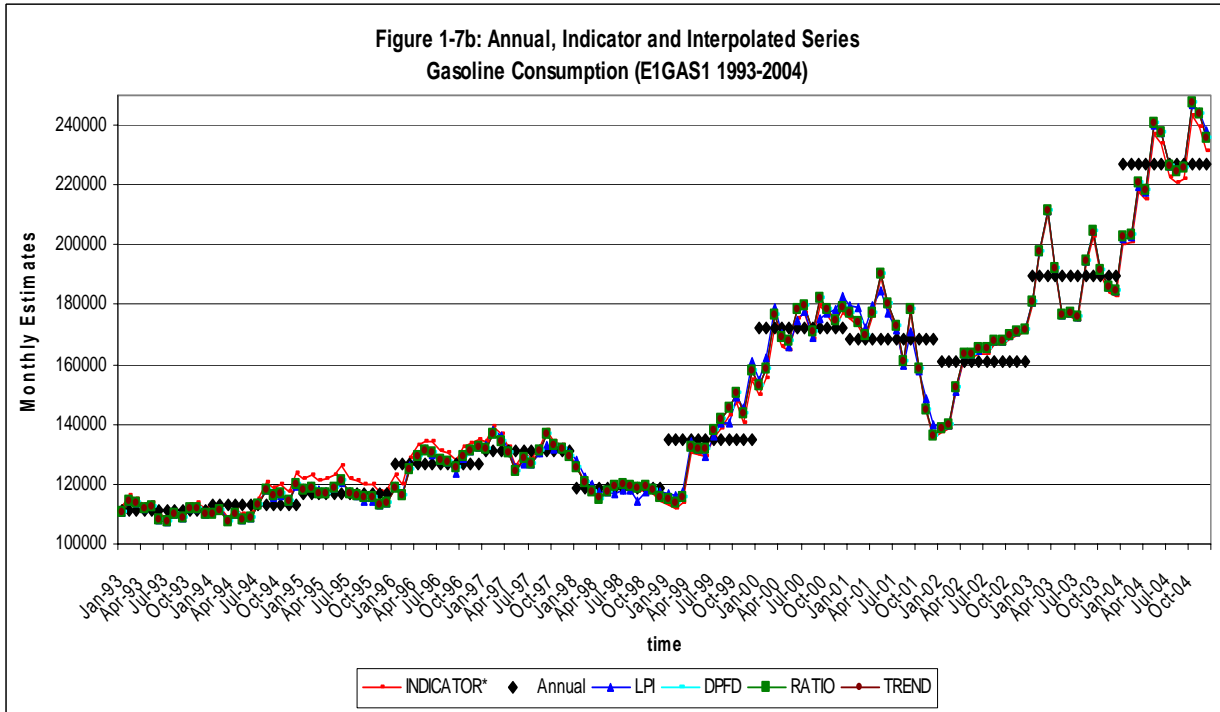
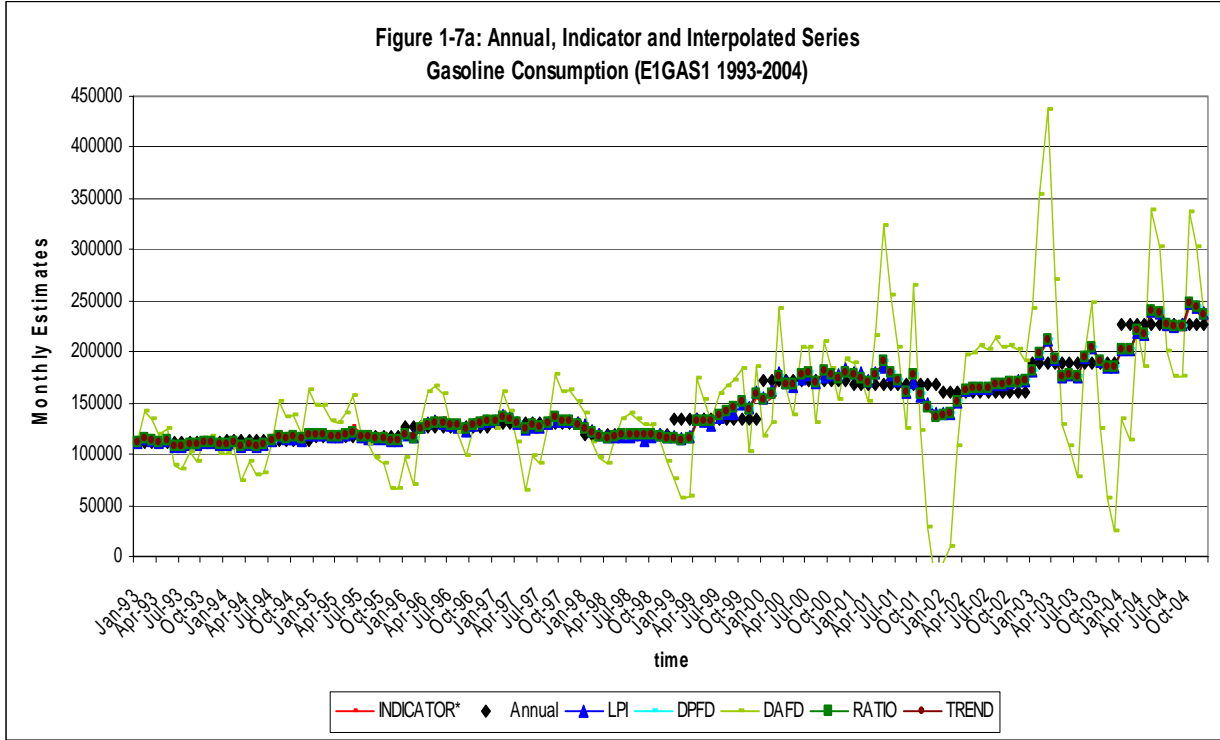
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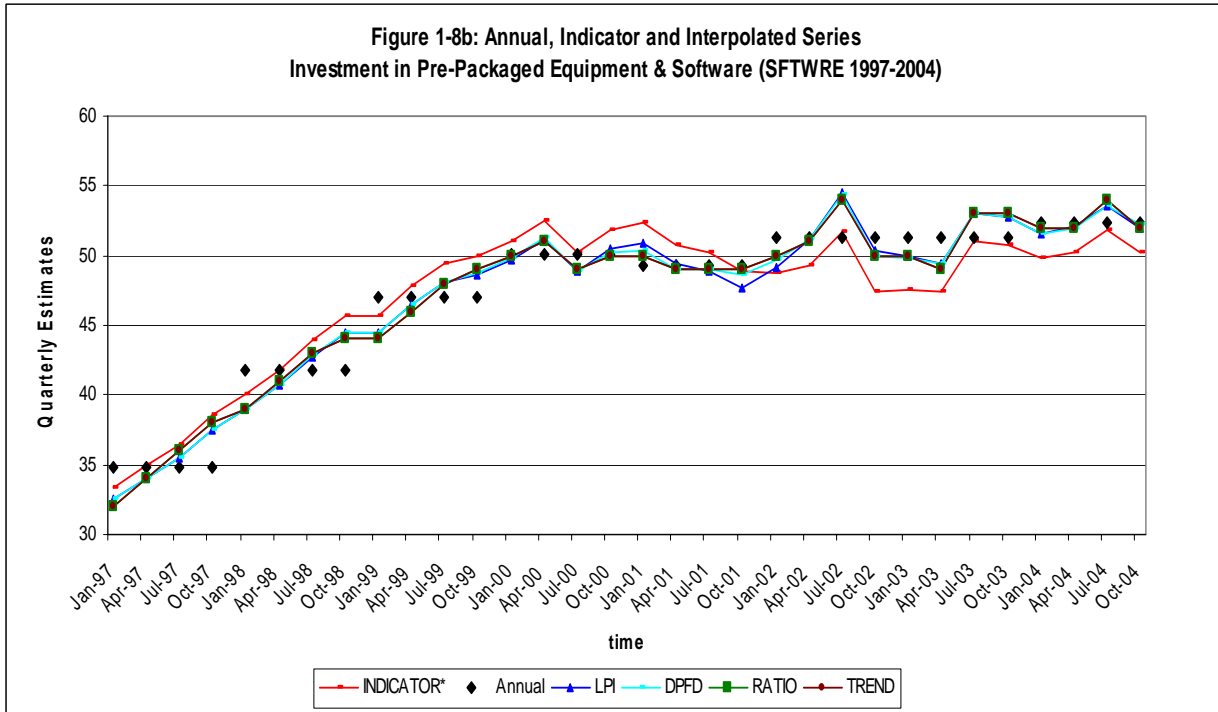
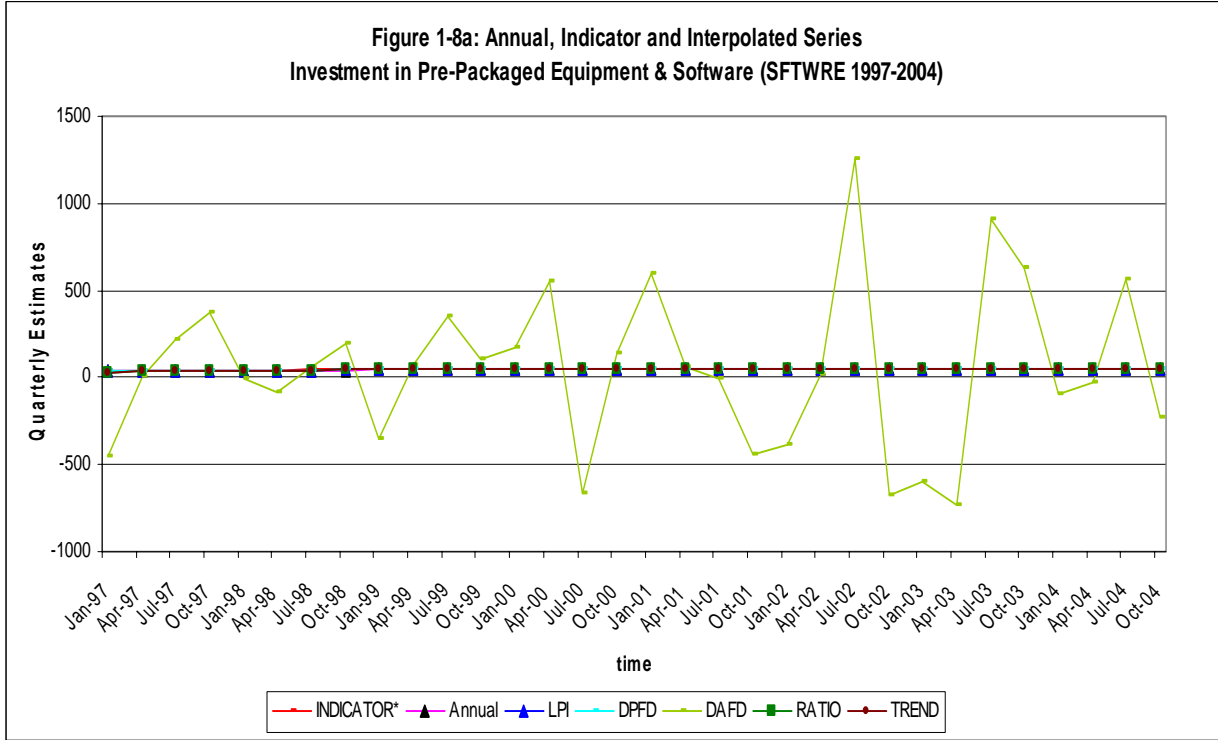
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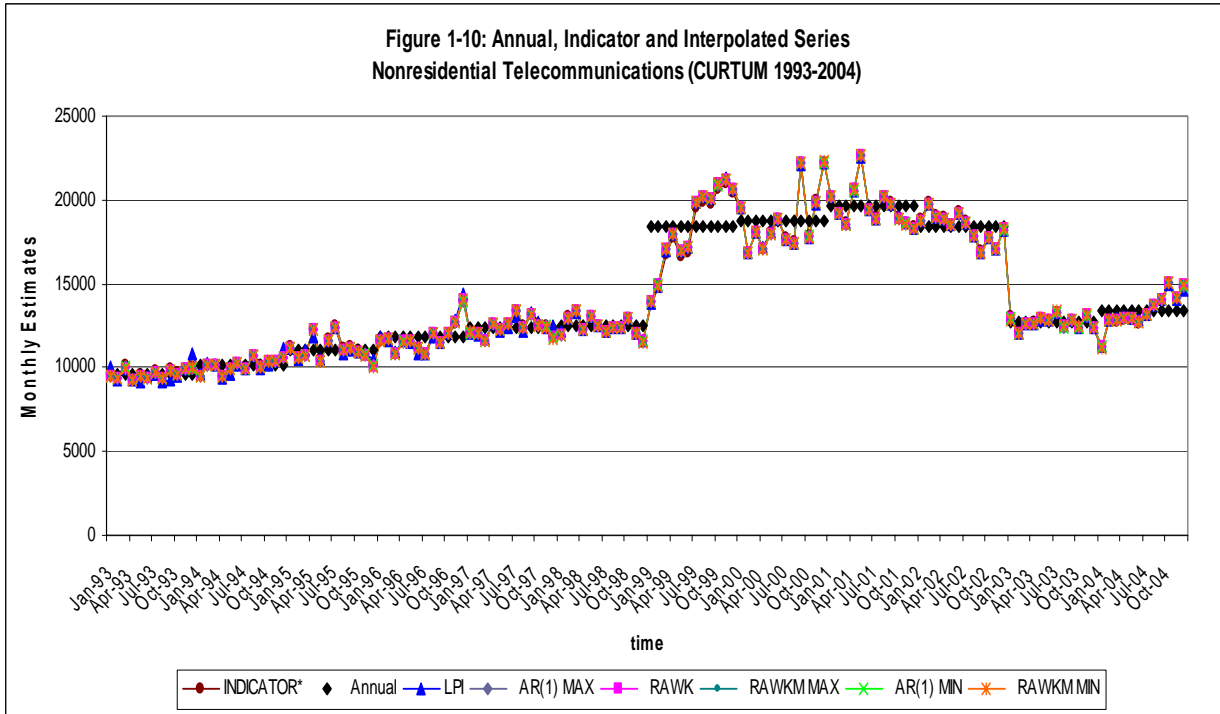
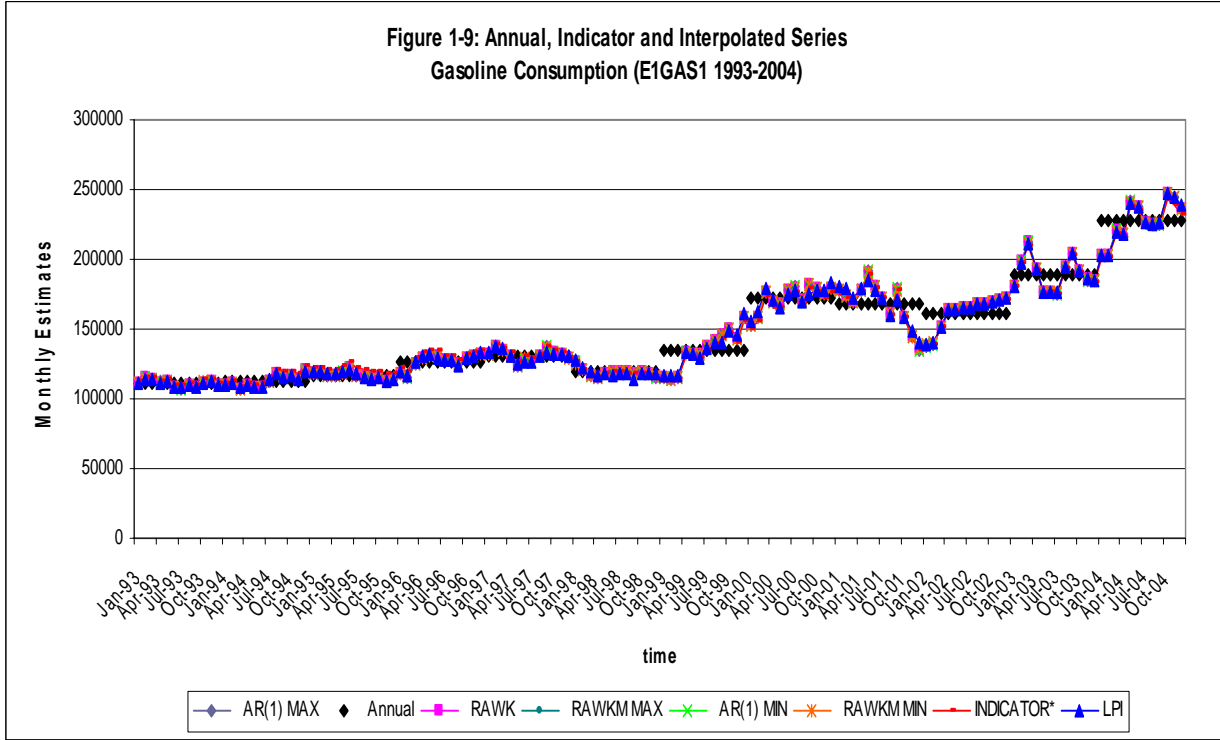


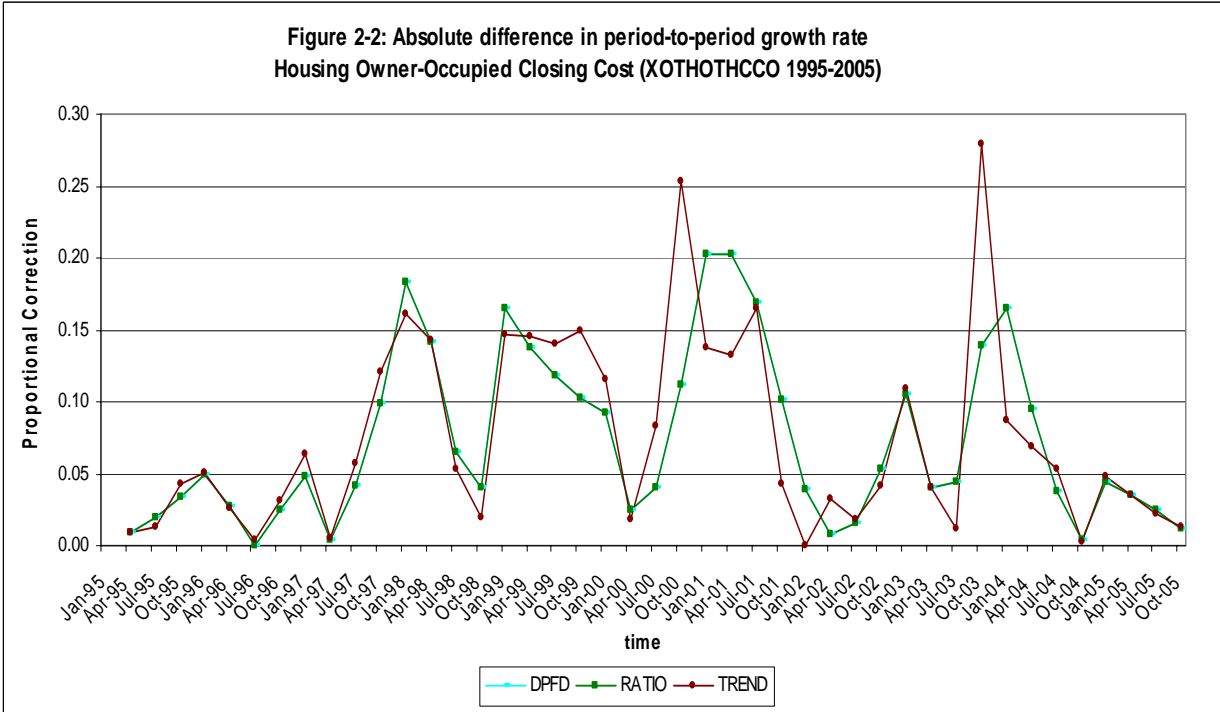
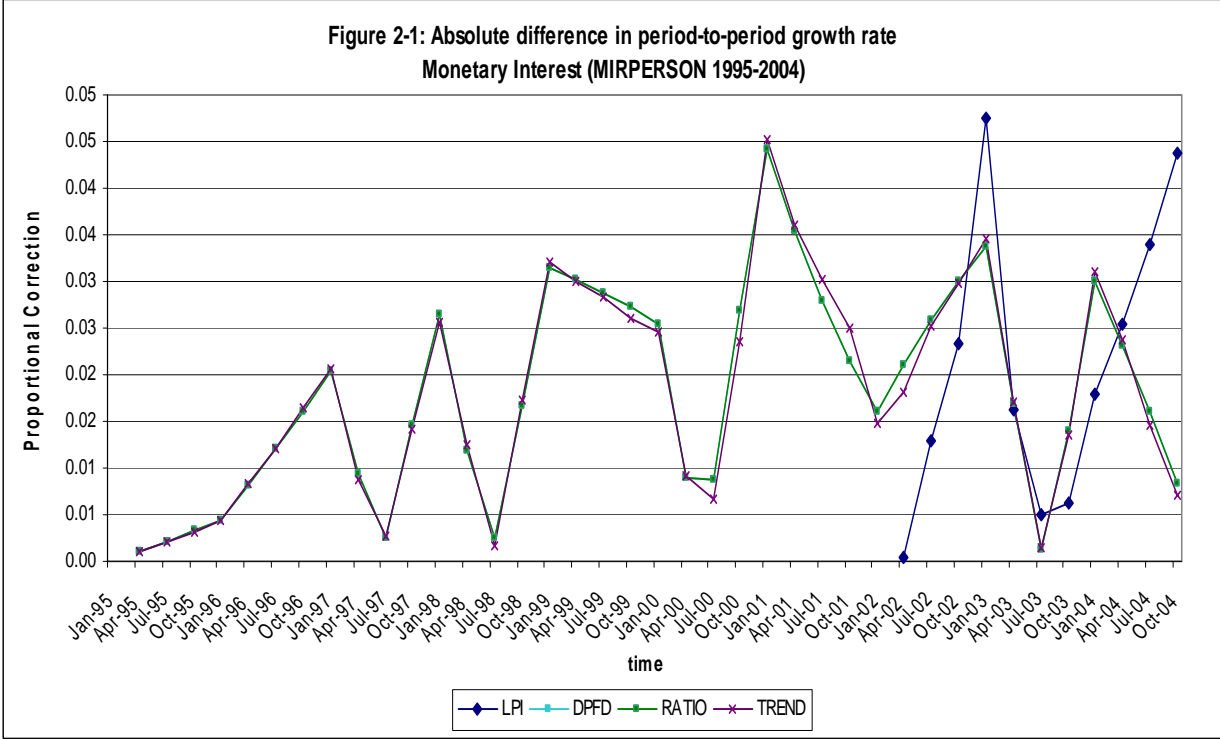


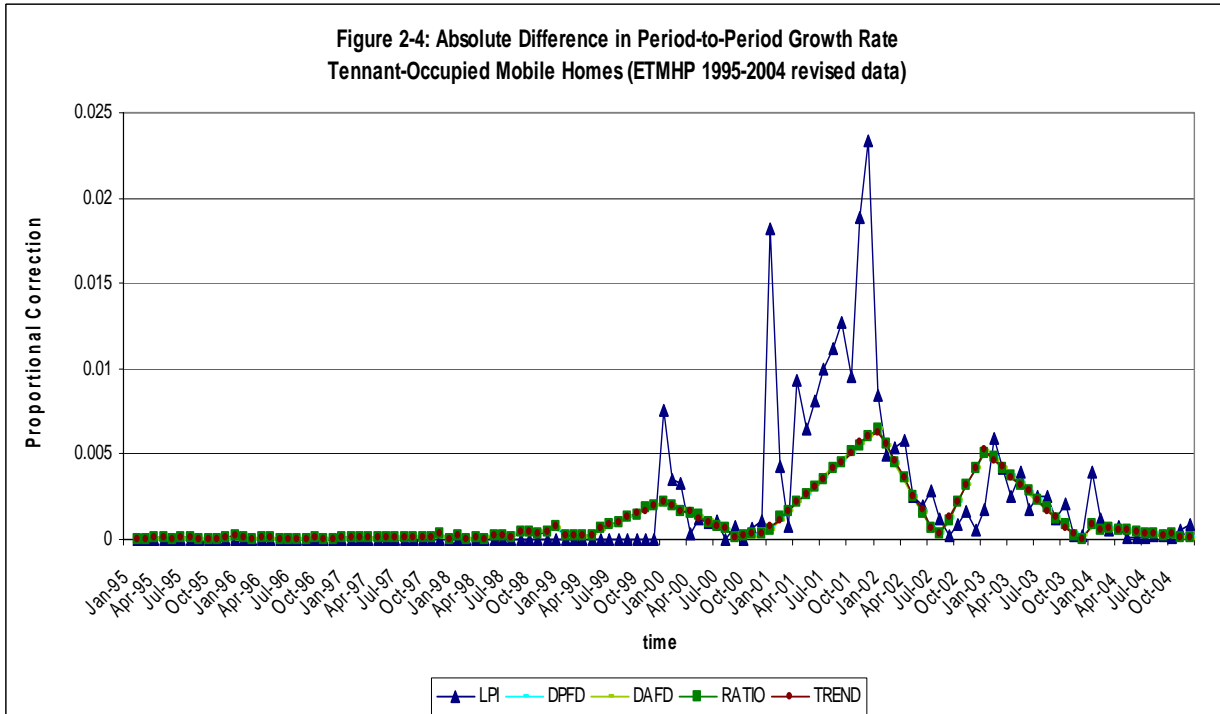
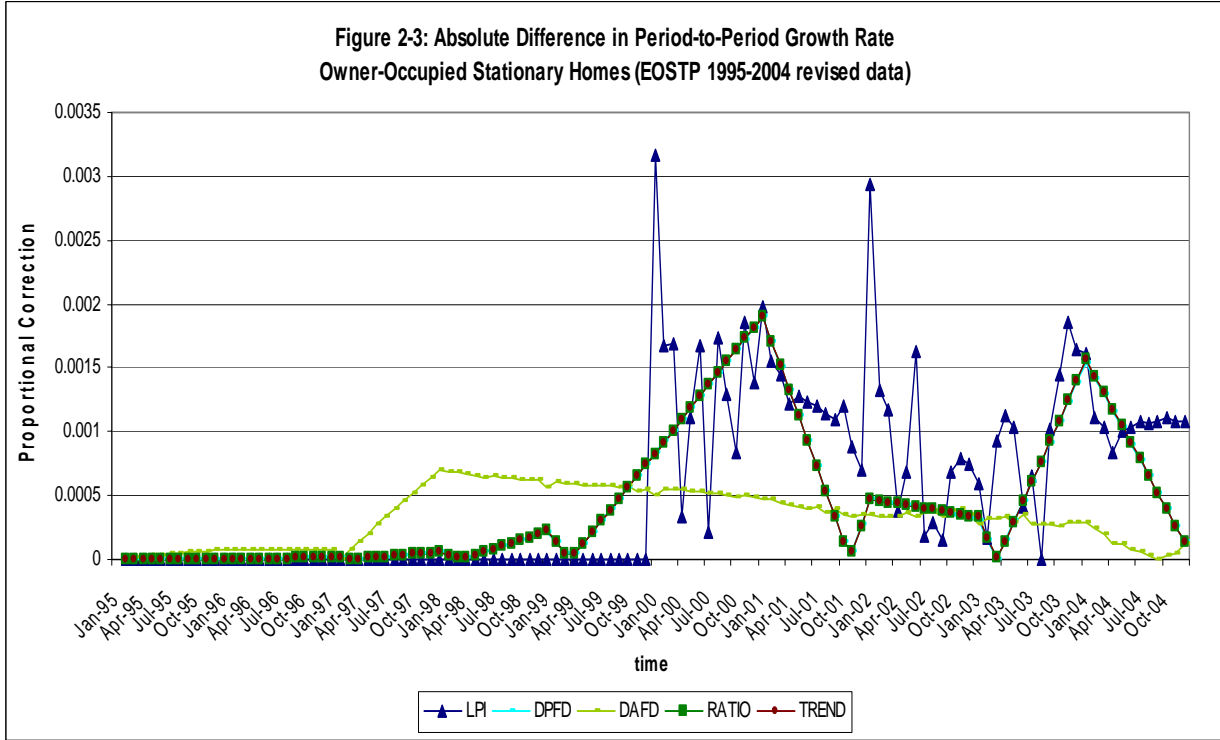


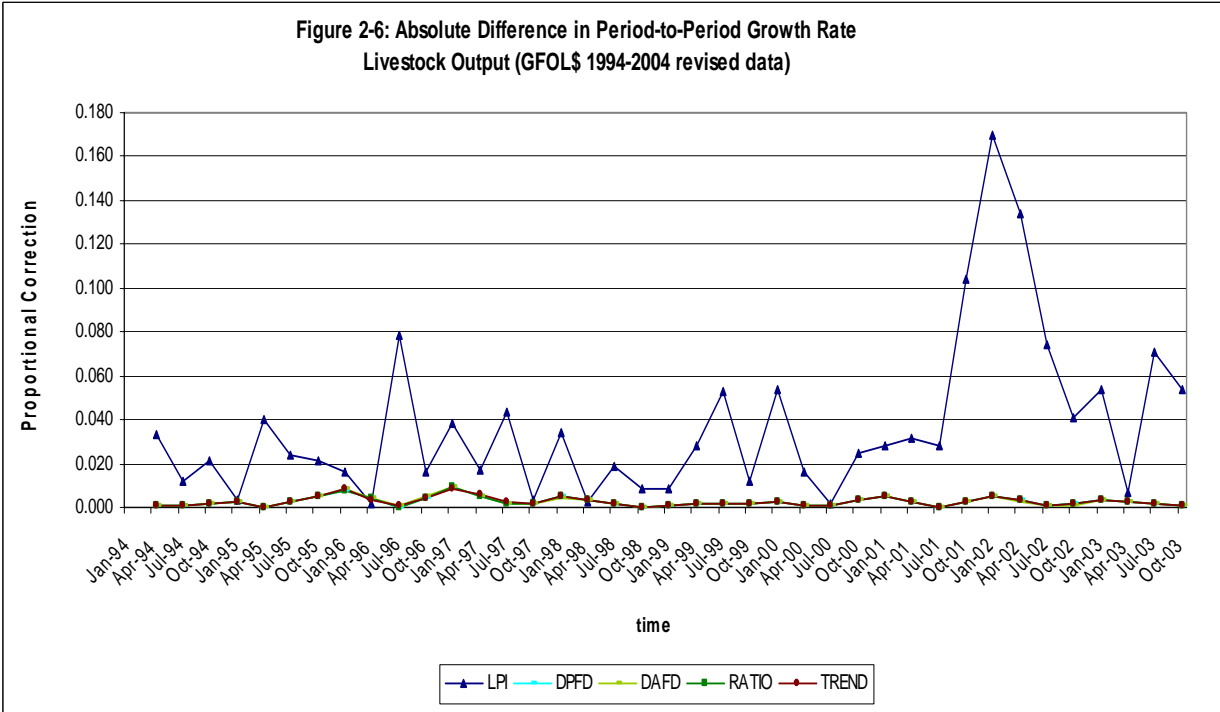
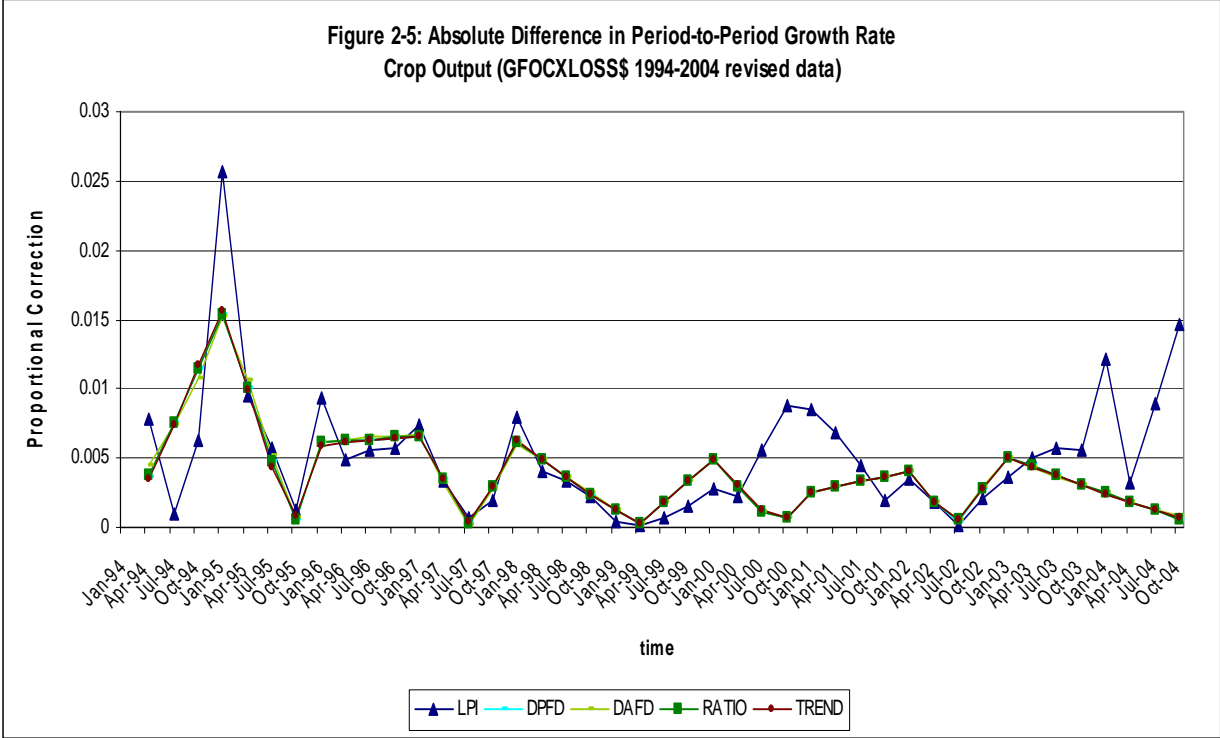


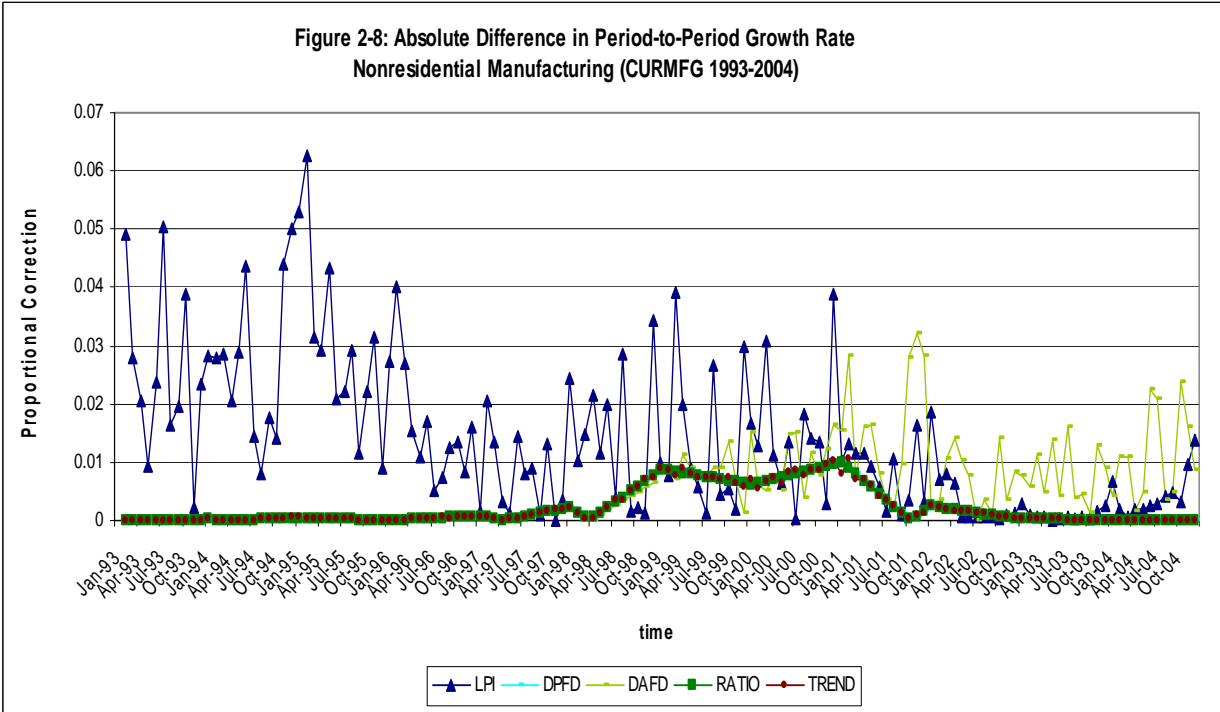
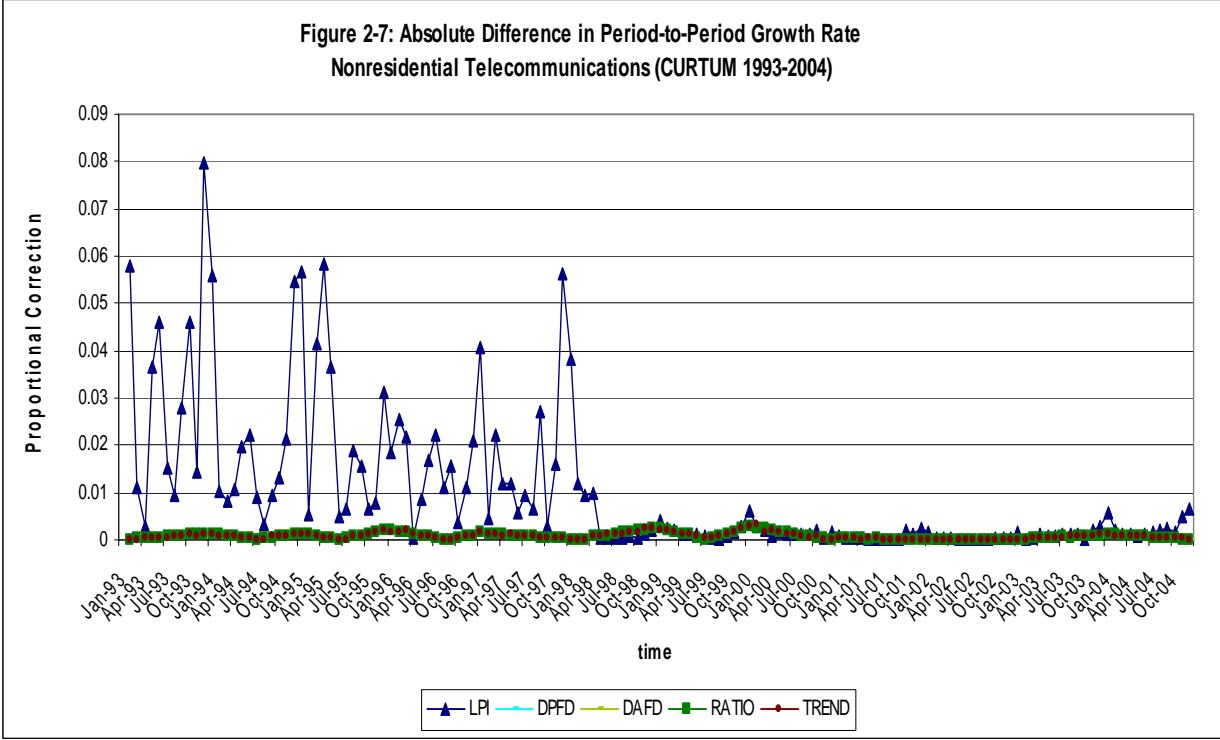


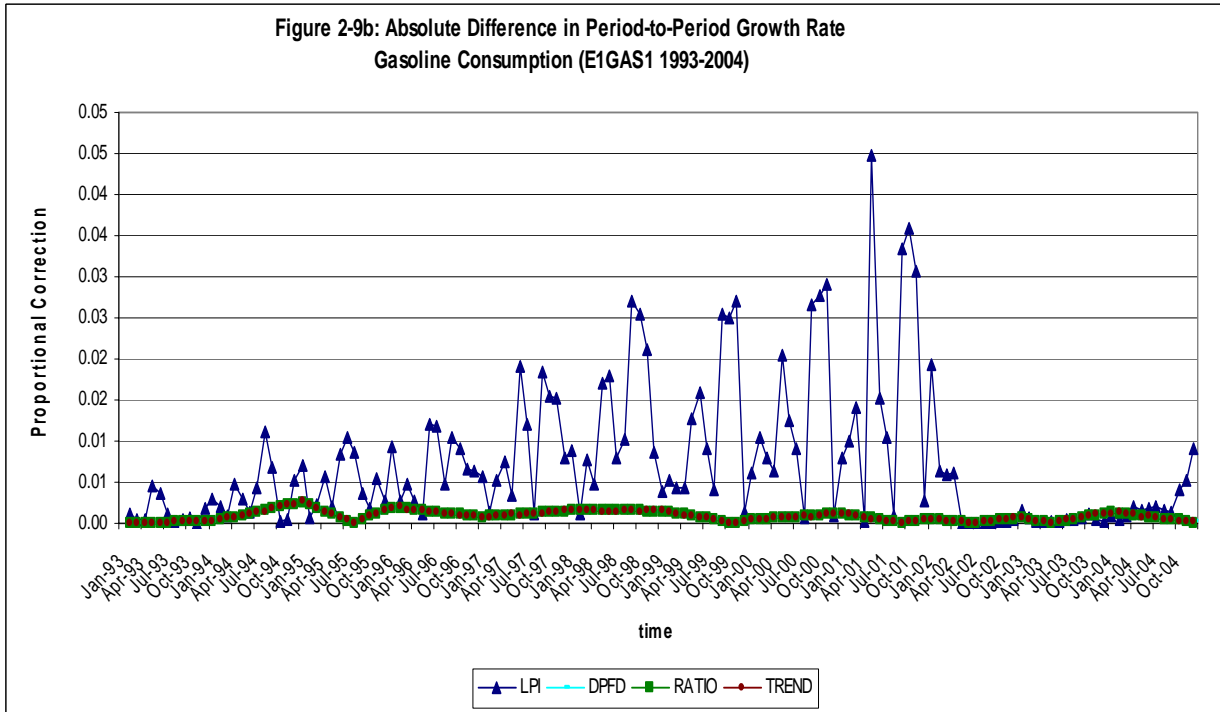
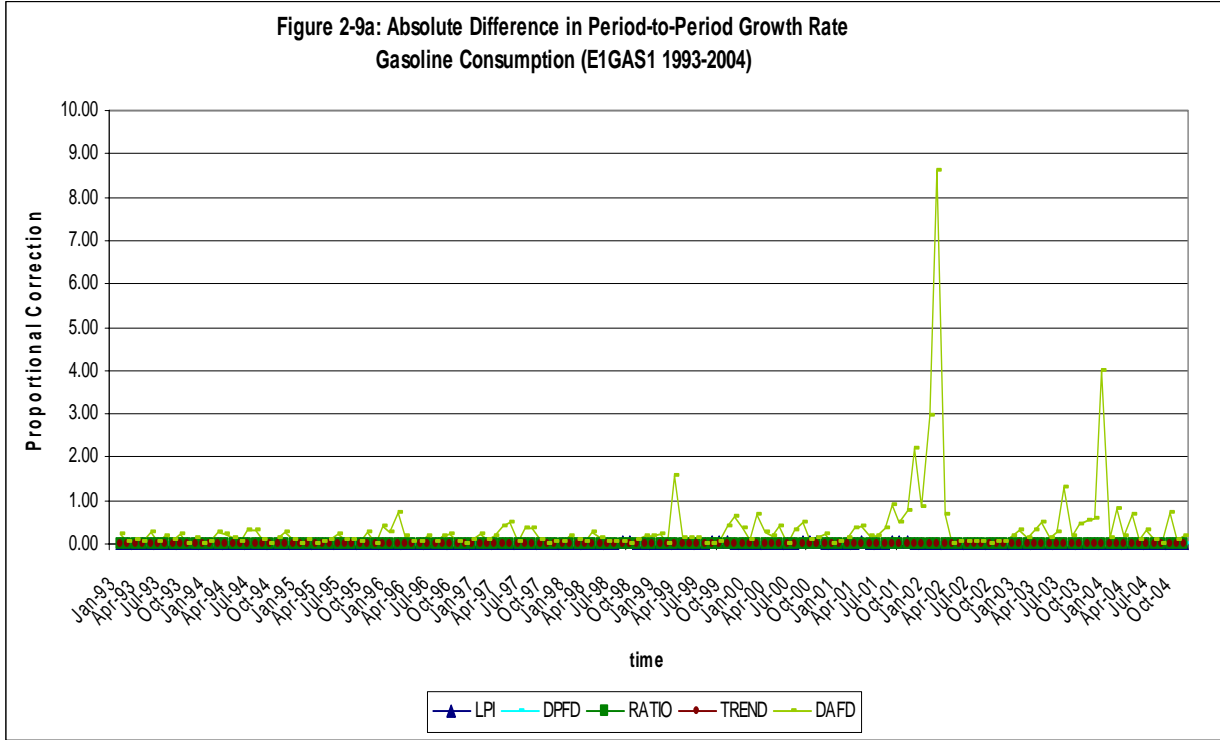


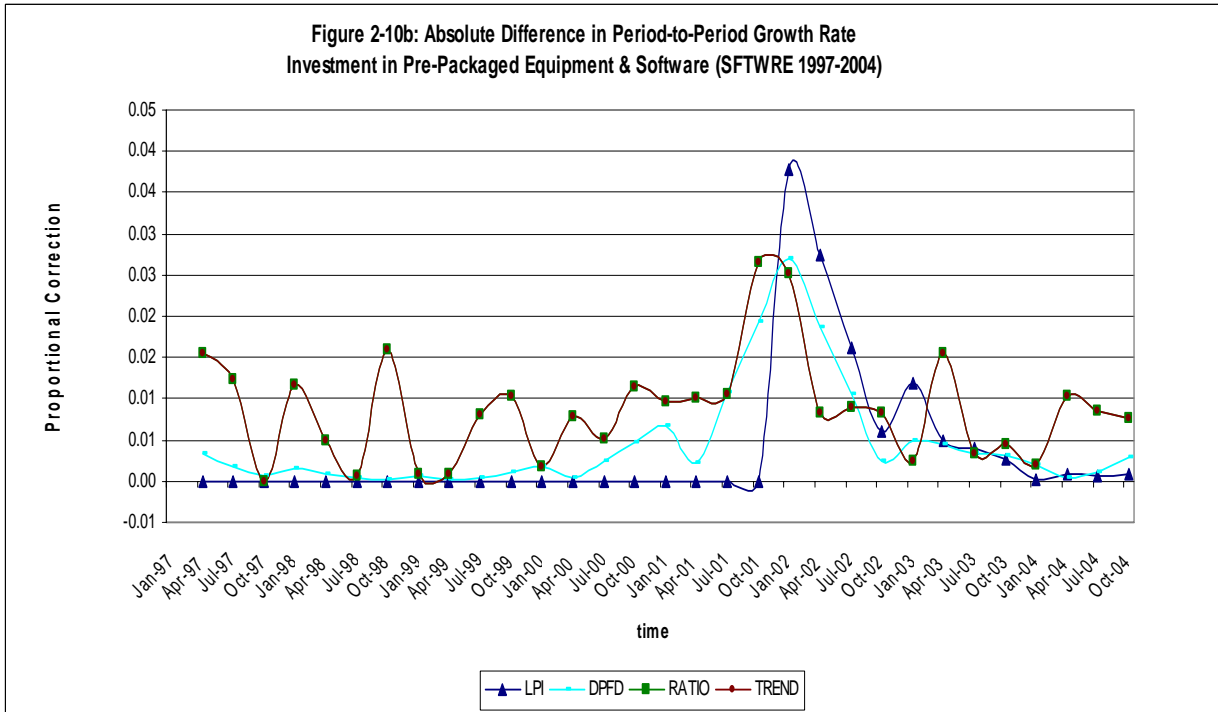
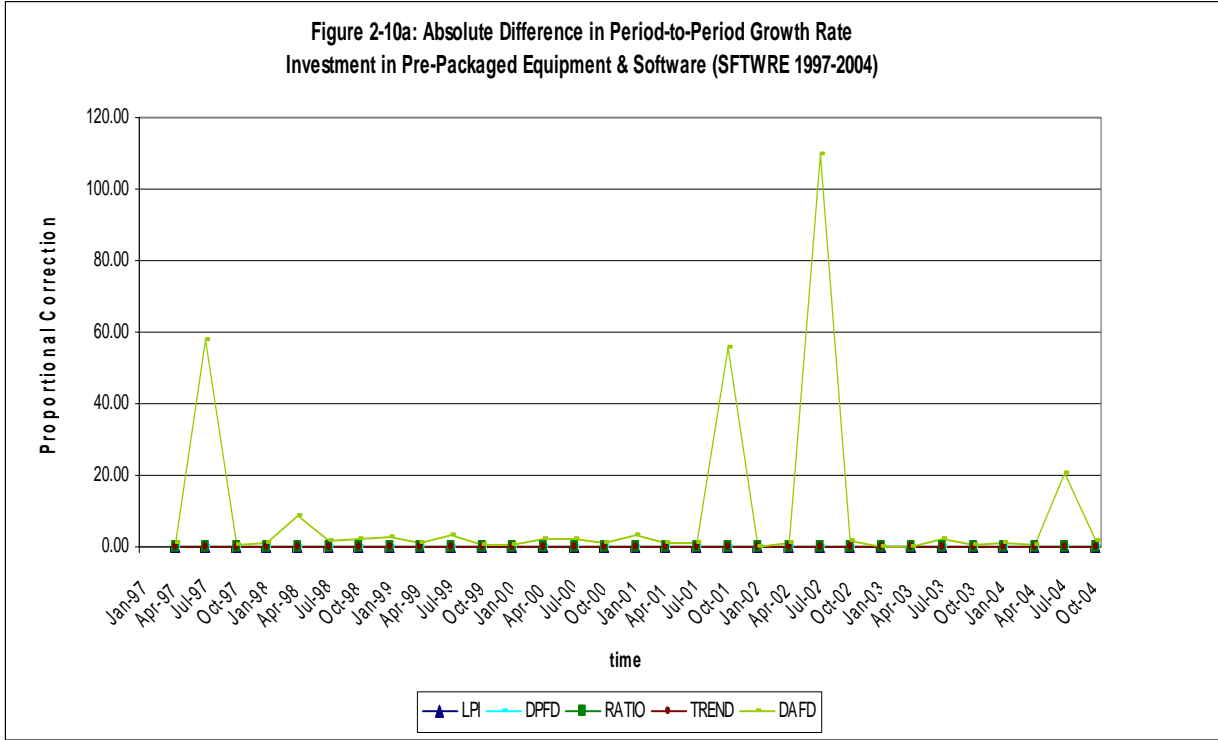


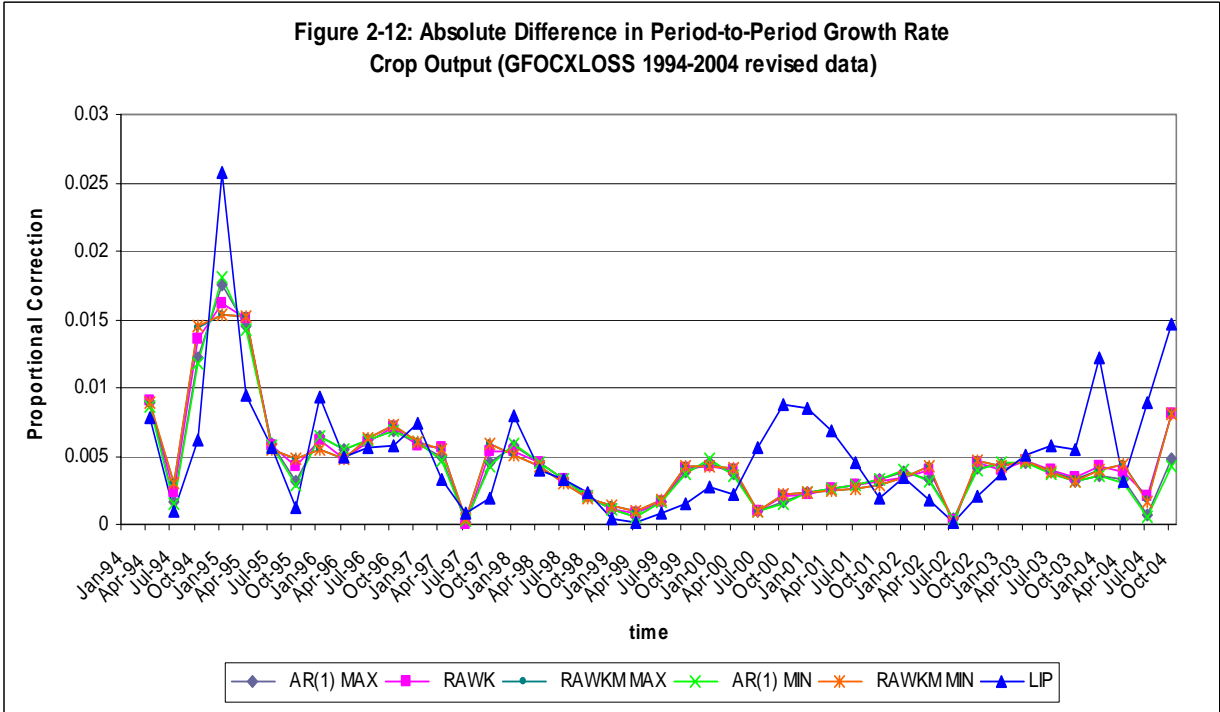
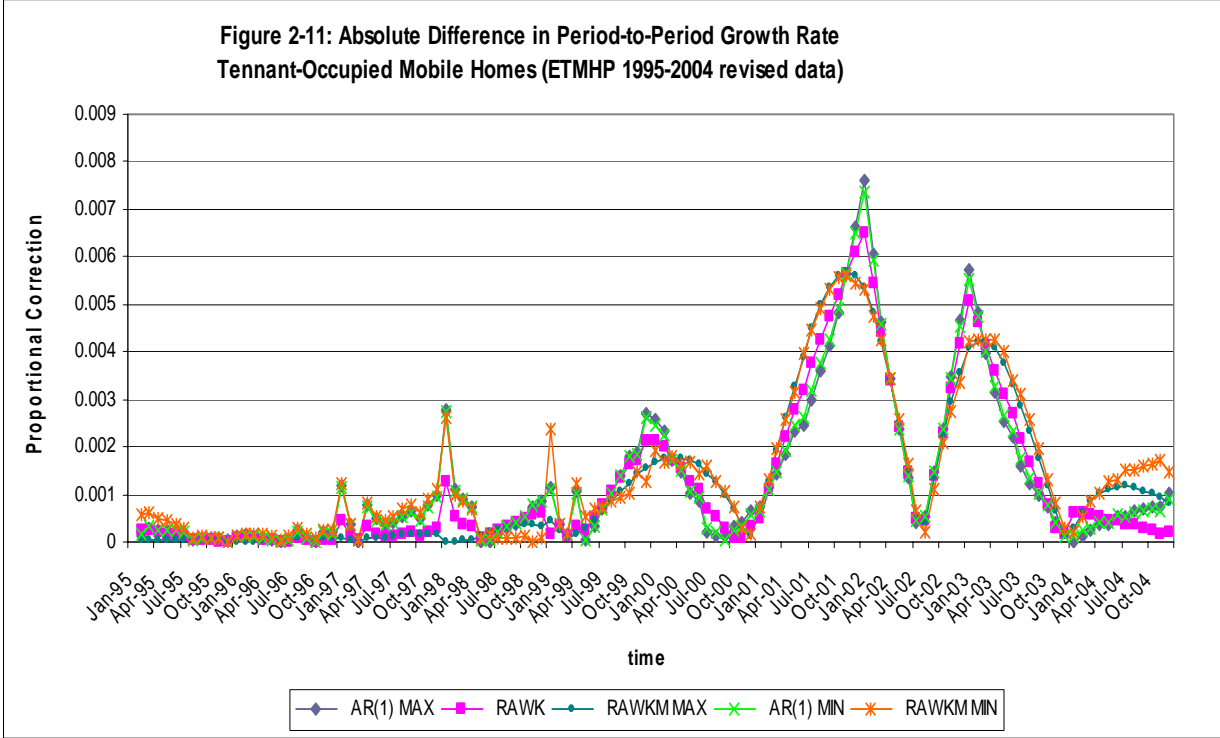


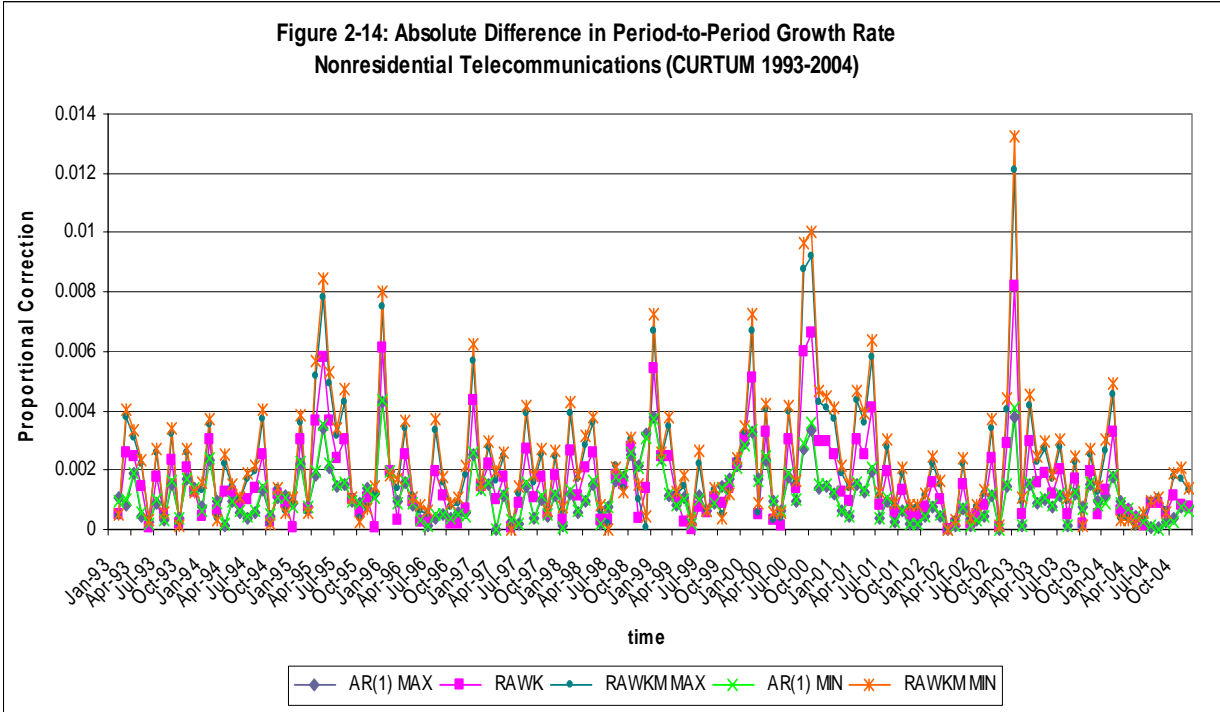
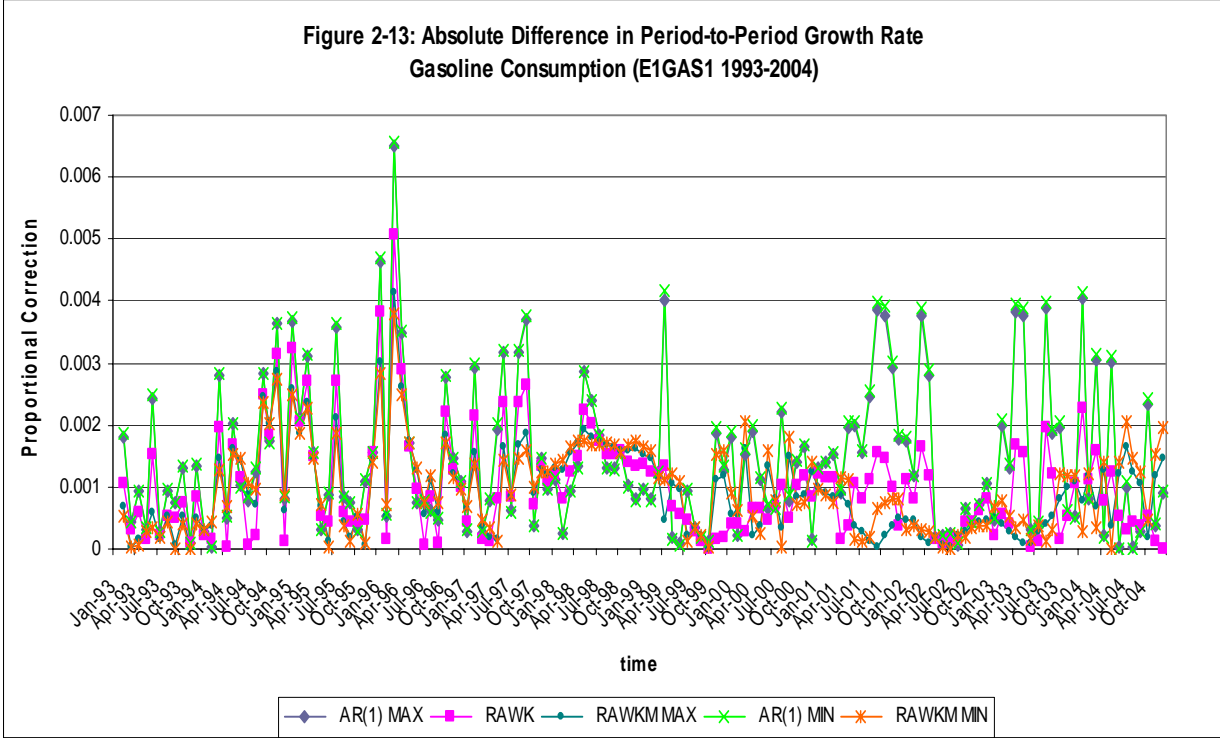


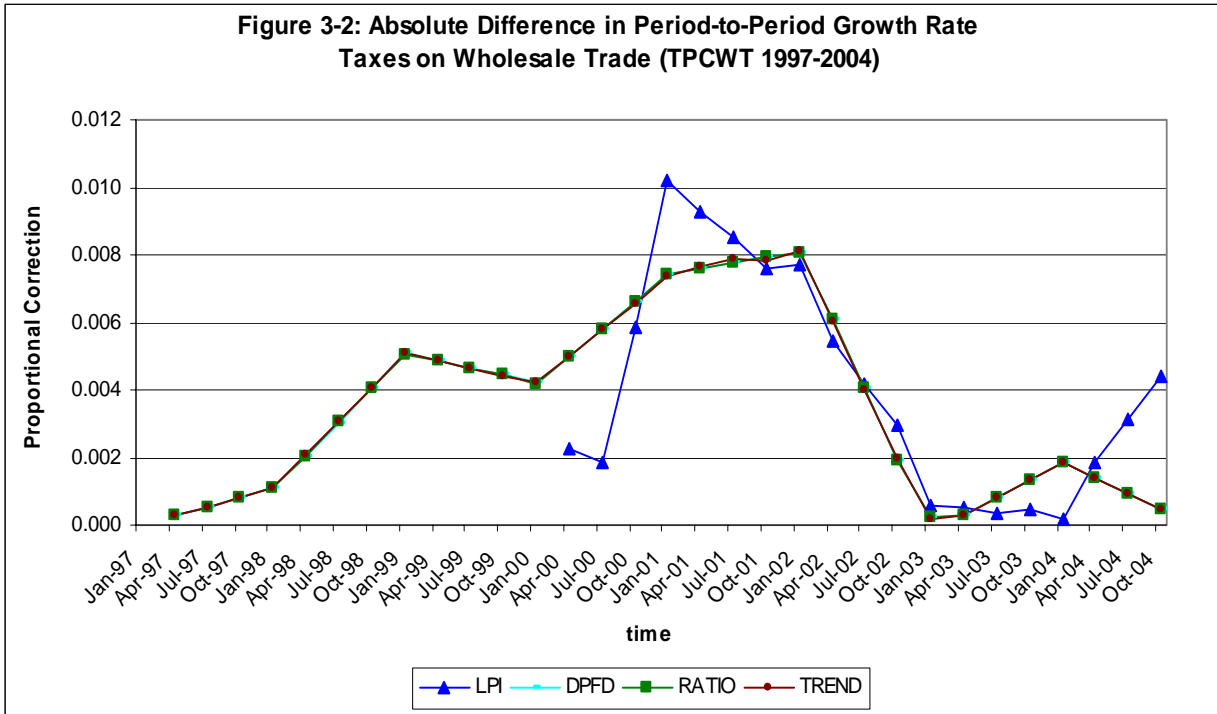
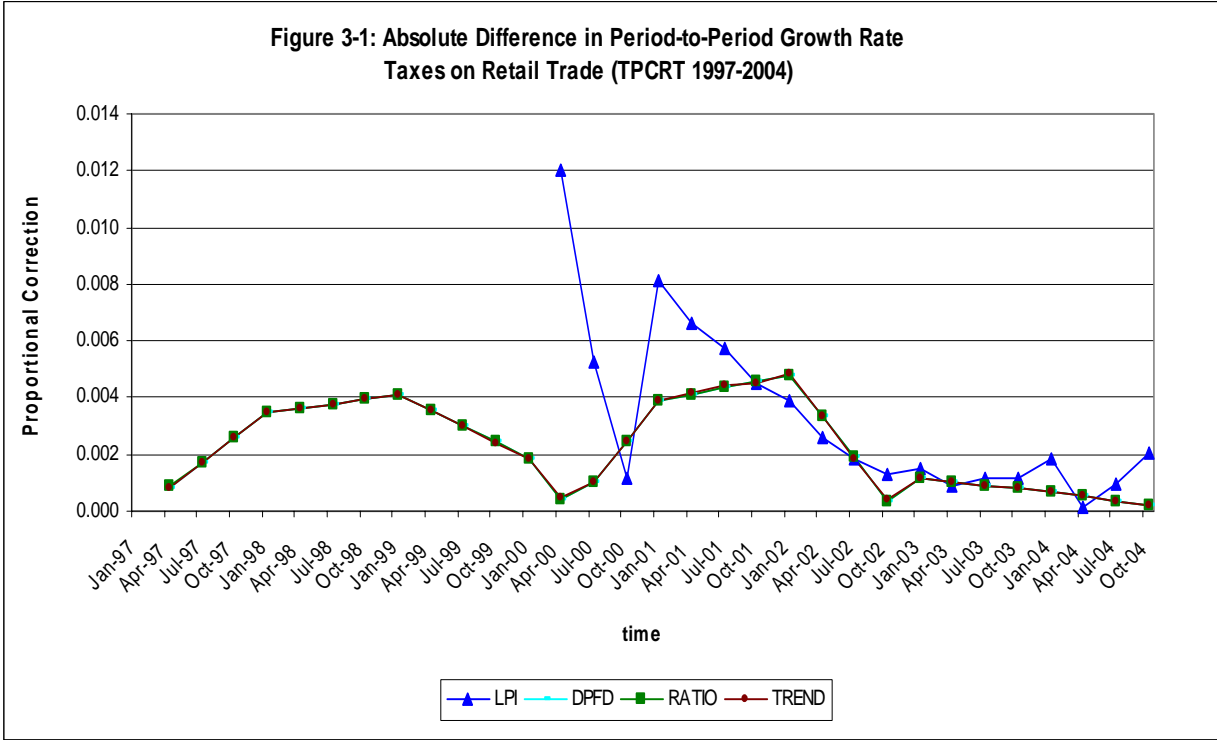












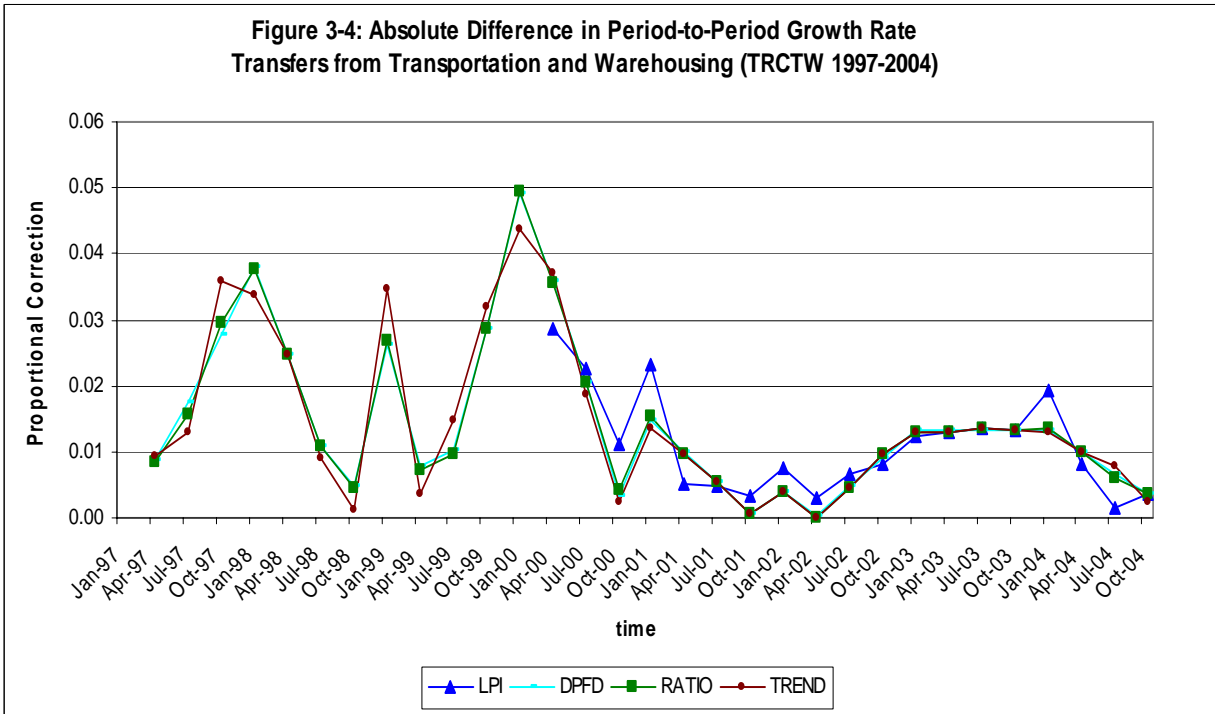
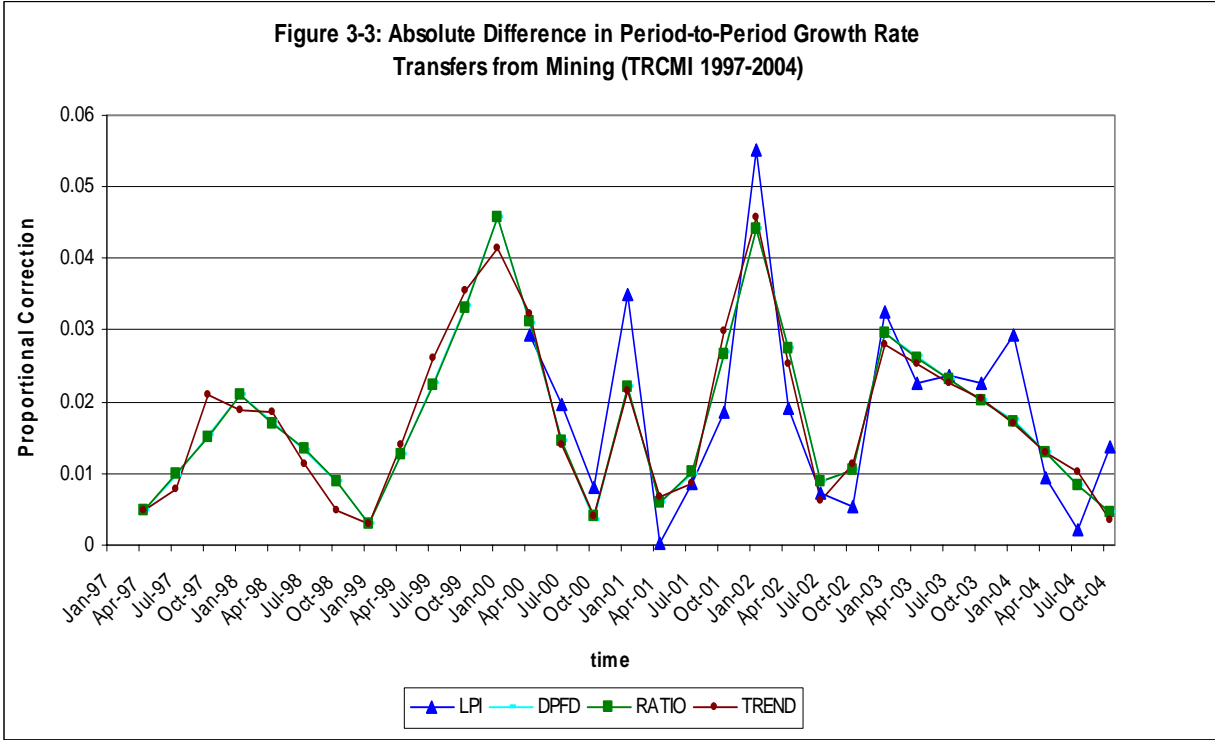


Figure 4-1: Final Sub-Annual Estimate
TDI Benefits (CGNTRTD 1993-2004)

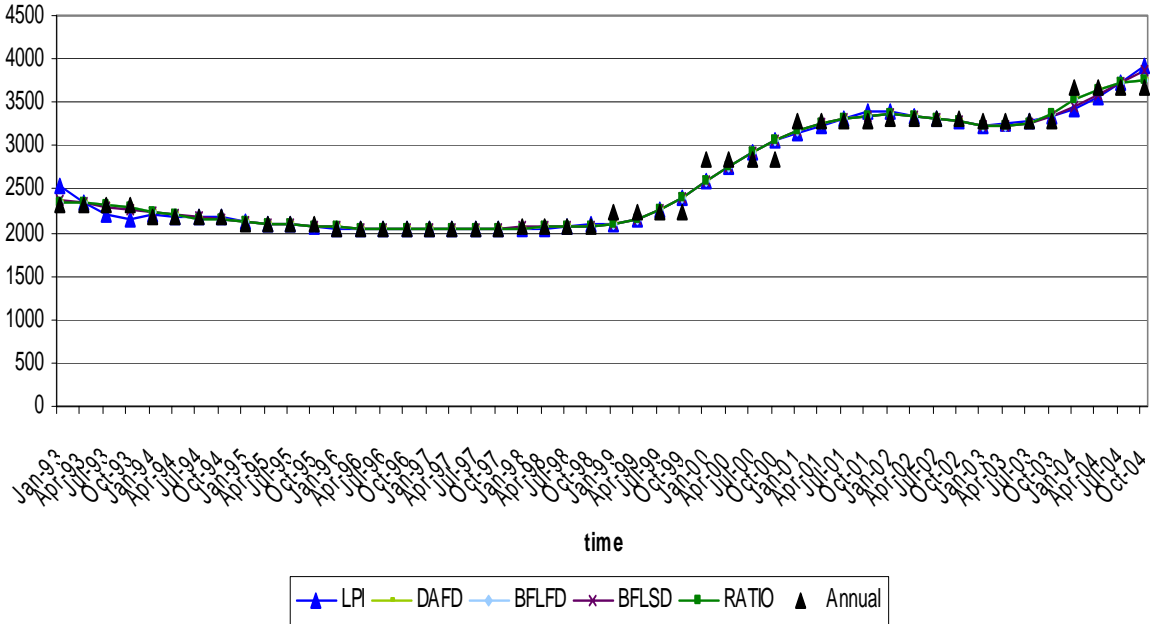
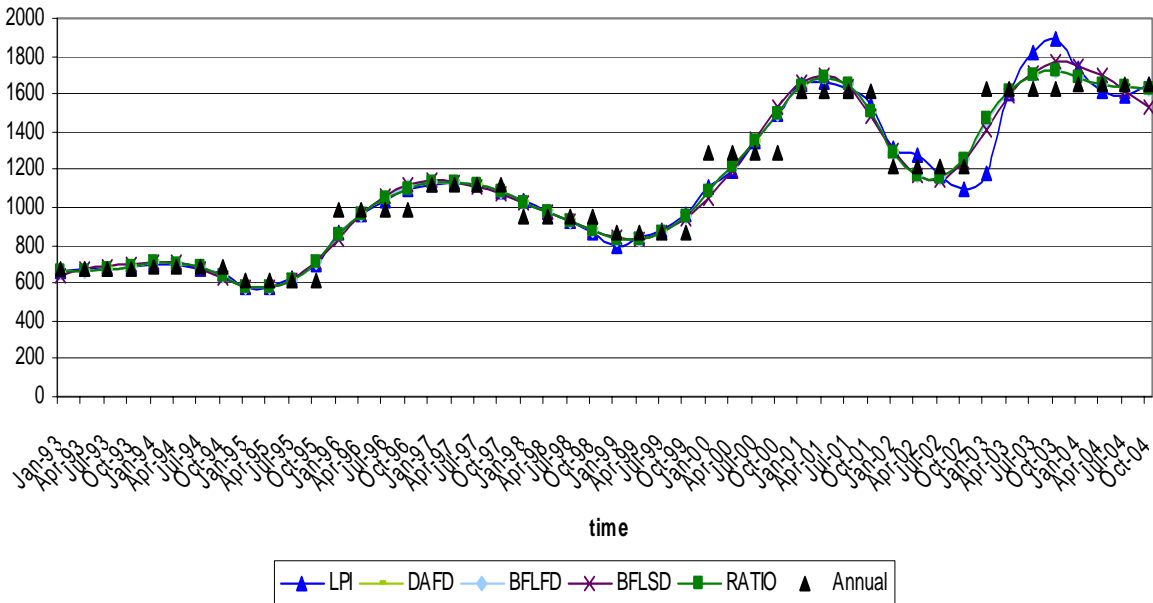
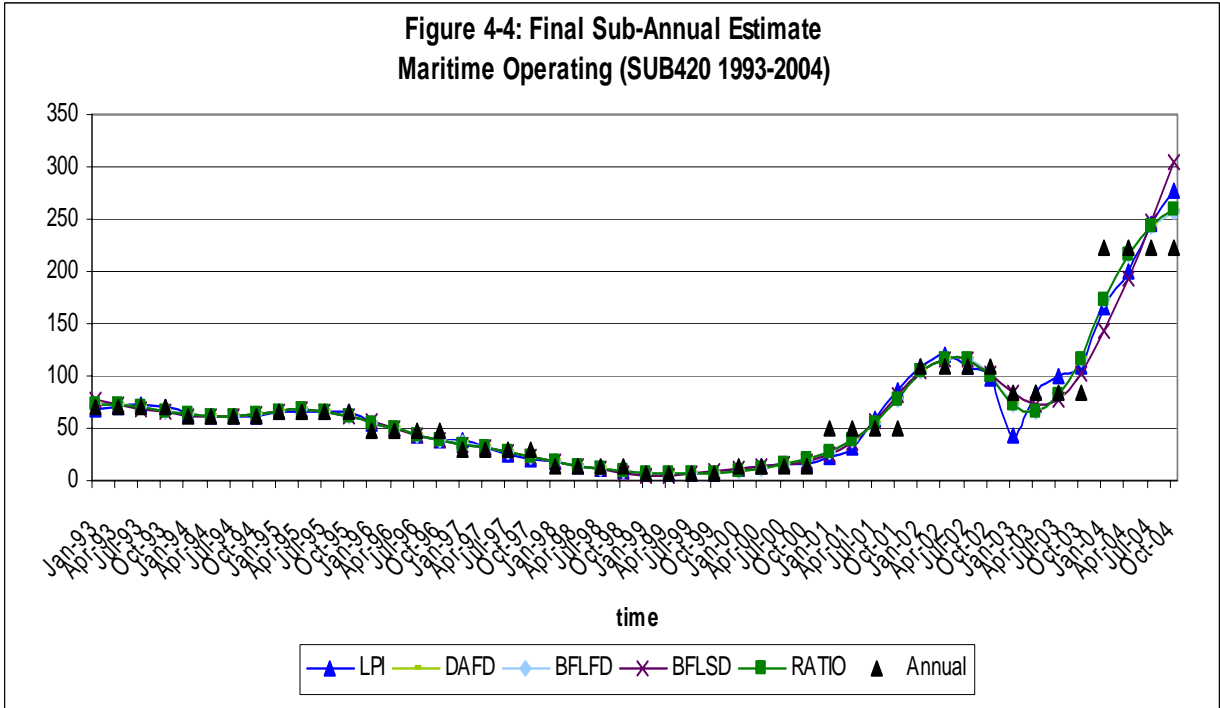
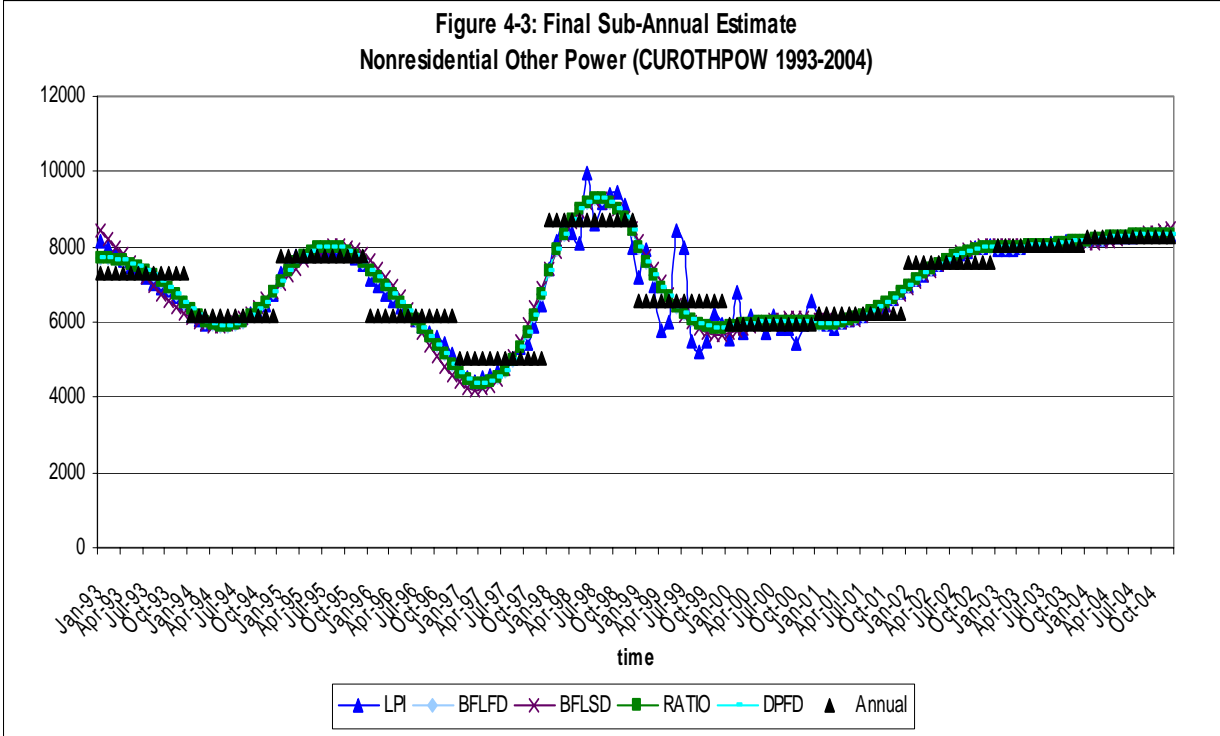
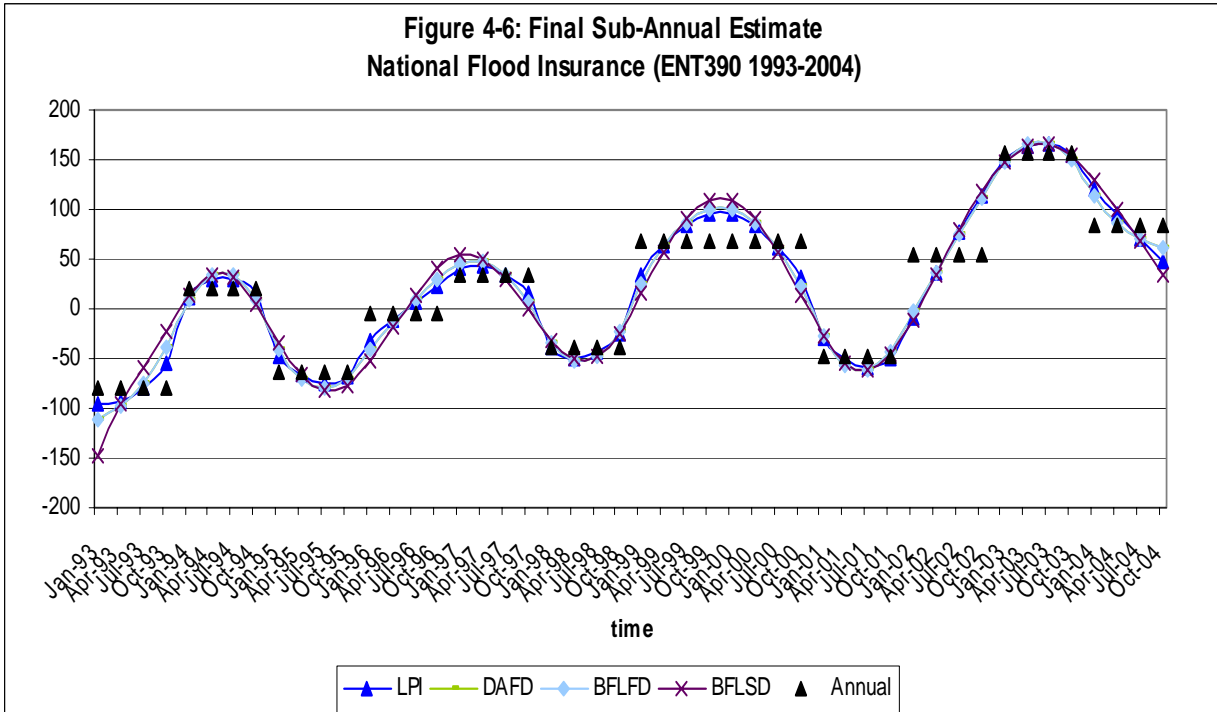
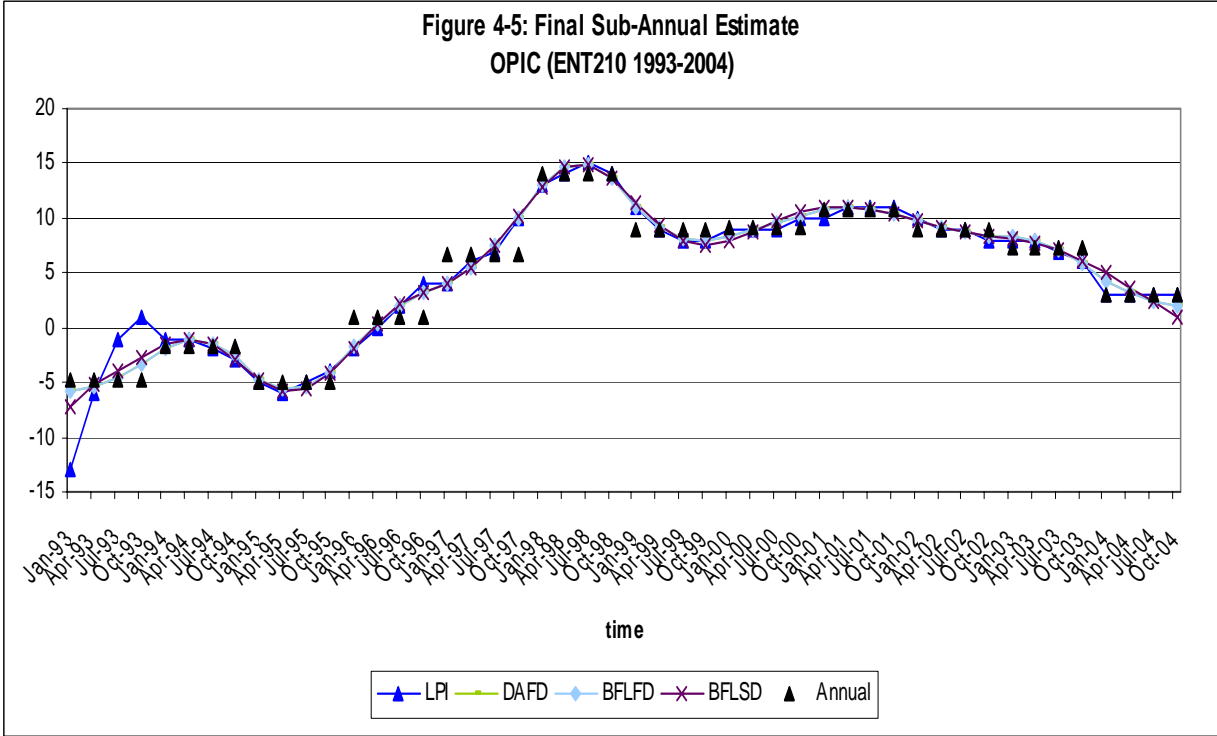
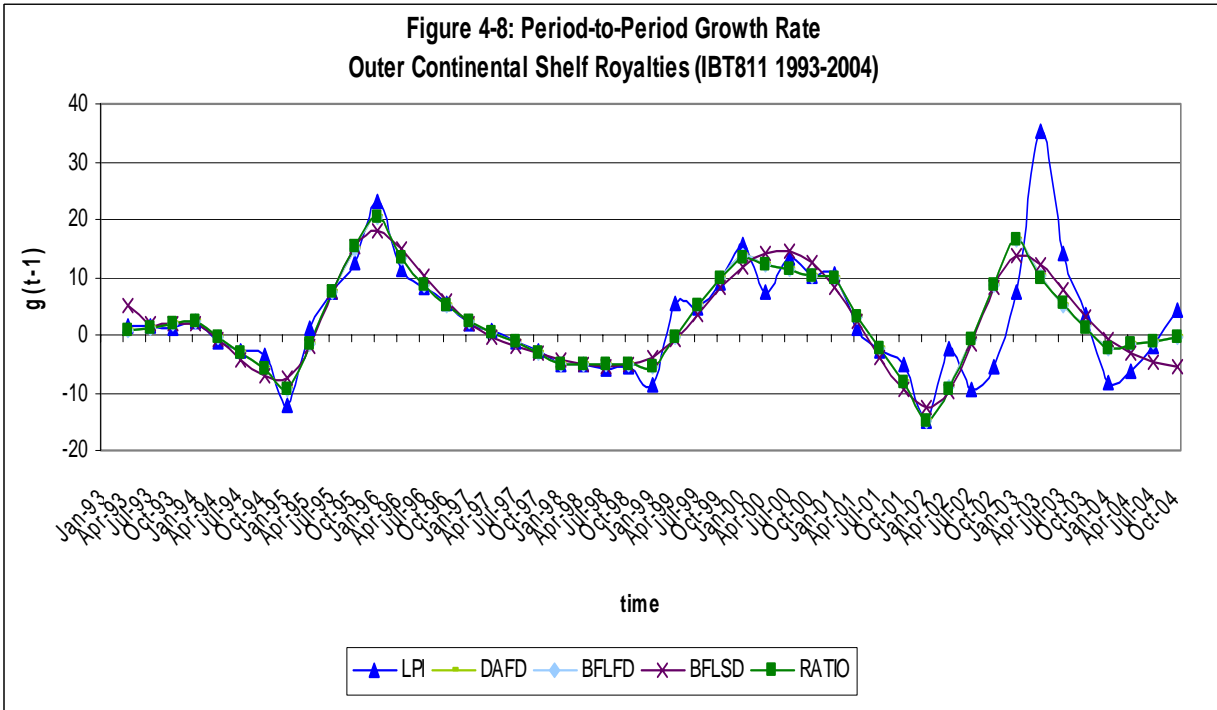
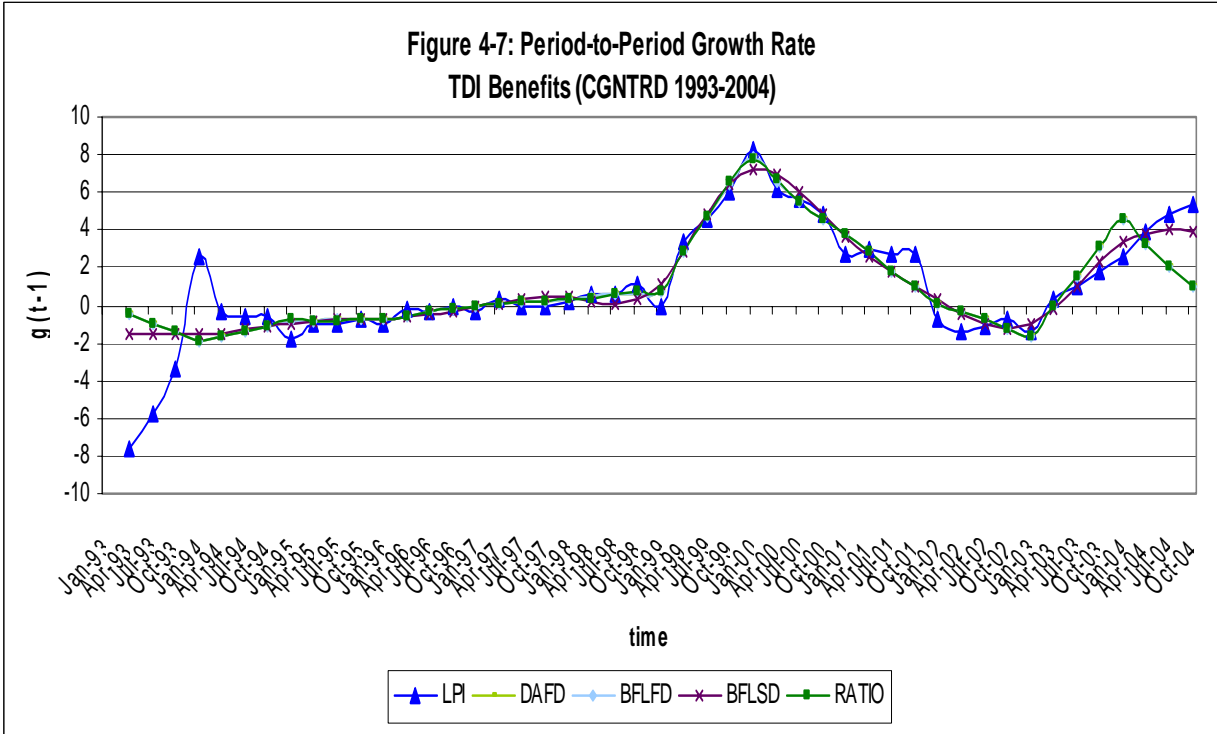


Figure 4-2: Final Sub-Annual Estimate
Outer Continental Shelf Royalties (IBT811 1993-2004)

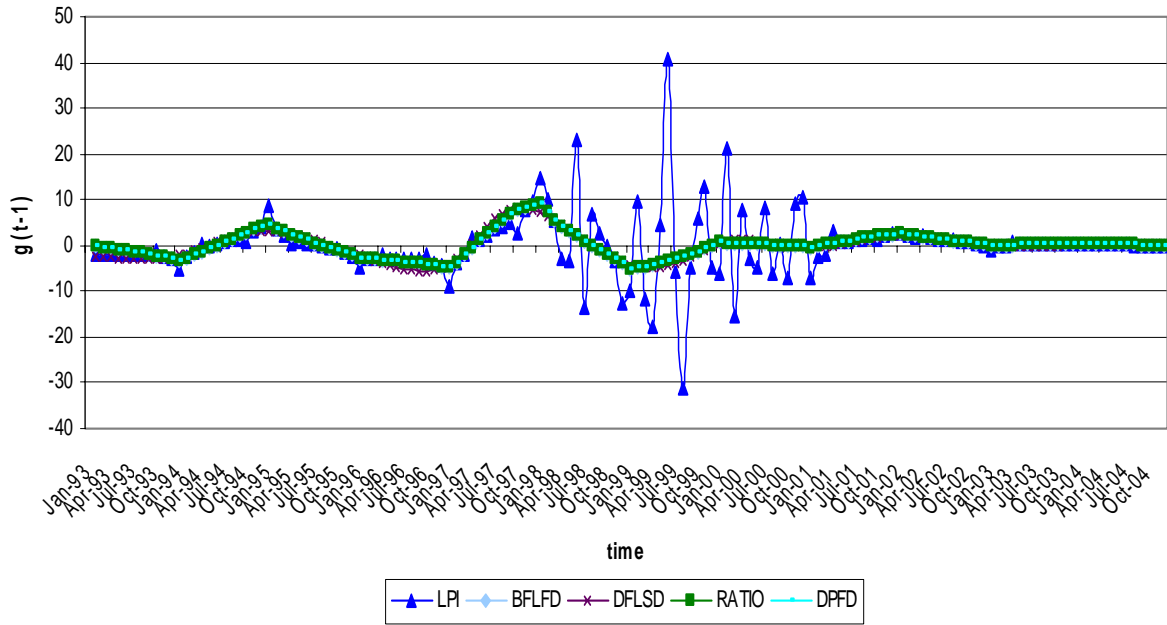




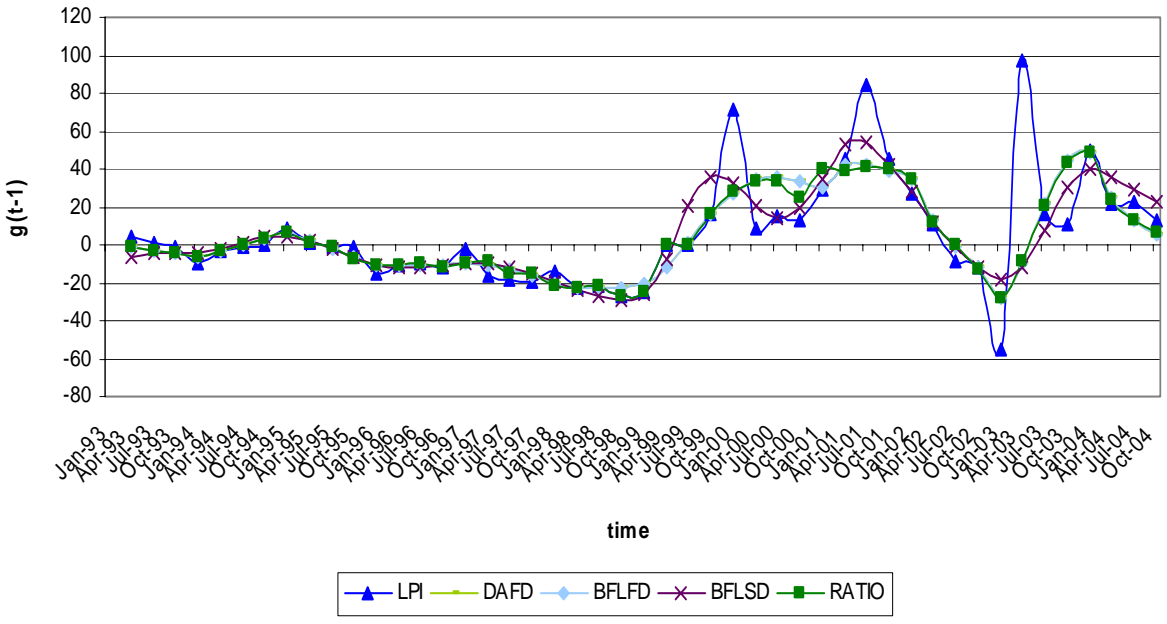




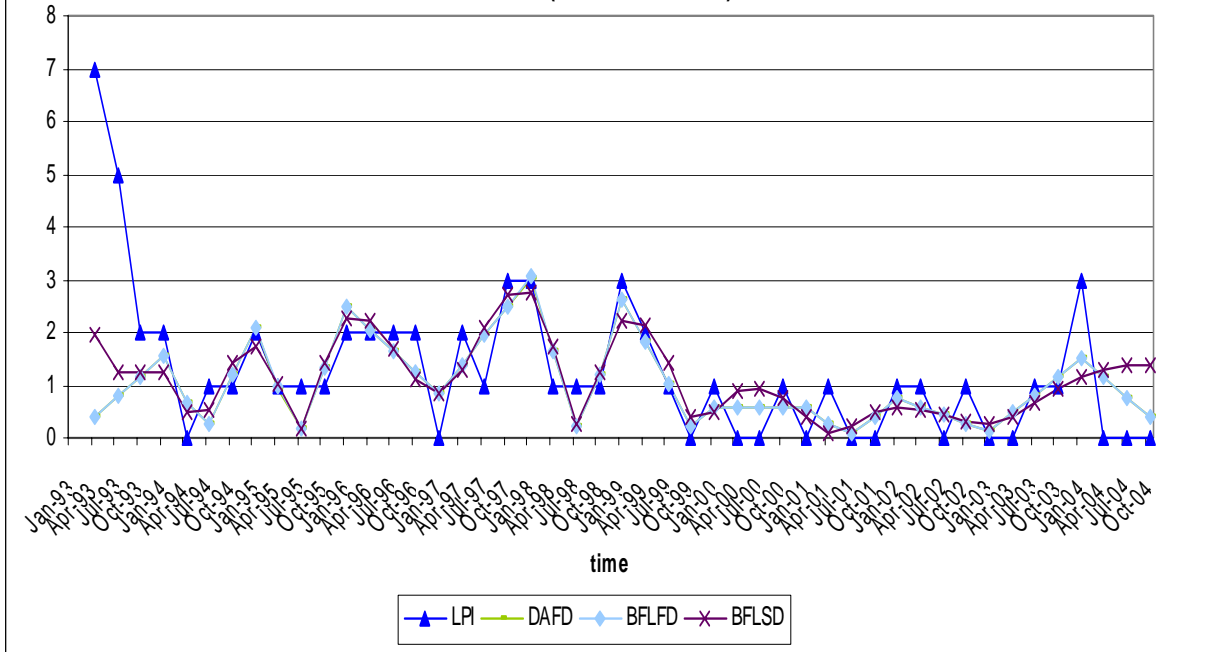
**Figure 4-9: Period-to-Period Growth Rate
Nonresidential Other Power (CUROTHPOW 1993-2004)**



**Figure 4-10: Period-to-Period Growth Rate
Maritime Operating (SUB420 1993-2004)**



**Figure 4-11: Absolute Period-to-Period Level Change
OPIC (ENT210 1993-2004)**



**Figure 4-12: Absolute Period-to-Period Level Change
National Flood Insurance (ENT390 1993-2004)**

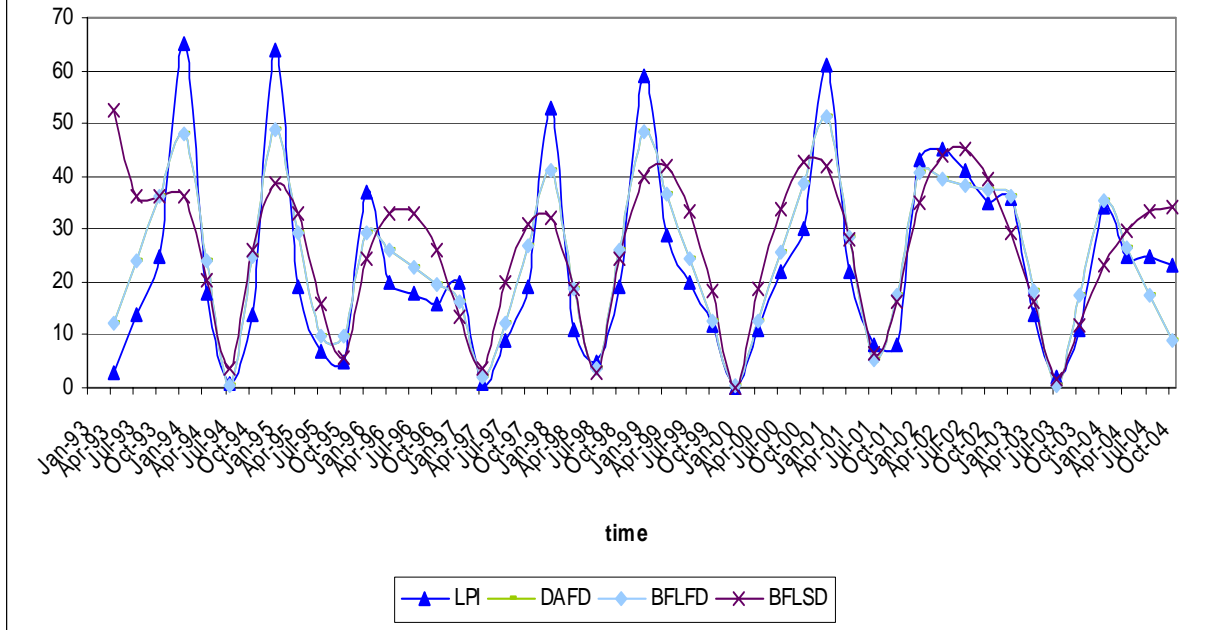


Table 1: Correlation Coefficient ρ between Annual and Sub-annual Indicators

NEA Series		Taxes		Transfer Payments	
Series	ρ	Series	ρ	Series	ρ
MIRPERSON	0.8130	TPCAG	0.9516	TRCAG	0.9443
XOTHOTHCOST	0.9236	TPCMI	0.8579	TRCMI	0.8968
XOTHOTHCCO	0.9273	TPCUT	0.9844	TRCUT	0.9124
GFOCXLOSS	0.9729	TPCAE	0.9918	TRCAE	0.9535
EOMHP	0.8711	TPCCO	0.9970	TRCCO	0.9647
EOSTP	0.9994	TPCDG	0.9971	TRCDG	0.9349
ETMHP	0.9795	TPCED	0.9954	TRCED	0.9345
ETSPP	0.9804	TPCFR	0.9941	TRCFR	0.8286
EIGAS1	0.9988	TPCIN	0.9721	TRCIN	0.9698
GFOLS	0.9927	TPCNG	0.9773	TRCNG	0.9503
CURMFG	0.9320	TPCOS	0.9985	TRCOS	0.9536
CURTUM	0.9996	TPCPF	0.9854	TRCPF	0.9757
CURMFU	0.9999	TPCRT	0.9951	TRCRT	0.9473
SFTWRE	0.9654	TPCTW	0.9550	TRCTW	0.9519
		TPCWT	0.9981	TRCWT	0.9460
		TPCGV	-0.6598		

Table 2-1: Proportional Discrepancies of NEA Series from All Methods

No.	Series	D_z^p		D_x^p								
		IND	LPI	DPFD	DAFD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
1	MIRPERSON	1.0502	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	XOTHOTHCOST	0.0399	1.0409	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	XOTHOTHCCO	0.0339	1.0332	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	GFOCXLOSS	1.0018	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	EOMHP	1.0240	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	EOSTP	1.0125	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	ETMHP	1.0050	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	ETSPP	0.9768	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	E1GAS1	0.1296	0.8997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	GFOL	1.0003	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	CURMFG	0.9098	0.9167	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	CURTUM	1.0008	0.9167	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	CURMFU	1.0002	0.8997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	SFTWRE	0.0021	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	MEAN	0.7276	0.9791	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2-2: Proportional Discrepancies of Taxes and Transfers from All Methods

No.	Series	D_z^p		D_x^p				No.	Series	D_z^p		D_x^p			
		IND	LPI	DPFD	RATIO	TREND	IND			LPI	DPFD	RATIO	TREND		
1	TPCAG	0.0079	1.0000	1.0000	1.0000	1.0000	17	TRCAG	0.0051	1.0000	1.0000	1.0000	1.0000		
2	TPCMI	0.0181	1.0000	1.0000	1.0000	1.0000	18	TRCMI	0.0832	1.0000	1.0000	1.0000	1.0000		
3	TPCUT	0.0453	1.0000	1.0000	1.0000	1.0000	19	TRCUT	0.0528	1.0000	1.0000	1.0000	1.0000		
4	TPCAE	0.0565	1.0000	1.0000	1.0000	1.0000	20	TRCAE	0.0189	1.0000	1.0000	1.0000	1.0000		
5	TPCCO	0.0073	1.0000	1.0000	1.0000	1.0000	21	TRCCO	0.0363	1.0000	1.0000	1.0000	1.0000		
6	TPCDG	0.0193	1.0000	1.0000	1.0000	1.0000	22	TRCDG	0.0720	1.0000	1.0000	1.0000	1.0000		
7	TPCED	0.0122	1.0000	1.0000	1.0000	1.0000	23	TRCED	0.0718	1.0000	1.0000	1.0000	1.0000		
8	TPCFR	0.2877	1.0000	1.0000	1.0000	1.0000	24	TRCFR	0.1748	1.0000	1.0000	1.0000	1.0000		
9	TPCIN	0.0496	1.0000	1.0000	1.0000	1.0000	25	TRCIN	0.0429	1.0000	1.0000	1.0000	1.0000		
10	TPCNG	0.0381	1.0000	1.0000	1.0000	1.0000	26	TRCNG	0.2017	1.0000	1.0000	1.0000	1.0000		
11	TPCOS	0.0219	1.0000	1.0000	1.0000	1.0000	27	TRCOS	0.0156	1.0000	1.0000	1.0000	1.0000		
12	TPCPF	0.0299	1.0000	1.0000	1.0000	1.0000	28	TRCPF	0.0899	1.0000	1.0000	1.0000	1.0000		
13	TPCRT	0.1944	1.0000	1.0000	1.0000	1.0000	29	TRCRT	0.0598	1.0000	1.0000	1.0000	1.0000		
14	TPCTW	0.0230	1.0000	1.0000	1.0000	1.0000	30	TRCTW	0.0598	1.0000	1.0000	1.0000	1.0000		
15	TPCWT	0.1885	1.0000	1.0000	1.0000	1.0000	31	TRCWT	0.0444	1.0000	1.0000	1.0000	1.0000		
16	TPCGV	0.0002	1.0000	1.0000	1.0000	1.0000									
	MEAN	0.0625	1.0000	1.0000	1.0000	1.0000		MEAN	0.0686	1.0000	1.0000	1.0000	1.0000		

D_z^p and D_x^p : proportional discrepancies with respect to the indicator and to the final estimates.

Table 3-1: Average Absolute Change in Period-to-Period Growth Rates of the 14 NEA Series from All Methods Used in Estimation

No.	Series	LPI	DPFD	DAFD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
1	MIRPERSON	0.021167	0.019644	0.019731	0.019644	0.019583	0.021569	0.021677	0.021124	0.021570	0.020747
2	XOTHOTHCOST	0.133133	0.073193	0.074254	0.073196	0.075050	0.091518	0.122100	0.134704	0.091679	0.134655
3	XOTHOTHCCO	0.139077	0.072490	0.073897	0.072493	0.074296	0.090358	0.120557	0.133172	0.090591	0.133112
4	GFOCXLOSS	0.005332	0.004029	0.004055	0.004030	0.004022	0.005208	0.004395	0.004614	0.004636	0.004339
5	EOMHP	0.004309	0.004007	0.003991	0.004009	0.004002	0.004229	0.004312	0.004412	0.004235	0.004341
6	EOSTP	0.000562	0.000453	0.000335	0.000456	0.000456	0.000517	0.000493	0.000523	0.000517	0.000519
7	ETMHP	0.003792	0.002064	0.002042	0.002062	0.002068	0.001283	0.001202	0.001263	0.001284	0.001446
8	ETSPP	0.002032	0.001540	0.001539	0.001540	0.001540	0.000992	0.000932	0.000979	0.001012	0.000987
9	E1GAS1	0.007328	0.000842	0.338758	0.000840	0.000841	0.001474	0.001000	0.000919	0.001504	0.000962
10	GFOL	0.036586	0.002612	0.002607	0.002612	0.002617	0.002124	0.002591	0.002714	0.002264	0.002715
11	CURMFG	0.013898	0.002238	0.005462	0.002239	0.002241	0.005300	0.005270	0.005476	0.001294	0.001498
12	CURTUM	0.010083	0.000770	0.000770	0.000772	0.000771	0.001103	0.001602	0.002184	0.000131	0.000192
13	CURMFU	0.010561	0.000165	0.000174	0.000165	0.000165	0.000184	0.000279	0.000705	0.000185	0.000705
14	SFTWRE	0.003651	0.004529	9.256027	0.008688	0.008688	0.004369	0.004883	0.004966	0.004397	0.004921
	MEAN	0.027965	0.013470	0.698832	0.013768	0.014024	0.017374	0.022031	0.024061	0.016992	0.023555
	STEDV	0.046776	0.025616	2.464566	0.025530	0.026180	0.033123	0.044423	0.049064	0.033385	0.049261

Note: Average absolute change in period-to-period growth rate measures how strong the short-term movement preservation is from final estimates generated from each method. Both mathematical and regression methods are used in the temporal disaggregation of the 14 NEA series.

Table 3-2: Average Absolute Change in Period-to-Period Growth Rates of taxes and Transfer Payment Series from All Methods Used in Estimation

No.	Series	LPI	DPFD	RATIO	TREND	No.	Series	LPI	DPFD	RATIO	TREND
1	TPCAG	0.006431	0.011371	0.011368	0.011369	17	TRCAG	0.013351	0.014940	0.014993	0.015030
2	TPCMI	0.028513	0.028179	0.028153	0.028008	18	TRCMI	0.019062	0.017891	0.017911	0.017824
3	TPCUT	0.004098	0.004910	0.004919	0.004916	19	TRCUT	0.017733	0.017084	0.017081	0.017095
4	TPCAE	0.003437	0.003621	0.003622	0.003619	20	TRCAE	0.011737	0.016984	0.016982	0.016821
5	TPCCO	0.007254	0.008035	0.008061	0.008044	21	TRCCO	0.011490	0.016933	0.016879	0.016468
6	TPCDG	0.001573	0.001938	0.001945	0.001945	22	TRCDG	0.014070	0.019178	0.019176	0.019165
7	TPCED	0.004984	0.003499	0.003526	0.003526	23	TRCED	0.013455	0.018675	0.018663	0.018666
8	TPCFR	0.004669	0.003052	0.003051	0.003050	24	TRCFR	0.029007	0.020065	0.020062	0.019831
9	TPCIN	0.005971	0.004057	0.004063	0.004063	25	TRCIN	0.008618	0.013094	0.013120	0.012951
10	TPCNG	0.007865	0.007526	0.007527	0.007531	26	TRCNG	0.029719	0.049168	0.049170	0.049002
11	TPCOS	0.002999	0.001962	0.001965	0.001963	27	TRCOS	0.011520	0.016679	0.016801	0.016528
12	TPCPF	0.004389	0.004072	0.004079	0.004078	28	TRCPF	0.008145	0.012897	0.012914	0.012550
13	TPCRT	0.003302	0.002215	0.002315	0.002313	29	TRCRT	0.012417	0.017723	0.017709	0.017575
14	TPCTW	0.013122	0.014782	0.014782	0.014796	30	TRCTW	0.011007	0.014489	0.014456	0.014437
15	TPCWT	0.003304	0.002680	0.002680	0.002682	31	TRCWT	0.012330	0.017084	0.017073	0.016974
16	TPCGV	0.032408	0.039481	0.041658	0.039787						
	MEAN	0.008395	0.008836	0.008982	0.008856		MEAN	0.014911	0.018859	0.018866	0.018728
	STEDV	0.009045	0.010596	0.011012	0.010629		STEDV	0.006520	0.008635	0.008632	0.008631

Note: Average absolute change in period-to-period growth rate measures how strong the short-term movement preservation is from final estimates generated from each method. Only mathematical methods are used in the temporal disaggregation of taxes and transfer payment series. LPI estimates are from 2000 to 2004 and estimates from other methods are from 1997 to 2004.

Table 4-1a: Distortions at Breaks between Years (C_B) of NEA Series from All Methods Used in Estimation

No.	Series	LPI	DPFD	DAFD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
1	MIRPERSON	NA	0.023008	0.023009	0.023008	0.023104	0.024147	0.024050	0.023277	0.024120	0.022932
2	XOTHOTHCOST	NA	0.087255	0.087255	0.087260	0.092153	0.103276	0.142011	0.158695	0.103421	0.158572
3	XOTHOTHCCO	NA	0.086506	0.086607	0.086512	0.091278	0.101939	0.139940	0.156539	0.102142	0.156395
4	GFOCXLOSS	0.006340	0.004395	0.004387	0.004395	0.004417	0.005000	0.005246	0.005280	0.004983	0.005280
5	EOMHP	0.004603	0.004479	0.004532	0.004481	0.004460	0.004839	0.004894	0.004567	0.004911	0.004450
6	EOSTP	0.000740	0.000517	0.000357	0.000520	0.000520	0.000580	0.000543	0.000524	0.000581	0.000523
7	ETMHP	0.002839	0.001467	0.001455	0.001485	0.001485	0.001858	0.001517	0.001333	0.001810	0.001591
8	ETSPP	0.001236	0.000898	0.000910	0.000899	0.000899	0.001079	0.001007	0.000981	0.001104	0.000987
9	E1GAS1	0.006609	0.000998	0.385888	0.000997	0.001008	0.001340	0.001015	0.001033	0.001367	0.001104
10	GFOL	0.037537	0.003389	0.003403	0.003390	0.003391	0.003272	0.003095	0.003076	0.003216	0.003076
11	CURMFG	0.016333	0.002544	0.006457	0.002546	0.002555	0.005676	0.005555	0.005475	0.005845	0.005495
12	CURTUM	0.013909	0.001004	0.001004	0.001009	0.001015	0.001482	0.001914	0.002562	0.001482	0.002795
13	CURMFU	0.009057	0.000210	0.000213	0.000208	0.000208	0.000247	0.000300	0.000647	0.000243	0.000647
14	SFTWRE	0.003956	0.005271	4.795107	0.009219	0.009219	0.004939	0.006167	0.005808	0.004950	0.005362
	MEAN	0.009378	0.015853	0.473065	0.016138	0.016837	0.019595	0.025468	0.027999	0.019633	0.027988
	STEDV	0.010558	0.030632	1.438067	0.030546	0.032269	0.037360	0.051637	0.057835	0.037419	0.057769

Note: Distortion at breaks between years is measured by the average absolute change in period-to-period growth rate between final estimates and indicator series of all sub-annual periods at breaks (4th to 1st quarters for quarterly data, Nov. to Feb. for monthly data) of all sample years.

Table 4-1b: Average Absolute Change in Period-to-Period Growth Rates of All Sub-annual Periods in the Middle of Each Sample Year (C_M) from NEA Series

No.	Series	LPI	DPFD	DAFD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
1	MIRPERSON	0.015665	0.016433	0.016436	0.016433	0.016222	0.019109	0.019412	0.019068	0.019137	0.018660
2	XOTHOTHCOST	0.089314	0.059769	0.059779	0.059771	0.058725	0.080294	0.103094	0.111804	0.080471	0.111825
3	XOTHOTHCCO	0.098526	0.059110	0.059180	0.059111	0.058086	0.079303	0.102055	0.110867	0.079565	0.110887
4	GFOCXLOSS	0.004127	0.003497	0.003554	0.003498	0.003463	0.003818	0.004010	0.004021	0.003725	0.004020
5	EOMHP	0.004223	0.003833	0.003784	0.003836	0.003834	0.003939	0.004079	0.004390	0.003911	0.004343
6	EOSTP	0.000485	0.000428	0.000329	0.000431	0.000431	0.000480	0.000472	0.000524	0.000480	0.000520
7	ETMHP	0.001496	0.001029	0.001016	0.001047	0.001052	0.001024	0.001064	0.001245	0.001048	0.001389
8	ETSPP	0.000911	0.000811	0.000811	0.000811	0.000811	0.000938	0.000894	0.000976	0.000953	0.000984
9	E1GAS1	0.007737	0.000776	0.317487	0.000773	0.000769	0.001535	0.000992	0.000867	0.000272	0.000203
10	GFOL	0.035682	0.001873	0.001851	0.001872	0.001881	0.001028	0.002110	0.002369	0.001356	0.002370
11	CURMFG	0.012363	0.002114	0.005038	0.002115	0.002114	0.005089	0.005101	0.005431	0.005073	0.005470
12	CURTUM	0.007751	0.000665	0.000665	0.000666	0.000660	0.000922	0.001464	0.002021	0.000959	0.002192
13	CURMFU	0.011392	0.000145	0.000156	0.000146	0.000146	0.000152	0.000267	0.000722	0.000155	0.000722
14	SFTWRE	0.003365	0.003833	13.438140	0.008191	0.008191	0.003835	0.003678	0.004177	0.003880	0.004508
	MEAN	0.020931	0.011023	0.993445	0.011336	0.011170	0.015202	0.018847	0.020331	0.015162	0.020276
	STEDV	0.032251	0.020917	3.582809	0.020834	0.020463	0.029097	0.037496	0.040682	0.029217	0.040706

Note: Each entry contains the average absolute change in period-to-period growth rates of all sub-annual periods in the middle of a year (2nd to 3rd quarter for quarterly data and Mar. to Oct. for monthly data) of all sample years.

Table 4-2a: Distortion at Breaks between Years (C_B) of Taxes and Transfer Payments from All Methods Used in Estimation

No.	Series	LPI	DPFD	RATIO	TREND	No.	Series	LPI	DPFD	RATIO	TREND
1	TPCAG	0.004006	0.014711	0.014748	0.014736	17	TRCAG	0.009569	0.017161	0.017387	0.017973
2	TPCMI	0.017983	0.033139	0.033123	0.033240	18	TRCMI	0.014683	0.020406	0.020390	0.020362
3	TPCUT	0.002694	0.005509	0.005517	0.005495	19	TRCUT	0.013601	0.019511	0.019466	0.019541
4	TPCAE	0.001787	0.003891	0.003885	0.003876	20	TRCAE	0.006814	0.019793	0.019713	0.020096
5	TPCCO	0.004287	0.008569	0.008542	0.008477	21	TRCCO	0.006817	0.020173	0.020119	0.020319
6	TPCDG	0.000994	0.002303	0.002282	0.002293	22	TRCDG	0.008304	0.022325	0.022373	0.022963
7	TPCED	0.002905	0.003724	0.003729	0.003705	23	TRCED	0.007538	0.021346	0.021356	0.022007
8	TPCFR	0.002744	0.003491	0.003491	0.003474	24	TRCFR	0.020035	0.022859	0.022851	0.022560
9	TPCIN	0.003072	0.004344	0.004351	0.004343	25	TRCIN	0.004809	0.015646	0.015757	0.015716
10	TPCNG	0.005173	0.009191	0.009205	0.009199	26	TRCNG	0.017785	0.060204	0.060218	0.058322
11	TPCOS	0.001776	0.002375	0.002377	0.002366	27	TRCOS	0.006660	0.019262	0.019646	0.019645
12	TPCPF	0.001988	0.004352	0.004336	0.004336	28	TRCPF	0.005246	0.015206	0.015230	0.015187
13	TPCRT	0.001707	0.002487	0.002486	0.002482	29	TRCRT	0.007027	0.020437	0.020451	0.020827
14	TPCTW	0.010877	0.018076	0.018067	0.018105	30	TRCTW	0.006800	0.016670	0.016898	0.016860
15	TPCWT	0.002663	0.003697	0.003697	0.003691	31	TRCWT	0.007491	0.020202	0.020205	0.020418
16	TPCGV	0.018741	0.047274	0.047523	0.048617						
	MEAN	0.005212	0.010446	0.010460	0.010527		MEAN	0.009545	0.022080	0.022137	0.022186
	STEDV	0.005618	0.012724	0.012772	0.013003		STEDV	0.004695	0.010788	0.010766	0.010258

Note: Distortion at breaks between years is measured by the average absolute change in period-to-period growth rate between final estimates and indicator series of all sub-annual periods at breaks (4th to 1st quarters for quarterly data, Nov. to Feb. for monthly data) of all sample years. LPI estimates are from 2000 to 2004 and estimates from other methods are from 1997 to 2004.

Table 4-2b: Average Absolute Change in Period-to-Period Growth Rates of All Sub-annual Periods in the Middle of Each Sample Year (C_M) of Taxes and Transfers

No.	Series	LPI	DPFD	RATIO	TREND	No.	Series	LPI	DPFD	RATIO	TREND
1	TPCAG	0.003881	0.008239	0.008199	0.008213	17	TRCAG	0.006884	0.012858	0.012749	0.012271
2	TPCMI	0.017000	0.023228	0.023494	0.023093	18	TRCMI	0.008871	0.015533	0.015587	0.015445
3	TPCUT	0.002341	0.004367	0.004359	0.004373	19	TRCUT	0.008307	0.014808	0.014844	0.014801
4	TPCAE	0.002341	0.004367	0.004359	0.004373	20	TRCAE	0.007550	0.014350	0.014421	0.013751
5	TPCCO	0.004595	0.007563	0.007610	0.007638	21	TRCCO	0.007254	0.013895	0.013842	0.012858
6	TPCDG	0.000937	0.001596	0.001630	0.001620	22	TRCDG	0.008923	0.016227	0.016178	0.015604
7	TPCED	0.003195	0.003288	0.003335	0.003358	23	TRCED	0.008912	0.016171	0.016138	0.015535
8	TPCFR	0.002972	0.002641	0.002639	0.002652	24	TRCFR	0.015663	0.017446	0.017448	0.017274
9	TPCIN	0.004211	0.003807	0.003793	0.003800	25	TRCIN	0.005726	0.010700	0.010648	0.010359
10	TPCNG	0.004490	0.005965	0.005954	0.005968	26	TRCNG	0.018618	0.038822	0.038813	0.040264
11	TPCOS	0.001897	0.001576	0.001578	0.001585	27	TRCOS	0.007436	0.014258	0.014133	0.013606
12	TPCPF	0.003348	0.003830	0.003838	0.003836	28	TRCPF	0.004755	0.010733	0.010744	0.010077
13	TPCRT	0.002320	0.002153	0.002154	0.002156	29	TRCRT	0.008158	0.015178	0.015138	0.014525
14	TPCTW	0.005385	0.011694	0.011703	0.011694	30	TRCTW	0.006695	0.012445	0.012167	0.012166
15	TPCWT	0.002335	0.003447	0.003446	0.003450	31	TRCWT	0.007619	0.014161	0.014137	0.013746
16	TPCGV	0.020915	0.030505	0.036160	0.030524						
	MEAN	0.005135	0.007392	0.007766	0.007396		MEAN	0.008758	0.015839	0.015799	0.015485
	STEDV	0.005563	0.008165	0.009306	0.008146		STEDV	0.003638	0.006638	0.006654	0.007131

Note: Each entry contains the average absolute change in period-to-period growth rates of all sub-annual periods in the middle of a year (2nd to 3rd quarter for quarterly data and March to October for monthly data) of all sample years.

Table 5-1a: Absolute Change in Period-to-Period Growth Rate between Final and Indicator Series in the Second Period (C_2) of NEA Series from all Methods

No.	Series	LPI	DPFD	DAFD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
1	MIRPERSON	0.000443	0.001073	0.001073	0.001073	0.001039	0.017312	0.010937	0.006471	0.017072	0.004062
2	XOTHOTHCOST	0.045531	0.009303	0.009303	0.009186	0.008289	0.116648	0.091610	0.087212	0.116653	0.087183
3	XOTHOTHCCO	0.026249	0.009230	0.009230	0.009092	0.008624	0.116038	0.091041	0.086632	0.116044	0.086601
4	GFOCXLOSS	0.007853	0.003692	0.004403	0.003701	0.003475	0.008857	0.009002	0.008804	0.008647	0.008878
5	EOMHP	0.003782	0.001241	0.001470	0.001224	0.000747	0.004238	0.000760	0.000332	0.004480	0.000021
6	EOSTP	0.003167	0.000832	0.000505	0.000828	0.000828	0.001041	0.000322	0.000354	0.001048	0.000319
7	ETMHP	0.007607	0.002218	0.002198	0.002204	0.002204	0.000141	0.000231	0.000036	0.000169	0.000554
8	ETSPP	0.003847	0.001287	0.001283	0.001283	0.001278	0.001998	0.001070	0.001209	0.002205	0.001280
9	E1GAS1	0.001092	0.000078	0.212876	0.000011	0.000003	0.001804	0.001055	0.000699	0.001878	0.000524
10	GFOL	0.033026	0.000581	0.000576	0.000587	0.000575	0.000922	0.003670	0.005313	0.001176	0.005315
11	CURMFG	0.049258	0.000014	0.000403	0.000003	0.000003	0.008260	0.008403	0.009842	0.007807	0.010026
12	CURTUM	0.057952	0.000106	0.000106	0.000029	0.000136	0.001060	0.000496	0.000473	0.000969	0.000511
13	CURMFU	0.027046	0.000151	0.000221	0.000149	0.000149	0.000332	0.000492	0.001729	0.000335	0.001727
14	SFTWRE	0.000002	0.003444	1.007543	0.015540	0.015540	0.001229	0.001381	0.000069	0.001554	0.000929
	MEAN	0.019061	0.002375	0.111962	0.003208	0.003064	0.021435	0.016853	0.016085	0.021422	0.015923
	STEDV	0.020461	0.003145	0.303743	0.004710	0.004594	0.042400	0.033263	0.031619	0.042395	0.031668

Note: The statistics C_2 measures distortion at the beginning of the final sub-annual series.

Table 5-1b: Absolute Change in Period-to-Period Growth Rate between Final and Indicator Values in the Last Period (C_T) of NEA Series from All Methods

No.	Series	LPI	DPFD	DAFD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
1	MIRPERSON	0.043712	0.005955	0.005955	0.005956	0.005492	0.032176	0.044425	0.037230	0.032772	0.032371
2	XOTHOTHCOST	0.012632	0.011093	0.011093	0.011327	0.012817	0.010289	0.052239	0.042231	0.010444	0.040693
3	XOTHOTHCCO	0.012786	0.011717	0.011717	0.011993	0.013430	0.009981	0.050552	0.038861	0.010173	0.037164
4	GFOCXLOSS	0.014604	0.000599	0.000894	0.000601	0.000631	0.004792	0.008068	0.008126	0.004230	0.008034
5	EOMHP	0.007308	0.000617	0.000407	0.000613	0.000614	0.001133	0.004646	0.003704	0.001761	0.003943
6	EOSTP	0.001078	0.000130	0.000115	0.000132	0.000132	0.000134	0.000359	0.001018	0.000140	0.001163
7	ETMHP	0.000858	0.000061	0.000005	0.000152	0.000152	0.001022	0.000190	0.000815	0.000913	0.001466
8	ETSPP	0.001090	0.000019	0.000192	0.000019	0.000019	0.000471	0.000399	0.000718	0.000693	0.000729
9	E1GAS1	0.009177	0.000171	0.184967	0.000109	0.000126	0.000895	0.000002	0.001454	-0.000928	0.001946
10	GFOL	0.054080	0.000869	0.000871	0.000869	0.000859	0.000307	0.000505	0.001064	0.000427	0.001081
11	CURMFG	0.013799	0.000001	0.008517	0.000000	0.000000	0.007744	0.002638	0.002246	0.009797	0.002370
12	CURTUM	0.006496	0.000102	0.000102	0.000152	0.000085	0.000750	0.000766	0.001297	0.000621	0.001356
13	CURMFU	0.001781	0.000057	0.000053	0.000037	0.000086	0.000332	0.000492	0.001729	0.000335	0.001727
14	SFTWRE	0.000905	0.002961	1.414260	0.007532	0.007532	0.004255	0.003198	0.003590	0.004628	0.002967
	MEAN	0.012879	0.002454	0.146398	0.002821	0.002998	0.005387	0.012714	0.010807	0.005491	0.010311
	STEDV	0.016236	0.004131	0.424106	0.004425	0.004862	0.008900	0.020920	0.016473	0.009195	0.015282

Note: The statistics C_T measures distortion at the end of the final sub-annual series.

Table 5-2a: Absolute Change in Period-to-Period Growth Rate between Final and Indicator Values in the Second Period (C_2) of Taxes and Transfer Payments

No.	Series	LPI	DPFD	RATIO	TREND	No.	Series	LPI	DPFD	RATIO	TREND
1	TPCAG	0.018493	0.009110	0.008296	0.008305	17	TRCAG	0.031642	0.007879	0.007591	0.007591
2	TPCMI	0.072552	0.010632	0.010337	0.010510	18	TRCMI	0.029310	0.004831	0.004924	0.004927
3	TPCUT	0.002654	0.002573	0.002400	0.002365	19	TRCUT	0.029292	0.005503	0.005590	0.005595
4	TPCAE	0.012119	0.001877	0.002002	0.001971	20	TRCAE	0.022738	0.010853	0.011757	0.011757
5	TPCCO	0.008943	0.005186	0.006252	0.005979	21	TRCCO	0.016262	0.009734	0.009696	0.009696
6	TPCDG	0.008943	0.000145	0.000502	0.000502	22	TRCDG	0.025447	0.011808	0.011704	0.012568
7	TPCED	0.011180	0.000902	0.001524	0.001524	23	TRCED	0.028500	0.011293	0.011294	0.011722
8	TPCFR	0.007507	0.000319	0.000302	0.000296	24	TRCFR	0.023732	0.003871	0.003902	0.003703
9	TPCIN	0.014039	0.000513	0.000380	0.000380	25	TRCIN	0.015417	0.007985	0.008165	0.007409
10	TPCNG	0.022426	0.002826	0.002664	0.002582	26	TRCNG	0.121623	0.040431	0.060572	0.040703
11	TPCOS	0.012504	0.000835	0.001127	0.001127	27	TRCOS	0.019182	0.010779	0.011391	0.011391
12	TPCPF	0.019175	0.002064	0.002330	0.002211	28	TRCPF	0.019182	0.007345	0.007353	0.007348
13	TPCRT	0.011990	0.000821	0.000855	0.000821	29	TRCRT	0.026207	0.011188	0.010822	0.011344
14	TPCTW	0.017095	0.011550	0.011888	0.011775	30	TRCTW	0.028514	0.008688	0.008476	0.009471
15	TPCWT	0.002261	0.000292	0.000262	0.000262	31	TRCWT	0.024032	0.010570	0.009975	0.010667
16	TPCGV	0.129279	0.030823	0.030470	0.030647						
	MEAN	0.023198	0.005029	0.005100	0.005079		MEAN	0.030739	0.010851	0.012214	0.011059
	STEDV	0.032526	0.007860	0.007727	0.007773		STEDV	0.025625	0.008557	0.013615	0.008649

Note: The statistics C_2 measures distortion at the beginning of the final sub-annual series. LPI estimates are from 2000 to 2004 and estimates from other methods are from 1997 to 2004.

Table 5-2b: Absolute Change in Period-to-Period Growth Rate between Final and Indicator Values in the Last Period (C_T) of Taxes and Transfer Payments

No.	Series	LPI	DPFD	RATIO	TREND	No.	Series	LPI	DPFD	RATIO	TREND
1	TPCAG	0.010793	0.001024	0.001899	0.001899	17	TRCAG	0.009570	0.005301	0.001394	0.005049
2	TPCMI	0.012411	0.001644	0.001232	0.001174	18	TRCMI	0.013677	0.004520	0.004478	0.003420
3	TPCUT	0.000366	0.000668	0.000842	0.000791	19	TRCUT	0.012528	0.004429	0.004047	0.003193
4	TPCAE	0.001566	0.000328	0.000230	0.000190	20	TRCAE	0.007930	0.003557	0.002308	0.003221
5	TPCCO	0.003585	0.003445	0.002725	0.002579	21	TRCCO	0.006229	0.003172	0.003003	0.002934
6	TPCDG	0.002355	0.000610	0.000205	0.000205	22	TRCDG	0.005519	0.003129	0.003178	0.002372
7	TPCED	0.001004	0.000183	0.000371	0.000371	23	TRCED	0.004393	0.003063	0.002927	0.002345
8	TPCFR	0.002443	0.000359	0.000347	0.000343	24	TRCFR	0.026826	0.022630	0.022577	0.013893
9	TPCIN	0.004657	0.001246	0.001383	0.001359	25	TRCIN	0.001812	0.003684	0.004256	0.003320
10	TPCNG	0.001705	0.001062	0.001272	0.001208	26	TRCNG	0.008472	0.004066	0.004085	0.003191
11	TPCOS	0.001722	0.000700	0.000472	0.000366	27	TRCOS	0.006256	0.003355	0.002964	0.000610
12	TPCPF	0.002068	0.000313	0.000055	0.000055	28	TRCPF	0.003099	0.003303	0.003419	0.002556
13	TPCRT	0.002064	0.000202	0.000172	0.000172	29	TRCRT	0.005842	0.003085	0.002958	0.001965
14	TPCTW	0.004204	0.002021	0.001716	0.001716	30	TRCTW	0.003715	0.003657	0.003694	0.002372
15	TPCWT	0.004416	0.000441	0.000467	0.000467	31	TRCWT	0.006607	0.003163	0.002574	0.001663
16	TPCGV	0.011953	0.035845	0.001698	0.001820						
	MEAN	0.004207	0.003131	0.000943	0.000920		MEAN	0.008165	0.004941	0.004524	0.003474
	STEDV	0.003928	0.008765	0.000784	0.000775		STEDV	0.006106	0.004936	0.005060	0.003046

Note: The statistics C_T measures distortion at the end of the final sub-annual series.

Table 6-1: Absolute Change in Period-to-Period Growth Rate between the Final and Indicator Values in the Linking Period (C_L) Using Linking Alternative 1: Setting Linking As An Initial Condition in the Optimization Model

Alternative 1:	Linking to Benchmarked Series			Linking to Benchmarked Series		
	Obtained Using the Same Method			Obtained Using LPI		
Series	DPFD**	RATIO**	TREND**	DPFD*	RATIO*	TREND*
MIRPERSON	0.019145	0.019975	0.019144	0.029351	0.028715	0.029350
XOTHOTHCCO	0.101260	0.101252	0.108766	0.124225	0.124217	0.134156
XOTHOTHCOST	0.103711	0.103699	0.111862	0.133326	0.133320	0.145946
GFOCXLOSS	0.005340	0.005327	0.005389	0.003868	0.003868	0.003906
GFOL	0.006112	0.006102	0.006065	0.023099	0.023099	0.023506
EOMHP	0.011240	0.011241	0.011500	0.011225	0.011226	0.011225
EOSTP	0.001217	0.001304	0.001304	0.001373	0.001380	0.001380
ETMHP	0.007767	0.007776	0.006539	0.005021	0.005014	0.005014
ETSP	0.005756	0.005756	0.005756	0.005249	0.005249	0.005249
MEAN	0.029061	0.029159	0.030703	0.037415	0.037343	0.039970

Note: ** stands for linking to benchmarked sub-annual values estimated using the same method, and * stands for linking to benchmarked sub-annual values estimated using the LPI procedure.

Table 6-2: Absolute Change in Period-to-Period Growth Rate between the Final and Indicator Values in the Linking Period (C_L) Using Linking Alternative 2: Replacing Previous Estimates with Revised Final Estimates in the Whole Sample

Alternative 2:	Linking to Benchmarked Series			Linking to Benchmarked Series		
	Obtained Using the Same Method			Obtained Using LPI		
Series	DPFD^	RATIO^	TREND^	DPFD	RATIO	TREND
MIRPERSON	0.015850	0.015851	0.017117	0.033120	0.033121	0.033187
XOTHOTHCCO	0.096004	0.095995	0.106978	0.138122	0.138113	0.143138
XOTHOTHCOST	0.098423	0.098405	0.110324	0.152860	0.152852	0.158419
GFOCXLOSS	0.006296	0.006271	0.006336	0.003744	0.003742	0.003807
GFOL	0.006747	0.006731	0.006701	0.043753	0.043751	0.043780
EOMHP	0.025510	0.025503	0.025254	0.025449	0.025439	0.024142
EOSTP	0.003523	0.003836	0.003836	0.004163	0.004145	0.004145
ETMHP	0.049781	0.049899	0.044998	0.038944	0.039000	0.039172
ETSP	0.017140	0.017142	0.017133	0.015085	0.015086	0.015078
MEAN	0.035475	0.035515	0.037631	0.050582	0.050583	0.051652

Note: ^ stands for using newly revised final estimates to replace previous estimates estimated using the same method.

Table 7-1: Contemporaneous Discrepancy in Two-Stage Estimation for Taxes

Date	Stage 1=LPI		Stage 2=DPFD		Stage 1=DPFD		Stage 2=DPFD		Stage 1=RATIO		Stage 2=DPFD		Stage 1=TREND		Stage 2=DPFD	
	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %
1/1/1997	NA	NA	NA	NA	-66.66	0.0112	0.45	0.0000	-0.11	0.0000	0.02	0.0000	-47.22	0.0079	0.03	0.0000
4/1/1997	NA	NA	NA	NA	-4.25	0.0007	4.02	0.0000	-1.44	0.0002	-0.01	0.0000	-28.41	0.0047	0.07	0.0000
7/1/1997	NA	NA	NA	NA	31.71	0.0051	0.91	0.0000	0.27	0.0000	-0.08	0.0000	15.20	0.0025	0.11	0.0000
10/1/1997	NA	NA	NA	NA	39.21	0.0063	-1.28	0.0000	1.28	0.0002	0.17	0.0000	60.43	0.0097	0.15	0.0000
1/1/1998	NA	NA	NA	NA	16.80	0.0027	-4.52	0.0000	-1.74	0.0003	0.44	0.0001	67.92	0.0108	0.19	0.0000
4/1/1998	NA	NA	NA	NA	1.04	0.0002	5.94	0.0000	0.39	0.0001	0.42	0.0001	16.60	0.0026	0.24	0.0000
7/1/1998	NA	NA	NA	NA	-7.96	0.0012	10.68	0.0000	0.61	0.0001	0.38	0.0001	-36.21	0.0056	0.28	0.0000
10/1/1998	NA	NA	NA	NA	-9.85	0.0015	-13.02	0.0000	0.74	0.0001	0.56	0.0001	-48.30	0.0074	0.32	0.0000
1/1/1999	NA	NA	NA	NA	-4.29	0.0007	4.87	0.0000	-1.60	0.0002	0.37	0.0001	-4.55	0.0007	0.37	0.0001
4/1/1999	NA	NA	NA	NA	-0.30	0.0000	-18.88	0.0000	-0.10	0.0000	0.03	0.0000	-11.27	0.0017	0.44	0.0001
7/1/1999	NA	NA	NA	NA	1.99	0.0003	4.28	0.0000	1.83	0.0003	-0.68	-0.0001	-6.91	0.0010	0.51	0.0001
10/1/1999	NA	NA	NA	NA	2.57	0.0004	3.23	0.0000	-0.13	0.0000	0.41	0.0001	22.73	0.0033	0.57	0.0001
1/1/2000	7192.00	1.0310	1.96	0.0000	1.21	0.0002	3.43	0.0000	-0.28	0.0000	1.28	0.0002	55.85	0.0080	0.66	0.0001
4/1/2000	343.00	0.0485	4.20	0.0000	0.18	0.0000	1.01	0.0000	-0.32	0.0000	0.12	0.0000	9.86	0.0014	0.78	0.0001
7/1/2000	-3422.00	0.4805	6.58	0.0000	-0.50	0.0001	4.02	0.0000	-0.16	0.0000	0.14	0.0000	-29.75	0.0042	0.88	0.0001
10/1/2000	-4113.00	0.5723	9.12	0.0000	-0.89	0.0001	-5.81	0.0000	0.75	0.0001	0.02	0.0000	-35.96	0.0050	1.00	0.0001
1/1/2001	-1721.00	0.2373	11.37	0.0000	-0.88	0.0001	-10.20	0.0000	0.35	0.0000	0.11	0.0000	-4.08	0.0006	1.13	0.0002
4/1/2001	-78.00	0.0107	13.21	0.0000	-0.52	0.0001	-25.36	0.0000	-0.22	0.0000	-0.53	-0.0001	-23.32	0.0032	1.25	0.0002
7/1/2001	821.00	0.1132	14.75	0.0000	0.19	0.0000	14.36	0.0000	1.47	0.0002	1.31	0.0002	-13.91	0.0019	1.36	0.0002
10/1/2001	978.00	0.1326	16.15	0.0000	1.25	0.0002	-6.84	0.0000	-1.61	0.0002	0.28	0.0000	41.32	0.0056	1.41	0.0002
1/1/2002	324.00	0.0434	17.24	0.0000	2.76	0.0004	-11.96	0.0000	1.16	0.0002	0.12	0.0000	122.90	0.0165	1.47	0.0002
4/1/2002	-65.00	0.0086	21.55	0.0000	2.27	0.0003	14.98	0.0000	0.93	0.0001	0.08	0.0000	28.07	0.0037	1.57	0.0002
7/1/2002	-259.00	0.0336	23.83	0.0000	-0.19	0.0000	8.07	0.0000	-0.12	0.0000	0.87	0.0001	-69.49	0.0090	1.67	0.0002
10/1/2002	0.00	0.0000	23.68	0.0000	-4.82	0.0006	-7.34	0.0000	-1.97	0.0003	-1.13	-0.0001	-81.48	0.0105	1.76	0.0002
1/1/2003	120.00	0.0153	25.45	0.0000	-11.64	0.0015	-42.07	-0.0001	-1.37	0.0002	1.75	0.0002	3.43	0.0004	1.83	0.0002
4/1/2003	68.00	0.0086	28.21	0.0000	-9.98	0.0013	-4.19	0.0000	1.11	0.0001	0.61	0.0001	17.11	0.0022	1.93	0.0002
7/1/2003	-32.00	0.0040	32.96	0.0000	0.68	0.0001	-32.99	0.0000	-1.22	0.0002	-0.05	0.0000	-3.13	0.0004	2.01	0.0002
10/1/2003	-156.00	0.0190	35.58	0.0000	20.96	0.0026	-17.05	0.0000	1.48	0.0002	-0.04	0.0000	-17.41	0.0021	2.09	0.0003
1/1/2004	-845.00	0.1009	38.66	0.0000	51.65	0.0062	-10.29	0.0000	1.12	0.0001	0.78	0.0001	-4.80	0.0006	2.14	0.0003
4/1/2004	-591.00	0.0697	40.09	0.0000	44.20	0.0052	-22.88	0.0000	-1.84	0.0002	-0.50	-0.0001	0.18	0.0000	2.14	0.0003
7/1/2004	123.00	0.0144	40.74	0.0000	-3.22	0.0004	-11.31	0.0000	0.05	0.0000	-0.16	0.0000	1.01	0.0001	2.16	0.0003
10/1/2004	1313.00	0.1508	41.78	0.0000	-92.62	0.0106	1.64	0.0000	0.67	0.0001	-0.54	-0.0001	3.62	0.0004	2.19	0.0003
Mean	0.00	0.1547	22.35	0.0000	0.00	0.0019	-5.13	0.0000	0.00	0.0001	0.20	0.0000	0.00	0.0042	1.08	0.0001
Std dev	2153.22	0.2589	12.83	0.0000	26.28	0.0030	13.20	0.0000	1.10	0.0001	0.59	0.0001	40.97	0.0040	0.76	0.0001

Table 7-2: Contemporaneous Discrepancy in Two-Stage Estimation for Transfers

Date	Stage 1=LPI		Stage 2=DPFD		Stage 1=DPFD		Stage 2=DPFD		Stage 1=RATIO		Stage 2=DPFD		Stage 1=TREND		Stage 2=DPFD	
	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %	Diff	Diff %
1/1/1997	NA	NA	NA	NA	-3.05	0.0109	0.00	0.0000	0.00	0.0000	0.00	0.0000	-149.00	0.5337	0.00	0.0000
4/1/1997	NA	NA	NA	NA	-0.29	0.0010	0.00	0.0000	1.00	0.0035	0.00	0.0000	-88.00	0.3110	0.00	0.0000
7/1/1997	NA	NA	NA	NA	1.60	0.0049	0.00	0.0000	-2.00	0.0061	0.00	0.0000	0.00	0.0000	0.00	0.0000
10/1/1997	NA	NA	NA	NA	1.72	0.0058	0.00	0.0000	1.00	0.0034	0.00	0.0000	237.00	0.8050	0.00	0.0000
1/1/1998	NA	NA	NA	NA	0.86	0.0027	0.00	0.0000	0.00	0.0000	0.00	0.0000	317.00	1.0000	0.00	0.0000
4/1/1998	NA	NA	NA	NA	0.13	0.0004	0.00	0.0000	-1.00	0.0032	0.00	0.0000	130.00	0.4208	0.00	0.0000
7/1/1998	NA	NA	NA	NA	-0.34	0.0010	0.00	0.0000	0.00	0.0000	0.00	0.0000	-127.00	0.3884	0.00	0.0000
10/1/1998	NA	NA	NA	NA	-0.65	0.0015	0.00	0.0000	1.00	0.0022	0.00	0.0000	-320.00	0.7169	0.00	0.0000
1/1/1999	NA	NA	NA	NA	-0.22	0.0006	0.00	0.0000	-1.00	0.0029	0.00	0.0000	-7.00	0.0200	0.00	0.0000
4/1/1999	NA	NA	NA	NA	-0.02	0.0001	0.00	0.0000	0.00	0.0000	0.00	0.0000	-51.00	0.1447	0.00	0.0000
7/1/1999	NA	NA	NA	NA	0.09	0.0003	0.00	0.0000	-1.00	0.0028	0.00	0.0000	-28.00	0.0783	0.00	0.0000
10/1/1999	NA	NA	NA	NA	0.13	0.0003	0.00	0.0000	2.00	0.0053	0.00	0.0000	86.00	0.2281	0.00	0.0000
1/1/2000	752.00	1.7735	0.00	0.0000	0.07	0.0002	0.00	0.0000	0.00	0.0000	0.00	0.0000	200.00	0.4717	0.00	0.0000
4/1/2000	36.00	0.0834	0.00	0.0000	0.01	0.0000	0.00	0.0000	0.00	0.0000	0.00	0.0000	75.00	0.1738	0.00	0.0000
7/1/2000	-357.00	0.8122	0.00	0.0000	-0.04	0.0001	0.00	0.0000	0.00	0.0000	0.00	0.0000	-114.00	0.2594	0.00	0.0000
10/1/2000	-431.00	0.9527	0.00	0.0000	-0.06	0.0001	0.00	0.0000	0.00	0.0000	0.00	0.0000	-161.00	0.3559	0.00	0.0000
1/1/2001	-179.00	0.3865	0.00	0.0000	-0.05	0.0001	0.00	0.0000	-1.00	0.0022	0.00	0.0000	26.00	0.0561	0.00	0.0000
4/1/2001	-8.00	0.0170	0.00	0.0000	-0.02	0.0000	0.00	0.0000	2.00	0.0043	0.00	0.0000	23.00	0.0490	0.00	0.0000
7/1/2001	86.00	0.1733	0.00	0.0000	0.02	0.0000	0.00	0.0000	1.00	0.0020	0.00	0.0000	-24.00	0.0484	0.00	0.0000
10/1/2001	101.00	0.2136	0.00	0.0000	0.07	0.0001	0.00	0.0000	-2.00	0.0042	0.00	0.0000	-25.00	0.0529	0.00	0.0000
1/1/2002	34.00	0.0727	0.00	0.0000	0.18	0.0004	0.00	0.0000	-2.00	0.0043	0.00	0.0000	26.00	0.0556	0.00	0.0000
4/1/2002	-6.00	0.0129	0.00	0.0000	0.13	0.0003	0.00	0.0000	0.00	0.0000	0.00	0.0000	12.00	0.0258	0.00	0.0000
7/1/2002	-29.00	0.0622	0.00	0.0000	-0.01	0.0000	0.00	0.0000	2.00	0.0043	0.00	0.0000	-15.00	0.0322	0.00	0.0000
10/1/2002	1.00	0.0021	0.00	0.0000	-0.28	0.0006	0.00	0.0000	0.00	0.0000	0.00	0.0000	-23.00	0.0493	0.00	0.0000
1/1/2003	30.00	0.0631	0.00	0.0000	-0.67	0.0014	0.00	0.0000	0.00	0.0000	0.00	0.0000	-19.00	0.0400	0.00	0.0000
4/1/2003	16.00	0.0330	0.00	0.0000	-0.58	0.0012	0.00	0.0000	0.00	0.0000	0.00	0.0000	-16.00	0.0330	0.00	0.0000
7/1/2003	-8.00	0.0162	0.00	0.0000	0.05	0.0001	0.00	0.0000	1.00	0.0020	0.00	0.0000	2.00	0.0040	0.00	0.0000
10/1/2003	-38.00	0.0760	0.00	0.0000	1.19	0.0024	0.00	0.0000	-1.00	0.0020	0.00	0.0000	33.00	0.0660	0.00	0.0000
1/1/2004	-191.00	0.3719	0.00	0.0000	2.98	0.0058	0.00	0.0000	0.00	0.0000	0.00	0.0000	59.00	0.1149	0.00	0.0000
4/1/2004	-134.00	0.2571	0.00	0.0000	2.55	0.0049	0.00	0.0000	-1.00	0.0019	0.00	0.0000	24.00	0.0461	0.00	0.0000
7/1/2004	23.00	0.0481	0.00	0.0000	-0.19	0.0004	0.00	0.0000	1.00	0.0021	0.00	0.0000	-26.00	0.0544	0.00	0.0000
10/1/2004	302.00	0.5518	0.00	0.0000	-5.38	0.0098	0.00	0.0000	0.00	0.0000	0.00	0.0000	-57.00	0.1042	0.00	0.0000
Mean	0.00	0.2990	0.00	0.0000	0.00	0.0018										

Table 8-1a: Average Absolute Period-to-Period Growth Rates of 10 Nonnegative Series with (with no Indicators)

No	Series	LPI	DAFD	BFLFD	BFLSD	RATIO
1	CGNTRMVGM	0.023405	0.021686	0.021686	0.022544	0.021698
2	CGNTRD	0.022476	0.018115	0.018115	0.019001	0.018124
3	CGNTRWC	0.012288	0.011966	0.011966	0.012215	0.011964
4	CGNTRMVCHP	0.147960	0.152658	0.153884	0.136917	0.153983
5	IBT811	0.070604	0.061159	0.061159	0.065522	0.061168
6	SUB230	0.008720	0.007409	0.007407	0.007485	0.007364
7	SUB270	0.021909	0.017439	0.017439	0.017856	0.017534
8	SUB420	0.196998	0.170361	0.172380	0.178834	0.169732
9	SUB370	0.084699	0.078003	0.078002	0.082908	0.078017
10	CUROTHPOW	0.039285	0.019513	0.019513	0.021267	0.019513
	MEAN	0.062834	0.055831	0.056155	0.056455	0.055910
	STEDV	0.063841	0.060243	0.060889	0.059506	0.060345

Table 8-1b: Average Absolute Period-to-Period Change in Level of 5 Series with Negative Values (with no Indicators)

No	Series	LPI	DAFD	BFLFD	BFLSD	RATIO
11	ENT210	1.276596	1.032340	1.032843	1.115596	NA
12	ENT230	30.446809	31.502340	31.502088	33.923819	NA
13	ENT260	1.000000	0.680000	0.684605	2.539333	NA
14	ENT330	6.680851	6.362979	6.362781	6.788759	NA
15	ENT390	84.454545	83.853636	83.852493	86.886250	NA
	MEAN	24.771760	24.686259	24.686962	26.250751	NA
	STEDV	35.49169395	35.42170396	35.42037586	36.42909631	NA

Table 8-2a: Period-to-Period Growth Rate in the Second and Last Periods for 10 Nonnegative Series (with no indicators)

No	Series	Period-to-period growth rate in second period					Period-to-period growth rate in period T				
		LPI	DAFD	BFLFD	BFLSD	RATIO	LPI	DAFD	BFLFD	BFLSD	RATIO
1	CGNTRMVGM	0.032407	0.008242	0.008234	0.033392	0.008301	0.009831	0.001589	0.001594	0.001853	0.001581
2	CGNTRD	0.076803	0.004719	0.004727	0.015832	0.004689	0.053591	0.010363	0.010368	0.039420	0.010215
3	CGNTRWC	0.010700	0.002202	0.002208	0.005663	0.002197	0.006607	0.002997	0.003004	0.009008	0.002891
4	CGNTRMVCHP	0.980769	0.855564	0.867715	0.705519	0.872928	0.031250	0.008663	0.008668	0.028051	0.008653
5	IBT811	0.016692	0.006208	0.006200	0.051918	0.006024	0.044795	0.005834	0.005828	0.053421	0.005511
6	SUB230	0.000000	0.002712	0.002688	0.006793	0.000000	0.000000	0.000258	0.000267	0.001478	0.000000
7	SUB270	0.022989	0.003803	0.003750	0.005436	0.011236	0.015000	0.006155	0.006165	0.020230	0.005000
8	SUB420	0.044118	0.013564	0.013675	0.061061	0.013699	0.130081	0.058041	0.058059	0.229874	0.061728
9	SUB370	0.054217	0.012591	0.012593	0.064276	0.011457	0.026087	0.000289	0.000289	0.032888	0.000000
10	CUROTHPOW	0.020180	0.002322	0.002331	0.025108	0.002331	0.000241	0.000269	0.000278	0.004725	0.000278
	MEAN	0.125887	0.091193	0.092412	0.097500	0.093286	0.031748	0.009446	0.009452	0.042095	0.009586
	STEDV	0.301219	0.268604	0.272445	0.214827	0.273975	0.039076	0.017457	0.017461	0.068245	0.018670

Table 8-2b: Period-to-Period Change in Level in the Second and Last Period for 5 Series with Negative Values (with no indicators)

No	Series	Period-to-period level change in second period					Period-to-period level change in period T				
		LPI	DAFD	BFLFD	BFLSD	RATIO	LPI	DAFD	BFLFD	BFLSD	RATIO
11	ENT210	7.000000	0.390000	0.392000	1.950306	NA	0.000000	0.380000	0.379384	1.381385	NA
12	ENT230	21.000000	11.250000	11.242637	33.943410	NA	2.000000	1.750000	1.750832	1.334403	NA
13	ENT260	1.000000	1.490000	1.489087	5.028955	NA	1.000000	0.680000	0.684605	2.539333	NA
14	ENT330	10.000000	0.740000	0.744938	3.643498	NA	10.000000	5.090000	5.094136	18.364176	NA
15	ENT390	3.000000	12.050000	12.051963	52.603882	NA	23.000000	8.850000	8.849186	34.293185	NA
	MEAN	8.400000	5.184000	5.184125	19.434010	NA	7.200000	3.350000	3.351629	11.582496	NA
	STEDV	7.861298	5.922743	5.920239	22.766691	NA	9.679876	3.598173	3.597543	14.599927	NA

Table 9-1: Comparative Evaluation of the Methods Used in the Estimation of the 14 NEA Series According to the Test Criteria

Mean of Statistics	LPI	DPFD	DPAD	RATIO	TREND	AR(1)MAX	RAWK	RAWKM MAX	AR(1)MIN	RAWKM MIN
D_X^P	0.979053	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
C^P	0.027965	0.013470	0.874160	0.013768	0.014024	0.017374	0.022031	0.024061	0.016992	0.023555
C_B	0.009378	0.015853	0.473065	0.016138	0.016837	0.019595	0.025468	0.027999	0.019633	0.027988
C_2	0.019061	0.002375	0.111962	0.003208	0.003064	0.021435	0.016853	0.016085	0.021422	0.015923
C_T	0.012879	0.002454	0.146398	0.002821	0.002998	0.005387	0.012714	0.010807	0.005491	0.010311
C_L	NA	0.029061	NA	0.029159	0.030703	NA	NA	NA	NA	NA

Note: Each entry contains the mean of the test statistics of the 14 series from a particular method used in estimation.

Table 9-2: Comparative Evaluation of the Methods Used in Estimation of Series on Taxes and Transfer Payments According to the Test Criteria

Mean of Statistics	16 series of taxes on production and imports				15 series of transfer payments			
	LPI	DPFD	RATIO	TREND	LPI	DPFD	RATIO	TREND
D_X^P	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
C^P	0.008395	0.008836	0.008982	0.008856	0.014911	0.018859	0.018866	0.018728
C_B	0.005212	0.010446	0.010460	0.010527	0.009545	0.022080	0.022137	0.022186
C_2	0.023198	0.005029	0.005100	0.005079	0.030739	0.010851	0.012214	0.011059
C_T	0.004207	0.003131	0.000943	0.000920	0.008165	0.004941	0.004524	0.003474
D_X^C (%)	0.154714	0.024831	0.024834	0.024855	0.296904	0.001800	0.001835	0.000397

Note: Each entry in the left panel contains the mean of the test statistics of the 16 series on taxes from a particular method used in estimation, and each entry in the right panel contains the corresponding mean statistics of the 15 series on transfer payments. Estimates from LPI are from 2000 to 1997 and estimates from other methods are from 1997 to 2004.

Table 9-3: Comparative Evaluation of the Methods Used in Estimation of Series without Sub-annual Indicators According to the Test Criteria

Mean of Statistics	Average statistics for 10 non-negative series					Average statistics for 5 series with negative values				
	LPI	DAFD	BFLFD	BFLSD	RATIO	LPI	DAFD	BFLFD	BFLSD	RATIO
g_t	6.283447	5.583084	5.615520	5.645487	5.590992	24.771760	24.686259	24.686962	26.250751	NA
g_2	12.588737	9.119256	9.241199	9.749978	9.328617	8.400000	5.184000	5.184125	19.434010	NA
g_T	3.174829	0.944566	0.945199	4.209480	0.958577	7.200000	3.350000	3.351629	11.582496	NA

Note: Each entry in the left panel contains the mean of the test statistics of the 10 nonnegative series without indicators, and each entry in the right panel contains the corresponding test statistics of the 5 series with negative values.

Appendix: Information on Series and Software Used in the Experiment

Table A1: Series Used in Estimation Experiment

	Series	Definition
		Series with Indicators
1	MIRPERSON.a <i>JTOTN.Q</i>	Monetary Interest Paid by Persons
2	XOTHOTHCOST.A <i>MTGVALORIG.Q</i>	Total Closing Cost <i>Mortgage Originations, seasonally adjusted</i>
3	XOTHOTHCCO.A <i>MTGVALORIG.Q</i>	Owner-Occupied Closing Cost <i>Mortgage Originations, seasonally adjusted</i>
4	GFOCXLOSS\$.A <i>JGFOCXLOSS\$.Q</i>	Crop Output <i>Constant-Dollar Gross Crop Output Excluding Losses</i>
5	EOMHP <i>EOMH</i>	Owner-Occupied Mobile Homes <i>Extrapolated using Mobile Home Housing Stock</i>
6	EOSTP <i>EOST</i>	Owner-Occupied Stationary Homes <i>Extrapolated using Stationary Unit Housing Stock</i>
7	ETMHP <i>ETMH</i>	Tenant-Occupied Mobile Homes <i>Extrapolated using Mobile Home Housing Stock</i>
8	ETSPP <i>ETSP</i>	Tenant-Occupied Stationary Homes <i>Extrapolated using Stationary Unit Housing Stock</i>
9	E1GAS1.A <i>E1GASOA.M</i>	Consumption of Gasoline <i>Monthly Indicator of Gasoline Consumption</i>
10	GFOL\$.A <i>JGFOL\$.Q</i>	Livestock output <i>Constant-Dollar Gross Livestock Output</i>
11	CURMFG.A <i>VIPMFG.M</i>	Nonresidential Manufacturing <i>Monthly Value Estimates for Nonresidential Const.</i>
12	CURTUM.A <i>VIPTCM.M</i>	Nonresidential Telecommunications <i>Monthly Value Estimates for Telecommunications Const.</i>
13	CURMFU.A <i>VIPMFU.M</i>	Residential Multi-Family <i>Monthly Value Estimates for Multi-Family Const.</i>
14	SFTWRE <i>QIND_PRE_SA.Q</i>	Investment in Pre-Packaged Equipment & Software <i>Quarterly Indicator</i>
15	TPCAG <i>TPCTL</i>	Taxes on Ag., Forestry, Fishing, Hunting <i>Total Taxes</i>
16	TPCMI <i>TPCTL</i>	Taxes on Mining <i>Total Taxes</i>
17	TPCUT <i>TPCTL</i>	Taxes on Utilities <i>Total Taxes</i>
18	TPCAE <i>TPCTL</i>	Taxes on Arts, Entertainment, etc. <i>Total Taxes</i>
19	TPCCO <i>TPCTL</i>	Taxes on Construction <i>Total Taxes</i>
20	TPCDG <i>TPCTL</i>	Taxes on Durable Goods <i>Total Taxes</i>
21	TPCED <i>TPCTL</i>	Taxes on Education, Health, Social Assistance <i>Total Taxes</i>
22	TPCFR <i>TPCTL</i>	Taxes on Fin., Ins., Real Est., Rental, Leasing <i>Total Taxes</i>
23	TPCIN <i>TPCTL</i>	Taxes on Information <i>Total Taxes</i>
24	TPCNG <i>TPCTL</i>	Taxes on Nondurable Goods <i>Total Taxes</i>
25	TPCOS <i>TPCTL</i>	Taxes on Other Services, Except Government <i>Total Taxes</i>
26	TPCPF <i>TPCTL</i>	Taxes on Professional & Business Services <i>Total Taxes</i>
27	TPCRT <i>TPCTL</i>	Taxes on Retail Trade <i>Total Taxes</i>
28	TPCTW <i>TPCTL</i>	Taxes on Transportation & Warehousing <i>Total Taxes</i>
29	TPCWT <i>TPCTL</i>	Taxes on Wholesale Trade <i>Total Taxes</i>
30	TPCGV <i>TPCTL</i>	Taxes on Government Enterprises <i>Total Taxes</i>
31	TRCAG <i>TRCTL</i>	Transfers from Ag., Forestry, Fishing, Hunting <i>Total Transfers</i>
32	TRCMI <i>TRCTL</i>	Transfers from Mining <i>Total Transfers</i>

Table A1: Series Used in Estimation Experiment (Cont.)

	Series	Definition
		Series with Indicators
33	TRCUT <i>TRCTL</i>	Transfers from Utilities <i>Total Transfers</i>
34	TRCAE <i>TRCTL</i>	Transfers from Arts, Entertainment, etc. <i>Total Transfers</i>
35	TRCCO <i>TRCTL</i>	Transfers from Construction <i>Total Transfers</i>
36	TRCDG <i>TRCTL</i>	Transfers from Durable Goods <i>Total Transfers</i>
37	TRCED <i>TRCTL</i>	Transfers from Education, Health & Social Assistance <i>Total Transfers</i>
38	TRCFR <i>TRCTL</i>	Transfers from Fin., Ins., Real Est., Rental, Leasing <i>Total Transfers</i>
39	TRCIN <i>TRCTL</i>	Transfers from Information <i>Total Transfers</i>
40	TRCNG <i>TRCTL</i>	Transfers from Nondurable Goods <i>Total Transfers</i>
41	TRCOS <i>TRCTL</i>	Transfers from Other Services, Except Government <i>Total Transfers</i>
42	TRCPF <i>TRCTL</i>	Transfers from Professional & Business Services <i>Total Transfers</i>
43	TRCRT <i>TRCTL</i>	Transfers from Retail Trade <i>Total Transfers</i>
44	TRCTW <i>TRCTL</i>	Transfers from Transportation & Warehousing <i>Total Transfers</i>
45	TRCWT <i>TRCTL</i>	Transfers from Wholesale Trade <i>Total Transfers</i>
		Series without Indicators
46	CGNTRMVG	State & Local Transfers: General Medical Assistance
47	CGNTRTD	TDI Benefits
48	CGNTRWC	Workers Compensation Benefits
49	CGNTRMVCHP	State & Local Transfers: Child Health Program
50	ENT210	OPIC
51	ENT230	Bonneville Power
52	ENT260	Colorado River Basins
53	ENT370	TVA
54	ENT390	National Flood Insurance
55	IBT811	Outer Continental Shelf Royalties
56	SUB230	Housing-235-236-Rent Supplement
57	SUB270	Rental Assistance
58	SUB310	Amtrak
59	SUB420	Maritime Operating
60	CUROTHPOW.A	Nonresidential Other Power

Note: The indicators and their definitions are in italic letters.

TPC-series: Taxes on production & imports. TRC-series: Business transfers to government.

ENT-series: Current surplus of government enterprise. SUB-series: Subsidies.

IBT-series: Indirect business Taxes.