A Public Finance Analysis of Multiple Reserve Requirements

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Federal Reserve Bank of Atlanta
Working Paper 98-1
January 1998

Abstract: This paper analyzes multiple reserve requirements of the type that have been imposed by a number of developing countries. We show that previous theoretical work on this topic has not succeeded in providing a social welfare rationale for the existence of multiple reserve requirements: in the basic reserve requirements model, any allocation that can be supported by a multiple-reserves regime can also be supported by a single-bond reserve requirement. We go on to present extended versions of the model in which it is possible for a multiple-reserves regime to improve social welfare relative to any single-reserve (currency or bond) and/or deposit-tax regime. We demonstrate the empirical plausibility of our approach by providing a case study of Mexico, a country with extensive historical experience with multiple reserve requirements.

JEL classification: E5, E6, F3

Key words: multiple reserve requirements, developing countries, monetary policy

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The authors are grateful for comments from Peter Rangazas, Bruce Smith, participants in seminars at the Federal Reserve Bank of Atlanta and the Chinese University of Hong Kong, and sessions at the 1996 annual meetings of the Society for Economic Dynamics and the Latin American Econometric Society. Finally, the authors are indebted to Rodolfo Padilla of the Banco de Mexico for his assistance in providing data. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System. Any remaining errors are the authors' responsibility.

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1 Introduction

1.1 Motivation

A multiple-reserve-requirements regime is a monetary regime in which the government imposes two types of reserve requirements on the banking system: a currency reserve requirement, which can be satisfied by holdings of government currency, and a bond reserve requirement, which can be satisfied by holdings of government bonds that return below-market rates of interest. Multiple-reserves regimes have been adopted by a number of developing countries at various times in recent years; examples include Chile, Korea, Mexico, and Pakistan. In each case, the country had a large public sector deficit and was attempting to finance a substantial portion of it via seigniorage. This observation suggests that the principal motives for the imposition of these regimes were considerations of public finance rather than monetary control or liquidity.[1]

A second observation about multiple-reserves regimes is that the real rate of return on reservable government bonds has invariably been higher than the real rate of return on government currency — that is, the bonds have always yielded positive nominal interest. We will refer to regimes like this as "conventional," and to regimes involving negative-nominal-interest bonds as "unconventional". While the fact that the nominal interest rates on *private* bonds are always positive makes conventionality seem natural, nothing about the structure of multiple-reserves regimes appears to require positive government bond rates. In a typical regime of this type, the government designates a particular class of bonds as reservable and forces the banks to hold these bonds and no others. Since the government is free, if it wishes, to offer different and higher-yielding bonds to nonbank lenders, it should also be free to impose any reservable bond rate it chooses.[2] The fact that the nominal bond rate is

[1] Jimenez (1968), discussing Argentina's regulations allowing banks to satisfy a portion their reserve requirements by holding low-yield government bonds, comments that "These bonds no longer represent a monetary policy but a fiscal policy by means of which the public sector absorbs part of the banking sector's legal reserve requirements in order to finance its expenditures."

[2] In some multiple-reserves regimes the government gives banks the option of holding currency or reservable government bonds to satisfy the second reserve requirement. In this case, it is clear that the banks will not

positive implies that bond reserve requirements produce less revenue per dollar of reservable assets than currency reserve requirements — in other words, that the implicit tax rate on reservable bonds is lower than the implicit tax rate on currency (the "inflation tax" rate). There is no obvious reason why this must be the case.

The purpose of this paper is to identify the properties of a plausible formal model that can explain these two observations about multiple-reserves regimes. We begin by assuming that the regimes have indeed been imposed for public-finance reasons, and that the governments that imposed them chose them over alternative seigniorage-augmentation strategies that would have been simpler to formulate and administer — strategies involving single currency or bond reserve requirements and/or direct taxation of bank deposits. It follows, in our view, that these governments believed multiple-reserves regimes were economically or politically preferable to these alternatives — that is, that they would produce efficiency gains for the economy as a whole and/or social-welfare gains for important socioeconomic groups.[3] Thus, our goal is to construct a formal model in which conventional multiple-reserves regimes can produce efficiency or social-welfare gains over related regimes that are simpler in nature.

1.2 Previous work

In recent years, authors such as Wallace (1984), Romer (1985), Freeman (1987) and Mourmouras and Russell (1992) have used general equilibrium models to analyze the role of currency reserve requirements in monetary policy and public finance. Espinosa (1995) has recently provided the first analysis of this type that is specifically concerned with multiple reserve requirements. Espinosa's principal concern is to explain why a developing country with a monetary regime involving a single currency reserve requirement (a regime which it might

purchase reservable bonds unless their nominal interest rate is non-negative. The question then becomes why the government chooses to offer banks this option.

[3] For our purposes, a policy is efficient, relative to another policy, if the consumption allocation it supports Pareto dominates the allocation supported by the alternative policy. A policy improves social welfare, relative to another policy, if the consumption allocation it supports has higher social utility, as measured by a social-utility function, than the alternative policy. Our use of terms such as "optimal" or "social-welfare-maximizing" should be understood as restricted to the context of a particular class of policies. None of the policies we study in this paper are first-best optimal.

have inherited from a previous government, or even from a colonizing power) might choose to adopt a multiple-reserves regime with a positive nominal bond rate. He answers this question by showing that when a single-currency-reserve scheme satisfies a certain elasticity condition, switching to a "conventional" multiple-reserves scheme can reduce both the steady-state inflation rate and the initial price level, as well as producing a Pareto improvement. Espinosa also considers the question of why a country might switch to a multiple-reserves regime from a regime which includes both a single currency reserve requirement and a proportional tax on bank deposits.[4] He demonstrates (in his Corollary 2) that a regime switch of this sort can reduce the initial price level, which will improve the welfare of some of the agents in his model.

As we have indicated, our goal in this paper is to identify conditions under which multiple reserve requirements may improve efficiency and/or social welfare relative to related but simpler monetary regimes. While it may appear at first glance that Espinosa has already accomplished this objective, there are two reasons why this is not really the case. First, Espinosa's analysis takes the initial monetary-policy regime as given: he does not ask the question of whether the welfare improvements accomplished by a switch to a multiple-reserves regime could also be acheived by simply changing the policy settings (reserve ratios or deposit tax rates) of the existing regime. We show in another paper [Espinosa and Russell (1996)] that in situations like those described in Espinosa's Proposition 1, the monetary authority can produce an allocation that Pareto-dominates the allocation supported by the original single-currency-reserves regime simply by changing the required reserve ratio; this allocation, moreover, cannot be Pareto-dominated by any multiple-reserves regime. We also show that in many cases where the initial regime involves a single currency reserve ratio plus a deposit tax, the potentially social-welfare-improving allocation produced by a switch to a multiple-reserves regime can also be supported simply by increasing the required reserve ratio.[5]

[4] Espinosa's Proposition 2 establishes that if the government is not concerned about these agents, then the optimal multiple-reserves scheme involves government bonds with a zero gross real rate of interest. This sort of scheme is identical to a single-currency-reserves/deposit-tax scheme.

[5] In certain other cases, no single-reserve/deposit-tax tax allocation can duplicate the socially optimal allo-

A second limitation of Espinosa's analysis is that he does not consider a third type of monetary regime that is also considerably simpler than a multiple-reserves regime — a regime in which there is a single reserve requirement, but reserves must be held in the form of *bonds* rather than currency. We show in Proposition 1 of this paper that in Espinosa's model, a single bond reserve requirement can duplicate the allocation supported by any multiple reserve requirement, conventional or unconventional. In fairness to Espinosa, we should emphasize that his decisions about the types of seigniorage-based deficit-finance regimes to analyze were dictated by the historical evidence: while there have been many examples of multiple-reserves regimes (see above), we do not know of any examples of regimes that have imposed single bond reserve requirements.[6] In addition, some of the duplicating single-bond-reserve regimes whose existence is guaranteed by our Proposition 1 will be unconventional in nature, and we do not know of any historical examples of negative-nominal-yield bonds. It seems to us, however, that any thoroughgoing explanation for the historical observations concerning single and multiple reserve requirements must attempt to explain *why* we have not observed single bond reserve requirements, and also why we have not observed reservable bonds with negative nominal yields.

1.3 Alternative models

The situation just described seem to leave the literature without a completely convincing theoretical explanation for the existence of multiple reserve requirements. We address this problem by modifying Espinosa's model in a way that allows multiple reserve requirements to produce social-welfare improvements over three different and less complex seigniorage-based deficit-finance strategies. Interestingly, it turns out that we can accomplish this goal without increasing the degree of agent heterogeneity that is present in Espinosa's model: in fact,

cation supportable by a multiple reserve requirement. Even in these cases, however, the optimal allocation can be supported by a single bond reserve requirement (see below).

[6] In Espinosa's model, if the nominal interest rate on bonds is positive then a single-bond-reserve regime is equivalent to a regime in which the government imposes a single currency reserve requirement and pays interest on the reserves at a less-than-market rate (see below). However, we do not know of any developing country that has used this type of regime.

there is a sense in which the modified model reduces the degree of diversity among agents. Propositions 2-4 establish the validity of these assertions. Proposition 2 is particularly important because it establishes that *conventional* multiple-reserves regimes are the only type that can improve on the allocations supportable by one of these alternative strategies.

We conclude our analysis of the revised model by providing an example in which a conventional multiple-reserves regime can improve on all three of the alternatives described above. It turns out, however, that the conditions necessary to produce examples like these are fairly restrictive. We respond to this potential objection by discussing the properties of a more general model that combines Espinosa's original model and our modified version. In this model, the conditions necessary for the existence of social-welfare-maximizing multiple-reserves regimes are considerably weaker.

1.4 Organization of the analysis

In the next section (Section 2) of this paper, we begin our formal analysis by presenting an abbreviated description of Espinosa's (1995) model. We use this model to show that any allocation supportable by a multiple-reserves regime can also be supported by a single bond reserve requirement. In Section 3 we construct a pair of modified versions of Espinosa's model and use these models to compare the properties of multiple-reserves regimes to those of simpler seigniorage-based deficit-finance regimes. In Section 4 we present some evidence from Mexico, a country that has used multiple reserve requiements quite extensively, that supports the empirical plausibility of our basic assumptions. Section 5 offers some concluding remarks. The proofs of several of the paper's propositions are presented in the appendix, along with a number of illustrative examples.

2 The model

Espinosa (1995) analyzes a two-period overlapping generations model with limited intra-generational heterogeneity and a number of legal/technological constraints on intertemporal trades. The model extends Freeman's (1987) reserve-requirements model by adding intra-generational heterogeneity of the type described by Sargent and Wallace (1982). Economic activity occurs at discrete dates $t = 1, 2, \ldots$. At each date t a generation of agents is born; these "members of generation t" live during dates t and $t{+}1$. Each generation of agents consists of N_p "poor savers" and N_r "rich savers." Rich savers differ from poor savers only in the magnitude and time-distribution of their endowments of the single consumption good. The endowment patterns of rich and poor savers are invariant to the dates at which these agents are born. At each date an arbitrary number of competitive banks are operating in the economy. These banks may hold one or more of the following types of assets:

- private one-period bonds, which are available on the international credit market at an exogenously-determined gross real interest rate $R > 1$.[7]
- government currency, which yields a gross real return rate $R_m(t) \geq 0$ that is determined by the government through its ability to control the growth rate of the stock of currency.
- government one-period bonds, which yield a gross real return rate $R_b(t) \geq 0$ that is specified by the government.[8]

The liabilities of the banks consist of deposits that are offered to the public at a competitively-determined gross real interest rate $R_d(t)$. The banks are assumed to have zero operating costs and to maximize their date-t profits, which must be zero in equilibrium.

The government is assumed to have imposed a legal minimum denomination on the real market value of a bank deposit. The individual endowments of the poor savers are assumed to be too small to permit them to purchase bank deposits; it is further assumed to be illegal and/or infeasible for them to pool their funds to purchase deposits, or to finance deposit purchases with unsecured credit.[9] Rich savers' endowments are assumed to be large enough

[7] Note that $R > 1$ implies that the net rate of return in the international credit market exceeds the net rate of growth of the economy, which under these assumptions is zero.

[8] Since the government sets the nominal interest rate on bonds and has perfect foresight regarding the currency inflation rate (see below), it effectively sets the real interest rate on bonds.

[9] For earlier examples of the use of minimum denomination restrictions of this type to generate demand for government currency, see Sargent and Wallace (1982) and Bryant and Wallace (1984).

that this minimum denomination is irrevelant to them. Private and government bonds are assumed to have larger minimum denominations that make them inaccessible to any agents except banks. Thus the only asset available to poor savers is government currency, while rich savers may purchase government currency and/or bank deposits.

The aggregate real savings functions of the poor and rich savers are denoted $m(R_m(t))$ and $d(R_k(t))$, respectively, where $R_k(t) \equiv \max\{R_m(t), R_d(t)\}$. These functions are assumed to be positive, continuous, and nondecreasing over relevant ranges of $R_m(t)$ and $R_k(t)$.[10]

The government is assumed to finance a fixed real deficit of G per period by issuing bonds and/or currency. The aggregate nominal stock of currency in circulation at date t is denoted $M(t)$; the date t price of a unit of the consumption good in terms of government currency (the date t price level) is denoted $p(t)$. Thus $R_m(t) \equiv p(t)/p(t{+}1)$. Government bonds are payable in government currency: a bond is a title to a quantity of currency next period equal to its nominal price plus its net nominal interest (which may be negative). The aggregate nominal price of the government bonds issued at date t is denoted $B(t)$ and the gross nominal interest rate on these bonds is denoted $R_{nom}(t)$; note that $R_b(t) = R_{nom}(t)R_m(t)$. Government seigniorage revenues at dates $t{\geq}2$ are given by

$$[M(t) - M(t\text{-}1)]/p(t) + [B(t) - R_{nom}(t\text{-}1)B(t\text{-}1)]/p(t)\,.$$

The welfare of the poor and rich members of any generation t is assumed to be strictly increasing in $R_m(t)$ and $R_k(t)$, respectively. It is assumed that at date 1 there are an arbitrary number of "initial old" agents (the members of "generation 0") who live for one period and are endowed, in aggregate, with a stock of government currency M_0 and a stock of government bonds B_0. The welfare of these agents is assumed to be strictly increasing in $1/p(1)$, the inverse of the initial price level, which determines the purchasing power of the nominal assets these agents are endowed with.[11]

[10]Espinosa assumed that these functions were strictly increasing in R_m and R_d. This assumption is not essential, however, and it is convenient to be able to use constant savings functions in examples.

[11]Our treatment of government bonds differs slightly from that of Espinosa (1995). Espinosa defined $B(t)$ as the nominal *face value* of the bonds outstanding at date t and $P_b(t)$ as the nominal price of those bonds, which

The government is assumed to impose bond and/or currency reserve requirements on the banks. The fractions of a banks' assets that it is required to hold in the form of currency and government bonds are denoted θ_m and θ_b, respectively. We assume $\theta_m, \theta_b \in [0,1]$ and $\theta \equiv \theta_m + \theta_b \in [0,1]$. Each reserve ratio is the minimum ratio of the market value of a bank's holdings of one of the reservable liabilities (currency or bonds) to the market value of its entire portfolio of liabilities.

Following Espinosa (1995) we will confine ourselves to the study of *binding stationary equilibria*. Binding stationary equilibria are equilibria in which [1] the rate of return on private credit exceeds the rates of return on government currency and bonds, so that the banks will hold government liabilities only to meet the reserve requirements, and [2] the values of all real variables are constant, while the values of all nominal variables grow at fixed rates (which may be zero).[12] Given R, G, and $M(0) + B(0)$, a binding stationary equilibrium can be characterized as values of R_m, R_b, θ_m, θ_b, R_d, and p_1 that satisfy

$$R_d = (1 - \theta_m - \theta_b)R + \theta_b R_b + \theta_m R_m \tag{1}$$

$$G = (1 - R_m)[m(R_m) + \theta_m d(R_d)] + (1 - R_b)\theta_b d(R_d) \tag{2}$$

and

$$[M_0 + B_0]/p_1 = m(R_m) + (\theta_m + \theta_b)\, d(R_d) - G\,. \tag{3}$$

The first equation expresses the relationship between the interest rate on bank deposits, the two reserve ratios, and the rates of return on the three nonbank assets that is implied by the requirement that banks earn zero profits. The second and third equations ensure that the government meets its budget constraint at dates $t \geq 2$ and $t=1$, respectively. We also

is the inverse of the gross nominal interest rate. He further assumed that the initial old were endowed with an initial stock of government currency $M(0)$ and an initial stock of government bonds with fixed face value $B(0)$. It follows that the real value of the total asset endowment of the initial old is $[M(0)+P_b(1)B(0)]/p(1)$. Espinosa, however, identifies this value as $[M(0) + B(0)]/p(1)$. This identification implies that the welfare of the initial old depends entirely on the value of $p(1)$ — a property that figures importantly in many of his results. Our revised formulation, in which B_0 is the fixed nominal *market value* of the bond endowment, delivers this property.

[12] The model is easily generalized to cover situations in which the values of real variables grow at fixed, exogenously-determined rates.

require $\theta_m, \theta_b \in (0,1)$ and $\theta \equiv \theta_m + \theta_b \in (0,1)$. It follows that $R_m < R_d < R$ in any binding stationary equilibrium; the first inequality implies that rich savers' asset portfolios will be composed entirely of bank deposits.

In what follows, it is useful to define

$$A \equiv m(R_m) + \theta\, d(R_d), \tag{4}$$

which represents aggregate real balances of government liabilities, and to note that equations 1-3 imply

$$[M_0 + B_0]/p_1 = A - G = R_m[m(R_m) + \theta_m d(R_d)] + R_b \theta_b d(R_d). \tag{5}$$

In a binding stationary equilibrium we have $p(t)/p(t{+}1) = R_m$, $R_{nom}(t) = R_{nom} \equiv R_b/R_m$, $M(t)/p(t) = m(R_m){+}\theta_m d(R_d)$ and $B(t)/p(t) = \theta_m d(R_d)$ for all $t{\geq}1$ [with $p(1) \equiv p_1$]. These equations imply $M(t{+}1)/M(t) = B(t{+}1)/B(t) = 1/R_m$ for all $t{\geq}2$.

We define a "reserve-requirements policy setting" as a vector ($\overline{\theta}_m$, $\overline{\theta}_b$, $\overline{R}_m$, $\overline{R}_b$), and an associated "private and public allocation" as values ($\overline{R}_m$, $\overline{R}_d$, $\overline{p}_1$, $\overline{G}$) that this vector of policy settings supports as a binding stationary equilibrium.

We begin our analysis by establishing that in this model, any allocation that can be supported by a multiple reserve requirement — including allocations that require positive nominal interest rates on reservable bonds — can be supported by a single *bond* reserve requirement. Thus, the model does not succeed in providing an efficiency or social-welfare rationale for the existence of multiple reserve requirements.

Proposition 1 *Any public and private allocation that can be supported as a multiple reserve requirement equilibrium with $\overline{\theta}_m > 0$ can be supported as an equilibrium with $\widehat{\theta}_m = 0$ — that is, by a single bond reserve requirement.*

In this model, the only difference between currency and bonds that is relevant to prospective bond holders is that two assets may yield different rates of return. Thus when $R_b > R_m$ a single-bond-reserve-requirement regime amounts to a single-currency-reserve regime in which the government pays interest, at a below-market rate, on the currency reserves.[13]

Espinosa and Russell (1996) show that when $R_b < R_m$, a multiple-reserves equilibrium allocation can be supported by a combination of a single currency reserve requirement and a proportional tax on deposits. Thus, a government that had access to both of these financing strategies (interest on currency reserves and direct taxation of deposits) would not need to impose either a bond reserve requirement or multiple reserve requirements. We will return to the question of deposit taxation in the next section of this paper.

3 A model in which multiple reserve requirements may be optimal

As we indicated in the introduction, the model laid out by Espinosa (1995) can explain why a government might wish to impose a multiple reserve requirement instead of a single currency reserve requirement. We have just seen, however, that the model cannot explain why a government would prefer a multiple reserve requirement to a single bond reserve requirement, or, equivalently, to the payment of interest on currency reserves.[14] In this section we explore generalizations of the basic model that may allow us to explain why some governments have chosen multiple reserve requirements.

3.1 Class B economies

We will call the first class of generalized versions of the model the "Class B" economies. (We will think of Espinosa's economies as the "Class A" economies.) In Class B economies Espinosa's "poor savers," whose choice of assets was restricted to fiat currency, are replaced with "middle-class savers" who can deposit funds in banks that can invest in a domestic

[13]Sargent and Wallace (1985) study a model in which the government pays interest on reserves at the market interest rate. We are not aware of any other models in which the government pays below-market interest on reserves.

[14]As we have noted above, complete equivalence would require the ability to duplicate multiple-reserves equilibria in which the nominal interest rate on reserveable bonds was negative. In regimes without reservable bonds this could be accomplished by paying zero interest on currency reserves and imposing a proportional tax on deposits.

storage technology with rate of return X. For purposes of simplicity, we assume $X = R$.[15] One can think of these economies as versions of Espinosa's economy in which the government has authorized the establishment of "national banks" that issue liabilities in denominations small enough to be accessible to poor savers but are permitted to acquire only domestic assets. We will refer to the banks that may invest in foreign securities as "international banks."

We will let $R_d^r(t)$ denote the gross rate of return on deposits in the international banks and $R_d^p(t)$ the gross rate of return on deposits in the national banks. The savings function of the rich savers will be denoted $d^r(R_d^r(t))$ and that of the middle-class savers $d^p(R_d^p(t))$.[16] The government, we suppose, may wish to treat rich and middle-class savers differently in the sense of confronting them with different deposit interest rates. We will also assume that rich savers have the option of holding deposits in national banks (but not vice-versa). Thus $R_d^r(t) \geq R_d^p(t)$ in any equilibrium in which the deposits of international banks are positive at date t.

Given R, G, and $M_0 + B_0$, a binding stationary equilibrium in a Class B economy can be characterized as values of θ_m^p, θ_m^r, θ_b^p, θ_b^r, R_m, R_b, R_d^p, R_d^r and $1/p_1$ that satisfy

$$R_d^p = (1 - \theta_m^p - \theta_b^r)R + \theta_b^p R_b + \theta_m^p R_m \tag{6}$$

$$R_d^r = (1 - \theta_m^r - \theta_b^r)R + \theta_b^r R_b + \theta_m^r R_m \tag{7}$$

$$G = (1 - R_m)[\theta_m^p d^p(R_d^p) + \theta_m^r d^r(R_d^r)] + (1 - R_b)[\theta_b^p d^p(R_d^p) + \theta_b^r d^r(R_d^r)] \tag{8}$$

and

$$G = (\theta_m^p + \theta_b^p)\, d(R_d^p) + (\theta_m^r + \theta_b^r)\, d(R_d^r) - [M_0 + B_0]/p_1\,. \tag{9}$$

We will refer to reserve requirement regimes in which each of the four reserve ratios θ_m^p, θ_m^r, θ_b^p and θ_b^r is positive as "general multiple-reserve-requirements regimes." The following result about these regimes greatly simplifies our analysis of Class B economies:

[15] In the concluding section we discuss some of the implications of permitting $X < R$, which may seem like a more reasonable assumption.

[16] Since we already use m as a subscript, we will continue to use p as the superscript for non-rich (now middle-class) savers and their banks.

Lemma 1 *In a Class B economy, any allocation that can be supported by a general multiple-reserve-requirements regime can be supported by a regime in which the government levies a single currency reserve requirement on the national banks and a single bond reserve requirement on the international banks.*

This lemma follows immediately from Proposition 1 for Class A economies. Note that in Class B economies there is no non-reserve demand for currency, and we are consequently free to think of currency as a second type of government bond.

On the basis of the lemma, we can rewrite the multiple reserves equilibrium conditions as

$$R_d^p = (1 - \theta_m)R + \theta_m R_m \tag{10}$$

$$R_d^r = (1 - \theta_b)R + \theta_b R_b \tag{11}$$

$$G = (1 - R_m)\theta_m d^p(R_d^p) + (1 - R_b)\theta_b^r d^r(R_d^r) \tag{12}$$

and

$$G = \theta_m\, d(R_d^p) + \theta_b\, d(R_d^r) - [M_0 + B_0]/p_1\,. \tag{13}$$

We then describe a "multiple-reserves policy setting" as a vector $(\overline{\theta}_m, \overline{\theta}_b, \overline{R}_m, \overline{R}_b)$, and an associated "public and private allocation" as values ($\overline{R}_m$, $\overline{R}_d^p$, $\overline{R}_d^r$, $1/\overline{p}_1$, G) that this vector of policy settings supports as a binding stationary equilibrium.

It should be clear that in Class B economies a financing regime involving a single government liability (currency or bonds) and a single reserve ratio will not in general be socially optimal, since such a regime does not allow the government to confront rich and middle-class savers with different rates of return. An interesting question, however, is whether allocations achievable via multiple reserve requirements can also be supported by simpler regimes. The lemma establishes that in Economy B, supporting any allocation achievable by a general multiple-reserves regime requires the government to impose a maximum of two different reserve ratios, in addition to issuing liabilities that yield two different rates of return. Is it possible for the government to support the same equilibria with regimes that require two reserve ratios, but only one rate of return? What about regimes that involve two rates of return, but only one reserve ratio?[17]

The first result in this section examines the first of these two possibilities. The question is whether it possible for the government to support a multiple-reserve-requirement equilibrium with an equilibrium involving a currency reserve requirement with two different reserve ratios — one for national banks, and the other for international banks. Proposition 2 establishes that this may not be possible if the original multiple-reserve-requirements equilibrium involves government bonds that yield positive nominal interest.

Proposition 2 *In Class B economies, any allocation supported by a multiple reserve requirement with $R_b < R_m$ can also be supported by a currency reserve requirement with two reserve ratios. A multiple reserve requirement equilibrium with $R_b > R_m$ can be supported by a single reserve requirement iff $1/\overline{p}_1 \leq \overline{AR_d^p}$.*

Proposition 2 establishes that in Class B economies, the sorts of multiple reserve requirement equilibria that may not be supportable by single currency reserve requirements are precisely the sort that we observe in practice — equilibria in which reservable bonds yield positive nominal interest. In the corollary to this proposition we show, by example, that some "conventional" equilibria can be supported by by a single currency reserve requirement, while others cannot.

Corollary 1 *There are Class B economies that have multiple-reserve-requirement equilibria that cannot be supported by a single currency reserve requirement. The condition $\overline{R}_b > \overline{R}_m$ is necessary but not sufficient for this sort of unsupportability.*

Note that the necessity of $\overline{R}_b > \overline{R}_m$ for unsupportability follows immediately from Proposition 2. We complete the proof of Corollary 1 by constructing two examples of multiple-reserves equilibria in which $\overline{R}_b > \overline{R}_m$. In Example 1 the multiple-reserves equilibrium cannot be supported by a single currency reserve requirement, but in Example 2 it can be so

[17] One reason we might suspect this is possible is that in Economy B there are usually many multiple-reserves policy settings that will support a given allocation. These settings produce different values of the currency and bond return rates R_m and R_b. See Section 3.2 below.

supported. [See the appendix.] There is nothing pathological about either of these examples: in the supporting equilibria, the seigniorage revenue functions $(1-R_m)\theta_m^p d^p(R_d^p)$ and $(1-R_m)\theta_m^r d^r(R_d^r)$ are increasing in θ when R_m is fixed at its equilibrium level (these are reserve ratio Laffer curves) and are decreasing in R_m when θ is fixed at its equilbrium level (these are conventional Laffer curves).

Our second result involves the question of whether a multiple-reserve-requirement equilibrium can be supported by a single reserve ratio applied to two different government liabilities — currency for the national banks, and bonds for the international banks. Proposition 3 establishes that this, too, is not always possible.

Proposition 3 *In Class B economies, equilibria produced by a multiple reserve requirement with two reserve ratios can be supported by a regime with only one reserve ratio if and only if* $(\theta_m - \theta_b)R\, d^r(R_d^r) \leq \theta_m R_m d$, *where* $d \equiv d^p(R_d^p) + d^r(R_d^r)$.

The corollary to this proposition establishes, by example, that it is nonvacuous:

Corollary 2 *There are multiple-reserves equilibria that cannot be supported as two-liability, one-reserve-ratio equilibria.*

The proof of this corollary takes the form of Example 3 (see the appendix).

Note that unlike the situation described in Proposition 2, a multiple-reserves equilibrium that is not supportable as a one-ratio, two-return-rates equilibrium need not satisfy $R_b > R_m$. Indeed, since $R_d^r > R_d^p \Leftrightarrow (\theta_m - \theta_b)R > \theta_m R_m - \theta_b R_b$, if we choose $d^p(R_d^p)$ sufficiently small we can easily construct examples in which $R_b < R_m$.

The final question we investigate in this section is whether it is always possible to duplicate a multiple-reserves equilibrium by means of a one-ratio currency reserve requirement, applied to both types of banks, plus a deposit tax applied to one of the banks. Deficit-finance regimes involving the direct proportional taxation of the returns on bank deposits have attracted a good deal of attention in recent years. While deposit taxation may seem conceptually different from seigniorage, Fama (1980) has argued that the two financing strategies are in fact equivalent. An example of this sort of equivalence is presented by

Freeman (1987), who studies the optimal level of a single currency reserve requirement in a model that is essentially identical to Espinosa's except that the agents are intragenerationally homogeneous.[18] Freeman assumes that the government is unconcerned or the welfare of the initial old agents, and acts to maximize the steady-state utility the members of generations $t \geq 1$. He shows that it is optimal for a government with this objective to choose the smallest reserve ratio consistent with financing its deficit — a ratio at which the *gross* real rate of return on currency is zero. This policy, he notes, is equivalent to replacing the currency reserve requirement with a proportional tax on deposit returns levied at a rate equal to the required reserve ratio.

Freeman's conclusion that a deposit tax is a special case of a currency reserve requirement can also be applied to a bond reserve requirement: in our model, adding a deposit tax to a financing regime that relies on a single currency reserve requirement is formally equivalent to adding a second reserve requirement with $R_b = 0$. Thus, there is a sense in which this sort of financing regime involves two reserve ratios and two rates of return, and thus violates our relative-simplicity criterion. Since the rate of return on bonds is fixed at zero, however, this regime is slightly simpler than a true multiple-reserves regime. We examine it partly for this reason and partly for consistency with Espinosa (1995) and Espinosa and Russell (1996) — both of which discuss deposit-tax regimes in Class A economies.

For purposes of analytical convenience, our notation for the deposit-tax regime allows for a different proportional tax to be levied on the deposits of each bank. We then require that one of the tax rates is zero and that the other is positive. It follows that the tax will always be imposed on the national banks.

Proposition 4 *In Class B economies, a multiple-reserves equilibrium is supportable as an equilibrium with a common currency reserve requirement imposed on both banks, plus a deposit tax imposed on one of the banks, if and only if* $1/p_1 \leq \overline{R}_d^p[\overline{A} - d^p(\overline{R}_d^p)(\overline{R}_d^r - \overline{R}_d^p)/R]$.

Unlike Class A economies [see Espinosa and Russell (1996)], in Class B economies multiple-reserves equilibria in which bonds do not yield positive nominal interest may not be support-

[18] In Freeman's model, every agent is a "rich saver."

able by a combination of a single currency reserve requirement and a deposit tax. Note that as $d^p(\overline{R}_d^p) \to 0$ the unsupportability condition in Proposition 4 approaches $1/\overline{p}_1 > \overline{R}_d^p\overline{A}$; it is readily seen that $\overline{R}_b > \overline{R}_d^p$, and thus $\overline{R}_b > \overline{R}_m$, is necessary for this to hold. When $d^p(\overline{R}_d^p)$ is relatively large, however, the second term on the right-hand-side of this condition is also relatively large, and we can have unsupportability even when $\overline{R}_b < \overline{R}_m$. A situation of this type is described in Example 4 (see the appendix).

The reader may verify that the multiple-reserves equilibrium in Example 1 cannot be supported by a currency reserve requirement with two reserve ratios, or by a common currency reserve requirement plus a deposit tax. Thus there are multiple-reserves equilibria that cannot be supported by any of these three alternative financing regimes. It follows that in some versions of Class B economies, and for some social welfare functions, a multiple reserve requirement can achieve a higher level of social welfare than any of these regimes.

3.2 Class C economies

We have succeeded in constructing a class of economies in which there may be a social-welfare case for the imposition of multiple reserve requirements rather than simpler reserve-requirement-based financing regimes. The circumstances in which this is possible, however, seem fairly restrictive in nature, involving as they do a proper subset of the conventional (positive-nominal-bond-rate) multiple-reserves equilibria in these economies. A question that consequently seems worth asking is whether some further generalization of the basic reserve-requirements model can widen the class of multiple-reserve equilibria that cannot be supported by one or more of these alternative regimes. One answer to this question is suggested by the following facts about Class B economies: (1) the equilibrium allocation produced by a given multiple-reserves policy setting can typically be supported by a range of alternative policy settings, each of which involves a different value of the rate of return on currency, and (2) for each of the alternative financing regimes, there will be at most one value of R_m consistent with supporting a given multiple-reserves allocation as an equilibrium. Fact 2 has been established by the proofs of Propositions 2, 3, and 4. Fact 1 is readily seen by using the multiple-reserves equilibrium conditions to solve for R_m, θ_b, and R_b in terms of θ_m :

$$R_m = \frac{\theta_m R - (R - R_d^p)}{\theta_m} \tag{14}$$

$$\theta_b = \frac{A - \theta_m d^p(R_d^p)}{d^r(R_d^r)} \tag{15}$$

and

$$R_b = \frac{R\left[A - d^r(R_d^r) - \theta_m d^p(R_d^p)\right] + R_d^r d^r(R_d^r)}{A - \theta_m d^p(R_d^p)} \ . \tag{16}$$

It follows that we can duplicate the values $\overline{R}_d^p$, $\overline{R}_d^r$, $1/\overline{p}_1$, and $\overline{G}$ from a given multiple-reserves equilibrium with different policy settings simply by choosing $\widehat{\theta}_m \neq \overline{\theta}_m$ and choosing $\widehat{\theta}_b$, $\widehat{R}_m$, $\widehat{R}_b$ by substituting the values $\overline{R}_d^p$, $\overline{R}_d^r$, and $\overline{A}$ into the three formulas presented above.[19]

While we must take care to choose values of $\widehat{\theta}_m$ that are consistent with $\widehat{R}_m \in [0, \overline{R}_d^p]$, $\widehat{\theta}_b \in [0, 1]$ and $\widehat{R}_b \in [0, R]$, if the original policy settings are not at the ends of these ranges there will be closed intervals of values of $\widehat{\theta}_m$ and $\widehat{R}_m$ that will support $\overline{R}_d^p$, $\overline{R}_d^r$, $1/\overline{p}_1$, and $\overline{G}$. (See Example 5 in the appendix.)

It should be emphasized that each of the three "simpler" types of reserve-requirements regimes discussed in the previous subsection is a special case of the general multiple-reserve-requirements regime described in Lemma 1. Consequently, we know that any equilibrium supported by one of these regimes (including the equilibrium value of R_m) can be supported as a multiple-reserves equilibrium.[20] However, general multiple-reserves regimes differ from these alternative regimes in providing the monetary authority with a menu of choices (as opposed to a unique choice) for the inflation rate. As a result, in "Class C" economies which include both "middle class savers" and Espinosa's "small savers," and in which the government cares about the welfare of the small savers while at the same time relying on

[19] These solutions can be obtained using the two deposit-rate equations and the equation that defines A. It is readily seen that if R_m, R_b, and θ_b are assigned these values, the solutions for p_1 and G are the values of these variables in the initial equilibrium.

[20] In the case of a deposit-tax scheme this may involve imposing a bond reserve ratio with $R_b < R_m$ on middle-class savers while leaving the currency rate of return and reserve ratio facing rich savers unchanged. While we usually think of middle-class savers as facing a currency reserve ratio, this duplication scheme poses no formal problems because we know that $R_m \leq R_d^p$ in the equililibrium to be duplicated.

them for a portion of its seigniorage revenues, multiple-reserves regimes can, in general, produce higher levels of social welfare than the alternatives.

4 Empirical plausibility of our approach

Is there empirical support for our public-finance/price-discrimination approach to explaining the existence of multiple reserve requirements? A thoroughgoing analysis of this question would be a major project extending far beyond the boundaries of this paper. However, we have managed to collect a good deal of information about one country — Mexico — that employed multiple reserve requirements during much of the post-World War II period. We can use this information to determine whether the approach we have used in this data seems generally plausible, at least as applied to this particular case.[21]

4.1 Nonbank currency demand

A basic assumption of our model is that there is a group of agents in the economy (our "small savers") who hold substantial amounts of fiat currency for purposes for which good substitutes are not readily available. As a result, increases in the inflation rate will not produce significant decreases in the quantity of fiat currency demanded by these agents and will also have large, adverse welfare effects on them.

The most obvious empirical counterpart of demand for fiat currency on the part of small savers is currency held by the nonbank public. Thus, in order to make a *prima facie* case for the empirical plausibilty of our assumption we need to show that during the period when Mexico had a multiple-reserves regime [1] a substantial amount of currency was held by the nonbank public and [2] the level of nonbank-public currency holdings was not very sensitive to changes in the rate of inflation. This is certainly not immediately obvious: one might imagine, for example, that the economy of a relatively-high-inflation country so close to the

[21]Data on monetary aggregates, government expenditures and GDP for Mexico were provided by the *Banco de Mexico*. Mexican reserve requirement data come from Sánchez-Lugo (1976), Subdirreción de Investigación Economica y Bancaria (1976), several issues of the Informe Anual of the *Banco de Mexico,* and Padilla (1996). We are grateful to Rodolfo Padilla for his assistance.

United States has become thoroughly "dollarized," so that nonbank holdings of currency are low under the best of circumstances and tend to drop very precipitously when the inflation rate increases.

During 1960-1995, the period for which we have complete data, Mexico's average inflation rate was 27.1 percent — more than six times higher than the average U.S. inflation rate.[22] Nevertheless, the average ratio of Mexican nonbank-public currency holdings to Mexican GDP was 4.4 percent, which is identical to the comparable U.S. figure. Moreover, evidence collected by Porter (1996) and others suggests that more than half of the U.S. currency outstanding is actually held abroad, while presumably few pesos are held outside Mexico. Consequently, *domestic* non-reserve currency demand has probably been considerably larger in Mexico than in the U.S.

In addition, nonbank currency demand in Mexico seems to have been remarkably insensitive to changes in the inflation rate — changes that were often very large by U.S. standards. For the purpose of describing Mexican currency demand we can divide 1960-1995 into two very distinct periods: 1960-1982 and 1983-1995. There were large, persistent inflation rate changes within each of these two periods. During 1960-1972, for example, Mexico's annual inflation rate was quite moderate, averaging 4.1 percent and never exceeding 7 percent; from 1973-1981 the inflation rate was much higher, averaging 22 percent. Nevertheless, the ratio of nonbank currency holdings to GDP was remarkably stable across the 1960-1982 period: the 4.7 percent average ratio that prevailed during 1960-1972 was almost identical to the 4.8 percent average ratio recorded during 1973-1982, and during both subperiods the variation in the annual values of the ratio was confined almost entirely to a range of 4.5 to 5 percent.

During 1982-1988, Mexico experienced very high inflation, with an average inflation rate of 83 percent. After 1988 the inflation rate declined, averaging 21 percent during 1989-1995. However, the average ratio of nonbank currency holdings to GDP was again very stable during the 1983-1995 period, averaging 3.9 percent during 1983-1988 and 3.8 percent during

[22] It is worth noting that while this is a high inflation rate by U.S. standards, it is quite moderate by the standards of developing countries.

1989-1995. In addition, the variation in the annual values was again confined almost entirely to a range with a width of 0.5 percentage points (3.5-4 percent).[23]

4.2 Seigniorage revenues

A second basic assumption of our model is that the government earns significant revenues from seigniorage, and that reserve requirements play an important role in generating these revenues.

4.2.1 Currency seigniorage

During 1960-1995, earnings from currency seigniorage averaged roughly 2.6 percent of Mexican GDP — more than 7 times the comparable U.S. percentage. Moreover, this figure underestimates the relative importance of currency seigniorage earnings to the Mexican government. Since the government spending share of GDP has been roughly half as large in Mexico (15 percent) as in the United States, currency seigniorage revenue, which has financed 17.5 percent of Mexican government expenditures, has been roughly 15 times more important to the Mexican government than to the U.S. government.[24]

The fact that Mexico's average inflation rate has been much higher than the U.S. rate has allowed it to earn relatively large amounts of currency seigniorage revenue from all sources of currency demand. Seigniorage earnings from nonbank holdings of currency, for example have averaged almost one percent of GDP and five percent of total government expenditures

[23] As this account indicates, Mexico's nonbank currency to GDP ratio experienced an abrupt, permanent decline of roughly 1.1 percent during 1982-1983. The ratio dropped from 5.1 percent in 1982, which was 0.3 percent above its very stable 1960-1982 average, to 3.8 percent in 1983, which was 0.1 percent above its equally stable average for 1983-1995. This decline in the currency ratio roughly coincided with an abrupt, persistent increase in the inflation rate, which rose from 26 percent in 1981 to 61 percent in 1992 and remained above 50 percent until 1989. As we have seen, however, there were large, persistent changes in the inflation rate both before and after 1982-1983, and these changes do not appear to have had any significant effect on the value of the currency ratio. This observation suggests that the permanent decline in the ratio was probably not an purely endogenous response to the contemporary increase in inflation rate. In fact, it appears to have been the result of a basic change in the legal restrictions governing depository institutions: decontrol of deposit interest rates, which was enacted in 1982 and took effect in 1983.

[24] For both the U.S. and Mexico, these data are constructed by dividing the change in the nominal money stock (nonbank currency, reserves, or both) during the year by the level of nominal GDP for the year.

— almost two and four times the comparable U.S. percentages, respectively. In Mexico, however, seigniorage earnings from bank reserves have been almost twice as large as earnings from nonbank currency holdings. The reason for this, of course, is that Mexico's relatively high currency reserve requirements (see below) have caused bank reserves to average almost twice the size of nonbank currency holdings — a very different situation from the United States, where nonbank currency holdings have averaged roughly four times larger than bank reserves.

4.2.2 Bond seigniorage

Giovannini and de Melo (1993) provide some evidence in favor of the proposition that the government of Mexico has earned substantial amounts of revenue from bond seigniorage. These authors attempt to measure the revenues earned by the governments of developing countries from "financial repression" — that is, from imposing regulations that allow the government to pay lower interest rates on debt sold to its citizens than on its borrowings from foreign coutries. Their estimate of this revenue is the difference between the interest rates on foreign-held and domestically-held government debt, multiplied by the stock of domestically-held debt. In the case of Mexico, they are able to obtain the necessary data only for the years 1984-1987. Their estimate of Mexican average annual financial-repression revenue for this period is remarkably high — almost 6 percent of GDP and almost 40 percent of total government tax revenue.

The Giovannini-de Melo estimates are probably inflated by two special features of the years 1984-1987. The most important of these the fact that the country's inflation rate was extraordinarily high — averaging more than 80 percent. Another special feature of this period is that the Mexican government had recently been forced to reschedule a substantial portion of its large foreign debt, which made the interest rates it had to pay on any new debt relatively high. Nevertheless, even if in normal times the annual revenues from financial repression have been only half as large, relative to GDP, as the Giovannini-de Melo estimates for 1984-1987, then the volume of Mexico's revenues from this source has normally been similar to that of its revenues from currency seigniorage. This would seem to be a conservative estimate: although high inflation also increased Mexico's currency seigniorage revenues, its average ratio of currency seigniorage to GDP during the entire 1960-1995 period was less than 30 percent smaller than the 3.6 percent average ratio recorded during 1984-1987.

For the purposes of our analysis, only part of Mexico's financial-repression revenue can be considered bond seigniorage. Our model abstracts from government borrowing, and our bond seigniorage tax rate is properly interpreted as the difference between the growth rate of real output and the real rate of return on currency — which is presumably somewhat smaller than the difference between the latter and the real interest rate on foreign-held government debt. We have not been able to construct direct estimates of the volume of revenue the Mexican government has earned from bond seigniorage, largely because we have not been able to obtain data on the interest rates on reservable bonds. We do know that Mexico's bond reserve requirements have been relatively high — on the same order as its currency reserve requirements. For demand deposits at deposit banks in the Federal District (see below), for example, the average bond reserve ratio during 1960-1990 was almost identical to the average currency reserve ratio — almost 25 percent in each case.[25] We can use this fact to construct estimates of the volume of bond seigniorage earnings that would have been produced by different average nominal interest rates on reservable bonds. If we assume, for example, that overall holdings of reservable bonds were in fact equal to currency reserve holdings, and that the average nominal interest rate on these bonds was 10 percent — so that the average bond seigniorage tax rate was three-fifths as large as the average inflation tax rate — then Mexican bond seigniorage revenues would have averaged three-fifths of the country's currency seigniorage revenues, which would be 7.5 percent of Mexican government expenditures.

[25]Additional information on the magnitude and distribution of Mexico's bond reserve requirements is presented in the next subsection.

4.3 Reserve requirements

Another set of fundamental assumptions of our model involves the level and variability of the reserve requirements the government imposes on the banking system. Our model implicitly assumes that the government imposes, or feels free to impose, both currency and bond reserve requirements, that the aggregate reserve ratio currency and bonds is relatively high, and that the government is in a position to change the currency and/or bond reserve ratios in order to achieve its revenue and welfare goals. Finally, the model assumes that the government is able to impose different currency and bond reserve requirements on different banks, or on different types of accounts within banks — accounts that might be held by different groups of depositors.

As the account below indicates, all these things seem to have been true of Mexico. The Mexican government has imposed large currency and bond reserve requirements on its banks and related financial institutions. It has also imposed different reserve requirements on banks in different locations (inside and outside the Federal District — see below), on different types of depository institutions (deposit banks and savings banks) and on different types of accounts (demand deposits and time deposits).

4.3.1 History

In 1936 the Bank of Mexico was granted essentially complete authority to impose reserve requirements on Mexican financial institutons; this included the authority to select different types of reserve assets, to impose different reserve requirements on different institutions and/or classes of deposits, and to impose different requirements on institutions in different regions. Mexico began employing multiple reserve requirements in October of 1948, when it imposed a 25 percent government-bond reserve ratio on banks of deposit (*bancos de deposito*), which constituted by far the largest group of banks.[26] The currency reserve ratio imposed on these banks was simultaneously reduced from 50 percent to 20 percent. Bond reserve

[26]See Sanchez-Lugo (1976).

requirements were extended to savings banks (*bancos de ahorro*) in August 1955, when a 37.5 percent bond requirement was added to the 10 percent currency reserve requirement already imposed on these banks. However, the bond reserve requirements on savings banks were reduced to 2 percent in 1963 and were eliminated entirely in 1973. (Savings and deposit banks were merged in 1981.)

From 1960 through 1976, the government bond reserve requirements on demand deposits at banks in the Federal District (Mexico City and its environs, which dominate Mexico's economy) were 35 percent, while the currency reserve requirements were 15 percent. Banks outside the Federal District faced the same currency reserve requirements, but bond reserve requirements of only 10 percent. Currency and bond reserve requirements were also imposed on time deposits at deposit banks in both regions. These requirements were changed somewhat more frequently. The currency reserve requirements on time deposits ranged from 10-15 percent from 1960-1971. During the same period, the government bond reserve requirements on time deposits ranged from 0-50 percent, although they were set at 15-20 percent in most years. During the 1970s, reserve requirements on time deposits were eliminated — the bond requirements in 1973 and the currency requirements in 1977.

In 1977, the Mexican government essentially eliminated its remaining bond reserve requirements on deposits denominated in domestic currency, though it retained bond reserve requirements on foreign-currency-denominated deposits (see below).[27] At the same time, it increased its currency reserve requirements in a way that kept the aggregate reserve ratio on banks inside the Federal District essentially unchanged (it rose from 50 to 54 percent), but increased the aggregate reserve ratio on banks outside the district (from 25 to 38 percent). Two years later, the currency reserve ratios for the two different types of banks were equalized at 38 percent. During the next few years the common currency reserve ratio was gradually increased, reaching 48 percent in 1984. In 1985, bond reserve requirements were reinstated

[27]It eliminated the bond reserve requirements on deposits at deposit banks and mortgage credit societies, but retained them on deposits at finance societies (see below). Bond reserve requirements at deposit banks actually began to be phased out in August 1972, when they were dropped on deposits accepted after August 21.

and the bond reserve ratio was set at 38 percent. The currency reserve ratio was reduced to 10 percent, so the aggregate reserve ratio remained unchanged. The 10 percent currency reserve ratio remained in place through 1990, but the bond reserve ratio was gradually decreased to 30 percent. In 1991, however, Mexico reversed course again and eliminated both currency and bond reserve requirements entirely — with the exception, again, of deposits denominated in foreign currency.

Beginning in 1958, Mexico also imposed currency and bond reserve requirements on two smaller groups of financial institutions: finance societies (*sociedades financieras*) and mortgage credit societies (*sociedades de credito hipotecario*). These institutions, like the savings banks, offered noncheckable deposits with longer average maturities than those of the deposit banks. The reserve requirements imposed on these two types of institutions were different from each other and from the reserve requirements imposed on deposit and savings banks. Compared to the mortgage credit societies, the requirements on the finance societies were higher in aggregate and were more heavily weighted towards bond reserves. The reserve requirements on these two types of banks were changed quite frequently, particularly during the 1970s, before the two types of institution were merged into the deposit banks in 1981.

As we have indicated, Mexico has allowed banks to accept deposits denominated in foreign currency (principally U.S. dollars), and has imposed both currency and bond reserve requirements on these deposits. The aggregate reserve ratios on these deposits have usually been quite high, and they have typically been heavily weighted towards bond reserve requirements. For deposits of this type, the analogue of the "inside or outside the Federal District" distinction is "along or below the northern frontier" — the norther frontier being the Mexican states along the border with the U.S. During 1972-1980 the reserve requirements imposed on foreign-currency-denominated deposits at deposit banks along the northern frontier were different from the reserve ratios imposed on these deposits at deposit banks located elsewhere in the Mexico. Typically, the reserve ratios imposed on northern-frontier banks were a bit less severe, involving slightly lower aggregate reserve ratios and/or somewhat heavier reliance on bond reserves.

4.3.2 Implications

From the point of view of the specifics of our analysis, perhaps the most interesting aspect of this record is the fact that prior to 1977 the Mexican government used bond reserve requirements to impose differentially high (indirect) tax rates on demand deposits at deposit banks inside the Federal District. In addition, the government used both currency and bond reserve requirements to impose different tax rates on deposits at savings banks, finance societies and mortgage credit societies. Since economic activity and income in Mexico are disproportionately concentrated in Mexico City, it does not seem unreasonable to think of the deposit banks inside the Federal District as serving "rich savers" and of the banks outside the district as serving "poor savers."[28] Similarly, holdings of noncheckable, longer term deposits are concentrated among the relatively wealthy Mexicans.

Shortly after the Mexican government eliminated bond reserve requirements on domestic-currency deposits it abandoned its entire differential deposit-tax system, first by equalizing the reserve requirements on the two types of deposit banks and later by merging the other three types of financial institution into the deposit banks. This behavior suggests that differential deposit taxation may have been the government's principal motive for imposing bond reserve requirements — precisely the assumption that forms the basis of our analysis.[29]

[28]Given Mexico's relatively high inflation rates, the high reserve ratios on foreign-currency-denominated deposits have probably been necessary to reduce their attractiveness relative to deposits denominated in pesos. However, the geographic differentiation of these reserve requirements requirements may have been motivated by price discrimination/social-welfare considerations. While holdings of deposits denominated in foreign currency are probably concentrated among relatively wealthy Mexicans, this may be somewhat less true along the northern frontier because of the relatively high percentage of the local population engaged in trade with the United States.

[29]One problem with this argument is that bond reserve requirements were not permanently dropped — they were reimposed in 1985. When this happened, however, the currency reserve requirements were drastically reduced, so that the aggregate reserve ratio was unchanged. Thus, in this case the government seems to have used the reimposition of bond reserve requirements as a device to reduce the implicit tax rate on deposits without changing the aggregate reserve ratio. It seems likely that politics played a role in this decision. At the time bond reserve requirements were reimposed Mexico's inflation rate was extremely high, and the public was undoubtedly resistant to changes in government policy that seemed to benefit the banks at the expense

5 Concluding remarks

This paper has attempted to provide a social-welfare-based explanation for the existence of seigniorage-based deficit-finance regimes that involve multiple reserve requirements. Its point of departure was the analysis conducted by Espinosa (1995), who sought to construct a model that would explain why a number of developing countries have adopted multiple-reserves regimes. We began by demonstrating that in Espinosa's model, any allocation that can be supported by a multiple-reserves regime can also be supported by a single bond reserve requirement. Thus, this model does not really explain why a government would find it useful to impose two reserve requirements. We have gone on to construct two closely related models that improve on Espinosa's model by providing a clear social-welfare rationale for a particular type of multiple-reserves regime: a regime in which the government imposes a currency reserve requirement on banks that serve one class of depositors, and a bond reserve requirement on banks that serve another class. More specifically, we have shown that this type of multiple-reserves regime — a regime that requires the government to impose two reserve ratios and to issue liabilities with two rates of return — may allow the government to achieve a higher level of social welfare than simpler deficit-financing regimes that require it to impose fewer reserve ratios and/or issue fewer types of liabilities. The multiple-reserves regimes that are potentially social-welfare improving, moreover, are conventional in nature.

A potentially interesting extension of our analysis involves economies in which $X < R$ — that is, economies in which the rate of return on the domestic investments available to "national banks" is lower than the rate that prevails in the international credit market. In these economies there is an efficiency rationale for deposit taxation even when the government cares about the initial old: it may make sense to minimize the bond reserve ratio on international banks, since they hold relatively productive assets, and to compensate for the

of nonbank holders of currency. Changes in the aggregate reserve ratio are probably easier for the public to understand and interpret than changes in the nature and distribution of reserve assets. Consequently, the government may have viewed reimposing bond reserve requirements as a way to reduce the burden of reserve costs on banking system without making it obvious that it was doing so.

resulting loss of money demand (which hurts the initial old by increasing the initial price level) by increasing the currency reserve ratio imposed on the national banks. If the government is to avoid harming middle-class savers, however, it must compensate them for the increased currency reserve requirement by increasing the rate of return on currency; if there are poor savers in the model, this may produce an unacceptable loss of seigniorage revenues.

The lemma established in Section 3, which greatly simplified the analysis of multiple-reserves regimes in economies with heterogeneous depositors, also identifies what is perhaps the biggest limitation of our analysis. While it is true that we frequently observe the governments of developing countries imposing different currency and bond reserve requirements on different types of banks, it is also true that we frequently observe them imposing both currency and bond reserve ratios on a given type of bank. Our analysis does not explain this behavior. We believe the basic reason for this is that we have followed Freeman (1987) and Espinosa (1995) by studying economies in which [1] the government imposes reserve requirements only for the purpose of deficit finance and [2] government liabilities differ from each other only in their rates of return and in their accessibility to different types of agents, and not in less-readily-definable characteristics such as "liquidity" or "usefulness as media of exchange."[30] We suspect that constructing a model in which imposing currency and bond reserve requirements on the same bank improves welfare may require assuming that the government uses reserve requirements for additional purposes, and that government currency is inherently superior to government bonds for these purposes. We also suspect that it will be very difficult to do this in a manner that is both rigorous and plausible.

[30] These assumptions seem very reasonable to us. Goodfriend and Hargraves (1983) note that in the modern U.S., at least, reserve requirements have played a very minor role in liquidity regulation and monetary control, but a fairly significant role in deficit finance. Government bonds, moreover, are simply default-free claims to future government currency. [For an extended discussion of the implications of the latter fact, see Wallace (1983).]

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Appendix A

Proof of Proposition 1: As in the proof of Proposition 2, $1/\widehat{p}_1 = 1/\overline{p}_1$ and $\overline{G} = \widehat{G}$ imply $\overline{A} = \widehat{A}$, and this equality, together with $\widehat{R}_d = \overline{R}_d$ and $\widehat{R}_m = \overline{R}_m$, implies $\widehat{\theta} = \overline{\theta}$. Since $\widehat{\theta}_m = 0$, $\widehat{R}_d = \overline{R}_d$ and $\widehat{\theta} = \overline{\theta}$ imply

$$\widehat{\theta}_b \widehat{R}_b = \overline{R}_d - (1 - \overline{\theta})R;$$

since $\widehat{\theta}_b = \widehat{\theta} = \overline{\theta}$, we must have $\widehat{R}_b = [\overline{R}_d - (1 - \overline{\theta})R]/\overline{\theta}$. This setting for R_b is feasible if and only if $[\overline{R}_d - (1 - \overline{\theta})R]/\overline{\theta} < R$. This condition will be satisfied iff $R > \overline{R}_d$, an inequality which holds in any binding stationary equilibrium. □

Proof of Proposition 2: In a supporting single-double currency reserve requirement equilibrium, we must have $\widehat{G} = \overline{G}$ and $1/\widehat{p}_1 = 1/\overline{p}_1$. Since $[M(0) + B(0)]/p_1 = A - G$, it follows that we must have $\widehat{A} = \overline{A}$. Thus any duplicating equilibrium satisfies $(1 - \widehat{R}_m)\overline{A} = \overline{G}$, and, consequently,

$$\widehat{R}_m = \frac{\overline{A} - \overline{G}}{\overline{A}} = \frac{1/\overline{p}_1}{\overline{A}}.$$

Note that $0 < \widehat{R}_m < 1$.

In a duplicating equilibrium we also have

$$\overline{R}^p_d = (1 - \widehat{\theta}^p_m)R + \widehat{\theta}^p_m \widehat{R}_m$$

and

$$\overline{R}^r_d = (1 - \widehat{\theta}^r_m)R + \widehat{\theta}^r_m \widehat{R}_m$$

where θ^p_m and θ^r_m denote reserve ratios imposed on the national and international banks, respectively. It follows that

$$\widehat{\theta}^p_m = \overline{\theta}_m \frac{R - \overline{R}_m}{R - \widehat{R}_m} = \overline{\theta}_m \frac{R - \overline{R}_m}{R - \frac{1/\overline{p}_1}{\overline{A}}}$$

and

$$\widehat{\theta}^r_m = \overline{\theta}_b \frac{R - \overline{R}_b}{R - \widehat{R}_m} = \overline{\theta}_b \frac{R - \overline{R}_b}{R - \frac{1/\overline{p}_1}{\overline{A}}}.$$

Since $[M(0) + B(0)]/p_1 = A - G$, we have

$$[M(0) + B(0)]/\overline{p}_1 = \overline{\theta}_m \overline{R}_m d^p(\overline{R}^p_d) + \overline{\theta}_b \overline{R}_b d^r(\overline{R}^r_d)$$

and

$$[M(0)+B(0)]/\widehat{p}_1 = \widehat{\theta}^p_m \widehat{R}_m d^p(\overline{R}^p_d) + \widehat{\theta}^r_m \widehat{R}_m d^r(\overline{R}^r_d).$$

Thus $1/\widehat{p}_1 = 1/\overline{p}_1 \Leftrightarrow \widehat{\theta}^p_m \widehat{R}_m d^p(\overline{R}^p_d) + \widehat{\theta}^r_m \widehat{R}_m d^r(\overline{R}^r_d) = \frac{\overline{p}_1(R-\overline{R}_m)}{R\overline{A}-1/\overline{p}_1}\overline{\theta}_m d^p(\overline{R}^p_d) + \frac{(1/\overline{p}_1)(R-\overline{R}_b)}{R\overline{A}-(1/\overline{p}_1)}\overline{\theta}_b d^r(\overline{R}^r_d) = \frac{1/\overline{p}_1}{R\overline{A}-1/p_1}[(R-\overline{R}_m)\overline{\theta}_m d^p(\overline{R}^p_d) + (R-\overline{R}_b)\overline{\theta}_b d^r(\overline{R}^r_d)]$. Recalling that $\overline{A} = \overline{\theta}_m d^p(\overline{R}^p_d) + \overline{\theta}_b d^r(\overline{R}^r_d)$, we have $(R-\overline{R}_m)\overline{\theta}_m d^p(\overline{R}^p_d) + (R-\overline{R}_b)\overline{\theta}_b d^r(\overline{R}^r_d) = R\overline{A} - 1/\overline{p}_1$ and $\frac{1/\overline{p}_1}{R\overline{A}-1/p_1}[(R-\overline{R}_m)\overline{\theta}_m d^p(\overline{R}^p_d) + (R-\overline{R}_b)\overline{\theta}_b d^r(\overline{R}^r_d)] = 1/\overline{p}_1$.

We also need $\widehat{G} = \overline{G}$, which is equivalent, given $1/\widehat{p}_1 = 1/\overline{p}_1$, to $\widehat{A} = \overline{A}$, where $\widehat{A} = \widehat{\theta}^p_m d^p(\overline{R}^p_d) + \widehat{\theta}^r_m d^r(\overline{R}^r_d)$. Now $\widehat{\theta}^p_m d^p(\overline{R}^p_d) + \widehat{\theta}^r_m d^r(\overline{R}^r_d) = \overline{\theta}_m \frac{R-\overline{R}_m}{R-\frac{1/\overline{p}_1}{\overline{A}}} d^p(\overline{R}^p_d) + \overline{\theta}_b \frac{R-\overline{R}_b}{R-\frac{1/\overline{p}_1}{\overline{A}}} d^r(\overline{R}^r_d)$, so $\widehat{A} = \overline{A} \Leftrightarrow (R-\overline{R}_m)\overline{\theta}_m d^p(\overline{R}^p_d) + (R-\overline{R}_b)\overline{\theta}_b d^r(\overline{R}^r_d) = R\overline{A} - 1/\overline{p}_1$, which was established above.

Finally, we need to show that $0 < \widehat{\theta}^p_m < 1$ and $0 < \widehat{\theta}^r_m < 1$. Now $\widehat{\theta}^p_m$, $\widehat{\theta}^r_m > 0$ follows from $\widehat{R}_m < 1 < R$ (see above). For $\widehat{\theta}^p_m$, $\widehat{\theta}^r_m < 1$ we need $(R-\overline{R}_m)\overline{\theta}_m < R - \widehat{R}_m$ and $(R-\overline{R}_b)\overline{\theta}_b < R - \widehat{R}_m$, respectively; these are equivalent to $\widehat{R}_m < \overline{R}^p_d$ and $\widehat{R}_m < \overline{R}^r_d \Leftrightarrow 1/\overline{p}_1 < \overline{A}\overline{R}^p_d$ and $1/\overline{p}_1 < \overline{A}\overline{R}^r_d \Leftrightarrow$

$$\overline{\theta}_m \overline{R}_m d^p(\overline{R}^p_d) + \overline{\theta}_b \overline{R}_b d^r(\overline{R}^r_d) < [\overline{\theta}_m d^p(\overline{R}^p_d) + \overline{\theta}_b d^r(\overline{R}^r_d)]\overline{R}^p_d$$

and

$$\overline{\theta}_m \overline{R}_m d^p(\overline{R}^p_d) + \overline{\theta}_b \overline{R}_b d^r(\overline{R}^r_d) < [\overline{\theta}_m d^p(\overline{R}^p_d) + \overline{\theta}_b d^r(\overline{R}^r_d)]\overline{R}^r_d.$$

We know $\overline{R}^p_d > \overline{R}_m$ and $\overline{R}^r_d > \overline{R}_b$. We have also assumed $\overline{R}^r_d \geq \overline{R}^p_d$, which implies $\overline{R}^r_d > \overline{R}_m$. It follows that $\widehat{\theta}^r_m < 1$. We cannot guarantee $\widehat{\theta}^p_m < 1$, however, since it is conceivable that $\overline{R}^p_d < \overline{R}_b$. Note that in this case $\overline{R}_b > \overline{R}_m$. □

Proof of Corollary 1:

Example 1 *Let* $R = 1.02$, $d^p(R^p_d) = 1$, $d^r(R^r_d) = 10 - 4/R^r_d$, *and* $G \doteq 0.07010$. *Then* $(\overline{\theta}_m, \overline{\theta}_b) = (0.2, 0.5)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.8, 0.99)$ *are equilibrium multiple-reserve policy settings. These settings produce* $\overline{R}^p_d = 0.976$, $\overline{R}^r_d = 1.005$, $1/\overline{p}_1 \doteq 3.140$ *and* $\overline{A} \doteq 3.181$; *notice that* $\overline{R}_b > \overline{R}_m$. *Since* $1/\overline{p}_1 > \overline{A}\overline{R}^p_d$, *Proposition 5 shows that this multiple-reserve-requirement equilibrium cannot be supported as a single-reserve-requirement equilibrium. (A supporting equilibrium would require* $\widehat{R}_m = (1/\overline{p}_1)/\overline{A} \doteq 0.9872$, *and this implies* $\widehat{\theta}^r_m \doteq 1.052$.*)*

Example 2 *Again let* $R = 1.02$, $d^p(R^p_d) = 1$ *and* $d^r(R^r_d) = 10 - 4/R^r_d$, *but let* $G \doteq 0.06413$. *Then* $(\overline{\theta}_m, \overline{\theta}_b) = (0.2, 0.4)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.8, 0.99)$ *are equilibrium multiple-reserve policy settings. These settings produce* $\overline{R}^p_d = 0.976$, $\overline{R}^r_d = 1.008$, *and* $1/\overline{p}_1 \doteq 2.549$; *notice, again, that* $\overline{R}_b > \overline{R}_m$. *The supporting single-reserve-requirement equilibrium involves* $\widehat{R}_m \doteq 0.9754$ *and* $(\widehat{\theta}^p_m, \widehat{\theta}^r_m) \doteq (0.9879, 0.2694)$. □

Proof of Proposition 3: In a supporting equilibrium, we must have

$$\overline{R}_d^p = (1-\widehat{\theta}_m)R + \widehat{\theta}_m\widehat{R}_m,$$

$$\overline{R}_d^r = (1-\widehat{\theta}_m)R + \widehat{\theta}_b\widehat{R}_b$$

and

$$\overline{A} = \widehat{\theta}_m\overline{d}.$$

The last equation implies $\widehat{\theta}_m = \overline{A}/\overline{d}$; note that $0 < \overline{\theta}_m, \overline{\theta}_b < 1$ implies $0 < \widehat{\theta}_m < 1$. The first two equations then imply

$$\widehat{R}_m = \frac{R(\overline{A}-\overline{d}) + \overline{R}_d^p\overline{d}}{\overline{A}}$$

and

$$\widehat{R}_b = \frac{R(\overline{A}-\overline{d}) + \overline{R}_d^r\overline{d}}{\overline{A}}.$$

Note that $\widehat{R}_m > \overline{R}_d^p \Leftrightarrow R(\overline{A}-\overline{d}) + \overline{R}_d^p\overline{d} > \overline{R}_d^p A \Leftrightarrow R(\overline{A}-\overline{d}) > \overline{R}_d^p(\overline{A}-\overline{d}) \Leftrightarrow \overline{R}_d^p > R$, so we have $\widehat{R}_m \leq \overline{R}_d^p$. Also, $\widehat{R}_b < 0 \Leftrightarrow R(\overline{A}-\overline{d}) + \overline{R}_d^r\overline{d} < 0$. Now $R(\overline{A}-\overline{d}) = R(\overline{\theta}_m - 1)d^p(\overline{R}_d^p) + R(\overline{\theta}_b - 1)d^r(\overline{R}_d^r) = (\overline{\theta}_m\overline{R}_m - \overline{R}_d^p)d^p(\overline{R}_d^p) + (\overline{\theta}_b\overline{R}_b - \overline{R}_d^r)d^r(\overline{R}_d^r)$, so $R(\overline{A}-\overline{d}) + \overline{R}_d^r\overline{d} < 0 \Leftrightarrow (\overline{R}_d^r - \overline{R}_d^p)d^p(\overline{R}_d^p) + \overline{\theta}_m\overline{R}_m d^p(\overline{R}_d^p) + \overline{\theta}_b\overline{R}_b d^r(\overline{R}_d^r) < 0 \Leftrightarrow \overline{\theta}_m\overline{R}_m d^p(\overline{R}_d^p) + \overline{\theta}_b\overline{R}_b d^r(\overline{R}_d^r) < (\overline{R}_d^p - \overline{R}_d^r)d^p(\overline{R}_d^p)$; since $\overline{R}_d^r \geq \overline{R}_d^p$ by assumption, we have $\widehat{R}_b \geq 0$. It remains to be checked that $\widehat{R}_m \geq 0$. Now $\widehat{R}_m \geq 0 \Leftrightarrow \overline{\theta}_m\overline{R}_m d^p(\overline{R}_d^p) + \overline{\theta}_b\overline{R}_b d^r(\overline{R}_d^r) \geq (\overline{R}_d^r - \overline{R}_d^p)d^r(\overline{R}_d^r)$. Since $\overline{R}_d^r - \overline{R}_d^p = (\overline{\theta}_m - \overline{\theta}_b)R + \overline{\theta}_b\overline{R}_b - \overline{\theta}_m\overline{R}_m$, $\overline{\theta}_m\overline{R}_m d^p(\overline{R}_d^p) + \overline{\theta}_b\overline{R}_b d^r(\overline{R}_d^r) \geq (\overline{R}_d^r - \overline{R}_d^p)d^r(\overline{R}_d^r) \Leftrightarrow \overline{\theta}_m\overline{R}_m d^p(\overline{R}_d^p) \geq (\overline{\theta}_m - \overline{\theta}_b)Rd^r(\overline{R}_d^r) - \overline{\theta}_m\overline{R}_m d^r(\overline{R}_d^r) \Leftrightarrow \overline{\theta}_m\overline{R}_m d^r(\overline{R}_d^r) \geq (\overline{\theta}_m - \overline{\theta}_b)Rd^r(\overline{R}_d^r)$. □

Proof of Corollary 2:

Example 3 *Let $R = 1.02$, $d^p(R_d^p) = 1$, $d^r(R_d^r) = 10$, and $G = 0.34$. Then $(\overline{\theta}_m, \overline{\theta}_b) = (0.4, 0.2)$ and $(\overline{R}_m, \overline{R}_b) = (0.4, 0.95)$ are equilibrium multiple-reserve policy settings. These settings produce $\overline{R}_d^p = 0.772$, $\overline{R}_d^r = 1.006$, $1/\overline{p}_1 = 2.06$ and $\overline{A} = 2.4$. Note that $(\overline{\theta}_m - \overline{\theta}_b)Rd^r(\overline{R}_d^r) = 2.04 > \overline{\theta}_m\overline{R}_m\overline{d} = 1.76$. A supporting equilibrium would require $\widehat{\theta}_m = \overline{A}/\overline{d} \doteq 0.2182$, and this implies $\widehat{R}_m = -0.1167$.* □

Proof of Proposition 4: We have

$$\overline{R}_d^p = (1-\widehat{\theta}_m-\widehat{\theta}_b^p)R + \widehat{\theta}_m\widehat{R}_m,$$

$$\overline{R}_d^r = (1 - \widehat{\theta}_m - \widehat{\theta}_b^r)R + \widehat{\theta}_m \widehat{R}_m$$
$$\overline{A} = \widehat{\theta}_m \overline{d} + \widehat{\theta}_b^p d^p(\overline{R}_d^p) + \widehat{\theta}_b^r d^r(\overline{R}_d^r).$$

These equations collectively imply $\widehat{R}_m = \overline{p}_1/\widehat{\theta}_m\overline{d}$; the first two equations then imply

$$\widehat{\theta}_b^p = \frac{R(\overline{A} - \widehat{\theta}_m\overline{d}) + d^r(\overline{R}_d^r)[\overline{R}_d^r - \overline{R}_d^p]}{R\overline{d}}$$

and

$$\widehat{\theta}_b^r = \frac{R(\overline{A} - \widehat{\theta}_m\overline{d}) + d^p(\overline{R}_d^p)[\overline{R}_d^p - \overline{R}_d^r]}{R\overline{d}}.$$

Now $\widehat{\theta}_b^p = 0 \Leftrightarrow \widehat{\theta}_m = \frac{R\overline{A} + d^r(\overline{R}_d^r)[\overline{R}_d^r - \overline{R}_d^p]}{R\overline{d}}$. Substitution into the expression for $\widehat{\theta}_b^r$, and simplifying, yields

$$\widehat{\theta}_b^r = (\overline{R}_d^p - \overline{R}_d^r)/R.$$

Thus $\widehat{\theta}_b^p = 0 \Leftrightarrow \widehat{\theta}_b^r \leq 0$. But $\widehat{\theta}_b^r = 0 \Leftrightarrow \widehat{\theta}_m = \frac{R\overline{A} + d^p(\overline{R}_d^p)[\overline{R}_d^p - \overline{R}_d^r]}{R\overline{d}}$, which yields $\widehat{\theta}_b^p = (\overline{R}_d^r - \overline{R}_d^p)/R$. Thus $\widehat{\theta}_b^r = 0 \Leftrightarrow \widehat{\theta}_b^p \geq 0$.

We have $\widehat{\theta}_b^p = (\overline{R}_d^r - \overline{R}_d^p)/R \Leftrightarrow \hat{R}_m = (1/\overline{p}_1)/[\overline{A} - d^p(\overline{R}_d^p)(\overline{R}_d^r - \overline{R}_d^p)/R]$ (note that $R_m > (1/\overline{p}_1)/\overline{A} = \frac{\theta_m R_m d^p(\overline{R}_d^p) + \theta_b R_b d^r(\overline{R}_d^r)}{[\theta_m d^p(\overline{R}_d^p) + \theta_b d^r(\overline{R}_d^r)] - d^p(\overline{R}_d^p)(\overline{R}_d^r - \overline{R}_d^p)/R}$). So $\widehat{R}_m > \overline{R}_d^p \Leftrightarrow \overline{\theta}_m \overline{R}_m d^p(\overline{R}_d^p) + \overline{\theta}_b \overline{R}_b d^r(\overline{R}_d^r) > \overline{R}_d^p([\overline{\theta}_m d^p(\overline{R}_d^p) + \overline{\theta}_b d^r(\overline{R}_d^r)] - d^p(\overline{R}_d^p)(\overline{R}_d^r - \overline{R}_d^p)/R)$, which is $1/\overline{p}_1 > \overline{R}_d^p[\overline{A} - d^p(\overline{R}_d^p)(\overline{R}_d^r - \overline{R}_d^p)/R]$.
□

Example 4 *Let* $R = 1.02$, $d^p(R_d^p) = 2$, $d^r(R_d^r) = 1$, *and* $G = 0.115$. *Then* $(\overline{\theta}_m, \overline{\theta}_b) = (0.5, 0.1)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.9, 0.85)$ *are equilibrium multiple-reserve policy settings. These settings produce* $\overline{R}_d^p = 0.96$, $\overline{R}_d^r = 1.003$, $1/\overline{p}_1 = 0.985$ *and* $\overline{A} = 1.1$. *Note that* $1/\overline{p}_1 = 0.985 > \overline{R}_d^p[\overline{A} - d^p(\overline{R}_d^p)(\overline{R}_d^r - \overline{R}_d^p)/R] \doteq 0.975$. *A supporting equilibrium would require* $\widehat{\theta}_b^p \doteq 0.0422$, $\widehat{\theta}_m \doteq 0.3386$, *and* $\widehat{R}_m \doteq 0.9698 > \overline{R}_d^p$.

Example 5 *Suppose* $d^p(R_d^p) = 10 - 4/R_d^p$, $d^r(R_d^r) = 48 - 43/R_d^r$, *and* $\overline{G} \doteq 0.852288$. *Then* $\overline{\theta}_m = 0.4$, $\overline{\theta}_b = 0.25$, $\overline{R}_m = 0.75$ *and* $\overline{R}_b = 0.9$ *is an equilibrium policy setting. This setting produces* $\overline{R}_d^p = 1.02$, $\overline{R}_d^r = 1.125$, $1/\overline{p}_1 \doteq 4.02353$ *and* $\overline{A} \doteq 4.87582$. *The interval of supporting values of* $\widehat{\theta}_m$ *is approximately* $[0.15, 0.701613]$, *and the corresponding* $\widehat{R}_m$*-interval is approximately* $[0, 0.943448]$. *For values of* $\widehat{\theta}_m$ *on the former interval we have* $\widehat{\theta}_b \in [0, 1]$ *and* $\widehat{R}_b \in [0, R]$. *For values of* θ_m *below the lower bound of the interval* $\widehat{R}_m < 0$, *and for values above its upper bound* $\widehat{R}_b < 0$ *or* $\widehat{R}_b > R$.