CUADERNOS DE ECONOMÍA, VOL. 44 (NOVIEMBRE), PP. 141-150, 2007

AN AUCTION MECHANISM FOR THE COMMONS: SOME EXTENSIONS*

JUAN-PABLO MONTERO
Pontificia Universidad Católica de Chile

Efficient regulation of the commons requires information about the regulated firms that is rarely available to regulators (e.g., cost of pollution abatement). Montero (2008) proposes a simple mechanism for inducing firms to truthfully reveal their private information: a uniform price sealed-bid auction of an endogenous number of (transferable) licenses with a fraction of the auction revenues given back to firms. This paper discuses further properties of the mechanism including its extension to the possibility of private externalities and non-transferability of licences.

JEL: D44, D62, D82
Keywords: Externalities, Asymmetric Information, Uniform-Price Auction.

1. INTRODUCTION

Regulatory authorities generally find that part of the information they need for implementing an efficient regulation is in the hands of those who are to be regulated. Regulating externalities such as access to common resources (*e.g.*, clean air, water streams, fisheries, etc.) is not the exception. Environmental regulators, for example, know little about firms' pollution abatement costs, so without communicating with firms they would be unable to establish the efficient level of pollution. Different mechanisms have been proposed for inducing firms to reveal their private information but for different reasons, these mechanisms has been of limited use[1]. In a recent paper, Montero (2008) proposes a simpler and more

* Associate Professor of Economics at the Pontificia Universidad Católica de Chile and Research Associate at the MIT Center for Energy and Environmental Policy Research. I would like to thank seminar participants and anonymous referees for several comments and Fondecyt (Grant # 1070982) for financial support.
Email: jmontero@faceapuc.cl

[1] Some of these mechanisms include Roberts and Spence (1976), Kwerel (1977), Dasgupta et al. (1980), Spulber (1988), Varian (1994) and Duggan and Roberts (2002).

effective mechanism: a uniform price sealed-bid auction of an endogenous number of (transferable) licenses with a fraction of the auction revenues given back to firms[2]. The mechanism is developed under the additional assumption that firms know nothing about the other firms' characteristics (they may be even unaware of the number of firms being regulated).

The auction's main ingredients –endogenous supply of licenses and paybacks– enter into the uniform-price format in a way that the resulting auction mechanism is both ex-post efficient and strategy-proof (*i.e.*, telling the truth is a dominant strategy). The supply curve of licenses reflects the cost to society (other than firms) from allocating these licenses to firms. Paybacks, on the other hand, are such that the total payment for licenses of each firm is exactly equal to the "damage" it exerts upon all the other agents (*i.e.*, other regulated firms and the rest of society). Hence, the auction mechanism follows a Vickrey-Clarke-Groves (VCG) payoff rule in that it makes each firm to pay exactly for the externality it imposes on the other agents.

The purpose of this paper is to present some additional, yet important, properties of the mechanism not included in Montero (2008) and to show how the mechanism can be extended to other externality problems such as those involving non-uniformly mixed pollutants (*i.e.*, firms' pollutants are not perfect substitutes in the damage function) and private externalities. In so doing, the rest of paper is organized as follows. The next section provides a brief description of the auction mechanism of Montero (2008) and the following section presents the properties and extensions.

2. THE AUCTION MECHANISM

This section, which closely follows Montero (2008), introduces the auction mechanism for the case of a classical pollution externality (*i.e.*, homogeneous pollutant).

2.1 Notation and first-best allocation

Consider $n \geq 1$ firms ($i = 1,...,n$) to be regulated. All firms are assumed to have inverse demand functions for pollution of the form $P_i(x_i)$ with $P_i'(x_i) < 0$, where x_i is firm *i*'s pollution level that is accurately monitored by the regulator (In some cases I will work with the demand function, which is denoted by $X_i(p)$ with $X_i'(p) < 0$, where *p* is the price of pollution). Function $P_i(\cdot)$ is only known by firm *i*, neither by the regulator nor by the other firms. The aggregate demand curve for pollution is denoted by *P*(*x*), where $x = \sum_{i=1}^{n} x_i$ is total pollution. The social

[2] Licenses are generally refered to as permits or allowances in water and air pollution control, as rights in water supply management and as quotas in fisheries management. In this paper, I will use the term license throughout.

damage caused by pollution x is $D(x)$ with $D(0) = 0$, $D'(x) > 0$ and $D''(x) \geq 0$ $D'(x)$ can be interpreted more generally as the regulator's supply function for licenses $S(p)$, where $D'(S(p)) = p$. We may want to assume that $D(x)$ is publicly known but it is actually not necessary.

In the absence of regulation firm i would emit x_i^0, where $P_i(x_i^0) = 0$. Hence, firm i's cost of reducing emissions from x_i^0 to some level $x_i < x_i^0$ is $C_i(x_i) = \int_{x_i}^{x_i^0} P_i(z)dz$ —note that $-C_i'(x_i) \equiv P_i(x_i)$— and the minimum total cost of achieving pollution level $x < x^0$ is $C(x) = \int_x^{x^0} P(z)dz$.

The regulator's objective is to minimize the sum of clean-up costs and damages from pollution, *i.e.*, $C(x) + D(x)$. Therefore, the socially optimal or first-best pollution level $x^* < x^0$ satisfies

(1) $$P(x^*) = D'(x^*) = P_i(x_i^*) \quad \text{for all } i = 1,...,n$$

But the regulator cannot directly implement the first-best allocation because he does not know the demand functions $P_i(\cdot)$. He must then look for mechanisms in which it is in the firms' best interest to communicate their private information to him. Montero's (2008) auction scheme is one of such mechanisms.

2.2 The auction scheme

Consider $n \geq 1$ firms. The auction scheme operates as follows. Firms are informed in advance about the auction rules (including the way the auction clears and the paybacks are computed). Firm i (= 1, 2, …, n) is asked to bid a non-increasing inverse demand schedule $\hat{P}_i(x_i)$ (or, equivalently, a non-increasing demand schedule $\hat{X}_i(p)$). Based on this information, the regulator computes the residual supply function (*i.e.*, residual marginal damage function) for each firm i using the other firms' reported demand schedules, that is

(2) $$S_i(p) = S(p) - \hat{X}_{-i}(p)$$

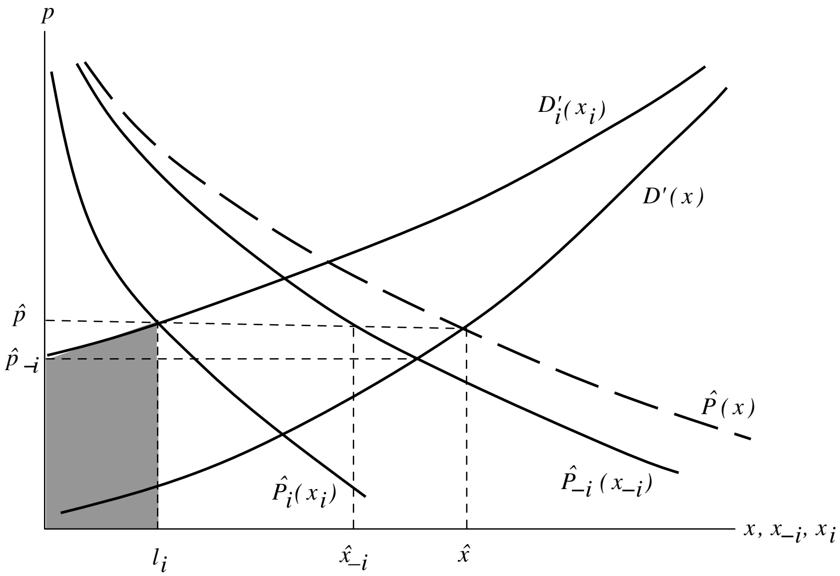
where $\hat{X}_{-i}(p) = \sum_{j \neq i} \hat{X}_j(p)$ and $D'(x) = S^{-1}(p)$. As shown in Figure 1, the residual marginal damage function $D_i'(x_i) = S_i^{-1}(p)$ is only defined at and above the point at which $D'(x) = \hat{P}_{-i}(x_{-i}) = \hat{p}_{-i}$. The regulator clears the auction by determining a price p_i and number licenses l_i for each bidder i according to

(3) $$p_i = \hat{P}_i(l_i) = D_i'(l_i)$$

or, equivalently, $l_i = S_i(p_i) = \hat{X}_i(p_i)$. Thus, firm receives l_i licenses and pay p_i for each license. Soon after the firm gets a fraction $\alpha_i(l_i)$ of the auction revenues back (*i.e.*, payback is $\alpha_i(l_i)p_i l_i$).

Since the efficient equilibrium price, given $\hat{X}_{-i}(p)$ and $\hat{X}_i(p)$, solves $\hat{X}_i(\hat{p}) = S(\hat{p}) - \hat{X}_{-i}(\hat{p})$, by making firm i face the marginal damage curve (2), we are basically informing the firm that for whatever demand report it chooses to submit to the regulator/auctioneer, its report, together with those of the other

FIGURE 1
AUCTION EQUILIBRIUM PRICES, LICENSES AND PAYMENTS



firms, will be used efficiently. In addition, Montero (2008) shows that if $\alpha_i(l_i)$ is equal to

(4) $$\alpha_i(l_i) = 1 - \frac{D_i(l_i)}{D_i'(l_i)l_i}$$

where $D_i(l_i) = \int_0^{l_i} D_i'(z)dz$ is *i*'s residual damage function, then it is optimal for each firm *i* to bid its true demand curve $P_i(x_i)$ regardless of what other firms bid. This efficient and strategy-proof result is not surprising in that the auction mechanism follows a VCG payoff rule: it makes each firm *i* pay for its (residual) damage $D_i(l_i)$ to all other agents. This residual damage, which is the shaded area in Figure 1, includes both the pecuniary externality imposed upon other regulated firms and the pollution externality imposed upon society.

Figure 1 also helps to see that the auction scheme implements the first-best with each firm facing the same price at the margin (*i.e.*, $p_i = p^*$ for all *i*) and getting exactly the first-best allocation of licenses (*i.e.*, $l_i = x_i^*$): if $\hat{P}_i(x_i) = P_i(x_i)$ and $\hat{P}_{-i}(x_{-i}) = P_{-i}(x_{-i})$, then $l_i = x_i^*$, $l = x^*$ and $\hat{p} = p^*$. Although in principle the regulator goes bidder after bidder determining individual prices p_i, these prices are all the same regardless of how truthful firms are (in terms of Figure 1: $p_1 = ... = p_n = \hat{p}$). But unless firms have identical demand curves, final prices, *i.e.*, $(1-\alpha_i)p$, will differ across firms (in and off equilibrium).

3. EXTENSIONS

This section presents some additional properties of the mechanism, not discused in Montero (2008), and then shows how the mechanism easily accommodates to other externality problems such as those involving non-uniformly mixed pollutants (*i.e.*, firms' pollutants are not perfect substitutes in the damage function) and private externalities.

3.1 Evolution of paybacks

As we increase the number of firms, firm i has virtually no effect on the equilibrium price, so $D_i'(x_i^*) \approx D_i'(0)$ and $\alpha_i(l_i) \approx 0$; hence, the auction scheme has converged to the Pigouvian principle for taxing externalities.

To illustrate how rapidly the auction's payment rule approaches Pigou, let us consider a numerical example. Suppose there are n symmetric firms with linear demand curves. The aggregate demand curve is $P(x) = \overline{p}(1 - x / x^0)$, where $\overline{p}$ is the choke price (*i.e.*, the price at which demand goes to zero) and x^0 is the unregulated level of pollution. The marginal damage function is $D'(x) = hx$. Solving as a function of the number of firms, we obtain

(5) $$\alpha(n) = \frac{1}{2}\frac{\overline{p}}{(n-1)hx^0 + n\overline{p}}$$

If we further let the slopes of the aggregate demand and marginal damage curves be the same (*i.e.*, $\hat{X}_1(p) = l_1^*$), then equation (5) reduces to $\alpha(n) = 1/(4n-2)$, where $n \geq 1$. The rebate for three firms is 10 percent, for ten firms is 2.6 percent, and for 100 firms is less than 0.3 percent.

3.2 Off-equilibrium behavior

If we have a single firm (i = 1) to be regulated and this firm knows $D(x)$, it should be noticed that the firm does not need to truthfully bid its entire demand schedule but only the portion relevant to the auction clearing. It could for instance submit the perfectly inelastic demand schedule around its first-best allocation, *i.e.*, $\hat{X}_1(p) = l_1^*$. In the context of multiple firms (i = 1, …, n), however, it is in each firm's best interest to bid truthfully not only that portion of the demand curve around its first-best allocation x_i^* but rather a large portion of its demand curve. Even if a firm knows $D(x)$, it can no longer anticipate $l_i^* = x_i^*$ with precision because it does not know other firms' demand curves $P_{-i}(x_{-i})$ (it may be even unaware of the number of firms being regulated). To be more precise, a firm will only find it strictly optimal to bid truthfully the portion of its demand curve that is relevant for the auction clearing. Thus, if firms assign zero probability to the event that the clearing price will fall below some value, say $\underline{p}$, firms can just bid an almost perfectly elastic (or inelastic for that matter) demand curve for $p \leq \underline{p}$. While this off-equilibrium behavior has no consequences on the clearing price, and hence, on implementing the first-best allocation, it does have an effect on firms' total payments. But because demand schedules are non-increasing in p, a firm's total payment will never be greater (and generally smaller) than the Pigouvian payment.

3.3 Budget balancing

The auction mechanism is, like any other VCG mechanism, a non-budget-balanced mechanism both in and off equilibrium (unless $\hat{X}_i(p)=0$ for all i). Although there is no efficiency reasons for balancing the budget there may be political economy reasons for doing so (Tietenberg, 2003)[3]. As first pointed out by Groves and Ledyard (1977), if there are at least three agents it should be possible to balance the budget for a variety of mechanisms. The basic idea is to distribute the surplus or deficit generated by each agent ($D_i(l_i)$ in our case) among the other agents in some lump-sum manner as to avoid any incentive effects. Behind this idea lies an implicit "separability" condition that in our case would allow us to either make the payment ($D_i(l_i)$) independent of some firm j 's report (*i.e.*, $\hat{P}_j(x_j)$), as in Duggan and Roberts (2002), or to perfectly disentangle the contribution of each firm $j \neq i$'s report to firm i's payment, as in Varian (1994). By construction, the auction mechanism lacks of such separability; hence, there is no way in which the mechanism can be modified to achieve perfect budget-balancing while retaining its first-best properties[4].

There exists, however, an approximate solution. Building upon the idea of Groves and Ledyard (1977), let denote by $D_j^{-i}(l_j^{-i})$ the total payment that firm j would have hypothetically faced under the same auction mechanism but in the absence of firm i's demand schedule, where l_j^{-i} is the corresponding number of licenses allocated to j. The regulator can thus fashion a lump-sum compensation refunding R_i for firm i using these influence-free hypothetical payments. For example,

$$(6) \qquad R_i = \frac{1}{n-1}\sum_{j\neq i} D_j^{-i}(l_j^{-i}) \text{ where } n \geq 2$$

This solution assures a perfectly balanced budget (*i.e.*, $\Sigma_{i=1}^{n} R_i = \Sigma_{i=1}^{n} D_i(l_i)$) only in the limiting case of a large number of firms; otherwise, ΣR_i could be smaller, greater or equal than $\Sigma D_i(l_i)$. The ratio $\rho \equiv \Sigma R_i / \Sigma D_i(l_i)$ will ultimately depend on the number of firms and shape of the demands and marginal damage curves. For linear curves, for example, it can be shown that for three (symmetric)

[3] We may also want to take into consideration the general equilibrium reasons of Bovenberg and Goulder (1996) for not balancing the budget.

[4] The reason why the mechanisms of Duggan and Roberts (2002) and Varian (1994) can balance the budget is because they are based on discrete announcements by firms. In the former firms announce quantities while in the latter they announce prices. In the auction mechanism firms announce a continuum of quantity-price pairs.

firms ρ can be anywhere between 0.60 and 1.50, for ten firms anywhere between 0.90 and 1.11 and for 100 firms anywhere between 0.99 and 1.01. Thus, a regulator that cannot run a deficit, *i.e.*, constrained to return at most $\sum D_i(l_i)$ to firms, can inform in advance that it will return only some fixed fraction of the total $\sum D_i(l_i)$ (in the case of 10 firms this fraction could be 90 percent).

3.4 Imperfect substitutability of licenses

Consider the case in which social damage is no longer a function of total pollution but, as in Dasgupta *et al.* (1980) and Duggan and Roberts (2002), of the firms' pollution vector. There are $n \geq 2$ firms with (privately known) demand and cost functions $P_i(x_i)$ and $C_i(x_i)$, respectively, where $i = 1, \ldots, n$. Pollution damages are denoted by the differentiable and convex function $D(\mathrm{x})$, where $\mathrm{x} = (x_1, \ldots, x_n)$ is the pollution vector. Without perfect substitutability of pollutants, and hence of licenses, we do not want to insist on a uniform-price auction because it may be socially optimal that each firm faces a different price for licenses at the margin. For the same reason the regulator wants to make licenses to be firm-specific as to prevent any trading of licenses after the auction.

Let $\mathrm{x}^* = (x_1^*, \ldots, x_n^*)$ be the first-best allocation vector (which is interior and unique); then x* satisfies the first-order conditions

$$(7) \qquad -C_i'(x_i^*) \equiv P_i(x_i^*) = \frac{\partial D(\mathrm{x}^*)}{\partial x_i} \text{ for all } i = 1, \ldots, n$$

For the auction mechanism to deliver the first-best allocation, the payment rule identified in Section 2 implies that firm i's residual damage curve as a function of x_i must be

$$(8) \qquad D_i(x_i) \equiv \int_0^{x_i} \frac{\partial D(x_1^*(y), \ldots, x_{i-1}^*(y), y, x_{i+1}^*(y), \ldots, x_n^*(y))}{\partial y} dy$$

where $x_j^*(y)$ is the first-best allocation to firm $j \neq i$ when y licenses are allocated to firm i. It is easy to see that if firm i's total payment is given by (8), the solution to firm i's problem, *i.e.*, find the number of licenses l_i that minimizes $C_i(l_i) + D_i(l_i)$, satisfies the first-order condition (7).

To compute firm i's residual damage curve the auctioneer/regulator will use the bids from the remaining $n - 1$ firms to solve a system of $n - 1$ irst-order conditions

$$(9) \qquad \hat{P}_j(x_j) = \frac{\partial D(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n)}{\partial x_j}$$

for $j = 1, \ldots, n$ and $j \neq i$. Solving the system of equations (9) leads to $n - 1$ functions of the form $x_j^*(x_i)$ for all $j \neq i$. These functions are then entered into $D(x_i, \mathrm{x}_{-i}^*(x_i))$ to finally obtain firm i's residual damage function (8).

Given definition (8), the auction works exactly as before. The regulator clears the auction by determining a price p_i and number licenses l_i for each bidder i according to

$$(10) \quad p_i = \hat{P}_i(l_i) = D_i'(l_i) \equiv \frac{\partial D(l_i, \mathrm{x}_{-i}(l_i))}{\partial l_i}$$

and soon after gives i a rebate of $\alpha_i(l_i)p_i l_i$, where $\alpha_i(l_i) = 1 - D_i(l_i) / l_i D_i'(l_i)$ with $0 \leq \alpha_i(l_i) \leq 1$.

3.5 Private externalities

Consider now the case in which firms not only impose costs on society but also impose costs (or benefits) on other firms. Fishing in open sea and grazing goats in public land are two "commons" examples but the analysis here applies more generally to any private externality problem. There are $n \geq 2$ firms. Firm i's production is denoted by x_i and its (differentiable) profit function by where $\Pi_i(x_1,...,x_i,...,x_n)$ where $\partial\Pi_i(\cdot)/\partial x_i > 0$ and $\partial^2\Pi_i(\cdot)/\partial x_i^2 < 0$. For concreteness, let us focus on the case of pure private negative externalities, *i.e.*, $\partial\Pi_i(\cdot)/\partial x_j < 0$ for all $j \neq i$ (it is relatively straightforward to generalize the scheme to the presence of both social and private externalities).

Let $\mathrm{x}^* = (x_1^*,...,x_n^*)$ be the first-best or joint-profit-maximizing allocation vector (which is interior and unique); then x* satisfies the first-order conditions

$$(11) \quad \frac{\partial\Pi_i(\mathrm{x}^*)}{\partial x_i} + \sum_{j\neq i}\frac{\partial\Pi_j(\mathrm{x}^*)}{\partial x_i} = 0 \text{ for all } i = 1,...,n$$

Had the regulator known the size of the externality exerted by each firm at the first-best level, *i.e.*, $\Sigma_{j\neq i}\partial\Pi_j(\mathrm{x}^*)/\partial x_i$, he would have just charged a Pigouvian tax equal to $\tau_i^* \equiv \Sigma_{j\neq i}\tau_{ij}^* \equiv \Sigma_{j\neq i}\partial\Pi_j(\mathrm{x}^*)/\partial x_i$ to firm i's output, where τ_{ij}^* measures the (first-best) marginal damage that i imposes on j. But since regulators generally do not have such information, Varian (1994) has provided them with the following simple multistage mechanism. First, all firms simultaneously announce the magnitude of the vector of Pigouvian taxes to be faced by each firm (including itself). Then the regulator uses firms' announcements to compute transfers from/to firms as a function of the production vector x. Finally, output x is decided. Varian shows that transfers can be structured in a way that the (unique) subgame-perfect equilibrium of this game is that each firm reports the first-best Pigouvian tax vector and that x = x*.As explained by Varian (1994) in the concluding paragraph of his paper, however, the main problem with this multistage mechanism is that it requires complete information by the firms.

The auction mechanism proposed in this paper does not require firms to possess any such information. It assumes that $\Pi_i(\mathrm{x})$ is firm i's private information. In the specific context of private externalities, the auction mechanism operates as follows. Firms are asked to submit (non-increasing) demand schedules $\hat{P}_i(x_1,...,x_i,...,x_n)$ for $i = 1, \ldots, n$[5]. The regulator/auctioneer uses that information to recover "reported" profit functions

$$(12) \qquad \hat{\Pi}_i(x_i, \mathrm{x}_{-i}) = \int_0^{x_i} \hat{P}_i(x_1,...,x_{i-1},y,x_{i+1},...,x_n)dy$$

which then he uses to compute the residual damage functions as dictated by Proposition 4

$$(13) \qquad D_i(x_i) \equiv \sum_{j\neq i} \hat{\Pi}_j(x_1^{**},...,x_{i-1}^{**},0,x_{i+1}^{**},...,x_n^{**}) - \sum_{j\neq i} \hat{\Pi}_j(x_1^{*}(x_i),...,x_i,...,x_n^{*}(x_i))$$

for all $j = 1, \ldots, n$ and $j \neq i$.

The first sum in (13) is the "reported" first-best profits of all firms but i in the absence of firm i and the second sum is the first-best profits of all firms but i when firm i is allowed to produced $x_i > 0$. As in the basic model, expression (13) tracks down the (first-best) profit losses that the presence of firm i, as measured by x_i, causes on all other agents. Again, it is not difficult to see that if firm i's total payment is given by (13), the solution to firm i's problem, *i.e.*, find the number of licenses $l_i = x_i$ that maximizes $\Pi_i(l_i, \mathrm{x}_{-i}) - D_i(l_i)$, satisfies the first-order condition (11). The computation of functions $x_j^*(x_i)$ for all $j \neq i$ is as in the previous section: the auctioneer will use the bids from the $j \neq i$ firms and solve the $n - 1$ first-order conditions as a function of x_i.

A simple example may help here (to make it more interesting I will allow for corner solutions). Consider two firms 1 and 2 (or j and i) with profit functions $\Pi_i(x_i, x_j) = (\theta_i - x_i - x_j)x_i \geq 0$, where the value of θ_i is firm i's private information. For $\theta_i > \theta_j$ the socially optimal solution is

$$(14) \qquad x_i^* = \frac{\theta_i}{2} \text{ and } x_j^* = 0$$

(and for $\theta_i = \theta_j = \theta$ the efficient solution is $x_i^* + x_j^* = \theta / 2$).

In the absence of regulation, firms will produce beyond this joint-profit maximizing level (we may have a total collapse of the resource in that $\theta_i < x_i + x_j$ for $i = 1, 2$)[6]. The auction mechanism corrects the externalities as follows. Firms are asked to report their demand curves or types to the auctioneer, say $\hat{\theta}_1$ and $\hat{\theta}_2$, knowing beforehand that the regulator/auctioneer will use this information to determine allocations

$$(15) \qquad l_i = \begin{cases} \hat{\theta}_i / 2 & \text{if } \hat{\theta}_i > \hat{\theta}_j \\ 0 & \text{if } \hat{\theta}_i < \hat{\theta}_j \end{cases}$$

[5] Note that in many commons problems these demand schedules will reduce to $\hat{P}_i(x_i, x_{-i})$, where $x_{-i} \equiv \sum_{j\neq i} x_j$.

[6] Suppose that is common information that θ_i 's are i.i.d. over the support $[\underline{\theta}_i, \overline{\theta}_i]$, the Bayesian Nash equilibrium is

$$x_i = \frac{\theta_i}{2} - \frac{2E[\theta_i] - E[\theta_j]}{6}$$

for $i = 1, 2$ and where $E[\cdot]$ is the expected value operator.

and total payments

$$(16) \quad D_i = \begin{cases} \hat{\theta}_j^2 / 4 & \text{if } \hat{\theta}_i > \hat{\theta}_j \\ 0 & \text{if } \hat{\theta}_i < \hat{\theta}_j \end{cases}$$

for $i = 1, 2$. If $\hat{\theta}_i = \hat{\theta}_j = \hat{\theta}$ the regulator flips a coin for deciding who gets the $\hat{\theta}/2$ licenses for a total payment of $\hat{\theta}^2/4$ (we assume that the winning firm opts to produce despite making zero profits).

By letting firm *i* face a payment equal to firm *j*'s (first-best) profits had firm *i* not existed (*i.e.*, $\Pi_j = \theta_j^2/4$), it is in firm *i*'s best interest to submit a truthful bid (*i.e.*, $\hat{\theta}_i = \theta_i$) regardless of what firm *j* bids. This is not surprising since the auction mechanism has collapsed to a single-object second-price auction[7].

REFERENCES

Bovenberg, L. and L. Goulder (1996), "Optimal environmental taxation in the presence of other taxes: general equilibrium analysis", *American Economic Review* 86, 985-1000.

Dasgupta, P., P. Hammond and E. Maskin (1980), "On imperfect information and optimal pollution control", *Review of Economic Studies* 47, 857-860.

Duggan, J. and J. Roberts (2002), "Implementing the efficient allocation of pollution", *American Economic Review* 92, 1070-1078.

Groves, T. and J. Ledyard (1977), "Optimal allocations of public goods: A solution to the free rider problem", *Econometrica* 45, 783-809.

Kwerel, E. (1977), "To tell the truth: Imperfect information and optimal pollution control", *Review of Economic Studies* 44, 595-601.

Montero, J.-P. (2008), "A simple auction mechanism for the optimal allocation of the commons", *American Economic Review*, forthcoming.

Roberts, M. and M. Spence (1976), "Effluent charges and licenses under uncertainty", *Journal of Public Economics* 5, 193-208.

Spulber, D. (1988), "Optimal environmental regulation under asymmetric information", *Journal of Public Economics* 35, 163-181.

Tietenberg, T. (2003), "The tradable-permits approach to protecting the commons: Lessons for climate change", *Oxford Review of Economics Policy* 19, 400-419.

Varian, H. (1994), "A solution to the problem of externalities when agents are well-informed", *American Economic Review* 84, 1278-1293.

[7] Since there are multiple socially optimal solutions for the case in which $\theta_i = \theta_j$, one may be inclined to replace the coin-flipping allocation by a more equitable allocation such as the following: if $\hat{\theta}_i = \hat{\theta}_j = \hat{\theta}$, then $l_i = l_j = \hat{\theta}/4$ and $D_i = D_j = \hat{\theta}^2/8$. This allocation rule still yields a truth-telling Nash equilibrium but no longer in dominant strategies. If, for example, firm *i* believes $\hat{\theta}_j > \theta_i > \theta_j$, it may be optimal for i to bid $\hat{\theta}_i = \hat{\theta}_j > \theta_i$.