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- A Representative Study –

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The Impact of Order Size on Stock Liquidity - A Representative Study

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Abstract

Liquidity, the ease of trading an asset, strongly varies between different sizes of stock positions. We analyze this aspect using the Xetra Liquidity Measure (XLM), which calculates daily, weighted spread for impatient traders transacting against the limit order book. For this measure, we have data for 160 German stocks over 5.5 years, which allows us a representative analysis of the order-size impact on liquidity cost and its main statistical characteristics.

We find that in the sample period average liquidity costs rose to over 100bp in large DAX and to 460bp in large SDAX positions. Over the last 5.5 years, liquidity has equally improved across all order sizes. Liquid position sizes, however, suffered less badly during the recent sub-prime crises, which represents another type of the flight-to-liquidity.

As the basis for further theoretical analysis, we find that trends in liquidity levels and inefficiencies in liquidity prices of large positions generate non-normality in the liquidity distribution. We also show that - as a rule of thumb - liquidity of an order size relative to market value and transaction volume is constant across stocks and time. While order size is not the most important liquidity determinant, doubling order size increases liquidity cost by 5-10% on average when accounting for other differences in stocks.

Keywords: Asset liquidity, liquidity cost, price impact, weighted spread, Xetra Liquidity Measure (XLM)
JEL classification: G10, G32
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Contents

1 Introduction

Liquidity has lately received much attention in the academic world and in practice. In fact, a stock position cannot be bought or sold without cost or delay in execution. The most important cost is the spread, the difference between the achievable transaction price and the fair price of a stock. This spread serves as important measure of the liquidity of an asset. Moreover, if volume traded in the stock is not large enough, the investor has to delay his trade, which induces further costs. From an investor perspective, the liquidity of an asset can be measured by the total cost required to trade a position in an asset.

How can one measure this cost of trading a position? Academic literature has brought forward a multitude of cost measures. Starting with Roll (1984) and Amihud and Mendelson (1986), many papers have analyzed variants of the bid-ask-spread, data which is easily available. But this measure neglects that spread differs for different order sizes. Only small positions, smaller than the bid-ask-depth, can be traded at such a cost.

Larger positions incur larger costs, the so called price impact (of the position's size), which results because supply and demand curves for stocks are not perfectly elastic. Initially the price impact was measured with proxies.[1] The problem with estimating liquidity cost ex-post from transaction prices is to distinguish between the informational and the liquidity component in the price change. Pastor and Stambaugh (2003) used a method based on price changes with subsequent reversals, but were not able to distill stock-specific liquidity measures.[2]

More recently, a direct method of measuring size-specific spread has been proposed. When order book data is available, the price of instant liquidity for a position of a certain size can be extracted as weighted spread in the limit order book. Under the assumption that a position is transacted as a market-order against available limit-orders, the difference between the realized price and the mid-point of the bid-ask-spread measures the price impact of the trade due to liquidity. As this is an ex-ante measure of committed liquidity, informational effects of a transaction cannot play a role here. Exchanges increasingly use transparent, electronic limit order books, for example the London Stock Exchange, the Nasdaq, the Frankfurt Xetra, the Euronext or the Australian Stock Exchange. They also start to make these weighted spread data available to researchers.

[1] Cp. for example Kyle (1985); Amihud (2002).

[2] Cp. Kyle (1985); Brennan and Subrahmanyam (1996); Amihud (2002) for other approaches.

Exchanges using electronic limit order books start to make these weighted spread data available, for example . Hence, the above method of calculating liquidity becomes more generally applicable.

Several papers have already used new methods of measuring liquidity costs. Irvine et al. (2000) use the cost of a round trip for trades of various sizes as a liquidity measure, which they compare to quoted and effective spread. Empirically, they show that the measure is correlated with other measures of liquidity and that it predicts the number of trades of a certain size. Coppejans et al. (2001) employ a similar measure to analyze the relation between market liquidity, returns and volatility in an intraday sample. They reveal a large inter-temporal variation and show that liquidity is concentrated on certain points in time. Coppejans et al. (2004) discuss the stochastic dynamics of liquidity with a measure similar to the cost of a round trip and find a negative relation to volatility and a high degree of resiliency, i.e. high mean reversion speed of liquidity prices after shocks. Domowitz et al. (2005) employ the cost of round trip to analyze liquidity commonality and show that market liquidity and returns can remain uncorrelated because they are caused by different economic forces. While liquidity is driven by liquidity supply and demand (i.e. cross-correlation between limit and market orders), returns are driven by correlation in order flow (i.e. order direction and size). Gomber et al. (2004) extract weighted spread from the limit order book to show that resiliency is generally high after liquidity shocks and public information has negligible impact on liquidity. They also show that large transactions are timed on periods with high liquidity.

Common to all papers above is the methodology to manually extract intraday time-series of size-specific spreads from the limit order book. As this involves large amounts of data, empirical samples are usually restricted to few months and few stocks.[3] In contrast, we have been provided with a more representative sample of weighted spread, size-specific liquidity costs for 160 stocks over 5.5 years. While all other papers use intraday data, we look into daily data, which in many applications are more relevant.

Our representative sample of over 320 thousand stock-days allows us to shed some light on the fundamental question, whether the size impact is substantial enough to receive dedicated attention from a theoretical and practical point-of-view. What are the benefits of using weighted spread, liquidity data differentiated by order size? What are the general statistical features of this type of data? This venue has not

[3] Cp. appendix 9 for an overview.

been tackled in research based on smaller samples available so far. In detail, we will cover the following three aspects:

1. How large are liquidity costs for a certain position size and how did they change over time? While this question is more descriptive of nature, representative empirical estimates provide a reference and can illustrate the usefulness of this measure for practical applications.

2. How is daily, order-size-differentiated liquidity distributed? A representative answer to this question can be directly used in risk management methods and is the basis for empirical analysis or theoretical models on size-differentiated liquidity.

3. What is the role of order-size in explaining size-specific liquidity cost when controlling for a variety of other stock characteristics? We will use the broadness of our sample to directly estimate the order-size impact on liquidity costs in an univariate analysis.

We therefore contribute to the existing literature by distilling stylized facts on order-size differentiated liquidity. This will clarify the usefulness of this new measure for practical applications, like asset allocation, asset pricing or risk management. It can also provide stimulus for further theoretical research such as the role of order-size-differentiated liquidity in asset pricing or the order book dynamics.

The reminder of the paper is organized as follows: Section 2 describes the theoretical framework to integrate our analysis in the existing context. Section 3 describes our data and introduces the XLM liquidity measure. Our empirical analysis can be found in section 4. Section 5 summarizes our main results and concludes.

2 Definition of size-adjusted liquidity

We define illiquidity as the cost of trading an asset relative to fair value.[4] The mid-point of the bid-ask-spread is assumed as fair price. We distinguish three cost components of the relative liquidity cost L(q) in percent of the mid-price[5] for an order quantity q

$$L(q) := T(q) + PI(q) + D(q) \tag{1}$$

[4] Cp. Amihud and Mendelson (2006); Loebnitz (2006); Buhl (2004).
[5] Mid-price is the mid-point of the bid-ask-spread.

where T(q) are direct trading costs, PI(q) is the price impact vs. mid-price due to the size of the position, D(q) are delay costs in the case a position cannot be traded immediately.[6]

Direct trading costs include exchange fees, brokerage commissions and transaction taxes. These are also often called explicit transaction costs, but their main feature is that they are deterministic.[7] The *price impact* is the difference between the transaction price and the mid-price, which result from imperfectly elastic demand and supply curve for stocks. For small volumes this is the bid-ask-spread, but for larger volumes price impact will be larger. *Delay costs* comprise costs for searching a counter-party and the cost imposed on the investor due to price risk and price impact risk during the delay.[8] For many assets like most stocks and bonds on an exchange the search costs are negligibly small.

This cost definition takes a practical, concrete investor's perspective and can integrate other definitions in the literature. First, liquidity is often abstractly defined as "the ease of trading an asset".[9] From an investor's point of view, ease is only a question of money. Kempf (1999) identifies a price and a time dimension. In the definition we suggest, price is specified as the liquidation price which is achieved by subtracting liquidity cost from the mid-price. The time dimension is also converted into a cost measure via delay cost.

Second, definition (1) can also easily integrate the commonly cited aspects of breadth, depth and resilience.[10] Breath is the tightness of the bid-ask-spread, i.e. the cost of transacting a position up to a certain size at short notice, which is included in the price impact PI(q). Depth is defined as the minimum quotation volume, i.e. the maximum volume q that can be traded at the bid-ask-spread PI(q). To be more precise, the bid-ask-spread does not represent the minimum price discount costs but the guaranteed minimum costs, because transactions can occur inside the bid-ask-spread. Resilience is the speed with which prices revert to their equilibrium level after a shock in the transaction flow.[11] It measures the change of liquidity over time dL(q)/dt in the specific situation after a shock in the transaction volume.

In above framework, liquidity is the effect a transaction has for an investor. This perspective integrates the multitude of liquidity aspects discussed in the literature.

[6] This definition closely follows Amihud and Mendelson (2006), but additionally differentiates by the size of the position.

[7] Cp. Loebnitz (2006), p.18 f.

[8] Almgren (2003) calls price impact risk "trading enhanced risk".

[9] Cp. for example Longstaff (1995).

[10] Cp. Garbade and Silber (1982), Kyle (1985) and Harris (1990).

[11] Put differently: "the extend of bearing large-order flow in one direction without affecting the market price", Amihud and Mendelson (2006).

While most other measures have been judged by their effect on asset prices, a cost perspective provides an intermediate conceptional step. If a liquidity aspect results in high liquidity costs in economic downturns, it will have a large effect on asset prices. This conceptional step represents the economic explanation for the validity of certain liquidity aspects in asset pricing, because liquidity aspects that generate substantial costs in poor states will price assets.

Assets can have different degrees of liquidity. Most stocks can be traded immediately without delay, but with a price impact. In this paper, we analyze this type of liquidity. We assume that all our stocks are at least immediately tradable. In many cases, an investor can deliberately trade-off between a larger price impact and higher delay costs. This aspect is detailed on a stream of literature on trading strategies.[12] We take the worst-case perspective of impatient traders and assume that all positions have to be traded immediately as market orders. In this case there is no deliberate trade-off and delay costs D(q) are zero. We neglect the possibility of issuing limit instead of market orders as well as up-floor trading opportunities, where positions are settled outside the limit order book. As a result, our result is an upper bound to real liquidity costs, which is valid for scenarios of forced liquidation, e.g. for investors with margin calls.

There is an additional theoretical argument, why the assumption of zero delay cost is realistic in a multitude of cases. If markets are efficient and liquidity prices are efficient, then the marginal gain from lower transaction cost by delaying a transaction will equal the marginal loss due to higher delay cost. If liquidity prices are efficient, than the average investor will be just as well of by liquidating immediately against the order book than from employing complicated optimal delay strategies.

For very large or institutional traders, T(q) can be considered negligible. On the Xetra system of the Deutsche Börse, for example, institutional traders pay a negligible amount of around 0.5bp as transaction fees.[13] Transaction cost T(q) can also be neglected if time variation of liquidity is of major interest. To simplify the analysis, we assume that transaction costs T(q) are zero.

Our analysis will focus focus on the price impact of a trade PI(q), which represents, as argued above the most substantial liquidity cost component.

[12] Cp. for example Almgren and Chriss (1999, 2000); Almgren (2003); Bertsimas and Lo (1998) and others.

[13] Cp. Deutsche Boerse (2008).

3 Description of data

We have obtained liquidity data from the Xetra system of the Frankfurt Stock Exchange, which covers the bulk of stock transactions in Germany. Deutsche Börse is among the top 10 largest stock exchanges in the world and Xetra is its electronic trading platform. Trading can be conducted from 9 a.m. to 5.30 p.m. and starts with an opening auction. It is interrupted by an intraday auction around 1 p.m. and ends with a closing auction. Between auctioning times, trading is continuously possible. An electronic order book collects all limit and market orders from market participants. Orders in the order book will be matched based on price and time priority.

In general, the limit order book is anonymous, but transparent to all participants. However, traders can also submit hidden, "iceberg" orders to trade large volumina, where traded volume is only revealed up to a certain size and a similar order of equal size will be initiated once the first limit order is transacted. Market makers post bid- and ask quotes up to a prespecified minimum quotation volume.

The Xetra system automatically calculates the Xetra Liquidity Measure (XLM) from the visible and invisible part of the limit order book. XLM is a weighted spread measure, calculating the cost of immediate execution of a round-trip order of a specific size q compared to its fair value.

Fair value is set at the mid-point P_{mid} of the bid-ask-spread. Mathematically, weighted spread and XLM can be calculated as the average limit-order-volume-weighted price of all limit orders, which are required for transacting a specific size, relative to the mid price.

$$XLM(q) := \left[\frac{\sum_i b_{i,t} v_{i,t}}{v} + \frac{\sum_j a_{j,t} v_{j,t}}{v} \right] \times \frac{100}{P_{mid}}$$

where $b_{i,t}$ and $v_{i,t}$ are the bid-prices and volumes of individual limit orders, where limit order volumes v_i add-up to $v = \frac{q}{P_{mid}}$. $a_{j,t}$ and $v_{j,t}$ are defined analogeously. Limit orders are sorted according to price priority. This simplifies to

$$XLM(q) = \frac{P(q)_{net,buy} - P(q)_{net,sell}}{P_{mid}} \tag{2}$$

where q is the size of the order in Euro, $P(q)_{net,buy}$ is the average net price achieved when buying an order of size q as market order and $P(q)_{net,sell}$ is the corresponding average net price for liquidating the position. Naturally, the net prices are lower than the bid- and ask-quotes in the order book for larger sizes, since a market order is executed by price priority, not by matching volumes. Thus the

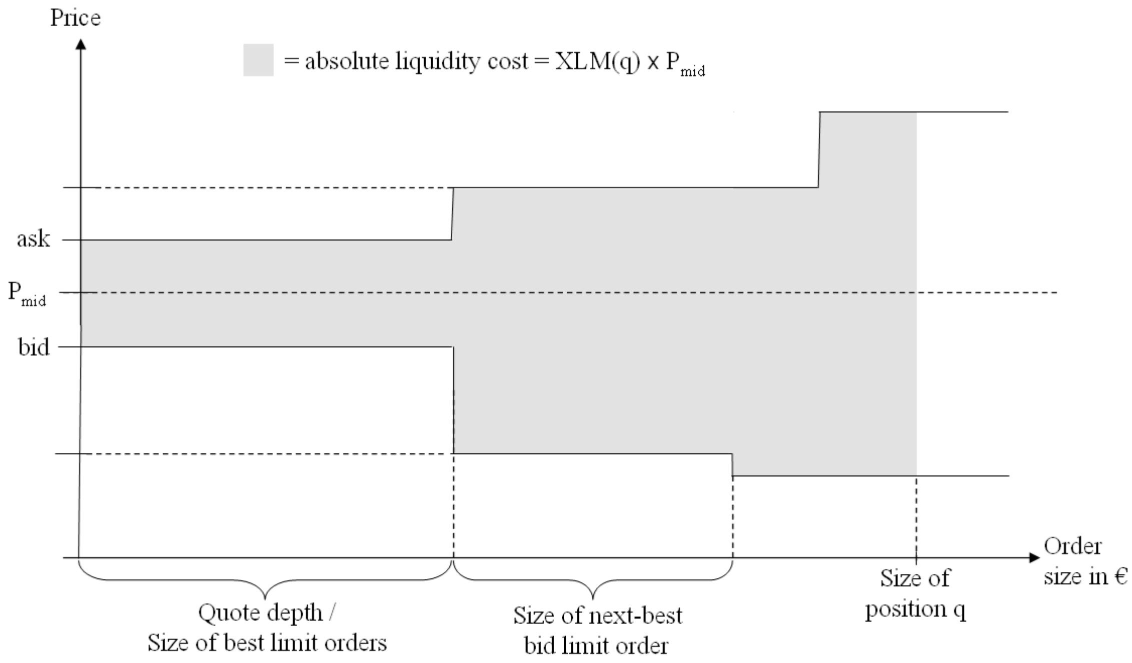

Figure 1: Weighted spread as area between limit order curves

net price equals the volume-weighted average of the best bid- and ask-quotes until their added volume reaches the volume of the market order.

Graphically, XLM is the area between the bid- and the ask-curve in the limit order book up to the order size q divided by the mid-price (see figure 1). XLM calculates the price impact of an order of size q in basis points. It can also be seen as the relative liquidity discount for a round-trip of an order of size q. Gomber and Schweickert (2002) provide some further theoretical background.

Our sample consists of 5.5 years of daily data (July 2002 to January 2008) for all 160 stocks in the four major German stock indices (DAX, MDAX, SDAX, TecDAX). The DAX contains the 30 largest publicly listed companies in Germany (by free-float market volume), the MDAX the subsequent 50 largest[14] and the SDAX the following 50 largest. The TecDAX, introduced during the sample period on 24.03.2003, comprises the 30 largest technology stocks. In total, we therefore cover a market capitalization of approximately € 1.2 trillion, which represents the largest part of the market capitalization in Germany.[15] As far as we know, this is the most representative sample on size-specific liquidity cost available to academia.

We received XLM data for all days, where a stock was included in one of the cited indices. Daily values are calculated by Xetra as the equal-weighted average

[14]MDAX contained 70 stocks before 24.03.2003 and 50 stocks thereafter.
[15]Values as of 1/2008.

of all available by-minute data points.[16] We break our total sample into four subsamples, each containing the stocks of one major index. For each stock we define the following variables:

- L(q): Cost for transacting a position of size q in basis points
- q: size of position in thousand Euro
- S: quoted bid-ask-spread at day closing relative to the mid-point in bp
- P: mid-point of the bid-ask quote at day closing in Euro
- MV: market value at day closing in million Euro
- VO: trading volume in number of traded shares in Euro, which we obtain by multiplying number of traded shares with mid-quote closing prices P

We obtained data for all data items but the first one from Thomson Financial Datastream. Three stocks could not be included in the analysis due to missing XLM or Datastream data.[17] This left 99.9% of the total 323,953 stock-days[18] in the sample.

With the data items above, we proceeded as follows. Liquidity costs L(q) were calculated from a transaction perspective. As a per-transaction figure has much more practical meaning, than a per-round-trip figure, we assume that the order book is symmetrical on average, i.e. the liquidity cost for buying and selling are equal. Therefore, we can calculate the price impact per transaction under the assumptions outlined in section 2 as

$$L(q) = PI(q) = \frac{XLM(q)}{2} \tag{3}$$

It is important to note that this measure captures the committed part of liquidity only, while there might be hidden liquidity in the market as well. Since we assume a worst case, however, where we transact immediately against the order book, there is no time for additional (hidden) liquidity to enter the market. This type of measure acts as an upper bound to liquidity cost, because it only measures part of the liquidity supply.[19]

[16] For liquid volume classes this comprises a maximum of 1,060 measurements during continuous trading.

[17] Procon Multimedia (in SDAX between 10/2002 and 03/2003) and Medisana (in SDAX between 12/2002 and 03/2003). Data could not be obtained for Sparks Networks (in SDAX between 06/2004 and 12/2005), because it was not available in Datastream anymore.

[18] 383 stock-days excluded.

[19] Cp. also Irvine et al. (2000), p.4f.

Liquidity costs were provided for each stock for 10 out of the 14 volume classes q of € 10, 25, 50, 75, 100, 150, 250, 500, 750, 1000, 2000, 3000, 4000 and 5000 thousand. Volume classes for DAX stocks went up to € 5000 thsd., but excluded € 10, 75, 150 and 750 thsd.. Stocks in the other indices had liquidity costs for all volume classes up to € 1 million.[20] In total, our sample contains 1.8 million observations for the 1424 trading days.

Quoted spread S measures the minimum ex-ante liquidity cost. While our liquidity measure $L(q)$ is standardized by size category, quoted spread is not. The largest order size tradable at the quoted spread, i.e. the spread depth, differs between stocks and changes over time. Spread measures different economic aspects for stocks which are covered by a market maker and for those stocks without coverage. Therefore spread depth differs between, but also within those categories.

On Xetra, coverage is required only for illiquid stocks - as defined by past XLM and order book volume criteria.[21] On 31.01.2008, 35% of our sample had coverage. In DAX and MDAX only one stock was covered, in SDAX 86% of the stocks were covered.[22] In the case with coverage spread is the quoted spread of the market maker. Spread depth can be freely selected by the market maker above the Xetra-regulated minimum, called minimum quotation volume (MQV), which varies depending on stock liquidity as measured by past-XLM. According to our data, minimum quotation volume for covered stocks was € 17.338.

In cases without coverage, spread is the minimum spread available in the order book. It corresponds to the order size of the limit order with the best price at a particular moment, which is naturally non-standardized. While the Xetra MQV is valid for liquid, non-covered stocks as well, the average minimum was € 27, i.e. non-existent for practical purposes. Spread depth for non-covered stocks therefore varies even more widely.

Two aspects should be kept in mind when comparing spread and the XLM liquidity measure. First, spread for covered stocks is likely to follow other dynamics, since the size of the spread has Xetra-regulated upper bounds.[23] In contrast, XLM liquidity prices result from free supply and demand behavior. Second, there is potential overlap between spread and the XLM. 51 stocks in our sample had minimum

[20] We had to exclude 408 (<0.01% of total) observations, where liquidity data were available outside the volume class structure described above. As these values were available for connected periods of less than seven days, we assume that the automatic calculation routine of the Xetra computer was extended during trial periods. This procedure ensures that our liquidity estimates remain representative.

[21] Market makers are called 'Designated Sponsors' on Xetra.

[22] While historic data was not available to us, it is plausible that similar differences existed during the whole sample period.

[23] Cp. Deutsche Boerse (2007), p. 5, 9.

quotation volume above € 10.000, 4 stocks between € 25.000 and € 30.000 (mostly in SDAX and TecDAX). As a consequence, L(q) measured quoted spread in small volume classes q of € 10 and 25 thousand for these 51 stocks. While we have no historic data on MQV, it is safe to assume that this was valid over the whole sample period.

We also had to adjust mid-price data P, because Datastream carries forward price data even if no transaction took place. We removed all price data at days, when no transaction volume was recorded. Data for market value MV and transaction volume VO were used as provided by Datastream.

4 Empirical results

Empirical results are presented along the main questions. Up-front in section 4.1, we provide some market background to our analysis. Section 4.2 provides estimates of liquidity cost by order size. Section 4.3 shows the time variation of liquidity in our sample period. Section 4.4 goes into detail on on order-size-differentiated liquidity distributions. The last section 4.5 compares liquidity in the cross-section controlling for a variety of different stock characteristics.

4.1 Market background

As background to our analysis, table 1 summarizes market conditions during the sample period. Markets were bullish in the largest part of the sample period. We also captured the downturns in the second half of 2002 and the first month of 2008. Due to beginning and end of period declines, overall return was rather average at 8% p.a.. Naturally, market capitalization increased similar to returns. Average market capitalization is several times larger in the DAX than in all other indices. MDAX contained the second largest average market capitalization stocks. Volatility exhibited a similar, but reversed pattern than returns. Consequently, our sample is rather positively biased.

Daily transaction volume strongly increased during the sample period, which is already a plausible indicator for improving liquidity. Transaction volume was largest in the DAX. Transaction volume in the other indices were several magnitudes smaller. Contrary to the general trend, transaction volume in the TecDAX remained rather steady after its initiation in 2003 and exhibits a level slightly lower than the MDAX. SDAX transaction volume was again several times smaller than in MDAX or TecDAX. The high diversity in transaction volumes underlines the representativeness of our sample.

Market segment overview	II/2002	2003	2004	2005	2006	2007	1/2008	Total period[a]
Average continuous period return [b]								
DAX	-52%	24%	6%	27%	20%	22%	-15%	6%
MDAX	-23%	39%	15%	25%	25%	-1%	-12%	12%
SDAX	-36%	35%	11%	28%	29%	4%	-14%	10%
TecDAX	n/a	52%	3%	26%	24%	32%	-25%	23%
Total	-35%	24%	10%	26%	24%	11%	-15%	8%
Average period return volatility (annualized) [c]								
DAX	64%	41%	22%	19%	23%	25%	51%	30%
MDAX	54%	39%	28%	26%	30%	35%	59%	35%
SDAX	65%	47%	35%	31%	36%	38%	58%	40%
TecDAX	n/a	54%	42%	31%	38%	44%	71%	42%
Total	60%	44%	32%	27%	32%	36%	59%	37%
Average free-float market capitalization in million Euro								
DAX	15,217	14,615	17,983	20,350	24,357	29,949	29,325	21,008
MDAX	1,043	1,330	1,940	2,537	3,734	3,797	3,121	2,453
SDAX	106	235	320	393	500	775	640	418
TecDAX	n/a	725	863	898	995	1,221	1,204	955
Total	3,639	3,483	4,319	4,998	6,154	7,379	7,009	5,160
Average daily transaction volume in thsd. Euro								
DAX	93,500	94,399	98,037	119,563	165,833	250,835	351,793	144,040
MDAX	1,384	2,297	4,035	6,242	11,034	18,243	22,351	7,557
SDAX	36	160	237	514	958	2,129	2,081	780
TecDAX	n/a	1,813	2,345	2,308	4,769	7,946	11,430	4,052
Total	20,431	19,543	20,268	25,206	35,797	54,891	75,739	31,020

Table 1: Market conditions during sample period

a. annualized; b. Includes dividend returns, because price series are adjusted for corporate capital actions; c. volatility estimated from daily cont. returns and annualized with $\sqrt{250}$; All values equal-weighted.

4.2 Descriptive statistics of order-size-differentiated liquidity costs

We start with looking at detailed descriptive statistics of liquidity cost L(q), which will serve as representative reference for practice and provide some structural insight. We calculate the cross sectional averages for a specific sub-sample over a specific period. Table 2 shows average liquidity cost over the whole sample period by index and order size.

The first columns presents average liquidity costs for different order sizes. The min-column contains the spread estimate for the minimum order size, the following columns the cost estimates for higher order sizes according to our liquidity measure (3). We report the cross-sectional mean, median and standard deviation in each sub-sample. Availability is available data in percentage of the theoretical maximum.[24] In the last column of table 2 we specifically estimated the impact of doubling order size in absolute basis points on liquidity costs and in percentage points on availability.[25]

Between 6/2002 and 1/2008, Investors had to pay between 0.09 bp and 460 bp on average for buying or selling a stock position, which already shows that liquidity costs varies largely between order sizes and can reach substantial amounts. While in the DAX average liquidity costs start at a negligible 0.09 bp for the minimum order size, they reach over 100bp when trading a position larger than of € 3 million. Liquidity costs at the smallest order size of € 10 and 20 thsd. respectively is several times the level of the spread. As many (institutional) investors rarely trade positions lower than € 10 thsd., the spread is therefore insufficient as an liquidity estimate.

Comparing average and median liquidity level between different indices, the DAX was the most liquid on average, followed by the MDAX, TecDAX and then the SDAX. A similar result shows when looking at the size impact on liquidity. The size impact was statistically significant at the 1%-level in all indices, smallest in the DAX and largest in the SDAX. When doubling order size, liquidity costs in the DAX increase by an absolute 28.28 bp in the average stock. In the SDAX liquidity impact in the average stock was almost three times as high at 82.41 bp.

Median liquidity was lower than the mean in all order sizes, which reveals a right-skewed liquidity cost distribution for all sizes and all indices. Size impact in

[24] As the sample comprises 1424 trading days, the maximum possible number of observations per volume class is 42.720 for the DAX, 74.900 for the MDAX, 71.200 for the SDAX and 37.170 for the TecDAX.

[25] The ordinary-least-squared regression specification for each statistic stat is $stat(q) = C + ln(q) + \epsilon_q$ with C being the intercept. Statistics of the minimum order size/spread do not enter the calculation, because corresponding minimum order size was not available. OLS regression with availability has limited validity, because the statistic is distributed between 0 and 1 and is non-normal, but has been included for sake of completeness.

Avg. liquidity cost (in bp)		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	Mean	0.09	n/a	6.06	7.10	n/a	9.25	n/a	16.07	28.54	n/a	56.10	97.50	116.90	136.91	153.60	56.75	28.28 ***
	Median	0.05	n/a	6.09	7.01	n/a	8.86	n/a	14.56	24.01	n/a	49.98	67.91	97.67	120.89	154.43	23.81	25.08 ***
	Std. Dev.	0.11	n/a	2.68	3.56	n/a	5.42	n/a	11.57	23.28	n/a	46.63	97.39	83.42	87.35	91.56	76.14	19.88 ***
	Availability	99%	n/a	100%	100%	n/a	100%	n/a	100%	100%	n/a	100%	98%	94%	90%	85%	97%	-0.02 **
MDAX	Mean	0.29	27.08	34.09	42.36	50.65	59.39	77.62	109.80	160.77	194.36	217.26	n/a	n/a	n/a	n/a	88.19	43.72 ***
	Median	0.17	20.32	25.11	33.30	40.27	48.40	68.18	100.24	155.58	189.58	205.36	n/a	n/a	n/a	n/a	52.80	43.81 ***
	Std. Dev.	0.37	38.84	60.12	63.92	55.84	51.31	58.04	71.17	86.30	99.90	105.71	n/a	n/a	n/a	n/a	92.70	12.99 ***
	Availability	100%	100%	100%	99%	98%	97%	95%	92%	81%	71%	62%	n/a	n/a	n/a	n/a	90%	-0.08 ***
SDAX	Mean	0.64	81.87	97.29	127.58	156.12	179.68	209.48	251.68	326.19	397.71	460.23	n/a	n/a	n/a	n/a	172.57	82.41 ***
	Median	0.41	61.87	81.20	109.15	134.32	150.80	201.72	233.58	293.86	342.42	412.83	n/a	n/a	n/a	n/a	133.12	75.62 ***
	Std. Dev.	0.88	100.29	82.88	94.51	111.74	145.43	181.29	166.41	137.31	199.11	247.46	n/a	n/a	n/a	n/a	159.55	29.71 ***
	Availability	98%	97%	91%	86%	81%	77%	69%	56%	33%	20%	13%	n/a	n/a	n/a	n/a	62%	-0.19 ***
TECDAX	Mean	0.26	31.68	41.92	58.87	77.32	96.61	128.30	174.31	245.30	291.33	331.65	n/a	n/a	n/a	n/a	125.20	68.82 ***
	Median	0.20	30.82	38.85	53.61	70.05	86.73	120.43	173.01	227.30	255.28	311.84	n/a	n/a	n/a	n/a	75.72	63.27 ***
	Std. Dev.	0.22	17.56	26.60	44.01	64.15	84.86	102.59	109.12	112.08	124.42	136.73	n/a	n/a	n/a	n/a	123.57	27.35 ***
	Availability	99%	100%	100%	100%	100%	99%	98%	92%	73%	56%	44%	n/a	n/a	n/a	n/a	86%	-0.12 ***
All	Mean	0.36	n/a	48.59	62.43	n/a	87.60	n/a	130.62	164.15	n/a	192.90	n/a	n/a	n/a	n/a	108.63	40.82 ***
	Median	0.19	n/a	30.22	39.03	n/a	59.12	n/a	103.05	153.73	n/a	161.66	n/a	n/a	n/a	n/a	69.23	39.96 ***
	Std. Dev.	0.58	n/a	67.15	78.98	n/a	107.83	n/a	129.46	135.74	n/a	168.71	n/a	n/a	n/a	n/a	122.48	26.61 ***
	Availability	98%	n/a	97%	95%	n/a	92%	n/a	82%	68%	n/a	51%	n/a	n/a	n/a	n/a	82%	-0.12 ***

Table 2: Liquidity cost by index and size

Discrete liquidity cost, average over sample period; equal weighting in categories; TecDAX from start 2003/03 only (values similar in magnitude for sample 03/2003-01/2008); Min-column measures spread for the minimum order size, all-column is average over all standardized order sizes, i.e. without minimum; Size impact is the coefficient in 10^2 of log order-size in a regression of the distribution statistic on log order-size including an intercept; * indicates 10% confidence level, ** 5% and *** 1% of being different from zero based on two-tailed test.

the median is very similar to the impact in the average. The dispersion of liquidity cost across stocks is of a similar order of magnitude as the liquidity level and increases with order size. Liquidity variation seems to be closely connected to liquidity level.

Generally, as order size increases, availability decreases, which is underlined by the statistically significant size-impact statistic.[26] This is due to the fact already mentioned above that larger orders could not be transacted against the limit order book for all stocks. Availability of spread was in some cases slightly below 100%, because Datastream did not provide data for all stock-days. For small order classes up to € 25 thousand, over 90% of all stock positions could be instantly liquidated. However, in the SDAX, for example, availability drops down to 13% of all stocks for the volume class of € 1 million. In the DAX, even large orders can be continuously executed against the limit order book as availability is below 90% for the largest volume class of € 5 million only. The pattern of availability for the TecDAX underlines the conclusion above that the TecDAX is much more liquid than the SDAX. Comparing the TecDAX with the MDAX with respect to availability, the MDAX is only very slightly more liquid in order sizes above € 500 thousand.

The TecDAX was created in March 2003. Therefore, TecDAX numbers are based on the mean from 3/2003 to 1/2008, in contrast to the rest of the sample, which ranges from 7/2002 to 1/2008. Statistics for the comparable sample (3/2003 to 1/2008) are similar in relative magnitude between the indices.[27]

All in all, the discussion shows that liquidity costs can be substantially underestimated when looking at spread only. The impact of size is quite substantial, especially in stocks with smaller market capitalization.

4.3 Order-size-differentiated liquidity over time

To provide a first picture on the different behavior of liquidity at different position sizes over time, we calculated pairwise-sample correlations between spread and liquidity at larger order sizes as presented in table 3. Correlation between spread and liquidity the rest of the order book are relatively low below 65%, correlations between adjoining measures of L(q) are very close to one. Correlations drop to 30 to 40% when looking at correlations between liquidity of very small and of very large sizes. While correlations continuously drop as the difference between order sizes gets larger, there is an increase in correlation between the volume class of € 750 thsd. and € 1 million. This is due to the fact that the sample at € 1 million is dominated

[26]Because spread data in the min-column comes from a different source than the liquidity data, availability between these two is not directly comparable.

[27]Summary results for the comparable sample (3/2003 to 1/2008) are available on request.

	Spread	L(10)	L(25)	L(50)	L(75)	L(100)	L(150)	L(250)	L(500)	L(750)	L(1000)	L(2000)	L(3000)	L(4000)	L(5000)
Spread	1.00	0.64	0.64	0.59	0.50	0.50	0.44	0.44	0.41	0.27	0.37	0.46	0.45	0.41	0.41
L(10)		1.00	0.88	0.81	0.73	0.70	0.65	0.59	0.56	0.48	0.50				
L(25)			1.00	0.92	0.82	0.79	0.70	0.67	0.67	0.52	0.65	0.78	0.73	0.69	0.67
L(50)				1.00	0.95	0.89	0.78	0.72	0.69	0.55	0.68	0.81	0.77	0.73	0.72
L(75)					1.00	0.97	0.88	0.79	0.65	0.57	0.62				
L(100)						1.00	0.95	0.86	0.72	0.58	0.71	0.83	0.79	0.75	0.74
L(150)							1.00	0.91	0.72	0.65	0.66				
L(250)								1.00	0.90	0.87	0.78	0.86	0.81	0.77	0.77
L(500)									1.00	0.95	0.87	0.88	0.83	0.79	0.78
L(750)										1.00	0.95				
L(1000)											1.00	0.89	0.76	0.66	0.61
L(2000)												1.00	0.93	0.83	0.78
L(3000)													1.00	0.95	0.91
L(4000)														1.00	0.97
L(5000)															1.00

Table 3: Correlation of liquidity across order size

Pairwise sample correlations between spread and size-adjusted liquidity L(q) of different order sizes q in thsd. Euro.

by DAX stocks. DAX stocks have generally higher correlations which explains the increase between € 750 thsd. and € 1 million in the full sample.[28]

This correlation analysis is an indicator that liquidity behaves very differently across order sizes. Liquidity cost at the left side of the order book, like spread, are a poor proxy for the real liquidity cost of larger position.

Figure 2 shows the daily development of liquidity cost L(q), averaged over all order sizes and cross-sections, during the whole sample period by index. While the equal average over all available volume classes is somewhat arbitrary, because it is strongly influenced by the selection of volume classes, it nevertheless gives a general picture on the overall liquidity trend. While the underlying stocks change over time as stocks move in and out of indices, the effect on the index mean should be negligible. The average can be interpreted as expected liquidity cost when trading a random position in the specific index.

All index averages have experienced a strong decline in the last 5.5 years with a recent strong increase in 1/2008. In a first phase from 7/2002 to 3/2003 liquidity was highly volatile and showed side-way movement. This phase corresponds to the end of the collapsing high-tech bubble. From 3/2003 on, liquidity steadily declined, interrupted by short, but substantial increases. Most of these increase spikes can be tracked to major disturbances at the stock market. Liquidity cost increased around August 2004 after the publication of low earnings forecasts in tech stocks and during the stock market crash of May 2006, which spilled over from emerging-market exchanges. The recent sub-prime crises is also apparent in the data. Upward spikes can be observed during the crash in February/March 2007 after bankruptcy declarations of sub-prime lenders, in August 2007, where the influence of sub-prime on bank portfolios became known, especially on the German IKB bank, and most

[28]Statistics on DAX correlations available on request.

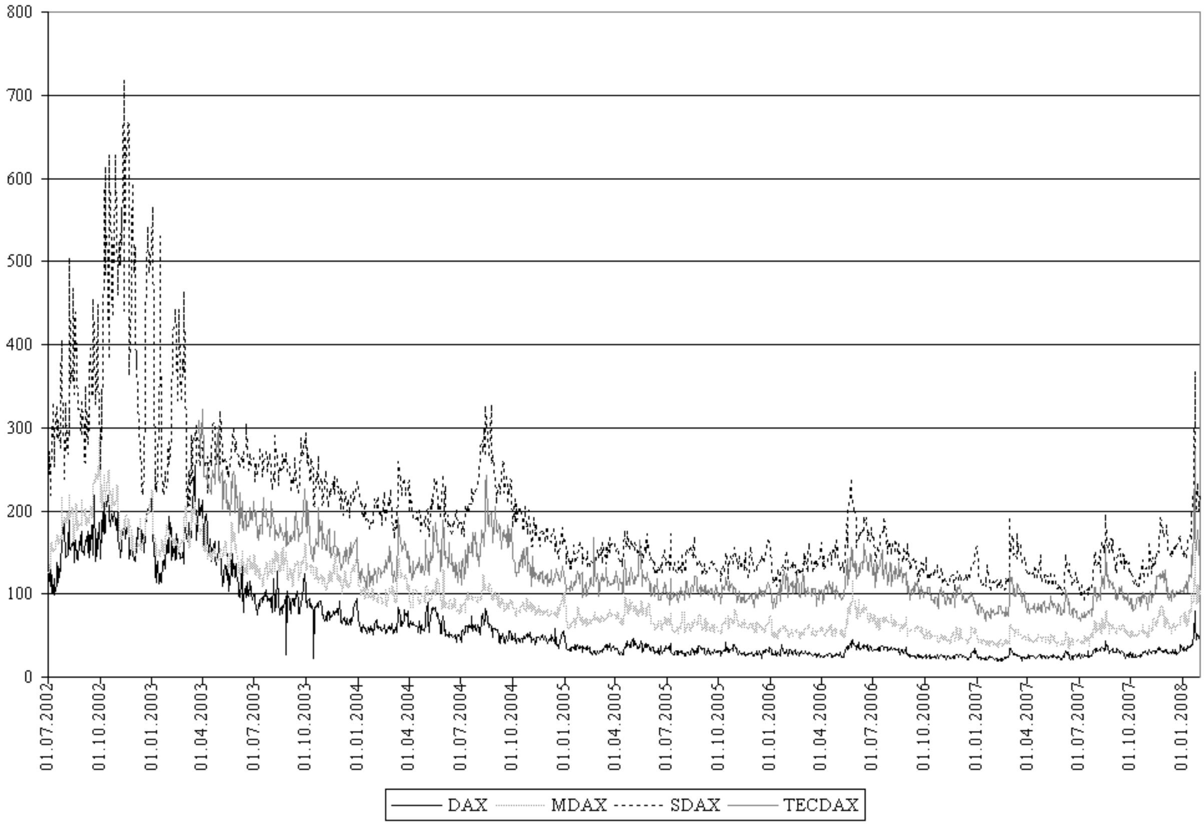

Figure 2: Development of average liquidity cost by index

Figure 2 shows the development of the cross-sectional mean of all order sizes by day over the sample period. As noted before, TecDAX was created in 3/2003.

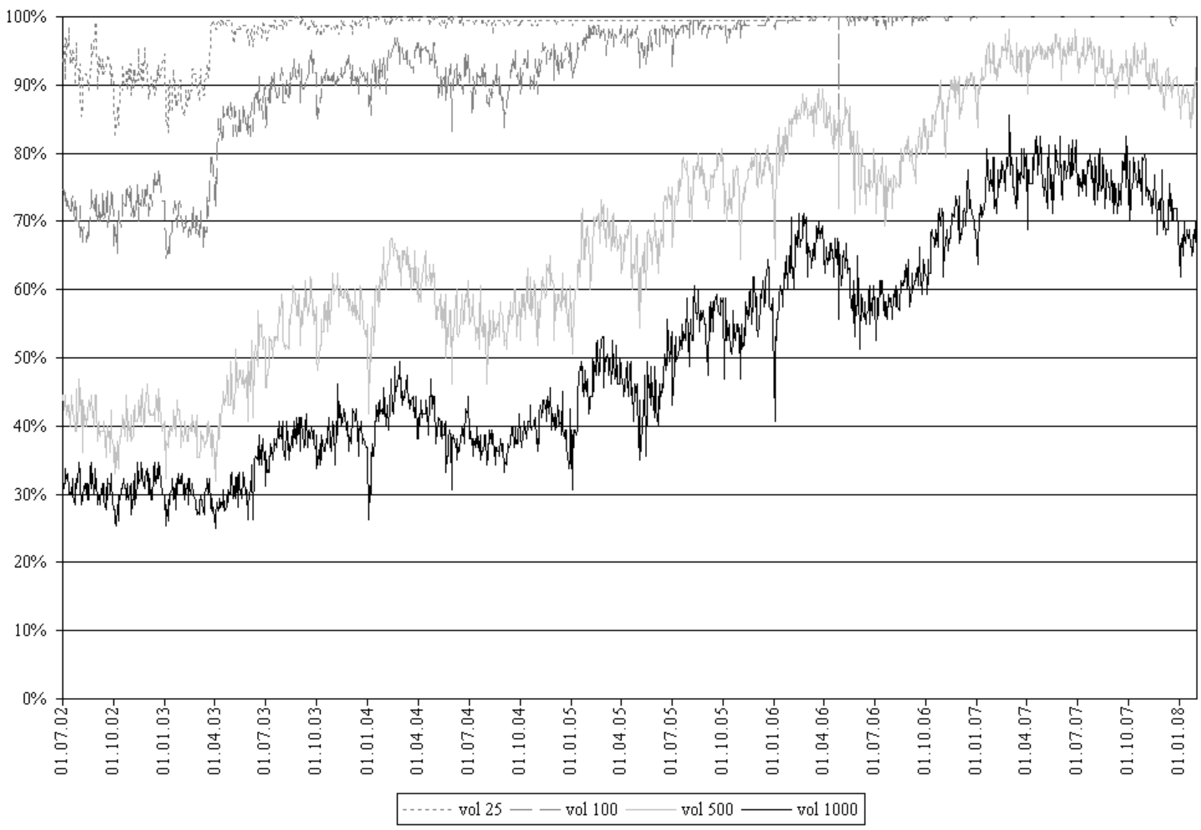

Figure 3: Development of availability by selected volume classes

recently during the crash of January 2008 after equity shortages of major banks around the world.

Increases occur over short periods of time, while decreases take place over calm periods of slightly positive market conditions. This asymmetry skews the distribution of liquidity changes to the right. The general negative trend explains the slight positive skewness in liquidity level distributions. Index means move relatively synchronous, while changes in liquidity seem to be connected to the liquidity level and are thus much less pronounced in the less liquid indices.

To investigate into the time variation of liquidity costs by size, we first look at the variation of availability over time. Figure 3 reveals that availability has strongly increased, especially in larger sizes. In 100% of the stocks, the volume classes of € 25 and € 100 thousand was tradable in recent months. Tradability of € 1 million strongly improved from around 30% of the stocks in 2002 to above 60% lately. Therefore, sample size increases with time for larger volumes.

Due to the changing sample, we observe two contrary effects. As liquidity improves, liquidity costs fall. At the same time, larger stock positions become tradable. Availability in these order sizes increase. The successive inclusion of comparatively

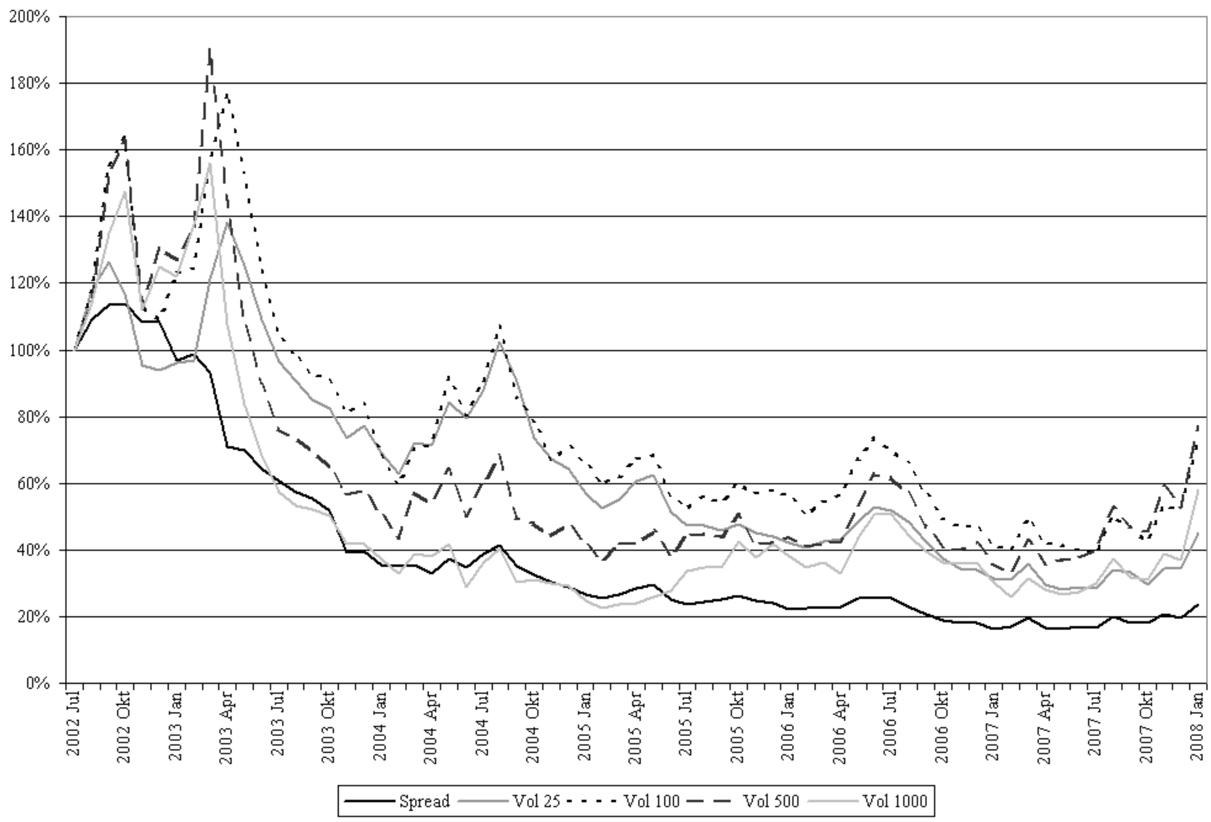

Figure 4: Development of average liquidity cost by selected volume classes (constant sample, indexed)

Monthly average of stocks with availability over 97% in volume class; values are indexed on 07/2002 mean.

illiquid stocks with high liquidity cost drives up the average. As a consequence, the development of average liquidity cost will not be representative for the development of liquidity cost for a specific stock position. Non-constant sample average are upward biased over time, especially in larger order sizes, where availability strongly increases.

To measure the development of liquidity cost for a specific stock position, we constructed a constant sample and recalculated the average liquidity cost over time. We included only those stocks and sizes, which were available at least 97% of the sample period.[29] The caveat of this type of analysis is that only very liquid stocks are included in the average and the average is taken on a less-representative fraction of the market. To make different order sizes more comparable, we also indexed liquidity cost levels on the July 2002 mean.

Results in figure 4 show that liquidity costs have decreased across all order sizes. Absolute reduction is larger for bigger positions, but relative decline was similar

[29]We chose 97% as cut-off, because it provided a good balance between non-distorted results and excluding too many stocks from the analysis.

across sizes. Relative reduction was larger in smaller order sizes over the whole sample period. Spread declined by 80%, liquidity of a € 25 thsd. position by about 50%. In contrast, liquidity in larger volumes have been brought up to near high, historic levels in the recent crises. Larger volumes seem to be affected more strongly in crises. This effect can be interpreted as another variant of flight to liquidity, where stocks positions that are liquid remain relatively liquid in crises, while less liquid stock positions suffer more.

The discussion shows that the dynamics of liquidity is similar in the general direction across order sizes. However, the absolute magnitude of change is different. Absolute improvement has been greatest in larger sizes. We have also revealed different crises behavior, where we uncovered a flight-to-liquidity asymmetry between the liquidity of small and large order sizes. This is a strong indication that liquidity risk will increase strongly with increasing position size. Applying time dynamics from liquidity measures of small positions such as the spread will be inappropriate for capturing the dynamics of the liquidity deeper in the order book.

4.4 Distributional characteristics of liquidity across order sizes

Since we have access to a very representative sample, we will dedicate some time and space to the distributional characteristics of such a type of liquidity measure. The analysis of the distributional characteristics is useful for several applications, for example in risk measurement and management, in asset pricing models or in theoretical models to assume appropriate liquidity processes.

As the selection of reported volume classes is arbitrary, it is important not to calculate aggregate distribution statistics across order classes. Fineness of the reported classes would directly impact distributional characteristics. We therefore present all distributional statistics separate for each order size. This also allows to investigate the impact of order size on the liquidity distribution.

From an economic perspective, it is difficult to aggregate liquidity cost by absolute order size across stocks. It can be argued that, for example, liquidating a € 100.000 position in a large-cap stock is not comparable to the same position in a small cap stock, as the position in the large cap stock represents a much smaller part of the market value and should therefore be more liquid and have consequently less liquidity costs. A similar argumentation goes for the Euro-position in relation to the prevailing transaction volume in the market. A position size relative to the market value of the stock and prevailing transaction volume would be more comparable across stocks.

While we do not want to empirically investigate into this argument further in this section[30] and to keep the provided statistics as simple as possible, we chose not to generate new relative size categories. We also wanted to avoid to reduce the generality of our results by using a specific method for re-categorizing liquidity data. To still account for the argument above, our distributional statistics will not be calculated on liquidity data aggregated across all stocks, but we calculate stock-specific distribution statistics and present their cross-sectional mean and median. As reference, we included spread in the distributional analysis. Because the order class of spread differs widely between stocks, we designated this order class as “min”.

Table 4 presents distributional statistics on liquidity cost and absolute liquidity cost change in bp. The size-impact statistic reveals that there is a statistically significant size-impact not only on the liquidity level, but also on its variance, skewness and kurtosis. Variance seems to be closely connected to the level of liquidity. The cost mean and also the variance at the spread level are much lower. Otherwise, the distribution of the spread behaves similar to the distribution at the € 10 tsd. volume class.

Looking at absolute liquidity changes removes the skewness, which reveals that trend is a major cause of the skewness. The negative mean and median reflect the overall negative trend in the sample period. The trend seems to be increasing with size, but only in the median stock. The absolute value of the trend is very small, below 0.5bp per day on average. But variance is large so changes in liquidity cost can be quite significant in certain times. There has been no overall trend in the spread. Even when trend is removed, the distribution remains heavily fat-tailed. Kurtosis also strongly increases with order size.

In order to create a distribution that is more closely normally distributed, we take the logarithm of absolute liquidity in basis points.[31] Table 5 shows that this removes most of the kurtosis and skewness. Distributions are now by tendency much more normal. Kurtosis is almost removed, while some skewness remains in the data. While the economic interpretation is more difficult, this conversion is helpful in statistical applications, for example mean-variance estimations. Size impact remains intact and statistically significant for practically all statistics at the 1-5% level.

To analyze the remaining kurtosis in more detail from an economic point-of-view, we concentrate on outliers as potential source. To identify outliers, we calculate standardized z values of log liquidity log(L(q)) by subtracting the monthly mean and

[30] Refer to 4.5 for a more detailed analysis.

[31] Please note that we take logarithm of liquidity cost in basis points, i.e. in 10^{-4}, not in decimal. Variation of liquidity in decimal is so small that the logarithm would be close to a linear transformation and therefore have no impact on distribution statistics.

Liquidity cost (in bp)

Distribution statistic	Cross-section statistic	Order size (in thsd. Euro where applicable) Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
Mean	Mean	1.06	48.94	48.59	62.38	92.62	87.37	130.38	130.25	163.60	249.10	192.25	97.51	116.90	136.30	153.17	108.36	17.76 **
	Median	0.48	32.38	30.22	39.03	67.61	59.12	104.77	103.05	153.73	216.98	161.66	67.91	97.67	120.89	154.43	69.23	18.74 **
Median	Mean	0.58	40.89	39.20	48.57	70.26	65.35	98.54	98.31	127.62	202.42	151.00	50.09	61.85	76.07	90.46	81.87	8.67
	Median	0.35	26.94	24.74	31.28	49.50	43.11	73.43	71.16	107.43	173.07	120.69	41.29	53.25	65.91	81.57	50.02	9.66 *
Variance[a]	Mean	0.66	34.03	38.08	68.53	145.03	159.11	233.70	228.40	269.32	429.36	426.32	349.77	407.01	436.76	487.97	194.64	77.50 ***
	Median	0.00	3.87	3.57	5.92	20.52	16.91	81.19	84.92	162.11	283.40	172.44	62.08	106.68	171.06	184.68	23.46	31.81 ***
Skewness	Mean	2.99	1.96	2.41	2.47	2.61	2.86	3.06	2.92	3.01	3.30	3.23	3.22	3.94	3.65	3.75	2.80	0.27 ***
	Median	2.07	1.65	1.79	1.92	2.24	2.24	2.62	2.44	2.48	2.19	2.43	2.54	2.80	2.50	2.58	2.18	0.14 ***
Kurtosis	Mean	31.90	13.65	23.07	18.98	17.39	22.87	23.42	22.75	23.71	32.38	27.43	24.86	44.18	35.14	37.39	22.93	3.58 ***
	Median	8.65	6.58	6.74	7.87	9.75	10.21	13.63	10.88	11.75	10.43	11.37	11.14	11.74	10.98	12.11	9.53	0.75 ***
Mean - Median	Mean	0.48	8.05	9.39	13.82	22.36	22.02	31.84	31.94	35.99	46.69	41.26	47.42	55.05	60.23	62.71	26.49	9.09 ***
	Median	0.09	4.72	4.95	6.99	13.65	12.15	25.04	23.58	31.40	41.08	37.66	28.71	45.06	53.81	49.52	14.32	8.08 ***

Absolute change in liquidity cost (in bp)

Distribution statistic	Cross-section statistic	Order size (in thsd. Euro where applicable) Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
Mean	Mean	0.00	0.03	-0.01	-0.03	-0.14	-0.13	-0.18	-0.17	-0.20	-0.06	0.00	-0.08	-0.16	-0.27	-0.35	-0.10	-0.03
	Median	0.00	-0.02	-0.02	-0.02	-0.04	-0.02	-0.06	-0.03	-0.03	-0.11	-0.02	-0.05	-0.08	-0.12	-0.17	-0.03	-0.02
Median	Mean	0.00	-0.07	-0.05	-0.17	-0.29	-0.17	-0.23	-0.03	-0.02	0.53	0.40	0.10	0.28	0.24	0.09	-0.04	0.08 **
	Median	0.00	-0.02	-0.01	-0.04	-0.11	-0.02	-0.11	0.01	0.01	0.18	0.13	0.06	0.16	0.11	0.08	-0.01	0.03 ***
Variance[a]	Mean	0.07	23.07	18.82	32.12	59.12	62.16	115.24	109.57	144.23	261.86	245.74	110.00	176.26	151.93	168.03	94.03	29.94 ***
	Median	0.00	1.27	1.10	2.09	6.92	7.42	27.94	42.87	64.45	144.82	75.64	15.24	31.11	54.52	66.68	9.55	11.55 **
Skewness	Mean	0.05	-0.01	-0.01	-0.01	0.23	0.05	0.01	-0.07	-0.31	-0.27	-0.06	-0.03	-0.12	-0.66	-0.34	-0.05	-0.07
	Median	0.01	0.05	0.02	0.04	0.15	0.01	-0.03	-0.05	-0.13	-0.10	-0.04	0.01	-0.13	-0.34	-0.31	0.00	-0.05
Kurtosis	Mean	27.26	19.09	34.33	37.14	30.36	43.26	37.27	41.54	43.13	42.57	49.63	64.40	78.86	73.49	79.45	40.20	8.72 ***
	Median	7.97	8.38	9.67	12.31	15.69	16.86	18.99	18.61	18.01	15.91	20.04	20.97	22.50	24.39	22.45	15.21	2.25 ***
Mean - Median	Mean	0.00	0.10	0.04	0.14	0.16	0.04	0.05	-0.14	-0.18	-0.59	-0.41	-0.18	-0.43	-0.50	-0.43	-0.06	-0.11
	Median	0.00	-0.01	0.00	0.01	0.04	-0.01	-0.03	-0.06	-0.04	-0.41	-0.16	-0.08	-0.24	-0.25	-0.23	-0.02	-0.05

Table 4: Distributional characteristics of liquidity

The "Min"-column contains the distribution statistic of the half-spread for a minimum order size, other order size columns contain the statistics for L(q) and dL(q); Size impact is the coefficient of log order-size in an OLS-regression of the distribution statistic on log order-size including an intercept; a. Values in 10^2; ***, ** and * indicate 1%, 5% and 10% confidence level of being different from zero based on a two-tailed test.

Distribution statistic	Cross-section statistic	Order size (in thsd. Euro where applicable) Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
Mean	Mean	-1.16	3.39	3.18	3.39	3.99	3.68	4.36	4.11	4.38	5.16	4.56	3.89	4.17	4.37	4.52	3.92	0.19 ***
	Median	-1.10	3.31	3.25	3.48	4.00	3.83	4.38	4.31	4.72	5.18	4.77	3.94	4.32	4.47	4.61	4.01	0.21 ***
Median	Mean	-1.12	3.34	3.11	3.31	3.92	3.59	4.29	4.02	4.29	5.13	4.46	3.66	3.94	4.15	4.32	3.83	0.16 **
	Median	-1.05	3.29	3.21	3.44	3.90	3.76	4.30	4.26	4.68	5.15	4.79	3.72	3.98	4.19	4.40	3.91	0.17 **
Variance	Mean	0.86	0.30	0.31	0.34	0.39	0.39	0.44	0.44	0.45	0.40	0.45	0.61	0.60	0.59	0.57	0.40	0.05 ***
	Median	0.78	0.26	0.26	0.29	0.34	0.33	0.38	0.36	0.36	0.33	0.37	0.50	0.54	0.55	0.52	0.33	0.05 ***
Skewness	Mean	-0.33	0.22	0.40	0.44	0.36	0.50	0.39	0.48	0.45	0.22	0.49	0.90	0.89	0.83	0.81	0.44	0.09 ***
	Median	-0.36	0.29	0.45	0.50	0.40	0.56	0.41	0.51	0.49	0.23	0.52	0.91	0.92	0.85	0.93	0.49	0.09 ***
Kurtosis	Mean	3.64	3.30	3.29	3.28	3.36	3.41	3.46	3.39	3.45	3.63	3.87	3.72	3.60	3.45	3.47	3.42	0.06 ***
	Median	3.14	3.01	3.07	3.08	3.19	3.14	3.21	3.19	3.19	3.26	3.24	2.99	3.08	2.98	3.06	3.13	0.00
Mean - Median	Mean	-0.03	0.05	0.07	0.08	0.07	0.09	0.07	0.09	0.09	0.04	0.10	0.23	0.23	0.22	0.21	0.09	0.03 ***
	Median	-0.05	0.04	0.07	0.07	0.05	0.08	0.06	0.08	0.08	0.02	0.07	0.20	0.20	0.22	0.20	0.07	0.03 ***

Table 5: Distributional characteristics of log liquidity

The "Min"-column contains the distribution statistic of the half-spread for a minimum order size, other order size columns contain the statistics for log(L(q)); Size impact is the coefficient of log order-size in an OLS-regression of the distribution statistic on log order-size including an intercept; ***, ** and * indicate 1%, 5% and 10% confidence level of being different from zero based on a two-tailed test.

dividing by the monthly standard deviation. Scanning of situations with absolute z-values above 3 (0.4% of all observations), reveals four types of outlier situations, which all present variants of market imperfections.

First, some records of L(q) exceed 100% (46 observations), i.e. transaction cost exceed the price. This could be due to data punching errors or due to highly asymmetrical order books, where limit orders on the ask-side in the depth of the book are much larger than 200% of the mid-price. If the limit order book is highly asymmetrical, our estimation procedure for a per-transaction liquidity cost in equation (3) produces economically meaningless results. It is also very plausible that liquidity prices were inefficient in these situations. We removed these meaningless records from further analysis.

Second, outliers occur after large changes in trading volume, i.e. either if trading volume was very large on that day or on the day before. Our explanation is that large trading volume consumes limit orders and will lead to large liquidity cost if resiliency for this particular stock is low. In this case, new limit orders do not refill the order book quickly enough. As a consequence, not all situations with exceptionally high volume exhibit large liquidity cost, but only those where resiliency was low.

Third, outliers occur after large price returns, because limit orders are fixed and do not necessarily adjust quickly to changing mid-prices.

Fourth, outliers can be identified near the maximum order book depth as measured by the maximum volume class available in the liquidity data. This is also consistent with the fact that kurtosis increases with order size. The higher the order size, the more stocks in the sample have reached their maximum depth. In these cases, it is plausible that the price priority rule does not lead to efficient liquidity prices, because single or very few limit orders determine liquidity cost. Because it is implausible that large, single orders underestimate liquidity cost, because this would generate losses to the liquidity provider, a reduction of the number of limit orders will inflate liquidity cost and cause outliers.

The exclusion of these outliers, however, only partially removes the kurtosis in the liquidity distributions.[32] In summary, the distributional analysis revealed that applications should use log versions of liquidity and respect liquidity trends that are inherent in the data. Despite the trends, daily fluctuations seem to be random over longer term.

[32]Statistics available on request.

4.5 Comparing order-size-differentiated liquidity between stocks

4.5.1 The role of relative order size

In this section, we want to follow up on the hypothesis that order size relative to market value and transaction volume is much more comparable across stocks than absolute order size. As argued in section 4.4, this is plausible using common sense. But it is also backed by analogous application of existing theory on the bid-ask-spread.

A market maker quoting the bid-ask spread and a trader initiating a limit order face a very similar situation.[33] A bid-ask-quote or a limit order commit to trade a certain quantity at a certain price. Both liquidity providers will want to get compensated for bearing two risks. First, they have to bear unwanted inventory risk that the price moves against them, e.g. through new, favorable information, while the limit order is in the order book. Second, they have to protect themselves against adverse information risks that traders only trade against limit orders when they are better informed. Liquidity costs, which are returns for liquidity providers, therefore compensate for price risk (i.e. inventory risk), informational asymmetry and possibly, in addition, the fixed cost for providing liquidity.[34] These risks get relatively more important for larger order sizes as capital restrictions aggravate the situation of the trader.

To analyze the impact of order size in the light of above consideration, we use the following ordinary least squared (OLS) regression specification in a pooled panel. It also mirrors our assumption that liquidity cost depend on relative order size. Sub-index t indicates time and super-index i the stock. Formulation in elasticities allows for smooth statistical properties.[35]

$$log(L_t^i(q)) = C + \sum_j \beta_j z_{jt}^i + \sum_{k=1}^{4} \alpha_k log(L_{t-k}^i(q)) + \epsilon_t \qquad (4)$$

$L(q)$ is liquidity cost to be explained. C is a constant capturing the fixed cost liquidity level. We use different combinations of explanatory variables z_j. We included four lags of log liquidity to remove autocorrelation in the error term. ϵ_t is the time-varying error term. The main dependent variables are as follows:

- $log(q_t^i)$ is the log of the size of the position in thsd. Euro,

[33] This has been modeled for example by Rosu (2003) and Beltran et al. (2005).
[34] Cp. Grossman and Miller (1988) and the overview in Stoll (2000).
[35] Cf. discussion in section 4.4.

- $log(VO)$ is the log of the trading volume in thsd. number of stocks,
- $log(MV)$ is the log of market value of the stock in million Euro,
- R is the continuous mid-price return of the day in percent,
- $log(\sigma_R)$ is the log of the daily return variance in percent, which we measures with the 10-day backward looking, moving variance.
- $log(P)$ is the log of the price level of the day in Euro.

Position size q is included to estimate the size impact. It proxies for the importance of capital restrictions. Transaction volume VO is a good proxy for low inventory risk due to higher participation in trading a particular stock. If transaction volume increases, the time until a limit order is executed is reduced, which in turn reduces unwanted price risk. Market value MV is a good proxy for both low inventory risk due to low price risk and low adverse information risk. High market value stocks experience higher coverage by analysts and traders. This reduces information asymmetries. In total, the same position in a high market value and high transaction volume stock should experience lower liquidity cost due to lower risks.

Continuous return R controls for market conditions and is also a proxy for increased trading and thus reduces inventory risk through shorter delay. Return variance σ_R directly captures inventory risk and is also a control for market conditions. Price level P captures the fix cost of liquidity provision as low price stock require a higher liquidity cost percentage if fix costs exist.

We will have two main lines of regression specification. One includes market value as determinant and the other includes return variance and price level. A combined specification leads to high multicollinearity.[36] We assume that this is because market value acts as proxy for differences in risk and will be correlated with the other risk factors. While the first specification line investigates into our hypothesis of order size, relative to market value and transaction volume, being a determinant of liquidity cost, the second specification analyzes liquidity cost when more finely accounting for differences in stock characteristics. We also employ different time-specific intercepts besides the constant intercept to account for time variation.

Table 6 shows results of the specification with market value. Model 1.0 reveals regression results with constant intercept. Coefficients are reported in percent. All variables are statistically significant at the 1% level. Adjusted-R^2 is high, the Durbin-Watson statistic indicates that autocorrelation has been successfully removed with four lags of the dependent variable.

[36] Variance inflation factors slightly above five. Tables available on request.

	Model 1.0			Model 1.1			Model 1.2			Model 1.3		
	Coef.	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**
Log(q)	5.27 ***	(0.00)	0.12	6.27 ***	(0.00)	0.17	6.43 ***	(0.00)	0.17	7.33 ***	(0.00)	0.19
Log(VO)	-3.15 ***	(0.00)	-0.05	-3.40 ***	(0.00)	-0.05	-3.46 ***	(0.00)	-0.05	-3.00 ***	(0.00)	-0.04
Log(MV)	-2.85 ***	(0.00)	-0.07	-3.67 ***	(0.00)	-0.10	-3.79 ***	(0.00)	-0.11	-2.86 ***	(0.00)	-0.07
R	-102.67 ***	(0.01)	-103.36	-99.16 ***	(0.01)	-98.87	-97.27 ***	(0.01)	-96.68	-99.43 ***	(0.01)	-98.22
Const. intercept	44.56 ***	(0.00)	10.42							53.69 ***	(0.00)	12.80
Log(SCOM(t))										5.63 ***	(0.00)	0.13
Intercept 2002				60.23 ***	(0.00)	17.96						
Intercept 2003				56.60 ***	(0.00)	15.54						
Intercept 2004				53.34 ***	(0.00)	14.01						
Intercept 2005				51.09 ***	(0.00)	12.87						
Intercept 2006				52.06 ***	(0.00)	13.33						
Intercept 2007				52.57 ***	(0.00)	13.53						
Intercept 2008				59.14 ***	(0.00)	17.40						
Intercept 2002 Q3							62.65 ***	(0.00)	19.88			
Intercept 2002 Q4							61.02 ***	(0.00)	19.20			
Intercept 2003 Q1							60.06 ***	(0.00)	18.62			
Intercept 2003 Q2							58.65 ***	(0.00)	17.51			
Intercept 2003 Q3							58.20 ***	(0.00)	16.73			
Intercept 2003 Q4							56.26 ***	(0.00)	15.87			
Intercept 2004 Q1							55.34 ***	(0.00)	15.68			
Intercept 2004 Q2							54.72 ***	(0.00)	15.43			
Intercept 2004 Q3							55.43 ***	(0.00)	15.42			
Intercept 2004 Q4							53.58 ***	(0.00)	14.53			
Intercept 2005 Q1							52.31 ***	(0.00)	13.98			
Intercept 2005 Q2							52.56 ***	(0.00)	13.99			
Intercept 2005 Q3							52.14 ***	(0.00)	13.76			
Intercept 2005 Q4							52.82 ***	(0.00)	13.98			
Intercept 2006 Q1							52.61 ***	(0.00)	14.03			
Intercept 2006 Q2							55.65 ***	(0.00)	15.22			
Intercept 2006 Q3							53.02 ***	(0.00)	14.31			
Intercept 2006 Q4							52.60 ***	(0.00)	13.85			
Intercept 2007 Q1							52.58 ***	(0.00)	13.97			
Intercept 2007 Q2							52.74 ***	(0.00)	14.05			
Intercept 2007 Q3							54.85 ***	(0.00)	14.87			
Intercept 2007 Q4							55.69 ***	(0.00)	15.08			
Intercept 2008 Q1							60.65 ***	(0.00)	18.14			
Log(L(q)_t-1)	50.20 ***	(0.00)	4.47	49.62 ***	(0.00)	4.42	49.50 ***	(0.00)	4.41	48.81 ***	(0.00)	4.31
Log(L(q)_t-2)	16.13 ***	(0.00)	1.44	15.80 ***	(0.00)	1.41	15.74 ***	(0.00)	1.41	15.47 ***	(0.00)	1.37
Log(L(q)_t-3)	11.39 ***	(0.00)	0.98	11.09 ***	(0.00)	0.95	11.05 ***	(0.00)	0.95	10.88 ***	(0.00)	0.93
Log(L(q)_t-4)	13.69 ***	(0.00)	1.04	13.25 ***	(0.00)	1.01	13.21 ***	(0.00)	1.01	12.98 ***	(0.00)	0.98
No. of obs.		1,772,853			1,772,853			1,772,853			1,764,198	
Adj. R-squared		0.95			0.95			0.95			0.95	
Durbin-Watson stat.		2.01			2.00			2.00			2.01	
Schwarz crit.		0.39			0.39			0.39			0.37	

Variance inflation factors (VIFs)				
Intercept	constant	yearly	quarterly	constant
Log(q)	3.77	4.29	4.38	4.32
Log(VO)	2.58	2.65	2.68	2.58
Log(MV)	4.62	**5.02**	**5.09**	4.61
R	1.00	1.00	1.01	1.00
SCOM(t)				2.05
Log(L(q)_t-1)	**18.13**	**18.24**	**18.26**	**18.26**
Log(L(q)_t-2)	**22.62**	**22.65**	**22.66**	**22.61**
Log(L(q)_t-3)	**22.61**	**22.64**	**22.64**	**22.59**
Log(L(q)_t-4)	**17.99**	**18.07**	**18.08**	**18.00**

Table 6: Regression results on relative order size

Dependent variable is log(L(q)), which is log liquidity cost of order size q in bp, q is order size in thsd. Euro, MV is market value in million Euro, VO is transaction volume in thsd. stocks, RSIGMA10 is the 10-day backward rolling variance, P is the mid-price, R is the cont. mid-price return, SCOM is the average log half-spread at time t.

Heteroskedasticity consistent coefficient errors and covariances (White (1980)) used; standard errors in brackets; ***, ** and * indicate significance at 1%, 5% and 10% level; Coef.* contains coefficients standardized by coefficient variance over dependent variable variance in 10^4.

Coefficient signs are as expected. Order size q is positively related to liquidity costs. Increases in market value MV and transaction volume VO decrease liquidity cost as does price return R. Liquidity is very persistent as can be seen from the high coefficients of the lagged variables. Standardized coefficients (reported in 10^4) reveal that return is, by far, the most influential factor. Cutting returns by half more than doubles liquidity cost (103%). Order size and market volume are more important than transaction volume.

Interesting is the absolute value of the coefficients. When order size, market value and transaction volume is proportionally increased, liquidity cost remain approximately constant.[37] This confirms our hypothesis that relative order size, i.e. order size relative to transaction volume and market value, is a decisive category when comparing liquidity across stocks and time. It is also a practical rule of thumb. The error of this rule of thumb remains below 1.5% between specifications.

Results are robust when controlling for time variation in liquidity cost with yearly intercepts in models 1.1 or even finer, quarterly intercepts in model 1.2. Only coefficient levels vary very slightly. There is, however, high multicollinearity as revealed by the variance inflation factors at the bottom of the table, which distort results.

Time varying intercepts reveal that the descriptive results of section 4.3 must be differentiated. Liquidity levels improved over the last years, but almost reached past levels in the recent crises when accounting for improved market values and transaction volumes.

In model 1.3, we use the prevailing spread level as daily intercepts. Spread level is measured as the average daily half-spread across all stocks SCOM, also dubbed liquidity commonality. When finely accounting for time variation, results remain unchanged.

We now turn to the second main specification, which precludes market value MV but includes return variance RSIGMA10 and price level P to control for stock characteristics in a more differentiated way. Table 6 shows regression statistics. Model 2.0 has been specified with constant intercept. The regression shows no autocorrelation and high adjusted-R^2. This specification is slightly preferable as shown with the lower Schwarz criterion compared to models 1.x.

All effects work in the expected direction. Liquidity cost is negatively related to transaction volume, price return and price level. It is positively correlated with order size and mid-price return volatility. Return keeps its dominant role and the coefficient is very similar to prior specifications of 1.x. In contrast, transaction

[37] With an error of only 0.74% (=5.27% - 3.15% - 2.85%).

	Model 2.0			Model 2.1			Model 1.2			Model 1.3		
	Coef.	Stdev.	Coef.'	Coef.	Stdev.	Coef.'	Coef.	Stdev.	Coef.'	Coef.	Stdev.	Coef.'
Log(q)	9.72 ***	(0.00)	0.28	10.13 ***	(0.00)	0.30	10.21 ***	(0.00)	0.30	10.66 ***	(0.00)	0.31
Log(VO)	-7.65 ***	(0.00)	-0.17	-7.98 ***	(0.00)	-0.19	-8.04 ***	(0.00)	-0.19	-7.13 ***	(0.00)	-0.16
R	-95.79 ***	(0.01)	-90.75	-93.29 ***	(0.01)	-87.52	-91.23 ***	(0.01)	-85.60	-94.72 ***	(0.01)	-88.97
log(RSIGMA10)	4.39 ***	(0.00)	0.09	3.98 ***	(0.00)	0.09	3.86 ***	(0.00)	0.09	3.96 ***	(0.00)	0.09
Log(P)	-9.24 ***	(0.00)	-0.32	-9.96 ***	(0.00)	-0.37	-10.09 ***	(0.00)	-0.37	-8.41 ***	(0.00)	-0.29
Const. intercept	117.14 ***	(0.01)	43.79							114.99 ***	(0.01)	42.78
Log(SCOM(t))										4.22 ***	(0.00)	0.10
Intercept 2002				121.81 ***	(0.01)	47.10						
Intercept 2003				119.60 ***	(0.01)	45.18						
Intercept 2004				117.32 ***	(0.01)	44.19						
Intercept 2005				115.82 ***	(0.01)	43.35						
Intercept 2006				117.91 ***	(0.01)	44.57						
Intercept 2007				119.78 ***	(0.01)	45.72						
Intercept 2008				127.91 ***	(0.01)	54.47						
Intercept 2002 Q3							123.11 ***	(0.01)	49.20			
Intercept 2002 Q4							120.64 ***	(0.01)	47.62			
Intercept 2003 Q1							122.15 ***	(0.01)	48.60			
Intercept 2003 Q2							119.17 ***	(0.01)	46.81			
Intercept 2003 Q3							119.76 ***	(0.01)	46.35			
Intercept 2003 Q4							117.91 ***	(0.01)	44.89			
Intercept 2004 Q1							118.68 ***	(0.01)	45.81			
Intercept 2004 Q2							116.52 ***	(0.01)	45.14			
Intercept 2004 Q3							117.48 ***	(0.01)	45.20			
Intercept 2004 Q4							115.71 ***	(0.01)	44.23			
Intercept 2005 Q1							115.96 ***	(0.01)	44.37			
Intercept 2005 Q2							115.12 ***	(0.01)	43.76			
Intercept 2005 Q3							115.52 ***	(0.01)	43.94			
Intercept 2005 Q4							115.64 ***	(0.01)	43.92			
Intercept 2006 Q1							117.15 ***	(0.01)	44.67			
Intercept 2006 Q2							119.84 ***	(0.01)	46.25			
Intercept 2006 Q3							117.26 ***	(0.01)	45.00			
Intercept 2006 Q4							117.28 ***	(0.01)	44.99			
Intercept 2007 Q1							118.89 ***	(0.01)	45.83			
Intercept 2007 Q2							117.85 ***	(0.01)	46.36			
Intercept 2007 Q3							120.29 ***	(0.01)	46.44			
Intercept 2007 Q4							120.58 ***	(0.01)	46.51			
Intercept 2008 Q1							128.03 ***	(0.01)	54.56			
Log(L(q)_t-1)	47.14 ***	(0.00)	4.38	46.85 ***	(0.00)	4.35	46.74 ***	(0.00)	4.34	46.44 ***	(0.00)	4.30
Log(L(q)_t-2)	14.90 ***	(0.00)	1.38	14.76 ***	(0.00)	1.36	14.72 ***	(0.00)	1.36	14.57 ***	(0.00)	1.34
Log(L(q)_t-3)	10.19 ***	(0.00)	0.90	10.06 ***	(0.00)	0.89	10.06 ***	(0.00)	0.89	9.99 ***	(0.00)	0.88
Log(L(q)_t-4)	12.09 ***	(0.00)	0.96	11.95 ***	(0.00)	0.95	11.97 ***	(0.00)	0.95	11.80 ***	(0.00)	0.93
No. of obs.		1,582,762			1,582,762			1,582,762			1,574,913	
Adj. R-squared		0.95			0.95			0.95			0.95	
Durbin-Watson stat.		1.98			1.97			1.97			1.98	
Schwarz crit.		0.34			0.34			0.34			0.33	

Variance inflation factors (VIFs)				
Intercept	constant	yearly	quarterly	quarterly
Log(q)	4.80	4.94	4.97	5.00
Log(VO)	4.51	4.67	4.69	4.61
R	1.00	1.01	1.01	1.00
log(RSIGMA10)	1.49	1.57	1.60	1.51
Log(P)	2.20	2.40	2.42	2.26
SCOM(t)				2.07
Log(L(q)_t-1)	**18.45**	**18.49**	**18.51**	**18.49**
Log(L(q)_t-2)	**22.56**	**22.57**	**22.57**	**22.53**
Log(L(q)_t-3)	**22.50**	**22.51**	**22.51**	**22.46**
Log(L(q)_t-4)	**17.97**	**17.99**	**18.00**	**17.95**

Table 7: Regression results on detailed stock characteristics

Dependent variable is l(q), which is liquidity cost of order size q in bp, q is order size in thsd. Euro, MV is market value in million Euro, VO is transaction volume in number of stocks, RSIGMA10 is the 10-day backward rolling variance, P is the mid-price, R is the continuous mid-price return, SCOM is the average log half-spread at time t. Heteroskedasticity coefficient errors and covariances (White (1980)); standard errors in brackets; ***, ** and * indicate significance at 1%, 5% and 10% level; Coef.* contains coefficients standardized by coefficient variance over dependent variable variance in 10^4.

volume VO takes a more important role. Increase of transaction volume by 100% decreases liquidity by 7.65% in model 1.0 compared to 3.15% in model 2.0. Return volatility's (RSIGMA10) influence is smallest. Absolute order size q has higher coefficients when more finely controlling for differences in stocks.

Effects are again robust when accounting for time variation in the various forms in models 2.1 to 2.3. Time coefficients show that time patterns are similar to the models 1.x, but more robust here because there is no multicollinearity.

In summary, we have shown that order size is a significant determinant of liquidity cost, even when controlling for different stock characteristics and time variation. We can also safely conduct that relative order size is a much better category for comparing liquidity across cross-sections than absolute order size depending on the question at hand. Liquidity of an absolute order size might be of interest when holding a similar position in different stocks. Liquidity of a relative order size will be more suitable when investing in a certain fraction of a company or when predicting liquidity cost across stocks. The rule-of-thumb of constant liquidity costs for relative order size (position volume relative to market value and transaction volume) is quite robust across specifications and has an approximation error of below 1.5%. The interrelations are astonishingly stable, which might provide an indication, that they are driven by fixed structures yet to be analytically described.

4.5.2 When to trade large stock positions

Chordia et al. (2001) have found a day-of-the-week effect in the quoted bid-ask-spread. Quoted spread is found to decline from Monday to Friday and be significantly lower on next to holidays. We retest this hypothesis on the liquidity cost of different order sizes by including weekday dummies and dummies for days before and after holidays in our regression specification. However, in contrast to Chordia et al. we control for all stock characteristics including trading volume. Table 8 on the following page shows the results. In all our specifications Monday and Fridays have significantly higher liquidity costs. Monday is the least liquid day of the week with liquidity cost around 5% higher than average, Tuesday is the most liquid day. Liquidity then continually deteriorates from Tuesday until the end of the week. Days adjoining holidays are similarly illiquid than start and end of the week.

This contrasts to Chordia et al., because we find Monday to be similarly illiquid than Fridays when looking at position size relative to transaction volume and market capitalization or relative to transaction volume alone. Investors should know that relative position size is more expensive to trade on Mondays, Fridays and on days adjoining holidays.

	Model 5.2			Model 5.3			Model 6.2			Model 6.3		
	Coef.	Stdev.	Coef.'	Coef.	Stdev.	Coef.'	Coef.	Stdev.	Coef.'	Coef.	Stdev.	Coef.'
Log(q)	6.38 ***	(0.00)	0.17	7.30 ***	(0.00)	0.19	10.13 ***	(0.00)	0.30	10.58 ***	(0.00)	0.31
Log(VO)	-3.42 ***	(0.00)	-0.05	-2.96 ***	(0.00)	-0.04	-7.98 ***	(0.00)	-0.19	-7.06 ***	(0.00)	-0.16
Log(MV)	-3.79 ***	(0.00)	-0.11	-2.87 ***	(0.00)	-0.07						
R	-98.20 ***	(0.01)	-97.88	-100.20 ***	(0.01)	-99.16	-91.57 ***	(0.01)	-85.97	-94.96 ***	(0.01)	-89.21
log(RSIGMA10)							3.84 ***	(0.00)	0.09	3.92 ***	(0.00)	0.08
Log(P)							-10.03 ***	(0.00)	-0.37	-8.36 ***	(0.00)	-0.29
Monday	5.11 ***	(0.00)	0.28	5.32 ***	(0.00)	0.29	4.29 ***	(0.00)	0.25	4.49 ***	(0.00)	0.26
Tuesday	-0.63 ***	(0.00)	-0.03	-0.50 ***	(0.00)	-0.03	-0.83 ***	(0.00)	-0.05	-0.75 ***	(0.00)	-0.04
Thursday	2.42 ***	(0.00)	0.13	2.41 ***	(0.00)	0.13	2.17 ***	(0.00)	0.12	2.17 ***	(0.00)	0.12
Friday	3.51 ***	(0.00)	0.19	3.38 ***	(0.00)	0.18	3.05 ***	(0.00)	0.17	2.95 ***	(0.00)	0.17
Before/after holiday	2.99 ***	(0.00)	0.70	3.07 ***	(0.00)	0.71	5.71 ***	(0.00)	1.54	5.49 ***	(0.00)	1.48
Const. intercept				51.53 ***	(0.00)	12.39				112.56 ***	(0.01)	42.22
Log(SCOM(t))				5.62 ***	(0.00)	0.13				4.22 ***	(0.00)	0.10
Intercept 2002 Q3	60.27 ***	(0.00)	19.21				120.76 ***	(0.01)	48.63			
Intercept 2002 Q4	58.65 ***	(0.00)	18.54				118.36 ***	(0.01)	47.10			
Intercept 2003 Q1	57.64 ***	(0.00)	17.98				119.73 ***	(0.01)	48.02			
Intercept 2003 Q2	56.33 ***	(0.00)	16.89				116.88 ***	(0.01)	46.24			
Intercept 2003 Q3	55.95 ***	(0.00)	16.18				117.53 ***	(0.01)	45.85			
Intercept 2003 Q4	53.97 ***	(0.00)	15.32				115.63 ***	(0.01)	44.38			
Intercept 2004 Q1	53.06 ***	(0.00)	15.13				116.41 ***	(0.01)	45.29			
Intercept 2004 Q2	52.51 ***	(0.00)	14.89				114.31 ***	(0.01)	44.60			
Intercept 2004 Q3	53.13 ***	(0.00)	14.87				115.19 ***	(0.01)	44.66			
Intercept 2004 Q4	51.33 ***	(0.00)	14.01				113.45 ***	(0.01)	43.71			
Intercept 2005 Q1	50.14 ***	(0.00)	13.50				113.72 ***	(0.01)	43.86			
Intercept 2005 Q2	50.29 ***	(0.00)	13.48				112.92 ***	(0.01)	43.26			
Intercept 2005 Q3	49.86 ***	(0.00)	13.26				113.25 ***	(0.01)	43.42			
Intercept 2005 Q4	50.61 ***	(0.00)	13.49				113.39 ***	(0.01)	43.41			
Intercept 2006 Q1	50.35 ***	(0.00)	13.53				114.93 ***	(0.01)	44.18			
Intercept 2006 Q2	53.46 ***	(0.00)	14.71				117.63 ***	(0.01)	45.75			
Intercept 2006 Q3	50.76 ***	(0.00)	13.80				115.00 ***	(0.01)	44.50			
Intercept 2006 Q4	50.35 ***	(0.00)	13.35				115.02 ***	(0.01)	44.48			
Intercept 2007 Q1	50.36 ***	(0.00)	13.49				116.66 ***	(0.01)	45.35			
Intercept 2007 Q2	50.46 ***	(0.00)	13.53				115.52 ***	(0.01)	45.73			
Intercept 2007 Q3	52.57 ***	(0.00)	14.35				118.01 ***	(0.01)	45.93			
Intercept 2007 Q4	53.39 ***	(0.00)	14.56				118.31 ***	(0.01)	46.01			
Intercept 2008 Q1	58.13 ***	(0.00)	17.55				125.77 ***	(0.01)	54.70			
Log(L(q)_t-1)	49.48 ***	(0.00)	4.41	48.75 ***	(0.00)	4.31	46.77 ***	(0.00)	4.35	46.45 ***	(0.00)	4.31
Log(L(q)_t-2)	16.04 ***	(0.00)	1.43	15.76 ***	(0.00)	1.40	14.97 ***	(0.00)	1.38	14.83 ***	(0.00)	1.37
Log(L(q)_t-3)	11.14 ***	(0.00)	0.96	10.97 ***	(0.00)	0.94	10.15 ***	(0.00)	0.90	10.07 ***	(0.00)	0.89
Log(L(q)_t-4)	12.87 ***	(0.00)	0.98	12.66 ***	(0.00)	0.96	11.66 ***	(0.00)	0.93	11.50 ***	(0.00)	0.91
No. of obs.	1,770,606			1,761,951			1,580,593			1,572,744		
Adj. R-squared	0.95			0.95			0.95			0.95		
Durbin-Watson stat.	2.00			2.01			1.98			1.98		
Schwarz crit.	0.38			0.36			0.33			0.33		

Variance inflation factors (VIFs)				
Intercept	constant	yearly	quarterly	quarterly
Log(q)	4.38	4.32	4.98	**5.01**
Log(VO)	2.68	2.59	4.71	4.63
Log(MV)	**5.09**	4.61		
R	1.01	1.00	1.01	1.00
log(RSIGMA10)			1.60	1.51
Log(P)			2.43	2.26
SCOM(t)		2.05		2.07
Log(L(q)_t-1)	**18.37**	**18.37**	**18.62**	**18.59**
Log(L(q)_t-2)	**22.77**	**22.72**	**22.68**	**22.64**
Log(L(q)_t-3)	**22.75**	**22.70**	**22.61**	**22.57**
Log(L(q)_t-4)	**18.17**	**18.09**	**18.10**	**18.05**

Table 8: Day-of-the-week and holiday effect

Dependent variable is log(L(q)), which is log liquidity cost of order size q in bp, q is order size in thsd. Euro, MV is market value in million Euro, VO is transaction volume in thsd. stocks, RSIGMA10 is the 10-day backward rolling variance, P is the mid-price, R is the cont. mid-price return, SCOM is the average log half-spread at time t. Heteroskedasticity consistent coefficient errors and covariances (White (1980)) used; standard errors in brackets; ***, ** and * indicate significance at 1%, 5% and 10% level; Coef.* contains coefficients standardized by coefficient variance over dependent variable variance in 10^4.

5 Conclusion and outlook

Based on a representative sample of weighted spread for over 320 thousand stock-days, we analyzed the impact of size on liquidity cost, its variation and generally its distributional characteristics. Our main finding is that the impact of order size on liquidity is substantial and cannot be neglected. Easily available bid-ask-spread data can only poorly proxy for cost level and its variation in larger position sizes.

Average liquidity costs varied greatly between order sizes and stocks, strongly increasing with order size up to 460bp. DAX was the most liquid with the lowest cost, followed by MDAX, TecDAX and than SDAX. Even in the DAX, liquidity cost surpassed 100bp for order sizes larger than € 2 million. The possibility of being able to liquidate a position against the order book also strongly declined with size and showed a similar cross-sectional rank than the cost level. Availability was $>$90% for small sizes, but dropped to 13% for € 1 million in the SDAX.

Liquidity strongly improved over the last 5.5 years. Liquidity costs continuously decreased during calm, positive market periods. Sudden increases occurred at stock market crashes such as the events of the sub-prime crises in 2007 and 2008. These spikes are especially pronounced in larger order sizes. The fact that illiquid, large order sizes suffered worse than liquid, small order sizes, presents another aspect of the flight-to-liquidity asymmetry. Trading against the order book was increasingly possible over the sample period. Availability of limit order book increased to 100% in small orders below € 100 thousand across all indices. DAX and MDAX of any size were almost 100% tradable in recent months.

Distributional characteristics of liquidity costs differ greatly between order sizes. Not only do mean liquidity costs increase with order size, so does its variance. In the last 5.5 years, liquidity experienced a steady decline. Outliers due to inefficient liquidity prices generate fat tails in the liquidity distribution, especially in large order sizes.

We also investigated into the fact that the liquidity of absolute-Euro order sizes shows very different behavior across stocks. Our explanation is that absolute order size is not very comparable across stocks. We show that order size relative to market volume and prevailing transaction volume has very stable liquidity cost across stocks and time. Liquidity of relative order size is therefore much better measure in cross-sectional analysis and can act as a rule-of-thumb in comparisons.

In summary, our main conclusions is that liquidity strongly differs across sizes. An impact of size is traceable in distributional characteristics and liquidity dynamics. The empirical evidence presented here can provide new impetus into theoretical modeling of liquidity. In addition, it has impact on practical applications, where liquidity cost and its variation play a role, especially risk management.

Empirical tests of size impact in other limit order book markets are a natural next steps to further generalize our results. From a theoretical point of view, we suggest to investigate into the differences in liquidity determinants across order sizes. It would be interesting to clarify what drives liquidity of large sizes in opposite to smaller sizes, which would provide insight into the different dynamics present in the order book.

Another next step addresses the strong assumption that delay costs are zero. In reality, this assumption has two distinct aspects. In the first case, an asset might not be liquid enough to be instantly tradable. In the XLM data we use, this shows in the number of available data points during the day or in the non-availability of an XLM value for a certain size class. This would lead to forced delay. In the second case, minimization of total liquidity cost might result in deliberate delay of (parts of) the position. Some work has been done here in the literature on optimal trading strategies. A more thorough analysis could extend in both directions.

We have also not touched on analyzing the size impact on liquidity risk. Is it substantial enough to be included in standard risk measures? A suitable method of integrating the size impact into risk calculations and the impact on portfolio correlations has not developed yet.

Recent availability of these rich data on order-size differentiated liquidity allows for a multitude of new research questions and can deepen the understanding of the order book as well as improve the preciseness of practical applications.

6 Appendix

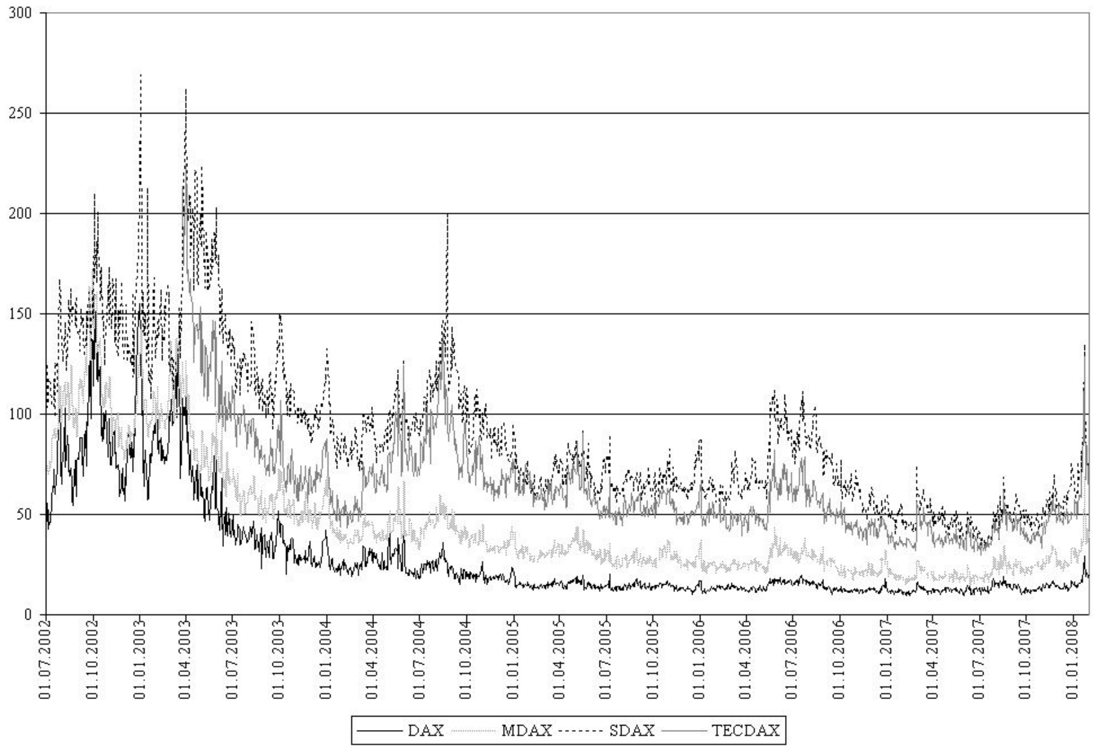

Figure 5: Development of average liquidity cost by index (constant sample)

Paper	Topic	Size of sample
Irvine et al. (2000)	Comparison of several measures	263 (TSE), 2 months 1996
Buhl (2004)	Liquidity in risk management	10 stocks (SMI), 3 months 2002
Giot and Grammig (2005)	Liquidity risk	3 stocks (DAX), 3 months 1999
Francois-Heude and Van Wynendaele (2001)	Liquidity risk	1 stocks (CAC40), 4 months 1999
Gomber et al. (2004)	Resiliency of liquidity	21 stocks (DAX and others), 1 month 2002
Beltran-Lopez et al. (2006)	PCA of the order book	30 stocks (DAX), 3 months 2004
Coppejans et al. (2001)	Market liquidity and volatility	Swedish futures, 7 months 1995-96
Biais et al. (1995)	Liquidity supply and demand	40 stocks (CAC40), 1 month 1991
Domowitz et al. (2005)	Liquidity and order flow	19 stocks (ASX), 10 months 2000

Table 9: Overview of samples in empirical papers on order-size-differentiated liquidity

Avg. liquidity cost (in bp since 24/3/03)		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	**Mean**	0.06	n/a	4.92	5.67	n/a	7.18	n/a	11.79	19.67	n/a	37.85	67.54	91.03	111.10	127.27	46.99	66.42 ***
	Median	0.04	n/a	5.09	5.83	n/a	7.34	n/a	11.49	19.05	n/a	35.81	64.83	96.06	106.61	111.87	20.21	64.73 ***
	Std. Dev.	0.07	n/a	1.85	2.34	n/a	3.39	n/a	6.75	12.87	n/a	26.86	50.46	56.72	62.31	70.60	58.70	75.29 ***
	Availability	99%	n/a	100%	100%	n/a	100%	n/a	100%	100%	n/a	100%	99%	97%	94%	90%	98%	
MDAX	**Mean**	0.20	18.63	23.02	30.35	38.01	45.67	61.36	91.35	145.83	182.26	207.95	n/a	n/a	n/a	n/a	78.41	57.34 ***
	Median	0.14	15.58	18.56	24.34	30.85	38.55	53.84	79.60	126.95	169.64	197.27	n/a	n/a	n/a	n/a	42.36	60.56 ***
	Std. Dev.	0.20	12.04	16.29	22.53	28.22	33.37	43.06	59.14	84.20	97.77	105.49	n/a	n/a	n/a	n/a	84.73	50.61 ***
	Availability	98%	100%	100%	100%	100%	100%	100%	99%	91%	82%	72%	n/a	n/a	n/a	n/a	94%	
SDAX	**Mean**	0.47	60.20	80.62	112.83	139.46	162.69	195.38	245.75	324.65	396.40	459.60	n/a	n/a	n/a	n/a	160.79	45.09 ***
	Median	0.36	52.27	67.84	97.56	125.18	150.78	184.46	227.33	293.86	342.42	412.83	n/a	n/a	n/a	n/a	121.94	46.04 ***
	Std. Dev.	0.39	34.10	55.01	69.41	77.18	86.10	97.16	130.71	137.11	198.68	247.50	n/a	n/a	n/a	n/a	134.53	39.46 ***
	Availability	99%	99%	99%	96%	92%	88%	79%	64%	38%	23%	15%	n/a	n/a	n/a	n/a	69%	
TECDAX	**Mean**	0.26	30.83	40.77	57.26	75.28	93.87	124.54	170.17	241.78	289.84	326.97	n/a	n/a	n/a	n/a	122.72	55.07 ***
	Median	0.20	28.51	38.64	53.61	70.05	86.73	120.43	173.01	227.30	255.28	288.88	n/a	n/a	n/a	n/a	73.87	54.36 ***
	Std. Dev.	0.22	17.18	26.17	43.37	63.24	83.93	101.26	108.03	112.80	125.08	137.66	n/a	n/a	n/a	n/a	122.76	45.15 ***
	Availability	99%	100%	100%	100%	100%	99%	98%	92%	73%	56%	44%	n/a	n/a	n/a	n/a	86%	
All	**Mean**	0.27	n/a	40.77	55.79	n/a	80.87	n/a	125.47	160.37	n/a	188.51	n/a	n/a	n/a	n/a	100.84	42.95 ***
	Median	0.17	n/a	25.88	33.62	n/a	50.75	n/a	93.51	133.15	n/a	163.86	n/a	n/a	n/a	n/a	61.91	53.36 ***
	Std. Dev.	0.30	n/a	44.51	60.42	n/a	85.15	n/a	120.25	138.09	n/a	173.51	n/a	n/a	n/a	n/a	111.43	36.65 ***
	Availability	98%	n/a	100%	99%	n/a	96%	n/a	87%	73%	n/a	54%	n/a	n/a	n/a	n/a	86%	

Table 10: Liquidity cost by index and size since 24/03/2003

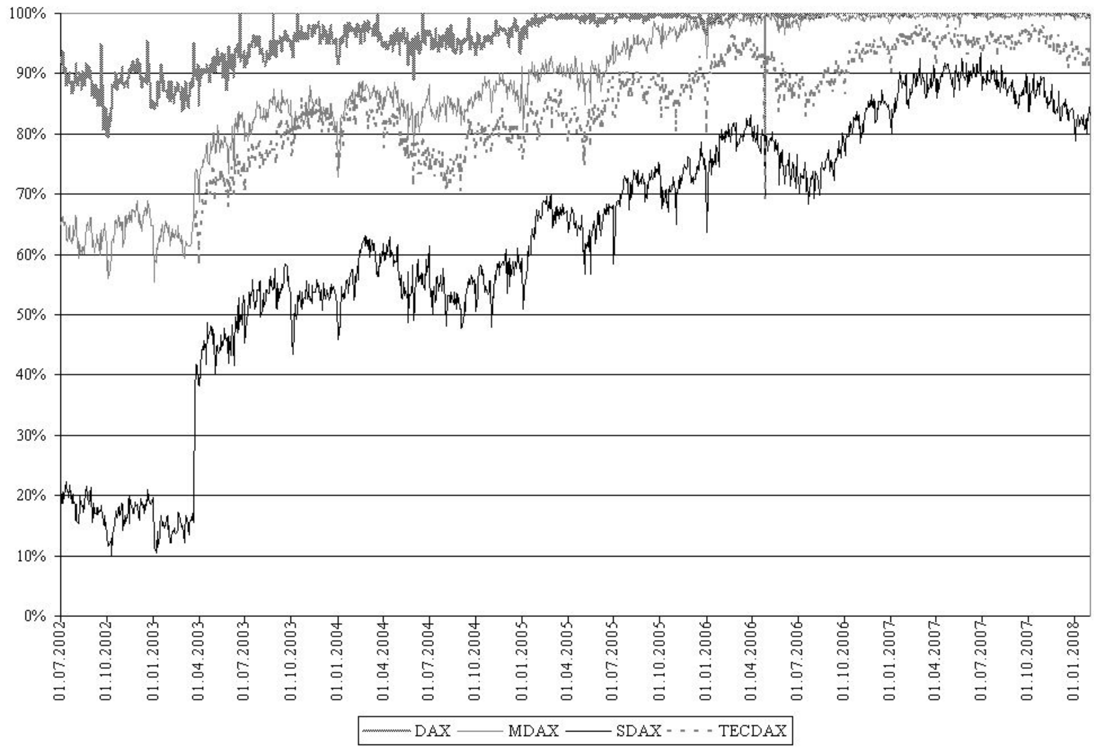

Figure 6: Development of average availability by index

Avg. liquidity cost		Order size (in thsd. Euro)															
(in bp)		**Min**	**10**	**25**	**50**	**75**	**100**	**150**	**250**	**500**	**750**	**1000**	**2000**	**3000**	**4000**	**5000**	**All**
DAX	**II/2002**	0.26	n/a	13.96	17.08	n/a	23.96	n/a	46.99	91.04	n/a	187.50	320.53	350.82	396.82	442.47	162.40
	2003	0.15	n/a	9.36	11.51	n/a	15.82	n/a	29.29	54.66	n/a	120.40	211.23	252.03	286.11	311.67	116.90
	2004	0.06	n/a	4.99	5.79	n/a	7.34	n/a	12.01	20.10	n/a	39.01	79.65	116.46	146.57	168.50	56.18
	2005	0.05	n/a	3.80	4.28	n/a	5.24	n/a	8.09	12.65	n/a	21.36	38.85	58.54	77.57	96.65	32.24
	2006	0.04	n/a	4.10	4.50	n/a	5.33	n/a	7.97	12.39	n/a	20.61	34.96	49.74	66.67	83.02	28.81
	2007	0.04	n/a	3.83	4.27	n/a	5.21	n/a	8.14	12.72	n/a	20.48	33.61	46.12	60.19	75.75	27.01
	01/2008	0.05	n/a	4.88	5.58	n/a	7.06	n/a	11.64	18.81	n/a	30.61	50.58	72.49	100.43	128.15	42.95
	All	0.09	n/a	5.99	7.04	n/a	9.22	n/a	16.10	28.56	n/a	56.79	97.51	116.90	136.30	153.17	60.15
MDAX	**II/2002**	0.72	85.00	110.60	134.10	157.38	179.16	229.36	301.14	415.67	452.24	460.99	n/a	n/a	n/a	n/a	189.32
	2003	0.47	43.70	56.48	73.38	93.12	112.71	151.13	215.18	292.46	342.23	365.64	n/a	n/a	n/a	n/a	141.79
	2004	0.22	21.52	26.38	34.89	43.98	53.62	75.14	116.36	197.07	242.91	275.82	n/a	n/a	n/a	n/a	92.87
	2005	0.17	14.74	18.03	23.29	28.49	33.71	44.38	66.64	124.13	169.20	198.75	n/a	n/a	n/a	n/a	66.69
	2006	0.13	11.87	14.02	18.01	22.22	26.52	35.18	52.63	98.00	137.95	171.71	n/a	n/a	n/a	n/a	57.78
	2007	0.13	10.79	12.89	16.76	20.79	24.78	32.49	47.05	82.13	117.52	152.18	n/a	n/a	n/a	n/a	51.39
	01/2008	0.18	15.69	19.17	25.68	32.86	40.10	54.21	80.90	146.36	211.70	279.26	n/a	n/a	n/a	n/a	89.70
	All	0.29	28.48	35.91	45.01	54.53	63.79	82.60	112.75	160.87	192.79	215.90	n/a	n/a	n/a	n/a	90.37
SDAX	**II/2002**	1.83	243.35	327.98	484.71	692.59	923.62	1,248.33	952.52	126.58	93.04	70.88	n/a	n/a	n/a	n/a	394.86
	2003	1.03	131.92	174.85	231.88	287.69	334.10	397.62	495.91	621.85	577.60	515.66	n/a	n/a	n/a	n/a	253.75
	2004	0.57	80.39	113.41	175.49	222.11	265.32	310.33	391.68	438.16	521.72	680.02	n/a	n/a	n/a	n/a	203.29
	2005	0.41	52.83	67.97	95.29	122.77	146.23	184.89	232.44	312.66	411.80	484.35	n/a	n/a	n/a	n/a	141.50
	2006	0.36	44.58	55.53	74.84	95.14	116.68	157.01	225.06	326.41	403.38	458.06	n/a	n/a	n/a	n/a	141.34
	2007	0.28	30.02	37.46	49.90	62.93	76.48	105.06	164.50	285.09	367.26	439.58	n/a	n/a	n/a	n/a	131.04
	01/2008	0.36	42.62	54.31	76.35	100.64	127.68	181.92	300.79	495.65	630.40	736.20	n/a	n/a	n/a	n/a	194.33
	All	0.64	80.80	95.90	125.21	151.96	174.88	203.74	248.41	326.41	398.92	464.20	n/a	n/a	n/a	n/a	169.94
TecDAX	**II/2002**		n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	
	2003	0.43	54.93	74.38	105.84	143.45	183.54	241.96	320.12	376.24	411.95	469.00	n/a	n/a	n/a	n/a	187.05
	2004	0.30	36.98	50.07	72.96	98.70	125.57	169.21	230.80	312.85	349.55	349.32	n/a	n/a	n/a	n/a	141.58
	2005	0.23	26.80	34.99	47.49	60.09	73.41	100.09	145.12	235.59	293.06	321.63	n/a	n/a	n/a	n/a	107.33
	2006	0.20	22.80	29.07	40.05	51.04	61.95	83.95	128.53	222.66	285.76	336.10	n/a	n/a	n/a	n/a	109.40
	2007	0.16	18.26	23.30	31.61	39.87	48.04	63.91	93.82	169.25	227.18	273.38	n/a	n/a	n/a	n/a	91.89
	01/2008	0.23	28.23	35.44	48.27	62.17	76.01	103.09	158.04	293.98	354.04	431.06	n/a	n/a	n/a	n/a	141.20
	All	0.26	30.83	40.77	57.26	75.28	93.87	124.54	170.17	241.78	289.84	326.97	n/a	n/a	n/a	n/a	122.72
All	**II/2002**	1.00	n/a	123.84	135.02	n/a	167.27	n/a	201.32	232.66	n/a	259.79	n/a	n/a	n/a	n/a	202.75
	2003	0.58	n/a	82.33	104.57	n/a	146.37	n/a	209.01	222.72	n/a	238.91	n/a	n/a	n/a	n/a	164.37
	2004	0.31	n/a	53.66	78.64	n/a	113.00	n/a	153.37	173.69	n/a	179.78	n/a	n/a	n/a	n/a	118.36
	2005	0.23	n/a	33.94	46.39	n/a	69.15	n/a	108.20	140.92	n/a	164.43	n/a	n/a	n/a	n/a	85.48
	2006	0.20	n/a	27.95	37.34	n/a	57.20	n/a	107.06	148.33	n/a	178.73	n/a	n/a	n/a	n/a	83.89
	2007	0.16	n/a	20.82	27.56	n/a	41.62	n/a	84.79	140.03	n/a	190.64	n/a	n/a	n/a	n/a	77.18
	01/2008	0.22	n/a	30.52	41.98	n/a	68.01	n/a	148.67	231.24	n/a	280.95	n/a	n/a	n/a	n/a	118.80
	All	0.36	n/a	48.59	62.38	n/a	87.37	n/a	130.25	163.60	n/a	192.25	n/a	n/a	n/a	n/a	108.36

Table 11: Liquidity cost by index, year and order size

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