On Government Credit Programs

Marco Espinosa-Vega, Bruce D. Smith, and Chong K. Yip

Federal Reserve Bank of Atlanta
Working Paper 98-2
February 1998

Abstract: Credit rationing is a common feature of most developing economies. In response to it, the governments of these countries often operate extensive credit programs and lend, either directly or indirectly, to the private sector. We analyze the macroeconomic consequences of a typical government credit program in a small open economy. We show that such programs increase long-run production if the economy is in a development trap and that such programs often lead to endogenously arising aggregate volatility. On the other hand, they may eliminate certain indeterminacies created by endogenous credit market frictions.

JEL classification: E5, O4

Key words: credit rationing, credit programs, financial intermediaries

---

Please address questions regarding content to Marco Espinosa-Vega, Research Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, NW, Atlanta, Georgia 30303-2713, 404/521-8630, marco.espinosa@atl.frb.org; Bruce D. Smith, Economics Department, University of Texas at Austin, Austin, Texas 78712, 512/475-8548, bsmith@mundo.eco.utexas.edu; or Chong K. Yip, Economics Department, Chinese University of Hong Kong, Fung King Hei Building, Shatin, NT, Hong Kong, 852/2609-7057, chongkeeyip@cuhk.edu.hk.

Questions regarding subscriptions to the Federal Reserve Bank of Atlanta working paper series should be addressed to the Public Affairs Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, NW, Atlanta, Georgia 30303-2713, 404/521-8020. The full text of this paper may be downloaded (in PDF format) from the Atlanta Fed's World-Wide Web site at http://www.frbatlanta.org/publica/work_papers/.

On Government Credit Programs

1. Introduction

In most economies, the government intervenes in the financial system to a considerable extent. This is particularly true in developing countries, where credit market frictions are typically perceived to be large, and where rationing of credit is wide-spread.[1] Government interventions in credit markets take a variety of forms, including directed credit programs, loan guarantees, interest rate controls or subsidies, and direct lending or rediscounting by the Central Bank.[2] In this paper we analyze the macroeconomic consequences of direct government lending or rediscounting by the central bank, although our main results can also be applied to other large-scale government credit programs.

Clearly, the main motivation of such government programs has been to extend credit to rationed agents, and thereby promote investment, capital accumulation, and an increase in productivity. However, in practice government credit market

[1] See Smith and Stutzer (1989) or Gale (1990) for a discussion of some interventions in the credit market by the government in the U.S. Khatkate (1982b), Chavez (1983), Johnson (1983), Stockhausen (1983), or Bhatt (1988) discuss a representative set of credit market interventions in developing countries. See McKinnon (1973), Khatkate (1982a,b), Diaz-Alejandro (1985), Bhatt (1988) and Carter (1988) for discussions of credit rationing in these countries.

[2] For example, in Turkey central bank rediscounting was the dominant monetary policy instrument into the late 1980s ("The Turkish Economy and the Financial Sector," 1995, p.12). Central bank rediscounting in Mexico is discussed, for example in Chavez (1983).

interventions have often been viewed as counterproductive, reducing aggregate credit extension and interfering with capital investment.[3] In this paper we undertake an analysis of the aggregate effects of large-scale government lending programs or of the rediscounting of private lending by a central bank. Among other things, we focus on the long-run output consequences of such programs. But we also consider the effects of such programs on equilibrium dynamics, and on the potential for the indeterminacy or multiplicity of equilibrium.

To that end, we rely on a conventional monetary growth model (Diamond, 1965), reformulated so that capital investment may require of external finance. In our model, credit rationing arises endogenously because of a costly state verification (CSV) problem[4] in credit markets. Motivated by the presence of this rationing, the government (central bank) utilizes seignorage income to rediscount bank lending or to subsidize private credit extension. Also, in keeping with the circumstances relevant to most developing countries, we focus on a small open economy. Furthermore, in our model, lenders are subjected to a reserve requirement, just as intermediaries are in most developing economies.

In this framework, we are able to obtain the following results. First, if there

[3] Khatkate (1982b) and Bhatt (1988) give illustrative examples of counterproductive government credit programs.

[4] See Townsend (1979), Diamond (1984), Gale and Hellwig (1985), or Williamson (1986, 1987). The specific formulation we utilize is similar to that in Boyd and Smith (1997a,b).

is any steady state equilibrium where credit is rationed domestically, then there are typically at least two such equilibria. One has relatively high capital stock and output level; we refer to this as the high activity steady state. The other has a relatively low capital stock and output level. Moreover, for a wide range of parameter values both steady states can be approached. Hence development traps are possible; here these are the consequence of a CSV problem in credit markets.

The long-run aggregate consequences of a government lending program turn out to depend very heavily on which of the steady state equilibria prevails. In the high activity steady state we show that an increase in the volume of government lending (rediscounting) – or an increase in the magnitude of the implied subsidy on government loans – necessarily reduces the long-run capital stock and output level, as well as the volume of private credit extended to domestic residents. Thus, government credit market interventions are counterproductive – at least with respect to long-run output – in the high activity state. This effect is reversed, however, whenever the economy is caught in a development trap. In essence, then, the case for subsidized government lending as a means of stimulating production may make sense only if the economy is stranded in a low activity equilibrium to begin with.

With respect to shorter-run phenomena, we also show that the magnitude of government lending substantially affects the properties of dynamical equilibria, the scope for indeterminacy of equilibrium and the potential for endogenously arising volatility. More specifically, when government credit programs are not too large, we show that the low activity steady state is a saddle, and the dynamical equilibrium paths approaching it do so monotonically. Matters are much different near the high activity steady state, however. In particular, we state conditions where extremely small government lending leads the high activity steady state to be a sink. Since our economy has only a single initial condition, the result is an indeterminacy; there are multiple equilibrium paths that approach the high activity steady state. Moreover, as the size of the of the government credit program increases, we state conditions under which dynamical equilibria approaching the high activity steady state must exhibit fluctuations. In effect, then, government lending programs become responsible for endogenously arising volatility. This volatility, coupled with some of the findings described above, supports the view that these programs may interfere with macroeconomic performance.

Interestingly, as government credit programs become very large, the properties of the dynamical equilibria near each steady state become very different. For large government interventions the low activity steady state can become a sink, while the high activity steady state becomes a saddle. When this is the case, the high activity steady state is no longer indeterminate. But, as we show, it is also the case that paths approaching the high activity steady state necessarily display endogenously arising fluctuations as they do so.

This last point deserves some emphasis. As a whole, our results imply that small government credit programs will be most conducive to large output levels, near the high activity steady state. But they can also easily lead to the indeterminacy of equilibrium. Large government programs have adverse consequences for long-run output (again, near the high activity steady state), and they can lead to endogenously generated volatility. But they are consistent with a determinate equilibrium. Thus here as elsewhere,[5] a tension between the determinacy and the "efficiency" of equilibrium can easily arise.

Intuitively speaking, what accounts for the steady state results we have described above? The answer has to do with the interaction between the CSV problem, the domestic reserve requirement, the operation of international capital markets and the way the government lending program is financed. In a small open economy, agents lending domestically must receive the prevailing world real return on their investments. In addition, the domestic reserve requirement forces such agents to hold a portfolio consisting of domestic loans and domestic real balances.

[5] See Smith (1989, 1991, 1994) or Woodford (1994).

It is the return on this portfolio that must match the world rate of interest.

For a given domestic rate of inflation, this return is clearly determined by the rate of return on loans. In the presence of the CSV problem, the return on domestic loans depends on two factors: the domestic marginal product of capital, and the amount of internal finance provided by domestic investors.[6] A higher domestic capital stock reduces the marginal product of capital, but it also increases the income level –and the quantity of internal finance – of domestic investors. As a result, there are two ways in which domestic borrowers can offer the necessary expected return on loans. They can either have a high capital stock and income level, a large volume of internal finance, and a low marginal product of capital, or they can have a low capital stock, a low volume of internal finance, and a large marginal product of capital. Hence there will typically be two steady state equilibria: one with a high, and one with a low capital stock.

Government credit programs affect these equilibria in two ways. When the government increases the magnitude of its lending – or the implied subsidy on a given volume of lending – it must finance this by printing money. Thus the steady state rate of inflation rises. At the same time, the cost of funds to agents lending

[6] The point that the quantity of internal finance affects the rate of return perceived by external investors in the presence of a CSV problem was first made by Bernanke and Gertler (1989).

domestically – net of implied government subsidies – falls. Under weak conditions the interaction of these two effects implies that the real return that borrowers must offer lenders declines as the magnitude of government lending increases. In the high (low) activity steady state this is accomplished via a decline in the level of internal finance (the marginal product of capital), and a corresponding reduction (increase) in the steady state capital stock. This change in the capital stock also partially accounts for the results we obtain regarding dynamical equilibria.

The remainder of the paper proceeds as follows. Section 2 lays out the model environment, while section 3 describes trade in credit and factor markets. As a point of reference, section 4 analyzes the general equilibrium of a closed economy, and section 5 then examines the steady state equilibria of a small open economy. Section 6 takes up dynamics, and section 7 offers some concluding remarks.

2. The Model

2.1. Environment

Consider a small open economy consisting of an infinite sequence of two-period lived, overlapping generations. Each generation is identical in size and composition, and contains a continuum of agents with unit mass. Within each generation, agents are divided into two types: "potential borrowers" and "lenders". A fraction $\alpha \in (0,1)$ of the population is potential borrowers. Throughout, we let $t = 0, 1, ...$ index time.

At each date a single final good is produced using a constant returns to scale technology with capital and labor as inputs. We assume the final good to be mobile across borders, while factors of production are immobile. Let K_t denote the time t capital input, and L_t denote the time t labor input of a representative firm. Then its final output is $F(K_t, L_t)$. F satisfies the following conditions: it is increasing in each argument, strictly concave, and $F(0, L) = F(K, 0) = 0$ holds, for all K, L. In addition, if $k \equiv \frac{K}{L}$ is the capital-labor ratio, and if $f(k) \equiv F(k, 1)$ denotes the intensive production function, then $f' > 0 > f''$ holds $\forall k$, and f satisfies the standard Inada conditions. Finally, we assume that the inherited capital stock at date t is used in production, and that thereafter it depreciates completely.

With respect to endowments, all young agents are endowed with one unit of labor, which is supplied inelastically, and agents are retired when old. Individuals other than the old of period zero have no endowment of capital or final goods, while the initial old agents have an aggregate capital endowment of $K_0 > 0$.

Agents of all types are assumed to care only about old age consumption and, in addition, all agents are risk neutral. Thus all young period income is saved.

Potential borrowers and lenders are differentiated by the fact that each potential borrower has access to a stochastic linear technology for converting date t final goods into date $t+1$ capital. Lenders have no access to this technology.

The capital investment technology has the following properties. First, the investment technology is indivisible: each potential borrower has one investment project which can only be operated at the scale q. In particular, $q > 0$ units of the final good invested in one project at t yield zq units of capital at $t+1$, where z is an iid (across borrowers and periods) random variable, which is realized at $t+1$. We let G denote the probability distribution of z, and assume that G has a differentiable density function g with support $[0, \bar{z}]$. We let $\hat{z}$ denote the expected value of z.

The amount of capital produced by any investment project can be observed costlessly by the project owner. Any agent other than the project owner can observe the return on the project only by bearing a fixed cost of $\gamma > 0$ units of capital.[7]

[7] That is, in verifying the project return, γ units of capital are used up. The assumption that capital is consumed in the verification process follows Bernanke and Gertler (1989).

3. Trade

3.1. Factor Markets

We assume that capital and labor are traded in competitive markets at each date. Thus if w_t denotes the time t real wage rate and ρ_t is the time t capital rental rate, the standard factor pricing relationships obtain:

$$\rho_t = f'(k_t) \tag{3.1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t). \tag{3.2}$$

Notice that $w'(k) > 0$ holds and, in addition, we will assume the following.

Assumption 1. $w''(k) < 0; \forall k \geq 0.$

Assumption 1 is satisfied if, for example, f is any CES production function with elasticity of substitution no less than one. Assumption 1 guarantees the uniqueness of a non-trivial steady state equilibrium in the model without money and without access to foreign capital markets.

3.2. Credit Markets

All young agents at t supply one unit of labor inelastically, earning the real wage rate w_t. For lenders this income is saved, either in the form of money, assets issued abroad, or loans to domestic borrowers. Under the assumption that the domestic country is small, domestic residents then take the world interest rate as given. We can think of all domestic credit extension as being intermediated in the manner described by Williamson (1986).

Potential borrowers also have young period income w_t, and we will assume that they must obtain external financing to operate their investment projects.

Assumption 2. *$q > w(k_t)$ for all "relevant" values of k_t.*

Let b_t denote the amount borrowed by the operator of a funded project (in real terms) at t; clearly

$$b_t = q - w(k_t). \tag{3.3}$$

If potential borrowers wish to obtain external funding - which is necessary to operate their projects - they do so by announcing loan contract terms. These announced contract terms are either accepted or rejected by intermediaries: borrowers whose terms are accepted then operate their projects. Following Williamson (1986, 1987), a loan contract consists of the following objects. First, there is a set of project return realizations A_t for which verification of the return occurs at t. Verification of project returns does not occur if $z \in B_t \equiv [0, \bar{z}] - A_t$.[8] Second, if $z \in A_t$, then the contractual repayment can meaningfully be made contingent on the project return. Thus if $z \in A_t$ we denote the promised payment (per unit borrowed) by $R_t(z)$. On the other hand, if $z \in B_t$ then the loan payment cannot meaningfully depend on the project return, and the only incentive compatible loan contract offers an uncontingent payment of x_t (per unit borrowed) for all $z \in B_t$. All payments specified by any loan contract are in real terms.

Loan contracts offered by borrowers are either accepted or rejected by intermediaries who - without loss of generality - we can think of as making all domestic loans. Thus intermediaries take deposits, make loans, and conduct monitoring of project returns as required by the contracts they accept. We assume that any lender can establish an intermediary. Then, in equilibrium, intermediaries will be perfectly diversified, earn zero profits, and have a nonstochastic return on their portfolios. Thus they need not be monitored by their depositors.[9]

Intermediaries accept deposits taking their gross opportunity cost- r^d_{t+1} be-

[8] We thus abstract from stochastic state verification. While this is a real restriction, Boyd and Smith (1994a) show that the welfare gains from stochastic monitoring are trivial when realistic parameter values are assumed.

[9] Intermediation in this context is discussed by Williamson (1986). See Krasa and Villamil (1992) for a consideration of intermediaries that cannot perfectly diversify risk.

tween t and $t+1$ - as given, and they act as if they can obtain any desired quantity of deposits at that rate. It follows that intermediaries are willing to accept loan contract offers yielding an expected return of at least r^d_{t+1}. Thus loan contract offers (that have any prospect of acceptance) must satisfy the expected return constraint

$$\int_{A_t} [R_t(z)b_t - \rho_{t+1}\gamma]g(z)dz + x_t b_t \int_{B_t} g(z)dz \geq r^d_{t+1} b_t. \tag{3.4}$$

In particular, expected repayments must at least cover the intermediary's cost of funds - $r^d_{t+1}b_t$ - plus the real expected monitoring cost

$$\rho_{t+1}\gamma \int_{A_t} g(z)dz.$$

The expected monitoring cost depends on ρ_{t+1} because γ units of *capital* are expended when project returns are verified. Finally, since only project owners directly observe project returns, they must have the proper incentives to correctly reveal when a monitoring state has occurred. The appropriate incentive constraint is

$$R_t(z) \leq x_t; \quad z \in A_{t.} \tag{3.5}$$

In addition, the repayments specified by any contract must be feasible for the borrower, so that

$$R_t(z) \leq \frac{\rho_{t+1} z q}{b_t}; \quad z \in A_t \tag{3.6}$$

$$x_t \leq \inf_{z \in B_t} [\frac{\rho_{t+1} z q}{b_t}]. \tag{3.7}$$

Equations (3.6) and (3.7) require that repayments never exceed the real value of the capital yielded by an investment project, which in state z is $zq\rho_{t+1}$ at $t+1$.

Borrowers, then, will maximize their own expected utility by choice of contract terms, subject to the constraints just described. Therefore, announced loan contracts at date t will be selected to maximize

$$\rho_{t+1}\hat{z}q - b_t \int_{A_t} R_t(z) g(z) dz - x_t b_t \int_{B_t} g(z) dz$$

subject to (3.4)-(3.7).

The solution to the borrower's problem is, as is well-known, to offer a standard debt contract (modified for the presence of internal finance). In particular, the borrower either repays x_t (principal plus interest) or else defaults. In the latter case the lender monitors the project, and retains the proceeds of the project net of monitoring costs. Formally,

Proposition 3.1. *Suppose $q > b_t$. Then the optimal contractual loan terms satisfy*

$$R_t(z) = \frac{\rho_{t+1} z q}{b_t}; \quad z \in A_t \tag{3.8}$$

$$A_t = [0, \frac{x_t b_t}{q \rho_{t+1}}) \tag{3.9}$$

$$r_{t+1}^d = \int_{A_t} \left[R_t(z) - \frac{\rho_{t+1}\gamma}{b_t} \right] g(z)dz + x_t \int_{B_t} g(z)dz. \tag{3.10}$$

The proof of Proposition 1 is standard,[10] and we omit it here.

For future reference, we substitute (3.8) and (3.9) into (3.10). Then, the expected return received by a lender under the optimal contract is given by

$$\int_{A_t} \left[R_t(z) - \frac{\rho_{t+1}\gamma}{b_t} \right] g(z)dz + x_t \int_{B_t} g(z)dz$$

$$= x_t - \frac{\rho_{t+1}\gamma}{b_t} G(\frac{x_t b_t}{\rho_{t+1} q}) - \frac{\rho_{t+1} q}{b_t} \int_0^{\frac{x_t b_t}{\rho_{t+1} q}} G(z)dz \equiv \pi \left[x_t; \frac{b_t}{\rho_{t+1}} \right]. \tag{3.11}$$

The function π gives the expected return to the lender as a function of the gross

[10] See Gale and Hellwig (1985) or Williamson (1986, 1987).

loan rate, x_t, the amount of external finance required, b_t, and the future relative price of capital, ρ_{t+1}.

It will be useful in what follows to put some additional structure on the function π. In particular, we will assume the following.

Assumption 3. $g(z) + (\frac{\gamma}{q})g'(z) \geq 0$; *for all* $z \in [0, \bar{z}]$.

Assumption 4. $\pi_1[0, (\frac{b_t}{\rho_{t+1}})] > 0$.

Assumption 3 implies that $\pi_{11} < 0$. Thus, if assumption 4 holds, the function π has the configuration depicted in Figure 1. Evidently, depending on the value of $\frac{b_t}{\rho_{t+1}}$, there is a unique value of x_t which maximizes the expected return that can be offered to any lender. We will denote this value by $\hat{x}(\frac{b_t}{\rho_{t+1}})$, where the function $\hat{x}$ is defined implicitly by

$$\pi_1\left[\hat{x}(\frac{b_t}{\rho_{t+1}}); \frac{b_t}{\rho_{t+1}}\right] \equiv 1 - (\frac{\gamma}{q})g\left[\hat{x}(\frac{b_t}{\rho_{t+1}})\frac{b_t}{\rho_{t+1}q}\right] - G\left[\hat{x}(\frac{b_t}{\rho_{t+1}})\frac{b_t}{q\rho_{t+1}}\right] \equiv 0. \quad (3.12)$$

Equation (3.12) and Assumption 3 imply that

$$\hat{x}(\frac{b_t}{\rho_{t+1}})\frac{b_t}{\rho_{t+1}q} \equiv \eta \quad (3.13)$$

where $\eta > 0$ is a constant satisfying $1 - (\frac{\gamma}{q})g(\eta) - G(\eta) \equiv 0$. When all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return, η is the critical project return for which a borrower's project income exactly covers loan principal plus interest. In other words, project return verification occurs iff $z \in [0, \eta)$.

3.3. Credit Rationing

One feature of the environment under consideration - which was originally noted by Gale and Hellwig (1985) and Williamson (1986, 1987) - is that it can allow for the existence of unfulfilled demand for credit. In particular, if all borrowers desire to operate their projects at date t, the total (per capita) demand for credit is αq. The total per capita supply of saving is $w(k_t)$ at t. Domestic credit demand exceeds domestic credit supply, and hence credit must be rationed in the closed economy, if the following assumption holds for all $t \geq 0$.

Assumption 5. $\alpha q > w(k_t)$.

When credit rationing exists, however, it also must be the case that

$$x_t = \hat{x}(\frac{b_t}{\rho_{t+1}}). \tag{3.14}$$

When equation (3.14) holds, all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return. Rationed (unfunded) potential borrowers cannot then obtain credit by changing loan contract terms, since doing so simply reduces the expected return perceived by (all) lenders. Thus if assumption 5 and equation (3.14) hold at date t, credit rationing is an equilibrium outcome for a closed economy. We henceforth focus on economies where credit rationing occurs at all dates.[11]

3.3.1. Payoffs Under Credit Rationing

We now describe the expected payoffs received by lenders and (funded) borrowers at t when credit is rationed. For lenders, equations (3.11) and (3.14) imply that the expected return on bank loans is given by

$$\begin{aligned}
\pi\left[\hat{x}(\frac{b_t}{\rho_{t+1}});\frac{b_t}{\rho_{t+1}}\right] &\equiv \frac{\rho_{t+1}q}{b_t}\left\{\hat{x}(\frac{b_t}{\rho_{t+1}})\frac{b_t}{\rho_{t+1}q}-(\frac{\gamma}{q})G\left[\hat{x}(\frac{b_t}{\rho_{t+1}})\frac{b_t}{\rho_{t+1}q}\right]-\int_0^{\frac{\hat{x}b_t}{\rho_{t+1}q}}G(z)dz\right\} \\
&\equiv \frac{\rho_{t+1}q}{b_t}\left[\eta-(\frac{\gamma}{q})G(\eta)-\int_0^{\eta}G(z)dz\right]\equiv r_t^l, \qquad (3.15)
\end{aligned}$$

[11] The assumption that credit rationing obtains is maintained because it results in a substantial technical simplification. However, credit rationing is clearly a widespread phenomenon in developing countries (McKinnon, 1973), and there is substantial evidence of significant rationing of credit even in the United States (Japelli, 1990). Therefore this does not seem to be an empirically unreasonable assumption.

where r_t^l denotes the gross real return on loans. Note that equation (3.15) asserts that the return to a lender between t and $t+1$ depends only on the ratio $\frac{\rho_{t+1}}{b_t}$ and parameters (and is proportional to that ratio) when credit rationing obtains.

It is also straightforward to demonstrate that the expected utility of a funded borrower under credit rationing is given by

$$\rho_{t+1}\hat{z}q - r_{t+1}^l b_t - \rho_{t+1}\gamma G\left[\hat{x}(\frac{b_t}{\rho_{t+1}})\frac{b_t}{\rho_{t+1}q}\right] \equiv \rho_{t+1}q\left[\hat{z} - (\frac{\gamma}{q})G(\eta)\right] - r_{t+1}^l b_{t.}$$

Since any potential borrower always has the option of foregoing his project and depositing his income in a bank, all potential borrowers can guarantee themselves the utility level $r_{t+1}^d w_t$, where r_{t+1}^d is the rate of return on deposits between t and $t+1$. Thus (potential) borrowers prefer borrowing to lending under credit rationing iff

$$\rho_{t+1}q\left[\hat{z} - (\frac{\gamma}{q})G(\eta)\right] - r_{t+1}^l b_t \geq r_{t+1}^d w_t. \tag{3.16}$$

We now define

$$\begin{aligned}\phi &\equiv \hat{z} - (\frac{\gamma}{q})G(\eta) \\ \psi &\equiv q\left[\eta - (\frac{\gamma}{q})G(\eta) - \int_0^\eta G(z)dz\right].\end{aligned} \tag{3.17}$$

The variable ϕ represents the expected project yield per unit invested, under credit rationing, net of capital consumed by monitoring.[12] Similarly, ψ determines the expected rate of return received by lenders under credit rationing, since

$$r^l_{t+1} \equiv \psi \frac{\rho_{t+1}}{b_t} \equiv \psi \frac{\rho_{t+1}}{[q - w(k_t)]}. \tag{3.18}$$

Then (3.16) reduces to $q\rho_{t+1}(\phi - \psi) \geq r^d_{t+1} w_t$.

4. General Equilibrium

As a benchmark, we begin by considering a closed economy confronted with the credit market friction just described. This section also provides a number of ingredients that are useful for the analysis which follows. We start first with a non-monetary economy, and then move on to consider the behavior of a monetary economy when a reserve requirement has been imposed on agents engaged in lending activity.[13]

[12] Of course, we assume that $\phi > 0$.

[13] For a discussion of a closed, monetary economy, see Boyd and Smith (1997b).

4.1. A Closed Economy

When credit is rationed, let μ_t denote the fraction of potential borrowers who do obtain credit at t. Since each funded borrower borrows $b_t = q - w(k_t)$, total (per capita) loans are $\alpha\mu_t[q - w(k_t)]$. The total supply of savings is $(1-\alpha)w(k_t)$ plus $\alpha(1-\mu_t)w(k_t)$, since unfunded borrowers simply deposit their income with an intermediary and, in effect, become lenders. Thus an equality between "sources" and "uses" of funds requires that

$$\alpha\mu_t[q - w(k_t)] = (1-\alpha)w(k_t) + \alpha(1-\mu_t)w(k_t). \tag{4.1}$$

Equation (4.1) reduces to

$$\alpha q\mu_t = w(k_t). \tag{4.2}$$

Assumption 5 implies that $\mu_t < 1$ holds, for all $t \geq 0$.

Under our assumption that returns on investment projects are iid across borrowers, the fact that there is a large number of borrowers implies that there is no aggregate randomness in this economy. In particular, the time $t+1$ per capita capital stock is simply $\hat{z}\alpha q\mu_t = \hat{z}w(k_t)$, less capital expended on monitoring at $t+1$. The amount of capital consumed by monitoring is simply

$\gamma\alpha\mu_t G(\frac{x_t b_t}{\rho_{t+1} q}) = \gamma\alpha\mu_t G(\eta) = \frac{\gamma}{q} G(\eta) w(k_t)$ under credit rationing. Thus

$$k_{t+1} = \left[\hat{z} - (\frac{\gamma}{q})G(\eta)\right] w(k_t) = \phi w(k_t) \tag{4.3}$$

gives the equilibrium law of motion for k_t when credit is rationed at all dates. We will adopt the following assumption.

Assumption 6. $\phi w'(0) > 1$.

When assumption 6 holds, equation (4.3) has the configuration depicted in Figure 2. More specifically, given assumption 1, k_{t+1} is an increasing, concave function of k_t. Assumption 6 implies that the locus defined by (4.3) lies above the 45° line for sufficiently small values of k_t, and standard arguments establish that it lies below the 45° line for large enough values of k_t. Thus assumptions 1 and 6 imply the existence of a unique value $k^* > 0$ that satisfies $k^* = \phi w(k^*)$. Clearly, this steady state is asymptotically stable.

In order for credit rationing to obtain in the steady state, it is necessary that k^* satisfy two conditions. First,

$$\alpha q > w(k^*) \tag{4.4}$$

must hold so that assumption 5 is satisfied. Second, in an economy with a single asset, the rates of return on deposits and loans must be equal. It is then easy to verify that (3.16) is satisfied iff

$$\phi[q - w(k^*)] \geq \psi. \tag{4.5}$$

If (4.4), (4.5), and $k_0 < k^*$ hold, then our economy does experience credit rationing, and potential borrowers prefer borrowing over lending at all dates. It is straightforward to produce examples where (4.4), (4.5), and our other assumptions are satisfied. For instance, here is one such example.

Example 1 Let $f(k) = k^{0.5}$, $q = 2.2$, $g(z) = \frac{1}{\bar{z}}$ with $\bar{z} = 20.302$, $\gamma = 24.174$, and $\alpha > 0.95$ hold. For these parameter values, $\phi = 5.11$ and $\psi = 4.7$. Then the unique (nontrivial) steady state capital stock for the closed, nonmonetary economy is $k^* = 6.528$, and it is easy to verify that (4.4) and (4.5) are satisfied.

4.2. A Closed Monetary Economy with a Reserve Requirement

In this section we introduce two additional factors; money, and a reserve requirement that is imposed on lenders. More specifically, in this section we assume that there is a constant stock of domestic currency, issued in the amount M per capita. In addition, we assume that all agents making loans are subject to a requirement that they hold a certain quantity of this currency per dollar lent. Such reserve requirements are very common in developing countries [see e.g. Brock (1989) and Espinosa (1994)].

Let p_t denote the (domestic) price level at time t, and let $m_t \equiv M/\ p_t$ be the per capita quantity of real balances. We assume that all lenders are subject to the requirement that the real balances they hold equal at least λ times the value of their loan portfolio, where the value λ is chosen by the government. Thus, in the aggregate

$$m_t \geq \lambda \alpha \mu_t \left[q - w\left(k_t\right)\right] \tag{4.6}$$

must hold. If $\frac{p_t}{p_{t+1}} < \psi \frac{f'(k_{t+1})}{[q-w(k_t)]} \equiv r_t^l$ holds, then the reserve requirement in (4.6) will be binding, and all lenders - who we can think of as intermediaries - will hold λ units of domestic real balances per unit lent. Hence the return on $1+\lambda$ units of funds deposited at t - of which one unit is lent and λ units are held as reserves - is given by

$$r_{t+1}^d = \frac{1}{1+\lambda}\left\{\psi \frac{f'(k_{t+1})}{[q-w(k_t)]} + \frac{p_t}{p_{t+1}}\lambda\right\}.$$

Or alternatively, defining $\theta \equiv 1/(1+\lambda)$ and $R_t^m \equiv \frac{p_t}{p_{t+1}}$,

$$r^d_{t+1} = \theta\psi \frac{f'(k_{t+1})}{[q - w(k_t)]} + (1-\theta)R^m_t. \tag{4.7}$$

Notice that $(1-\theta)$ is a conventional reserve requirement. Throughout we assume that $\theta > 1/2$, so that the reserve requirement is not too large.

As before, sources of funds in this economy are simply per capita savings, $w(k_t)$, while uses of funds are investments, $\alpha q \mu_t$, plus the accumulation of real balances, m_t. Hence sources and uses of funds are equal when

$$\alpha q \mu_t + m_t = w(k_t). \tag{4.8}$$

Assuming that the reserve requirement is binding, equations (4.6) and (4.7) imply that

$$\alpha q \mu_t = \frac{w(k_t)}{1 + \frac{\lambda}{q}[q - w(k_t)]}. \tag{4.9}$$

We now derive the equilibrium law of motion for the domestic capital stock. Clearly, at $t+1$, the domestic per capita capital stock is equal to $\hat{z}\alpha q \mu_t$ minus the capital expended on monitoring at $t+1$, $\gamma \alpha \mu_t G(\eta)$. It follows that the capital stock evolves according to

$$k_{t+1} = \left[\widehat{z} - \frac{\gamma}{q}G(\eta)\right] \frac{w(k_t)}{1 + \frac{\lambda}{q}[q - w(k_t)]} = \frac{\phi w(k_t)}{1 + \frac{\lambda}{q}[q - w(k_t)]}. \tag{4.10}$$

If $\phi w'(0) > 1 + \lambda$, then the law of motion given in (4.7) has the configuration depicted in Figure 3. Under this condition there necessarily exists at least one nontrivial capital stock $\overline{k}$ satisfying (4.10) when $k_t = k_{t+1} = \overline{k}$. Moreover, if we let k_l denote the largest steady-state level of capital stock, it is easy to show that $k_l < k^*$ holds. It follows that assumption 5 will be satisfied, in the steady state, if $\alpha q > w(k^*)$ obtains.

In this economy, the return on loans satisfies

$$r_{t+1}^l = \frac{\psi f'(k_{t+1})}{[q - w(k_t)]}.$$

It is easy to show that, when the reserve requirement binds, the return on real balances satisfies

$$R_t^m = \frac{m_{t+1}}{m_t} = \frac{k_{t+2}[q - w(k_{t+1})]}{k_{t+1}[q - w(k_t)]}.$$

The following proposition is a special case of Huybens and Smith's (1997) proposition 2:

Proposition 4.1. *(a) The reserve requirement binds at t along the equilibrium* *path defined by (4.10) iff*

$$\frac{\phi w(k_{t+1})}{1+\frac{\lambda}{q}[q-w(k_{t+1})]} \leq \psi \frac{k_{t+1}f'(k_{t+1})}{[q-w(k_{t+1})]}.$$

(b) The reserve requirement binds in any steady state with credit rationing if $\psi > f^{-1}(q)$ holds.

In addition, the steady state return on money in this economy is given by $R^m = 1$, while the steady state return on loans is given by $\frac{\psi f'(\overline{k})}{[q-w(\overline{k})]}$. Hence the reserve requirement binds in steady state iff $\psi H(\overline{k}) > 1$. As noted in part (b) of the proposition, this condition must be satisfied if $\psi H[f^{-1}(q)] = \frac{\psi}{f^{-1}(q)} > 1$ holds.

Finally, it remains to state conditions under which borrowers will prefer to borrow rather than lend. In a steady state equilibrium the appropriate condition is:

$$\frac{[\phi q-\psi]f'(\overline{k})}{w(\overline{k})} > \frac{\theta\psi f'(\overline{k})}{[q-w(\overline{k})]} + (1-\theta).$$

It is straightforward to check that this condition must be satisfied if (4.5) holds.

5. A Small Open Economy with Government Credit Subsidies

We now turn to our primary task: to examine how government credit subsidies (undertaken on a large scale) affect the equilibria of a small open economy. For obvious reasons, we focus on monetary economies. In addition we assume that the domestic government imposes a reserve requirement on all agents engaged in domestic lending. Such an assumption serves two purposes. First, for most developing countries it is realistic. Second, it enables the domestic currency to be held even though it is dominated in rate of return. Finally, we assume that government loan programs are financed via money creation. Again for many developing countries this is quite realistic. Moreover, it enables our model to capture an important aspect of government policy in many LDC's: the central bank is prominently engaged in domestic lending and in rediscounting the loans of private banks. The resources required to do this come from printing money.

Often, the rationale for such policies is that financial market frictions lead credit to be rationed domestically. One such friction is the CSV problem which arises endogenously. We now examine how this friction interacts with the operation of international capital markets, and with the lending programs conducted by the government.

5.1. International Capital Markets

We assume that the domestic residents can borrow and lend in world capital markets at the world rate of interest R.[14] Since the domestic country is assumed to be small, the activity of its residents or its government does not affect this rate.

As before, we assume that all agents lending domestically are subject to a reserve requirement that, for every dollar lent, λ dollars of domestic currency must be held. Hence, if $m_t \equiv M_t/p_t$ denotes the per capita supply of domestic real balances, in the aggregate m_t must continue to satisfy equation (4.6). If $R > 1$ holds, then clearly the domestic reserve requirement will be binding. In addition, we assume that the domestic country operates under a regime of flexible exchange rates, so that it is free to finance its expenditures via money creation.

5.2. Government Credit Policy

We assume that the domestic government conducts its lending policy by rediscounting the loans of private sector banks. More specifically, for every dollar of private sector lending, the government extends τ/θ dollars of credit to private

[14] R is a gross real rate. Also, foreigners who lend in the domestic economy are subject to domestic reserve requirements.

banks. Given the presence of binding reserve requirements in our economy, this is equivalent to the assumption that the government lends τ to each bank per dollar of assets held. Clearly, we impose $\tau \in [0, 1]$.

On each unit that the government lends, it charges the gross real rate of interest r. We assume that $r \leq 1$ holds and, in order for government loans to be attractive to banks, $R > r$ must hold in equilibrium as well. We do not impose that $r \geq R_t^m$ must be satisfied: if this condition fails the central bank charges negative nominal rates of interest at t. This can be interpreted as the government extends credit in combination with a subsidy. In the extreme case $r = 0$, the government is simply offering a lump-sum transfer to each bank. Note that both r and τ are policy parameters that are selected by the government.

In order to finance its lending activity, we assume that the central bank issues money. At date t, let μ_t denote the fraction of domestic borrowers who receive loans. Then the private sector extends loans with a real rate value of $\alpha\mu_t[q - w(k_t)]$ at t. The central bank finances a fraction τ of this lending; hence its loans at t equal $\tau\alpha\mu_t[q - w(k_t)]$. Similarly when the government extends loans of $\tau\alpha\mu_{t-1}[q - w(k_{t-1})]$ at t-1, its interest income at t equals $r\tau\alpha\mu_{t-1}[q - w(k_{t-1})]$. The difference between time t loans and time t interest income must be financed with seignorage income. Hence, the time t government budget constraint is given by

$$\frac{M_t - M_{t-1}}{p_t} = \tau\alpha\mu_t[q - w(k_t)] - r\tau\alpha\mu_{t-1}[q - w(k_{t-1})]. \tag{5.1}$$

Clearly, when $\tau = 0$, the government program is inactive.

5.3. General Equilibrium

When $R > 1$ holds, the reserve requirement necessarily binds. Hence (4.6) holds with equality. That condition, along with the government budget constraint implies that

$$R_t^m = r\tau\left(\frac{\theta}{1-\theta}\right) + \left[1 - \tau\left(\frac{\theta}{1-\theta}\right)\right]\frac{k_{t+2}[q - w(k_{t+1})]}{k_{t+1}[q - w(k_t)]}\,. \tag{5.2}$$

In addition, domestic lenders hold portfolios with a weight θ attached to loans, and a weight $(1-\theta)$ attached to reserves. Hence the return on a bank's assets is given by

$$\theta\psi\frac{f'(k_{t+1})}{[q - w(k_t)]} + (1-\theta)R_t^m.$$

Each bank raises funds by borrowing a fraction $(1-\tau)$ of its funds in private capital markets, and it borrows τ from the government. Hence any bank's cost of funds equals $(1-\tau)R+\tau r$. If there is free entry into domestic lending each bank must earn zero profits; hence in equilibrium,

$$\theta\psi\left[\frac{f'(k_{t+1})}{[q-w(k_t)]}\right]+(1-\theta)R_t^m=(1-\tau)R+\tau r;\ t\geq 1, \tag{5.3}$$

must obtain.

Equations (5.2) and (5.3) govern the evolution of the domestic capital stock, $\{k_t\}$. Having obtained this sequence, equation (5.3) gives the equilibrium inverse inflation rate, R_t^m. Finally, sources and uses of funds must be equal. Sources of funds are domestic per capita savings $w(k_t)$. Uses of funds are domestic investments, $\alpha q\mu_t$, plus real balances m_t, plus net claims acquired abroad, s_t. Hence

$$s_t=w(k_t)-\alpha q\mu_t-m_t=w(k_t)-(k_{t+1}/\phi)-m_t \tag{5.4}$$

Equation (5.4) allows one to deduce the net flows of capital into and out of the domestic country.

5.4. Steady State Equilibria

We begin our analysis of equilibria by examining steady states. Dynamical equilibria are studied in section 6.

In steady state, the government budget constraint reduces to

$$m(1 - R^m) = (1 - r)\tau\alpha\mu[q - w(k)]. \tag{5.5}$$

In addition when the reserve requirement binds, we have

$$m = \lambda\alpha\mu\left[q - w\left(k\right)\right] = \left(\frac{1-\theta}{\theta}\right)\alpha\mu[q - w(k)]. \tag{5.6}$$

Equations (5.5) and (5.6) imply that the steady state return on reserves is given by

$$R^m = 1 - (1 - r)\tau\left(\frac{\theta}{1-\theta}\right) \tag{5.7}$$

If we now define the function $H(k)$ by

$$H(k) = \frac{f'(k)}{[q - w(k)]},$$

then (5.3) and (5.7) imply that steady state capital stock satisfies the condition

$$\theta\psi H(k) + (1-\theta)\left[1 - (1-r)\tau\left(\frac{\theta}{1-\theta}\right)\right] = (1-\tau)R + \tau r \tag{5.8}$$

or equivalently,

$$\theta\psi H(k) = R - (1-\theta) + \tau[r - R + \theta(1-r)]. \tag{5.9}$$

Evidently, in order to characterize steady states, it will be useful to know more about the function $H(k)$. Some of its properties are stated in the following lemma.

Lemma 5.1. *The function H satisfies*

(a) $\lim_{k\to 0} H(k) = \infty$

(b) $\lim_{k\to \hat{k}} H(k) = \infty$, *where* $\hat{k} \equiv w^{-1}(q)$

(c)$H'(k) \leq 0; k \leq f^{-1}(q)$, *and*

(d)$H'(k) \geq 0; k \geq f^{-1}(q)$.

Proof. Part (a) is, of course, immediate from the Inada conditions and $w(0) = 0$. Part (b) is also obvious. For (c) and (d), straightforward differentiation yields

$$H'(k) = -f''(k)\frac{[f(k) - q]}{[q - w(k)]^2},$$

establishing the result. ■

Lemma 1 implies that the function H has the configuration depicted in Figure 4. The following proposition is now immediate.

Proposition 5.2. *(a) Suppose that*

$$\left[\frac{R-(1-\theta)+\tau[r+\theta(1-r)-R]}{\theta\psi}\right] > H[f^{-1}(q)]. \tag{5.10}$$

Then there are exactly two values of k, denoted by k_1 and k_2 in Figure 4, that satisfy (5.9).

(b) Suppose that

$$\left[\frac{R-(1-\theta)+\tau[r+\theta(1-r)-R]}{\theta\psi}\right] < H[f^{-1}(q)] \tag{5.11}$$

Then there is no monetary steady state with credit rationing in the presence of foreign asset flows and a reserve requirement.

When equation (5.10) is satisfied there are two candidate steady state equilibria. Their associated capital stocks are denoted by k_1 and k_2 in Figure 4. In addition, in order to verify that credit is rationed in each steady state, it is necessary to check that the solutions to (5.9) imply that $\mu < 1$ holds. It is easy to show that $\mu < 1$ in each steady state iff $k_2/\phi\alpha q < 1$. It is also necessary to verify that $R^m < R$ holds, so that the reserve requirement binds. This will be the case if $R > 1$, although clearly this is sufficient rather than necessary for the reserve requirement to bind. Finally, we must check that borrowers prefer borrowing to lending in each candidate steady state. It is straightforward to verify that this requirement of equilibrium will be satisfied at each candidate steady state if $\frac{(\phi q-\psi)f'(k_2)}{w(k_2)} > R$.

Parenthetically, one may ask how credit rationing can arise in a small open economy, which presumably can absorb large quantities of funds without affecting world capital markets? The answer derives from the presence of the CSV problem. At date t, the domestic economy has an inherited capital stock, k_t. Given k_t —and $w(k_t)$— and given the perfect foresight domestic price level sequence $\{p_t\}$, domestic borrowers must offer a return on loans no less than $R-(1-\theta)R^m$ in order to compete for funds. The highest expected return that domestic agents can offer at $t+1$ is $\psi f'(k_{t+1})/[q-w(k_t)]$. Thus, given k_t and R_t^m, there is a largest value of k_{t+1} that permits domestic borrowers to compete in international financial markets. Credit rationing arises, then, because any further domestic credit extension would be inconsistent with domestic borrowers offering a competitive rate of return on loans.[15]

[15] For a small open economy there may also be equilibria in which credit is not rationed [see Huybens and Smith (1997)]. It is easy to describe conditions under which equilibria without credit rationing have the feature that all domestic investment is self-financed. When this is the case, a government loan program will obviously be irrelevant. For this reason, we confine our attention to equilibria in which credit is rationed.

We close this section with an example of an economy that satisfies all of our assumptions. The example is identical to Example 1, except for the presence of subsidy scheme.

Example 2. Let $\lambda = 2.2$, so $(1 - \theta) = .6875$, $R = 1.061$, $r = .965$ and $\tau = .011$. Then $k_1 = 3.40028$ and $k_2 = 6.53322$. Clearly, $k_1 < k^* < k_2$. It is also worth emphasizing that this economy continues to experience credit rationing. This can be easily verified by verifying that (4.4) and (4.5) continue to be satisfied.

5.5. Comparative statics

We now examine the comparative static consequences of changes in world financial markets, and of changes in government policy. We focus particularly on the capital stock and, by implication, long-run real activity in our analysis.

The effect of an increase in the world interest rate, R, depends on which steady state obtains, as illustrated in Figure 5. In the low-capital-stock steady state, an increase in the world interest rate from R to R' reduces the steady state capital stock, while the same increase raises the capital stock in the high-capital-stock steady state. Intuitively, when the world interest rate rises, the return on domestic investments must also rise in order for domestic borrowers to compete in international capital markets. In the (low) high capital-stock steady state, an increase in the marginal product of capital (an increase in the level of internal finance) is required to accomplish this, so that the capital stock must fall (rise).

An increase in τ represents a policy of a higher level of credit subsidization. The effect of such an increase depends not only on which steady state prevails, but on the magnitude of reserve requirements, and on the magnitude of the implicit subsidy provided by the government when it extends credit to banks. We henceforth assume that[16]

$$R > \theta + (1 - \theta)r$$

Under this assumption, the effect of an increase in τ is depicted in Figure 6. An increase in τ has the effect of reducing (increasing) the capital stock in the high (low)-capital-stock steady state. In effect, the increase in the size of the government subsidy program gives banks access to cheaper sources of funds. As a result of competition, the zero profit condition implies that the return on bank loans must fall; in the high (low)-capital-stock- steady state this requires a reduction in the level of internal finance (an increase in the marginal product of capital). Thus the capital stock must fall (rise). As a corollary, giving banks access to low

[16] This condition is implied by our previous assumptions whenever $R > 1$.

cost funds can be counterproductive as many have argued unless the economy is in a low activity equilibrium.

The effect of an increase in r is depicted in Figure 7. An increase in r, for fixed values of τ, raises the cost of funds to banks, and hence has an effect opposite to that of an increase in τ.

6. Dynamic Equilibria

When the reserve requirement binds, equations (5.2) and (5.3) can be used to derive the following equilibrium law of motion for the capital stock:

$$k_{t+2} = B(\tau)^{-1} k_{t+1} \left[\frac{q - w(k_t)}{q - w(k_{t+1})} \right] \left[A(\tau) - \frac{\theta \psi f'(k_{t+1})}{q - w(k_t)} \right] \tag{6.1}$$

where $A(\tau) \equiv (1-\tau)R + (1-\theta)\tau r > 0$ and $B(\tau) \equiv 1 - (1+\tau)\theta$. Clearly the evolution of the capital stock is governed by a second order difference equation. We now proceed as follows.

Defining

$$k_{t+1} = y_t, \tag{6.2}$$

we may rewrite equation (6.1) as

$$y_{t+1} = B(\tau)^{-1} y_t \left[\frac{q - w(k_t)}{q - w(y_t)} \right] \left[A(\tau) - \frac{\theta \psi f'(y_t)}{q - w(k_t)} \right] \tag{6.3}$$

We henceforth work with the dynamical system consisting of equations (6.2) and (6.3) .

In order to analyze local dynamics, we linearize (6.2) and (6.3) in a neighborhood of any steady state, (k, y). We then have

$$(k_{t+1} - k, y_{t+1} - y)' = J(k_t - k, y_t - y)',$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial k_{t+1}}{\partial k_t} & \frac{\partial k_{t+1}}{\partial y_t} \\ \frac{\partial y_{t+1}}{\partial k_t} & \frac{\partial y_{t+1}}{\partial y_t} \end{bmatrix}$$

with all partial derivatives evaluated at the appropriate steady state. It is straightforward to show that

$$\frac{\partial k_{t+1}}{\partial k_t} = 0, \tag{6.4}$$

$$\frac{\partial k_{t+1}}{\partial y_t} = 1, \tag{6.5}$$

$$\frac{\partial y_{t+1}}{\partial k_t} = -A(\tau)B(\tau)^{-1}\left[\frac{kw'(k)}{q - w(k)}\right], \tag{6.6}$$

$$\frac{\partial y_{t+1}}{\partial y_t} = 1 + \left[\frac{kw'(k)}{q - w(k)}\right]\left[1 + \frac{\theta\psi}{kB(\tau)}\right]. \tag{6.7}$$

Let $T(k)$ and $D(k)$ be the trace and determinant of J respectively. Then we have

$$T(k) = 1 + \left[\frac{kw'(k)}{q - w(k)}\right]\left[1 + \frac{\theta\psi}{kB(\tau)}\right], \tag{6.8}$$

$$D(k) = A(\tau)B(\tau)^{-1}\left[\frac{kw'(k)}{q - w(k)}\right]. \tag{6.9}$$

Notice that (6.8) can be rewritten as

$$T(k) = 1 + A(\tau)^{-1}B(\tau)D(k)\left[1 + \frac{\theta\psi}{kB(\tau)}\right]. \tag{6.10}$$

Now, we would like to analyze equilibrium dynamics in a neighborhood of each steady state. The nature of these dynamics depend upon the signs of $T(k)$

and $D(k)$, which in turn depend upon the sign of $B(\tau) \equiv 1 - (1+\tau)\theta$. Clearly, $B(\tau) > (<) 0$ holds if $\tau < (>) \frac{1-\theta}{\theta}$. Thus, given the reserve requirement $(1-\theta)$, $B(\tau)$ is positive (negative) when government credit provision is relatively small (large). We next consider each case separately.

6.1. Case 1: $B(\tau) > 0$.

When $B(\tau) > 0$ holds, we have $D(k) > 0$ and $T(k) > 1$. Hence the eigenvalues of J are either both positive real numbers, or they are complex conjugates. In addition, it is possible to derive very definite conclusions about the local stability properties of each steady state. We begin with the following result.

Proposition 6.1. *When $B(\tau) > 0$ holds, $T(k) \gtrless 1 + D(k)$ iff $q \gtrless f(k)$.*

Proof. Using the definitions of $A(\tau)$ and $B(\tau)$, we rewrite (5.8) as

$$\theta\psi H(k) = A(\tau) - B(\tau). \qquad (6.11)$$

Equations (6.10) and (6.11) then imply that

$$1 + D(k) - T(k) = \frac{\theta\psi D(k)}{A(\tau)k}[kH(k) - 1]. \qquad (6.12)$$

holds. Since $kH(k) = \frac{kf'}{kf'+[q-f(k)]}$, it follows that $kH(k) \gtrless 1$ iff $q \lessgtr f(k)$. ■

Proposition 6.1 has an immediate corollary: the low-capital-stock steady state is necessarily a saddle, while the high-capital-stock steady state is either a source or a sink. Specifically, it is a sink (source) when $D(k) < (>) 1$.[17] In addition, local dynamics near the low-capital stock steady state are necessarily monotone.

In order to say more about the properties of dynamical equilibria in a neighborhood of the high-capital-stock steady state, we henceforth assume that the production function takes the Cobb-Douglas form $f(k) = Ak^{\delta}$, with $\delta \in (0, 1)$. Under this assumption, $D(k_2)$ has a particularly simple form:

$$D(k_2) = (1 - \delta)A(\tau)B(\tau)^{-1}k_2 H(k_2). \tag{6.13}$$

Then, in particular, $D(k_2) < (>) 1$ holds iff $(1 - \delta)A(\tau)B(\tau)^{-1}k_2 H(k_2) < (>) 1$ obtains. We can now state an immediate implication of this observation.

Proposition 6.2. *The high-capital-stock steady state is a source if*

$(1 - \delta)A(\tau)B(\tau)^{-1} > 1$.

Proof. Since $k_2 \geq f^{-1}(q)$, $k_2 H(k_2) \geq H[f^{-1}(q)]f^{-1}(q) \equiv 1$ must hold. Thus $D(k_2) \geq (1 - \delta)A(\tau)B(\tau)^{-1}$,which establishes the result. ■

[17] See Azariadis (1993), chapter 6.4.

Since as $\tau \uparrow \left(\frac{1-\theta}{\theta}\right)$, $A(\tau)B(\tau)^{-1} \to \infty$, it is apparent that the high-capital-stock steady state will be a source. Thus certain choices of the size of government credit programs will imply that the high-capital-stock steady state cannot be approached.

We now recall that there are two steady states equilibria with credit rationing iff (5.10) holds. Suppose that[18]:

$$(i)\ R > (1-\theta) + \theta \left[\frac{\psi}{f^{-1}(q)}\right]$$

holds, and define the value τ^* by

$$\tau^* \equiv \frac{R - (1-\theta) - \theta \frac{\psi}{f^{-1}(q)}}{R - (1-\theta)r - \theta}.$$

If we assume that

$$(ii)\ (1-\theta)(1-r) > \theta \left[1 - \frac{\psi}{f^{-1}(q)}\right],$$

[18] Condition (i) asserts that there must be two steady state equilibria with credit rationing when $R^m = 1$ holds.

then $\tau^* \in (0,1)$. It is then easy to verify that (5.10) [5.11] holds if $\tau < (>) \tau^*$. Stated differently, there are two nontrivial steady states equilibria with credit rationing iff $\tau < \tau^*$ is satisfied. We henceforth assume that this condition obtains. It is also useful to know when $\tau^* < (>) \frac{1-\theta}{\theta}$ holds, since if $\tau^* < \frac{1-\theta}{\theta}$, $B(\tau) > 0$ is satisfied for all $\tau \leq \tau^*$. It is straightforward to verify that $\tau^* < (>) \frac{1-\theta}{\theta}$ obtains iff

$$(iii)\ (2\theta - 1)R + r(1-\theta)^2 < (>) \frac{\psi}{f^{-1}(q)} \theta^2,$$

Of course if $\tau^* > \frac{1-\theta}{\theta}$, then the government can set τ in such a way that two steady state equilibria with credit rationing exist, and that $B(\tau) < 0$.

Armed with conditions on τ such that (5.10) holds, we are now prepared to state results concerning how the stability properties of the high-capital-stock steady state depend on that parameter. We begin with the following proposition.

Proposition 6.3. *Suppose $\tau^* < \frac{1-\theta}{\theta}$ holds. Then for τ sufficiently close to, but below τ^*, $D(k_2) < (>) 1$ holds if*

$$(iv)\ (1-\delta)R < (>) (1-\theta) + \{(1-\delta)[R - r(1-\theta)] - \theta\} \left[\frac{R - (1-\theta) - \theta \frac{\psi}{f^{-1}(q)}}{R - (1-\theta)r - \theta} \right].$$

The proofs of propositions 6.3, 6.4 and 6.5 can be found in the appendix. Proposition 6.3 describes what happens for large values of τ. The next proposition describes results for small values of τ.

Proposition 6.4. *(a) When $\tau = 0 < \tau^*$ holds, the high-capital-stock steady state is a sink if*

$$(v) \quad \left[\frac{(1-\delta)R}{(1-\theta)}\right]\left[\frac{R-(1-\theta)}{\theta\psi}\right]\phi\alpha q \leq 1$$

and if credit is rationed ($\mu < 1$).

(b) When $\tau = 0$, the high-capital-stock steady state is a source if $(1-\delta)R > (1-\theta)$.

When (v) holds, the high-capital-stock steady state is a sink whenever the government credit program is sufficiently small.

Finally, we state conditions under which $B(\tau) > 0$ holds, and under which dynamical equilibrium paths approaching the high-capital-stock steady state display damped oscillation. Our results also imply that, depending on the choice of τ, this endogenously arising volatility may dampen only arbitrarily slowly. Thus long-lived, endogenously generated volatility can easily be observed for some settings of government policy.

Proposition 6.5. *Suppose that condition (v) and $\tau^* > \frac{1-\theta}{\theta}$ hold. (a) Then there exists a value $\tau_c \in (0, \frac{1-\theta}{\theta})$ such that, when $\tau = \tau_c, D(k_2) = 1$ holds. (b) For τ in* *a neighborhood of τ_c, $T(k_2)^2 < 4D(k_2)$ holds. Consequently, the eigenvalues of J are complex conjugates.*

Obviously, all of the preceding results have been derived when $\tau < (\frac{1-\theta}{\theta})$ $[B(\tau) > 0]$. Assuming $\tau^* > \frac{1-\theta}{\theta}$ holds, we now consider the properties of dynamical equilibria for values of τ such that $\tau > \frac{1-\theta}{\theta}$.

6.2. Case 2: $B(\tau) < 0$.

Whenever $B(\tau) < 0$ holds, we have $D(k) < 0$. It follows that J has two real eigenvalues of opposite signs. The local stability properties of the two steady states are dramatically altered relative to the $B(\tau) > 0$ case. For notational purposes, let λ_1 and λ_2 denote the eigenvalues of J, and let $\lambda_1 > 0 > \lambda_2$. Then we have the following results.

Lemma 6.6. *Let $B(\tau) < 0$ hold. Then we have $1 + D(k_2) < T(k_2)$ so that $\lambda_1 > 1$.*

Proof. For $B(\tau) < 0$, equation (6.12) and $k_2 H(k_2) \geq 1$ imply that $1 + D(k_2) - T(k_2) < 0$. ■

The lemma implies that when $\tau > \frac{1-\theta}{\theta}$, the high-capital-stock steady state cannot be a sink. It must be either a source or a saddle. We can also establish that the low-capital-stock steady state cannot be a source. It must be either a sink or a saddle.

Lemma 6.7. *Let $B(\tau) < 0$ hold. Then we have $1 + D(k_1) > T(k_1)$ so that $\lambda_1 < 1$.*

Proof. For $B(\tau) < 0$, $1 + D(k_1) - T(k_1) > 0$ follows directly from (6.12) and $k_1 H(k_1) < f^{-1}(q) H[f^{-1}(q)] = 1$. ■

It remains to study the magnitude of the negative eigenvalues at each steady state. The following lemmas provide some useful information under the Cobb-Douglas production specification.

Lemma 6.8. *If $1 + A(\tau)B(\tau)^{-1} \geq 0$, then $\lambda_2 > -1$.*

Proof. From (6.10), we have

$$1 + D(k) + T(k) = (1 + A(\tau)^{-1} B(\tau))(1 - \delta)[1 + kH(k)] + 2\delta. \tag{6.14}$$

If $1 + A(\tau)B(\tau)^{-1} \geq 0$, then $1 + D(k) + T(k) > 0$ is satisfied, and the result follows [Azariadis (1993), chapter 6.4] ■

Lemma 6.8 states a condition under which the high-capital-stock steady state is a saddle ($-1 < \lambda_2 < 0 < 1 < \lambda_1$). Thus the local stability properties of the steady states can be reversed when $B(\tau) < 0$. In particular, there are indeterminacies in a neighborhood of the low-capital-stock steady state when $B(\tau) < 0$ and $1 + A(\tau)B(\tau)^{-1} \geq 0$. Under the same conditions, there is a unique equilibrium path approaching the high-capital-stock steady state. At the same time, paths approaching that steady state must display endogenously arising fluctuations as they do so. As we show below, for appropriate choices of τ, those fluctuations will dampen only arbitrarily slowly.

Lemma 6.8 implies that government credit policies can potentially be used to render equilibria in a neighborhood of the high-capital-stock steady state determinate. This is clearly an argument in favor of such policies. However, determinacy comes at the price of allowing endogenously arising volatility to emerge.

When exactly can government policy be used to induce determinacy of equilibrium in a neighborhood of the high-capital-stock steady state? The definitions of $A(\tau)$ and $B(\tau)$ imply that $1 + A(\tau)B(\tau)^{-1} \geq 0$ holds iff

$$(1 + \tau)\theta \geq 1 + (1 - \tau)R + (1 - \theta)\tau r \tag{6.15}$$

Equation (6.15) requires that both θ and τ be relatively large. In other words, the high-capital-stock steady state will be a saddle if the reserve requirement $(1-\theta)$ is sufficiently low, and if the size of the government program (τ) is relatively large.

Not surprisingly, it is also possible to show the converse: when θ and τ are relatively small, consistent with $B(\tau) < 0$, the high-capital-stock steady state is a source. We now state the relevant result.

Lemma 6.9. *If* $-[1 + A(\tau)B(\tau)^{-1}] > 2\delta/(1-\delta)$, *then* $\lambda_2 < -1$ *at the high-capital-stock steady state.*

Proof. Equations (6.14) and $k_2H(k_2) \geq 1$ imply that $1 + D(k_2) + T(k_2) < 0$ necessarily holds if $-[1 + A(\tau)^{-1}B(\tau)] \geq 2\delta/(1-\delta)$. The result then follows [Azariadis (1993), chapter 6.4]. ■

The fact that $\lambda_2 > (<) -1$ holds at the high-capital-stock steady state when τ is large (small) suggests the existence of a critical value, denoted $\widehat{\tau}$, such that, when $\tau = \widehat{\tau}$, $\lambda_2 = -1$ at the high-capital-stock steady state. The next lemma confirms this intuition. It is then immediate that, for some choices of τ the high-capital-stock steady state is a saddle, and that paths that approach it display oscillation that dampens arbitrarily slowly.

Lemma 6.10. *Suppose that* $\theta \geq \frac{1+r}{2+r}$ *holds. Then there exists a value* $\widetilde{\tau} \in \left(\frac{1-\theta}{\theta}, 1\right)$ *such that* $1 + A(\tau)B(\tau)^{-1} = 0$. *In addition, there exists a value* $\widehat{\tau} \in \left(\frac{1-\theta}{\theta}, \widetilde{\tau}\right)$ *such that, when* $\tau = \widehat{\tau}$, $\lambda_2 = -1$ *at the high-capital-stock steady state.*

Proof. Since

$$A(\tau)B(\tau)^{-1} \equiv \frac{(1-\tau)R + (1-\theta)\tau r}{1-\theta(1+\tau)},$$

we have

$$\frac{d[A(\tau)B(\tau)^{-1}]}{d\tau} = \frac{B(\tau)[(1-\theta)r - R] + \theta A(\tau)}{B(\tau)^2} > 0,$$

since $B(\tau) < 0$ and $R > r$. Moreover, it is straightforward to calculate that $1 + A(\tau)B(\tau)^{-1} = 0$ holds iff

$$\tau = \frac{R + (1-\theta)}{R + (1-\theta) + [2\theta - 1 - r(1-\theta)]} \equiv \tilde{\tau},$$

and that under our hypotheses, $\tilde{\tau} \in \left(\frac{1-\theta}{\theta}, 1\right)$. Finally, at $\tau = \tilde{\tau}$, Lemma 6.9 implies that $\lambda_2 > -1$. As $\tau \downarrow\in \frac{1-\theta}{\theta}$, $A(\tau)B(\tau)^{-1} \to -\infty$ and Lemma 6.10 implies that $\lambda_2 < -1$. The existence of the desired value $\hat{\tau}$ then follows from the intermediate value theorem. ■

Lemma 6.10 nearly allows us to state an even stronger result. The lemma implies that the dynamical system defined by equations (6.2) and (6.3) undergoes a flip bifurcation at $\tau = \hat{\tau}$. Thus there are solution sequences to those equations that display undamped oscillation. However, it is quite difficult to verify that such sequences will necessarily imply that the reserve requirement binds at each date, so we must remain agnostic as to whether undamped oscillation is a possibility.

7. Concluding Remarks

We have analyzed the effects of a government credit program (rediscounting) in a small open economy with a domestic credit market friction. We have seen that such a program can potentially have a positive effect on long-run output levels only when the economy is in a development trap; otherwise it is counterproductive. We have also seen that such programs can give rise to endogenous volatility. On the other hand, central bank rediscounting on a large scale can render the high activity steady state determinate. In this sense, there is a tension in central bank rediscounting between the determinacy of equilibria – on the one hand – and the level of output and the volatility of output and inflation, on the other.

Of course these results have been obtained only for one particular – albeit important – kind of program: the central bank conducts monetary policy by rediscounting private loans. An important issue for future research concerns whether similar results can be obtained for other types of credit programs, for example loan guarantees or interest rate controls.

8. Appendix

Proof of Proposition 6.3:

As $\tau \uparrow \tau^* < \frac{1-\theta}{\theta}$, $k_2 \downarrow f^{-1}(q)$. Then, we have $D(k_2) \rightarrow (1-\delta)A(\tau^*)B(\tau^*)^{-1}$. Hence, for τ near (but below) τ^*, $D(k_2) < (>) 1$ holds iff $1 > (<)\ (1-\delta)A(\tau^*)B(\tau^*)^{-1}$. Using the definitions of $A(\tau), B(\tau)$, and τ^*, this condition is equivalent to

$$\begin{aligned} &(1-\delta)R - (1-\delta)[R - r(1-\theta)]\left[\frac{R-(1-\theta)-\frac{\theta\psi}{f^{-1}(q)}}{R-\theta-(1-\theta)r}\right] \\ < \quad &(>)(1-\theta)+\theta\left[\frac{R-(1-\theta)-\frac{\theta\psi}{f^{-1}(q)}}{R-\theta-(1-\theta)r}\right]. \end{aligned}$$

Rearranging terms gives condition (iv). ■

Proof of Proposition 6.4:

(a) When $\tau = 0$, we have $A(0)B(0)^{-1} = \frac{R}{(1-\theta)}$, and –in addition– $\theta\psi H(k_2) = R-(1-\theta)$. Hence, when $\tau = 0$ holds,

$$D(k_2) = \left[\frac{(1-\delta)R}{1-\theta}\right]\left[\frac{R-(1-\theta)}{\theta\psi}\right]k_2.$$

Moreover, if credit is rationed $(\mu < 1)$, $k_2 < \phi\alpha q$ holds. Hence if

$$\left[\frac{(1-\delta)R}{1-\theta}\right]\left[\frac{R-(1-\theta)}{\theta\psi}\right]\phi\alpha q \leq 1,$$

we have $D(k_2) < 1$ when $\tau = 0$.[19]

(b) We have

$$\begin{aligned} D(k_2) &= (1-\delta)A(\tau)B(\tau)^{-1}k_2 H(k_2) \geq (1-\delta)A(\tau)B(\tau)^{-1}f^{-1}(q)H(f^{-1}(q)) \\ &= (1-\delta)A(\tau)B(\tau)^{-1}. \end{aligned}$$

Moreover, $A(0)B(0)^{-1} = \frac{R}{1-\theta}$. Hence if $\frac{(1-\delta)R}{(1-\theta)} > 1$, $D(k_2) > 1$ holds when $\tau = 0$. ■

Proof of Proposition 6.5:

(a) Condition (v) implies that, when $\tau = 0$, $D(k_2) < 1$ holds. If $\tau^* > \frac{1-\theta}{\theta}$ also holds, then as $\tau \uparrow \frac{1-\theta}{\theta}$, $A(\tau)B(\tau)^{-1} \uparrow \infty$. Since $D(k_2) = (1-\delta)A(\tau)B(\tau)k_2 H(k_2)$ $\geq (1-\delta)$ $A(\tau)B(\tau)^{-1}$ it follows that as $\tau \uparrow \frac{1-\theta}{\theta}$, $D(k_2) > 1$ must hold. Since $A(\tau)B(\tau)^{-1}$and k_2 vary continously with τ, for $\tau < \frac{1-\theta}{\theta}$, the existence of the desired value τ_c is implied by the intermediate value theorem.

(b) When $\tau = \tau_c$, $D(k_2) = 1$ holds. It follows that, at $\tau = \tau_c$, $T(k_2)^2 < 4D(k_2)$ is equivalent to $T(k_2) < 2$. But when $D(k_2) = 1$, equation (6.12) implies that

$$T(k_2) = 2 - \left[\frac{\theta\psi}{A(\tau_c)k_2}\right][k_2 H(k_2) - 1] < 2,$$

[19] $k_2 < \phi\alpha q$ holds at $\tau = 0$ iff $\theta\psi H[\phi\alpha q] > R - (1-\theta)$.

since for $\tau < \tau^*$, $k_2 H(k_2) > f^{-1}(q) H[f^{-1}(q)] = 1$. The claim then follows from continuity. ■

References

[1] Azariadis, Costas, 1993, *Intertemporal Macroeconomics*, Oxford: Blackwell.

[2] Bernanke, Ben S., and Mark Gertler, 1989, Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review* 79, 14-31.

[3] Boyd, John H., and Bruce D. Smith, 1994a, How Good Are Standard Debt Contracts?: Stochastic versus Nonstochastic Monitoring in a Costly State Verification Environment, *Journal of Business.*

[4] Boyd, John H., and Bruce D. Smith, 1997a, Capital Market Imperfections, International Credit Markets, and Nonconvergence, *Journal of Economic Theory* 73, 335-364.

[5] Boyd, John H., and Bruce D. Smith, 1997b, Capital Market Imperfections in a Monetary Growth Model, *Economic Theory.*

[6] Brock, Philip, 1989, Reserve Requirements and the Inflation Tax, *Journal of Money Credit and Banking* 21 (February), 106-121.

[7] Carter, Michael R., 1988, Equilibrium Rationing of Small Farm Agriculture, *Journal of Development Economics* 28, 83-103.

[8] Chavez, J. Jesus Romero, 1983, Centralized Rediscounting and Loan Guarantee Facilities in Mexico, in: J.D. Von Pischke, Dale W. Adams, and Gordon Donald, eds., Rural financial markets in developing countries (Johns Hopkins University Press, Baltimore, MD).

[9] Diaz-Alejandro, Carlos, 1985, Good-bye Financial Repression, Hello Financial Crash, *Journal of Development Economics* 19, 1-24.

[10] Espinosa-Vega, Marco, 1995, Multiple Reserve Requirements, *Journal of Money Credit and Banking* 27 (August), 762-776.

[11] Gale, William G., 1990, Federal Lending and the Market for Credit, *Journal of Public Economics* 42, 177-193.

[12] Gonzalez-Vega, Claudio, 1984, Cheap Agricultural Credit: Redistribution in Reverse, in: Dale W. Adams, Douglas H. Graham, and J.D. Von Pischke, eds., Undermining Rural Development with Cheap Credit (Westview Press, Boulder, CT).

[13] Huybens, Elisabeth and Bruce Smith, 1997, Financial Market Frictions, Monetary Policy and Capital Accumulation in a Small Open Economy." Forthcoming *Journal of Economic Theory.*

[14] Innes, Robert, 1991, "Investment and government intervention in credit markets when there is asymetric information," *Journal of Public Economics* 46 347-381.

[15] Japelli, Tullio, 1991, Who is Credit Constrained in the U.S. Economy?, *Quarterly Journal of Economics,* (105), 219-34.

[16] Johnson, Ometunde E.G., 1983, Credit Controls as Instruments of Development Policy in the Light of Economic Theory, in: J.D. Von Pischke, Dale W. Adams, and Gordon Donald, eds., Rural Financial Markets in Developing Countries (Johns Hopkins University Press, Baltimore, MD).

[17] Khatkate, Deena, 1982a, National and International Aspects of Financial Policy in LDC's: A Prologue, World Development 10, 689-694.

[18] Khatkate, Deena, 1982b, Anatomy of Financial Retardation in a Less Developed Country: The case of Sri Lanka, 1951-976, World Development 10, 829-840.

[19] McKinnon, Ronald I., 1973, Money and Capital in Economic Development, Brookings Institute, Washington, D.C.

[20] Sanchez-Lugo, Luis. Instrumentos de Politica Monetaria y Crediticia, in Cincuenta Años de Banca Central, Banco de Mexico, Fondo de Cultura Economica, 1975, 369-387.

[21] Smith, Bruce D. and Michael J. Stutzer, 1989, Credit Rationing and Government Loan Programs: A welfare analysis, *American Real Estate and Urban Economics Association Journal* 17, 177-193.

[22] Weil, Philippe, 1992, The Budgetary Arithmetics of Loan Guarantees and Deposit Insurance. Carnegie-Rochester Conference Series on Public Policy 37 97-122.

The figures for this working paper are not available in PDF format. To obtain a hard copy of the figures, please use the electronic publications order form on this Web site and request a copy of Working Paper 98-2.