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CONVENTIONAL VALUATION AND THE TERM STRUCTURE OF INTEREST RATES

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ABSTRACT

There does not appear to be a general tendency for long-term interest rates either to overreact or to underreact to short-term interest rates relative to a rational expectations model of the term structure. Rather, there appears to be some tendency for markets to set long-term interest rates in terms of a convention or rule of thumb that makes long rates behave as a distributed lag, with gradually declining coefficients, of short-term interest rates. People seem to remember the recent past but blur the more distant. In some monetary policy regimes this convention implies overreaction, in others underreaction.

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Introduction

It is plausible that prices of long-term bonds and other long-term assets would be heavily influenced by social conventions or "rules of thumb". Investors may assume that markets price long-term bonds roughly according to the convention that their yield to maturity will be equal to their recent memories of the level of short-term interest rates plus a constant "risk premium." Such an assumption might be largely self-fulfilling, since people who believe the convention characterizes market prices may act to make it do approximately so.

It is also plausible that prices of long-term bonds are influenced by changing perceptions unrelated to any such mechanistic rule of thumb. Changing attitudes, fashions, public confidence or beliefs would plausibly account also for some unpredictable drift in long-term interest rates.

Together, the conventional valuation rule and the unpredictable drift notions might be regarded as Keynesian. Keynes said at one point that the long-term interest rate is "a highly psychological phenomenon," and at another that it is "highly conventional....its actual value is largely governed by

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the prevailing view as to what its value is expected to be." 1

The literature on efficient markets is widely interpreted as providing evidence contrary to this conventional-psychological view. If people are guided exclusively by convention or by changing fashions or attitudes, then it seems likely that they should create "profit opportunities" for others not blinded by convention. The general impression in the profession from the large literature testing for market efficiency is that such profit opportunities do not exist. Ironically, this general impression persists even though there is no agreement about <u>which</u> efficient markets model is supported by the data.

The Rational Expectations theory of the term structure of interest rates with constant risk premium is the form of the efficient markets models most widely cited with regard to interest rates. The theory is a useful starting point from which to describe the behavior of interest rates.² Departures from the theory are usually referred to in terms of time-variation of risk premia, and showing how the expectations theory fails might also be described as describing the behavior through time of the risk premium. The expectations theory with constant risk premium has had its ups and downs when tested with data. Sutch [1968],

1. <u>General Theory</u>, pp.202-3. This possibility may also call to mind the literature on multiple rational expectations equilibrium, as for example in Cass and Shell [1983]. But here it will not be assumed that rules of thumb are strictly rational.

2. There are actually a number of variants of this theory with similar implications for data. They will be unified in a linearized model below.

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Modigliani and Shiller [1973] and Sargent [1979] have claimed evidence supportive of the theory. Later, however, it was claimed that the theory could be rejected: Shiller [1979], Hansen and Sargent [1981]. I then claimed that long-term interest rates appear to be too volatile to be in accordance with such simple expectations theories. Yet the evidence for the claimed excess volatility of long-term interest rates was itself criticized by Flavin [1983] [1984] and others. Moreover, it was claimed by Campbell and Shiller [1984] and Mankiw and Summers [1984] that recent U. S. long-term interest rates do not seem to overreact to short-term interest rates.

This paper will attempt to straighten out some of these apparently conflicting claims, as well as to point to directions for alternatives to rational expectations models. The simple linearized expectations model will be described and compared with the data. New in this paper are estimates of Modigliani-Sutch equations, characterizing in simple terms how long and short rates are related, for a number of sample periods and two countries. This gives us a better picture of the robustness of the relation, and enables us to view it under different monetary policy regimes. Some notions of "overreaction" of long-term interest rates to short rates will be studied, and estimates and standard deviations of the extent of overreaction or underreaction will be presented for the various sample periods. This puts on a surer footing some comparisons made by Sutch [1967] and Modigliani and Shiller [1973]. Finally, an attempt

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will be made here to determine whether the reliance on a conventional valuation formula or the component of long rate unrelated to lagged short rates might be considered the reason that the slope of the term structure gives wrong signals as to the course of future interest rates.

Description of the Historical Relation

Between Long and Short Rates

It is important first to clarify in what sense the long rate is actually described by the sort of convention mentioned above. Modigliani and Sutch [1966] were the first to show that the long-term interest rate might well be described as a simple distributed lag on short-term interest rates, or, in terms of the annual data used in this paper:³:

(1)
$$R_t^{(n)} = \sum_{i=0}^{4} \beta_i^{(n)} R_{t-i}^{(1)} + C^{(n)} + u_t^{(n)}$$

where $R_t^{(n)}$ is the n-period rate (yield to maturity in percent on

^{3.} In this paper with its annual data I shall estimate distributed lags that include the current and four lagged values, approximately the same total lag length as Modigliani and Sutch [1966] used with their quarterly data. Throughout this paper the term "short rate" and "one-period rate" will be used interchangeably, though in the data sets the short rates are not exactly one-year rates.

n-period coupon bonds) at time t, 4 R_t ${}^{(1)}$ is the one-period rate at time t, C is a constant term and u_t is an error term uncorrelated with current and lagged short rates. In their estimates of the quarterly analogue of (1), they imposed an "Almon" fourth-order-polynomial distributed lag on all coefficients except the first, which was unconstrained. In their estimates for 1952 first quarter to 1961 fourth quarter, the estimate of β_0 was 0.32, and the sum of all the coefficients β_i i = 0,...,16 was 0.99, or virtually one. The pattern of distributed lag coefficients after β_0 was hump shaped, with comparatively small values for interest rates corresponding to lags of less than a year or more than three years, and the largest values for lags of about two years.

Modigliani and Sutch interpreted this distributed lag as representing the combined effect of two different expectations mechanisms for future short-term interest rates. A regressive expectations mechanism would make expected future short rates a moving average (with positive weights that decline exponentially with increasing lag) of current and lagged short rates. An extrapolative expectations mechanism would make the expected change in short-term interest rates a moving average with positive weights (that decline with lag) of current and lagged <u>changes</u> in short-term interest rates. The combination of both

^{4.} Superfluous parentheses in superscripts are to indicate that the superscript is not to be interpreted as an exponent. In what follows, the (n) superscript will be omitted on coefficients and error terms except when necessary for clarity.

mechanisms might produce, they argued, the pattern of distributed lag coefficients that they found in their estimates. They did not refer to "rational expectations" (actually they referred to Keynes [1936]) in motivating these mechanisms, so it seems that they were at that time referring to habits of thought or conventions people use to formulate expectations.

The original Modigliani-Sutch relation was expanded further by Modigliani and Shiller [1973] to allow for a separate effect of real interest rates and of inflation on long-term interest rates. This two-distributed-lag equation was incorporated as the basic term structure equation in the MIT-Penn-SSRC Econometric Model of the United States. The out of sample performance of this equation has been good (see Shiller-Campbell-Schoenholtz [1983].)

Estimates of equation (1) for data sets other than those used by Modigliani and Sutch appear in Table 1. The various sample periods used here were chosen with the idea of looking separately at various monetary policy regimes (see Appendix) in two countries, the U.S. and the U.K. There is a very substantial amount of data used here that is out of the sample used by Modigliani and Sutch [1966]. Here, the estimates are produced by ordinary least squares, without the Almon constraint. With these annual data, the multicollinearity that necessitated a procedure like the Almon is less of a problem.

While these estimates do not show evidence of the

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extrapolative expectations hypothesized by Modigliani and Sutch⁵ it does appear indeed that for widely different sample periods and for two different countries there is some consistency in the pattern of response of long rates to short rates. In all cases, the estimated coefficients β_i are positive. In all cases the distributed lag has an exponential appearance, gradually tailing off. In several estimates, the last coefficient β_4 is larger than the rest, suggesting that the last coefficient is proxying for omitted further lags.

There are, however, some differences in response patterns across sample periods. The more recent data sets show a much higher R-squared than do the pre-depression data sets, that represented gold-standard monetary regimes. The pre-depression data for the U. K., where the dependent variable is the British Consol yield, are conspicuously different in that the sum of the β_i is less than .5, rather than over 1.00 as is the case with the U. S. data sets.

One might note that the Durbin-Watson statistic in these regressions is uniformly low, meaning that we ought not to trust the t-statistics from the regression. Phillips and Pippenger [1979] siezed upon this fact to criticize Modigliani and Sutch [1966] and Modigliani and Shiller [1973]. They found that with their quarterly U. S. data from 1955 to 1971 if one

5. That is, β_1 is not negative or small relative to the adjacent coefficients.¹ This may be due in part to the choice of annual rather than quarterly data.

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first-differences the data, both long and short rates, and runs a similar regression, the coefficients of lagged short rates are significant at the 1% level only if corporate yields are used. They reported that the lagged interest rates were not significant at the 5% level if treasury yields were used. However, their results with the treasury data still show a distributed lag pattern that was similar to that estimated with corporate yields. In all of the regressions shown in Table 1, the coefficient β_1 of the short rate lagged a year is significant at the 5% level whether a Cochrane-Orcutt serial correlation was used or whether the data were first differenced prior to running an ordinary least squares regression. Similarly, coefficient β_2 was significant at the 5% level in half of these regressions.

The original Modigliani-Sutch relation can also be interpreted in terms of the spread between the long interest rate and the short rate. Subtracting the current short rate from both sides of the Modigliani-Sutch equation, one finds that the spread depends negatively on the current short rate and positively on a distributed lag of short rates. The R squared in this transformed regression is usually quite high (see the R' squared shown in Table 1.) Thus, the spread shows a distinct tendency to be negative when the current short rate is below a sort of

^{5.} Ordinary least squares rather than Cochrance-Orcutt results were presented in Table 1 because the former allows us to make an argument in the context of a rational expectations model that the expected values of the coefficients are unaffected by omission in the regression of information in the market information set.

average of lagged short rates and to be positive when the current short rate is above the average of lagged short rates. The sum of the coefficients in the transformed regression is often about zero, indicating that the <u>level</u> of short interest rates has little effect on the spread. If we added a constant to all of the short rates over the last 5 years, the prediction for the spread would be nearly unchanged.

The changes across sample periods in the relation of long rates to short rates documented in Table 1 might be justified in terms of the rational expectations theory of the term structure if the time series properties of short rates had changed appropriately across sample periods. Whether the distributed lag coefficients like those in table 1 are consistent with the rational expectations theory of the term structure has been the subject of discussion for some time, starting with Richard Sutch's Ph.D. dissertation [1967], and my own Ph.D. dissertation [1972], and then with Modigliani and Shiller [1973], Sargent [1979], Hansen and Sargent [1981] and others. However, these authors did not investigate whether broad changes in the time series properties of the short rate across sample periods could account for the changes in the relation between the long rate and the short rate.

<u>The Linearized Expectations Theory of the Term</u> <u>Structure of Interest Rates</u>

A linearized version of the expectations theory of the term structure of interest rates for coupon bonds was presented in Shiller [1972] and Modigliani and Shiller [1973] and developed further in Shiller [1979], Shiller Campbell and Schoenholtz [1983] and Campbell and Shiller [1984]. The underlying assumption of this linearized expectations theory is that long-term interest rates (yields to maturity) on coupon bonds not far from par can be written as a weighted average of expected future short-term interest rates with more weight on the interest rates less far in the <u>f</u>uture. In the extreme case of a consol, whose maturity is infinite, the long rate is a weighted average of all future short-term rates, with weights which decline geometrically into the future. The conventional assumption in the literature testing the expectations theory of the term structure had been that the long-rate is an <u>unweighted</u> average of expected future short rates. This conventional assumption is really appropriate as an approximation only for relatively short-term bonds, and could of course not be used to study consols which are part of the data for this paper.

It is helpful to write the expectations theory of the term structure with the help of the concept of <u>duration</u> (Macaulay,

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[1938]). The duration of a bond is a

discount-factor-times-payment-weighted average of all the times to payments of a bond. It is supposed to give a better measure of how long-term a bond is than does the time to maturity. The formula gives less weight to the coupons and principal which occur far into the future because these contribute relatively less to price today, as they are heavily discounted. For par coupon bonds whose yield to maturity in percent is r, Macaulay's duration is:

(2)
$$D_n = (1-g^n)/(1-g)$$

Where g=1/(1+r) and where n is the number of periods to maturity of the bond.⁷ Thus, the duration of a consol (n= ∞) is not infinite but equals (1+r)/r. The duration of very long-term bonds is just less than (1+r)/r. For example, if r = 5% then a consol has a duration of 20 years and a 25 year bond has a duration of 15 years. Indeed, we would expect its price or yield to resemble somewhat those of consols.

The linearized expectations theory of the term structure of interest rates is then:

(3)
$$R_{t}^{(n)} = (1/D_{n})\sum_{j=0}^{n-1} (D_{j+1}-D_{j})E_{t}R_{t+j}^{(1)} + \Phi_{n}$$

^{7.} The rate r is expressed as a proportion per period, while interest rates in the data used in the tables are in percent per annum.

Where E_t denotes expectation conditional on all information publicly available at time t. $\underline{\sigma}_n$ is a risk or liquidity premium which is assumed constant through time.⁸ In this formula, each future one-period rate is given weight corresponding to the contribution to total duration of the time period to which it applies. Since time periods further into the future have less contribution to duration, the short rates corresponding to these time periods will be given less weight. Equation 3 can be motivated in a number of ways. One is by linearizing the present value formula for coupons and principal (discounted by $E_t R_t^{(1)}$, $E_t R_t^{(n)}$. . .) around r.

Accompanying the model are various expressions for linearized holding period yields and forward rates, so that (except for the constants \tilde{q}_n) all expected linearized holding period yields equal the spot rate of the corresponding maturity and all linearized forward rates equal the corresponding expected spot rates (see Shiller, Campbell and Schoenholtz [1983] where the accuracy of the linearizations was also studied). These linearizations allow us to interpret the expectations model in various ways, without encountering the "Jensen's inequality" problems emphasized by Cox, Ingersoll and Ross [1981], problems

^{8.} We might call Φ a "rolling risk premium" since it relates to the difference between the long rate and a rolling-over of short rates. This will distinguish it from the holding period risk premia or forward rate risk premia with which it is often confused. See Campbell and Shiller [1983].

that are for the most part inconsequential. For our purposes here, we need only the linearized one-period holding yield on n-period bonds:

(4)
$$h_t^{(n)} = R_t^{(n)} + (D_n^{-1})(R_t^n - R_{t+1}^{(n-1)})$$

This formula is a linearization around r of the one-period holding return on an n-period bond in terms of $R_t^{(n)}$, $R_{t+1}^{(n-1)}$ and the coupon. The one-period holding return is the return from buying an n-period bond at time t, receiving its coupon between t and t+1, and selling the bond at time t+1, when it is an (n-1)-period bond. The model (3) implies that $E_t h_t^n = R_t^{(1)}$ plus a constant, or, conversely, the latter (subject to a terminal condition) implies the model (3). When maturities are distant as with the long bonds in this paper there is no significant distinction between the yield $R_{t+1}^{(n)}$ and $R_{t+1}^{(n-1)}$. With consols, the two are of course identical. In each application of the formula (4) in this paper, a single long-term bond yield will be used for both $R_t^{(n)}$ and $R_t^{(n-1)}$.

Implied Behavior of The Long Rate for Various Subperiods

Let us consider an autoregressive forecasting equation for the short-term (one-period) interest rate $R_{+}^{(1)}$:

(5)
$$R_t^{(1)} = \sum_{i=1}^{5} \alpha_i R_{t-i}^{(1)} + C + u_t.$$

where u_t is an error term which is serially uncorrelated and uncorrelated with R_{t-i} (1) i > 0 and C is a constant term. Table 2 shows results from estimation of this fifth order autoregressive model for the short-term interest rate for each of the data sets.

If this is indeed the optimal forecasting equation that is based on 5 lagged values, and no other information is available that will help forecasting, then by the expectations model the long-term interest rate $R_t^{(n)}$ will be explained perfectly (that is with no error) as a distributed lag, depending on $R_{t-i}^{(1)}$ i=0,...,4:

(6)
$$R_t^{(n)} = \sum_{i=0}^4 \mu_i^{(n)} R_{t-i}^{(1)} + C.$$

The coefficients μ_i i = 0,...,4 in the distributed lag and the constant term C can be derived from those in equation (5) using (3) and the "chain principle of forecasting." These coefficients are related to those in (5) by:

(7)
$$0 = D_n \mu_i - (D_n - 1) (\mu_{i+1} + \mu_0 \alpha_{i+1}) + I(i)$$

$$i = 0, \dots, 4.$$

where I(i) = 1 if i = 0 and is zero otherwise, and $\mu_5=0$. If we replaced $R_t^{(n)}$ with $\Sigma \mu_i R_{t-i}^{(1)}$ and $R_{t+1}^{(n-1)}$ with $\Sigma \mu_i R_{t-i+1}^{(1)}$ in the formula (4) for the holding yield, then $h_t^{n}-R_t^{(1)}$ would be

uncorrelated with each of the current and five lagged short rates.

Of course, the assumption that market forecasts of future interest rates equal autoregressive forecasts is quite restrictive, and could easily be rejected since long-term interest rates cannot be explained perfectly by a distributed lag on short-term interest rates. However, if long rates are set in accordance with (3) with <u>more</u> information than is contained in the history of short rates, then it follows that a theoretical regression of long rates on a distributed lag of short rates will show μ_i as the coefficient of r_{t-i} , i=0,...,4.

One can thus evaluate the expectations model (3) by estimating the autoregression (5) for the one-period rate and then solving the system of equations (7) for the weights μ_i i=0,...,4 and comparing these with the estimates of β_i , i=0,...,4. Except for sampling error, the two must be the same. Such estimates of μ_i i = 0,...,4, for the data sets of this paper appear in Table 3 alongside the estimates of β_i . We may say that except for the very recent data sets, data set number one when estimated through 1983 and data set number 2, the long-term interest rate appear to overreact to short-term interest rates, $i.e., \mu_i$ tends to be less than the corresponding β_i , i=0,...,4.

There is however reason to suspect that this procedure may be biassed toward finding overreaction, at least in some of the sample periods. In both the recent U. S. and U. K. regressions the sum of the coefficients of the lagged interest rates in Table

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2 is about one, suggesting that the characteristic equation corresponding to the autoregression may have a root equal to one. It is well established that in the case of a simple autoregression, with one lag only, if the coefficient α_i equals 1.00 the ordinary least squares estimate of it will be biassed downward. In this case, there is a bias in the method toward finding spurious overreaction.⁹ There do not appear to be Monte Carlo results that would tell us the extent of the bias for the fifth-order autoregression used here.

Those who studied whether the μ_i equal the β_i dealt with this problem generally by imposing a unit root and estimating the forecasting equation for short-term interest rates in first-differenced form. The unit root was assumed in Modigliani and Shiller [1973] and Campbell and Shiller [1983].¹⁰ Sutch [1967] did not assume a unit root, but he proceeded the other way, computing α_i i=1,...,5 from (7) and comparing these with the estimates of α_i from an autoregression. His procedure appeared to have an effect on these comparisons similar to that of assuming the unit root.

The problem of assuming the unit root is that it forces the μ_i to sum to one. Imposing the unit root thus assumes the conclusion that there is no overreaction as defined here. It

9. A similar point was raised by Mankiw and Shapiro [1985] regarding Flavin's [1981] observation that consumption appears to overreact to income.

10. Sargent [1979] and Hansen and Sargent [1981] also imposed the unit root in their rather different procedures.

remains possible, however, that some other sort of overreaction might be revealed with this procedure. In Campbell and Shiller [1983] overreaction was defined as that the β_i showed relatively too much weight on the current short rate relative to short rates lagged more periods.

As a way of exploring this possibility, the autoregressive equations for the short rate in Table 2 were reestimated subject to the constraint that $\Sigma \alpha_i = 1$, i. e., a fourth order autoregressive model for the first difference of the short rate was estimated. Using (7) with μ_i ' in place of μ_i , the coefficients μ_i ' i=0,...,4 were computed, and these also appear in Table 3. Of course, we no longer find that there is overreaction as defined above since the $\Sigma \mu_i$ ' = 1 by construction. It probably makes sense to look at these estimated μ_i ' only for the twentieth century data, where short rates seem to show evidence of nonstationarity. Here, the long rate appears to underreact to the current short rate (confirming results in Campbell and Shiller [1984]) and to put relatively too much weight on the past.

None of these methods readily allows for any formal testing of the model. We can look to see whether the estimated coefficients $\beta_i^{\ }$ are similar in appearance to the implied coefficients $\mu_i^{\ }$ i=0,...,4, in Table 3, but we cannot tell directly whether the difference is statistically significant. This shortcoming of the procedure was rectified by Sargent [1979] who showed how a likelihood ratio test can test the cross-equation

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restrictions that were examined, subject of course to his imposition of the unit root.

Fortunately, it is easy to run such a test in the present context. One can merely regress the excess return $h_t^n - R_t^{(1)}$ on current and lagged short rates. That is, one estimates the model:

(8)
$$h_t^{(n)} - R_t^{(1)} = \sum_{i=0}^{4} h_i^{(n)} R_{t-i}^{(1)} + u_t^{(1)}$$

where f_i i=0,...,4 are coefficients and u_t is an error term uncorrelated with $R_{t-i}^{(1)}$ i=0,...,4. By the model (3) all coefficients f_i i = 0,...,4 should be zero (the short rates are in the public information set at time t) and moreover the error terms are serially uncorrelated.¹¹ As a test of the model (3), we may perform significance tests with the estimated values of f_i , i = 0,...,4. These tests may be regarded as "forward-filtered tests" as defined by Hayashi and Sims [1983] of the model (3). Such tests are much simpler to perform than the tests Sargent [1979] and Hansen and Sargent [1981] performed, tests that they described as involving complicated nonlinear cross-equation restrictions. Their tests were much more complicated because

^{11.} The t-test here may be unreliable in small samples, of course. A simple example will illustrate why this may be a problem. Suppose that the short rate is a first-order autoregressive process with autoregressive coefficient h just under one, and the long rate is equal to the short rate times (1-g)/(1-hg). If (8) were run truncating the distributed lag at zero, then in finite samples f will tend to be negative, falsely suggesting that long rates tend to overreact to short rates.

they in effect assumed that they had data on $R_{t+1}^{(n)}$ but not on $R_{t+1}^{(n-1)}$. They did not use consol data or make the approximation that these are the same as was done here. For their relatively short maturities, such a distinction may be more important.

The excess return regressions are shown in Table 4. The significance levels at which the expectations hypotheses can be rejected by an F-test vary from .01 for data set 3 to .26 for data set 2. There does seem to be some evidence against the expectations model here, although not always impressive evidence judged from the standpoint of conventional significance levels. There seems to be a pattern for the coefficients. Except for data set 1 when estimated through 1983, the sum of the coefficients f_i i=0,...,4 is positive. Moreover, for each data set the coefficient of the current short rate is negative, and the sum of the lagged coefficients is positive. This pattern of coefficients is crudely consistent (given the estimates of equation (1)) with the notion that the excess return is explained by the spread between the long rate and the short rate, as will be discussed below.

The results of the above regression can be interpreted in terms of an overreaction or underreaction of long rates to short rates. Call $j_i^{(n)} = \beta_i^{(n)} - \mu_i^{(n)}$. Thus, j_i is the amount by which the long rate "overreacts" to $R_{t-i}^{(1)}$. Then it can be shown that, assuming the error term u_t in (1) is uncorrelated with all current and lagged $R_t^{(1)}$, the following relation holds:

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(9)
$$f_{i} = D_{n^{j}i} - (D_{n^{-1}})(j_{i+1} + j_{0}\alpha_{i+1})$$
$$i=0,1,2,3,4$$

If we substitute (9) into (8) and consider this and equation (5) as a two equation system in the 12 parameters α_i i = 1,...,5, j_i i = 0,...,4 and constant terms, then we can derive joint estimates of the parameters and their standard errors using nonlinear multivariate regression (seemingly unrelated regression). Under the null hypothesis, equation (3), and under the assumption that the autoregression was not truncated too early, the error term in each equations is serially uncorrelated. The error terms will still be serially uncorrelated in both equations under an alternative hypothesis that makes the long rate equal to that given by (3) plus $\Sigma j_i r_{t-i}$. Of particular interest are the "overreaction" coefficients j_i i=0,...,4, and these are shown in Table 5. These were computed without constraining the sum of the α_i to be one.

In other words, the j_i i=0,...,4 in Table 5 were computed so that if one "corrected" the long rate $R_t^{(n)}$ by subtracting $\Sigma j_i R_{t-i}^{(1)}$ from it, and if one then computed the excess return $h_t^{(n)} - R_t^{(1)}$ using the corrected $R_t^{(n)}$ and $R_{t+1}^{(n)}$ in place of the actual values, then this excess return would be perfectly uncorrelated in the sample with each of $R_{t-i}^{(1)}$ i=0,...,4.¹²

^{12.} In the final column of Table 5 are estimates of what the distributed lag of long rates on short rates <u>should</u> have looked like in Table 1. These estimates are just $\mu_1^{-1} = \beta_1^{-1} - j_1^{-1}$. When these are compared with estimates derived from the α_1^{-1} in Table 2

It should be noted that the standard errors of the estimated j_i here may not be trustworthy. One factor not accounted for here is that while the error terms under the null are each serially uncorrelated, the assumption that cross correlations are zero at other than zero lag does not follow from the model. And of course, any assumption about error terms under the alternative hypothesis is lacking in motivation. Moreover, there is also the above-mentioned problem concerning applicability of asymptotic distribution theory.

The estimated forecasting equations for the short rate in Table 2 always have a negative value at one lag, and this tends to produce a small value of μ_1 compared to adjacent values of μ_1 . In other words, there ought to be the extrapolative expectations hypothesized by Modigliani and Sutch [1966] and the absence of evidence for it in Table 1 here stands in contradiction of the rational expectations model.

Another Characterization of the Eailure of the Expectations Model

One might say that the simplest and most fundamental

using equations (7), in Table 3, the estimated μ_i^{\wedge} i = 0,...,4 look reasonably similar except for data set number 2. Any differences between these two estimates of μ_i° i = 0,...,4 can come only from a nonzero correlation of the residual in the estimated equation 1 with R_{t-5}° or from differences in correlation R_t° with R_{t-i}° i=i=0,...,4 when the sample is shifted one period.

implication of the rational expectations theory of the term structure is that relatively upward sloping term structures (where the long rate is greater than the short rate by more than the usual term premium) ought to portend a subsequent increase in interest rates. Relatively downward sloping term structures (where the long rate is less than the short rate plus the usual term premium) ought to portend subsequent decreases in interest rates. The expectations model (3) allows us to say this more formally. If the excess return $h_{+}-r_{+}^{((1)}$ is to be uncorrelated with the spread $R_t^{(n)} - R_t^{(1)}$ then a regression of $R_{t+1}^{(n)} - R_t^{(n)}$ on the spread $R_{t}^{(n)} - R_{t}^{(n)}$ should yield a positive slope coefficient, equal to $1/(D_n-1)$.¹³ It is easy to see why the correct formulation must look like this. Let us suppose to simplify the argument that the term premium Φ_n in the model (3) is zero, so that expected returns on both long and short debt must be the same, and suppose that the bond is a consol. If the long rate is above the short rate, there must be a capital loss to offset the higher current yield on long bonds, if the high vield is not to indicate a relatively higher expected return on the long bond. A capital loss of course means a rise in long-term interest rates.

It was Franco Modigliani who first pointed out to me that the fact is just the opposite: when long-term interest rates are above short rates the long-term interest rate shows a tendency to

^{13.} As before, technically the dependent variable should be $R_{t+1} = R_t$

decline subsequently rather than rise. Thus, when long rates are relatively high there tends to be a subseqent capital gain on long bonds which further augments their higher current yield. As far as I can tell, this fact had not been documented before.¹⁴

I showed evidence for this fact for a number of sample periods (Shiller [1979]) and the fact was further confirmed in Shiller, Campbell and Schoenholtz [1983], Campbell and Shiller [1984], and Mankiw and Summers [1984]. Table 6 shows the regressions for the data sets used in this paper. The t-statistics presented in Table 6 are not the usual t-statistics but are for the null hypothesis that the coefficient of the spread is $1/(D_p-1)$.

This perverse behavior of the term structure relative to the expectations hypothesis could be due to the way the long rate responds to the short rate, as estimated in Table 1, or it could be due to noise in the long rate series that is unrelated to the history of short rates. To decide which, the spread $R_t^{(n)} - R_t^{(1)}$ was decomposed into two components: that corresponding to the fitted value in the regressions of Table 1 and the residuals of Table 1. Regressions of excess returns on these two variables appear in Table 7. We see that both variables play some role. The coefficients of the fitted spread greater than one indicate that the pattern of reaction of long rates to short rates is part of the reason the shape of the term structure gives wrong signals

^{14.} This observation was made by Macaulay [1938] p. 33, who, however, did not document it or emphasize it.

as to the future course of interest rates.

Mankiw and Summers [1984] looked at regressions of changes in long rates on the long-short spread for evidence for a different notion of overreaction. For them, overreaction occurs if long-term bounds are priced in accordance with (3) but with too short a duration, i. e, with a duration less than implied by the actual maturities and average levels of interest rates. They pointed out that this sort of overreaction could never explain the wrong sign of coefficient of the spread variable in regressions like those in Table 6 here.

In contrast, the wrong sign of the coefficient of the spread variable could in the recent sample periods instead be due to the sort of underreaction defined above in that the long rate reacts relatively too much to the past and too little to the current short rate. In those cases where $\Sigma\beta_i \approx 1$ the spread variable tends to be high when short rates are low relative to their average level over the preceding few years. If long rates were to tend to increase subsequently, as the expectations model would predict, then given the fact that long rates tend to be have like a moving average of short rates, it would have to be the case that the short rate tends to increase substantially at such a time. In fact it does not.

An extreme caricature for the U.S. data would be that the long-term interest rate is a moving average of the short rate with exponentially decaying weights that sum to one:

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$$R_{t}^{(n)} = (1-h) \sum_{i=0}^{\infty} h^{i} R_{t-i}^{(1)} \quad 0 < h < 1$$

If this is the case, then the change in the long rate $R_{t+1}^{(n)} - R_t^{(n)}$ equals (1-h) $(R_{t+1}^{(1)} - R_t^{(n)})$. For this to be positively correlated with the yield spread $R_t^{(n)} - R_t^{(1)}$, it would have to be the case that when the short rate is below the long rate (or equivalently below its recent average value as defined by the moving average) it would have to tend to be above it the following year. In fact, the short rate is more persistent than that and tends to stay on the same side of the long rate. The caricature would be more realistic if we added a transient error term (representing an exogenous drift of long rates unrelated to short rates) to the above equation, another factor which would tend to make for a wrong sign in the regression of the change in the long rate on the spread.

The Volatility of Long-Term Interest Rates

It was shown in Shiller [1979] that the model (3) for n= ∞ implies that, for a given variance of h_t there is a lower bound to the possible variance of $R_t^{(1)}$. A high variance of h_t can only be justified if there is enough variation in short term interest rates themselves.¹⁵ The variance inequality was extended

^{15.} Analogous variance inequalities were also used to evaluate the model that corporate stock prices equal the present value at

formally to the finite n case in Shiller [1981]. In the present notation this is:

(10) $\operatorname{var}(R^{(1)}) \ge \operatorname{var}(h^{(n)})/D_n(g^2)$

where $D_{p}(q^{2}) = (1-q^{2n})/(1-q^{2})$. When sample variances were substituted into (9) then the inequality was found generally to be violated for g in the relevant range, as is verified for all of the data sets of this paper, Table 8. Rejection of the expectations model for violation of this inequality was criticised by Flavin [1983] [1984] and others on the grounds that small sample properties of the estimates of these variances may be unreliable. This criticism of the use of this inequality is certainly valid, especially with regard to more recent interest rate data that seem more likely to show nonstationarity. The violation of the variance equalities only show, as I originally noted, that the variability of changes in long-term interest rates can only be reconciled with the expectations theory of anticipated variance of short rates was much higher than the historical variance. This is true as well for the 19th century data.

My concern here is merely to judge which component of the long rate accounts for the violation of this inequality in the data sets used here. Table 8 also shows standard deviations for the various data sets in this paper of the excess return computed

a constant discount rate of expected future dividends, Shiller [1981] and LeRoy and Porter [1981].

not from the actual $R_t^{(n)}$ and $R_{t+1}^{(n)}$ but using the fitted values of the regressions of Table 1. Clearly the fitted values violate the inequalities too.

Conclusion

This paper began with the plausible notion that a conventional valuation rule for long term bonds causes their yields to behave as a sort of moving average of lagged short rates. If people are relying on their memories to price bonds, then it is plausible that they would blur the past, and that the distributed lag would have a simple form, as the roughly exponential decay form estimated here. Whether this is an overreaction or underreaction depends on the stochastic properties of the short-term interest rate.

The sharpest contrasts are between the old historical data and the most recent data. For the British data 1828-1930, the short rate seems to be quickly mean-reverting. In this period, it appears that the long-term interest rate overreacted somewhat to short rates, and this caused excess volatility in long-term interest rates. With the recent U. S. data, on the other hand, (whether the sample ends in 1979 or 1984), there is not such evidence of quick mean reversion in the short rate. One cannot rule out either that there is substantial mean reversion or alternatively that the autoregressive forecasting equation has a unit root, should be estimated in first-differenced form, that short rates are unstationary and have no mean to revert to. If one assumes that short rates are mean reverting, as many of the estimated equations without the imposition of the unit root suggest, then the long-rate appears to have overreacted to short rates in all but the most recent sample period. If one makes the unit root assumption, it appears that the long rate was not excessivly volatile relative to the expectations theory of the term structure, and in fact <u>underreacted</u> to short rates. There was underreaction in the sense that the long rate than the past short rates.

The tendency of long rates to rely too heavily on the past rather than current short rates accounts partly for the dramatic failure of the slope of the term structure to predict changes in long-term interest rates. Thus, for example, the term structure tends to be upward sloping when the short rate has dropped below its average value for the past five or so years. Since the short rate does not revert quickly above its average value over the last five or so years, the result is that the long rate will be subsequently lower, not higher as the expectations theory would predict.

The dramatic failure in U. S. data of the slope of the term structure to predict the direction of future interest rates is not due only to the underreaction noted above. The failure is also due to a component of the long-term interest rate (one might

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suppose it a "fads" or "fashions" component or alternatively a "time-varying risk premium" component) that is mean-reverting and unrelated to the history of short rates. This component also contributes to the high volatility of short-term holding yields on long-term bonds.

The expectations theory, however, is not completely without value. The reaction of long-term interest rates to short term interest rates appears to show <u>some</u> relation to the stochastic properties of short rates. With the pre-depression British series, the short rate appeared quickly mean-reverting and the duration of the consol was much longer than with the other series studied. Indeed the consol yield showed much less reaction to short rates than did long yields in other periods. The notion that bonds yields are determined <u>strictly</u> by some conventional valuation formula without regard for the stochastic properties of short rates is contradicted by these data. Instead, conventional valuation seems to account for a bias in the behavior of long rates relative to the expectations model, and does not alone amount to a theory of the term structure.

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Data	Sample	Constant	Lag	Coeffi-	(Std. er)	R	Durbin
Set	Period	(Std. er)	(1)	cient of		Squared	Watsun
Coun	_			Short		Sum of	SER
try				Rate		Coeffi-	
LI Y						cients	
				ß,^		lags 0-4	
				1			
						R'	
						Squared	
1		0.793	Ŏ	0.380	(0.063)	0.970	1.269
		(0.271)	1 2	0.243	(0.095) (0.111)	1 026	0.600
U.S.			2 3	0.130 0.171		1.020	0.000
			_3 4	0.102		0.893	
			7	O, IVE	(0100//	0.0.0	
1	1956-	0.121	0	0.305	(0.077)	0.963	0.873
-		(0.297)	1	0.256	(0.079)		
U.S.			2	0.282	(0.082)	1.183	0.410
			3	0.180	(0.082)		
			4	0.160	(0.077)	0.919	
2	1940-	1.954	o	0.427	(0.127)	0 747	0.274
Ζ.		(1.110)	1	0.193	(0.151)		·····
U.K.			2	0.170	(0.147)	0.856	1.677
			3	0.008	(0.151)		
			4	0.058	(0.129)	0.580	
-	10/1	0.040	0	0.304	(0.068)	0.852	0.549
3		-0.248 (0.301)	1	0.253	(0.074)	0.002	0:017
U.S.		(0.001)	2	0.191	(0.067)	1.030	0.570
0.0.			3	0.052			
			4	0.231		0.664	
4	1828-	1.502	о	0.168	(0.047)	0.417	0.167
-	1920-	(0.225)	1	0.092	(0.055)	/	
	1,00	····	2	0.092	(0.056)	0.524	0.479
U. K.			Ī	0.046	(0.056)	·	
			4	0.126	(0.048)	0.803	

Table 1. Regressing the Long Rate on Current and Lagged Short Rates

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See appendix for source of data. All distributed lags were estimated with ordinary least squares. R' squared is the R squared in a regression with the long rate minus the current short rate as the dependent variable, and the same independent variables.

	Sample Period		Lag	Coeffi- cient of Short Rate	(Std. er)	Squared Sum of Coeffi- cients	Durbin Watson SER
						lags 1-5	
1	1957-	0.416	1	1.089	(0.213)	0.695	2.126
	1984	(0.960)	2	-0.616	(0.340)		
U.S.			3	0.217	(0.370)	1.057	1.957
			4	0.052			
			5	0.315	(0.315)		
1	1057	1 770		0 547	10 01 -		
1	1957- 1979	1.338 (1.031)	1	0.516	(0.267)	0.542	1.921
u.s.	17/7	(1.051)	2 3	-0.369	(0.274)	÷ • •	
0.5.			د. 4	0.051 0.419	(0.286)	0.868	1.421
			4	0.251	(0.284) (0.267)		
			ل	0.231	(0.267)		
2	1961-	0.351	1	0.714	(0.161)	0.733	2.349
	1984	(1.506)	2	-0.678	(0.200)	0.700	2.34/
U.K.		· - ·	3	0.735	(0.199)	1.106	2.112
			4	-0.689	(0.204)	11100	<u></u>
			5	1.024	(0.225)		
			_				
3	1862-	0.907	1	0 . 48 2	(0.127)	0.481	1.956
	1931	(0.568)	2	-0.007	(0.139)		
U.S.			3	0.269	(0.127)	0.813	1.075
			4	0.080	(0.130)		
			5	-0.011	(0.118)		
4	1829-	1.807	1	0.564	(0.102)	0.221	1.992
	1931	(0.491)	2	-0.215	(0.119)		
U.K.			3	0.184	(0.122)	0.479	1.045
			4	-0.055	(0.121)		
			5	0.001	(0.105)		

Table 2. Regressing the Short Rate on Lagged Short Rates

See appendix for source of data. All distributed lags were estimated with ordinary least squares.

$\begin{array}{c ccc} (i) & \beta_1 & \mu_1 & \mu_1 & \mu_1' \\ from & from (7) & from (7) \\ rry & Table 1 & Table 2 & first-difference autoregression \\ \hline \\ 1984 & 1 & 0.243 & 0.086 & -0.074 \\ 2 & 0.130 & 0.415 & 0.247 \\ U.S. & 3 & 0.171 & 0.266 & 0.152 \\ 4 & 0.102 & 0.242 & 0.136 \\ \hline \\ 1978 & 1 & 0.256 & 0.051 & 0.104 \\ 2 & 0.282 & 0.136 & 0.223 \\ 1978 & 1 & 0.256 & 0.051 & 0.104 \\ 2 & 0.282 & 0.136 & 0.223 \\ U.S. & 3 & 0.148 & 0.134 & 0.223 \\ U.S. & 4 & 0.160 & 0.052 & 0.085 \\ \hline \\ 2 & 1960 & 0 & 0.427 & 0.479 & 0.334 \\ 1984 & 1 & 0.193 & 0.090 & 0.044 \\ U.K. & 2 & 0.170 & 0.423 & 0.269 \\ 3 & 1861 & 0 & 0.304 & 0.235 & 0.212 \\ 1970 & 1 & 0.253 & 0.067 & 0.203 \\ \hline \\ 3 & 1861 & 0 & 0.304 & 0.235 & 0.512 \\ 1970 & 1 & 0.253 & 0.067 & 0.203 \\ \hline \\ 4 & 0.058 & 0.450 & 0.292 \\ \hline \\ 3 & 1861 & 0 & 0.304 & 0.235 & 0.512 \\ 1970 & 1 & 0.253 & 0.067 & 0.203 \\ \hline \\ 4 & 0.251 & 0.015 & 0.012 \\ \hline \\ 4 & 1828 & 0 & 0.168 & 0.062 & 0.507 \\ 1970 & 1 & 0.092 & -0.005 & 0.129 \\ \hline \\ 0.K. & 3 & 0.046 & -0.003 & 0.071 \\ \hline \\ 0.K. & 3 & 0.046 & -0.003 & 0.071 \\ \hline \\ 0.K. & 3 & 0.046 & -0.003 & 0.071 \\ \hline \\ 0.K. & 3 & 0.046 & -0.003 & 0.071 \\ \hline \end{array}$		Sample Period	Lag	Actual	Theoretical	Theoretical
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			(i)	ß.^	μ, ^	μ, '
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Coun-					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	try			Table 1	& Table 2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1	1956-	0	0.380	0.823	0.541
U.S. $\begin{array}{cccccccccccccccccccccccccccccccccccc$	1					
U.S. 3 0.171 0.266 0.152 1 1956- 0 0.305 0.242 0.136 1 1978 1 0.256 0.051 0.104 2 0.282 0.136 0.235 U.S. 3 0.148 0.134 0.2235 U.S. 3 0.148 0.134 0.223 2 1960- 0 0.427 0.479 0.334 U.K. 2 0.170 0.423 0.269 3 0.008 0.110 0.044 U.K. 2 0.170 0.423 0.269 3 0.008 0.110 0.061 0.061 4 0.058 0.450 0.292 3 $1861 0$ 0.304 0.235 0.512 1930 1 0.253 0.067 0.203 2 0.191 0.073 0.209 U.S. 3 0.052 0.015 0.063 1930 <td></td> <td>1,0.</td> <td></td> <td></td> <td></td> <td></td>		1,0.				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	u.s.					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						
U.S. $\begin{array}{cccccccccccccccccccccccccccccccccccc$	1	1956-	0	0.305	0.220	0.352
U.S. $3 \\ 4$ 0.1480.1340.22321960-00.4270.4790.334198410.1930.0900.044U.K.20.1700.4230.26930.0080.1100.06140.0580.4500.29231861-00.3040.235193010.2530.0670.20320.1910.0730.209U.S.30.0520.01541828-00.1680.06290.092-0.0050.129U.K.30.046-0.0030.071		1978	1	0.256	0.051	0.104
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			2	0.282	0.136	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	U.S.					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			4	0.160	0.052	0.085
U.K.20.1700.4230.26930.0080.1100.06140.0580.4500.29231861-00.3040.2350.512193010.2530.0670.20320.1910.0730.209U.S.30.0520.0150.06341828-00.1680.0620.507193010.092-0.0050.12941828-00.1680.0620.50790.092-0.0050.1290.0080.K.30.046-0.0030.071	2					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		1984				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	U.K.					
3 1861- 1930 0 0.304 0.235 0.512 1930 1 0.253 0.067 0.203 2 0.191 0.073 0.209 U.S. 3 0.052 0.015 0.063 4 1828- 1930 0 0.168 0.062 0.507 2 0.092 -0.005 0.129 2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071						
1930 1 0.253 0.067 0.203 2 0.191 0.073 0.209 U.S. 3 0.052 0.015 0.063 4 0.231 0.015 0.012 4 1828- 1930 0 0.168 0.062 0.507 2 0.092 -0.005 0.129 2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071			4	0.058	0.450	0.292
U.S. 2 0.191 0.073 0.209 U.S. 3 0.052 0.015 0.063 4 0.231 0.015 0.012 4 1828- 0 0.168 0.062 0.507 1930 1 0.092 -0.005 0.129 2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071	3	1861-	о			
U.S. 3 0.052 0.015 0.063 4 0.231 0.015 0.012 4 1828- 0 0.168 0.062 0.507 1930 1 0.092 -0.005 0.129 2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071		1930				
4 0.231 0.015 0.012 4 1828- 1930 0 0.168 0.062 0.507 2 0.092 -0.005 0.129 2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071						
4 1828- 0 0.168 0.062 0.507 1930 1 0.092 -0.005 0.129 2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071	U.S.					
193010.092-0.0050.12920.0920.0080.217U.K.30.046-0.0030.071			4	0.231	0.015	0.012
2 0.092 0.008 0.217 U.K. 3 0.046 -0.003 0.071	4					
U.K. 3 0.046 -0.003 0.071		1930				
4 0.126 0.000 0.076	U.K.					
			4	0.126	0.000	0.076

Table 3. Actual and Theoretical Reaction of Long Rates to Short Rates.

t	t						
Data	Sample	Constant	Lag	Coeffi	- (Std. er)	R Bar	Durbin
Set		(Std. er)				Squared	
				of		·	
Coun-	Assumed	j		Short		Sum of	SER
trv	Duratio	n		Rate		Coeffi-	
1						cients	
				f _i ^		lags 1-4	
				1		2	
						F	Prob>F
1	1956-	-0.892	0	-3.976	(1.459)	0.217	2.952
	1983	(6.571)	1	4.617	(2.332) (2.535)		
			2	-1.882	(2.535)	3.467	13.403
U. S.	15			2.892			
			4	-2.160	(1.967)	2.493	0.062
1	1956-	-4.277	0	-1.635	(1.358)	0.121	2.279
_	1978	(5.236)	1	1.735			
				1.187		1.911	7.219
U. S.	15		3	-0.250			
01 01	10		4	-0.761	(1.358)	1.607	0.212
2	1960-	10.809	0	-1.249	(1.083)	0.086	2.320
		(10.155)		2.518			
			2	-0.215		2.246	14.232
υ. κ.	12		3	1.484	(1.372)		
			4	-1.541	(1.516)	1.432	0.260
3	1861-	-3.500	o	-1.270	(0.650)	0.142	2.150
	1930	(2.897)	1		(0.708)		
			2	1.229	(0.645)	2.063	5.480
U. S.	15		3	-1.247			
				0.930	(0.600)	3.279	0.011
4	1828-	- 1.094	o	-1.111	(0.448)	0.052	1.521
	1930	(2.154)		1.172	(0.524)		
			2	-0.047		1.305	4.586
U. K.	30		3	-0.321	(0.533)		
			4	0,501	(0.459)	2.124	0.068

Table 4_{11} , Regressing the Linearized Excess Return $h_t = R_t$ on Current and Lagged Short Rates

See appendix for source of data. All distributed lags were estimated with ordinary least squares. Durations are approximately from (2) using the sample mean for the long rate, n=25 for U. S. data, n = ∞ for U. K. data.

	Sample Period	Lag	Discrepancy	(Standard Error of	Actual	Theoretical
		(i)	j_^	Discrep-	ß, ^	۳,^
Coun-			from sys-	ancy)	from	from j ^
try			tem Esti-		Table 1	and
			mation			ß_^
	· · · · · ·					
1	1956-	0	-0.490	(1.264)	0.380	0.870
	1983	1	0.293	(0.179)	0.243	-0.050
		2	-0.318	(0.756)	0.130	0.448
U.S.		3	-0.100	(0.527)	0.171	0.271
		4	-0.288	(0.487)	0.102	0.390
1	1956-	0	0.052	(0.162)	0.305	0.253
	1978	1	0.146	(0.084)	0.256	0.110
		2	0.052	(0.127)	0.282	0.230
U.S.		3	-0.032	(0.116)	0.148	0.180
		4	-0.038	(0.074)	0.160	0.198
2	1960-	0	0.447	(0.244)	0.427	-0.020
	1983	1	0.282	(0.085)	0.193	-0.089
U.K.		2	0.383	(0.255)	0.170	-0.213
		3	0.108	(0.874)	0.008	-0.100
		4	0.292	(0.262)	0.058	-0.234
3	1861-	0	0.133	(0.096)	0.304	0.171
-	1930	1	0.169	(0.055)	0.253	0.084
		2	0.100	(0.056)	0.191	0.091
U.S.		ŝ	-0.016	(0.039)	0.052	0.068
0		4	0.061	(0.034)	0.231	0.170
4	1828-	0	0.008	(0.034)	0.168	0.160
-7	1930	1	0.042	(0.015)	0.092	0.050
	1700	2	0.005	(0.021)	0.072	0.087
U.K.		$\frac{2}{3}$	0.005	(0.015)	0.046	0.041
LJ s 1∿. s		4	0.017	(0.014)	0.126	0.109

Table 5. Discrepancies between Actual and Theoretical Reaction of Long Rates to Short Rates.

Data Set Coun- try	Sample Period	Constant (Std. er)	Coeffi- cient of Spread (Std. Er.)	R Bar Squared T ¹	Durbin Watson SER
		·····			
1	1956-	0.459	-0.293	0.193	2.439
U.S.	1983	(0.180)	(0.107)	-3.406*	0.917
1	1956-	0.357	-0.128	0.055	2.057
U.S.	1978	(0.120)	(0.084)	-2.374*	0.501
2	1960	0.209	-0.049	-0.038	1.850
U.K.	1983	(0.280)	(0.119)	1.175	1.342
3	1861-	-0.067	-0.133	0.088	2.233
U.S.	1930	(0.045)	(0.048)	-4.519*	0.378
4	1 828	0.010	0.004	-0.009	1.480
U.K.	1 9 30	(0.004)	(0.016)	-2.418*	0.160

Table 6, Regressing the Change in the Long-Term Interest Rate $(R_{t+1} - R_{t})$ on the Spread between the Long Rate and the Short Rate $(R_{t} - R_{t})$

¹ T statistic for hypothesis that coefficient equals $1/1(D_{-1})$. *Signifcant at 5% level. See appendix for source of data.

ı.

	Period	Constant (Std. er)	cient of Fitted	dual	Squared	Durbin Watson SER
1 U.S.	1956- 1 98 3		3.661 (1.303)		0.269 5.977*	
1	1956-		2.563	5.336	0.147	1.913
U.S.	1978		(1.252)	(4.211)	2.898	7.112
2	1960-	-2.323	1.604	1.446	-0.031	1.863
U.K.	1983	(3.178)	(1.814)	(2.124)	0.658	15.112
3	1861-	0.928	2.494	3.590	0.193	2.167
U.S.	1930	(0.635)	(0.829)	(1.166)	9.264 *	5.313
4 U.K.	1828- 1930		0.940 (0.490)	0.694 (0.988)	0.021 2.084	

Table 7. Regressing the Excess Return of Long Bonds over the Short Rate on The Fitted Value of the Long-Short Spread and Residual from the Regression of Table 1.

* Significant at the 1% level. See appendix for source of data.

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Data Set Coun- try	Sample Period	σ(h) σ(h)/√D _n (g ²)	σ(hf)	σ(hf)/√(D _n (g ²)
1	1956-	14.942	12.569	3.563
U.S.	1983	5.738	4.827	
1	1956-	7.683	6.822	1.796
U.S.	1978	2.950	2.620	
2	1960-	15.714	14.252	3.623
U.K.	1983	6.710	6.085	
3	1861-	6.105	6.681	1.497
U.S.	1930	2.345	2.566	
4	1828-	4.469	6.476	1.184
U.K.	1930	1.227	1.778	

Table 8. Sample Standard Deviations of Actual and Fitted Linearized Holding-Period Returns and Variance Inequality

Note: h is defined as in expression (4) in text, while hf is as in expression (4) but with the fitted values of equation (1) as estimated in Table 1 in place of R_t and R_{t+1} . The inequality (10) in the text implies that $\sigma(h)/J(D_{t}(q^2))$ be less than $\sigma(R_{t})$, if the model (3) is valid and sample standard deviations equal population standard deviations. Duration D_t is as shown in Table 3.

See appendix for source of data.

Appendix: Data Sources

Data Set 1: 1948-1984. The long-term interest rate for the United States is Moody's Aaa Corporate Bond Yield Average for January, from Moody's Investor Service, <u>Bond Survey</u>. The short-term interest rate is the bond-equivalent yield on 6-month (150-179 day prior to November 1979) commercial paper rate for January, from the Board of Governors of the Federal Reserve System of the United States, as reported in the <u>Federal Reserve</u> <u>Bulletin</u>. January figures are monthly averages of daily figures. Bond equivalent yield is computed by the transformation r = D/(1-D/200) where D is the rate on a discount basis.

Data Set 2: 1956 to 1984. The long-term interest rate for the United Kingdom is the flat yield on 2½ percent British Consols Observations are taken on the last Friday of March. The short-term interest rate is the three-month local authorities temporary loan rate for the last Friday of the March starting in 1961 and for the last Saturday of the March before that, as reported in the <u>Bank of England Statistical Abstract</u>, No. 1 (1970), table 29, and subsequent issues of the <u>Bank of England</u> <u>Quarterly Bulletin</u>.

Data Set 3: 1857 to 1930, Macaulay [1938]. The long-term interest rate is the unadjusted index number for January of yields of American Railroad Bonds. The short-term interest rate is the January commercial paper rate in New York City: for 1857 to 1923 'choice 60-90 day two name paper'; 1924 to 1930 '4 to 6

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month prime double and single name paper.' These data are in columns three and five of Macaulay's Table 10, pp. A142-60.

Data Set 4: 1828 to 1930. The long-term interest rate is the annual average of 3 percent British Consols through 1888 and on 2½ percent government annuities starting in 1889 (Homer [1963] Table 19, col. 2 and Table 57, col. 2.) The short rate is, for 1824-44, Dverend and Gurney's annual average of first class 3-month bill rates and, after 1844, the annual average rates (averaging maximum and minimun) for 3-month bank bills, both from Mitchell and Deane [1962, p. 460.] This data set was data set number 6 in Shiller [1979].

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