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Conceptions of Freedom and Ranking Opportunity Sets – A Typology

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Abstract A wide diversity of rankings of opportunity sets are characterized through what is now commonly called the freedom of choice literature. An opportunity set is better ranked when it provides more freedom. This survey is organized as a typology of the rankings, according to the specific conception of freedom they capture: freedom of choice, freedom as autonomy, freedom as exercise of significant choices, negative freedom. The role of preferences in freedom rankings is discussed in the conclusion.

Keywords opportunity sets, freedom of choice, well-being, typology

Introduction

The literature of ranking opportunity sets (Barberá et al. (2004)), tackles a wide range of problems such as choosing in an uncertain environment or valuing freedom of choice *per se*. This article focuses on the normative side of this literature, now commonly called the freedom of choice literature and in which the range and/or the content of opportunity sets is at stake.

To define an opportunity set, one can represent individual choice as a two stages problem. At the first stage, an individual makes decisions which will constrain the scope of later feasible choices over options. This amounts to picking one set of options among many, or, even, to rank sets of options. At the second stage, the individual chooses one option out of the previous set. Let us say an opportunity set is a restaurant menu: you first choose the restaurant among all available restaurants in the city, then you pick a meal from the chosen restaurant's menu. The aim of the freedom of choice literature is to rank opportunity sets according to some normative criterion such as utility, freedom of choice, specific notions of freedom, or individual overall well-being.

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Let X be the universal set of alternatives. Let $x, y, ... \in X$ be the alternatives (commodities, options, actions...) the agent may be faced with at second stage. Let $\sharp X$ be the cardinality of the set X. As in the traditional microeconomic framework, individuals have preferences over the alternatives, standing for his utility over the options. Let R be a preorder, i.e. a reflexive and transitive binary relation over X. xRy means that the option x is at least considered as good as y according to preferences R. The symmetric and asymmetric part of R are respectively denoted I and P. Let $\Pi(X)$ be the set of all non-empty subsets of X. The elements of $\Pi(X)$, denoted as $A, B, ...$ are the opportunity sets the agent may be faced with in the first stage. Let $\sharp A$ be the cardinality of the set A. Let $\succeq$ be a preorder, i.e. a reflexive and transitive binary relation defined over $\Pi(X)$. For all $A, B \in \Pi(X)$, ($A \succeq B$) is to be interpreted as 'the set A is considered at least equally good than B according to the specific value at stake'. It could mean, for instance: 'A provides at least as much freedom as B'. The symmetric part and assymetric part of $\succeq$ are respectively $\sim$ and $\succ$. In the freedom of choice literature, the preorder $\succeq$ is characterized axiomatically, i.e. the set of basic conditions under which a specific ranking of opportunity sets hold is specified.

This paper is a survey of the freedom of choice literature.[1] In almost every paper of this literature, we find the following interpretation of the characterized rankings: if the opportunity set A is ranked higher than B, A provides more freedom of choice. The diversity of the rankings calls for another look at the actual definition of freedom of choice. The concept of freedom captured by each ranking deserves an explicit definition, rather than the vague 'freedom of choice'. A formal representation of a ranking is attached to a specific notion of freedom. Hence this survey provides a typology of the formal rankings according to their normative interpretation.

As a consequence of this focus on the normative interpretation, I will ignore those rankings which are extensions of each conception of freedom; I will restrict myself to presenting one ranking for each of them. I will especially focus on the results consistent with most standard formal representation in the literature, as in Pattanaik and Xu's seminal paper of 1990; the interpretation of the results based on distinct frameworks (for instance Klemish-Alhert (1993)) will be referred to but will not be given an extensive presentation. Furthermore, some contributions to the freedom of

[1] To my knowledge, there does not exist any specific survey on the freedom of choice literature. For a general survey on the rankings of opportunity sets, see Barberá et al. (2004). For a critical presentation of the freedom of choice literature, see Bavetta (1995), (2004). For a survey of a closely related literature, equal opportunity, see Peragine (1999).

choice literature do not focus specifically on the issue of normative interpretation but rather on the extension of existing results to a different framework, such as the corresponding results in economic context (see Pattanaik and Xu (2000b) or Arlegi et al. (2005)); for this reason, they will not figure in this survey.

It should also be remarked that other interpretations of the rankings discussed here are certainly possible. I do not pretend to close the discussion in this regard.[2] Some authors might even find my interpretation of their ranking controversial: for instance, some authors consider that the idea of preference for freedom is a conception of freedom of choice while, here, it will be taken as an element of global well-being. The attribution of a normative tag to each ranking is intended to be meaningful as a whole to be able to see where the freedom of choice literature is going.

How is the typology organized? It could straightforwardly be based on the corresponding axioms: each ranking is characterized by axioms, which can also be called 'conditions', or 'definitions', which contribute to capture some specific value. A typical presentation of the resulting typology could be summarized through a large table, with axioms in columns and specified rankings in rows. A ranking, characterized by a set of axioms, each of them being associated with some specific and stable interpretation, would be clearly presented. Unfortunately, the freedom of choice literature is not sufficiently unified to build such a coherent table. The problem is that the axioms, which are meant to capture a specific meaning, are formulated in quite different ways in different papers (e.g. the strong monotonicity axiom). Hence a typology of the rankings from the freedom of choice literature cannot be based on axioms but rather on the desired meaning of the axiom, such as the intrinsic importance of a wider of choice (e.g. any formulation of the monotonicity axiom) or the value of utility of achieved choices (e.g. dominance axioms).

The richness of this literature indeed yields to distinctions between different notions of freedom, even different nuances of the concept of freedom of choice. Note I will neither discuss the social aspects of freedom (see Sugden (1985), Carter (2004), Oppenheim (1995), (2004)), nor political liberties or freedom of the will (see Hayek (1960: 11–21)). Instead, I will concentrate on freedom of choice as opportunity (section 2), freedom of choice as autonomy (section 3), freedom as the exercise of choice (section 4), and negative freedom (section 5). As concluding remarks, I will discuss the role of the main axioms used to characterize the rankings (subsection 6.1). Then, challenging the indifference condition, I will imagine

[2] See, for instance, the discussion of the concept of freedom under the cardinal ranking by I. Carter (2004).

what a ranking of opportunity sets based on utility or overall freedom would look like (subsection 2). Finally, I will discuss the special links between preferences and freedom in the freedom of choice literature and I will argue for a strong distinction between two cases: a certain type of freedom ranking, or an individual overall well-being ranking, based on different prudential values among which freedom (subsection 3).

1. Freedom as opportunity: the role of the scope of choice

The 'freedom of choice literature' was initiated by Pattanaik and Xu's (1990) seminal article. They maintained that freedom of choice is captured by the combination of three conditions: indifference to no-choice situations, independence and simple monotonicity.

The first condition is a way to express the value of freedom of choice because it captures the fact that there is choice (Carter 2004: 72). Presenting no-choice situations is likely to capture the idea of no-choice. The idea is that there is no reason why one singleton would provide more choice than another singleton insofar as there is no choice in each. Here the difference between ranking according to choice or freedom of choice (represented by $\succ$) on the one hand and utility (represented by P) on the other is made very clear. If utility was at stake, the two singletons would not be indifferent but their ranking would be based on the preference over the options.

Condition 1 [Indifference between no-choice situations] $\forall x, y \in X$, $\{x\} \sim \{y\}$.

The second condition refers to the intrinsic importance of freedom of choice, which is defined by Carter (1999: 41) as follows: 'One phenomenon X has an intrinsic value if and only if X is one end in itself, i.e., if X has a positive total value which is not reducible with the value of any other phenomenon'. Carter proposes a test to determine whether or not an intrinsic importance to freedom of choice is expressed in some judgment. Let there be two situations S_1 and S_2. S_2 is composed by the options contained in S_1 and eight others. The preferred options are already included in S_1. One eventually must choose only one option. Which of the two sets seem to be more valuable? If I find S_2 more valuable, freedom has an intrinsic importance to me. The options that render the set valuable are not only the preferred options but also the others that are not preferred. I find valuable to be able to push back the nine other options. My 'yes' to one option is meaningful because I was also able to say 'no' to other options; I do choose the kind of life I do wish to live, and eventually reveal my identity, thanks to these nine 'no'. More choice is thus always consid-

ered better than less choice, whatever the options. The valuation of 'more choice' or 'more freedom of choice' *per se* then requires a strict monotonicity condition.

Condition 2. [Strict monotonicity] $\forall x, y \in X, x \neq y, \{x, y\} \succ \{y\}$.

A third condition is necessary to induce a ranking. Pattanaik and Xu propose an independence condition, requiring that if A and B are two possible available sets and if x does not belong either to A or to B, then the ranking of A and B in terms of freedom[3] corresponds to the ranking of $A \cup \{x\}$ and $B \cup \{x\}$.

Condition 3. [Independence] $\forall A, B \in \Pi(X), \forall x \in X (A \cup B)$

$$\left[A \succeq B \text{ if and only if } A \cup \{x\} \succeq B \cup \{x\} \right].$$

According to the cardinal ranking,[4] the more options in a set, the more choice is provided by the set. The scope of choice is simply assessed by counting the number of options contained in the set.

Rule 1. [Cardinal ranking] For all $A, B \in \Pi(X)$, $A \succeq B$ if and only if $\#A \geq \#B$.

Proposition 1. [Pattanaik and Xu 1990] Preorder $\succeq$ satisfies simple monotonicity (condition 2), indifference between no choice situations (condition 1) and independence (condition 3) if and only if $\succeq$ is the cardinal ranking.

The cardinal ranking rule is characterized by conditions capturing the ideas of freedom, choice and freedom of choice. Nevertheless, the simplicity of this result has given rise to much debate. Several papers, written in direct reaction to Pattanaik and Xu (1990)'s proposition, had considerable importance renewing debates about freedom: what it means, how to represent it formally, and what relevant elements should be taken into account. Puppe (1995) raised the problem of incompatibility of valuing utility – as experience for preference satisfaction – and freedom – as opportunities for preference satisfaction – which generated a discussion about the role of

[3] The fact that this version of the independence condition captures the idea of freedom of choice is controversial. See Carter (2004).

[4] For a presentation and justification of this rule, see Jones et Sugden (1982), Sen (1985), and van Hees (1998).

utility (see also Gravel (1994), (1998)). This discussion lead to the consideration of reasonable preferences by the valuation of autonomy (see section 3), and to the literature on global well-being based on several prudential values (see subsection 3). In addition, Klemish-Alhert (1993) brought to light the importance of diversity in valuing choice, which lead to the development of several frameworks likely to capture the valuation of the exercise of significant choices (see section 4). As we notice, the literature generated by Pattanaik and Xu's (1990) paper tackles different issues, wider than just freedom of choice. While they introduced a simple way to represent freedom of choice as a pure opportunity concept, the so-called 'freedom of choice literature' has spread in diverse directions to include other conceptions of freedom such as freedom as autonomy, exercice or negative freedom, as well as including diverse conceptions of global well-being, in which freedom and utility are ingredients of the overall judgement.

2. Freedom as autonomy: the role of reasonable preferences

An individual is *autonomous* if she makes choices entirely according to her will, meaning her choices are independent of her conditionings or the will of other people.

Jones and Sugden (1982) point to a tension between the economic theory of choice according to which the preferences are given, such as the standard framework of microeconomic theory and the valuation of autonomy. Take the standard representation of choice in economics, which relies on the strong model of preferences (Haslett 1990). Whenever i chooses x rather than y among the opportunity set $\{x, y\}$, he reveals his preference for x over y. For i, x is better than y. Imagine now a second case: i is presented with opportunity set and is forced to have x by force, coercion, pity, etc., but in any case, without even asking i what he desired. i derives the exact same satisfaction in both cases since he will anyways get his preferred option. Jones and Sugden (1982: 59) propose to challenge the assumption of given preferences to avoid this annoying consequence:

> To suppose that the act of choice requires the exercise of mental powers is to suppose that the chooser is in some considerable measure an autonomous agent; whatever he chooses, he might have chosen something else. There is a tension between this assumption and the idea implied in the economic theory of choice, that preferences are given. What makes significant choice possible is that preferences are not just part of a person's physiology or psychology like the color of his eyes or a tendency to depression. […] The concept of significant choice can best be understood by considering the various preferences that a person might have, rather than merely the preferences that he actually reveals when he finally makes a choice. Preferences only catch the idea of the

satisfaction we get from different options, whatever chosen or imposed. Preferences are therefore insensitive to the idea of autonomy. In the example, *i* has definitely more autonomy in the first case than in the second case. His choice is indeed autonomous if he chooses something while he could have chosen something else; he is autonomous if his preference relation could actually have been different. Accounting for other possible preferences excludes cases where he is obliged to get his preferred option. Therefore, the introduction of a wide scope of possible preferences in the assessment of opportunity sets, which we will now call 'reasonable preferences', is necessary to capture the idea of autonomy: the wider the range of reasonably chosen options[5] from an opportunity set is, the more autonomous the person who faces it is.

Following Jones and Sugden (1982) and Puppe (1998), the options that are likely to be chosen, according to reasonable preferences, will be called essential options. Denote $E(A)$ as the subset of essential alternatives from the set A. This also means that an option is essential if extracting it from the set strictly decreases the freedom derived from a set:

$$E(A) := \{x \in A : A \succ A \setminus \{x\}\}.$$

An option is ineligible if it could never be chosen, e.g. by any reasonable preference. The fact that certain option may never be chosen has an important consequence for the formulation of monotonicity condition. If you add another option to a set, you can no longer believe that autonomy is increased no matter what the option is. However, adding an option will never *decrease* the autonomy provided by the set if it is ineligible. Hence, while a strict monotonicity condition is appealing for freedom of choice, it is less so for the autonomy ranking, which only needs to respect a weak monotonicity condition.

Condition 4. [Weak monotonicity] For all $x, y \in X, \{x, y\} \succeq \{x\}$.

The weak monotonicity condition is actually a very significant weakening, because all that is necessary here is a 'conditional' monotonicity: the new option is likely to raise autonomy in the only case in which it is likely to be chosen. Indeed, the kind of additional alternatives that will increase

[5] Note that it is equivalent, in the opportunity set framework, to think in terms of possibly chosen option in a set or in terms of possible preference. As a matter of fact, all options are considered initially before picking one single option (from the set), which is the best element according to a preference relation. If there are several possible preference relations, there might be several possible chosen options.

the autonomy has to be specified. Another monotonicity condition, called I-monotonicity and proposed by Pattanaik and Xu (1998), captures the idea according to which only essential alternatives may increase the autonomy provided by an opportunity set. Considering all reasonable preferences to identify the best elements, for all $x \in X$ and all $A \in Z$, we write $x[I]A$ if and only if $max(A \cup \{x\}) = A \cup \{x\}$; $x[P]A$ if and only if $\{x\} = \max(A \cup \{x\})$; $A[P]x$ if and only if $x \notin \max(A \cup \{x\})$.

Condition 5. [I-monotonicity] For all $A, B \in \Pi(X)$, and for all $x \in X \setminus A$, $(x[I]A \text{ and } A \succeq B)$ implies $[A \cup \{x\} \succ B]$.

The assessment of autonomy will be different according to the breadth of possible preferences. But now, we need to distinguish between the nature of restrictions of the set of possible preferences to a set of reasonable preferences on the one hand, and the design of such restrictions on the other.

2.1 *All rational preferences are reasonable: Valuing the scope of choice*

At the one extreme, all possible rational preference may be considered as reasonable. Even very odd and eccentric preferences should be accepted as reasonable. J. S. Mill (1859: 83) for instance would defend this idea according to which there is no justification to restrict the scope of reasonable preferences. Therefore, each option from a set is essential, no matter which it is. The required monotonicity condition here is exactly equivalent to strict monotonicity. We are then brought back to cardinal ranking and Pattanaik and Xu (1990)'s result.

2.2 *Just one preference ordering is reasonable: Valuing indirect utility*

At the other extreme, if just one preference is considered as reasonable, the actual individual preference, then we are back to the traditional rule based on the comparison of indirect utilities. Hence,

Rule 2. For all $A, B \in \Pi(X)$, $A \succeq B$ if an only if $[\max(A) R \max(B)]$.

2.3 *Some preferences are reasonable, some are not: Valuing autonomy*

Restrictions on the family of acceptable preferences may be justified. For instance, 'no reasonable person may ever consider that being beheaded at dawn' is an acceptable restriction. Although to my knowledge not much in the freedom of choice literature describes formally different *objects* of preferences, such as this counter example suggests – the choice of being

killed is indeed not an object of reasonable desire. The focus is rather on the *design of the restriction*, specifying whether or not the fact that an option is essential is given. In other words, these autonomy rankings rely on information on the best or worse elements of reasonable preferences. An intuitive rule based on the cardinality of essential options represents a mere conception of autonomy, focusing on essential options. An annoying counter example generated by this previous rule is avoided when ineligible options, or a wider range of elements than just essential options, are taken into account.

2.3.1 *Simple cardinality of essential options*

If, for all reasonable preferences, x is strictly worse than at least one option from A, e.g if x is ineligible, then adding x to A does not raise autonomy.

Condition 6. [Type 1 irrelevance of dominated alternatives] For all $A,B \in \Pi(X)$ and for all $x \in X$, if $A[P]x$, then $[A \succeq B$ if and only if $A \cup \{x\} \succeq B]$ and $[B \succeq A$ if and only if $B \succeq A \cup \{x\}]$.

The basic composition condition, corresponding to an extension of the independence condition to inclusions of any sets rather than a singleton, is the following: for all $A,B,C \in \Pi(X)$, we have : $A \sim A \cup B$ implies that, for all $C, A \cup C \sim A \cup B \cup C$. If $A \succeq B$ and $C \succeq D$, we could expect that $A \cup C \succeq B \cup D$. This formulation is not used in this setting for it is likely to induce counter-examples. If A and C share too numerous common elements, it is possible than $B \cup D$ provides less freedom nevertheless. The composition condition proposed by Sen (1991) is designed to avoid this annoying consequence : we further assume that the sets do not have common elements, e.g. $A \cap C = B \cap D = \varnothing$. But even this new formulation is likely to induce counter-examples in this settings for some alternatives from the sets could be ineligible. The specific composition condition proposed by Pattanaik and Xu (1998) requires then that each distinct alternative from a set is an essential alternative.

Condition 7. [Composition] For all $A,B,C,D \in \Pi(X)$, such that

$(A \cap C = B \cap D = \varnothing$, *and* $\max(A \cup C) = A \cup C$, *and* $\max(B \cup D) = B \cup D)$.

$[A \succeq B$ and $C \succeq D]$ implies $[A \cup C \succeq B \cup D]$, and $[A \succeq B$ and $C \succ D]$ implies $[A \cup C \succ B \cup D]$.

Cardinal ranking of essential alternatives compares opportunity sets according to the number of essential alternatives in the sets.

Rule 3. [Cardinal ranking of essential alternatives] For all $A,B \in \Pi(X)$, $\left[A \succeq B \text{ if and only if } \#\max(A) \geq \#\max(B)\right]$, where $\#\max(A)$ stands for the number of time a reasonable preference considers an element from A is optimal.

Proposition 2. [Pattanaik and Xu 1998] The relation $\succeq$ satisfies indifference between no choice situations (condition 1), I-monotonicity (condition 5), type 1 irrelevance of dominated alternatives (condition 6), and composition (condition 7) if and only if $\succeq$ is the cardinal ranking of essential alternatives.

Example 1. Consider the following example. Take two sets. $A=\{x,y\}$ and $B=\{z,w\}$ and two possible reasonable orderings of preferences: $xPyPzPw$ and $yP'xP'wP'z$. The set of essential elements from A is therefore $\{x,y\}$ and from B is $\{z,w\}$. In both cases, the cardinality of essential elements is the same, which induces that they provide exactly the same autonomy according to the rule 3. It is quite difficult to swallow when considering than a reasonable person would never choose an option from B if he was given the choice to pick one from A as well. Intuitively, the fact that some elements are more likely to be chosen is important, not just the fact that they are likely to be chosen in some set. Therefore, possessing more essential elements is not necessarily better; counting essential elements is not sufficient to capture the idea of "more autonomy" because it does not consider relevant information on the reasonable preferences.

2.3.2 *Weighted cardinality rule of essential options*

Another form of restriction consists in taking into account the role of ineligible actions in the set.

We first formulate dominance and non-dominance conditions in a similar framework.

Condition 8. [Simple non-dominance] For all $x,y \in X$, if $x[I]y$, then $\{x\} \sim \{y\}$.

Condition 9. [Simple dominance] For all $x,y \in X$, if $x[P]y$, then $\{x\} \succ \{y\}$.

If, for all reasonable preferences, x is strictly worse than at least one option from A, i.e. if x is ineligible, then the status of the set $B \cap \{x\}$ vis-à-vis A should not be better nor worse that the status of B vis-à-vis A.

Condition 10. [Type 2 irrelevance of dominated alternatives] For all

$A,B \in \Pi(X)$ and for all $x \in X$, if $A[P]x$, then $[A \succeq B$ if and only if $A \succeq B \cup \{x\}]$ and $[B \succeq A$ if and only if $B \cup \{x\} \succeq A]$.

A weaker composition condition adds some further conditions to the composition condition 7: every alternative from $A \cap D$ can be considered by a reasonable person to be at least as good as all alternatives in $A \cap D$, and every alternative in $B \cap C$ can be considered by a reasonable person to be at least as good as all alternatives in $B \cap C$.

Condition 11. [Weak composition] For all $A,B,C,D \in \Pi(X)$, such that

$(A \cap C = B \cap D = \varnothing$, *and* $\max(A \cup C) = A \cup C$, *and* $\max(A \cup D) = A \cup D$,
and $\max(B \cup C) = B \cup C$, *and* $\max(B \cup D) = B \cup D)$,

$[A \succeq B$ and $C \succeq D]$ implies $[A \cup C \succeq A \cup D]$, and $[A \succeq B$ and $C \succ D]$ implies $[A \cup C \succ B \cup D]$.

Let A^B refers for all elements $a \in A$ such that reasonable person may never consider a to be at least as good as all the elements of B. Of course, these elements are not necessarily identical with B^A. Pattanaik and Xu (1998) characterize a weighted cardinality rule of essential options. In this ranking of different sets, what is at stake is not just the number of essential alternatives in each sets but also the fact that these sets lead to *more* valuable choices, which rules out the paradox raised in example 1.

Rule 4. [Weighted cardinal ranking of essential alternatives] For all

$A,B \in \Pi(X), \left[A \succeq B \text{ if and only if } \#[\max(A) - A^B] \geq [\#\max(B) - B^A]\right]$.

Proposition 3. [Pattanaik et Xu 1998] The relation $\succeq$ satisfies simple dominance (condition 9), simple non-dominance (condition 8), I-monotonicity (condition 5), type 1 irrelevance of dominated alternatives (condition 6), type 2 irrelevance of dominated alternatives (condition 10) and weak composition (condition 11) if and only if $\succeq$ is the weighted cardinal ranking of essential alternatives.

2.3.3 *Lexicographic version of the cardinality rule*

Another way to avoid the problem raised by example 1 is given by the lexicographic approach of the cardinal ranking. Romero-Medina (2001: 185) proposes to 'sequentially remove the first element in all the reasonable persons' preferences and compare the available sets of newspapers according to this new set of preferences. This new reference set where alternatives are sequentially eliminated […] is a compromised idea of

freedom that can only be justified when the set of reasonable preferences cannot discriminate'.

3. Freedom as the exercise of significant choices: the role of diversity

The independence condition (see condition 3) used in the cardinal ranking (rule 1) is likely to raise some problems, well illustrated by this example (Pattanaik and Xu 1990: 389).

Let's take the following universal set of options: {train, car 1, car 2}. By indifference, we get: {train} ~ {car 1}. By independence, we then get: {train, car 2} ~ {car 1, car 2}, meaning that these two sets offers exactly the same freedom of choice. Appealing to intuition, this result raises substantial problems. The first set allows for a choice among a wide scope of diverse transportation modes. The choice of options among the first set is then harder and more substantial because choosing train rather than car expresses the kind of life you decided to lead. In the second set, the act of choosing just exercises on the kind of car, e.g. the color of the car, which is less significant. If we imagine that this second car is exactly similar in brand, type, series, color..., this result is even more annoying. It comes into conflict with the idea of valuing significant choices. To express this idea, the independence condition has to be weakened so that diversity or similarity of options are now taken into account.

To speak of significant choice refers to two kinds of situations: according to Jones and Sugden (1982: 57), 'an option is not significant in relation to a choice set if it is either indistinguishable from another option or ineligible.' A choice between indistinguishable options is not significant because one may as well pick and not choose (Ullmann-Margalit and Morgenbesser 1977). The more diverse the options in the set, the more significant is the choice. Indeed, diversity induces a harder choice, meaning the exercise of choice is more meaningful. J. S. Mill values significant choices and, through it, the exercise of choice in itself, whatever it happens to be. It is through the act of choice that human faculties develop: 'The human faculties of perception, judgment, discriminative feeling, mental activity, and even moral preference, are exercised only in making a choice. [...] The mental and moral, like the muscular powers, are improved only by being used' (Mill (1859: 74–75)). And these faculties are one of the elements of well-being (Mill 1859: 72).

The issue at stake is therefore how to introduce the notion of similarity or diversity of options. At the very least this requires some more information about the options. The only required information concerns whether or not options are similar, or, at most, how diverse they are. This will affect the very definition of monotonicity, in the same way that the account for reasonable preferences affected autonomy orderings, as well as the formu-

lation of indifference as well.

3.1 *Extreme alternatives*

A first way to take into account diversity is to focus on the most extreme alternatives in the opportunity set.

Klemish-Alhert (1993) represents opportunity sets as convex hulls in the space of universal set of goods. The conditions used to compare sets are re-expressed in this framework. Convex hull monotonicity for instance is a condition that permits us to value the scope of alternatives in the hull. This hull will be larger if its boundary alternatives will be far from one another.

Rosenbaum (2000) assesses freedom from the scope of choices according to certain characteristics, requiring a wide amount of information at this point. Formally, freedom, as the exercise of significant choices, is a function of the mathematical distance between extreme points in the space of characteristics. The more distinct these characteristics, the more free the individual facing the set. If some characteristic reveal to be more important than others, this unequal importance is expressed by unequal weight for this characteristic.

We are then led to focus on the extreme positions. Klemish-Alhert (1993: 197) finds this result controversial because it is likely to induce undesirable consequences. For instance, enjoying freedom in a country where it is just allowed to express extreme opinions will be considered, in this framework, as more valuable than in any country where all opinions can be expressed except extreme ones. It is hard to swallow that it is more significant to choose between extreme opinions than between a diversity of mild opinions. To avoid these counter-intuitive consequences, it seems interesting to take into account how each option contribute to the diversity of the set, rather than focusing on extreme options.

3.2 *Binary judgements of similarity*

Diversity or similarity of options can be just a question of binary judgments : either two options are similar, either they are not.

Pattanaik and Xu (2000a) introduce on a similarity relation, written S, and defined over X is reflexive and symmetric (but not transitive). We read xSy as: 'x is similar to y' and $\neg xSy$ as: 'x is not similar to y'. For all $A \in \Pi(X)$, we say that A is homogeneous if and only if, for all $a,a' \in A$, $a'Sa$. For all $A \in \Pi(X)$, a similarity based partition of A is defined as a class $\{A_1,...,A_m\}$ such that: (1) $A_1,...,A_m$ are all non-empty subsets of A; (2) $A_1 \cup ... \cup A_m = A$; (3) $A_A,...,A_m$ are pairwise disjoint; and (4) for all $k \in \{1,\ldots;,m\}$, A_k is homogeneous. The similarity based parti-

tion will be denoted by $\varphi(A), \varphi'(A), \varphi"(A)$, etc.

Monotonicity and composition are re-formulated in this new setting.

Condition 12. [S-monotonicity] For all $A \in \Pi(X)$ such that A just contains similar alternatives according to S, and, for all $x \in X \setminus A$, $[xSA \Rightarrow A \cup \{x\} \sim A]$ and $[\neg xSA \Rightarrow A \cup \{x\} \succ A]$.

Condition 13. [S-composition] For all $A, B, C, D \in \Pi(X)$, if $[A \cup C = B \cup D = \varnothing$, C and D contain similar alternatives, and C is not similar to $A]$, then $[(A \succeq B$ and $C \succeq D)$ implies $A \cup C \succeq B \cup D]$ and $[(A \succ B$ and $C \succ D)$ implies $A \cup C \succ B \cup D]$.

Under the simple similarity based rule, opportunity sets are ranked according to the cardinalities of their smallest similarity-based partitions.

Rule 5. [Simple similarity based ordering] For all $A, B \in \Pi(X)$, $A \succeq_{\#S} B$ if and only if $\#\varphi(A) \geq \#\varphi(B)$.

Proposition 4. [Pattanaik and Xu 2000a] $\succeq$ satisfies indifference between no-choice situations (condition 1), S-monotonicity (condition 12) and S-composition (condition 13) if and only if $\succeq$ is the simple similarity based ordering.

This result answers the objection raised against the cardinal ordering, in which freedom of choice is growing even in the case clones are added to existing alternatives. But a different conception of freedom appears then. The repairing of the cardinality ordering means that we move from one conception of freedom, as a mere and vague conception of opportunity, to another conception of freedom, as the valuation of the exercise of significant choices.

3.3 *Approximations of similarity*

It might be difficult to definitely hold that 'alternatives are similar' or 'they are not'. Bavetta and Del Seta (2001) use the concept of rough approximations of the sets. They consider, in the universal set of options, first, the inner (or lower) set of options that is just composed by similar options, and second, the outer (or upper) set of options, whose complement does not include any options that are similar to options from the set. This induces two kinds of possible orderings, based on the cardinality of each of these rough approximations of the sets.

3.4 *Assessing degrees of similarity*

The next step is to take into account degrees of similarity rather than binary judgements about similarity. Van Hees (2004) raises a problem linked with this ambition. He establishes that the characterization of similarity based orderings is often impossible because of the definition of distance between alternatives and sets.

Bervoets and Gravel (2003) propose an ordering of opportunity sets based on an ordinal notion of diversity, while Bossert et al. (2003) propose a cardinal approach of diversity. The study of bio-diversity, by Nehring and Puppe (2002), illustrates another way to take into account the problem of diversity, considering the diversity of attributes characterizing options.

Lastly, Baujard and Gaspart (2006) consider the issue of diversity in the framework of equal opportunity. They capture the idea of valuing the exercise of 'hard choice' through a specific trade-off between utility and choice. The role of diversity between the alternatives is taken into account through a trigger value where utility and diversity seem indifferent.

4. Freedom as negative freedom: the role of identifying constraints

Freedom is about the absence of constraints. The differences between the conceptions of freedom are due to the focus on different kinds of constraints. 'Negative freedom' refers to the absence of coercion (see Berlin 1969). As a relational concept, freedom is defined relative to the constraints that other individuals impose on individual choices. It is a space in which an individual can act and choose without being prevented by others. In this sense, freedom, or unfreedom, does not depend on personal abilities to do what we want, but on other people's interference, coercion, or oppression. Measuring negative freedom supposes to measure the absence of this kind of constraints, rather than any other constraints.

If freedom were to be defined only by this criterion, as soon as there is no coercion in both situations, the two following situations would be indifferent: there are many available options in the first one, just one option is available in the second one. Yet, freedom is greater when non-prevented opportunities are more numerous. Measuring negative freedom therefore has then two aspects: the identification of the origin of constraints and the situation of opportunity.

Constraints may be of different kinds. An action is an opportunity if it is doable. But the reason why it is doable is not just that there is no coercion against it realization. Twenty years ago, people could not phone from the top of the Mont-Blanc. This opportunity did not exist for technical reason. This non-existence did not affect negative freedom but just a tech-

nical ability. It is necessary to distinguish between different kinds of constraints and focus on the negative aspect of freedom when assessing opportunity sets.

Following van Hees (1998), an opportunity situation consists of the ordered pair of a feasible set and an opportunity set, (A,G), describing the set of actions A doable from the fact that there exists an adapted technology, and the opportunity set G composed by actions that nobody is preventing us from doing by coercion, and that we will call by the general term 'external constraints'.

4.1 *1. External constraints and negative freedom*

In a first approach, we focus on the external constraints, considering all actions as possible, as far as technological possibilities are concerned. The basic conditions to capture the idea of freedom are then re-formulated in the framework of opportunity situations.

Condition 14. [Strict monotonicity]
$\forall x, y \in A, x \neq y, (A,\{x,y\}) \succ (A,\{y\})$.

Condition 15. [Indifference between no-choice situations]
$\forall x, y \in A, (A,\{x\}) \sim (A,\{y\})$.

The next condition captures the role of external constraints in the definition of negative freedom. A person's negative freedom depends on the things she is not free to choose. A situation in which an option has become technologically feasible and can be chosen without constraints yields the same amount of freedom as the one in which it was not yet feasible. In other words, as long as there are no further external constraints, there is no reason why negative freedom should change.

Condition 16. [Immunity to opportunities deriving from new technology] $\forall G \in \Pi(X)$ and $\forall x \in X : (A,G) \sim (A \cup \{x\}, G \cup \{x\})$.

Rule 6. [Constraints-based cardinality ordering] $(A,G) \succeq (B,F)$ if and only if $\#(B-F) \geq \#(A-G)$.

According to the constraints based cardinality quasi-ordering, the less external constraints to realize technically possible actions, the more a person is free, when freedom is taken as negative freedom.

Proposition 5. [Van Hees 1998] Preorder $\succeq$ satisfies indifference between no-choice situations (condition 15), strict monotonicity (condition

14), and immunity to opportunities deriving from new technology (condition 16) if and only if $\succeq$ is the constraints based cardinality quasi-ordering.

4.2 *2. Technological constraints and negative freedom*

Even though technological constraints may be independent from external constraints, they may affect the actual negative freedom derived from an opportunity situations. As a matter of fact, if some new technology now exists but if it does not change the opportunity set, then the freedom derived from the set is reduced. Innovations are likely to increase freedom to the only condition that it is available to the person.

Condition 17. [Decreasing with new technology] $\forall G \in \Pi(X)$ and $\forall x \in X - A : (A,G) \succ (A \cup \{x\},G)$.

As in condition 16, both the feasible set and the opportunity set is changing. The opportunity situation resulting from the combination of two should be ranked exactly between the two situations of which it is the combination. This captures the idea that an increase in available options does not necessarily outweigh the increase in the number of forbidden options.

Condition 18. [Independence with variable technology] If A and B are disjoint, then

$$\forall G,F \in \Pi(X) : (A,G) \sim (B,F) \Rightarrow (A,G) \sim (A \cup B, G \cup F) \sim (B,F)$$
$$\text{and } (A,G) \succ (B,F) \Rightarrow (A,G) \succ (A \cup B, G \cup F) \succ (B,F).$$

Condition 19 generalizes indifference between no-choice situations by considering both the feasible set and the opportunity set rather than just the allowed alternatives.

Condition 19. [Neutrality to any permutation] $\forall G \in \Pi(X)$ and for any permutation π of X, $(A,G) \sim (\pi A, \pi G)$, where πA and πG stand for the images of A and G under π, respectively.

At this point, van Hees considers the ordering previously proposed by Steiner (1983), which depends, not any more on the difference, but on the ratio between opportunity set cardinality sets of doable actions.

Rule 7. [Steiner's ordering] $(A,E) \succeq (B,F)$ iff $\#E/\#A \geq \#F/\#B$.

When technological innovations increase opportunity set, freedom is increasing.

Proposition 6. [Van Hees 1998] Preorder $\succeq$ satisfies decreasing with new technology (condition 17), independence with variable technology (condition 18), and neutrality to any permutation (condition 19) if and only if $\succeq$ is the Steiner's ordering.

5. Concluding remarks on the freedom of choice literature

5.1 *Freedom rankings*

I have presented rankings of opportunity sets according to different notions of freedom: freedom of choice, freedom as autonomy, freedom as the valuation of exercise of choice, and negative freedom. It is clearly evident that there is no unity in the freedom of choice literature. In particular, it can be seen that the axioms do not necessarily belong to a single concept of freedom and this is a source of confusion.

It is nevertheless interesting to see what role the axioms play in our conception of freedom. Pattanaik and Xu's (1990) seminal paper, for instance, captures some confusing idea of freedom of choice by valuing the scope of choice, where indifference between no choice situations, strict monotonicity and independence hold. Most contributions are then considered as a discussion of this first framework. When strict monotonicity is questioned and replaced by some conditional monotonicity, that pays attention to reasonable preferences rather than actual preferences, *autonomy* is at stake. When independence is questioned, it becomes possible to take into account diversity and the *value of significant choices per se*. When indifference between no choice situations is questioned and related to distinct kinds of constraints, we capture the idea of *negative freedom*.

5.2 *Open tracks for other types of rankings according to one value*

The indifference condition, that seemed relevant for freedom of choice *per se* raises a disturbing problem. With indifference between no choice situations, we should place : {education} ~ {a glass of champagne}. Yet, the first set (or option) leads to many different, valuable, and lasting opportunities – such as jobs and consumption... The second set, in contrast, provides a short pleasure and does not open up many other opportunities. In other words, to speak about freedom in a more intuitive meaning, it is important to give up the assumption, imposed by condition 1 of indifference between no-choice situations, according to which options are considered to be homogeneous.

There are two ways to differentiate between options. The first consists in considering the actual utility or welfare provided by each option (see Sen (1990: 470)). In this case, the obtained ranking captures the idea of utility or global well-being. The second way is to count the actual number of doable actions for an individual in a given situation, which leads to a conception of overall freedom (discussed below).

How can the idea of utility in an ranking of opportunity set be captured? This is certainly not straightforward. According to a subjective conception of utility, or according to the experience requirement (Glover (1977), Haslett (1990)), an individual should experience the use of an option to derive utility from it. Yet from an opportunity set, he will just experience one option, the one he has chosen among all. If this model of utility is retained, this means that an opportunity set is not an object of utility in itself, it can just be the object of indirect utility. If the criterion to rank the sets is utility, what we will be studying is the indirect utility of opportunity sets. In this welfarist point of view, choice has no other intrinsic value but that of the chosen option. Comparing two sets amounts to comparing the utility of the sets, which is captured by the dominance condition:

Condition 20. [Simple dominance] For all $x, y \in X$, if xPy, then $\{x\} \succ \{y\}$.

Besides, weak monotonicity (condition 4) has to apply. The indirect utility derived from the set will depend on the satisfaction the chosen option will provide. We then have to imagine which option might be chosen from the set. The description of the ranking will depend on the relevant preference relation: the actual preference relation when it is known and posted; the probable preference relations when they are flexible. When preferences are given, the ranking of best elements capture the idea of utility of an opportunity sets. When preferences are risky, there is no unique individual preference relation. Individual preferences depend on a parameter whose probability distribution is well-known. Arrow (1995) proposes an ordering based on this principle.[6] When preferences are uncertain, Kreps (1979) proposes a ranking that captures the idea of preference for flexibility, by valuing indirect utility (Arlegi and Nieto 2001a, Arlegi and Nieto 2001b). The maximal elements of the sets according to each preference (or utility function) is the only important information to compare sets. Of course, if the possible preferences are now interpreted as

[6] It is interesting to note there is no need here to use a different framework than the traditional microeconomic framework for utility.

reasonable preferences an individual may have – rather that actual preferences an individual is likely to have – the right interpretation of the obtained ordering does not concern flexibility and indirect utility but autonomy, as we have seen above. Thus to capture the idea of utility of opportunity sets, at least weak monotonicity and dominance should hold.

The second way to differentiate between options consists in considering that freedom derives from the number of actions that are eventually possible to achieve once all constraints and interaction have been taken into account (Carter 1999): the idea of overall freedom is expressed in this case, i.e. 'how free people are *tout court*' (Carter (1999: 14)). The substantial part of freedom stands in the actual possibility or impossibility to do the things. On this regards, it is important to incorporate the impact of *all* constraints on the ability to take advantages of the options. Firstly, people may not be prevented by anyone to buy some expensive jewelry, but most of them could never afford it, even though they would be willing to invest their whole budget in it. As it seems now important to describe the system of constraints, whatever their nature, that limit the scope of doable actions, we are required to give up the independence condition. Secondly, there also exist some constraints at the collective level on individual actions. 'McEnroe and Becker playing together can both win, but not all together; they do not have the collective freedom to both win the match' (Carter 1999: 258). It is indeed important to take into account the effect of the system of individual interactions on the freedom enjoyed by each individual. Thus overall freedom could be captured in ranking opportunity sets with some alteration of indifference between no choice situation and independence conditions, and by considering the system of interactions with other people.

5.3 *Freedom and preferences*

I now turn to a discussion of the special links between preferences and freedom in the freedom of choice literature. I claim that the role of preferences is distinct when we take into account just reasonable preferences or actual utility.

Firstly, the introduction of reasonable preferences induces a certain type of freedom ranking as we have seen above, interpreted as freedom as autonomy. Note that the ignorance of the actual individual preferences was required to be able to capture the idea of autonomy in the rankings. The introduction of preferences is then a way to define the idea of freedom.

Secondly, information about actual individual preferences (given and stable, or flexible) is introduced in the framework. The interpretation of the resulting ranking will obviously be very distinct. Freedom should then

be considered as another element of the rankings, while it has been considered as the key element in the previous sections. I claim that it induces rankings that capture individual overall well-being, based on different prudential values among which freedom is just an input, just as utility. In these rankings, utility can be interpreted with the experience model in the sense of indirect utility actually derived from the chosen objects from the set; freedom is considered mostly as a basic concept of freedom of choice. Global well-being, in contrast, is based on these two values, utility and freedom. Therefore, what is commonly called preferences and linked with utility would just be an ingredient in addition to freedom to induce some individual well-being. In a wide welfarist point of view, there is no reason why all these ingredients should not be relevant for well-being.[7] There also could be others, such as flourishing for instance[8] but the rankings of opportunity sets we know of focus on these two prudential values: freedom and utility.

The first attempts to introduce utility in the freedom of choice literature came up against impossibilities (Puppe (1995), Gravel (1994), Gravel (1998)). To be able to account for utility in ranking opportunity sets, as we have seen above, we must adopt a weak, rather than the strong version of the monotonicity condition. In contrast, valuing the intrinsic value of freedom requires a strict monotonicity. It is, therefore, quite straightforward that one cannot value utility and freedom at the same time.

As utility and freedom are distinct values to be taken into account in a single ranking, it is then necessary to think of ways to make these commensurable. Bossert et al. (1994) for instance provide three distinct rankings: one is a lexical order of preferences over freedom; the other one a lexical order of freedom over preferences; and, when both values are given an equal importance, the latter ranking reveals to be very incomplete. Baujard and Gaspart (2006), in a economic environment framework, have used the notion of discontinuity introduced by Griffin (1986: 85-89) to capture the idea of a specific commensurability between utility and freedom, taken as the value of the exercice of significant hard choices.

Another way to introduce freedom in global well-being rankings is to consider, in a *wide* welfarist manner, that well-being result from some trade-off between freedom and utility. Puppe (1996) considers the role of freedom through the options of opportunity sets, but he restricts his at-

[7] A confusion can be made in that that the resulting well-being may be called utility in certain context, when utility is taken as a wide preference concept of what is good for the person. To avoid the risk of confusion, we here chose to use distinct word to call these two concepts : the experience of a subjective mental state is called utility, while the judgement of what is good for a person according to herself is called well-being.

[8] See Griffin (1986) for a thorough discussion of well-being and its determinants.

tention to essential alternatives, those that contribute to utility. In the Puppe's rankings, the importance of freedom does not depend on how it adds up to utility but rather on how it contributes to utility. Utility is, therefore, all what eventually matters. Bossert (1997) considers that well-being derives from freedom of choice and utility. He proposes a family of rankings, in which utilities from each option contribute to the overall result. With certain assumptions, it is even possible to assign a specific weight to each option according to their desirability.

Thus, utility and freedom may contribute to some overall well-being, that might be represented by suitable preferences. It is though important to keep in mind the actual interpretation of the resulting preferences, distinct from the preferences standing for utility only. As I have noted above, there is clearly no unity in the freedom of choice literature: some part is devoted to capture some specific conceptions of freedom – which was the subject of this paper – but some other part is focused on the problem of global well-being, in which freedom is an input of the resulting judgment.

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