Jump Risk, Time-Varying Risk Premia, and Technical Trading Profits

Chenyang Feng and Stephen D. Smith

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Abstract: In this paper we investigate the recently documented trading profits based on technical trading rules in an asset pricing framework that incorporates jump risk and time-varying risk premia. Following Brock, Lakonishok, and LeBaron (1992), we apply popular technical trading rules to the daily S&P 500 index over a long period of time. Trading profits are examined using bootstrap simulation to address distributional anomalies. We estimate a variety of asset pricing models, including the random walk, autoregressive models, a combined jump diffusion model, and a combined model of jump-diffusion and autoregressive conditional heteroscedasticity. Technical trading profits are shown to be statistically significant for the pure diffusion models and autoregressive models, yet become less significant when jump risk is incorporated into the model and virtually disappear for an asset pricing model that incorporates both jump risk and time-varying risk premia. The empirical evidence suggests that technical trading profits could be fair compensation for the risk of price discontinuity as well as time-varying risk premia of asset returns. Alternatively, technical trading profits provide a test of specification of asset pricing models; in this vein the evidence provides support for the incorporation of jump risk into asset pricing models.

JEL classification: G12, G14

Key words: asset pricing, information and market efficiency

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Introduction

Empirical evidence on short term stock and foreign exchange data suggests that stock returns or exchange rate changes are not stationary diffusion processes. They display leptokurtosis and skewness. Finding the right distributional assumption for these series is important since the underlying return generating process for equity returns and foreign exchange rates are critical in pricing financial products. Theoretically, the deviation from normality of the return process can be accounted for by assuming either a mixture of stationary distributions, such as two normal distributions with different means and variances, or a mixture of a normal and jump process, or alternatively, a distribution such as the normal distribution with time-varying parameters.

Jarrow and Rosenfeld (1984) and Ball and Torous (1985) have found evidence that daily stock returns are characterized by lognormally distributed jumps. Akgiray and Booth (1988) suggest a mixed jump-diffusion process for exchange rate changes. On the other hand, models with autoregressive conditional heteroscedasticity, first proposed by Engle (1982), are popular alternatives for specifying the return processes. Jorion (1988) uses a combination of a mixture of jump-diffusion process and ARCH process to estimate weekly and monthly US stock index and US $/DM exchange rate series, using maximum likelihood estimation. He finds that exchange rate changes exhibit systematic jump risk. More recently, Brorsen and Yang (1994) use combinations of a jump-diffusion process and GARCH type processes to estimate US stock index series. They find that nonlinear dependency is not removed after all the specification adjustments.

A common feature of the above-mentioned tests is that, to determine the goodness of fit of the specifications, for either nested or nonnested tests, the authors use either the likelihood ratio test, or adjusted forms of the test such as the AIC (Akaike Information Criterion) or the SBC (Schwartz Baysian Criterion). While these parametric test techniques may be rigorous, other means of testing the model's consistency with the sample data may shed more light on this issue, especially considering the residual nonlinear dependency observed in the sample data under the above specifications.

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1. Technical trading rules.

Recently, a considerable amount of work has provided support for the value of technical trading rules. In Brown and Jennings (1989), for instance, it is demonstrated that under a noisy rational expectations equilibrium model in which a current price does not fully reveal private information due to heterogeneously informed market participants, past and current prices together enable uninformed price-taking investors to make more precise inferences about the signals. In empirical work, Brock, Lakonishok and LeBaron (1992) use bootstrap simulations of various null asset pricing models and find that simple technical rule profits cannot be explained away by the popular statistical models of stock index returns. Levich and Thomas (1993) use the same bootstrap simulation technique to present evidence on the profitability and statistical significance of technical trading rules in the foreign exchange market with currency futures data.

The existing evidence on the profitability of technical rules challenges the traditional version of the efficient market hypothesis. However, given the fact that jump risk and time varying volatility are observed in both equity and currency markets, in order to obtain a proper evaluation of technical trading rules, we need to reexamine profits in the context of these more complex processes. It is consistent with the efficient market hypothesis to have trading profit that compensates for risk within the correctly specified asset pricing model. On the other hand, if the technical trading profits, adjusted for risk, are no longer significant, then the profits are explainable by the null model. This would indicate that the null model under which the trading rule is examined is correctly specified. This approach provides such a specification test of asset pricing models.

2. Bootstrap simulation.

Brock, Lakonishok and LeBaron (1992) provide the first work that combines technical analysis, evaluation of asset pricing models, and the bootstrap methodology. Bootstrap methods (Efron 1979) provide simulated empirical distributions for the profits under various null models, which can be compared with actual profits. This strategy addresses the distributional anomalies of equity returns and exchange rate changes discussed before, such as leptokurtosis, autocorrelation, conditional heteroscedasticity, and changing conditional means. In addition, we can develop a

joint test of significance for a set of trading rules by utilizing bootstrap distributions for these tests.

3. The mixed jump-diffusion model.

In Brock, Lakonishok and LeBaron (1992), technical trading profits are found to be statistically profitable when applied to daily DJIA series over a period of 90 years. Furthermore, these profits are not explainable under a variety of null models, including the random walk model, AR(1), GARCH-M model, and exponential GARCH model. More recently, Kho (1996) provides evidence on time varying risk premia using similar tests on weekly currency futures data over a period of 11 years. He finds that significant technical trading profits on currency futures can be explained away by a GARCH-M model of the currency futures price process. While GARCH type models capture time varying risk premia of asset returns, they do not address the possibility of discontinuities in prices, or jump risk.

Apart from the empirical evidence on the existence of jump risk, the mixed jump-diffusion model describes certain institutional features of the asset markets quite well. For stocks, for instance, strategic trading of information traders are analyzed by Kyle (1985). In this model, information is gradually incorporated into prices through trading in a specialist market. This process is generally continuous. There are also unanticipated news announcements about major macroeconomic indicators, such as the interest rates, that have a direct impact on the stock index, or firm-specific "events", which affects individual stock prices. The price adjustment in the latter case would be discontinuous. In a typical jump-diffusion model, the Wiener (diffusion) process captures continuous fluctuations in stock prices due to strategic trading by informed traders, trading by liquidity traders, and market microstructure effects; an independent compound Poisson (jump) process models the discrete jumps in stock prices due to unanticipated information released to the public.

For foreign exchange markets, the jump factor has even stronger institutional appeal. The diffusion process captures the speculative and liquidity trading of the currency, whereas the jump process captures two possibilities: first, when major macroeconomic news break out; second, when the central bank revalues the currency for reasons such as realignments of parity.

II. The null models and moving average trading rules.

In light of the previous literature and institutional attributes of stock and currency markets discussed in the previous section, we consider the following model to address both jump risk and time varying risk premia:

In a standard jump diffusion process, let $x_t = \ln(P_t/P_{t-1})$, where P is either the dollar price of the foreign currency or the dollar price of the normalized stock market index. We assume that

$dP_t/P_t = \alpha dt + \sigma dz_t + dq_t$. Then in discrete time, $\ln(P_t/P_{t-1}) = \mu + \sigma z + \sum_{i=1}^{q_t} \ln Y_i$ where $\mu \equiv \alpha - \sigma^2/2$ and ln Y

$\sim N(\theta, \delta^2)$. The Poisson process q has parameter λ, and Y is the jump size.

To combine ARCH(1) into the model, we respecify x_t as

$$x_t \mid t-1 = \mu + \sqrt{h_t} z + \sum_{i=1}^{q_t} \ln Y_i,$$

where $h_t = \alpha_0 + \alpha_1(x_{t-1} - \mu)^2$.

To test the profitability of technical trading rules, we use the commonly used moving average oscillator rule. This technical trading rule is applied to a daily stock index series (the S&P 500). The moving average rule is intended to smooth out the noise in a price series. Since a long-term moving average of past prices responds more slowly than the price series, it is intended to be below the price during the early stage of a bull market and above the price during the early stage of a bear market. To exploit this pattern, signals are generated based on recent price levels and the long-term moving average, computed as $(1/L)\Sigma_{i=0}^{L-1}P_{t-i}$, where P is the asset price at time t. A moving average rule is denoted by (short, long, band). For example, a (2, 150, 1) rule generates a buy (sell) signal when the two most recent daily prices cross the band of the 150 day long term moving average from below (above), with a band of 1% centered around the long term average.

III. Results.

The results are based on the data of S&P 500 daily index series from July 4, 1962, to December 30, 1994. Descriptive statistics in Table 1 show that the daily log price changes exhibit excessive kurtosis and skewness, while the first order serial correlation coefficient is significant and positive. We use a (1, 50, 1) rule in our tests. Thus the length of the moving average is 50 days, the current day price is compared with the mean of the moving average, and the band is 1%.

Table 2 shows the trading profits based on the (1, 50, 1) rule. Over the 32 year period, 3907 buy signals are generated and 2325 sell signals are generated. The "buy-sell" portfolio has a mean value of one day return of 0.047% . The t statistic, which tests the difference of the mean (buy - sell) from the unconditional difference, which is zero, equals 2.07, significant at the 5% level.

The estimation-based bootstrap method (Freedman and Peters 1984) simulates empirical error distributions generated from a null model. To adjust for heteroscedasticity, the resampling algorithm is applied to the standardized residuals. The i.i.d. standardized residuals are sampled with replacement. We estimate the following three null models: AR(1), jump-diffusion, and jump-diffusion with ARCH(1), using maximum likelihood estimation. The parameters are reported in Table 4. The result shows that the jump factor is significant.

To illustrate the simulation process for the jump diffusion model, bootstrap simulations are conducted as follows. With the initial value of a price series, we obtain unobservable errors recursively. We then obtain the simulated return series as follows. First, we generate a random number q from a Poisson distribution with a parameter λ. Second, we obtain q independent normal variates, each with a mean of θ and variance δ^2. Third, the sample for the diffusion process is generated using the normal random number generator with a mean of $(\alpha - \sigma^2/2)$ and variance σ^2. Fourth, the normal random variables from the compound Poisson process and the diffusion processes are added to obtain a single observation. The simulated returns are then exponentiated back to a simulated price series (since the trading rule is based on prices, not returns). We replicate the return series 1000 times. The fraction of the replications which generates a return larger than that from the actual series is considered a simulated p-value.

Table 3 reports the major finding of the paper. For the AR(1) model, the fraction of simulated (buy-sell) returns that are greater than the actual returns is 0.009. This indicates that the actual trading profits, with a simulated p-value of 0.009, is highly significant under the AR(1) model. For the jump-diffusion model, the fraction (simulated p-value) is 0.090, with indicates that under a jump-diffusion model, trading profits are no longer significant at conventional levels. Finally, for the combined jump-diffusion-ARCH(1) model, the fraction becomes 0.425. Thus under this combined model the significance of technical trading profits disappears.

IV. Concluding Remarks.

Both time varying risk premia and jump risk are potential explanations for what looks like excess profits to technical analysis. We utilize technical trading profits and bootstrap simulation to test alternative null models of asset returns. When the technical trading profits become insignificant in a simulated distribution under the null model, then the profits may as well be interpreted as fair compensation for the time varying risk or jump risk incorporated in the null models. This in turn provides a test of the specifications of the null models. Using stock index data, we show that technical trading rules generate significant trading profits, assuming stock returns are a simple diffusion process. However, when we generate a simulated return distribution under a jump-diffusion model and a combined jump-diffusion and ARCH(1) model, we find that the actual trading profits are explained away. This suggests that jump risk, or a combination of jump risk and time varying risk premia, may be appropriate specifications for asset return generating processes for major stock index series.

References

Akgiray, Vedat and G. Geoffrey Booth, 1988, "Mixed Diffusion-Jump Process Modeling of Exchange Rate Movements", *Review of Economics and Statistics*, 70, 631-637.

Ball, Clifford A., and Walter N. Torous, 1985, "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing", *Journal of Finance*, 40, 155-173.

Brock, William, Josef Lakonishok, and Blake LeBaron, 1992, "Simple Technical Trading Rules and the stochastic properties of stock returns", *Journal of Finance*, 47, 1731-1764.

Brorsen, B. and S. Yang, 1994, "Nonlinear dynamics and the distribution of daily stock index returns", *Journal of Financial Research*, Vol 42, No 2, 187-203.

Engle, Robert F., 1982, " Autoregressive Conditional Heteroscedasticity with Estimates for the Variance of United Kingdom Inflation", *Econometrica,* 50, 987-1007.

Effron, B. 1979, "Bootstrap methods: Another look at the jackknife", *The Annals of Statistics*, 7, 1-26.

Jarrow, Robert and Eric R. Rosenfeld, 1984, "Jump Risk and the Intertemporal Capital Asset Pricing Model", *Journal of Business*, 57, 337-351.

Jorion, P., 1988, "On jump processes in the foreign exchange and stock markets", *Review of Financial Studies,* 1, 427-445.

Kho, Bong-Chan, 1996, "Time-varying risk premia, volatility, and technical trading rule profits: evidence from foreign currency futures markets", *Journal of Financial Economics*, 41, 249-290.

Kyle, Albert S., 1985, "Continuous Auctions and Insider Trading", *Econometrica,* 53, 1315-1335.

Levich, Richard, and Lee Thomas, 1993, "The significance of technical trading rule profits in the foreign exchange market: a bootstrap approach", *Journal of International Money and Finance*, 12, 451-474.

Table 1. Descriptive Statistics for the Sample of S&P 500 Return Series. Return is obtained by the log difference of prices. From 620704 to 941230. The total sample size is 8180.

mean	standard deviation	skewness	kurtosis
0.000257	0.008823	-2.17	61.09

Autocorrelations:

lag 1	lag 2	lag 3	lag 4	lag 5
0.109*	-0.02	-0.00	-0.02	0.02

 * denotes significance at 5% level.

Table 2. Profits from moving average (1,50,1) rule applied to daily S&P 500 stock index from 1962 to 1994. The number of buy and sell signals are generated over the entire time period. The T-statistic is calculated in accordance with Brock, Lakonishok and Lebaron (1993).

No. of Buys	No. of Sells	Percent of positive buy trades	Percent of positive sell trades	buy - sell mean returns per day	T-value for buy - sell return
3907	2325	53.1	50.2	0.000476	2.076

Table 3. Simulation tests from null models bootstraps for 1000 replications. The "buy-sell fraction" is the fraction of trading profits based on simulated returns that are greater than the actual profits. It is the simulated p-value for the actual trading profits.

Models	Buy-Sell fraction
AR(1)	0.009
Jump diffusion	0.090
Jump diffusion with ARCH(1)	0.425

Note: the parameter estimates for the models are provided in Table 4.

Table 4. Maximum likelihood estimates of the jump-diffusion model and the jump-diffusion ARCH(1) model on the S&P 500 index returns from 1962 to 1994. We use Bernoulli approximation to estimate the jump-diffusion model. B(q) is the Bernoulli variable that equals 1 with probability q. The BHHH optimization method is used to obtain the ML estimates.

Jump diffusion model:

$$x_t = \mu + \sigma z + B(q) * Y \qquad \text{where } \ln Y \sim N(\theta, \delta^2).$$

$\mu = 0.243 * 10^{-3}$, $(t=2.79)$; $\sigma^2 = 0.043$, $(t=46.22)$; q=0.08968, (t=13.26);
$\delta^2 = 0.379$, $(t=50.12)$; $\theta = 0.00015$, $(t=0.158)$. Log likelihood = 27872.

Jump diffusion - ARCH(1) model:

$$x_t = \mu + \sqrt{h_t} z + B(q) * Y \qquad \text{where } h_t = \alpha_0 + \alpha_1 (x_{t-1} - \mu)^2, \text{ and lnY same as}$$

above.

$\mu = 0.000462$, $(t=5.84)$, $\alpha_0 = 0.02857$, $(t=27.21)$;
$\alpha_1 = 199.12$, $(t=25.59)$; $\theta = 0.00081$, $(t=1.71)$; $\delta^2 = 0.164$, $(t=23.65)$; q=0.171, (t=12.48).
Log likelihood=28048.