Hanck, Christoph

Working Paper

The Error-in-Rejection Probability of Meta-Analytic Panel Tests

Technical Report / Universität Dortmund, SFB 475 Komplexitätsreduktion in Multivariaten Datenstrukturen, No. 2006,46

Provided in cooperation with:
Technische Universität Dortmund

The Error-in-Rejection Probability of Meta-Analytic Panel Tests

Christoph Hanck*

Preliminary.

November 13, 2006

Abstract

Meta-analytic panel unit root tests such as Fisher's χ^2 test, which consist of pooling the p-values of time series unit root tests, are widely applied in practice. Recently, several Monte Carlo studies have found these tests' Error-in-Rejection Probabilities (or, synonymously, size distortion) to increase with the number of series in the panel. We investigate this puzzling finding by modelling the finite sample p-value distribution of the time series tests with local deviations from the asymptotic p-value distribution. We find that the size distortions of the panel tests can be explained as the cumulative effect of small size distortions in the time series tests.

Keywords: Panel Unit Root Tests, Meta-Analysis, Error-in-Rejection Probability

1 Introduction

Meta-analysis is a useful tool to efficiently combine related information.[1] In recent years, the meta-analytic testing approach has been fruitfully applied to nonstationary panels: Consider the testing problem on the panel as consisting of N testing problems for each unit of the panel. That is, conduct N separate time series tests and obtain the corresponding p-values of the test statistics. Then, combine the p-values of the N tests into a single panel test statistic. Among others, Maddala and Wu [1999], Choi [2001] and Phillips and

*Research supported by DFG under Sonderforschungsbereich 475. I would like to thank Sebastian Herr for very helpful discussion. Universität Dortmund/Ruhr Graduate School in Economics. Contact Information: Tel. (+49) 0231-7553127, christoph.hanck@uni-dortmund.de.

[1]See Hedges and Olkin [1985] for an excellent introduction to the topic.

Sul [2003] propose meta-analytic panel unit root and cointegration tests. The tests are intuitive, relatively easy to compute and allow for a considerable amount of heterogeneity in the panel.

Via Monte Carlo experiments, the above-cited authors show that their meta-analytic tests can be substantially more powerful than separate time series tests on each unit in the panel, justifying the use of panel tests. Disturbingly, however, Choi [2001], Hlouskova and Wagner [2006] and Hanck [2005], inter alia, find the Error-in-Rejection Probability (ERP) (or, synonymously, size distortion) to be increasing in N. That is, the (absolute) difference between the estimated rejection probability (or type I error rate) $R(\alpha, N)$ and the nominal significance level α, $ERP_N(\alpha) := |R(\alpha, N) - \alpha|$, gets larger with N. A priori, this finding is counterintuitive, since, more information should improve the performance of the panel tests.

We argue that this behavior may be explained as the cumulative effect of arbitrarily small ERPs in the underlying time series test statistics composing the panel test statistics. Under a simple H_0, assuming continuous distribution functions of the test statistics, p-values of test statistics should be distributed uniformly on the unit interval, denoted $\mathcal{U}[0, 1]$ [see, e.g., Bickel and Doksum, 2001, Sec. 4.1]. We model size-distorted time series tests by deviations from the null distribution of the test statistics' p-values. The analytical and simulation evidence reported in the following sections corroborate our conjecture.

2 The P-Value Combination Test

We briefly review the p-value combination test whose ERP is investigated subsequently.[2] We discuss the example of a panel unit root test. The conclusions might however be valid also for other applications of the meta test. Denote by p_i the marginal significance level, or p-value, of a time series unit root test applied to the ith unit of the panel. Let θ_{i,T_i} be a unit root test statistic on unit i for a sample size of T_i . Let F_{T_i} denote the null distribution function of the test θ_{i,T_i}. Since the tests considered here are one-sided, $p_i = F_{T_i}(\theta_{i,T_i})$ if the test rejects for small values of θ_{i,T_i} and $p_i = 1 - F_{T_i}(\theta_{i,T_i})$ if the test rejects for large values of θ_{i,T_i}. We only consider time series tests with the null of a unit root.

[2] Similar results for other widely used meta-analytic tests such as the inverse normal test are available upon request.

We test the following null hypothesis:

$$H_0: \text{ The time series } i \text{ is unit-root nonstationary} \qquad (i \in \mathbb{N}_N), \tag{1}$$

against the alternative

$$H_1: \text{ For at least one } i, \text{ the time series is stationary.}$$

($(i \in \mathbb{N}_N)$ is shorthand for $i = 1, \ldots, N$.) The N p-values of the individual time series tests, p_i $(i \in \mathbb{N}_N)$, are combined as follows to obtain a test statistic for panel (non-)stationarity:

$$P_{\chi^2} = -2 \sum_{i=1}^{N} \ln p_i \tag{2}$$

The P_{χ^2} test, via pooling p-values, provides a convenient test for panel (non-)stationarity by imposing minimal homogeneity restrictions on the panel. For instance, the panel can be unbalanced. For further discussion see Choi [2001] or Hanck [2005]. The following lemma gives the asymptotic distribution of the test.

Lemma 1 *(Distribution of the P_{χ^2} test).*
Under the null of panel nonstationarity and assuming continuous distribution functions of the θ_{i,T_i}, the P_{χ^2} test is, as $T_i \to \infty$ $(i \in \mathbb{N}_N)$, asymptotically distributed as

$$P_{\chi^2} \to_d \chi^2_{2N}$$

Proof. The proof is an application of the transformation theorem for absolutely continuous random variables (r.v.s) [see, e.g., Bierens, 2005, Thm. 4.2]. Under H_0 and as $T_i \to \infty$ $(i \in \mathbb{N}_N)$, $p_i \sim \mathcal{U}[0,1]$. Let $y = g(p_i) := -2 \ln p_i$. Then, $p_i = g^{-1}(y) = e^{-\frac{1}{2}y}$ and

$$f_{-2\ln p_i}(y) = f_{p_i}(g^{-1}(y))|g^{-1'}(y)|.$$

Hence, $g^{-1'}(y) = -\frac{1}{2}e^{-\frac{1}{2}y}$. Thus, $|g^{-1'}(y)| = \frac{1}{2}e^{-\frac{1}{2}y}$. We have $f_{p_i}(g^{-1}(y)) = 1 \ \forall \ g^{-1}(y) \in [0,1]$. This implies $f_{-2\ln p_i}(y) = \frac{1}{2}e^{-\frac{1}{2}y}$. The density of a χ^2_R r.v. is $f_{\chi^2_R}(y) = \frac{1}{2^{R/2}\Gamma(R/2)} y^{\frac{R}{2}-1} e^{-\frac{y}{2}}$. With $R = 2$, $f_{\chi^2_2}(y) = \frac{1}{2\Gamma(1)} e^{-\frac{y}{2}}$. Recall that $\Gamma(1) = \int_0^\infty t^{1-1} e^{-t}\, dt = 1$. So,

$$f_{\chi^2_2}(y) = \frac{1}{2} e^{-\frac{y}{2}}.$$

We have shown that $f_{-2\ln p_i}(y) = f_{\chi^2_2}(y)$. The proof is completed by noting that the sum of N independent χ^2_R r.v.s is distributed as χ^2_{NR}. □

The decision rule is to reject the null of panel nonstationarity when P_{χ^2} exceeds the critical value from a χ^2_{2N} distribution at the desired significance level. We see from the proof that the test has a well-defined asymptotic distribution (for $T \to \infty$) for any finite N. This feature is attractive because in many applications, the assumption that N, the number of units in the panel, goes to infinity may not be a natural one.

Table I—Simulated Type I Error Rates for the P_{χ^2} Test.

	N	5	10	25	30	50	60	100
Maddala and Wu [1999]				.044		.107		.131
Choi [2001]		.050	.070	.090		.090		.130
Hanck [2005]			.035	.031		.021		.014
Hlouskova and Wagner [2006]			.090	.110		.120		.145
Choi [2006]					.051		.042	.037

Note: All results are for the nominal 5% level.

3 The Error-in-Rejection Probability of the Combination Test

As should be clear from the previous discussion, any unit root for which p-values are available can be used to compute the P_{χ^2} test statistic. Popular choices include the Augmented Dickey-Fuller test [Dickey and Fuller, 1979]. It is well-known that using the (first-order) asymptotic approximation F, a functional of Brownian Motions and possibly nuisance parameters, to the exact, finite T_i null distribution of the test statistics, F_{T_i}, need not be accurate. This is because the null hypothesis (1) is not a simple one (and the available test statistics are not pivotal). H_0 is satisfied by all unit-root nonstationary processes

$$y_{i,t} = y_{i,t-1} + u_{i,t}, \qquad (i \in \mathbb{N}_N)$$

where the errors $u_{i,t}$ can be from a wide class of dependent and heterogeneous sequences. See, for instance, the fairly general strong mixing conditions on $u_{i,t}$ of Phillips [1987]. Hence, the p-values of the test need no longer be uniformly distributed on the unit interval, even if the true Data Generating Process (DGP) of the time series is from the null hypothesis set of unit-root nonstationary processes. Thus, the assumptions required for validity of Lemma 1 need no longer be met.

As we argue in this section, this fact can explain the counterintuitive finding of a deteriorating performance of the P_{χ^2} test with increasing N. Table I summarizes selected Monte Carlo results on the ERP of the P_{χ^2} test reported in the literature.[3] Most authors find $R(\alpha, N) - \alpha$ to increase with N, while Hanck [2005] and Choi report an inverse relationship. All find $ERP_N(\alpha) := |R(\alpha, N) - \alpha|$ to increase with N.

[3]The differences stem from the length of the underlying time series, the type of non-stationarity test applied to the time series, as well as the design of the DGP.

We propose the following modelling assumption to investigate this behavior.

Assumption 1 *(Generalized p-value distribution).*
For finite T_i, the p-values are distributed as $\tilde{p}_i \sim \mathcal{U}[a, b]$, where $a \geqslant 0$, $b \leqslant 1$ and $a < b$, $(i \in \mathbb{N}_N)$.

Since the exact, finite T_i distribution of the test statistics is generally unknown, so is the exact p-values' distribution. The assumption is, however, convenient for modelling purposes. First, letting $a \to 0$ and $b \to 1$, it comprises the asymptotic result as a limiting case. Second, it is easy to characterize the ERP of a single time series test in terms of a and b. More precisely, since a rejection at level α is equivalent to a p-value $p < \alpha$,

$$\mathsf{P}(F_{\tilde{p}_i} < \alpha) = R(\alpha, 1) = \begin{cases} 0 & \text{for} \quad a > \alpha \\ \frac{\alpha - a}{b - a} & \text{for} \quad a < \alpha \text{ and } b > \alpha \\ 1 & \text{for} \quad b < \alpha \end{cases}$$

In particular, it is possible to model "oversized" unit root tests by taking $\tilde{p}_i \sim \mathcal{U}[0, b]$, where $b < 1$. Intuitively, we remove the p-values corresponding to the test statistics speaking most strongly in favor of H_0. Conversely, $\tilde{p}_i \sim \mathcal{U}[a, 1]$, $a > 0$ represents an "undersized" test. The following lemma derives the density function of $-2\ln\tilde{p}_i$ under Assumption 1.

Lemma 2 *(Distribution of $-2\ln\tilde{p}_i$).*
Under $\tilde{p}_i \sim \mathcal{U}[a, b]$, the density of $-2\ln\tilde{p}_i$ is given by

$$f_{-2\ln\tilde{p}_i}(y) = \begin{cases} 0 & \text{for} \quad y \in (-\infty, -2\ln b) \\ \frac{1}{2(b-a)} e^{-\frac{y}{2}} & \text{for} \quad y \in [-2\ln b, -2\ln a] \\ 0 & \text{for} \quad y \in (-2\ln a, \infty), \end{cases}$$

taking $-\ln a = \infty$ for $a = 0$.

Proof. Again, we can apply the transformation theorem for absolutely continuous r.v.s. Using the notation from the proof of Lemma 1, we still have $\tilde{p}_i = g^{-1}(y) = e^{-\frac{1}{2}y}$ and hence $|g^{-1\prime}(y)| = \frac{1}{2}e^{-\frac{y}{2}}$. $f_{\tilde{p}_i}$ follows immediately from Assumption 1 as $f_{\tilde{p}_i}(g^{-1}(y)) = \frac{1}{b-a}$ for $g^{-1}(y) \in [a, b]$ and 0 otherwise. The support of the r.v. $-2\ln\tilde{p}_i$ follows from solving g^{-1} for the lower and upper bounds of $\tilde{p}_i$. It is verified elementarily that $f_{-2\ln\tilde{p}_i}(y)$ satisfies $\int_{\mathbb{R}} f_{-2\ln\tilde{p}_i}(\tilde{y})d\tilde{y} = 1$. □

$f_{-2\ln\tilde{p}_i}(y)$ contains the density of the χ^2_2 distribution as a special case with $a = 0$ and $b = 1$. We now study the ERP of the P_{χ^2} test for the case $N = 1$, denoted $ERP_1(\alpha)$.

Let c_{α_2} be the critical value of the χ^2_2-distribution at nominal level α, i.e. $\int_0^{c_{\alpha_2}} \frac{1}{2} e^{-\frac{1}{2}\tilde{y}} d\tilde{y} = 1 - \alpha \Rightarrow c_{\alpha_2} = -2\ln\alpha$. Then,

$$\begin{aligned} R(\alpha, 1) &= 1 - \int_{-\infty}^{-2\ln\alpha} f_{-2\ln\tilde{p}_i}(\tilde{y}) d\tilde{y} \\ &= \frac{\alpha}{b} \end{aligned}$$

Let us consider a specific example. We investigate the "oversized" case, $\tilde{p}_i \sim \mathcal{U}[0, 0.9]$, and $\alpha = 0.05$. Then, $ERP_1(0.05) = \left|\frac{0.05(1-0.9)}{0.9}\right| = \frac{0.05(1-0.9)}{0.9} \approx 0.005$, yielding an ERP which would be considered small in most Monte Carlo analyses.

For $N = 2$, we derive the following lemma in the appendix:

Lemma 3 *(Density function of $f_{-2\sum_{i=1}^2 \ln\tilde{p}_i}(y)$).*

$$f_{-2\sum_{i=1}^2 \ln\tilde{p}_i}(y) = \begin{cases} 0 & \text{for} \quad y \in (-\infty, -2\ln b) \\ \frac{1}{4(b-a)^2} e^{-\frac{y}{2}} (y + 4\ln b) & \text{for} \quad y \in [-4\ln b, -2\ln a - 2\ln b] \\ \frac{1}{4(b-a)^2} e^{-\frac{y}{2}} (-y - 4\ln a) & \text{for} \quad y \in (-2\ln a - 2\ln b, -4\ln a] \\ 0 & \text{for} \quad y \in (-4\ln a, \infty), \end{cases}$$

taking $-\ln a = \infty$ for $a = 0$.

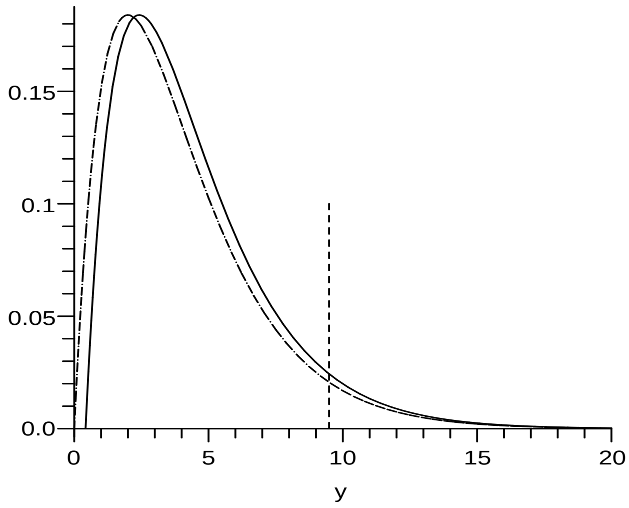
Continuing the above example, we compute $ERP_2(0.05)$ as

$$\begin{aligned} ERP_2(0.05) &= |R(0.05, 2) - 0.05| = R(0.05, 2) - 0.05 \\ &= 1 - \int_{-\infty}^{c_{\alpha_4}} f_{-2\sum_{i=1}^2 \ln\tilde{p}_i}(\tilde{y}) d\tilde{y} - 0.05 \\ &\approx 0.009 \end{aligned}$$

Note that $ERP_2(0.05) > ERP_1(0.05)$. To illustrate, Figure I displays $f_{-2\sum_{i=1}^2 \ln\tilde{p}_i}(y)$ and the density function of the χ^2_4 distribution. The generalized p-value distribution lies to the right of the χ^2_4 distribution, as expected. The dashed line indicates the 0.95 quantile of the χ^2_4 distribution. $f_{-2\sum_{i=1}^2 \ln\tilde{p}_i}(y)$ has probability mass of more than 0.05 to the right of c_{α_4}.

To analyze the ERP of the P_{χ^2} test for general N, we require the cumulative distribution function of the r.v. $-2\sum_{i=1}^N \ln\tilde{p}_i$ under Assumption 1. In keeping with most applications in the literature, we assume independence across i. It is then possible to write the density of $-2\sum_{i=1}^N \ln\tilde{p}_i$ as the convolution of $f_{-2\ln\tilde{p}_i}$ ($i \in \mathbb{N}_N$) [see, e.g., Shiryaev, 1996, pp. 241]

$$\begin{aligned} f_{-2\sum_{i=1}^N \ln\tilde{p}_i}(y) &= f_{-2\ln\tilde{p}_1} * \cdots * f_{-2\ln\tilde{p}_N}(y) \\ &= \frac{e^{-\frac{y}{2}}}{2^N(b-a)^N} \varphi_{-2\ln b, -2\ln a} * \cdots * \varphi_{-2\ln b, -2\ln a}, \end{aligned}$$



Dotted: χ^2_4-density, Solid: $f_{-2\sum_{i=1}^{2}\ln\tilde{p}_i}(y)$, Dashed: $\chi^2_{4,.95}$ critical value

Figure I—The density functions for the case $N = 2$

where φ is the indicator function of y on the interval $I = [-2\ln b, -2\ln a]$. Introducing a suitable standardization factor r_N, the convolution for general N can be written as a function of the indicator functions of y on the unit interval,

$$f_{-2\sum_{i=1}^{N}\ln\tilde{p}_i}(y) = r_N \frac{e^{-\frac{y}{2}}}{2^N(b-a)^N}\varphi_{0,1} * \cdots * \varphi_{0,1}.$$

By a Central Limit Theorem, the sum of N centered and standardized uniform r.v.s converges to a standard normal r.v. Using a further scaling constant s_N, we expect that the density of $f_{-2\sum_{i=1}^{N}\ln\tilde{p}_i}(y)$ can be well approximated for N sufficiently large by an expression of the form $r_N s_N \frac{e^{-\frac{y}{2}}}{2^N(b-a)^N}\phi_N(y)$. Here, $\phi_N(y)$ is the density function of the standard normal distribution (whose argument also depends on N).

The exact computation however quickly becomes cumbersome for large N. We shall therefore rely on simulation to further illustrate the effect of increasing N. For each $N \in \{1, 6, 11, \ldots, 246\}$ we generate p-values according to $\mathcal{U}[0.02, 1]$, $\mathcal{U}[0, 1]$ and $\mathcal{U}[0, 0.9]$, corresponding to under-, correctly, and oversized tests. Based on $R = 50,000$ replications, Figure II displays the rejection rates of the P_{χ^2} test as a function of N when using the 5% critical values of the appropriate χ^2_{2N} distribution.

The figure confirms the conjectures resulting from the theoretical analysis. When the p-values are, as they should be under H_0, distributed uniformly on the unit interval, the ERP of the P_{χ^2} test is excellent uniformly in N. Conversely, even for small deviations from the nominal size of the time series tests, the ERPs clearly increase in N. Reassuringly,

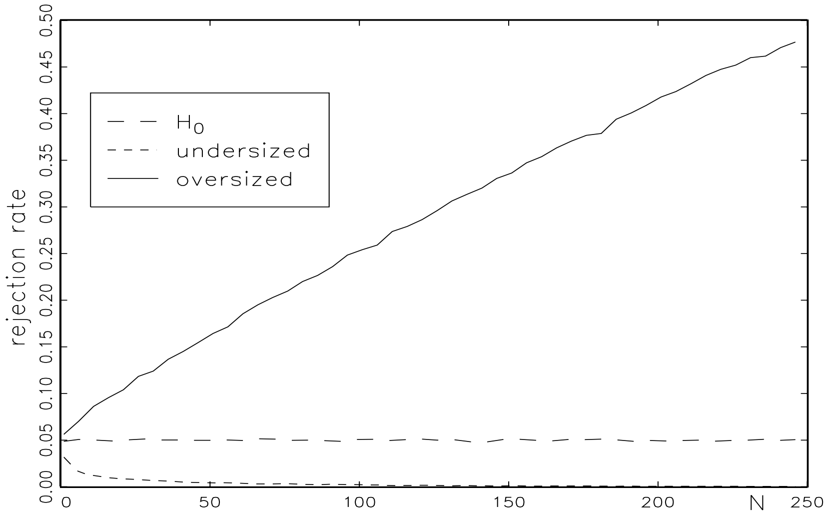


Figure II—Rejection Rates of P_{χ^2} test at 5% as a Function of N

the simulation result for $N = 1$ is virtually indistinguishable from the analytical result above. Similarly, Figure III reveals that the ERP of the panel test is the higher the stronger the component test statistics are oversized, as one would expect.

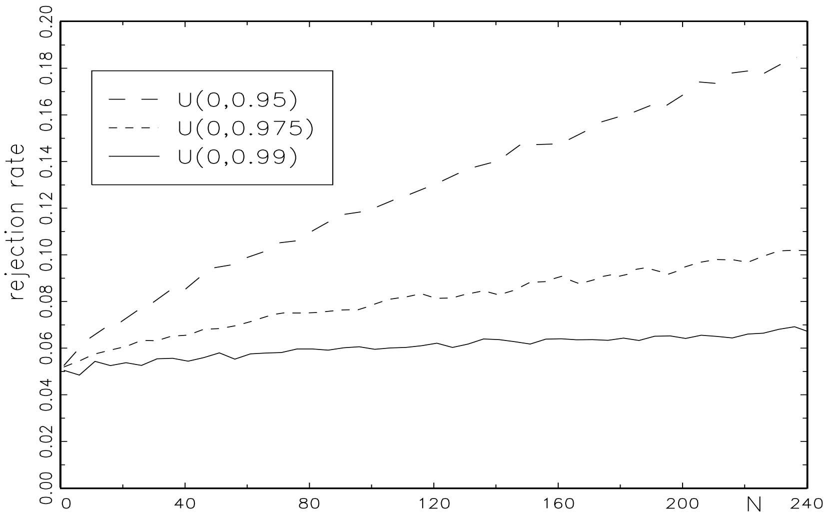


Figure III—Rejection Rates of P_{χ^2} test at 5% for differing degrees of "oversizedness"

4 Conclusion

We show that meta-analytic panel tests can have arbitrarily large Errors-in-Rejection Probabilities (size distortions) even when the underlying time series tests have only slight

Errors-in-Rejection Probabilities. The recommendation for empirical practice therefore is to use critical values which take into account as well as possible the shape of the exact (but generally unknown) finite sample distribution of the test statistics. One way to achieve this is to compute correction factors depending on T using response surface regressions [MacKinnon, 1991]. Even though we discuss the application of the P_{χ^2} test to testing problems for nonstationary panel data, the conclusions may hold more generally for other applications and other meta-analytic test statistics.

Appendix

Proof of Lemma 3

The convolution integral is given by

$$\begin{aligned} f_{-2\sum_{i=1}^{2}\ln\tilde{p}_i}(y) &= f_{-2\ln\tilde{p}_i} * f_{-2\ln\tilde{p}_i} \\ &= \int_{\mathbb{R}} \frac{1}{4(b-a)^2} e^{-\frac{1}{2}x} \varphi_{-2\ln b,-2\ln a} e^{-\frac{1}{2}(y-x)} \varphi_{-2\ln b,-2\ln a} dx \\ &= \frac{1}{4(b-a)^2} e^{-\frac{1}{2}y} \int_{\mathbb{R}} \varphi_{-2\ln b,-2\ln a} \varphi_{-2\ln b,-2\ln a} dx \end{aligned} \tag{A.1}$$

Since we consider the convolution of two densities with support $I = [-2\ln b, -2\ln a]$, the arguments have to satisfy $y-x, x \in [-2\ln b, -2\ln a]$, implying the following weak inequalities: $x \leqslant y+2\ln b$, $x \geqslant y+2\ln a$, $y \leqslant -4\ln a$ and $y \geqslant -4\ln b$. Together, these require that

$$\begin{aligned} x &\leqslant \min\{y+2\ln b, -2\ln a\} =: M(y) \quad \text{and} \\ x &\geqslant \max\{-2\ln b, y+2\ln a\} =: m(y) \end{aligned}$$

That is, we distinguish the following cases in (A.1):

1. For $y \in (-\infty, -4\ln b)$, we have $f_{-2\sum_{i=1}^{2}\ln\tilde{p}_i}(y) = 0$.
2. We have $-2\ln b \geqslant y + 2\ln a$ for $y \in [-4\ln b, -2\ln a - 2\ln b]$ and hence $m(y) = -2\ln b$ and $M(y) = y + 2\ln b$. Thus,

$$\begin{aligned} f_{-2\sum_{i=1}^{2}\ln\tilde{p}_i}(y) &= \frac{1}{4(b-a)^2} e^{-\frac{1}{2}y} \int_{-2\ln b}^{y+2\ln b} dx \\ &= \frac{1}{4(b-a)^2} e^{-\frac{y}{2}} (y + 4\ln b). \end{aligned}$$

3. We have $-2\ln b \leqslant y + 2\ln a$ for $y \in (-2\ln a - 2\ln b, -4\ln a)$ and hence $m(y) = y + 2\ln a$ and $M(y) = -2\ln a$. Thus,

$$\begin{aligned} f_{-2\sum_{i=1}^{2}\ln\tilde{p}_i}(y) &= \frac{1}{4(b-a)^2} e^{-\frac{1}{2}y} \int_{y+2\ln a}^{-2\ln a} dx \\ &= \frac{1}{4(b-a)^2} e^{-\frac{y}{2}} (-y - 4\ln a). \end{aligned}$$

4. For $y \in (-4\ln a, \infty)$, we have $f_{-2\sum_{i=1}^{2}\ln\tilde{p}_i}(y) = 0$. □

References

Bickel, P. J. and K. A. Doksum: 2001, *Mathematical Statistics*, Vol. 1. Upper Saddle River, New Jersey: Prentice Hall, 2nd edition.

Bierens, H. J.: 2005, *Introduction to the Mathematical and Statistical Foundations of Econometrics*. Cambridge: Cambridge University Press.

Choi, I.: 2001, 'Unit Root Tests for Panel Data'. *Journal of International Money and Finance* **20**(2), 249–272.

Choi, I.: 2006, 'Combination Unit Root Tests for Cross-Sectionally Correlated Panels'. In: D. Corbae, S. N. Durlauf, and B. E. Hansen (eds.): *Econometric Theory and Practice: Frontiers of Analysis and Applied Research: Essays in Honor of Peter C. B. Phillips*. Cambridge University Press, Chapt. 11, pp. 311–333.

Dickey, D. and W. Fuller: 1979, 'Distribution of the Estimators for Autoregressive Time Series with a Unit Root'. *Journal of the American Statistical Association* **74**(366), 427–431.

Hanck, C.: 2005, 'Mixed Signals Among Panel Cointegration Tests, Including Some New Ones'. Technical report, RGS.

Hedges, L. and I. Olkin: 1985, *Statistical Methods for Meta-Analysis*. San Diego: Academic Press.

Hlouskova, J. and M. Wagner: 2006, 'The Performance of Panel Unit Root and Stationarity Tests: Results from a Large Scale Simulation Study'. *Econometric Reviews* **25**(1), 85–116.

MacKinnon, J. G.: 1991, 'Critical Values for Cointegration Tests'. In: R. F. Engle and C. W. Granger (eds.): *Long-run Economic Relationships: Readings in Cointegration*. Oxford: Oxford University Press, Chapt. 13, pp. 267–276.

Maddala, G. and S. Wu: 1999, 'A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test'. *Oxford Bulletin of Economics and Statistics* **61**(S1), 631–652.

Phillips, P. and D. Sul: 2003, 'Dynamic Panel Estimation and Homogeneity Testing under Cross Section Dependence'. *Econometrics Journal* **6**, 217–259.

Phillips, P. C.: 1987, 'Time Series Regression with a Unit Root'. *Econometrica* **55**(2), 277–301.

Shiryaev, A. N.: 1996, *Probability*. Berlin: Springer, 2nd edition.