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CURRENT INCOME:  
LIQUIDITY CONSTRAINTS OR MYOPIA?

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ABSTRACT

Almost all of the recent empirical tests of the rational expectations - permanent income hypothesis (RE-PIH) have rejected the hypothesis. The null hypothesis in this empirical literature typically consists of the joint hypothesis that 1) agents' expectations are formed rationally, 2) desired consumption is determined by permanent income, and 3) capital markets are "perfect" in the sense that agents can lend or borrow against expected future income at the same interest rate. This paper attempts to determine whether the excess sensitivity of consumption to current income can be attributed to a failure of the third component of the joint hypothesis -- the assumption of "perfect" capital markets -- as opposed to a failure of one or both of the first two assumptions.

The paper examines, as a specific alternative to the PIH, a simple "Keynesian" consumption function in which the behavioral MPC out of transitory income is different from zero. Interpreting the unemployment rate as a proxy for the proportion of the population subject to liquidity constraints, the paper uses a generalized version of the econometric model in my earlier paper (1981) to conduct a specification test of the "Keynesian" consumption function. The finding that the estimate of the MPC out of transitory income is dramatically affected, in both magnitude and statistical significance, by the inclusion of the proxy for liquidity constraints suggests that liquidity constraints are an important part of the explanation of the observed excess sensitivity of consumption to current income.

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Almost all of the recent empirical tests of the rational expectations - permanent income hypothesis (RE-PIH) have rejected the hypothesis.<sup>1</sup> Using a rich variety of data sets and estimation methods, the empirical work indicates that the failure of the RE-PIH is significant, both statistically and quantitatively. The null hypothesis in this empirical literature typically consists of the joint hypothesis that 1) agents' expectations are formed rationally, 2) desired consumption is determined by permanent income, and 3) capital markets are "perfect" in the sense that agents can lend or borrow against expected future income at the same interest rate. Most previous studies have concluded that consumption is more sensitive to current income than is consistent with the joint hypothesis stated above, without attempting to determine which component of the joint hypothesis is responsible for the rejection.

This paper attempts to determine whether the excess sensitivity of consumption to current income can be attributed to a failure of the third component of the joint hypothesis -- the assumption of "perfect" capital markets -- as opposed to a failure of one or both of the first two assumption. The first two assumptions simply reflect the basic postulates that economic agents form expectations rationally and make decisions based on intertemporal utility maximization. If these postulates fail in the context of consumption behavior, some doubt is cast on their validity in other macroeconomic applications. If, on the other hand, capital market imperfections prevent agents from following their desired, permanent-income consumption path, the observed excess sensitivity of consumption to current income may be fully consistent with the basic postulate of the rational, forward looking agent.

This paper examines, as a specific alternative to the permanent income hypothesis, a simple "Keynesian" consumption function in which the behavioral MPC out of transitory income is different from zero. Empirical evidence that

the MPC out of transitory income is significantly different from zero is interpreted as evidence of myopic behavior; that is, evidence of a violation of one or both of the basic postulates of rational expectations and intertemporal utility maximization. As a specification test of the "Keynesian" consumption function, the consumption equation is modified by the inclusion of an additional variable; if the coefficient of the additional variable is different from zero, then the simple "Keynesian" consumption function is incorrect or incomplete. In principle, the specification test could be executed for an arbitrary choice of the additional variable. However, because the purpose of the paper is to investigate whether the observed excess sensitivity of consumption to current income arises because capital market imperfections (liquidity constraints) prevent agents from realizing their desired consumption paths, the specification of the simple "Keynesian" consumption function is tested with respect to a variable which is selected as a proxy for the severity or prevalence of liquidity constraints. Interpreting the unemployment rate as a proxy for the proportion of the population subject to liquidity constraints, the paper uses a generalized version of the econometric model in my earlier paper (1981) to conduct the specification test.

#### The model

Permanent income,  $y_t^p$ , is defined as the current resource flow which, conditional on expectations in period  $t$ , can be sustained for the remainder of the individual's lifetime:

$$(1) \quad y_t^p = k(T) \left( w_t + \sum_{s=0}^T \delta^{s+1} E_t x_{t+s} \right)$$

where  $\delta = 1/(1+r)$ . In this discrete time formulation,  $w_t$  represents real (nonhuman) wealth at the beginning of period  $t$ ;  $x_t$  represents labor income, assumed to be paid at the end of the period; and  $r$  denotes the real discount rate.  $E_t x_{t+s}$  denotes the mathematical expectation of  $x_{t+s}$  conditional on

information available in period  $t$ . The length of the individual's remaining lifespan is denoted  $T$ ; and the annuity rate, which is a function of  $T$ , is denoted  $k(T)$ .

In an infinite horizon version of the model,  $T = \infty$  and  $k(T)$  has a constant value equal to the discount rate. In the finite horizon case, the annuity rate obeys the non-linear difference equation

$$(2) \quad k(T-1) = k(T)/(1+r-k(T))$$

and the terminal condition  $k(0) = 1$ .

Following Hall (1978) and Flavin (1981), the permanent income consumption hypothesis is represented as:

$$(3) \quad c_t = \beta_p y_t^P$$

where  $\beta_p$  is the marginal propensity to consume out of permanent income.<sup>2</sup>

Substituting (3) into the definition of permanent income, the evolution of permanent income over time is given by:

$$(4) \quad y_t^P = (1+k(T)(1-\beta_p))y_{t-1}^P + k(T)[w_t - E_{t-1}w_t + \sum_{s=0}^T \delta^{s+1}(E_t - E_{t-1})x_{t+s}] .$$

If expectations are rational, both the expectation of next period's revision in expected future labor income,  $(E_t - E_{t-1})x_{t+s}$ , and the expectation of unanticipated capital gains,  $w_t - E_{t-1}w_t$ , conditional on information available in  $t-1$ , are equal to zero. Thus if  $\beta_p = 1$  permanent income, as defined in equation (1), has the random walk property

$$(5) \quad E_{t-1}y_t^P = y_{t-1}^P$$

which was stressed by Hall (1978). If  $\beta_p < 1$ , then permanent income is a nonstationary first order autoregressive process.

### Tests of the PIH against alternative hypotheses

In a series of nested tests, the permanent income hypothesis (PIH) is tested against two alternative hypotheses. In the first alternative, the MPC out of transitory income,  $\beta_T$ , is nonzero. The hypothesis is stated in first differences rather than levels in anticipation of the specification used in the empirical work:

$$(6) \quad \Delta c_t = \beta_p \Delta y_t^P + \beta_T (\Delta y_t - \Delta y_t^P).$$

Equation (6) includes as special cases both the PIH (with  $\beta_T = 0$ ) and an extreme form of the Keynesian consumption function in which consumption simply responds to current income ( $\beta_T = \beta_p$ ). If  $0 < \beta_T < \beta_p$ , consumption is determined by a blend of the permanent income and Keynesian hypotheses. Because a nonzero value of  $\beta_T$  indicates that there is some element of truth to the Keynesian view of consumption, equation (6) will be referred to as the "Keynesian" consumption function.

In addition to testing the PIH against the simple "Keynesian" alternative hypothesis by testing the restriction  $\beta_T = 0$ , the empirical work reported in this paper tests the validity of the Keynesian consumption function as specified by equation (6). As a specification test, equation (6) is modified by including another variable, denoted  $z_t$ :

$$(7) \quad \Delta c_t = \beta_p \Delta y_t^P + \beta_T (\Delta y_t - \Delta y_t^P) + \gamma \Delta z_t.$$

If  $\gamma$  is nonzero, the consumption function specified in equation (6) is incorrect or incomplete. Of the many possible sources of misspecification, this paper focuses on the potential misspecification of equation (6) arising from the exclusion of liquidity constraints as a determinant of consumption. That is, the specification of equation (6) is tested with respect to a variable which is selected as a proxy for the severity or prevalence of liquidity constraints.

Of the potential explanations of the observed excess sensitivity of consumption to current income, perhaps the most obvious is that individuals are "myopic" in the sense that the behavioral MPC out of transitory income,  $\beta_T$ , is substantially different from zero. If the excess sensitivity of consumption to current income arises because of this type of myopic behavior on the part of consumers, equation (6) is a valid specification of the consumption function. Alternatively, even if individuals are rational and forward looking, as posited by the PIH, and the behavioral MPC out of transitory income is zero, the observed excess sensitivity of consumption to current income could arise if liquidity constraints prevent the individual from realizing his desired consumption plan. An individual who is unable to borrow against expected future income and whose stock of nonhuman wealth is insufficient to maintain the desired consumption plan for the duration that transitory income is expected to be negative is considered to be liquidity constrained. If consumption is constrained by current income, actual consumption and transitory income will be positively correlated even though desired consumption may be determined by the PIH.

Of the variables which might proxy for the prevalence of liquidity constraints, the specification test focuses on the rate of unemployment. A person becoming unemployed in a given year can expect to spend 3 to 5 months in one or more spells of unemployment that year.<sup>3</sup> The unemployment spell or spells will reduce the individual's permanent income, although not by as much as the reduction in current income. Thus individuals who are unemployed will typically have negative transitory income.

Whether or not a significant fraction of the population has net worth insufficient to smooth their consumption path in the face of the negative transitory income realizations associated with a spell or spells of unemploy-

ment is an empirical question. According to survey data on the financial characteristics of consumers compiled by the Federal Reserve System in 1963 (Projector (1964)), 25% of all families<sup>4</sup> and 40% of all families with income below the median had net worth less than \$3,000 (in current dollars). Further, these families held the bulk of their assets in illiquid form (equity in home, auto, business or profession, and life insurance, annuities, or retirement plans); the average holdings of liquid assets, stocks, and bonds was about \$300 for this group. Thus the survey data indicates that a significant fraction of the population had net worth which was insufficient to completely insulate their consumption path from the negative transitory income realizations associated with a moderate spell of unemployment.

Even individuals with little or no nonhuman wealth could achieve their permanent income consumption path if they are able 1) to borrow against future expected labor income or 2) to insure against individual-specific risk in their labor income. While markets for collateralized consumer borrowing are well-developed, the personal bankruptcy laws would tend to prevent or distort the development of a market for non-collateralized consumer borrowing. Further, the problems of adverse selection and moral hazard may explain the absence of a private market for insurance against individual-specific risk in labor income.<sup>5</sup> Thus there seem to be important microeconomic factors which indicate that the Arrow-Debreu contingent claims markets which would eliminate liquidity constraints in an ideal world might not actually exist.

If liquidity constraints are responsible, at least in part, for the excess sensitivity of consumption to current income, an increase in the prevalence or severity of liquidity constraints should reduce consumption, ceteris paribus. Because the coefficient  $\gamma$  in equation (7) represents the effect on consumption of an increase in unemployment, the estimated value of  $\gamma$  is expected to be negative if liquidity constraints affect consumption. It should be emphasized that



the coefficient  $\gamma$  is estimated for the purpose of constructing a test statistic to check the specification of the consumption function given by equation (6);  $\gamma$  should not be interpreted as a structural parameter.

Combining equations (3) and (4), one obtains:

$$(8) \quad c_t - (1+k(T)(1-\beta_p))c_{t-1} = \beta_p k(T) \left\{ w_t - E_{t-1}w_t + \sum_{s=0}^T \delta^{s+1} (E_t - E_{t-1})x_{t+s} \right\}.$$

In order to simplify the estimation, the marginal propensity to consume out of permanent income,  $\beta_p$ , is assumed to be exactly unity. With  $\beta_p = 1$ , the left hand side of equation (8) is simply the first difference of consumption,  $\Delta c_t$ .

Because the implications of equation (8) will be tested using aggregate data, it is necessary to consider the effect on aggregate consumption of a systematic trend in per capita income. Even though an individual's permanent income is defined (assuming  $\beta_p = 1$ ) so that its movement over time is trendless, the presence of a trend in per capita income will induce a trend in aggregate permanent income and therefore in aggregate consumption. If per capita income has an exponential trend, due to advances in technical knowledge, for example, later generations will have greater lifetime wealth than earlier ones. As the older generations die and are replaced by the younger generations, aggregate consumption will trend upward at the same rate of growth as per capita income. Since the model is intended to explain revisions in planned consumption which are caused by changes in expectations about future income, equation (8) applies to the movement of consumption around a trend attributable to the trend in per capita income.

If an accurate measure of nonhuman wealth were available for the sample period under consideration (1929-1981), one could eliminate  $E_{t-1}w_t$  from equation (8) by using the relation:

$$(9) \quad E_{t-1}w_t = (1+r)w_{t-1} + x_{t-1} - c_{t-1}.$$

This is the basic approach taken by Hayashi (1982). However, in the absence of a reliable and consistent series on nonhuman wealth for the 1929-81 sample period, I take a different approach to measuring unanticipated capital gains ( $w_t - E_{t-1}w_t$ ) in this paper.

Unanticipated capital gains on nonhuman wealth can be defined as the present value of the revision in the expected earnings associated with the current asset holdings:<sup>6</sup>

$$(10) \quad w_t - E_{t-1}w_t = \sum_{s=0}^{\infty} \delta^{s+1} (E_t - E_{t-1}) r_{t+s} w_t.$$

The actual path of nonlabor income,  $r_{t+s} w_{t+s}$ , will reflect both the movements in the return to capital,  $r_{t+s}$ , which we want to capture, and the endogenous changes in earnings flows which result from the individual's decisions to accumulate or decumulate nonhuman wealth,  $w_{t+s}$ . I assume that the time series properties of aggregate nonlabor income are dominated by fluctuations in the rate of return to capital,  $r_{t+s}$  rather than the endogenous changes in  $w_{t+s}$ .<sup>7</sup> With this assumption, unanticipated capital gains are approximated as the present value of the revision in expected nonlabor income. Replacing  $w_t - E_{t-1}w_t$  with the present value of the revision in expected nonlabor income, equation (8) becomes:

$$(11) \quad \Delta c_t = k(T) \sum_{s=0}^T \delta^{s+1} (E_t - E_{t-1}) y_{t+s}$$

where  $y_{t+s}$  denotes total (labor plus nonlabor) disposable income in period  $t+s$ , stated in deviations from an exponential trend.

The information content of various observable variables for predicting future income is modeled using an autoregression which includes disposable income as one variable:

$$(12) \quad \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} AA(L) & AB(L) \\ BA(L) & BB(L) \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}.$$

In equation (12), AA(L), AB(L), BA(L), and BB(L) are polynomials in the lag operator, L; for example, AA(L) = a<sub>1</sub>(L) + a<sub>2</sub>(L) + ... + a<sub>n</sub>L<sup>n</sup>. Constant terms are included in the autoregression but are suppressed to simplify the notation. The disturbances, ε<sub>1t</sub> and ε<sub>2t</sub>, are independently distributed across time, but may be contemporaneously correlated.

Inverting (12) to obtain the moving average representation of {y<sub>t</sub>, z<sub>t</sub>}<sup>'</sup> and evaluating the moving average polynomial at L = δ = (1+r)<sup>-1</sup>, one obtains the 2 x 2 matrix which gives the partial derivative, with respect to each innovation, of the present discounted value of the revision in the expected path of each forecasted variable:

$$(13) \quad \begin{bmatrix} \frac{\partial(\text{PDV of } y_{t+s})}{\partial \epsilon_{1t}} & \frac{\partial(\text{PDV of } y_{t+s})}{\partial \epsilon_{2t}} \\ \frac{\partial(\text{PDV of } z_{t+s})}{\partial \epsilon_{1t}} & \frac{\partial(\text{PDV of } z_{t+s})}{\partial \epsilon_{2t}} \end{bmatrix} = \frac{1}{\text{Det}} \begin{bmatrix} 1-\text{BB}(\delta) & \text{AB}(\delta) \\ \text{BA}(\delta) & 1-\text{AA}(\delta) \end{bmatrix}$$

where "Det" is defined as the determinant of the matrix on the right hand side of equation (13); Det = (1-BB(δ))(1-AA(δ))-AB(δ)BA(δ). Since we are not interested in the path of the variable z<sub>t</sub> except to the extent that it provides information about the path of disposable income, only the top row of the matrix is pertinent. Using the notation Φ<sub>y</sub> for  $\frac{\partial(\text{PDV of } y_{t+s})}{\partial \epsilon_{1t}}$  and Φ<sub>z</sub> for  $\frac{\partial(\text{PDV of } y_{t+s})}{\partial \epsilon_{2t}}$ ,

$$(14) \quad \Phi_y = \frac{1-\text{BB}(\delta)}{\text{Det}} \quad \text{and} \quad \Phi_z = \frac{\text{AB}(\delta)}{\text{Det}} .$$

As the innovations in the bivariate representation of {y<sub>t</sub>, z<sub>t</sub>}<sup>'</sup>, ε<sub>1t</sub> and ε<sub>2t</sub> are unforecastable on the basis of observations on y<sub>t</sub> and z<sub>t</sub> dated t-1 and earlier. If, however, agents use other variables in addition to y<sub>t</sub> and z<sub>t</sub> to forecast future income, ε<sub>1t</sub> and ε<sub>2t</sub> will be partially forecastable on the basis of lagged values of these other informational variables. Thus the

innovations in the bivariate autoregression,  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$ , can each be decomposed into a component which is predictable on the basis of lagged information available to agents,  $\eta_{1t-1}$ , and a component which is orthogonal to the agent's entire information set,  $\tilde{\varepsilon}_{1t}$ :

$$(15) \quad \begin{aligned} \varepsilon_{1t} &= \tilde{\varepsilon}_{1t} + \eta_{1t-1} \\ \varepsilon_{2t} &= \varepsilon_{2t} + \eta_{2t-1} . \end{aligned}$$

The  $\eta_{1t-1}$  terms are assumed to be serially uncorrelated and uncorrelated with lagged values of  $y_t$  and  $z_t$ . By definition, the innovations perceived by agents,  $\tilde{\varepsilon}_{1t}$  and  $\tilde{\varepsilon}_{2t}$ , are uncorrelated with  $\eta_{1t-1}$ ,  $\eta_{2t-1}$ , and lagged values of  $y_t$  and  $z_t$ .

In revising their estimates of permanent income, agents respond to all available information, not just the new information contained in  $\tilde{\varepsilon}_{1t}$  and  $\tilde{\varepsilon}_{2t}$ :

$$(16) \quad \Delta y_t^p = k(T)[\Phi_y \tilde{\varepsilon}_{1t} + \Phi_z \tilde{\varepsilon}_{2t}] + \theta_t .$$

where the error term  $\theta_t$  is included to reflect the fact that agents are responding to many sources of new information in addition to the current observations on  $y_t$  and  $z_t$ . As a component of the revision in permanent income,  $\theta_t$  is uncorrelated with all lagged variables in the agent's information set, including its own lagged values. Substituting equations (15) and (16) into the permanent income consumption function yields:

$$(17) \quad \Delta c_t = k(T)[\Phi_y \varepsilon_{1t} + \Phi_z \varepsilon_{2t}] - \{k(T)[\Phi_y \eta_{1t-1} + \Phi_z \eta_{2t-1}] + \theta_t\} .$$

To simplify the notation, denote the consumption error term as  $\varepsilon_{3t}$ :

$$(18) \quad \varepsilon_{3t} = k(T)[\Phi_y \eta_{1t-1} + \Phi_z \eta_{2t-1}] + \theta_t$$

In summary, the model consists of the 3-equation system:

$$(19.1) \quad y_t = \mu_1 + AA(L)y_t + AB(L)z_t + \varepsilon_{1t}$$

$$(19.2) \quad z_t = \mu_2 + BA(L)y_t + BB(L)z_t + \varepsilon_{2t}$$

$$(19.3) \quad \Delta c_t = \mu_3 + k(T)[\Phi_y \varepsilon_{1t} + \Phi_z \varepsilon_{2t}] + \varepsilon_{3t}$$

In the empirical application of the model, the polynomials  $AA(L)$ ,  $AB(L)$ ,  $BA(L)$ , and  $BB(L)$  were each specified as third order. Note, from equations (15) and (18), that because agents form their forecasts of future income using an information set which contains variables in addition to  $y_t$  and  $z_t$ ,  $\varepsilon_{3t}$  is negatively correlated with  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$ . Under the assumptions specified above, the structural disturbances,  $\varepsilon_{1t}$ ,  $\varepsilon_{2t}$ , and  $\varepsilon_{3t}$ , are each serially uncorrelated.

Equation system (19) represents the restricted version of the model, in the sense that current  $y_t$  and  $z_t$  affect consumption only to the extent that they induce agents to revise their estimate of permanent income. In the unrestricted version of the model, the consumption equation (19.3) is replaced with a more general specification in which consumption responds directly to changes in  $y_t$  and  $z_t$  in addition to the response consistent with the permanent income hypothesis:

$$(20) \quad \Delta c_t = \mu_3 + k(T)[\Phi_y \varepsilon_{1t} + \Phi_z \varepsilon_{2t}] + \beta_0 \Delta y_t + \beta_1 \Delta y_{t-1} \\ + \beta_2 \Delta y_{t-2} + \gamma_0 \Delta z_t + \gamma_1 \Delta z_{t-1} + \gamma_2 \Delta z_{t-2} + \varepsilon_{3t} .$$

Equation (20) is a generalization of equation (8) in the sense that lagged as well as current changes in  $y_t$  and  $z_t$  are included. (Note that the subscript  $T$  has been dropped from the notation for the marginal propensity to consume out of transitory income,  $\beta_0$ .) Since the role of current income in signaling changes in permanent income has been explicitly included in the model, the  $\beta_i$

will differ from zero only if consumption is correlated with transitory income. Similarly, non-zero values of the  $\gamma_1$  indicate that the variable  $z_t$  has some effect on consumption beyond its role in providing new information about future income.

### Estimation of the model

The three equation system consisting of the bivariate autoregression and the consumption equation was estimated by Full Information Maximum Likelihood (FIML). Estimates of the model were obtained for four different specifications of the consumption equation — a fully unconstrained version with no restrictions on the  $\beta_1$  and  $\gamma_1$ , a fully constrained version with the restriction  $\beta_1 = \gamma_1 = 0$  imposed, and two partially constrained versions:  $\beta_1$  unconstrained with the restriction  $\gamma_1 = 0$  imposed, and vice versa.

In interpreting the empirical results, it is important to understand the relationship between the empirical approach pursued here, a single equation instrumental variables approach, and the approach used by Hall (1978).

Consider the reduced form of the system. The reduced form of equation (20) is:

$$(21) \quad \Delta c_t = \mu_3 + \beta_0(\mu_1 + (AA(L)-L)y_t + AB(L)z_t) \\ + \gamma_0(\mu_2 + BA(L)y_t + (BB(L)-L)z_t) + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} \\ + \gamma_1 \Delta z_{t-1} + \gamma_2 \Delta z_{t-2} + v_{3t}$$

where  $v_{3t} = (k(T)\Phi_y + \beta_0)\epsilon_{1t} + (k(T)\Phi_z + \gamma_0)\epsilon_{2t} + \epsilon_{3t}$ . Equations (19.1) and (19.2) are already reduced forms. Since  $\beta_1 = \gamma_1 = 0$  under the null hypothesis, the fully restricted reduced form consumption equation is simply:

$$(22) \quad \Delta c_t = \mu_3 + v_{3t} .$$

Instrumental variables estimates of the excess sensitivity parameters, the  $\beta_i$  and  $\gamma_i$ , could be computed by specifying the consumption equation as:

$$(23) \quad \Delta c_t = \mu_3 + \beta_0 \Delta y_t + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \gamma_0 \Delta z_t + \gamma_1 \Delta z_{t-1} + \gamma_2 \Delta z_{t-2} + \omega_t$$

where  $\omega_t = (1-\beta_0)\Delta y_t^p$ , and estimating the equation using lagged values of  $y_t$  and  $z_t$  as instruments. Since the revision in permanent income, which constitutes the disturbance in equation (23), is uncorrelated with previously available information, the lagged values of  $y_t$  and  $z_t$  are valid instruments.

Referring to the reduced form of the fully unconstrained system (equations (19.1), (19.2), and (21)) it is easy to show that, in the absence of any further restrictions, the structural parameters are just identified. When the system is just identified, estimating the multivariate system by FIML provides estimates of the excess sensitivity parameters, the  $\beta_i$  and  $\gamma_i$ , which are numerically identical to the estimates obtained by estimating equation (23) by instrumental variables, using  $y_{t-i}$  and  $z_{t-i}$ ,  $i=1,2,3$ , as instruments. That is, in the just identified case, FIML is equivalent to Two Stage Least Squares (2SLS) equation-by-equation, which in turn is equivalent to instrumental variables, using the predetermined variables, in this case  $y_{t-i}$  and  $z_{t-i}$ ,  $i=1,2,3$ , as instruments.

Next, consider the restricted reduced form consumption equation (22). By definition the reduced form expresses the conditional expectation of the endogenous variable,  $\Delta c_t$ , given the predetermined variables. Thus the empirical content of the restriction  $\beta_i = \gamma_i = 0$  is that the conditional expectation of  $\Delta c_t$  given lagged values of  $y_t$  and  $z_t$  is zero, which is an example of the basic empirical implication of the permanent income hypothesis tested by Hall (1978). To test this implication of the permanent income hypothesis, Hall estimated by OLS the conditional expectation of  $c_t$  given  $c_{t-1}$  and an array of variables dated  $t-1$  or earlier, then tested the hypothesis that the

lagged variables (other than  $c_{t-1}$ ) had no predictive content for current consumption. As an application of Hall's test, one could estimate by OLS equation (24):

$$(24) \quad \Delta c_t = \mu_3 + \pi_1 y_{t-1} + \pi_2 y_{t-2} + \pi_3 y_{t-3} + \pi_4 z_{t-1} + \pi_5 z_{t-2} + \pi_6 z_{t-3} + v_{3t}$$

and conduct an F-test of the hypothesis  $\pi_i = 0$ . Since the multivariate system consisting of equations (19.1), (19.2), and (21) is just identified, there are no over-identifying restrictions on the consumption reduced form, equation (21); thus equations (21) and (24) both represent the identical reduced form consumption equation. Further, imposing the restriction  $\beta_i = \gamma_i = 0$  implies that all of the reduced form coefficients (except the constant) should be zero, which is exactly the restriction tested by Hall.<sup>8</sup> Although both Hall's approach and the approach pursued here lead to the same specification of the reduced form consumption equation, under both the alternative and null hypotheses, the test procedure used here differs from Hall's in that the restriction is tested by computing the likelihood ratio statistic for the multivariate system (equations (19.1), (19.2), and (21)) while Hall's test statistic is computed by estimating the consumption equation alone. As proved in Flavin (1981), however, the two tests yield numerically identical values of the likelihood ratio test statistic.<sup>9</sup>

The purpose of this discussion is to establish that both the estimates of the excess sensitivity parameters, the  $\beta_i$  and  $\gamma_i$ , and the likelihood ratio test statistic for the hypothesis  $\beta_i = \gamma_i = 0$  are robust with respect to the approximation introduced to measure capital gains on nonhuman wealth when the multivariate model (equations (19.1), (19.2), and (21)) is just identified. Without even attempting to measure unanticipated capital gains, one could obtain consistent estimates of the  $\beta_i$  and  $\gamma_i$  by estimating equation (23) by instrumental variables and test the restrictions imposed by the permanent



income hypothesis by using Hall's reduced form approach. Since the point estimates of the excess sensitivity parameters and the value of the associated likelihood ratio test statistic obtained by estimating the multivariate model by FIML are numerically identical to the estimates and test statistic obtained by the procedures which do not involve the approximation, these empirical results do not depend on the approximation.<sup>10</sup>

### Data

In contrast to my earlier paper (1981), the models examined in this paper were estimated with annual rather than quarterly data. One advantage to using annual data is that the annual series are available for a longer sample period, 1929 to present, than quarterly data. In addition, it is important to know whether the results of the earlier paper are confirmed when the sampling interval is changed from quarterly to annual. Observations for the years 1942- 1949 were excluded from the sample on the grounds that rationing during World War II probably caused departures of actual from desired consumption. After allowing for the construction of lags, the basic sample period was 1933-41 and 1950-81.

Assuming that the utility function is separable in the major categories of consumption -- durables, nondurables, and services -- the model was estimated using real consumption of nondurable goods alone as the consumption concept. In principle, a series on the service flows from durable goods could be constructed. However, the model incorporates the assumption that consumption adjusts fully to a revision in permanent income within the current time period. Because of the costs associated with adjusting stocks of durable goods, a more complicated adjustment model would be required to explain consumption of services of durable goods. Consumption of services, which is dominated by the imputed service flow of housing, was excluded for the same reason. Consumption of

nondurables accounts for about 45% of total personal consumption expenditure. Both the consumption and income series, which was personal disposable income, were stated in 1972 dollars per capita.

Empirical results for the model with univariate income forecasting equation

The model considered in my 1981 paper is a special case of the model developed in the previous section. For comparison with the results of the earlier paper, the basic tests of the simpler model with the univariate income forecasting equation were repeated with the longer, annual data set.

Based on series of F-tests of the exclusion of higher order autoregressive parameters, the per capita disposable income series was modeled as a third order autoregression with a deterministic exponential trend. The estimated exponential growth in disposable income was 2.15482 percent annually, with a standard error of .114 percent. The precision of the estimate of the trend, even when estimated simultaneously with the autoregressive parameters, supports the view that income can be modeled as a stationary process around a positive trend. To simplify the estimation, the trend of 2.15482 percent was removed from the income series, and the tests were conducted using the detrended data. As mentioned above, the presence of an exponential trend in per capita income will induce an exponential trend in per capita consumption. Because the consumption variable is expenditures on nondurable goods rather than total consumption, the trend in nondurable goods consumption will not be equal to the trend in income unless the income elasticity of nondurables consumption is unity. For this reason, the consumption series was detrended by its own estimated exponential trend of 1.59896 percent.

With the univariate income forecasting equation, the model consists of equations (19.1) and (21) with the parameters of  $AB(L)$ ,  $BA(L)$ , and  $BB(L)$ , and  $\mu_2$ ,  $\gamma_0$ ,  $\gamma_1$ , and  $\gamma_2$  constrained to equal zero. The estimates of the struc-

tural parameters and their standard errors are reported in Table 1.

Table 1

$\mu_1$	$a_1$	$a_2$	$a_3$	$\mu_3$	$\beta_0$	$\beta_1$	$\beta_2$
436. (124.)	1.071 (.130)	-.362 (.184)	.004 (.121)	-2.11 (2.27)	.370 (.085)	.006 (.042)	-.011 (.036)

where  $a_i$  denotes the coefficient of  $L^i$  in the lag polynomial  $AA(L)$ .

Based on the estimated autoregressive parameters of the income process and an assumed real interest rate of 5%, the present discounted value of the revision in expected future income associated with an innovation in current income,  $\Phi_y$ , is 3.28. Assuming that the horizon of the representative individual is 20 years,  $T=20$ , the annuity rate,  $k(T)$ , is 0.079. Thus, the change in permanent income induced by an innovation in current income,  $k(T)\Phi_y$ , is .26. With a likelihood ratio statistic of 23.786 for the restriction  $\beta_0 = \beta_1 = \beta_2 = 0$ , the null hypothesis of no excess sensitivity of consumption to current income can be rejected at the .5% level.

In summary, the empirical results based on the model with a univariate income forecasting equation indicate that the excessive sensitivity of consumption to current income is significantly greater than zero, both statistically and quantitatively. Keeping in mind that the parameter  $\beta_0$  measures the excess sensitivity of nondurables consumption alone, a point estimate of .37 for  $\beta_0$  represents a very large departure from the permanent income hypothesis. Further, the estimate of  $\beta_0$  is reasonably precise, according to the standard error of .09. These results are in very close agreement with the 1981 paper based on post war quarterly data. With the quarterly data set, the point estimate of  $\beta_0$  was .355 (standard error .275) and the restriction  $\beta_1 = 0$  could be rejected at the .5% level.

Empirical results using the unemployment rate to help forecast disposable income

In this section the general model with a bivariate income forecasting equation is estimated, using the overall unemployment rate as the variable  $z_t$ . Under the permanent income hypothesis, the current observation on the unemployment rate affects consumption only to the extent that the unemployment innovation contains new information on future income. In the unrestricted version of the model, equation (20), current and lagged changes in the unemployment rate also have a direct impact on consumption. Results from the FIML estimation of the unconstrained system consisting of equation (19.1), (19.2), and (21) are reported in lines 5, 6, and 1 of Table 2.

When the  $\beta_i$  are included without constraint and the  $\gamma_i$  are constrained to be equal to zero, the specification of the alternative hypothesis is comparable to the alternative hypothesis in the model with the univariate income forecasting equation. In this version of the model, the unemployment rate is included as part of the information set used to predict future income but assumed to have no direct impact on consumption. The resulting estimates of the excess sensitivity of consumption to current income, the  $\beta_i$ , are reported in line 2 of Table 2. The estimate of  $\beta_0$  is .368, with a standard error of .070; the estimates of  $\beta_1$  and  $\beta_2$  are numerically small and insignificantly different from zero. Further, the estimates of the  $\beta_i$  are in very close agreement with the estimates obtained from the model with the univariate income forecasting equation.

When the unemployment rate is included in the model as possibly having a direct impact on consumption, the estimate of  $\beta_0$  drops from .368 to .146 and becomes statistically insignificant. Estimates of the  $\beta_i$  and  $\gamma_i$  obtained from the fully unrestricted version of the model are reported in line 1 of Table 2. Consistent with the interpretation of the unemployment rate as a proxy for

TABLE 2  
Consumption Equation

line	constraints imposed	parameter estimates (standard errors in parentheses)						value of log likelihood <sup>1</sup>	SER	DW
		$\mu_3$	$\beta_0$	$\beta_1$	$\beta_2$	$\gamma_0$	$\gamma_1$			
1.	none	-2.89 (2.49)	.146 (.196)	-.177 (.129)	.050 (.075)	-11.0 (8.07)	-4.64 (3.33)	1.02 (2.65)	\$12.65	2.04
2.	$\gamma_1=0$	-2.08 (2.20)	.368 (.070)	.007 (.039)	-.012 (.034)				\$12.73	1.96
3.	$\beta_1=0$	-1.50 (2.19)				-11.6 (1.90)	.024 (1.15)	.074 (1.05)	\$12.77	2.13
4.	$\beta_1=\gamma_1=0$	1.92							\$18.94	1.21

Bivariate Autoregression of  $y_t$  and  $z_t$

constraints imposed	parameter estimates <sup>2</sup> (standard errors in parentheses)						SER	DW		
	$\mu_1$	$aa_1$	$aa_2$	$aa_3$	$ab_1$	$ab_2$			$ab_3$	
5.	none	387. (232.)	1.54 (.204)	-.842 (.260)	.048 (.220)	19.2 (6.73)	-22.6 (8.10)	4.04 (6.27)	\$40.97	2.01
6.	none	$\mu_2$	$ba_1$	$ba_2$	$ba_3$	$bb_1$	$bb_2$	$bb_3$	1.23	1.99
		.965 (6.97)	-.026 (.006)	.029 (.008)	-.003 (.007)	.309 (.202)	.559 (.243)	.020 (.188)		

Notes

1. In each case, the three equation system consisting of the consumption equation and the bivariate autoregression was estimated simultaneously. This column reports the log likelihood of the three equation system. Estimates of the autoregressive parameters associated with the constrained consumption equations are omitted in order to save space.

2.  $aa_1$  denotes the coefficient of  $L^1$  in AA(L), etc.

the prevalence of liquidity constraints, the excess sensitivity of consumption to the unemployment rate is negative:  $\gamma_0 = -11.0$ . The point estimate of  $\gamma_0$  of  $-11.0$  indicates that a one percentage point increase in the unemployment rate decreases consumption by \$11.00 per capita (in 1972 dollars). The economic significance of the estimate of  $\gamma_0$  is more easily interpreted if the decrement to consumption is stated in terms of the dollar change in consumption per additional person unemployed as the unemployment rate increases by one percentage point, rather than the change in consumption per capita. Renormalizing the estimate of  $\gamma_0$  by multiplying by the ratio of total population to 1% of the labor force (232,060/1,123.83 in 1982) the excess sensitivity of consumption to unemployment amounts to a decrement of \$2,271.50 per additional person unemployed (1972 dollars). In assessing the magnitude of this estimate of the excess sensitivity of consumption to the unemployment rate, it is important to keep in mind that the estimate of  $\gamma_0$  of  $-11.0$  was obtained from the fully unconstrained version of the model. Since the parameters measuring the excess sensitivity of consumption to current income, the  $\beta_i$ , were included in the specification of the alternative hypothesis, the estimated decrement of \$2,271.50 per additional person unemployed measures the effect of unemployment on consumption, holding current income constant. With a likelihood ratio statistic of 33.04, the full set of restrictions imposed by the permanent income hypothesis ( $\beta_i = \gamma_i = 0$ ) can be rejected at the .5% level. As explained above, the likelihood ratio test statistic for the hypothesis  $\beta_i = \gamma_i = 0$  and the estimates of the  $\beta_i$  and  $\gamma_i$  in the fully unrestricted version of the model are robust with respect to the approximation introduced to measure capital gains on nonhuman wealth.

In addition to testing the permanent income hypothesis, a primary purpose of the paper was to conduct a specification test of the simple "Keynesian"

consumption function in which the behavioral marginal propensity to consume out of current income is greater than zero. Interpreting the alternative hypothesis with the  $\beta_i$  unconstrained and the restriction  $\gamma_i = 0$  imposed as the "Keynesian" consumption function, the specification test is executed by testing the restriction that the  $\gamma_i$  are zero. With a likelihood ratio statistic of 6.282, the hypothesis that the  $\gamma_i$  are each equal to zero is marginally rejected at the 10% level of significance.

At this juncture, the statistical results concerning the importance of liquidity constraints appear to be mixed: in both magnitude and statistical significance, the estimates of the  $\beta_i$  are dramatically affected by the inclusion of the  $\gamma_i$ , the estimates of the  $\gamma_i$  are of the predicted sign and indicate a quantitatively large direct effect of unemployment on aggregate consumption, but the formal specification test of the restriction  $\gamma_i = 0$  cannot be rejected at conventional levels of significance, such as the 5% level, although it can be rejected at the 10% level. Given the high correlation between aggregate income and the unemployment rate, however, one might suspect that the formal specification test has relatively low power against the restriction  $\gamma_i = 0$  when the  $\beta_i$  are included in the consumption equation without restriction.

To check this conjecture, the symmetric hypothesis test was executed: If the  $\gamma_i$  are included in the consumption equation without constraint, can the restriction  $\beta_i = 0$  be rejected? When the  $\gamma_i$  are included in the consumption equation, the likelihood ratio statistic for the hypothesis  $\beta_i = 0$  is 5.066, which has a marginal significance level of .167. Estimates of the  $\gamma_i$  obtained with the restriction  $\beta_i = 0$  imposed are reported in line 3 of Table 2. In summary, when either the  $\beta_i$  or the  $\gamma_i$  are included in the consumption equation without constraint, the restriction that the other set of excess sensitivity parameters is zero cannot be rejected at the 5% level of significance. Based

on this finding, I would interpret the failure of the specification test to reject the restriction  $\gamma_1 = 0$  at a high level of significance as an indication that the test has relatively low power. Since the symmetric hypothesis  $\beta_1 = 0$  cannot be rejected even at the 10% level, the empirical results cannot be interpreted as empirical confirmation of the simple "Keynesian" specification.

In addition to the estimates of the excess sensitivity parameters, the model provides estimates of the summary measures of the information content of innovations in income and the unemployment rate for the prediction of future income. Based on the estimated parameters of the bivariate autoregression and a real interest rate of 5%, the value of  $\Phi_y$  was 3.72; the value of  $\Phi_z$  was 27.9. It is important to keep in mind that in the context of the bivariate income forecasting model, the parameters  $\Phi_y$  and  $\Phi_z$  have the interpretation of partial rather than total derivatives. In order to directly compare the information content of an innovation in  $y_t$  in the bivariate autoregression to the information content of an innovation in the univariate income autoregression, one can calculate the total derivative  $\Phi_y^T = \frac{d(\text{PDV of } y_{t+s})}{d\epsilon_{1t}}$  :

$$(25) \quad \Phi_y^T = \Phi_y + \Phi_z \frac{\text{cov}(\epsilon_1, \epsilon_2)}{\text{var}(\epsilon_1)} .$$

Using the estimated covariance matrix of the disturbances,<sup>11</sup> the value of  $\Phi_y^T$  is 3.12, which is in close agreement with the value of  $\Phi_y$  of 3.28 for the univariate income forecasting model.

Similarly, the total derivative of the present discounted value of future income with respect to an innovation in the unemployment rate is:

$$(26) \quad \Phi_z^T = \Phi_z + \Phi_y \frac{\text{cov}(\epsilon_1, \epsilon_2)}{\text{var}(\epsilon_2)} = -60.60 .$$

Renormalizing the estimate of  $\Phi_z^T$  by multiplying by the ratio of total population to 1% of the labor force, as before, the value of  $\Phi_z^T$  implies a reduction in the present discounted value of future income of \$12,514 (or a reduction in permanent income,  $k(T)\Phi_z^T$ , of \$988.61) per additional person unemployed.



### Conclusions

When the unemployment rate is included as part of the information set used to predict future income but assumed to have no direct impact on consumption, the estimated excess sensitivity of consumption to current income is quantitatively large and statistically significant. When the unemployment rate, which is interpreted as a proxy for the severity and prevalence of liquidity constraints, is included in the model as possibly having a direct impact on consumption, the estimate of the excess sensitivity of consumption to current income drops from .368 to .146 and becomes statistically insignificant. Consistent with the interpretation of the unemployment rate as a proxy for liquidity constraints, the estimated excess sensitivity of consumption to the unemployment rate is negative. Further, the estimate of the excess sensitivity of consumption to the unemployment rate implies that consumption declines by \$2,271.50 per additional person unemployed, holding current income constant.

If the observed excess sensitivity of consumption to current income arises because agents are myopic in the sense that the behavioral MPC out of transitory income is nonzero, the proxy for liquidity constraints is an extraneous variable in the consumption equation. The finding that the estimate of the MPC out of transitory income is dramatically affected, in both magnitude and statistical significance, by the inclusion of the proxy for liquidity constraints indicates that the simple "Keynesian" consumption function with nonzero MPC out of transitory income is an incomplete model, and suggests that liquidity constraints are an important part of the explanation of the observed excess sensitivity of consumption to current income.

## Footnotes

1. Studies rejecting the RE-PIH include Blinder (1981), Flavin (1981), Hall and Mishkin (1982), Hayashi (1982) and Sargent (1978). In his original paper, Hall (1978) did not reject the RE-PIH decisively. However, Flavin (1981) showed that with a minor alteration in the specification of the alternative hypothesis, the RE-PIH could be rejected using Hall's test and data set. Bernanke (1981), studying panel data on automobile expenditures, did not find evidence against the hypothesis. For an extensive survey of the empirical literature on the RE-PIH, see King (1983).
2. Because the model is estimated using aggregate data, the assumption implicit in equation (3) that transitory consumption is identically zero can be restated as an assumption that realizations of transitory consumption at the micro level are distributed independently across the population. Thus even if the variance of the individual's transitory consumption is nonnegligible relative to the variance of permanent income, aggregate transitory consumption will be negligible as long as the individual realizations are mutually independent.
3. Akerlof and Main (1980) argue compellingly that statistics on the average duration of completed unemployment spells substantially underestimate the effective duration of unemployment because about half of all spells of unemployment in a given year are incurred by persons suffering multiple spells. Thus the average number of weeks spent in unemployment by a person who becomes unemployed in a given year is substantially larger than the average duration of an unemployment spell: persons experiencing one spell of unemployment during the year spent an average of 11.5 weeks unemployed, persons with 2 spells spent an average of 15.5 weeks unemployed, and persons with more than 2 spells were unemployed an average of 18.3 weeks (Akerlof and Main (1980) p. 888).

4. "Family" here refers to a family as defined by the Census Bureau, plus the Census Bureau concept of "unrelated individuals".
5. The public unemployment compensation program provides partial insurance against individual-specific risk in labor income.
6. In an exact specification, the rate at which future income is discounted,  $\delta$ , would vary with the ex ante real interest rate. To keep the model tractable, the discount factor is approximated as a constant.
7. In recent years aggregate annual saving flows (defined as personal saving plus undistributed corporate profits) have been in the range \$100-\$150 billion, or about 1% to 3% of aggregate nonhuman wealth (assuming, based on the data in Hayashi (1982, p. 914) that the value of aggregate nonhuman wealth in the U.S. is in the \$5,000-\$10,000 billion range.) Thus even if the real interest rate is as high as 10%, the change in nonlabor income due to endogenous savings flows is on the order of .2% per year.
8. While Hall's approach and the approach followed here lead to the same restrictions on the reduced form of the consumption equation, the methodology used in this paper has the advantage that it permits the recovery of the point estimates of the parameters measuring the excess sensitivity of consumption to current income and other variables, the  $\beta_i$  and  $\gamma_i$ , and their individual standard errors. With Hall's approach, the quantitative importance of a departure from the predicted behavior of consumption is difficult to assess since only reduced form coefficients are estimated.
9. In Flavin (1981) the result is proven in the context of the simpler model considered in that paper. An appendix proving the more general version of the result appropriate for the model in this paper is available from the author.
10. Since just-identification of the multivariate model was required to establish that the estimates and the test statistics were robust with respect to the approximation concerning capital gains on nonhuman wealth, the point

estimates and test statistics associated with the partially constrained versions, which are over-identified, do depend on the validity of the approximation.

11. The reduced form covariance matrix was estimated as:

$$\hat{\Omega} = \begin{bmatrix} 1,678.31 & & & \\ -35.85 & 1.507 & & \\ 318.05 & -8.612 & 160.11 & \end{bmatrix}$$

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Appendix to "Excess Sensitivity of Consumption to Current Income: Liquidity Constraints or Myopia?" by Marjorie Flavin

Proof that the likelihood ratio test statistic for the hypothesis  $\beta_1 = \gamma_1 = 0$  in the multivariate system consisting of equations (19.1), (19.2), and (21) is numerically identical to the likelihood ratio test statistic for the hypothesis  $\pi_1 = 0$  in equation (24).

Write the system in stacked form:

$$(A1) \quad Y = X\Pi + V$$

where  $Y$  is the  $T \times 3$  matrix of observations on the endogenous variables,  $Y = [y_t \ z_t \ \Delta c_t]$ ;  $X$  is the  $T \times 7$  matrix of observations on the predetermined variables,  $X = [1 \ y_{t-1} \ y_{t-2} \ y_{t-3} \ z_{t-1} \ z_{t-2} \ z_{t-3}]$ ;  $V$  is the  $T \times 3$  matrix of realizations of the reduced form disturbances,  $V = [v_{1t} \ v_{2t} \ v_{3t}]$ ; and  $\Pi$  is the  $7 \times 3$  matrix of reduced form coefficients.

Let  $\Omega$  denote the  $3 \times 3$  covariance matrix of the reduced form disturbances and let  $P$  denote the lower triangular matrix  $P$  such that  $PP' = \Omega^{-1}$ . By post-multiplying equation (A1) by  $P$ , one obtains the GLS transformation of the system:

$$(A2) \quad YP = X\Pi P + VP$$

Since the covariance matrix of the transformed disturbances is diagonal, the value of the log likelihood function of the three equation system (A2) is equal to the sum of the values of the log likelihood functions of the three individual equations. Consider the second equation in the transformed system. The vector of coefficients of the transformed second equation (given

by the second column of  $\Pi P$ ) consists of a linear combination of the second and third columns of  $\Pi$ .

The log likelihood of the transformed second equation is not affected by restricting the reduced form coefficients of the consumption equation (given in the third column of  $\Pi$ ) because each of the parameters in the second column of  $\Pi$  is a free parameter. By an analogous argument, one can show that the log likelihood of the transformed first equation is not affected by restricting the reduced form coefficients of the consumption equation. Thus all of the decrease in the log likelihood function of the three equation system (A2) associated with the restriction  $\beta_1 = \gamma_1 = 0$  is attributable to the decrease in the log likelihood function of the consumption equation alone.