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Efficient Labor Force Participation with Search and Bargaining

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Abstract

A fixed wage is inefficient in a standard search model when workers endogenously separate from employment. We derive an efficient employment contract that involves agents paying a hiring fee (or bond) upon the formation of a match. We estimate the fixed wage and efficient contract assuming the hiring fee is unobservable, and find evidence to reject the efficient contract in favor of the fixed wage rule. A counterfactual experiment reveals the current level of labor force participation to be 9% below the efficient level, and a structural shift to the efficient contract improves welfare by nearly 4%.

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1 Introduction

Transitions in and out of the labor force represent an important component of labor market activity. The movement of agents from employment to non-participation represents one example of these transitions. For the U.S. economy, these transitions are significant and vary across the wage distribution, with those at the lower end exiting at a faster rate than those at the higher end. Despite the importance of these flows for labor market outcomes, the literature studying equilibrium labor markets has devoted little attention to labor market participation decisions, and even less to the transition from employment to non-participation. Recent work in this area, while limited, has developed along two dimensions: one attempting to develop models to explain the transitions into and out of non-participation, and another that includes these transitions in policy experiments, allowing researchers to determine the effect of policies on the composition of the labor force.

We develop a model that contributes to the literature on both fronts. First, our model is capable of explaining how the transition from employment to non-participation changes over the wage distribution, a feature that remains relatively unexplored. Second, we find that when building a model to explain these flows, the policy implications remain unclear. Specifically, our analysis characterizes a potentially large negative externality arising from the endogenous transition from employment to non-participation. After recognizing the potential inefficiency, we determine if U.S. data remains consistent with employment contracts that internalize the externality, or with standard employment contracts that do not. Thus, the outcome of our analysis has important implications for the efficacy of labor market policies, especially those directed towards altering the composition of the labor force. In other words, we ask the question: Is the current level of labor force participation optimal? If the answer is no, then what are the potential welfare gains from achieving the optimal participation rate?

To capture the large transitions between labor market states, we develop a search and matching model with two important features. First, we extend the standard Pissarides (2000) model to include an endogenous labor force participation decision where agents receive idiosyncratic shocks affecting the value of non-market activities. Second, we include match specific productivity, resulting in a distribution of wages, transition rates that vary conditional on the wage, and thus transition rates depending on the employment contract determining the wage.

Modeling the decision to exit employment, however, introduces theoretical complications. Specifically, when agents take actions affecting the duration of the match, the nature of the employment contract affects the efficiency properties of the models' equilibria. In exploring this fact, we demonstrate what an efficient employment contract looks like, and show that the standard surplus splitting rule is not efficient.

The endogenous transition from employment to non-participation implies inefficiency because it introduces a negative externality. In our model, while employed in a productive match, agents receive opportunities for non-market activities. In some periods, the value of accepting these opportunities exceeds the value of retaining the match, and as a result, an agent may decide to quit and leave the labor force. The externality arises because agents fail to internalize the cost the separation imposes on their firm (who payed a fixed cost to post the vacancy and hire the worker). When the employment contract consists of only a fixed wage, determined by a surplus splitting rule, the aforementioned externality obtains; therefore, any equilibria produced by the model fail to be efficient. This occurs because ameliorating the externality requires firms to pay a wage equal to the productivity of the match. Of course, when the employment contract consists of only a fixed wage, no active equilibrium retains this feature. We derive an alternative employment contract, and demonstrate it is efficient. It specifies agents receive a wage equal to the productivity of the match and pay a "hiring fee" to the firm upon the formation of the match. The introduction of the hiring fee allows the externality to be internalized and efficiency is achieved.

In this paper, we focus on determining if the efficient contract or the fixed wage rule is more consistent with U.S. data. To make this comparison, we estimate both models using wage and duration data, assuming the hiring fee is unobservable. From the estimates, we compare the fit of the two models. While we do not observe an actual hiring fee (or other features of wage contracts resembling one, e.g. wage-tenure contracts) in the data, the two models do predict different combinations of wages and durations. Thus, despite not explicitly observing a hiring fee, observed wages and durations could be consistent with the model featuring such wage contracts. The idea is to determine whether the observed wages and durations are more consistent with the implications of the efficient contract or the inefficient fixed wage rule.

Using NLSY data, we estimate both models using maximum likelihood estimation. We find that under certain parametric restrictions, the likelihood function that incorporates the efficient contract is *nested* within the likelihood function that features the inefficient fixed wage rule. Given this fact, we are then able to perform a formal specification test to determine which contract best describes the U.S. data. The results indicate that the model with an inefficient fixed wage rule provides a better fit than the model with the efficient contract.

Given evidence of an inefficient contract, we then quantify its consequences. Specifically, we perform a counterfactual exercise to determine the welfare gains of a structural shift from the inefficient to the efficient contract. We find labor force participation would rise by 9%, employment by 17%, and unemployment would fall 11%. Overall, the adoption of the efficient contract increases welfare by 3.75%.

This paper contributes to several strands of the labor literature, and in particular the research on search.[1] First, several authors (for example Stevens (2004), Shimer (2005), and Bonilla & Burdett (2005)) show that if workers take actions affecting the duration of the match, then inefficiencies can arise depending on the nature of the employment contract. A few authors have also attempted to incorporate a labor market decision into search models. Garibaldi & Wasmer (2005) develop a search model with stochastic participation costs to analyze the labor force participation decision. In their model however, the wage distribution remains degenerate; as a result, it is unable to capture the flow of employed workers to non-participation across the wage distribution. Pries & Rogerson (2004) also develop a search model with matching frictions to study labor force participation decisions. Their model focuses on the dynamic nature of participation decisions, and qualitatively captures the correlation between participation rates and participation flows. Our model extends their analysis along two dimensions. First we perform a general equilibrium analysis, allowing for job creation decisions by firms. Second, we allow for a continuous distribution of idiosyncratic shocks to the relative value of non-participation. Pries & Rogerson (2004) use a two-shock case, which does not capture the variation in exit rates across the wage distribution. Finally, our work is also related to Yip (2003), who develops and estimates a search model with endogenous labor force participation decisions, driven by idiosyncratic shocks to the relative value of non-participation. Yip (2003) performs some counterfactual policy experiments, but does not derive Pareto optimal allocations. The counterfactual exercises in Yip (2003) focus on how changes in policy variables (such as unemployment benefits) affect the composition of the labor market as well as welfare. In contrast, we focus first on determining which employment contract best describes the data, and our

[1] The literature on search models of the labor market is extensive; consequently, we only discuss those papers most closely related to ours.

counterfactual exercise calculates the gains from achieving efficiency.

This work also contributes to the literature investigating the role of various labor market policies. Flinn (2006) represents one recent example of this discussion. Flinn analyzes the role of minimum wage policies in a search model with matching frictions similar to the one we use here. The focus of his analysis rests on determining whether the Hosios condition (Hosios (1990)) holds when the parameters are estimated using U.S. data. When the resulting parameters do not satisfy the aforementioned condition, the paper suggests there may exist a role for minimum wage policies in improving welfare. In contrast, our analysis abstracts from inefficiencies arising from parametric restrictions, focusing instead on structural inefficiencies. Flinn & Mabli (2008) represents another example studying the effects of minimum wage policies in a similar model with on-the-job search. This work is similar to ours in that the specific employment contract affects the welfare properties of equilibria.

Finally, the paper contributes to the efficiency wage literature (Carmichael (1990) contains a critical survey). For instance, we find firms pay higher wages due to job turnover. However, we do not find efficiency wages because of a strict assumption about the job turnover to wage relationship.[2] Rather, we deduce higher wages due to the increased bargaining position of the worker. More importantly, we contribute to the bonding critique. The efficient contract we study involves agents paying a "hiring" fee upon the formation of the match; this represents a similar concept to the "bonding" approach used in the efficiency wage literature. Much of that literature has assumed bonds do not exist and argued for theoretical reasons why this might be true. In contrast, we assume the bonds are unobservable and test for their presence.

The remainder of the paper proceeds as follows. Section 2 introduces the environment. Section 3 describes the agent and firm decision problems, and Section 4 characterizes the resulting equilibrium. Section 5 outlines the data, the estimation procedure, and discusses the results. Finally, Section 6 discusses the counterfactual welfare exercises and Section 7 concludes.

[2]This could be assumed and would result in even higher wages.

2 Model

2.1 The Environment

The economy consists of a unit-measure of infinitely-lived agents and a large measure of firms. Time, t, is continuous and goes on forever. Each agent has an endowment of one indivisible unit of time with three alternative, mutually exclusive uses: search for a job, work for a firm, or do not participate in the formal labor market.[3] Agents and firms are risk-neutral and discount at rate $r > 0$.

An unemployed agent searching for a job enjoys utility flow b, interpreted as the utility from not working. Upon receiving a job offer when unemployed, agents and firms observe a match specific productivity, y, drawn from a distribution $F(y)$ with support $[0, \bar{y}]$, and decide whether to accept or reject the match. Firms are composed of a single job, either filled or vacant, and discount future profits at rate r. Vacant firms are free to enter and pay a flow cost, $\gamma > 0$, to advertise a vacancy. Vacant firms produce no output and filled firms produce one final good.

The labor market is subject to search-matching frictions. The aggregate matching function, $\zeta(U,V)$, describes the flow of job offers, where U represents the measure of unemployed workers actively looking for jobs, and V the measure of vacant jobs. The matching function, ζ, is continuous, strictly increasing, strictly concave (with respect to each of its arguments), and exhibits constant returns to scale. Furthermore, $\zeta(0,\cdot) = \zeta(\cdot,0) = 0$ and $\zeta(\infty,\cdot) = \zeta(\cdot,\infty) = \infty$. Following Pissarides' terminology, define $\theta \equiv V/U$, referred to as labor market "tightness." Each vacancy is filled according to a Poisson process with arrival rate $\frac{\zeta(U,V)}{V} \equiv q(\theta)$. Similarly, each unemployed worker finds a job according to a Poisson process with arrival rate $\frac{\zeta(U,V)}{U} = \theta q(\theta)$. Filled jobs receive negative idiosyncratic productivity shocks rendering the match unprofitable with a Poisson arrival rate s. Denote by $n_e(y)$, n_u, and n_o the measure of workers employed with productivity y, unemployed, and not in the labor force, respectively.

The existence of opportunities to exit the labor force represents the distinguishing feature of our model. Employed and unemployed agents receive opportunities to exit the labor force at rates λ_e and λ_u, respectively. Upon receiving an opportunity, the agent observes the instantaneous payoff ε, drawn from a distribution $G(\varepsilon)$ on the support $[0, \overline{\varepsilon}]$. If the agent takes the opportunity, then the agent enjoys the instantaneous utility ε and exits the labor market. When outside the labor force,

[3] Non-participation can include a range of activities, including home-production, leisure, etc.

they enjoy a utility flow x and re-enter the labor market at an exogenous rate δ.

3 Bellman equations

Agent and firm decisions depend, among other factors, on the wage paid to workers. To determine wages for each match, we assume agents and firms bargain over their joint surplus. We consider two different types of contracts. The first contract specifies that upon the formation of a match, the worker pays a hiring fee, ϕ, and then receives a constant wage, w, thereafter. We establish below that this type of contract is Pareto-optimal, and thus refer to it as the "efficient contract." Alternatively, we consider an employment contract featuring only a wage determined by an exogenous surplus splitting rule, similar to the one used in Pissarides (2000). In either case, w, or the pair (ϕ, w), are determined through a bargaining solution.[4] This section begins with the case of the efficient contract. We focus on steady-state equilibria, where market tightness, θ, matching probabilities, and exit rates remain constant over time. In the next two subsections, we describe the flow Bellman equations for agents and firms, and characterize the employment contracts.

3.1 Agents

Denote the value of being an agent in state $i \in \{u, e, o\}$ by $\mathscr{V}_u$, $\mathscr{V}_o$ and $\mathscr{V}_e(y)$, respectively. An agent in state $i = u$ is considered unemployed, $i = e$ employed, and $i = o$ not in the labor force (non-participation). The flow Bellman equations for individuals' value functions follow

$$
\begin{aligned}
r\mathscr{V}_u &= b + \theta q(\theta) \int [\mathscr{V}_e(y) - \mathscr{V}_u - \phi(y)]^+ \, dF(y) + \lambda_u \int [\varepsilon + \mathscr{V}_o - \mathscr{V}_u]^+ \, dG(\varepsilon) && (1) \\
r\mathscr{V}_e(y) &= w(y) + s\,(\mathscr{V}_u - \mathscr{V}_e(y)) + \lambda_e \int [\varepsilon + \mathscr{V}_o - \mathscr{V}_e(y)]^+ \, dG(\varepsilon) && (2) \\
r\mathscr{V}_o &= x + \delta\,(\mathscr{V}_u - \mathscr{V}_o) && (3)
\end{aligned}
$$

where $[z]^+ = \max(z, 0)$. Equation (1) has the following interpretation. An unemployed worker enjoys a utility flow of b defined as the utility from not working. She finds a job with an instantaneous probability $\theta q(\theta)$. Each job has a match specific productivity, y, drawn from the distribution

[4] Implicit in this formulation is the firm commits to the terms of the employment contract. In particular, once the worker pays the hiring fee the firm does not renege on the promised future wage.

$F(y)$. Upon formation of the match, the agent pays a hiring fee, $\phi(y)$, and enjoys a surplus gain $\mathscr{V}_e(y)-\mathscr{V}_u$. When unemployed, the agent receives an opportunity to exit the labor force with instantaneous probability λ_u. If a worker chooses to exit, then the agent enjoys flow utility ε, drawn from the distribution $G(\varepsilon)$, and gains the capital $\mathscr{V}_o-\mathscr{V}_u$. From (2), an employed worker receives a wage w, loses the job with an instantaneous probability s, and has the opportunity to exit the labor force with an instantaneous probability λ_e. According to (3), an agent outside the labor force receives flow utility x, and enters unemployment with an instantaneous probability δ.

An agent accepts a job offer if $w \geq w(y_R)$, where y_R defines the point where workers remain indifferent between becoming employed or remaining unemployed, which satisfies

$$\mathscr{V}_e(y_R)=\mathscr{V}_u. \tag{4}$$

From (1) and (2) an individual in state i chooses to exit the labor market whenever $\varepsilon \geq \varepsilon_i$ where

$$\begin{aligned} \varepsilon_u &= \mathscr{V}_u-\mathscr{V}_o, & (5)\\ \varepsilon_e(y) &= \mathscr{V}_e(y)-\mathscr{V}_o, & (6)\end{aligned}$$

From (5)-(6) the value of the opportunity that makes an individual in a given state indifferent between exiting or not, ε_i, is equal to the expected cost of leaving, $\mathscr{V}_i(y)-\mathscr{V}_0$. Subtracting (5) from (6), we have

$$\mathscr{V}_e(y)-\mathscr{V}_u=\varepsilon_e(y)-\varepsilon_u \tag{7}$$

3.2 Firms

Firms participating in the market can be in one of two states: they either hold a vacant job (v) or a filled job (f). The following summarize firms' flow Bellman equations.

$$\begin{aligned} r\mathscr{V}_v &= -\gamma+q(\theta)\int_{y_R}\left(\phi(y)+\mathscr{V}_f(y)-\mathscr{V}_v\right)dF(y) & (8)\\ r\mathscr{V}_f(y) &= y-w(y)-s\left(\mathscr{V}_f(y)-\mathscr{V}_v\right)-\lambda_e\left[1-G(\varepsilon_e)\right]\left(\mathscr{V}_f(y)-\mathscr{V}_v\right) & (9)\end{aligned}$$

According to (8), a vacancy incurs an advertising cost γ; the firm finds an unemployed worker with an instantaneous probability $q(\theta)$ in which case it receives a hiring fee $\phi(y)$ and enjoys a surplus gain $\mathscr{V}_f(y) - \mathscr{V}_v$. According to (9), a filled job enjoys flow profit $y - w(y)$ and is destroyed when a negative idiosyncratic productivity shock occurs with instantaneous probability s, or if the worker exits, an event occurring with instantaneous probability $\lambda_e[1 - G(\varepsilon_e)]$. Free-entry of firms implies $\mathscr{V}_v = 0$ and therefore, from (8),

$$\int_{y_R} \left(\phi(y) + \mathscr{V}_f(y)\right) dF(y) = \frac{\gamma}{q(\theta)}. \tag{10}$$

From (10), the firms' surplus from a match (the sum of the expected value of a filled job and the expected value of the hiring fee) equals the average recruiting cost incurred by the firm. Note, if $\lambda_e = \lambda_u = 0$ and $F(y)$ remains degenerate, the model is exactly Pissarides (2000).

3.3 Employment contract with a hiring fee

We now discuss in more detail the employment contract specified thus far, and explore its efficiency properties. To determine the details of the pair-wise efficient employment contract, we define $\mathscr{S}(y) \equiv \mathscr{V}_e(y) - \mathscr{V}_u + \mathscr{V}_f(y)$, which represents the total surplus of a match (recall $\mathscr{V}_v = 0$). From (2) and (9),

$$r\mathscr{S}(y) = y - r\mathscr{V}_u - s\mathscr{S}(y) + \lambda_e \int_{\varepsilon_e}^{\bar{\varepsilon}} \left[\varepsilon - \mathscr{S} - (\mathscr{V}_u - \mathscr{V}_o)\right] dG(\varepsilon) \tag{11}$$

Equation (11) has the following interpretation. A match generates a flow surplus, $y - r\mathscr{V}_u$, composed of the output of the job and the permanent income of an unemployed person, $r\mathscr{V}_u$. The match is destroyed if either an exogenous shock occurs (at Poisson rate s), or if the worker chooses to separate. In the latter case, the value $\mathscr{S}$ of the match is lost and the worker exits the labor force, which generates an additional capital loss $\mathscr{V}_u - \mathscr{V}_o$. The value of the match also incorporates the value of the outside option undertaken by the employed worker.

We assume a worker and a firm can *jointly* determine the exit opportunities undertaken by the worker. It can be seen from (11), that the surplus of the match is maximized if

$$\varepsilon_e(y) = (\mathscr{S}(y) + \mathscr{V}_u - \mathscr{V}_o) = \left(\mathscr{V}_e(y) + \mathscr{V}_f(y) - \mathscr{V}_o\right). \tag{12}$$

Comparison of (6) and (12) reveals that if $\mathscr{V}_f > 0$, the worker's choice of which outside opportunities to accept and the choice that maximizes the match surplus differ; i.e. the total surplus of the match is not maximized. Employed workers separate "too frequently" because they do not internalize the negative externality they impose on the firm.[5]

We show that by allowing the employment contract to include the upfront fee, $\phi(y)$, the worker and the firm can reach a pairwise-efficient outcome. The employment contract (ϕ, w) is determined by the generalized Nash solution, where $\beta \in (0,1)$ denotes the worker's bargaining power. Specifically, the contract satisfies

$$(\phi, w) = \arg\max\left(\mathcal{V}_e(y) - \mathcal{V}_u - \phi(y)\right)^{\beta}\left(\mathcal{V}_f(y) - \mathcal{V}_v + \phi(y)\right)^{1-\beta}, \tag{13}$$

The following Lemma describes the properties of the Pareto optimal, or efficient employment contract.

Lemma 1 *The pair-wise efficient contract is*

$$w(y) = y, \tag{14}$$

$$\phi(y) = (1-\beta)\left(\mathcal{V}_e(y) - \mathcal{V}_u\right). \tag{15}$$

Proof. See Appendix A

According to Lemma 1, the pair-wise efficient contract sets the wage equal to the worker's productivity.[6] Since the worker gets the entire output generated by the match, $V_f = 0$, and the worker internalizes the effect of her exit decision on the total surplus of the match. The up-front payment splits the surplus of the match according to each agent's bargaining power.[7] To fully describe the equilibrium in the next section, it is useful to write the first order condition of (13) with respect to $\phi(y)$:

$$(1-\beta)[\mathcal{V}_e(y) - \mathcal{V}_u - \phi(y)] = \beta[\mathcal{V}_f(y) + \phi(y)] \tag{16}$$

In modeling the decision to leave the labor force, we have assumed agents receive idiosyncratic "shocks" which produce instantaneous flow utility before exiting the current state. Once they enter non-participation, $\mathcal{V}_o$ remains constant. Alternatively, one could assume the value of non-participation, $\mathcal{V}_o$ remains stochastic; i.e. the idiosyncratic shocks affect $\mathcal{V}_o$ directly. In this case,

[5] See Engelhardt, Rocheteau & Rupert (2008) for more discussion of this externality.

[6] Since the firm makes no profit after the hiring fee has been paid, it has no incentive to fire the worker as the value of a vacancy is no greater than the value of a filled job, i.e., $\mathcal{V}_f(y) = \mathcal{V}_v = 0$.

[7] Alternatively, the optimal contract could take the form of a constant wage, $w(y)$, and a payment from the worker to the firm (a fine) if the worker exits. This transfer would exactly compensate the firm for its lost surplus, and is equivalent to the current contract.

the Bellman equation for non-participation becomes

$$r\mathscr{V}_o(\varepsilon) = \varepsilon + \delta(\mathscr{V}_u - \mathscr{V}_o)$$

We use the stochastic flow value approach because it provides analytic convenience, allowing us to use integration by parts and write the equilibrium conditions cleanly. However, either structure leads to the same efficiency results discussed in Lemma 1; the optimal contract must include a hiring fee. Given this equivalence, we chose the analytically transparent approach as it plays a central role when comparing the two models.

3.4 Fixed wage employment contract

We now consider the case of a fixed (or exogenous) wage. In this case, $\phi(y) = 0, \forall y$ (i.e. the employment contract consists of only the wage), and the wage is determined using the following surplus splitting rule:

$$(1-\beta)\left[\mathscr{V}_e(y) - \mathscr{V}_u\right] = \beta\left(\mathscr{V}_f(y)\right), \tag{17}$$

which corresponds to the standard search and bargaining surplus splitting wage equation, as discussed in Pissarides (2000). Using (7), (17), and

$$\mathscr{V}_f(y) = \frac{y - w(y)}{r + s + \lambda_e[1 - G(\varepsilon_e(y))]} \tag{18}$$

we can write the wage in this case as

$$w(y) = y - \frac{1-\beta}{\beta}\left(r + s + \lambda_e[1 - G(\varepsilon_e(y))]\right)\left(\varepsilon_e(y) - \varepsilon_u\right) \tag{19}$$

Note, since $\beta \in (0,1)$, $y > w(y)$ and from (18) $\mathscr{V}_f(y) > 0$, efficiency is not achieved in this model.[8]

4 Equilibrium

This section characterizes the equilibrium for the aforementioned economy. To facilitate the presentation of both models simultaneously, we introduce an indicator function, χ, where $\chi = 1$

[8] Again, note if $\lambda_e = 0$, $w(y)$ follows exactly from Pissarides (2000).

when the employment contract includes the hiring fee, $\phi(y)$, and $\chi = 0$ for the fixed wage case. The idea is to show the equilibrium can be represented by three equations in three unknowns, $\varepsilon_e(y), \varepsilon_u$, and θ. Towards this end, the free entry condition implies

$$\frac{\gamma}{q(\theta)} = \int_{y_R} \left(\mathscr{V}_f(y) + \chi\phi(y)\right) dF(Y). \tag{20}$$

Using (7) and (16) for the efficient contract case, and (7) and (17) for the fixed wage contract case, to solve for $\mathscr{V}_f(y)$, substituting into (20) gives

$$\frac{\gamma}{q(\theta)} = \frac{(1-\beta)}{\beta}\left(\int_{y_R} (\varepsilon_e(y) - \varepsilon_u - \chi\phi(y))\, dF(Y)\right). \tag{21}$$

This represents the first relationship between $\varepsilon_e(y)$, ε_u, and θ. Then, combining (1), (3), (5) and (20), and integrating by parts, we have

$$(r+\delta)\varepsilon_u = b - x + \frac{\beta\gamma\theta}{(1-\beta)} + \lambda_u \int_{\varepsilon_u} [1 - G(\varepsilon)]\, d\varepsilon \tag{22}$$

This provides the second relationship between $\varepsilon_e(y)$, ε_u, and θ. Similarly, for $y \geq y_R$, using (2) and (6), and integration by parts, we have

$$r\varepsilon_e(y) + \delta\varepsilon_u + x = \chi y + (1-\chi)w(y) + s(\varepsilon_u - \varepsilon_e) + \lambda_e \int_{\varepsilon_e(y)} [1 - G(\varepsilon)]\, d\varepsilon, \tag{23}$$

where for $\chi = 0$, $w(y)$ follows (19). Equations (22) and (23) determine the values of ε_u and $\varepsilon_e(y)$ as functions of θ. These can then be substituted into (21) to determine θ.

To complete the description of equilibrium, consider the steady state distribution of agents across states, $(n_e(y), n_u, n_o)$, where n_i denotes the number of agents in state $i \in \{u, e, o\}$. This distribution is characterized by the following three equations.

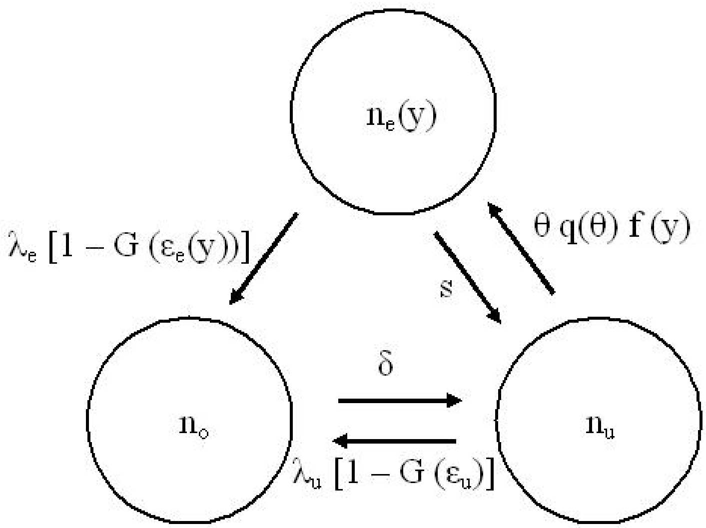
$$s\int_{y_R} n_e(y)dF(y) + n_o\delta = \{\theta q(\theta)\,[1 - F(y_R)] + \lambda\,[1 - G(\varepsilon_u)]\}\, n_u \tag{24}$$

$$\theta q(\theta)\,[1 - F(y_R)]\, f(y|y \geq y_R) n_u = (\lambda\,[1 - G(\varepsilon_e(y))] + s)\, n_e(y), \forall y \in [y_R, \bar{y}] \tag{25}$$

$$\int_{y_R} n_e(y)dF(y) + n_u + n_o = 1 \tag{26}$$

According to (24) the flows into and out of unemployment must be equal. The measure of agents entering unemployment is the sum of the employed workers who lose their jobs, sn_e, and the non-participants who enter the labor market, δn_o. The flow of agents exiting unemployment corresponds to those finding jobs, $\theta q(\theta)[1-F(y_R)]n_u$, or unemployed agents exiting the labor market, $\lambda_u[1-G(\varepsilon_u)]n_u$. Similarly, (25) prescribes that the flows into and out of employment must be equal in the steady state for all $y \in [y_R, \bar{y}]$. Equation (26) constrains agents to be either employed, unemployed, or out of the labor force. Figure 1 depicts the flows.

Figure 1: Worker Flows



We can now define an equilibrium for the model.

Definition 1 *A steady state equilibrium is a list, $\{\theta,\, \varepsilon_u,\, \varepsilon_e(y),\, n_e(y),\, n_u,\, n_o\}$, such that θ, ε_u, and $\varepsilon_e(y)$ solve (21), (22), and (23); and $n_e(y)$, n_u, n_o solve (24)-(26).*

Determining an equilibrium represents a fixed point problem in θ. Specifically, for an initial value of θ, (22) and (23) determine values for ε_u and $\varepsilon_e(y)$. Given these values, (21) determines a new value of θ. This process continues until (21) holds. In general, a unique equilibrium may not exist without further restrictions on the parameter space, or the functional forms of G, F and q. However, Engelhardt et al. (2008) show an equilibrium exists and is unique under the efficient contract given F is degenerate. Alternatively, an equilibrium can be shown to exist with a non-degenerate distribution F given certain conditions on λ_u and λ_e. In the simulations run below, we find an equilibrium exists and appears to be unique.

To this point, we have described two models, each of which makes predictions about labor market behavior and outcomes. Given the specified employment contract, each model has equilibrium flows into and out of the various labor market states, as well as the steady state levels of employment and labor market participation. The question we now pursue is which model best describes the behavior observed in the U.S. labor market? To perform this comparison, we estimate each model, and compare their respective predictions with observed behavior.

5 Estimation

We use Maximum Likelihood Estimation (MLE) to identify the parameters for each model specification. In this section we describe the data, derive the likelihood function, discuss identification, and provide the estimates of the two models' parameters. With these estimates, we then perform a likelihood ratio test to determine which model remains more appropriate for describing the observed behavior.

5.1 Data

The data comes from the National Longitudinal Survey 1997 (NLSY97), a panel data set of individuals in the U.S. between the ages of 12 and 16 in 1997. The NLSY97 observes when an individual is employed, unemployed, or out of the labor force, in addition to an individual's wage if employed. We estimate each model from the duration and wage data.

We start the sample on January 1, 2002; as a result, the individuals we observe are at least seventeen years of age. We also limit the sample to individuals who have exited school with less than a college degree.[9] We adopt a stock sampling approach. Specifically, we record what state an individual is initially in at a particular point in time, we then record the duration of time spent in the original state (unemployed, employed or out of the labor force), where the time spent is t_i for $i = \{e, u, o\}$. After a span of time, an individual transitions from the original state "i" to the new state "j." Denote by $N_{i,j} = 1$ the number of agents who have transitioned from state i to state j and zero otherwise, where $i, j \in \{e, u, o\}$.[10] Finally, the wage w_e is observed for those whom

[9]We follow Wolpin (1987), Bowlus, Kiefer & Neumann (1995) and others in determining who should be included in the sample.

[10]If we observe an agent transition from employment to employment, we assume they spend a short time, less than

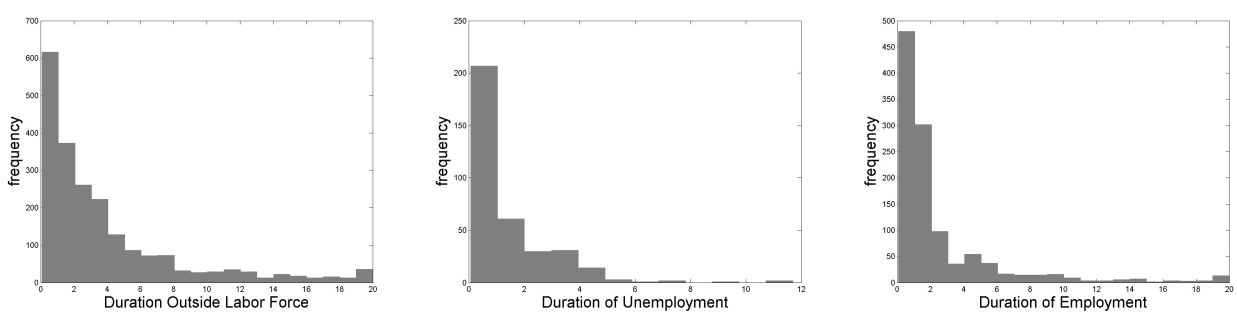
are initially employed as well as the wage w_u for those whom transition from unemployment to employment (wages are recorded as the hourly wage). Table 5.1 contains the descriptive statistics. Figure 2 presents the duration data graphically. The wage data are presented graphically in Section 5.5 when we compare the fit of the efficient and inefficient contracts against the observed data.

Table 1: Descriptive Statistics

Variable	Mean	Std. Dev.
Transitions from u to o ($N_{u,o}$)	0.043	0.202
Transitions from u to e ($N_{u,e}$)	0.055	0.229
Transitions from o to u ($N_{o,u}$)	0.31	0.463
Transitions from e to u ($N_{e,u}$)	0.321	0.467
Transitions from e to o ($N_{e,o}$)	0.259	0.438
Duration Unemployed (t_u)	1.363	1.644
Duration Outside Labor Force (t_o)	2.537	3.625
Duration Employed (t_e)	3.872	4.459
Wage if initially employed (w_e)	8.858	4.147
Wage if inititally unemployed (w_u)	8.542	4.431
Observations	3593	

Note: Durations are quarterly and wage statistics are hourly. Transitions do not sum to one due to right censoring and the wage distribution is trimmed at 2.5% level. Missing wage and duration data is assumed to occur randomly and are excluded.

Figure 2: Duration of Employment, Unemployment, and Outside the Labor Force



the interval of measure, unemployed. In addition, if we observe an agent transition from out of the labor force to employment, we assume they are unemployed for a period of time less than the frequency of the data, which is a week.

5.2 Likelihood Function, fixed wage employment contract

We begin by presenting the estimation of the fixed wage employment contract. The likelihood function consists of three multiplicative components derived from those we initially observe to be employed, unemployed, or not participating. In constructing the likelihood function, each component includes the length of time individuals spend in the initial state. Once individuals transition, we observe and use the information on where they transitioned. For those employed at some point during the sample, we observe and incorporate their wage into the likelihood function.

Non-participants represent a straightforward component of the likelihood function. In particular, individuals transition according to a Poisson process with rate δ. As a result, an agent's duration of time outside the labor force has an exponential distribution. Refer to Chapter 5 in Lancaster (1990) for the derivation of the probability distribution from stock sampling. Given this, non-participants contribute to the likelihood function as

$$P(N_{o,u}, t_o) = n_o \delta e^{-\delta t_o},$$

where n_o is the probability of being outside the labor force and is a function of the Poisson arrival rates as described in the flow equations (24)-(26).

The unemployed contribute to the likelihood function in three ways. Given they are initially unemployed with probability n_u, their duration of unemployment factors in as

$$f_u(t_u|u) = \{h_u + k_u\} e^{-\{h_u + k_u\} t_u},$$

where $h_u = \theta q(\theta)[1 - F(y_R)]$ and $k_u = \lambda_u[1 - G(\varepsilon_u)]$. Second, they contribute the probability of transitioning to employment or non-participation as

$$P(N_{u,e}|u) = \frac{h_u}{h_u + k_u},$$
$$P(N_{u,o}|u) = \frac{k_u}{h_u + k_u}.$$

Finally, if they transition to employment, then they draw a match specific productivity and in turn wage according to

$$f_w(w_u|N_{u,e}) = \frac{\partial y(w_u)}{\partial w_u} \frac{f(y(w_u))}{1 - F(y_R)}.$$

where the productivity of the match, $y(w_u)$, is derived from (19). The unemployed's complete contribution to the likelihood function is

$$P(t_u, N_{u,e}, N_{u,o}, w_u) = n_u f_u(t_u|u) P(N_{u,o}|u)^{N_{u,o}} \left[P(N_{u,e}|u) f_w(w_u|N_{u,e})\right]^{N_{u,e}}.$$

The employed contribute to the likelihood function in two ways. Given they are initially employed and paid w_e with probability $n_e(y(w_e))$, their duration of employment enters as

$$f_e(t_e|e, w_e) = \{s + k_e(y(w_e))\} e^{-\{s+k_e(y(w_e))\}t_e},$$

where $k_e(y(w_e)) = \lambda_e[1 - G(\varepsilon_e(y(w_e)))]$. Second, their probability of transitioning to unemployment or out of the labor force is

$$P(N_{e,u}|e, w_e) = \frac{s}{s + k_e(y(w_e))},$$
$$P(N_{e,o}|e, w_e) = \frac{k_e(y(w_e))}{s + k_e(y(w_e))}.$$

As a result, the complete contribution of the employed is

$$P(t_e, N_{e,u}, N_{e,o}, w_e) = n_e(y(w_e)) f_e(t_e|e, w_e) P(N_{e,u}|e, w_e)^{N_{e,u}} P(N_{e,o}|e, w_e)^{N_{e,o}},$$

The complete likelihood function can thus be written as[11]

$$lnL = \sum_{i \in o} log(P(N_{o,u}, t_o)) + \sum_{i \in u} log(P(t_u, N_{u,e}, N_{u,o}, w_u)) + \sum_{i \in e} log(P(t_e, N_{e,u}, N_{e,o}, w_e)) \qquad (27)$$

Since the distributions $G(\varepsilon)$ and $F(y)$ will take on an assumed parametric form, they can be characterized by the parameter vectors τ_g and τ_f, respectively.

5.3 Identification, fixed wage contract

This section describes identification of the fixed wage contract case. Identification of the efficient contract is similar and described in the next section.

The likelihood function in (27) leaves the following parameters to be determined: δ, s, λ_u,

[11] Note, although not described in (27), right censoring has been accounted for in the usual manner.

λ_e, r, b, x, γ, β, τ_g, τ_f, and the parameters of the matching function. Ignoring the parameters that determine n_o, n_u, and $n_e(y)$, identification of δ, s, h_u and k_u follows directly from the first order conditions of the likelihood function. More specifically, the simple exponential form for the duration data implies

$$
\begin{aligned}
\delta &= \frac{N_{o,u}}{\sum_{i\in o} t_i}, \\
s &= \frac{N_{e,u}}{\sum_{i\in o} t_i}, \\
k_u &= \frac{N_{u,o}}{\sum_{i\in o} t_i}, \\
h_u &= \frac{N_{u,e}}{\sum_{i\in o} t_i}.
\end{aligned}
$$

Although k_u and h_u are not parameters of interest, they are used to deduce λ_u and the parameters of the matching function, which we discuss in more detail below. Estimating n_o, n_u, and $n_e(y)$ directly is a simple multinomial problem. More robust estimates are provided, however, by substituting in the steady state probability of being in each state, as defined by the flow equations. We take the latter approach.

We take a reduced-form approach to estimating $k_e(y(w_e))$ (which we use to infer λ_e), fitting it according to $\kappa_1 \exp(\kappa_2 w)$. Specifically, we plug $\hat{k}_e(w) = \kappa_1 \exp(\kappa_2 w)$ into (27) and maximize the likelihood with respect to κ_1 and κ_2. Note, if we find $\kappa_2 > 0$, then individuals exit at a faster rate the more they earn. Alternatively, if $\kappa_2 < 0$, then the more individuals earn the less likely they are to exit their job. This approach is similar to Bontemps, Robin & van den Berg (2000) where they use a non-parametric procedure to estimate the wage distribution, which they then use to deduce other parameters from the model. We have also tried a nonparametric procedure, where we use a constant rate of exit, $k_e(w) = \kappa_i$, for different intervals of the wage distribution, $w \in [w_i, w_{i+1})$, and found similar results to our reduced-form approach. Comparison of the model's exit rates and the reduced-form estimates are provided in Figure 4 below.

We use the wage distribution to identify the parameter vector τ_f. In doing so, a complication arises as $y(w_u)$ and $y(w_e)$ are functions of the difference $\varepsilon_e(y) - \varepsilon_e(y_R)$ as seen in (19). To circumvent this difficulty, we assume the distribution of outside opportunities is exponential, so

$G(\varepsilon) = 1 - exp(-\frac{1}{\mu_g}\varepsilon)$. Under this parametric assumption, the difference can be derived in a relatively straightforward manner. First, from (2) and (6) we can write

$$(r+s)(\varepsilon_e(y) - \varepsilon_e(y_R)) = (r+s)(\mathscr{V}_e(y) - \mathscr{V}_e(y_R)) = w(y) - y_R + \lambda_e \left[- \int_{\varepsilon_e(y_R)}^{\varepsilon_e(y)} [1 - G(\varepsilon)] d\varepsilon \right], \quad (28)$$

then from the distributional assumption on $G(\varepsilon)$, (28) can be further simplified to

$$\varepsilon_e(y) - \varepsilon_e(y_r) = \frac{w - y_R + (k_e(y(w)) - k_e(y_R))\mu g}{r+s}. \quad (29)$$

As a result, plugging (29) into (19), and substituting in the reduced-form estimates $\hat{k}_e(w_i)$ and $\hat{k}_e(y_R)$, we can write the following:

$$y_i = w_i + \alpha \left(\frac{w_i - y_R + (\hat{k}_e(w_i) - \hat{k}_e(y_R))\mu g}{r+s} \right) [r + s + \hat{k}_e(w_i)], \quad (30)$$

for $i \in \{u, e\}$ and where $\alpha = (1-\beta)/\beta$. We refer to the inefficient wage splitting rule as defined by (30) as Specification I.

Given the difference in $\varepsilon_e(y)$ and $\varepsilon_e(y_R)$, we are able to discuss identifying the wage distribution. Following Flinn & Heckman (1982), we assume a log-normal distribution for wages, or

$$f_w(w_i) = \frac{\partial y(w_i)}{\partial w_i} \frac{1}{y(w_i)\sigma_f\sqrt{2\pi}} exp\left\{ -\frac{1}{2} \left(\frac{lny(w_i) - \mu_f}{\sigma_f} \right)^2 \right\} / (1 - F(y_R),$$

for $i \in \{u, e\}$.

Although we do not provide a proof of identification for κ_1, κ_2, α, μ_g, μ_f, and σ_f, we have run Monte Carlo experiments to test our optimization procedure. We find 1500 observations is sufficient in providing reliable estimates for the aforementioned parameters. In other words, we simulate data from the model, plug the data into likelihood function, and estimate the parameters. The results show the parameters used to generate the data are identified with the likelihood function even though the function can have all the standard issues when maximizing a function of many variables.

By imposing identifying restrictions, the parameters λ_u, λ_e, x, b, γ, r, along with the parameters of the matching function can be deduced conditional on the estimates from the likelihood function. Specifically, since we assume a time period of one quarter, we fix $r = 0.01$, consistent with a riskfree interest rate of 4% per annum. We also assume $b = x$ and the Hosios condition ($\beta = \eta$), where η is defined by the Cobb-Douglas matching function $m(u,v) = Au^{\eta}v^{1-\eta}$.[12]

Given α from the maximization of the likelihood function, β follows directly from $\alpha = (1-\beta)/\beta$, which determines η, given the Hosios condition. In determining the remaining parameters of the matching function, we follow Engelhardt et al. (2008) and recognize θ can be normalized to one, given γ remains free to be used to equate the free entry of firms, and $\theta q(\theta)$ has been identified. Hence, we set $\theta = 1$ and deduce $A = h_u/(1-F(y_R))$ as estimated in the likelihood function.

To obtain the arrival rates of outside opportunities, λ_u and λ_e, we use the fact $\lambda_u/\lambda_e = k(e)(y_R)/k_u$ as $V_e(y_R) = V_u$.[13] Therefore, $\lambda_u = \lambda_e \kappa_1 exp^{\kappa_2 \min(w_e)}/k_u$ and λ_e is set so the equilibrium exit rate of the median wage equals the reduced-form estimate found from maximizing the likelihood function.[14]

Finally, we plug the known parameters into the reservation wage equation, $V_e(y_R) = V_u$, to deduce b, and use the free-entry condition

$$\gamma = q(\theta)\int_{y_R} \frac{y - w(y)}{r + s + \lambda_e(1 - G(\varepsilon_e(y)))} f(y)dy.$$

to deduce γ.

5.4 Estimation of the efficient contract

Estimation of the efficient contract follows the estimation of the fixed wage contract. The likelihood function in (27) remains unchanged except that now $w_i = y_i$. This is accomplished by constraining $\alpha = 0$. As a result, the efficient contract is nested within the inefficient contract. Results from the estimation of the efficient contract are denoted as Specification E.

Our ability to nest the likelihood functions and thus perform a formal specification test depends on our reduced-form approach to estimating $k_e(y)$. Under this assumption, the exit rate is given by $\kappa_1 \exp(\kappa_2 w)$ for both contracts; however, calculating $k_e(y)$ from the model's equilibrium conditions would result in different functions, $\varepsilon_e(y)$, and thus the likelihood functions would not be nested.

[12] As an alternative to assuming $b = x$, we could take the number of outside opportunities as a fixed number per quarter, for example $\lambda_u = \lambda_e = 1$. Also, the Hosios condition can be relaxed if a CRS matching function is assumed or JOLTS data is incorporated as discussed by Engelhardt & Rupert (2009).

[13] As discussed above, if λ_e and λ_u were assumed to be a constant, then b and x could be deduced by plugging ε_u into (22) and the reservation wage equation, $V_e(y_R) = V_u$.

[14] It could be of interest to estimate $\lambda_e(y)$. However, we take a more conventional approach and assume one exit rate.

This can be seen from equation (23). While the reduced-form approach does represent a simplification, it does not appear to affect the results. In Figure 4, the left of the two plots compares the reduced-form estimates of $k_e(y)$ against those implied in the model (for the parameters estimated under Specification I). From this figure it can be seen that the reduced-form approximates the more complicated function arising from the model. In addition to providing the means for a formal specification test, the reduced-form approach also significantly reduces the computational burden of the estimation procedure.

5.5 Model comparison

We have developed and estimated two models of the U.S. labor market. The analysis now focuses on evaluating how well each model explains the observed outcomes. The comparison is strengthened by the fact that the efficient contract is nested within the inefficient contract. If $\alpha = 0$, then the likelihood functions of the two models coincide; therefore, if the constraint $\alpha = 0$ is rejected in favor of $\alpha \neq 0$, then we have evidence in favor of the inefficient contract.

Before going further, we stress the comparison is made by examining the fit of each models' predicted wage distribution to the observed wage distribution. While in theory it would be interesting to incorporate additional types of data including any up-front fees charged to the employee, the cost of forgone wages due to a probationary period, or penalties from quitting such as the loss of a retirement plan, it remains practically impossible as these variables are difficult to measure. Moreover, the inclusion of the hiring fee can have many alternative interpretations; it remains equivalent to a "quitting" fee, or could be interpreted as capturing some wage-tenure features of employment contracts, such as increasing wages. Therefore, one of the key contributions of this paper is to be able to deduce the type of contract with a limited amount of information. In other words, the important question is not whether we observe a hiring fee (or a "quitting" fee, etc.). Rather, what matters is whether the observed wages, durations, and transitions remain consistent with agents internalizing the negative externality. From (12) we know for any observed wage what the corresponding exit rate should be (or the corresponding duration). If observed wages, transitions, and durations are consistent with the efficient exit rates, then we would fail to reject the hypothesis that $\alpha = 0$. Therefore, we do not need to observe the specific employment contract to make this comparison.

Table 2: Parameter Estimates

	Specification	
Parameter	E	I
s	0.128	0.127
	(0.121, 0.135)	(0.121, 0.134)
δ	0.437	0.438
	(0.413, 0.463)	(0.415, 0.463)
k_u	0.244	0.243
	(0.215, 0.275)	(0.214, 0.277)
κ_1	0.309	0.35
	(0.283, 0.337)	(0.329, 0.365)
κ_2	-0.035	-0.048
	(-0.044, -0.026)	(-0.052, -0.043)
$h_u/(1-F(y_R))$	1.029	1.049
	(0.953, 1.125)	(0.967, 1.136)
μ_f	2.061	2.362
	(2.046, 2.076)	(2.237, 2.504)
σ_f	0.406	0.657
	(0.391, 0.42)	(0.612, 0.701)
$(1-\beta)/\beta$	–	3.078
		(2.282, 4.163)
μ_g	–	71.123
		(65.707, 82.645)
lnL	-20,701.546	-20,637.029
LR test		129.034
p value		$6.66x10^{-30}$

Note: Arrival rates are quarterly. In the brackets are the 5% and 95% percentiles from bootstrapping with 500 draws.

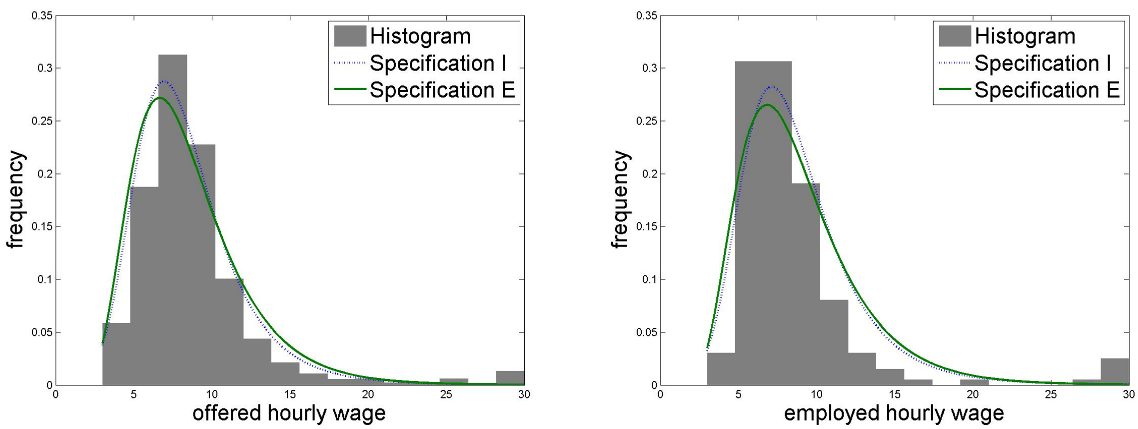
Table 2 presents the results from the estimation of the two likelihood functions under their respective specifications. In general, we find the unemployed exit the labor force at one-quarter the rate they enter employment. Also, those in the lower quartile of the wage distribution exit employment for non-participation 20% faster than those in the upper quartile. Note the transition rates remain relatively high compared to other studies; however, our sample consists of young individuals who experience higher transition rates. In particular, the exit rate δ states the average amount of time spent outside the labor force is 2.28 quarters. To some, this may seem high. However, the average duration outside the labor force (t_o) is 2.5 quarters (the estimate of δ is slightly different due to stock sampling and right censoring). Figure 2 provides a closer look at t_o.

For our analysis, α represents the key parameter estimate. Recall, in our formulation of the likelihood function, the value of the likelihood function under inefficient contract must be greater than the value of the likelihood function of the efficient contract. This occurs because the likelihood functions only differ in terms of the wage distributions, and the wage distributions remain identical if $\alpha = 0$; therefore the likelihood function of the efficient contract is nested in the likelihood function of the inefficient contract. This connection between the two likelihood functions allows for a clear comparison based on the fit of the wage distribution, and more specifically allows for a formal specification test (a likelihood ratio test). Regarding the fit, the likelihood ratio test, presented at the bottom of Table 2, rejects the efficient contract in favor of the inefficient contract. Thus, we are able to conclude that observed wages, transitions, and durations are not consistent with the predictions of the efficient contract, and instead remain more consistent with the inefficient contract.

Alternatively, it could be argued the market is efficient and the basic search model is misspecified. For instance, the incorporation of on-the-job search would change the wage contract and imply the estimation is misspecified. In terms of on-the-job search, we do observe job-to-job transitions which the current model does not capture. On the other hand, including it would not eliminate the negative externality discussed above. Actually, the similarities between the decision to exit employment for non-participation or another job is similar. In leaving the current job for a new and more productive job, agents still impose a negative externality on their current firm. Thus, while on-the-job search (and other extensions) represent critical additions to the understanding of whether the labor market is efficient, we believe our analysis highlights why such a discussion is important.

To further examine the results of the statistical test, we plot the key difference in Figure 3. This figure compares the two wage distributions (wages offered at the time of hire (w_u) and employed wage distribution (w_e)) observed in the data against the predicted distributions. Although the distinctions are subtle in the figure, the inefficient contract appears to fit the distribution better than the efficient contract. In particular, the inefficient contract has a larger mass of individuals at the lower end of the distribution, matching a noticeable feature of the observed distribution. Analytically, this arises from the second term in (30). If the term was constant or a linear function of w, then the inefficient and efficient distributions would be hard to differentiate as the location and scale parameters of the productivity distribution would compensate this difference. Therefore, the key is the way in which the last term of (30) changes. In particular, the term grows with y, implying the proportion of the surplus going to workers shrinks as y grows. This occurs because the rates of exit, $k_e(y)$, fall as output, y, and wages, w, grow. As a result, the inefficient contract produces a larger weight at the lower end of the distribution. Intuitively, the agents are losing their ability to bargain as productivity increases because their outside option is becoming less appealing. Therefore, the inefficient contract predicts that the wage rises as productivity rises, but at a slower rate. Given this, the inefficient contract has less dispersion relative to the efficient contract, in addition to predicting the larger mass of individuals in the lower end of the distribution.

Figure 3: Offered and Employed Hourly Wage Distributions



Given the employment contract featuring only a fixed wage best describes the data, we explore the differences in welfare between the fixed wage contract and the efficient contract. To demonstrate the differences, we use the parameters from Specification I (discussion of the identification of the estimates is provided in Section 5.3). Table 3 presents the list of the parameters and their values.

6 Welfare & efficiency

Based on the analysis thus far, we have concluded that the fixed employment contract represents the more appropriate model for capturing the observed labor market equilibrium. To quantitatively assess the implications of this, we investigate the available welfare gains of moving to the efficient

Table 3: Parameters

r	0.01	discount rate
b	0.002	unemployed utility flow
x	0.002	outside the labor force utility flow
A	1.049	efficiency of matching function
η	0.245	elasticity of matching function
β	0.245	bargaining power of workers
γ	21.442	recruiting cost
s	0.127	job destruction rate
μ_f	2.362	mean of log normal distribution of job productivity
σ_f	0.657	s.d. of log normal distribution of job productivity
μ_g	71.122	mean of exponential distribution of outside options
λ_u	0.475	unemployed flow of outside options
λ_e	0.591	employed flow of outside options
δ	0.438	non-participant separation rate

equilibrium. To accomplish this, we perform the following counterfactual exercise. We take the parameters estimated for the fixed wage contract, and calculate the equilibrium of the model using the efficient contract. From these calculations, we compare how the levels of unemployment, employment, labor force participation, and ultimately welfare would change if the market moved from the inefficient contract to the efficient one.

To perform the counterfactual exercise, we use the following welfare function

$$W = n_u(b + \lambda_u \int_{\varepsilon_u} \varepsilon dG(\varepsilon) - \theta\gamma) + \int_{y_R} \left\{ n_e(y) \left[y + \lambda_e \int_{\varepsilon_e(y)} \varepsilon dG(\varepsilon) \right] \right\} dy + n_o x. \tag{31}$$

From Equation 31, welfare equals the sum of unemployed workers' consumption (b), the average output (y) produced in each match, the utility flow of those of non-participants (x), and the utility from taking the outside option and exiting the labor force, minus the recruiting expenses incurred by firms to find unemployed workers (γ).[15]

Table 4 presents the increase in welfare and the change in labor market outcomes, given a structural shift from a fixed wage contract to the efficient contract. From Table 4, labor force participation would rise by 9%, unemployment would decrease by 10%, and employment would increase by 17%. When employed agents internalize the negative externality imposed by their

[15]The number of vacancies in equilibrium is equal to the product of market tightness, θ, and the measure of unemployed workers, n_u.

Table 4: Labor Market Outcomes from Shift in Contract

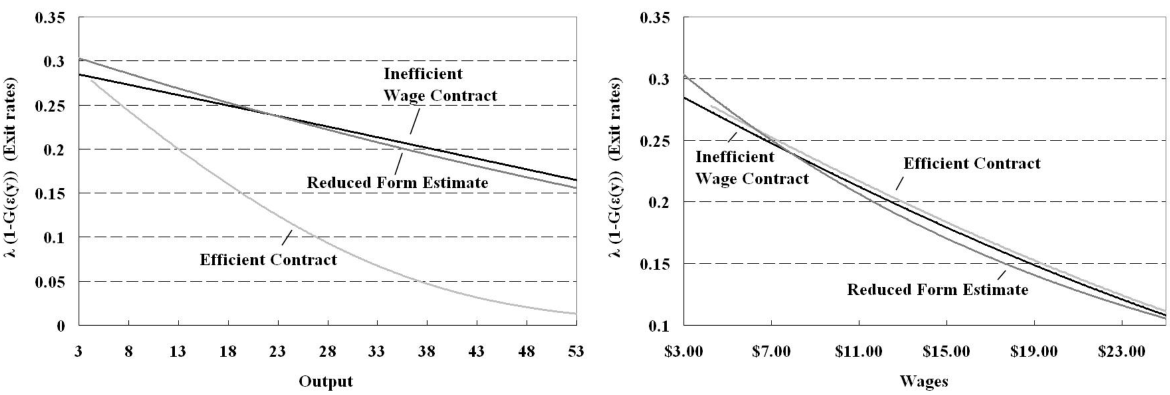
Parameters	Estimated Contract (Inefficient)	Counterfactual Contract (Efficient)	Percentage change
Not in Labor Force (%)	36.4	30.5	-16.21
Unemployed (%)	17.4	15.5	-10.92
Employed (%)	46.2	54	16.88
Welfare	23.2	24.07	3.75

decisions to quit, they exit at a slower rate than in the fixed wage contract. As a result, labor force participation rises. Also, the number of vacancies opened by firms increases as matches become longer and more valuable. Therefore, we observe a lower unemployment rate as well. Overall, we see a shift to the efficient contract would result in an increase in welfare of 3.75%.

The rate employed agents exit the labor force represents a key component of the model. It also represents the primary difference between the efficient employment contract and the contract featuring only a fixed wage. The rate at which an agent exits the labor force from employment is given by $\lambda_e[1-G(\varepsilon_e(w(y)))]$. One prediction of this model is whether exit rates from employment to non-participation vary across the distribution of wages. Since $\varepsilon_e(w(y))$ increases with the wage, depending on the distribution of $G(\varepsilon)$, employed agents in the upper-end of the wage distribution may exit at a slower rate than those in the lower end of the distribution. Indeed, we observe different exit rates over the wage distribution. Figure 4 provides the reduced-form exit rates given by the likelihood function. They are plotted by wages and output. In addition, it displays the rate workers exit under each contract assuming Specification I and the parameter estimates it implies are correct. Thus, the efficient contract estimates that are displayed in the figure are the counterfactual results found when running the model with the parameter estimates from Specification I.

Figure 4 is important for two reasons. First, it provides support that the exit rates from employment to non-participation decline as wages rise. In other words, it supports the idea that workers are influenced by their wage when choosing to exit. Second, internalizing the cost of exiting represents the key to improving welfare. If one looks at the exit rates by the output of a job, then the efficient contract implies a much smaller exit rate as workers internalize the full cost of exiting. However, it might be surprising that workers exit at a slightly faster rate under the efficient contract when looking at it by the wage. The reason is in the inefficient contract, the wage does not coincide with the output of a job as it does in the case of the efficient contract. Moreover, in the inefficient contract, the wage moves further from output as the output of a job increases. Examining exit rates at a wage of \$15.00 for example, is the same as output of 15 in the efficient contract, while it corresponds to output of much more that 15 in the inefficient contract. Thus, that the exit rates for the efficient contract eventually lie above those for the inefficient contract when plotted by wages highlights both the nature of the inefficiency in wages, and that this inefficiency grows as the output of the job increases.

Figure 4: Employed Exit Rate by Output and Wage



7 Conclusion

We develop an equilibrium search model capable of accounting for observed transitions in and out of the labor market. When agents receive idiosyncratic shocks affecting the value of non-market activity, an employment contract consisting of only a fixed wage remains inefficient. In the efficient employment contract, agents receive a wage equal to the productivity of the match, and pay a hiring fee upon its formation. The analysis in this paper highlights the difficulties facing researchers attempting to study labor market outcomes, wage contracts, and the implications of various policies on these outcomes. Based on our conclusions, the efficient employment contract does not appear to be consistent with the data, suggesting a role for labor market policies, specifically those aimed at reducing the flow of agents from employment to inactivity.

In this paper we do not discuss potential policies that could provide a structural shift from the observed equilibrium to one more closely resembling the efficient contract. These alternatives provide an interesting direction for future research. For example, a tax and transfer scheme could be designed in a manner that produces wage contracts similar to the efficient contract. In addition, future research should also focus on determining what factors may be preventing the efficient outcome from occurring.

A Appendix. Proof of Lemma 1

Proof of Lemma 1 According to Nash's axioms, (ϕ, w) must be pairwise Pareto-efficient. Since the up-front payment ϕ allows the worker and the firm to transfer utility perfectly, the wage, w, must be chosen to maximize the total surplus of the match. The comparison of (6) and (12) shows that the match surplus is maximized iff $\mathscr{V}_f = 0$. From (9), $\mathscr{V}_f = 0$ requires $w = y$. Finally, the first-order condition of (13) with respect to ϕ yields (15).■

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