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A Public Finance Analysis of Multiple Reserve Requirements

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Abstract: This paper analyzes multiple reserve requirements of the type that have been imposed by a number of developing countries. We show that previous theoretical work on this topic has not succeeded in providing a social welfare rationale for the existence of multiple reserve requirements. We go on to present a model in which it is possible for a multiple reserves regime to improve social welfare relative to simpler regimes involving reserve requirements and/or deposit taxes. We demonstrate the empirical plausibility of our approach by providing a case study of Mexico, a country with extensive historical experience with multiple reserve requirements.

JEL classification: E5, E6, F3

Key words: multiple reserve requirements, public finance

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1 Introduction

1.1 Motivation

A multiple reserve requirements regime is a monetary regime in which the government imposes two types of reserve requirements on the banking system: a currency reserve requirement, which can be satisfied by holdings of government currency, and a bond reserve requirement, which can be satisfied by holdings of government bonds that return below-market rates of interest. Multiple reserves regimes have been adopted by a number of developing countries at various times in recent years; examples include Chile, Korea, Mexico, and Pakistan. In each case, the country had a large public sector deficit and was attempting to finance a substantial portion of it via seigniorage. This observation suggests that the principal motives for the imposition of these regimes were considerations of public finance rather than monetary control or liquidity.[1]

A second observation about multiple reserves regimes is that the real rate of return on reservable government bonds has invariably been higher than the real rate of return on government currency — that is, the bonds have always yielded positive nominal interest. We will

[1] Jimenez (1968), discussing Argentina's regulations allowing banks to satisfy a portion of their reserve requirements by holding low-yield government bonds, comments that "These bonds no longer represent a monetary policy but a fiscal policy by means of which the public sector absorbs part of the banking sector's legal reserve requirements in order to finance its expenditures."

refer to regimes like this as "conventional," and we will describe regimes involving negative-nominal-interest bonds as "unconventional." While the fact that the nominal interest rates on *private* bonds are always positive makes conventionality seem natural, nothing about the structure of multiple reserves regimes appears to require positive nominal rates on reservable government bonds. In a typical regime of this type, the government designates a particular class of bonds as reservable and forces the banks to hold these bonds and no others. Since the government is free, if it wishes, to offer different and higher-yielding bonds to nonbank lenders, it should also be free to impose any reservable bond rate it chooses.[2] The fact that the nominal bond rate is positive implies that bond reserve requirements produce less revenue per dollar of reservable assets than currency reserve requirements — in other words, that the implicit tax rate on reservable bonds is lower than the implicit tax rate on currency (the "inflation tax" rate). There is no obvious reason why this must be the case.

The purpose of this paper is to identify the properties of a plausible formal model that can explain these two properties of multiple-reserves regimes. We begin by assuming that the regimes have indeed been imposed for public finance reasons, and that the governments that

[2] In some multiple reserves regimes the government gives banks the option of holding currency or reservable government bonds to satisfy the second reserve requirement. In this case, it is clear that the banks will not purchase reservable bonds unless their nominal interest rate is non-negative. The question then becomes why the government chooses to offer banks this option.

imposed them chose them over alternative seigniorage-augmentation strategies that would have been simpler to formulate and administer — strategies involving single currency or bond reserve requirements and/or direct taxation of the returns on bank deposits. It follows, in our view, that these governments believed multiple reserves regimes were economically or politically preferable to these alternatives — that is, that they would produce efficiency gains for the economy as a whole and/or social welfare gains for important socioeconomic groups.[3] Thus, our goal is to construct a formal model in which conventional multiple reserves regimes can produce efficiency or social welfare gains over related regimes that are simpler in nature.

1.2 Previous work

In recent years, authors such as Wallace (1984), Romer (1985), Freeman (1987), Brock (1989), Smith (1991), Mourmouras and Russell (1992), Cothren and Waud (1994), Freeman and Haslag (1996) and Bhattacharya and Haslag (1999) have used general equilibrium models to analyze the role of currency reserve requirements in monetary policy and public finance.

[3] For our purposes, a policy is efficient, relative to another policy, if the consumption allocation it supports Pareto dominates the allocation supported by the alternative policy. A policy improves social welfare, relative to another policy, if the consumption allocation it supports has higher social utility, as measured by a social utility function, than the alternative policy. Our use of terms such as "optimal" or "social-welfare-maximizing" should be understood as restricted to the context of a particular class of policies. None of the policies we study in this paper are first-best optimal.

The strand of this literature that is of most interest to us includes Freeman (1987), Brock (1989), Mourmouras and Russell (1992) and Bhattacharya and Haslag (1999). These papers address the question of whether a government that must cover a public sector deficit using revenue from seigniorage might have welfare-oriented reasons for preferring a seigniorage regime featuring a currency reserve requirement to simpler seigniorage regimes.

Espinosa (1995) provides the first theoretical analysis of multiple reserve requirements. He adopts the basic assumptions and methodology of Freeman (1987), but he augments Freeman's model by introducing intragenerational diversity. Espinosa studies the welfare and other effects of imposing a supplementary bond reserve requirement in an economy that already has a currency reserve requirement. His principal result [Proposition 1] is that under certain conditions, the government can increase efficiency (that is, it can produce a Pareto improvement) by requiring banks to replace part of their currency reserves with reserves of government bonds that pay positive nominal interest. He also shows [Corollary 1] that the consumption allocation produced by a conventional multiple reserves regime cannot be duplicated by a single currency reserve regime with a different reserve ratio.

Espinosa's success in identifying the potential welfare benefits of conventional multiple reserve requirements represents an important contribution. However, he does not show that policy regimes with two reserve requirements are the only way to achieve these benefits. His Proposition 1 assumes that the aggregate reserve ratio (initially currency, later currency plus bonds) is exogenously fixed. This assumption leaves open the possibility that the government could achieve similar efficiency gains by changing the reserve ratio but imposing only one reserve requirement (currency or bond).[4] Similarly, Espinosa does not show that the welfare benefits he describes can be achieved only by multiple reserves regimes that are conventional in nature. His Corollary 1 does not rule out the possibility that the relevant consumption allocations could also be supported by regimes with negative nominal interest bonds.

1.3 Our contribution

In this paper, we attempt to provide a more robust theoretical explanation for the existence of conventional multiple reserve requirements. Our principal goal is to explain why a central bank with complete flexibility in setting monetary and reserve policy — complete freedom

[4] Espinosa and Russell (1999) show that the efficiency improvements described in Espinosa's Proposition 1 (1995) are possible only because the single currency reserve allocations that satisfy his hypothesis can be Pareto dominated by allocations supported by alternative single currency reserve policy settings. Thus, the only policy settings from which switching to multiple reserves can increase efficiency are settings that the central bank should never have chosen in the first place. On the other hand, Espinosa's Corollary 1 (1995) establishes that allocations supported by conventional multiple reserve policy settings cannot be supported by single currency reserve regimes. This result demonstrates that multiple reserves regimes can increase social welfare over single currency reserve regimes, even if they cannot increase efficiency. It does not, however, rule out the possibility that these social welfare improvements could also be achieved by a single bond reserve requirement or by an unconventional multiple reserve requirement (see below).

to choose the number of different reserve ratios, the identities of the reservable assets and the return rates on those assets — might choose a multiple reserve requirement instead of an alternative seigniorage-based financing regime that would be less complex and easier to administer. The natural starting point for our investigation is Espinosa's (1995) model. We begin by investigating a basic question about that model: Does it have specifications in which multiple reserve requirements can be used to support consumption allocations that [1] cannot be supported by a single reserve requirements and [2] cannot be Pareto dominated by single-reserve allocations? We know that if multiple reserve requirements can support allocations with these two properties, then they can produce higher social welfare, for some social utility functions, than single reserve requirements. Thus, specifications of this type would provide a relatively robust theoretical explanation for the existence of multiple reserve requirements.

We find, however, that there are no such specifications. In Espinosa's model, any allocation that can be supported by a multiple reserve requirement can also be supported by a single bond reserve requirement. This result, which is our Proposition 1, implies that we will not be able to explain the existence of multiple reserve requirements unless we modify Espinosa's model. We proceed, in the spirit of Occam's Razor, by identifying the feature of the model that make currency reserve requirements superfluous and revising this feature in the simplest possible way. The result is a parsimonious model in which multiple reserve requirements can improve social welfare relative to a single reserve requirement of either type.

Showing that multiple reserve requirements can produce higher social welfare than single reserve requirements is not enough to explain their existence. A genuine multiple reserves regime is more complex than a single reserve regime along two dimensions: the government must impose two different reserve ratios (at least), and it must issue liabilities with two different rates of return. A truly satisfying explanation for multiple reserve requirements must explain why they are superior to alternative seigniorage regimes that might involve less dramatic increases in complexity, relative to single reserve regimes. Consequently, we extend our analysis by studying several alternative regimes of this sort. We show that that there are specifications of our model in which true multiple reserves regimes can produce higher social welfare than any of these alternative regimes. We also show that the only multiple reserves regimes that have this property are those in which reservable bonds yield positive nominal interest. This finding explains why the multiple reserves regimes we actually observe are conventional in nature.

1.4 Organization

In the next section (Section 2) of this paper, we begin our formal analysis by presenting an abbreviated description of Espinosa's (1995) model. We use this model to show that any allocation supportable by a multiple reserves regime can also be supported by a single bond reserve requirement. In Section 3 we construct a modified version of Espinosa's model and use it to compare the properties of genuine multiple reserve requirements regimes to those of simpler regimes involving a smaller number of government liabilities or a smaller number of reserve ratios. In Section 4 we present some evidence from Mexico, a country that has used multiple reserve requirements quite extensively, that supports the empirical plausibility of our basic assumptions. Section 5 offers some concluding remarks. The proofs of the paper's formal results are presented in the appendix, along with a number of illustrative examples.

2 The basic reserve requirements model

2.1 Specification

We analyze a two-period overlapping generations model with limited intragenerational heterogeneity and a number of legal/technological constraints on intertemporal trades. Economic activity occurs at discrete dates $t = 1, 2, \ldots$. At each date t a generation of households is born; these "members of generation t" live during dates t and $t+1$. Each generation of households consists of N_p "poor savers" and N_r "rich savers." Rich savers differ from poor savers in the magnitude of their endowments of the single consumption good and (possibly) in the distribution of these endowments across the two peiods of their lives. The endowment patterns of rich and poor savers are invariant to the dates at which these households are born.

At each date an arbitrary number of competitive private banks are operating in the economy. These banks may hold one or more of the following types of assets:

- private one-period bonds, which are available on the international credit market at an exogenously-determined gross real interest rate $R > 1$.[5]
- government currency, which yields a gross real return rate $R_m(t) \geq 0$ that is determined by the government through its ability to control the growth rate of the stock of currency.
- government one-period bonds, which yield a gross real return rate $R_b(t) \geq 0$ that is specified by the government.[6]

[5] Note that $R > 1$ implies that the net rate of return in the international credit market exceeds the net rate of growth of the economy, which under these assumptions is zero.

[6] Since the government sets the nominal interest rate on bonds and has perfect foresight regarding the currency inflation rate (see below), it effectively sets the real interest rate on bonds.

The liabilities of the banks consist of deposits that are offered to the public at a competitively-determined gross real interest rate $R_d(t)$. The banks are assumed to have zero operating costs and to maximize their date t profits, which must be zero in equilibrium.

The government is assumed to have imposed a legal minimum denomination on the real market value of a bank deposit. The endowments of the poor savers are assumed to be too small to permit them to purchase bank deposits. In addition, it is assumed to be illegal and/or infeasible for them to pool their funds to purchase deposits or to finance deposit purchases with unsecured credit. Thus, the only asset available to poor savers is government currency.

Rich savers' endowments are assumed to be large enough to make the minimum denomination on bank deposits irrelevant. However, private and government bonds are assumed to have larger minimum denominations that make them inaccessible to any households except banks. Thus, the assets available to rich savers are government currency and bank deposits.[7]

The market activities of the rich and poor savers can be completely described by their

[7] As we have indicated, this model is essentially identical to the model constructed by Espinosa (1995). Espinosa augments Freeman's (1987) model by adding intragenerational heterogeneity of the type described by Sargent and Wallace (1982). Another example of the use of minimum denomination restrictions to generate demand for government currency is Bryant and Wallace (1984).

aggregate real saving (first-period asset demand) functions. These functions are denoted $m(R_m(t))$ and $d(R_k(t))$, respectively, where $R_k(t) \equiv \max\{R_m(t), R_d(t)\}$. They are assumed to be continuous and nondecreasing for $R_m(t) \geq \underline{R}_m$ and $R_k(t) \geq \underline{R}_k$, with $0 \leq \underline{R}_m < 1$ and $0 \leq \underline{R}_k < R$.[8] If $\underline{R}_m > 0$ then we assume $m(R_m(t)) = 0$ for $0 \leq R_m(t) \leq \underline{R}_m$, and similarly for $\underline{R}_k > 0$ and $d(\cdot)$.[9]

The government is assumed to finance a fixed real deficit of $G > 0$ per period by issuing bonds and/or currency. The aggregate nominal stock of currency in circulation at date t is denoted $M(t)$. The date t price of a unit of the consumption good in terms of government currency (the date t price level) is denoted $p(t)$. Thus $R_m(t) \equiv p(t)/p(t+1)$. The government is assumed to increase the stock of currency at a constant gross rate $z \geq 1$, so that $M(t) = z\,M(t\text{-}1)$ for all $t \geq 1$.

Government bonds are payable in government currency: a bond is a title to a quantity of currency next period. The aggregate face value of the government bonds issued at date t

[8] Espinosa assumed these functions were strictly increasing in R_m and R_d. This assumption is not essential, however, and it is convenient to be able to use constant saving functions in examples. A common assumption that will produce constant saving functions is that households' preferences are log-linear and that they have no endowments in the second period of their lives.

[9] The purpose of these assumptions is to allow us to use examples involving asset-demand functions that have negative values at low positive gross return rates. We are implicitly assuming that agents may borrow on the international credit market, unintermediated, at gross rate R. For example, if $R_m \leq \underline{R}_m$ then poor savers may choose to consume their endowments or to borrow on the international credit market at R; in either case their real currency balances will be zero. Similarly, if $R_k \leq \underline{R}_k$ then rich savers may choose to consume their endowments or borrow at R; in either case, the assets of domestic intermediaries will be zero and their real reserves of fiat currency will also be zero.

is denoted $B(t)$. The currency price of a bond issued at date t, which is chosen by the government, is denoted $P_b(t)$. The gross nominal interest rate on such bonds is $R_{nom}(t) \equiv 1/P_b(t)$; note that $R_b(t) = R_{nom}(t)R_m(t)$.

Government seigniorage revenue at dates $t \geq 2$ is given by

$$[M(t) - M(t\text{-}1)]/p(t) + [B(t) - R_{nom}(t\text{-}1)B(t\text{-}1)]/p(t)\,.$$

The welfare of the poor and rich members of any generation $t \geq 1$ is assumed to depend uniquely on $R_m(t)$ and $R_k(t)$, respectively, and to be strictly increasing in these variables. It is assumed that at date 1 there are an arbitrary number of "initial old" households (the members of "generation 0") who live for one period and are endowed, in aggregate, with an stock of government currency $M(0) > 0$. We will sometimes refer to the members of generations $t \geq 1$ as the "full lived" households. The welfare of the initial old households is assumed to be strictly increasing in $1/p(1)$, the inverse of the initial price level, or, equivalently, in $m_0 \equiv M(0)/p(1)$, which is the aggregate real value of the initial currency endowment.

The government is assumed to impose bond and/or currency reserve requirements on the banks. The fractions of a banks' assets that it is required to hold in the form of currency and government bonds are denoted θ_m and θ_b, respectively. We assume $\theta_m, \theta_b \in [0,1]$ and $\theta \equiv \theta_m + \theta_b \in [0,1]$. Each reserve ratio is the minimum ratio of the market value of a bank's holdings of one of the reservable liabilities (currency or bonds) to the market value of its entire portfolio of liabilities.

2.2 Multiple reserves equilibria

We confine ourselves to the study of *binding stationary multiple reserves equilibria* (or simply *multiple reserves equilibria*), which are competitive equilibria in which [1] the rate of return on private bonds exceeds the rates of return on government currency or bonds, so that banks will hold government liabilities only to meet the reserve requirements, [2] the rate of return on bank deposits exceeds the rate of return on government currency, so that rich savers will hold only bank deposits, and [3] the values of all real variables and all nominal return rates are constant, while the values of all other nominal variables grow at the same fixed rate.[10] Given R, G, and $M(0)$, a binding stationary multiple reserves equilibrium can be characterized as values of the central bank policy variables z, P_b, θ_m, and θ_b that satisfy $z \geq 1$, $P_b > 0$, $\theta_m, \theta_b \in [0,1]$ and $\theta \equiv \theta_m + \theta_b \in (0,1]$, plus values of the endogenous variables R_m, R_b, R_d, and $p(1)$ that satisfy $p(1) > 0$,

$$R_m = \frac{1}{z}, \tag{1}$$

[10]The model can be generalized easily to cover situations in which the values of real variables grow at fixed, exogenously-determined rates.

$$R_b = \frac{R_m}{P_b}\,, \tag{2}$$

$$R_b < R\,, \tag{3}$$

$$R_d = (1 - \theta_m - \theta_b)\,R + \theta_b R_b + \theta_m R_m\,, \tag{4}$$

$$R_m < R_d < R\,, \tag{5}$$

$$G = (1 - R_m)\,[m(R_m) + \theta_m\,d(R_d)] + (1 - R_b)\,\theta_b\,d(R_d)\,, \tag{6}$$

and

$$m_0 = m(R_m) + (\theta_m + \theta_b)\,d(R_d) - G\,, \tag{7}$$

where $m_0 \equiv M(0)/p(1)$. The first equation follows from the fact that in a steady state equilibrium, aggregate real balances $M(t)/p(t)$ must be constant. The second equation rules out profitable arbitrage in the government currency and bond markets. Inequalities (3) and (5) guarantee that banks hold government liabilities only as legal reserves and that rich savers hold only bank deposits. The fourth equation expresses the relationship between the interest rate on bank deposits, the two reserve ratios, and the rates of return on the three nonbank assets that is implied by the assumption that banks earn zero profits. The sixth and seventh equations ensure that the central bank meets its budget constraint at dates $t \geq 2$ and $t = 1$, respectively.

Note that the value of R_m depends entirely on the value of z, which has no independent influence the value of any other variable. Similarly, the value of R_b depends on the values of R_m and P_b, and the value of P_b has no independent influence the value of any other variable. Thus, we are free to think of the central bank as choosing R_m and R_b directly, subject to the requirements $R_m \geq 0$ and $R_b \geq 0$.[11]

In what follows, it is useful to define

$$A \equiv m(R_m) + \theta\, d(R_d)\,, \tag{8}$$

which represents aggregate real balances of government liabilities, and to note that equations (4), (6) and (7) imply

$$m_0 = A - G = R_m[m(R_m) + \theta_m\, d(R_d)] + R_b\theta_b\, d(R_d)\,. \tag{9}$$

In a binding stationary multiple reserves equilibrium we have $p(t)/p(t{+}1) = R_m = 1/z$, $R_{nom}(t) = R_{nom} \equiv R_b/R_m$, $M(t)/p(t) = m(R_m) + \theta_m d(R_d)$ and $B(t)/p(t) = \theta_m d(R_d)$ for all $t{\geq}1$. These equations imply $M(t{+}1)/M(t) = B(t{+}1)/B(t) = 1/R_m = z$ for all $t{\geq}2$. Note that under our assumptions the central bank can use its control over z and P_b to set R_m and R_b at any values between zero and R. We will define a "multiple reserve requirements policy

[11]Note that we will allow the limiting cases $R_m = 0$ and/or $R_b = 0$, which correspond to arbitrarily large values of z and/or P_b.

setting" as a vector $(R_m, R_b, \theta_m, \theta_b)$, and we will define the associated "public and private allocation" (or sometimes "multiple reserves allocation") as the values (G, m_0, R_m, R_d) that this vector of policy settings supports as a binding stationary multiple reserves equilibrium.

Note that a stationary equilibrium without reserve requirements would consist of values of R_m and $p(1)$ such that $G = (1-R_m)\, m(R_m)$ and $M(0)/p(1) = m(R_m) - G$. For simplicity, we assume that the value of G is large enough to rule out equilibria of this type. A multiple reserves equilibrium is a *single currency reserve equilibrium* if $\theta_b = 0$ and/or $P_b = 1$, or, equivalently, if $P_b = 1$ so that $R_b = R_m$. A multiple reserves equilibrium is a *single bond reserve equilibrium* if $\theta_m = 0$. A single currency reserve policy setting can be defined as a vector (R_m, θ_m); a single bond reserve policy setting is a vector (R_m, R_b, θ_b). We define a "single reserve allocation" (currency or bond) as a public and private allocation supportable as a single reserve equilibrium. Finally, we refer to multiple reserves equilibria in which $\theta_m > 0$ and $\theta_b > 0$ with $\theta_m \neq \theta_b$, and $R_m > 0$ and $R_b > 0$ with $R_b \neq R_m$, as *genuine multiple reserves equilibria* (formalizing a definition from Section 1.3).

2.3 Redundancy of currency reserve requirements

We begin our analysis by establishing that currency reserve requirements are superfluous in this model. More specifically, we show that any public and private allocation — including allocations supported by genuine multiple reserves regimes, and including genuine multiple reserves regimes with positive nominal interest rates on reservable bonds — can be supported by a regime with a single *bond* reserve requirement.

Proposition 1 *Any public and private allocation that can be supported as a binding stationary multiple reserve requirement equilibrium with $\theta_m > 0$ can be supported as an equilibrium of the same type with $\theta_m = 0$ — that is, by a single bond reserve requirement.*

In this model, the only difference between currency and bonds that is relevant to prospective bond holders (that is, banks) is that the two assets may yield different rates of return. Thus, if $R_b > R_m$ then a single bond reserve requirement regime amounts to a single currency reserve regime in which the government pays interest on reserves at a below-market rate.[12] Espinosa and Russell (1999) show that if $R_b < R_m$ then any multiple-reserves equilibrium allocation in this model can be supported by a combination of a single currency reserve requirement and a proportional tax on deposits. It follows that a government that had access to both of these policy devices — interest on currency reserves and direct taxa-

[12]Smith (1991) and Freeman and Haslag (1996) study models in which the government may pay below-market interest on reserves.

tion of deposits — would not need to impose either a bond reserve requirement or multiple reserve requirements. (We will return to the question of deposit taxation at the end of the next section of this paper.)

Proposition 1 makes it clear that this model does not provide a robust explanation for the existence of multiple reserve requirements. Finding such an explanation is the principal goal of the remainder of this paper. Since we have adopted this model as our starting point, the question that now confronts us is: which of its feature(s) make currency reserve requirements superfluous when bond reserve requirements are available?

The offending feature turns out to be the assumption that the poor savers cannot hold any assets other than currency. This assumption implies that although the model includes two distinct types of full-lived households (poor and rich savers), binding reserve requirements of either sort can be imposed on only one type of household (rich savers). It turns out, moreover, that from policymakers' point of view, these two types of reserve requirements are virtually perfect substitutes.

In models of this type, the government uses reserve requirements to augment savers' demand for its currency and/or bond liabilities. Increases in the demand for these liabilities accomplish two purposes: they increase the volume of seigniorage revenue, and they increase the real value of the nominal asset endowments of the initial old households. Since both currency and bond reserve requirements are binding on the same households, they are equally useful for these purposes. *Ceteris paribus*, an aggregate reserve ratio of θ will produce the same reserve demand and the same initial old real balances, regardless of how it is divided across θ_m and θ_b. Of course, in a multiple reserves regime the amount of seigniorage revenue produced by a currency or bond reserve requirement will depend, in part, on the values of R_m and R_b respectively. However, if the government wishes to switch to a single bond reserve requirement without gaining or losing revenue, it can set the new bond reserve ratio at the same level as the old aggregate reserve ratio while setting the new real bond rate at the appropriate weighted average of the old real bond rate and the old real currency reserve rate. If it does this then there will be no change in either the deposit rate facing rich savers or the amount of seigniorage earned from them. Thus, any allocation supportable by a multiple reserves regime can be supported by a single bond reserve regime.

The converse, however, is not true: there may be allocations that can be supported by single bond reserve regimes but not by single currency reserve regimes. The reason for that in a single currency reserve regime, the central bank cannot set the rate of return on deposits, which controls the welfare of the rich savers, independently from the rate of return on currency, which controls the welfare of poor savers. Suppose, for example, that the central bank is happy with the current level of the aggregate reserve ratio, and thus the current welfare of the initial old households, but wishes to increase the welfare of the rich savers at the expense of that of the poor savers. Under a single reserve regime this is not possible. Given that the central bank does not wish to change the currency reserve ratio, its only remaining policy tool is the real currency return rate R_m. If the bank wants to increase the welfare of rich savers then it must increase R_m in order to increase R_d, which controls their welfare. However, an increase in R_m will also benefit the poor savers, and it will result in a loss of seigniorage revenue that will force an increase in the reserve ratio. The adverse effect of this reserve ratio increase on rich savers will more than offset the beneficial effect of the increase in R_m, leaving the rich savers worse off rather than better. Conversely, the central bank can help the rich savers by reducing the aggregate reserve ratio, but this move will hurt the initial old, not the poor savers. (See Example 1.)

Under a single bond reserve regime, in contrast, if the government wishes to benefit the rich savers at the expense of poor savers then it can simply increase the real bond return rate R_b, which will cause R_d to rise, and offset the revenue loss by reducing R_m. There is no net change in revenue, which means there is no need to change the reserve ratio and with it the welfare of the initial old. Thus, replacing a currency reserve requirement with a bond reserve requirement gives the central bank the ability to price discriminate between the poor savers and the rich savers. As we have seen, however, adding a currency reserve requirement to a bond reserve requirement does not materially increase the central bank's flexibility in this or any other regard.[13]

3 An alternative model

3.1 Overview

In this section, we explore a plausible alternative version of the basic reserve requirements model. This new version of the model allows us to explain why some governments have chosen to impose genuine multiple reserve requirements instead of opting for simpler reserve requirements regimes.

Before we begin our analysis, it may be helpful to clarify certain aspects of our analytical approach. Our explanation for the existence of multiple reserve requirements is based on the assumption that the central bank chooses policies that maximize social welfare, given

[13]García de Paso (1997) demonstrates that Romer's (1985) model is still more limited: it does not provide a role for a second reserve requirement, or even a second government liability. In Romer's model, any allocation supported by a multiple reserve requirement can be supported by a single currency reserve requirement. This result stems from the fact that Romer introduces a form of intragenerational diversity that is much more limited than the forms introduced by Espinosa (1995) or in our alternative model.

the policy tools it has available. Since our analysis is designed to identify the advantages of genuine multiple reserve requirements relative to simpler seigniorage regimes, we assume that the central bank has access to exactly the tools it needs to impose a multiple reserves regime with any policy settings (reserve ratios and return rates) it desires, subject to some constraints imposed by the underlying economic environment. The alternative seigniorage regimes we consider are special cases of multiple reserves regimes, so this assumption also gives the central bank the tools it needs to impose any of these regimes.

As different central banks may have different social welfare functions, it would not make sense to base our analysis on a single function or a single class of functions. Instead, we adopt an agnostic approach by deriving results that identify the distinctive characteristics of "unsupportable" allocations — allocations that can be supported by multiple reserve requirements, but not by simpler seigniorage regimes. We know these allocations will maximize social welfare for some social welfare functions.[14] ,[15] We provide examples to demonstrate

[14]An exception would be cases in which these allocations were Pareto dominated by other allocations supportable by multiple reserves regimes. In models of this general type, reserve requirements regimes will support allocations that cannot be Pareto dominated by other such regimes as long as they avoid overtaxation — that is, as long as they avoid policy settings in which increasing the implicit seigniorage tax rate, by reducing return rates on reservable assets and/or increasing reserve ratios, would reduce the revenue from seigniorage. This point is made by Romer (1985), and it is discussed at length by Espinosa and Russsell (1999). Our analysis implicity assumes that overtaxation is not occurring, and we take care to avoid it in our examples.

[15]Since there may be many specifications that give rise to allocations with the indicated characteristics, it would be more precise to say that for each specification that supports an allocation with the indicated characteristics, there is some social utility function under which that allocation (and the policy setting that supports it) maximizes social welfare.

that our results are nonvacuous. These examples consist of particular model specifications that have multiple reserves equilibria that support allocations with the indicated characteristics. In each case, we provide a particular specification of an indirect social utility function of a simple, standard type under which the equilibrium allocation maximizes social welfare. This function is described at the beginning of the appendix.

After we state our major results, we provide discussions in which we link the allocation characteristics we identify to [1] the features of the specifications and the policy settings that might generate allocations with these characteristics and [2] the nature of the social welfare functions under which allocations with these characteristics might be optimal. These discussions are intended to provide insights into a number of important questions: what kinds of economies might be expected to have multiple reserves regimes, what kinds of multiple reserve regimes these economies might be expected to have, and what kinds of motives might lead central banks to impose multiple reserves regimes in these economies.

3.2 Specification

Our alternative model eliminates the feature of the original model that accounts for its inability to provide a useful role for currency reserve requirements when bond reserve requirements are available: the assumption that fiat currency is the only asset that can be held by poor savers. We will refer to specifications of our alternative model as "Class B economies," while specifications of the original model will sometimes be called "Class A economies." In Class B economies, poor savers retain the option to hold currency, but they can also deposit funds in banks that invest in a domestic storage technology with gross rate of return X. For purposes of simplicity, we assume $X = R$.[16] We think of these economies as versions of the original economies in which the government has authorized the establishment of "national banks." These banks issue liabilities in denominations small enough to be accessible to poor savers, but they are permitted to acquire only domestic assets. We will refer to the banks that may invest in foreign securities as "international banks." The liabilities issued by these banks are accessible to rich savers but not poor savers, as in the original model. Rich savers also have the option of holding national bank deposits and/or currency.

We think our Class B economies capture an important aspect of the financial system in much of the developing world. Although the common people have some access to the banking system, the banking institutions that serve them are often quite different from the institutions that serve wealthier segments of the population, and the range of bank liabilities

[16] A note in the concluding section discusses some of the implications of permitting $X < R$, which may seem like a more reasonable assumption.

available to them is typically narrower and less attractive than the range of liabilities available to wealthier groups. In addition, the assumption that both types of savers have access to currency, which was also made by Espinosa (1995), reflects the fact that currency continues to play an indispensable role as an intratemporal medium of exchange.

In the most general version of our alternative model, the central bank would have the power to impose different currency and bond reserve requirements on both types of banks. Thus, it would wield six policy instruments: two return rates (on currency and bonds) and four reserve ratios. This version of the model is laid out in the appendix. It turns out, however, that any allocation supportable by a seigniorage regime of this type can also be supported by a regime in which the government imposes only two reserve requirements: a single currency reserve requirement on the national banks and a single bond reserve requirement on the international banks. This result, which we state as Lemma 1, follows rather trivially from Proposition 1 for Class A economies.[17]

In view of Lemma 1, we confine ourselves to studying multiple reserves regimes of the two-reserve-ratio type. We will continue to let R_m represent the rate of return on currency and to let R_b represent the rate of return on government bonds. We will let θ_m represent

[17] In Class B economies there is no non-reserve demand for currency, so for purposes of formal analysis we are free to think of currency as a second type of government bond that is available in relatively small denominations.

the currency reserve ratio imposed on national banks and θ_b the bond reserve ratio imposed on international banks. We will let $R_d^r(t)$ denote the gross rate of return on deposits in the international banks at date t and $R_d^p(t)$ the gross rate of return on deposits in the national banks at that date. The savings functions of the rich and poor savers will be denoted $d^r(R_d^r(t))$ and $d^p(R_d^p(t))$. Our assumptions about the properties of these functions will be identical to those we made about $d(R)$ in the previous section. We will occasionally use the abbreviations $d^p = d^p(R_d^p)$, $d^r = d^r(R_d^r)$, and $d = d^p(R_d^p) + d^r(R_d^r)$.

3.3 Multiple reserves equilibria

Given $R > 1$, $G > 0$, and $M(0) > 0$, a binding stationary multiple reserves equilibrium in this model can be characterized as nonnegative values θ_m, θ_b, R_m, R_b, R_d^p, R_d^r and a positive value $p(1)$ that satisfy $\theta_m \in (0, 1]$, $\theta_b \in (0, 1]$,

$$0 < R_b < R \tag{10}$$

$$0 < R_m < R_d^p < R_d^r < R \tag{11}$$

$$R_d^p = (1 - \theta_m)R + \theta_m R_m \tag{12}$$

$$R_d^r = (1 - \theta_b)R + \theta_b R_b \tag{13}$$

$$G = (1 - R_m)\theta_m d^p(R_d^p) + (1 - R_b)\theta_b^r d^r(R_d^r) \tag{14}$$

and

$$G = \theta_m\, d^p(R_d^p) + \theta_b\, d^r(R_d^r) - m_0. \tag{15}$$

where $m_0 \equiv M(0)/p(1)$.

Note that in (11) the inequality $R_m < R_d^p$ is a consequence of our assumption that poor savers have access to currency, while $R_d^p < R_d^r$ is a consequence of our assumption that rich savers have access to national bank deposits. These inequalities impose constraints on central bank policy that will play an important role in the analysis presented below.

We can now define a reserve requirements policy setting $(\theta_m, \theta_b, R_m, R_b)$ and the associated "public and private allocation" (G, m_0, R_d^p, R_d^r) in essentially the same manner as in the previous section. It should be noted, however, that in Class B economies the currency return rate R_m is an element of the policy setting, but it is no longer an element of the associated allocation: the welfare of the poor savers now depends uniquely on R_d^p, the rate of return on national bank deposits. We will continue to use the term "genuine multiple reserves regimes" to describe multiple reserves regimes in which both the currency and bond reserve ratios and return rates are positive and different from each other.

In Class B economies we can redefine

$$A \equiv \theta_m\, d^p(R_d^p) + \theta_b\, d^r(R_d^r)\,, \tag{16}$$

and we have

$$m_0 = A - G = R_m\theta_m\, d^p(R_d^p) + R_b\theta_b\, d^r(R_d^r)\,. \tag{17}$$

3.4 Prospects for simpler regimes

The Class B economies provide a plausible formal environment in which the simplest type of reserve requirements regime — a single reserve requirement (currency or bond) imposed on all banks at the same ratio — will not maximize social welfare, except in special cases. The central bank can no longer earn seigniorage from poor savers without forcing them to hold currency using a reserve requirement, and a simple currency reserve requirement would leave both types of savers facing the same rate of return.[18] The central bank consequently would lose the ability to use differential return rates to conduct social welfare-improving price discrimination.

[18]This statement would also be true of a single bond reserve requirement imposed on all banks at the same reserve ratio. For reasons described below, however, we will not consider regimes that do not feature a currency reserve requirement imposed on at least one type of bank. Since a simple single currency reserve requirement (imposed on all banks at the same ratio) is a special case of two of the alternative regimes we study below, the result that multiple reserves regimes can improve social welfare relative to these alternative regimes implies that it can can improve social welfare relative to a simple single reserve regime. Consequently, we do not prove the latter result separately.

3.4.1 Types of regimes

On the other hand, Class B economies support strategies for producing differential return rates that are more complex than a simple currency reserve requirement, but less complex than a genuine multiple reserve requirement. For example, it is still possible for a regime with a single reservable liability — presumably, currency — to confront different types of savers with different deposit return rates. The central bank can accomplish this by imposing a different currency reserve ratio on each type of bank. Regimes of this type are common in both developing and developed countries, including the United States, and it would not seem reasonable to describe them as featuring "multiple reserves." Alternatively, the government could impose a common reserve ratio on both types of banks while requiring each type of bank to hold a reservable liability with a different return rate. Although some might argue that a regime with two different reservable liabilities qualifies as a multiple reserves regime, the common reserve ratio makes regimes of this type materially simpler than the multiple reserves regimes we actually observe (see Section 4 below).

3.4.2 Preliminary analysis

The next section of this paper tests the ability of Class B economies to explain the existence of genuine multiple reserves regimes. It accomplishes this task by determining whether, and under what conditions, simpler regimes of the types just described can support the same public and private allocations as genuine multiple reserves regimes. Before we begin this process, we present a preliminary analysis which confirms that this is a question of sufficient depth to merit further investigation.

One feature of Class B economies which suggests that genuine multiple reserve requirements may continue to be superfluous is that the number of welfare targets facing the central bank has not increased. In the Class A economies, there are three welfare targets: the return rates facing the poor and rich savers and the real balances of the initial old. We showed in Proposition 1 that the central bank needs three policy instruments in order to hit these targets: two government liability return rates and one (bond) reserve ratio. There is no need for a fourth policy instrument in the form of a second (currency) reserve ratio, so there is no need for multiple reserves regimes. Class B economies differ from Class A economies in that reserve requirements are binding on both types of full lived household, so that a simple single reserve requirement no longer provides the central bank with three distinct policy instruments. But there are still only three welfare targets, so it seems reasonable to suppose that there may still be no need for more than three policy instruments.

There is, however, a more subtle analogy between the Class A and Class B economies that raises doubt that three policy instruments will always be sufficient. In order to develop this analogy, we need to say more about the nature of the policy-setting indeterminacy that characterizes multiple reserves regimes in Class A economies. Equations (4), (6) and (7) above can be shown to have the following implication: in a Class A economy, the public and private allocation $(G, \overline{m}_0, \overline{R}_m, \overline{R}_d)$ can be supported as a multiple reserves equilibrium by any values $\theta_b \in [0, \overline{\theta}]$, $\theta_m = \overline{\theta} - \theta_b$, and $R_b \in [0, R)$ that satisfy the single equation

$$\overline{R}_m \left(\overline{\theta} - \theta_b \right) + R_b \theta_b = \overline{R}_d - (1 - \overline{\theta}) R \,, \tag{18}$$

where $\overline{\theta}$ is the unique value of $\theta \equiv \theta_m + \theta_b$ that satisfies

$$\overline{m}_0 = \overline{R}_m \, m(\overline{R}_m) + \left[\overline{R}_d - (1 - \theta) R \right] \, d(\overline{R}_d) \,. \tag{19}$$

Thus, even after the central bank has selected a particular allocation — which, in Class A economies, fixes the value of R_m — it retains a good deal of flexibility in choosing the three remaining policy variables. It happens, moreover, that there is one simple strategy for choosing values of these variables that will always always support the desired allocation. This strategy, which involves setting $\theta_b = \overline{\theta}$ and thus $\theta_m = 0$, is a single bond reserve requirement.

On the other hand, many alternative strategies for reducing the dimensionality of the policy vector — that is, for constructing simpler reserve requirements regimes — that will *not* always succeed in supporting desired multiple reserves allocations. As we have seen, for example, the central bank usually cannot choose $\theta_b = 0$ — a single currency reserve requirement — because if it does then no value of R_b will satisfy equation (18). In fact, many values of θ_b that exceed zero may be ruled out by the fact that the required value of R_b will exceed R.[19] The fact that the central bank cannot make policy choices that cause R_m to exceed R_d — otherwise, the rich savers will abandon bank deposits in favor of currency — makes situations of this type more likely. Another simple policy choice the central bank cannot always make is $R_b = 0$, which amounts to replacing the bond reserve requirement with a tax on deposits. Under a regime of this sort, the required value of θ_b may exceed $\overline{\theta}$, which means θ_m is negative.

Thus, many strategies for supporting multiple reserves allocations by setting particular policy variables at values that produce simpler seigniorage regimes are not always feasible in Class A economies. These strategies are frustrated by the fact that they may require the central bank to set other policy variables at values that are inconsistent with constraints

[19]The government can issue reservable bonds with gross real return rates in excess of unity by covering the resulting interest losses with revenue from currency seigniorage. However, the government cannot issue bonds with gross real rates higher than R because the banks would prefer them to privately issued bonds.

imposed by the economic environment. It happens, as we have indicated, that there is a one relatively simple supporting regime that [1] never requires violating these constraints, [2] is clearly simpler than a genuine multiple reserves regime and [3] has a natural economic interpretation. This is the single bond reserve requirement regime. But is the existence of a relatively simple regime with properties [1]-[3] a characteristic feature of economies of this general type, or is it an artifact of the special features of Class A economies? The answer does not seem obvious.

Turning to Class B economies, suppose we have a public and private allocation $(\overline{G}, \overline{m}_0, \overline{R}^p_d, \overline{R}^r_d)$. Equilibrium conditions (12)-(15) yield

$$\overline{m}_0 = R_m \left[\frac{R - \overline{R}^p_d}{R - R_m}\right] d^p(\overline{R}^p_d) + R_b \left[\frac{R - \overline{R}^r_d}{R - R_b}\right] d^r(\overline{R}^r_d)\,. \tag{20}$$

Conditions (12) and (13) imply that for any vector $(R_m, R_b) \in [0, \overline{R}^p_d) \times [0, \overline{R}^r_d)$ that satisfies this single equation, there will be a legitimate vector (θ_m, θ_b) — that is, a vector inside $(0, 1] \times (0, 1]$ — that supports $(\overline{R}^p_d, \overline{R}^r_d)$. It follows that any vectors (R_m, R_b) that satisfy equation (20) can be part of multiple reserves policy settings that support our allocation.[20]

However, most of these vectors will produce different (though equivalent) specifications of genuine multiple reserves regimes, rather than regimes of any simpler type. Simpler reserve

[20]A supporting result that is useful here and elsewhere, and which is stated and proved in the appendix as Lemma 2, is that if $(\overline{G}, \overline{m}_0, \overline{R}^p_d, \overline{R}^r_d,)$ is a public and private allocation, then any multiple reserves policy setting that supports its "private component" $(\overline{m}_0, \overline{R}^p_d, \overline{R}^r_d)$ also supports $\overline{G}$.

requirements regimes can be characterized in terms of additional restrictions on R_m and R_b. For example, a regime in which currency is the only government liability, but there are two reserve ratios, is the special case of a multiple reserves regime in which $R_b = R_m$. It does not seem immediately obvious that equation (20) will always have a solution $\widehat{R}_m$ satisfying $R_m = R_b = \widehat{R}_m$, and it is even less obvious that such a solution will necessarily satisfy $\widehat{R}_m \in [0, \overline{R}_d^p)$. If not, then our allocation cannot be supported by a simple policy of this type.[21]

Another relatively simple seigniorage regime would feature a common reserve ratio for currency and bonds held by national and international banks, respectively, so that there would be two reservable liabilities with different return rates but only one reserve ratio. A regime of this sort can be characterized by a vector $(\widehat{R}_m, \widehat{R}_b)$ that satisfies equation (20) plus

$$\frac{R - \overline{R}_d^p}{R - R_m} = \frac{R - \overline{R}_d^r}{R - R_b}\,, \tag{21}$$

since equations (12) and (13) imply $\theta_m = (R - R_d^p) \,/\, (R - R_m)$ and $\theta_b = (R - R_d^r) \,/\, (R - R_b)$, respectively. Again it does not seem clear that equation (20) will always [or ever] have a solution that satisfies equation (21), and it seems even less clear that such a solution will necessarily satisfy both $\widehat{R}_m \in [0, \overline{R}_d^p)$ and $\widehat{R}_b \in [0, \overline{R}_d^p)$.

[21]Espinosa and Russell (1999) show that in Class A economies, an allocation supportable by a multiple reserves regime with $R_b > R_m$ can *never* be supported by a regime with $R_b = R_m$.

3.5 Properties of simpler regimes

As we have indicated, one implicit assumption that underlies our analysis of Class B economies is that government currency plays an indispensable role as an intratemporal medium of exchange. We draw some additional implications from this assumption and use them to narrow the range of alternative seigniorage regimes we study. In particular, we assume that the central bank is constrained to issue at least one liability — currency — that is accessible to all types of households, and that it is also contrained to take steps to ensure that there is a stock demand for this liability. In Class A economies, the existence of a stock demand for currency is ensured by the relatively extreme assumption that currency is the only asset poor savers can hold. In Class B economies, where this is no longer the case, demand for government liabilities can be ensured only through reserve requirements. Thus, we assume that the government must impose a currency reserve requirement on at least one of the two types of bank. As a result, when we study relatively simple seigniorage regimes with a single reservable liability we assume that the single liability must be currency, not bonds. Thus, we study regimes in which each type of bank must hold currency reserves at a different ratio, but we do not study regimes in which each bank must hold bond reserves at a different ratio.

Later, when we extend our analysis to include deposit taxes, we study regimes in which the central bank imposes a deposit tax on one type of bank plus a currency reserve requirement on one or both types of banks, but we do not study regimes in which it imposes a deposit tax on one bank plus a bond reserve requirement, or a deposit tax on both banks but no reserve requirement.

3.5.1 Regimes with a single reservable liability

We begin by investigating the extent to which it is possible for the central bank to support public and private allocations with seigniorage regimes involving a single currency reserve requirement with two different reserve ratios — one for national banks and the other for international banks. We will refer to regimes of this type as "dual currency reserves" (DCR) regimes.

A dual currency reserves equilibrium can be defined as a multiple reserves equilibrium in which $R_b = R_m$. The conditions for a dual currency reserves equilibrium are thus identical to those for a multiple reserves equilibrium except that condition (10) can be omitted, conditions (12) and (13) can be replaced by

$$R_d^p = (1 - \theta_m)R + \theta_m R_m \tag{22}$$

$$R_d^r = (1-\theta_b)R + \theta_b R_m \,, \tag{23}$$

respectively, and condition (14) can be replaced by

$$G = (1-R_m)\left[\theta_m\, d^p(R_d^p) + \theta_b\, d^r(R_d^r)\right]. \tag{24}$$

Since a dual currency reserves regime can be defined as a special case of a multiple reserves regime, any DCR equilibrium allocation is a public and private (that is, a multiple reserves) allocation. However, the converse is not always true. Proposition 2 identifies necessary and sufficient conditions under which allocations supportable by genuine multiple reserve requirements cannot be supported by dual currency reserve requirements. A distinctive feature of these allocations is that the only multiple reserves regimes that can support them are conventional in nature. That is, these allocations can be supported only by multiple reserves regimes in which reservable government bonds yield positive nominal interest.

Proposition 2 *In Class B economies, a public and private allocation is not supportable by a dual currency reserves regime if and only if*

$$\frac{\overline{m}_0}{\overline{A}} \geq \overline{R}_d^p \,. \tag{25}$$

If an allocation satisfies this condition, then any multiple reserves regime that supports it must involve $\overline{R}_b > \overline{R}_m$.

To obtain some intuition about Proposition 2, imagine a central bank trying to support a particular multiple reserves allocation with a dual currency reserves regime. The bank must issue a single reservable liability: currency. It must select the currency return rate, plus two reserve ratios, in order to raise the required seigniorage revenue $\overline{G}$, keep the real balances of the initial old households at $\overline{m}_0$, and keep the deposit rates facing the poor and rich savers at $\overline{R}_d^p$ and $\overline{R}_d^r$, respectively.

Keeping the real balances of the initial old at $\overline{m}_0$ requires fixing the aggregate demand for currency reserves at a level equal to the aggregate demand for total reserves in the original multiple reserves equilibrium. Given that total currency demand is fixed, there is only one currency return rate that will produce the required level of seigniorage revenue. This unique currency return rate $\widehat{R}_m$ is a weighted average of the currency and bond return rates in the original multiple reserves equilibrium. More specifically,

$$\widehat{R}^m = \frac{\overline{m}_0}{\overline{A}} = \overline{\alpha}_p \overline{R}_m + \overline{\alpha}_r \overline{R}_b, \tag{26}$$

where

$$\overline{\alpha}_p = \frac{\overline{\theta}_m \, \overline{d}^p}{\overline{\theta}_m \, \overline{d}^p + \overline{\theta}_b \, \overline{d}^r} \quad \text{and} \quad \overline{\alpha}_r = 1 - \overline{\alpha}_p = \frac{\overline{\theta}_b \, \overline{d}^r}{\overline{\theta}_m \, \overline{d}^p + \overline{\theta}_b \, \overline{d}^r},$$

so that the weights depend on the relative importance of the reserves held by poor vs. rich savers. Every multiple reserves policy setting that supports the allocation $(\overline{G}, \overline{m}_0, \overline{R}_d^p, \overline{R}_d^r)$ produces the same weighted average.

The central bank must now try to adjust the two reserve ratios θ_m and θ_b in order reconcile the fixed currency return rate $\widehat{R}_m$ with the target deposit rates $\overline{R}_d^p$ and $\overline{R}_d^r$. This will be possible as long as $\widehat{R}_m$ does not exceed $\overline{R}_d^p$, the rate of return on the deposits of the poor savers. If it does, then there will be no national bank reserve ratio large enough to produce the target deposit rate: even if the central bank imposes 100 percent reserves on national bank deposits ($\theta_m = 1$) it cannot drive the national bank deposit rate below $\widehat{R}_m$. Thus, the dual currency reserve regime does not always provide the central bank with the policy flexibility it needs to price discriminate against poor savers.

Equation (26) explains why $\overline{R}_b > \overline{R}_m$ — a positive nominal bond rate in any supporting multiple reserves regime — is a necessary condition for an allocation to fail to be supportable by a dual currency reserves regime. Too see why, suppose there is a supporting multiple reserves regime in which $\overline{R}_b < \overline{R}_m$. We have seen that $\widehat{R}_m$, the unique currency return rate in a supporting DCR regime, is a weighted average of these two return rates. Thus, $\widehat{R}_m$ must be below $\overline{R}_m$. And since the national bank deposit rate $\overline{R}_d^p$ must equal or exceed $\overline{R}_m$, we know $\widehat{R}_m$ must be lower than $\overline{R}_d^p$, which means the supportability problem described above will not arise. On the other hand, if $\overline{R}_b > \overline{R}_m$, then $\widehat{R}_m$ must exceed $\overline{R}_m$, making it possible that it will also exceed $\overline{R}_d^p$.

What kinds of economic structures and central bank preferences will produce public and private allocations that cannot be supported by dual currency reserve regimes? Unsupportability is most likely when $\overline{R}_d^p$ is close to $\overline{R}_m$ and when $\overline{R}_b$ is high relative to both. Since $\overline{R}_d^r$ must exceed $\overline{R}_b$, it is clear that unsupportability is most likely when the two deposit rates are far apart. This result is not very surprising. Multiple reserve requirements provide central banks with increased flexibility to price discriminate against poor savers. They should be most valuable to central banks who wish to engage in policies of this type — presumably, central banks that are quite concerned about the welfare of rich savers but relatively unconcerned about the welfare of poor ones.

Given that the central bank desires to price discrminate, unsupportability is more likely when $\overline{d}^r$ is large relative to $\overline{d}^p$: when this is the case, the weighted average $\widehat{R}_m$ is closer to $\overline{R}_b$ than to $\overline{R}_m$. Thus, the potential usefulness of multiple reserve requirements depends importantly on economic demography. Central bankers are most likely to find multiple reserve requirements useful in economies in which financial resources are concentrated in the hands of the rich savers. (The source of this concentration could be that rich savers are relatively numerous, or that they have relatively large endowments, or that they have a relatively high propensity to save.) Note that if $d^r(R_d^r)$ and $d^p(R_d^p)$ are increasing functions, as will be true in many plausible specifications, then a wide gap between $\overline{R}_d^r$ and $\overline{R}_d^p$ will increase the divergence between $\overline{d}^r$ and $\overline{d}^p$.

Finally, if an economy has the aforementioned characteristics then multiple reserve requirements are most likely to be useful when the central bank is relatively concerned about the welfare of the initial old. Concern about the welfare of the initial old will cause a central bank to choose relatively high reserve ratios, which will increase the value of the nominal liabilities the initial old are endowed with. In economies in which $\overline{d}^r$ is large relative to $\overline{d}^p$, the key reserve ratio will be θ_b, because the rich savers are the principal source of reserve demand. And it should be clear from equation (26) that higher values of $\overline{\theta}_b$ will produce values of $\widehat{R}_m$ that are closer to $\overline{R}_b$, and thus that are more likely to exceed $\overline{R}_d^p$.

The corollary to Proposition 2 establishes that it is nonvacuous:

Corollary 1 *There are Class B economies that have public and private allocations that cannot be supported by multiple single reserve regimes.*

We prove Corollary 1 via Example 2, which has all the features just described. The deposit rate received by rich savers is more than 30 basis points higher than the rate received by poor savers, the rich savers account for roughly 85 percent of the equilibrium financial resources, and the bond reserve ratio in the supporting multiple reserves policy setting is quite high (50 percent). Note that in the latter policy features a reservable bond return rate almost

20 basis points higher than the rate of return on currency.[22] We also provide an example (Example 3) of a public and private allocation that can be supported by a multiple single reserve regime. The economies of these two examples are identical except for the objectives of the central bank. In Example 3, the central bank is less concerned about the welfare of the initial old, so it chooses a lower bond reserve ratio. As a result, $\widehat{R}_m$ falls slightly below R_d^p, allowing the central bank to suppport the allocation with a dual currency reserve policy setting involving a relatively high currency return rate and a very high currency reserve ratio.

Proposition 2 establishes that Class B economies can have public and private allocations that can be supported only by values (R_b, R_m) with $R_b > R_m$ — that is, allocations that can be supported only by conventional multiple reserves regimes. It also establishes that these economies *cannot* have allocations that are supportable only by unconventional multiple reserves regimes. If an allocation is supportable by some values (R_b, R_m) featuring $R_b < R_m$, then Proposition 2 guarantees that it will also be supportable by values featuring $R_b \geq R_m$. This result may go a long way towards explaining why we do not observe reservable bonds that yield negative nominal interest: there is never any need for them.[23]

[22]There are alternative equivalent policy settings that involve lower bond reserve ratios and lower nominal bond rates. However, the bond reserve ratio cannot be set below 0.267, and the nominal bond rate must always be positive.

[23]Espinosa and Russell (1999) show that negative nominal interest bonds are also unnecessary in Class A economies.

3.5.2 Regimes with a common reserve ratio

Our second result involves regimes in which both types of banks face the same reserve ratio, but must hold different government liabilities — currency for the national banks and bonds for the international banks — in order to meet the reserve requirement. We will refer to regimes of this type as "dual reservable liability" (DRL) regimes.

A dual reservable liability equilibrium can be defined as a multiple reserves equilibrium in which $\theta_m = \theta_b$. For simplicity, we shall call the common reserve ratio θ. The conditions for a single multiple reserve are then identical to the multiple reserves equilibrium conditions except that the restriction $\theta \in (0, 1]$ replaces the analogous restrictions on θ_m and θ_b, equations (12) and (13) are replaced by

$$R_d^p = (1 - \theta)R + \theta\, R_m \tag{27}$$

$$R_d^r = (1 - \theta)R + \theta\, R_b\,, \tag{28}$$

respectively, and equations (14) and (15) are replaced by

$$G = \theta\left[(1 - R_m)\, d^p(R_d^p) + (1 - R_b)\, d^r(R_d^r)\right] \tag{29}$$

and

$$G = \theta\left[d^p(R_d^p) + d^r(R_d^r)\right] - m_0, \tag{30}$$

respectively.

Again, any dual reservable liability allocation is a public and private allocation, but again, the converse is not always true. Proposition 3 identifies necessary and sufficient conditions under which a public and private allocation cannot be supported by a dual reservable liability regime.

Proposition 3 *In Class B economies, a public and private allocation will fail to be supportable as a dual reservable liability regime if and only if*

$$R\left(1 - \frac{\overline{A}}{\overline{\overline{d}}}\right) \geq \overline{R}_d^p. \tag{31}$$

If an allocation satisfies this condition, then any multiple reserves regime that supports it must involve $\overline{\theta}_m > \overline{\theta}_b$.

We can explain the intuition behind Proposition 3 using the same kind of thought experiment we used in the preceding subsection. Suppose the central bank wants to support a public and private allocation with an DRL regime. If this new regime is to produce the same level of welfare for the initial old, then it must generate the same level of aggregate demand for reservable liabilities. In order to generate the required level of reserve demand, the central bank must set the common reserve ratio at a unique weighted average of the reserve ratios from the multiple reserves policy setting. (Every multiple reserves policy setting that supports a given allocation yields the same weighted average.) The common reserve ratio is

$$\widehat{\theta} = \overline{\beta}_p \overline{\theta}_m + \overline{\beta}_r \overline{\theta}_b = \frac{\overline{A}}{\overline{d}}, \tag{32}$$

where $\beta_p = \overline{d}^p / \overline{d}$ and $\beta_r = 1 - \beta_p = \overline{d}^r / \overline{d}$. Thus, condition (31) can be rewritten

$$\left(1 - \widehat{\theta}\right) R \geq \overline{R}_d^p. \tag{33}$$

It is immediately clear from the national bank deposit rate equation (27) that condition (33) implies $\widehat{R}_m < 0$ (see below).

To resume the heuristic account, suppose that central bank wishes to conduct aggressive price discrimination, keeping the return rate received by the rich savers high but driving the rate facing the poor savers low in order to force them to bear a disproportionate share of the burden of financing government expenditures. Under a genuine multiple reserves regime, the central bank could use differential reserve ratio policy to help it accomplish this task, setting the national bank reserve ratio at a high level and the international bank ratio at a low level. However, under a dual reservable liability regime it is constrained to impose the same reserve ratio on both types of banks. If a public and private allocation satisfies inequality (31), however, then a common reserve ratio low enough to enable the central bank to offer a high deposit rate to the rich savers will be too low to permit it to earn the needed amount of seigniorage revenue from poor savers. The low common reserve ratio will not force the national banks to hold large enough reserves, which means that even a very low currency return rate — that is, a very high inflation rate — will fail to produce the needed seigniorage revenue.[24]

The necessity of the condition $\overline{\theta}_m > \overline{\theta}_b$ for unsupportability follows immediately from an alternative form of condition (31), which is

$$R\,(\overline{\theta}_m - \overline{\theta}_b)\,\overline{d}^r \geq \overline{\theta}_m \overline{R}_m\,\overline{d}\,. \tag{34}$$

The logic here is straightforward. As we have seen, supportability problems may arise if the unique DRL currency return rate $\widehat{R}_m$ must be lower than any of the currency return rates under supporting multiple reserves regimes. This can happen only if the common DRL

[24]Why can't the central bank simply increase the reserve ratio on international banks and compensate the rich savers by increasing the rate of return on reservable bonds? The problem with this strategy involves an insight due to Freeman (1987). In models of this type, seigniorage is a particularly inefficient form of asset taxation because the assets that constitute its tax base — unbacked government liabilities — are inefficient assets. The source of this inefficiency is that unbacked government liabilities represent intergenerational transfers. The social rate of return on international transfers is the output growth rate, which is lower, by assumption, than the rate of return on private investment. Thus, allocating saving to currency or government bonds, rather than to private investment, results in a loss of consumption. Using reserve requirements to increase the seigniorage tax base increases the stock of currency and government bonds, which increases the quantity of consumption goods that must be given up in order to provide the government with a given amount of revenue. It follows that if the central bank tries to increase the reserve requirement without losing seigniorage revenue, it will have to reduce either the international bank deposit rate, which will reduce the consumption and welfare the rich savers, or the national bank deposit rate, which will have the same effect on the poor savers. Of course, an increase in the average reserve ratio would benefit the initial old, but the central bank may not interested in increasing the welfare of the initial old at the expense of the rich and/or poor savers.

reserve ratio $\widehat{\theta}$ is higher than the any of the currency reserve ratios under these regimes — which can happen, in turn, only if the bond reserve ratios in these regimes are invariably higher than the corresponding currency reserve ratios, so that the weighted average reserve ratio $\widehat{\theta}$ is higher than any of the currency reserve ratios.

What kind of economies and central bank preferences will produce situations in which a public and private allocation cannot be supported by a dual reservable liability regime? As we have already indicated, unsupportability is most likely in situations where the central bank is trying to price discriminate by setting the national bank deposit rate much lower than the international bank deposit rate. This strategy indicates that the central bank considers the welfare of the rich savers considerably more important than the welfare of the poor savers.

One very clear formal indication of the role of the spread between the two deposit rates in producing unsupportable allocations is the fact that another alternative form of condition (31) is

$$\overline{m}_0 \leq \left(\overline{R}_d^r - \overline{R}_d^p\right) \overline{d}^r. \tag{35}$$

Condition (35) also establishes that unsupportability is most likely when $\overline{d}^r$ is large relative to $\overline{d}^p$: note that $\overline{m}_0 = (\overline{R}_m \overline{\theta}_m) \overline{d}^p + (\overline{R}_b \overline{\theta}_b) \overline{d}^r$. When the central bank wishes to use a multiple reserves regime to price discriminate against the poor savers, it is likely to set the bond reserve ratio at a relatively low level and the currency reserve ratio at a relatively high level (see above). As we have seen, the common reserve ratio in a supporting DRL regime is an average of the original bond and currency reserve ratios weighted by the quantities of rich and poor saver deposits, respectively. As $\overline{d}^r$ gets larger relative to $\overline{d}^p$, the common reserve ratio gets closer to the relatively low bond reserve ratio. This increases the likelihood that the new currency return rate will need to be reduced to the point where the nonnegativity constraint become binding.

So far, the central bank preference and economic demography conditions that lead to DRL-unsupportability have been the same, qualitatively, as the conditions that lead to DCR-unsupportability. There is, however, an important difference between the two sets of conditions. When a public and private allocation meets the conditions already described, it is most likely to fail to be supportable by a dual reservable liability regime if the central bank is *not* very concerned about the welfare of the initial old, so that $\overline{m}_0$ is relatively low. This fact is obvious from condition (35). The intuition here is that if the central bank cares little about the welfare of the initial old then the bond reserve ratio is likely to be quite low, since this is the reserve ratio imposed on the group that is both economically dominant and politically favored. A low bond reserve ratio increases the likelihood that the new common reserve ratio will be lower than the original currency reserve ratio, forcing a decrease in the currency return rate R_m that violates its nonnegativity constraint.

Finally, how likely is it that DRL-unsupportable allocations will require genuine multiple reserves policy settings that feature positive nominal interest rates? We have seen that that public and private allocations are most likely to be unsupportable by DRL regimes when $\overline{R}_d^r$ is high relative to $\overline{R}_d^p$, and when the supporting multiple reserves regimes feature relatively low values of $\overline{R}^m$ (so that it does not have to fall far to reach zero), suggests that most DRL-unsupportable allocations will be supported by multiple reserves regimes in which nominal bond rates are necessarily positive. However, positive nominal bond rates are not an absolute prerequisite for DRL-unsupportability. The reason for this is that when the bond reserve ratio $\overline{\theta}_b$ is low and the currency reserve ratio $\overline{\theta}_m$ is high [see the discussion of condition (34) above], the international bank deposit rate $\overline{R}_d^r$ can be high relative to the national bank deposit rate $\overline{R}_d^r$ even when $\overline{R}_b$ is lower than $\overline{R}_m$.[25]

The corollary to Proposition 3 establishes, by example, that it is nonvacuous:

Corollary 2 *There are public and private allocations that cannot be supported by dual reservable liability regimes.*

[25]Situations of this sort are most likely when the central bank is not too concerned about the welfare of the initial old — which tends to produce a low value of $\overline{\theta}_b$ — and has a relatively large deficit to finance, so that a low value of $\overline{\theta}_b$ requires a low value of $\overline{R}_b$. They are also more likely when $\overline{d}^r$ is quite large relative to $\overline{d}^p$, since in these cases if $\overline{R}_b$ is low then $\overline{m}_0$ will be low relative to $\overline{d}^r$.

The proof of this corollary takes the form of Example 4. In this example, any multiple reserves regime that supports the optimal public and private allocation involves a positive nominal bond rate. As in the preceding examples, the rich savers account for most of the demand for assets and the spread between the national and international bank deposit rates is wide, with the national banks offering a very low rate. In this case, however, the average reserve ratio is relatively low, reflecting the fact that the central bank is less concerned about the welfare of the initial old and more concerned about the welfare of the rich savers.

In the supporting multiple reserves regime, the currency (national bank) reserve ratio is much higher than the bond (international bank) reserve ratio. In addition, the financial dominance of the rich savers ensures that the relevant weighted-average reserve ratio is much closer to the bond reserve ratio. Thus, a shift to a dual reservable liability regime requires a large decline in the currency reserve ratio — which means the currency return rate must fall to restore the original deposit rate. But since the currency return rate was already quite low, the required decline would drive it below zero.

3.5.3 Regimes with a deposit tax

The final question we investigate in this section is whether it is always possible to duplicate a public and private allocation by means of a currency reserve requirement on the national banks plus a proportional tax on funds deposited in the other type of bank. We will refer to regimes of this type as "single reserve/deposit tax" (SR/DT) regimes.[26]

Deficit finance regimes involving taxation of bank deposits have attracted a good deal of attention in recent years. While deposit taxation may seem conceptually different from seigniorage, Fama (1980) has argued that the two financing strategies are roughly equivalent. Freeman (1987) describes a special case in which this equivalence is precise. As we have noted, Freeman studies the optimal level of a single currency reserve requirement in a model that is essentially identical to Espinosa's (1995) except that the households are intragenerationally homogeneous. He assumes that the government is unconcerned about the welfare of the initial old households and acts to maximize the steady-state utility of the full lived households. He shows that under this assumption, it is optimal for the government to choose the smallest

[26]In an earlier version of this paper, we studied regimes in which the central bank imposed a currency reserve requirement on both types of banks, at a common ratio, plus a tax on funds deposited one of the banks. [The requirement $R_d^p \leq R_d^r$, which is part of equilibrium condition (11), implies that in a regime of this sort, the deposit tax must be imposed on the national banks.] Another alternative regime would feature a currency reserve requirement on the international banks plus a tax on funds deposited at national banks. It can be shown, however, that any public and private allocations supportable by regimes of this type can also be supported by dual currency reserve regimes.

reserve ratio consistent with financing its deficit — a ratio at which the *gross* real rate of return on currency is zero. This policy is equivalent to replacing the currency reserve requirement with a proportional tax on deposits levied at a rate equal to the required reserve ratio.[27]

In our model, a deposit tax can also be viewed as a special case of a bond reserve requirement. In particular, a single reserve/deposit tax regime is formally equivalent to a multiple reserves regime in which $R_b = 0$. The fact that the rate of return on reservable bonds is fixed at zero arguably makes these regimes simpler than other multiple reserves regimes. We examine them partly for this reason, and partly for consistency with Espinosa (1995) and Espinosa and Russell (1999) — both of which discuss deposit taxes in Class A economies.

The conditions for single reserve/deposit tax equilibria are identical to the multiple reserves equilibrium conditions except that condition (10) is omitted, condition (13) is replaced by

$$R_d^r = (1 - \theta_b)\, R\,, \tag{36}$$

[27] It is important to distinguish between a deposit tax, which is a tax on funds deposited, and a tax on deposit returns. Freeman (1987) demonstrates that in models of this type, deposit-returns taxes are not special cases of reserve requirements. For this reason, we do not study deposit-returns taxes in this paper. However, we think expanding our analysis to include these taxes would be an interesting extension of our research.

and condition (14) is replaced by

$$G = (1 - R_m)\,\theta_m\, d^p(R_d^p) + \theta_b\, d^r(R_d^r)]\,. \tag{37}$$

Proposition 4 describes the conditions under which it may be possible for genuine multiple reserves regimes to produce higher levels of social welfare than single reserve/deposit tax regimes.

Proposition 4 *A public and private allocation will fail to be supportable by a single reserve/deposit tax regime if and only if*

$$\frac{\overline{m}_0}{\overline{d}^p} \geq \overline{R}_d^p\,. \tag{38}$$

In order to grasp the intuition behind Proposition 4 it is necessary to understand the key difference between a deposit tax and a "normal" bond reserve requirement. Under a deposit tax, the government simply takes funds (goods) deposited by current young households and uses them to cover its deficit: none of these goods are transferred to the current old households in the form of debt repayments. To see this, note that total bond reserve demand $\theta_b\, d^r$, which is the quantity of goods the government obtains from the current young households by imposing bond reserve requirements, can be divided into bond seigniorage revenue $(1 - R_b)\,\theta_b\, d^r$, which is used to help finance the government deficit, and residual revenue $R_b\theta_b\, d^r$, which finances government debt service payments. The goods devoted to debt service are used to retire government liabilities held by the the current old households

— including, at date 1, the initial old households. But if $R_b = 0$, which is the case of a deposit tax, then none of the revenue goes to debt service.

Thus, when the international banks must pay a deposit tax instead of meeting a normal bond reserve requirement, none of the funds generated by "reserve taxation" of the initial (date 1) rich savers are transferred to the initial old households. This situation creates supportability problems if the central bank tries to use a deposit tax to support a public and private allocation in which funds obtained from the initial rich savers through a bond reserve requirement make a large contribution to the consumption of the initial old. It may not be possible to replace these funds with funds derived from taxation of the initial poor savers via a currency reserve requirement. In particular, since $\widehat{R}_m$, the currency return rate under the supporting single reserve/deposit tax regime, cannot exceed the national bank deposit rate $\overline{R}^p_d$, if initial-old real balances $\overline{m}_0$ exceed gross national bank deposit returns $\overline{R}^p_d \overline{d}^p$ then there will be no feasible currency reserve ratio (no $\widehat{\theta}_m \leq 1$) that produces a large enough transfer from the initial poor savers to the initial old.

What kind of economies and central bank preferences are likely to produce public and private allocations that cannot be supported by single reserve/deposit-returns tax regimes? The most important factor is certainly the fraction of the economy's financial resources that are controlled by the rich savers. If this fraction is large, then $\overline{d}^p$ is likely to be small relative to $\overline{m}_0$ and the allocation will not be supportable. Of course, if the central bank is unconcerned about the welfare of the initial old, so that $\overline{m}_0$ is quite small, then a small value of $\overline{d}^p$ may not be a problem. Thus, unsupportable allocations will tend to be observed in economies where the welfare of the initial old is important to the central bank.

An alternative form of condition (38) is

$$\frac{\overline{R}_d^r}{R}\frac{\overline{d}^r}{\overline{d}} \geq 1 - \overline{\theta}\,, \tag{39}$$

where $\overline{\theta} \equiv \overline{A}/\overline{d}$ is the unique deposit-volume-weighted average of the reserve ratios from the supporting multiple reserves regimes. This expression makes it clear that allocations are most likely to be unsupportable when $\overline{A}$ is close to $\overline{d}$, so that the average reserve ratio is relatively high, when $\overline{d}^r$ is large relative to $\overline{d}$ (and thus to $\overline{d}^p$), and when $\overline{R}_d^r$ is relatively high (close to R). Since the original SR/DT-unsupportability condition (38) indicates that allocations are most likely to be unsupportable when $\overline{R}_d^p$ is relatively low, we can conclude that a desire to price discriminate against poor savers — that is, to set $\overline{R}_d^p$ considerably lower than $\overline{R}_d^r$ — is also an important factor in making genuine multiple reserves regimes more useful to central banks than SR/DT regimes.

The characteristics that tend to produce public and private allocations that cannot be supported by single reserve/deposit tax regimes are qualitatively similar to the characteristics that tend to produce allocations that cannot be supported by dual currency reserve regimes. In view of the similarity between condition (25) and condition (38), this similarity should not be very surprising. Indeed, if $\overline{d}^p < \overline{A}$, which is likely in economies with these characteristics, then any allocation supportable by a SR/DT regime will also be supportable by a dual currency reserve regime.[28] Stated differently, if an economy has characteristics that cause its central bank to prefer a genuine multiple reserves regimes to a dual currency reserve regime, then the bank is unlikely to find a single reserve/deposit tax regime to be a useful alternative.

The only remaining question concerns the sign of the nominal interest rate on reservable bonds in multiple reserves regimes that support SR/DT-unsupportable allocations. Condition (38) indicates that allocations of this type will tend to have relatively high values of $\overline{m}_0$ and relatively low values of $\overline{R}_d^p$. A high value of $\overline{m}_0$ requires a high average return rate on government liabilities, while a low value of $\overline{R}_d^p$ requires a low value of $\overline{R}_m$. If $\overline{R}_m$ is low, however, then a high average government-liability return rate will require a high value of $\overline{R}_b$. Thus, allocations that are not supportable by single reserve/deposit tax regimes will

[28] If the central bank favors the initial old then required reserves $\overline{A}$ are likely to be relatively large. If rich savers account for the lion's share of the financial resources then $\overline{d}^p$ will be small relative to $\overline{A} \equiv \overline{\theta}_m \overline{d}^p + \overline{\theta}_b \overline{d}^r$.

usually be supportable only by multiple reserves regimes with positive nominal bond rates. As in the case of single multiple reserves regimes, however, positive nominal bond rates are not absolutely required for unsupportability. If $\overline{d}^p$ is small enough, so that currency reserve requirements can never provide much support for the initial old, then allocations involving relatively low values of $\overline{m}_0$ and/or relatively high values of $\overline{R}_d^p$ may be unsupportable.

The corollary to Proposition 4 establishes that it is not vacuous:

Corollary 3 *There are public and private allocations that cannot be supported by single reserve/deposit tax regimes.*

We prove the corollary via Example 5, which describes an economy and a social utility function under which the optimal public and private allocation is not supportable by a single reserve/deposit tax regime. In the economy of this example, any multiple reserves policy setting that supports the optimal allocation produces a positive nominal interest rate on reservable bonds. We also present another example — Example 6 — in which the optimal public and private allocation is not supportable by an SR/DT regime but can be supported by an unconventional multiple reserves regime. As in the case of Examples 2 and 3, the only difference between Examples 5 and 6 is that the central bank has different objectives. In the economy of these examples $\overline{d}^p$ is quite small relative to $\overline{d}^r$, which explains why both public and private allocations are SR/DT-unsupportable. In Example 6, however, the central bank is less concerned about the welfare of the rich savers and more concerned about that of the poor savers. As a result, the spread between R_d^r and R_d^p falls substantially, allowing the central bank to increase R_m and reduce R_b to the point where the former exceeds the latter.

3.6 An important example

We conclude our discussion of Class B economies by demonstrating, by example, that there are public and private allocations that cannot be supported by any of these three relatively simple types of seigniorage regime. Stated differently, there are Class B economies in which central banks with certain preferences will prefer multiple reserves regimes to any of these simpler regimes. Our results imply that the multiple reserves regimes these central banks will impose will be conventional in nature.

Proposition 5 *There are public and private allocations that cannot be supported by dual currency reserves regimes, dual reservable liability regimes, or single reserve/deposit tax regimes. Any multiple reserves regime that supports such an allocation must involve reservable bonds with positive nominal yields.*

We prove the first part of Proposition 5 by demonstrating that the allocation from Example 4 satisfies conditions (25), (31) and (38). The second part of the proposition follows trivially from Proposition 2. Since a public and private allocation that cannot be supported by the first regime we studied can be supported only by a multiple reserves regime with positive nominal bond rates, an allocation that cannot be supported by any of the three regimes must have the same characteristic.

The economy and equilibrium allocation described in Example 4 possess both of the features we have identified as generating a useful role for multiple reserve requirements. First, $\overline{R}_d^p$, the national bank deposit that faces poor savers, is substantially lower than $\overline{R}_d^r$, the international bank deposit rate that faces rich savers. Second, $\overline{d}^r$, the total real deposits of rich savers, is substantially larger than $\overline{d}^p$, the corresponding value for poor savers.[29]

Multiple reserve requirements have most often been employed by developing countries in which income/wealth and political power are divided quite unevenly across the population. (This has certainly been true of Mexico, whose reserve requirements history we study in the next section.) In these countries, people who are much wealthier than average control a very high percentage of the total income and wealth. These relatively wealthy people also dominate the political system, so government policies tend to be formulated in ways that protects their economic status. In the admittedly abstract context of our model, economic dominance by the relatively wealthy corresponds to poor savers' deposits comprising a small

[29]Note that $\overline{m}_0$, the total real balances of the initial old households, is not large relative to total deposits, reflecting a central bank that is not too concerned about the initial old. As we have seen, small values $\overline{m}_0$ are characteristic of public and private allocations that are not supportable by a single multiple reserves regimes, but tend to work in favor of supportability by the other two regimes we have studied. In this case, however, the large spread between R_d^r and R_d^p and the large scale difference between d^r and d^p is enough to offset the low value of $\overline{m}_0$.

share of total deposits, while central bank policies that protect the wealthy would be expected to produce a poor saver (national bank) deposit rate that is substantially lower than the rich saver (international bank) rate. Thus, the predictions of our model are broadly consistent with the characteristics of countries that have imposed multiple reserve requirements. Our model also predicts that the only type of multiple reserves regime that will be useful to central bankers is a conventional regime. This prediction is consistent with the fact that this is the only type of multiple reserves regime we seem to observe.

4 Empirical plausibility of our approach

Is there empirical support for our public-finance/price-discrimination approach to explaining the existence of multiple reserve requirements? A thoroughgoing analysis of this question would be a major project extending far beyond the boundaries of this paper. However, we have managed to collect a good deal of information about one country — Mexico — that employed multiple reserve requirements during much of the post-World War II period. We can use this information to determine whether the approach we have used in this model seems generally plausible, at least as applied to this particular case.[30]

[30]

4.1 Seigniorage and reserve requirements

A basic assumption of our model is that the government earns significant revenue from currency and bond seigniorage. Another key assumption is that reserve requirements play an important role in generating this revenue.

4.1.1 Currency seigniorage

In Mexico, base money demand been much larger, as a fraction of GDP, than in the United States. During 1960-1990, the period for which we have complete data,[31] Mexican demand for base money averaged 13 percent of GDP, compared to a U.S. figure of 4.5 percent. The composition of base money demand has also been very different across the two countries. In Mexico, bank reserves have accounted for roughly two-thirds of total base money balances. In the United States, in contrast, bank reserves have accounted for only about one-fifth of total holdings of base money.

Data on monetary aggregates, government expenditures and GDP for Mexico were provided by the *Banco de Mexico.* Mexican reserve requirement data come from Sánchez-Lugo (1976), Subdirreción de Investigación Economica y Bancaria (1976), several issues of the Informe Anual of the *Banco de Mexico,* and Padilla (1996). We are grateful to Rodolfo Padilla for his assistance.

[31]We have data for Mexico from 1960 to 1995, but since the Mexican goverment stopped using reserve requirements in 1991, we have used data for 1960-1990 in most cases.

During 1960-1990, Mexico's average inflation rate was 27.1 percent — almost 6 times higher than the average U.S. inflation rate.[32] This combination of a high inflation rate and large bank reserves has allowed the Mexican government to obtain large amounts of revenue from currency seigniorage. Currency seigniorage revenue has averaged almost 3 percent of Mexican GDP — almost 10 times higher than the comparable U.S. percentage. The latter figure, moreover, underestimates the relative importance of currency seigniorage earnings to the Mexican government. Since the government spending share of GDP has been almost twice as large in the United States as in Mexico (where it has averaged 17.7 percent), currency seigniorage revenue, which has financed 16 percent of Mexican government expenditures, has been at least 15 times more important to the Mexican government than to the U.S. government.[33]

4.1.2 Bond seigniorage

Giovannini and de Melo (1993) provide some evidence in favor of the proposition that the government of Mexico has earned substantial amounts of revenue from bond seigniorage.

[32] Although this is a high inflation rate by U.S. standards, it is quite moderate by the standards of developing countries.

[33] For both the U.S. and Mexico, these data are constructed by dividing the change in the nominal money stock (nonbank currency, reserves, or both) during the year by the level of nominal GDP for the year.

These authors attempt to measure the revenues earned by the governments of developing countries from "financial repression" — that is, from imposing regulations that allow the government to pay lower interest rates on debt sold to its citizens than on its borrowings from foreign coutries. Their estimate of this revenue is the difference between the interest rates on foreign-held and domestically-held government debt, multiplied by the stock of domestically-held debt. In the case of Mexico, they are able to obtain the necessary data only for the years 1984-1987. Their estimate of Mexican average annual financial-repression revenue for this period is remarkably high — almost 6 percent of GDP and almost 40 percent of total government tax revenue.

The Giovannini-de Melo estimates are probably inflated by two special features of the years 1984-1987: the extraordinarily high inflation rate — averaging more than 80 percent — and the fact that the Mexican government had recently been forced to reschedule a substantial portion of its large foreign debt, which made the average interest rate on the debt relatively high. However, even if the government's average annual revenue from financial repression has been only half as large, relative to GDP, as the Giovannini-de Melo estimates for 1984-1987, then the amount of revenue from this source has been similar to the amount of revenue from currency seigniorage. This is probably a very conservative estimate: although high inflation during 1984-1987 also increased revenue from currency seigniorage, the average ratio of currency seigniorage revenue to GDP for the entire 1960-1990 period is less than 20 percent smaller than the 3.6 percent average ratio recorded during 1984-1987.

For the purposes of our analysis, only part of Mexico's financial-repression revenue can be considered bond seigniorage. Our model abstracts from government borrowing, and our bond seigniorage tax rate is properly interpreted as the difference between the growth rate of real output and the real interest rate on reservable bonds — which is presumably somewhat smaller than the difference between the latter and the real interest rate on foreign-held government debt. We have not been able to construct direct estimates of the volume of revenue the Mexican government has earned from bond seigniorage, largely because we do not have data on the interest rates on reservable bonds. We do know that Mexico's bond reserve requirements have been relatively high — on the same order as its currency reserve requirements. For demand deposits at deposit banks in the Federal District (see below), for example, the average bond reserve ratio during 1960-1990 was almost identical to the average currency reserve ratio — almost 25 percent in each case. To obtain a rough estimate of the size of bond seigniorage revenue, we might assume that total bond reserves were equal to total currency reserves, and that the average nominal interest rate on reservable bonds was 10 percent, so that the average bond seigniorage tax rate was three-fifths as large as the average inflation tax rate. Under this assumption, Mexico's bond seigniorage revenue would have made up almost 10 percent of its total government revenue, and total seigniorage revenue would have provided more than a quarter of total government revenue.

4.2 Reserve requirements policy

Another set of fundamental assumptions of our model involves the level and variability of the reserve requirements the government imposes on the banking system. Our model implicitly assumes that the government imposes, or feels free to impose, both currency and bond reserve requirements, that the aggregate reserve ratio for currency and bonds is relatively high, and that the government is in a position to change the currency and/or bond reserve ratios in order to achieve its revenue and welfare goals. Finally, the model assumes that the government is able to impose different currency and bond reserve requirements on different banks, or on different types of accounts within banks — accounts that might be held by different groups of depositors.

As the account below indicates, all these things seem to have been true of Mexico. The Mexican government has imposed large currency and bond reserve requirements on its banks and related financial institutions. It has also imposed different reserve requirements on banks in different locations (inside and outside the Federal District — see below), on different types of depository institutions (deposit banks and savings banks) and on different types of accounts (demand deposits and time deposits).

4.2.1 History

In 1936 the Bank of Mexico was granted essentially complete authority to impose reserve requirements on Mexican financial institutions; this included the authority to select different types of reserve assets, to impose different reserve requirements on different institutions and/or classes of deposits, and to impose different requirements on institutions in different regions. Mexico began employing multiple reserve requirements in October of 1948, when it imposed a 25 percent government-bond reserve ratio on banks of deposit (*bancos de deposito*), which constituted by far the largest group of banks.[34] The currency reserve ratio imposed on these banks was simultaneously reduced from 50 percent to 20 percent. Bond reserve requirements were extended to savings banks (*bancos de ahorro*) in August 1955, when a 37.5 percent bond requirement was added to the 10 percent currency reserve requirement

[34] See Sanchez-Lugo (1976).

already imposed on these banks.[35]

From 1960 through 1976, the government bond reserve requirements on demand deposits at banks in the Federal District (Mexico City and its environs, which dominate Mexico's economy) were 35 percent, while the currency reserve requirements were 15 percent. Banks outside the Federal District faced the same currency reserve requirements, but bond reserve requirements of only 10 percent. Currency and bond reserve requirements were also imposed on time deposits at deposit banks in both regions. These requirements were changed somewhat more frequently. The currency reserve requirements on time deposits ranged from 10-15 percent from 1960-1971. During the same period, the government bond reserve requirements on time deposits ranged from 0-50 percent, although they were set at 15-20 percent in most years. During the 1970s, reserve requirements on time deposits were eliminated — the bond requirements in 1972 and the currency requirements in 1977.

In 1977, the Mexican government eliminated the remaining bond reserve requirements on deposits denominated in domestic currency, though it retained bond reserve requirements on foreign-currency-denominated deposits (see below).[36] It increased the currency reserve

[35]The bond reserve requirements on savings banks were reduced to 2 percent in 1963 and were eliminated entirely in 1973.

[36]It eliminated the bond reserve requirements on deposits at deposit banks and mortgage credit societies, but retained them on deposits at finance societies (see below). Bond reserve requirements at deposit banks actually began to be phased out in August 1972, when they were dropped on deposits accepted after August 21.

requirements in a way that kept the aggregate reserve ratio on banks inside the Federal District essentially unchanged (it rose from 50 percent to 54 percent) but increased the aggregate ratio on banks outside the district (which rose from 25 percent to 38 percent). Two years later, the currency reserve ratios for the two different types of banks were equalized at 38 percent. During the next few years the common currency reserve ratio was gradually increased, reaching 48 percent in 1984. In 1985, bond reserve requirements were reinstated: the bond reserve ratio was set at 38 percent. The currency reserve ratio was reduced to 10 percent, so the aggregate reserve ratio remained unchanged. The 10 percent currency reserve ratio remained in place through 1990, but the bond reserve ratio was gradually decreased to 30 percent. In 1991, however, Mexico reversed course again and eliminated both currency and bond reserve requirements entirely — with the exception, again, of deposits denominated in foreign currency.

Beginning in 1958, Mexico also imposed currency and bond reserve requirements on two smaller groups of financial institutions: finance societies (*sociedades financieras*) and mortgage credit societies (*sociedades de credito hipotecario*). These institutions, like the savings banks, offered noncheckable deposits with longer average maturities than those of the deposit banks. The reserve requirements imposed on these two types of institutions were

Reserve Requirements in Mexico, 1936-1991

1936	Bank of Mexico is granted authority to impose reserve requirements
1941	Bank of Mexico begins aggressive use of RRs policy
1945	Differential currency RRs on deposits (peso and dollar) at savings banks
1948	Bond RRs on peso deposits at deposit banks
1950	Bond RRs on dollar deposits at deposit banks
	Differential RRs (currency and bond) on peso deposits at F.D. banks
1955	Bond RRs on savings banks (peso and dollar deposits)
1958	Currency and bond RRs on finance and mortgage credit societies
1960	Bond RRs on time deposits
1963	Bond RRs on savings banks reduced to minimal level
1972	Differential RRs for dollar deposits at northern-border banks
	Bond RRs on time deposits eliminated
1973	Bond RRs on savings banks eliminated (peso deposits only)
1977	Currency RRs on time deposits eliminated
	Bond RRs on peso deposits eliminated
1980	Differential RRs on dollar deposits at northern-border banks eliminated
	Currency RRs on dollar deposits eliminated
1981	Deposit banks and savings institutions merged
1983	Differential RRs on Federal District banks eliminated
1986	Currency RRs on time deposits reinstated

different from each other and from the reserve requirements imposed on deposit and savings banks. Compared to the mortgage credit societies, the requirements on the finance societies were higher in aggregate and were more heavily weighted towards bond reserves. The reserve requirements on these two types of banks were changed quite frequently, particularly during the 1970s.[37]

As we have indicated, the government of Mexico has allowed banks to accept deposits denominated in foreign currency (principally U.S. dollars), and it has imposed both currency and bond reserve requirements on these deposits. The aggregate reserve ratios on these deposits have usually been quite high, and they have typically been heavily weighted towards bond reserve requirements. For deposits of this type, the analogue of the "inside or outside the Federal District" distinction is "along or below the northern frontier" — the northern frontier being the Mexican states along the border with the U.S. During 1972-1980 the reserve requirements imposed on foreign-currency-denominated deposits at deposit banks along the northern frontier were different from the reserve ratios imposed on these deposits at deposit banks located elsewhere in the Mexico. Typically, the reserve ratios imposed on northern-frontier banks were a bit less severe, involving slightly lower aggregate reserve ratios and/or

[37] In 1981, the finance societies, the mortgage credit societies and the savings banks were merged into the deposit banks.

somewhat heavier reliance on bond reserves.[38]

4.2.2 Implications

From the point of view of the specifics of our analysis, perhaps the most interesting aspect of this record is the fact that prior to 1977 the Mexican government used bond reserve requirements to impose differentially high (indirect) tax rates on demand deposits at deposit banks inside the Federal District. In addition, the government used both currency and bond reserve requirements to impose different tax rates on deposits at savings banks, finance societies and mortgage credit societies. Since economic activity and income in Mexico are disproportionately concentrated in Mexico City, it does not seem unreasonable to think of the deposit banks inside the Federal District as serving "rich savers" and of the banks outside the district as serving "poor savers."[39] Similarly, holdings of noncheckable, longer term deposits are concentrated among the relatively wealthy Mexicans.

[38]Mexico's policy regarding reserve requirements on foreign currency deposits provides an additional example of its use of reserve requirements as a device for price discrimination. However, we cannot claim that our formal model explains this behavior in any direct way, because the model does not include foreign currency. Similarly, our model does not allow us to distinguish between demand deposits and time deposits (see above).

[39]Given Mexico's relatively high inflation rates, the high reserve ratios on foreign-currency-denominated deposits have probably been necessary to reduce their attractiveness relative to deposits denominated in pesos. However, the geographic differentiation of these reserve requirements requirements may have been motivated by price discrimination/social-welfare considerations. While holdings of deposits denominated in foreign currency are probably concentrated among relatively wealthy Mexicans, this may be somewhat less true along the northern frontier because of the relatively high percentage of the local population engaged in trade with the United States.

Shortly after the Mexican government eliminated bond reserve requirements on domestic-currency deposits it abandoned its entire differential deposit-tax system, first by equalizing the reserve requirements on the two types of deposit banks and later by merging the other three types of financial institution into the deposit banks. This behavior suggests that differential deposit taxation may have been the government's principal motive for imposing bond reserve requirements — precisely the assumption that forms the basis of our analysis.[40]

5 Concluding remarks

5.1 Summary

This paper has attempted to provide a social-welfare-based explanation for the existence of seigniorage-based deficit finance regimes that involve multiple reserve requirements. Its point of departure was the analysis conducted by Espinosa (1995), who sought to construct

[40]One problem with this argument is that bond reserve requirements were not permanently dropped — they were reimposed in 1985. When this happened, however, the currency reserve requirements were drastically reduced, so that the aggregate reserve ratio was unchanged. Thus, in this case the government seems to have used the reimposition of bond reserve requirements as a device to reduce the implicit tax rate on deposits without changing the aggregate reserve ratio. It seems likely that politics played a role in this decision. At the time bond reserve requirements were reimposed Mexico's inflation rate was extremely high, and the public was undoubtedly resistant to changes in government policy that seemed to benefit the banks at the expense of nonbank holders of currency. Changes in the aggregate reserve ratio are probably easier for the public to understand and interpret than changes in the nature and distribution of reserve assets. Consequently, the government may have viewed reimposing bond reserve requirements as a way to reduce the burden of reserve costs on banking system without making it obvious that it was doing so.

a model that would explain why a number of developing countries have adopted multiple reserves regimes. We began by demonstrating that in Espinosa's model, any allocation that can be supported by a multiple reserves regime can also be supported by a single bond reserve requirement. Thus, this model does not really explain why a government would find it useful to impose two reserve requirements. We have gone on to construct an alternative model that succeeds in providing a clear social welfare rationale for a particular type of multiple reserves regime: a regime in which the government imposes a currency reserve requirement on banks that serve one class of depositors, and a bond reserve requirement on banks that serve another class. More specifically, we have shown that this type of multiple-reserves regime — a regime that requires the government to impose two reserve ratios, and to issue liabilities with two rates of return — may allow the government to achieve a higher level of social welfare than simpler deficit-financing regimes that require it to impose fewer reserve ratios and/or issue fewer types of liabilities. The multiple-reserves regimes that are potentially social-welfare improving, moreover, are conventional in nature.

5.2 Extensions

As we have noted, Espinosa (1995) introduces intragenerational heterogeneity into Freeman's (1987) reserve requirements model by assuming that there is a group of households who are shut out of the banking system and cannot hold any assets except currency. Our analysis has shown that this assumption has serious limitations as the basis for a theoretical explanation for the existence and nature of multiple reserve requirements. On the other hand, our examination of empirical evidence from Mexico has turned up a good deal of evidence consistent with the assumption.

Although nonbank currency demand has been much smaller relative to reserve demand in Mexico than in the United States, the reason for this is that reserve demand has been much larger, not that nonbank currency demand has been smaller. In fact, the average ratio of nonbank currency demand to GDP has been almost identical across the two countries.[41] In Mexico, moreover, this ratio has been extremely stable in the face of variation in the inflation rate that has been quite large relative to the variation experienced in the United States.[42]

[41]Evidence collected by Porter and Judson (1996) suggests that more than half of the U.S. currency outstanding may be held abroad, while presumably few pesos are held outside Mexico. Consequently, *domestic* non-reserve currency demand has probably been considerably larger in Mexico than in the United States.

[42]For additional details, see Espinosa and Russell (1998).

These observations seem consistent with Espinosa's (1995) assumption that there are households who provide a captive market for currency. However, the alternative model we have constructed does not provide any source of currency demand other than reserve requirements. Given the large size of Mexico's bank reserves relative to its nonbank currency holdings, we think a model that abstracts from them is a reasonable first approximation. In addition, there are alternative explanations for nature of Mexico's nonbank currency holdings whose implications may be very different from those of Espinosa's explanation. It is possible, for example, that currency is being held directly by all groups in the population in roughly the same proportions — presumably, for transactions purposes. If this is the case, then the existence of nonbank currency demand probably does not provide the central bank with opportunities for price discrimination, which means that abstracting from this source of currency demand is unlikely to affect our basic results.

Under the circumstances, however, a natural extension of our analysis would be to study the role of reserve requirements in "Class C" economies that include both our "moderately poor savers," who have access to national bank deposits, and "very poor savers" who cannot hold any assets other than currency. We have conducted a preliminary analysis of this type. Its results indicate that Class C economies may greatly broaden the range of circumstances under which multiple reserve requirements are likely to be useful to central banks. On the other hand, they may make it more difficult to account for our failure to observe unconventional multiple reserves regimes.[43]

Our analysis has succeeded in explaining the existence of seigniorage regimes in which the central bank imposes currency and bond reserve requirements with different reserve ratios. However, there is one aspect of observed multiple reserves regimes that it has not suceeded in explaining, which is that banks are often required to hold reserves of both currency and bonds on deposits of a particular type.

Lemma 1 establishes that in our model, there is never a need for more than one type of reserve requirement on each type of bank. The form of this result derives from our simplifying assumption that each type of bank serves a different type of depositors and issues a different type of deposit liability. There is, however, an equivalent interpretation of our model under which a single bank might serve both types of depositors and issue deposits of both types, with reserve requirements being levied on different types of deposits rather than different

[43]Another potentially interesting extension of our analysis involves economies in which $X < R$ — that is, economies in which the rate of return on the domestic investments available to national banks is lower than the rate that prevails in the international credit market. In these economies, there will be an efficiency rationale for deposit taxation even when the government cares about the initial old. It will make sense to minimize the bond reserve ratio on international banks, since they hold relatively productive assets, and to compensate for the resulting loss of money demand by increasing the currency reserve ratio imposed on the national banks. However, the feasibility of this policy will be limited by the fact that the currency reserve ratio cannot exceed unity.

types of banks. Thus, our model can explain why currency and bond reserve requirements might be levied on a particular type of bank.[44] Under this alternative interpretation, however, Lemma 1 would establish that there is never a need for more than on type of reserve requirement on each type of *deposit*. In practice, both currency and bond reserve requirements are often levied on a single type of deposit at a particular type of bank.[45]

Why has our model failed to capture this aspect of observed multiple reserves regimes? We believe the basic reason is that we have followed Freeman (1987) and Espinosa (1995) by studying economies in which [1] the government imposes reserve requirements only for the purpose of deficit finance and [2] government liabilities differ from each other only in their rates of return and in their accessibility to different types of households, and not in less-readily-definable characteristics such as "liquidity" or "usefulness as media of exchange."[46] We suspect that constructing a model in which imposing currency and bond reserve requirements on the same bank improves welfare may require assuming that the government uses

[44]We thank an anonymous referee for pointing out this alternative interpretation and its implication.

[45]Mexico, for example, has imposed currency and bond reserve requirements on demand deposits at banks inside the Federal District, and different currency and bond reserve requirements on demand deposits at banks outside the Federal District. Our model can explain why the reserve requirements on the two types of banks are different, but not why both types of reserve requirement are imposed on both types of banks.

[46]These assumptions seem very reasonable to us. Goodfriend and Hargraves (1983) note that in the modern U.S., at least, reserve requirements have played a very minor role in liquidity regulation and monetary control, but a fairly significant role in deficit finance. Government bonds, moreover, are simply default-free claims to future government currency. For an extended discussion of the implications of the latter fact, see Wallace (1983).

reserve requirements for additional purposes, and that government currency is inherently superior to government bonds for these purposes. We also suspect that it will be very difficult to do this in a manner that is both rigorous and plausible.

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Appendix A

Social utility functions: In Examples 2-6, which involve Class B economies, we use the indirect social utility function

$$W(m_0, R_d^r, R_d^p) = a_0 \log\left(1 + \frac{m_0}{d_1}\right) + a_1 \log\left(1 + \frac{R_d^r}{R}\right) + a_2 \log\left(1 + \frac{R_d^p}{R}\right),$$

where $a_0 \geq 0$, $a_1 \geq 0$, $a_2 \geq 0$, $\sum_{i=0}^{2} a_i = 1$, and $d_1 = d^p(1) + d^r(1)$. Note that R is the maximum possible value of R_d^r or R_d^p in a multiple reserves equilibrium, and in most specifications the maximum possible value of m_0 is d_1. In Example 1, which involves a Class A economy, we use the same function except that R_d replaces R_d^r, R_m replaces R_d^p, and $m(1)$ replaces $d^p(1)$ in the definition of d_1.

Proof of Proposition 1: Let $(\overline{G}, \overline{R}_m, \overline{R}_d^r, \overline{m}_0)$ represent a public and private allocation, and let $(\overline{R}_m, \overline{R}_b, \overline{\theta}_m, \overline{\theta}_b)$ be a policy setting that supports this allocation as a multiple reserves equilibrium. We wish to show that there is a policy setting $(\widehat{R}_m, \widehat{R}_b, \widehat{\theta}_b)$ that supports $(\overline{G}, \overline{R}_m, \overline{R}_d^r, \overline{m}_0)$ as a single bond reserve $(\widehat{\theta}_m = 0)$ equilibrium.

Clearly, we must choose $\widehat{R}_m = \overline{R}_m$. It follows that $\widehat{d}^p = \overline{d}^p$. Suppose further that we choose $\widehat{\theta}_b = \overline{\theta}$, which ensures $\widehat{\theta}_b \in (0, 1]$, and that we choose $\widehat{R}_b$ to solve $\overline{R}_d = \widehat{R}_d = (1 - \widehat{\theta}_b)R + \widehat{\theta}_b \widehat{R}_b$. It follows from equation (4) that

$$\widehat{R}_b = \frac{\overline{\theta}_m \overline{R}_m + \overline{\theta}_b \overline{R}_b}{\overline{\theta}}.$$

Note that $\min(\overline{R}_m, \overline{R}_b) \leq \widehat{R}_b \leq \max(\overline{R}_m, \overline{R}_b)$, so $\widehat{R}_b$ satisfies equation (3), and there is some $P_b > 0$ such that it satisfies equation (2). Since $\widehat{R}_d = \overline{R}_d$, we have $\widehat{d}^p = \overline{d}^p$. Thus, $\widehat{m}_0 = \widehat{d}^p + \widehat{\theta}_b \widehat{R}_b \widehat{d}^r = \overline{d}^p + (\overline{\theta}_m \overline{R}_m + \overline{\theta}_b \overline{R}_b) \overline{d}^r = \overline{m}_0$, using equation (9). In addition, $\widehat{A} = \widehat{d}^p + \widehat{\theta}_b \widehat{d}^r = \overline{d}^p + \overline{\theta}\, \overline{d}^r = \overline{A}$, so equation (9) implies $\widehat{G} = \overline{G}$. □

Example 1 *Let $R = 1.2$, $m(R_m) = 2 - 1/R_m$, $d(R_d) = 5$, $M(0) = 1$ and $\overline{G} = 0.0366667$; let the coefficients of the social welfare function be $a_0 = 0.122425$, $a_1 = 0.87593$, and $a_2 = 0.00164519$. The optimal private allocation is $\overline{m}_0 \doteq 2.30$, $\overline{R}_d = 1.08$, and $\overline{R}_m = 0.75$; it yields social utility $\overline{W} = 0.604724$. The single bond reserve policy setting $\overline{\theta}_b = 0.4$, $\overline{R}_b = 0.9$ and $\overline{R}_m = 0.75$ supports this allocation.*

If the central bank wishes to adopt a single currency reserve policy that supports $\widehat{m}_0 = \overline{m}_0$ then it must choose $\widehat{\theta}_m = \overline{\theta}_b = 0.4$ and $\widehat{R}_m = 0.865$, producing $\overline{R}_d = 1.065$ and $\widehat{W} = 0.599034$. Notice the enforced reduction in the spread between R_m and R_d.

The optimal single currency reserve policy setting is $\widetilde{\theta}_m = 0.220894$ and $\widetilde{R}_m = 0.802732$, which yields $\widetilde{R}_d = 1.11225$, $\widetilde{m}_0 \doteq 1.49206$, and $\widetilde{W} = 0.603589$. Notice that this allocation is very different from the optimal private allocation under a single bond reserve (or multiple reserves) regime.

General multiple reserve requirements regimes in Class B economies: Let θ^p_m and θ^r_m denote the currency reserve ratios on national and international banks, respectively, while θ^p_b and θ^r_b denote the bond reserve ratios on national and international banks, respectively. Then given $R > 1$, $G > 0$, and $M(0) > 0$, a binding stationary equilibrium in a Class B economy can be characterized as nonnegative values θ^p_m, θ^r_m, θ^p_b, θ^r_b, R_m, R_b, R^p_d, R^r_d and a positive value $p(1)$ that satisfy θ^p_m, $\theta^r_m \in [0,1]$, θ^p_b, $\theta^r_b \in [0,1]$, with $\theta^p = \theta^p_m + \theta^p_b \in (0,1]$, $\theta^r = \theta^r_m + \theta^r_b \in (0,1]$, equations (10) and (11),

$$R^p_d = (1 - \theta^p_m - \theta^p_b)R + \theta^p_b R_b + \theta^p_m R_m\,, \tag{.40}$$

$$R^r_d = (1 - \theta^r_m - \theta^r_b)R + \theta^r_b R_b + \theta^r_m R_m\,, \tag{.41}$$

$$G = (1 - R_m)[\theta^p_m d^p(R^p_d) + \theta^r_m d^r(R^r_d)] + (1 - R_b)[\theta^p_b d^p(R^p_d) + \theta^r_b d^r(R^r_d)] \tag{.42}$$

and

$$G = (\theta_m^p + \theta_b^p)\, d(R_d^p) + (\theta_m^r + \theta_b^r)\, d(R_d^r) - m_0\,, \tag{.43}$$

where $m_0 \equiv M(0)/p(1)$.

The following result about these regimes greatly simplifies our analysis of Class B economies:

Lemma 1 *In a Class B economy, any allocation that can be supported by a general multiple reserve requirements regime can be supported by a regime of the type described in Section 3.3 of the text.*

Proof of Lemma 1: Equations (42) and (43) imply

$$m_0 = \; (\theta_m^p R_m^p + \theta_b^p R_b^p)\, d^p(R_d^p) + \; (\theta_m^r R_m^r + \theta_b^r R_b^r)\, d^r(R_d^r)\,. \tag{.44}$$

Define $G^p = [(1 - R_m)\,\theta_m^p + (1 - R_b)\,\theta_b^p]\, d^p(R_d^p)$ and $m_0^p = \; (\theta_m^p R_m^p + \theta_b^p R_b^p)\, d^p(R_d^p)$. Similarly, define $G^r = [(1 - R_m)\,\theta_m^r + (1 - R_b)\,\theta_b^r]\, d^p(R_d^p)$ and $m_0^r = \; (\theta_m^r R_m^r + \theta_b^r R_b^r)\, d^r(R_d^r)$. Now let $(\overline{G}, \overline{m}_0, \overline{R}_d^r, \overline{R}_d^p)$ represent a public and private allocation supportable by a general multiple reserves regime. By Proposition 1, there are values $\widehat{\theta}_b^p \in (0,1]$ and $\widehat{R}_b^p \in [0, \overline{R}_d^p]$ that will produce $\widehat{R}_d^p = \overline{R}_d^p$, $\widehat{G}^p = \overline{G}^p$ and $\widehat{m}_0^p = \overline{m}_0^p$ when $\theta_m^p = 0$. [Note that $\widehat{R}_b^p \leq \overline{R}_d^p$ follows from equation (4) and the magnitude restrictions on the reserve ratios.] Proposition 1 also guarantees that there are values $\widetilde{\theta}_b^r \in (0,1]$ and $\widetilde{R}_b^r \in [0, R]$ that will produce $\widetilde{R}_d^r = \overline{R}_d^r$, $\widetilde{G}^r = \overline{G}^r$ and $\widetilde{m}_0^r = \overline{m}_0^r$ when $\theta_m^r = 0$. Now let $\theta_m^* = \widehat{\theta}_b^p$, $\theta_b^* = \widehat{\theta}_b^p$, $R_m^* = \widehat{R}_b^p$ and $R_b^* = \widehat{R}_b^r$ be policy settings for a multiple reserves regime of the type described in Section 3.3. By equation (4), we have $R_d^{p*} = \widehat{R}_d^p = \overline{R}_d^p$ and $R_d^{r*} = \widehat{R}_d^r = \overline{R}_d^r$. Note that $\widehat{R}_b^p \leq \overline{R}_d^p$ gives us $R_m^* \leq \overline{R}_d^p$. Since equations (42) and (44) imply $G = G^p + G^r$ and $m_0 = m_0^p + m_0^r$, respectively, we have $G^* = \widehat{G}^p + \widetilde{G}^r = \overline{G}^p + \overline{G}^r = \overline{G}$ and $m_0^* = \widehat{m}_0^p + \widetilde{m}_0^r = \overline{m}_0^p + \overline{m}_0^r = \overline{m}_0$. □

Proof of Lemma 2: Let $(\overline{R}_d^p, \overline{R}_d^r, \overline{m}_0)$ represent the private component of a public and private allocation in a Class B economy. Let $(\overline{R}_m, \overline{R}_b, \overline{\theta}_m, \overline{\theta}_b)$ be a multiple reserves policy setting that supports this component of the allocation. Equations (16) and (17) imply

$$\overline{\theta}_m \overline{R}_m = \overline{R}_d^p - \left(1 - \overline{\theta}_m\right) R$$

and

$$\overline{\theta}_b \overline{R}_b = \overline{R}_d^p - \left(1 - \overline{\theta}_b\right) R \,,$$

respectively. Substituting these equations into equation (21) produces

$$\overline{m}_0 = \overline{R}_d^p \overline{d}^p + \overline{R}_d^r \overline{d}^r + R\left(\overline{A} - \overline{d}\right). \tag{.45}$$

Equation (19) implies $\overline{A} = \overline{m}_0 + \overline{G}$. This equation can be substituted into the preceding equation and solved for $\overline{G}$, producing

$$\overline{G} = \overline{d} - \left(\overline{R}_d^p \overline{d}^p + \overline{R}_d^r \overline{d}^r\right) - \frac{R-1}{R}\overline{m}_0. \ \square$$

Proof of Proposition 2: Let $(\overline{G}, \overline{R}_d^p, \overline{R}_d^r, \overline{m}_0)$ represent a public and private allocation in a Class B economy, and let $(\overline{R}_m, \overline{R}_b, \overline{\theta}_m, \overline{\theta}_b)$ be a policy setting that supports this allocation as a multiple reserves equilibrium. We wish to identify conditions under which there is a policy setting $(\widehat{R}_m, \widehat{\theta}_b, \widehat{\theta}_m)$ that supports $(\overline{G}, \overline{R}_d^p, \overline{R}_d^r, \overline{m}_0)$ as a dual currency reserve (DCR) equilibrium.

We must have $\widehat{G} = \overline{G}$ and $\widehat{m}_0 = \overline{m}_0$. Equation (9) then requires $\widehat{A} = \widehat{\theta}_m \, \widehat{d}^p + \widehat{\theta}_b \, \widehat{d}^r = \overline{A}$, and equation (28) requires $\overline{G} = (1 - \widehat{R}_m) \, \widehat{A}$. It follows that

$$\widehat{R}_m = \frac{\overline{A} - \overline{G}}{\overline{A}} = \frac{\overline{m}_0}{\overline{A}}$$

in any supporting DCR equilibrium. Note that $0 \leq \widehat{R}_m \leq 1$.

In a supporting equilibrium, we must have $\widehat{R}_d^p = \overline{R}_d^p$ and $\widehat{R}_d^r = \overline{R}_d^r$. Equations (26) and (27) then require

$$\widehat{\theta}_m = \frac{R - \overline{R}_d^p}{R - \widehat{R}_m}$$

and

$$\widehat{\theta}_b = \frac{R - \overline{R}_d^r}{R - \widehat{R}_m}.$$

We need to show that $0 < \widehat{\theta}_m < 1$ and $0 < \widehat{\theta}_b < 1$. The inequalities $\widehat{\theta}_m > 0$ and $\widehat{\theta}_b > 0$ follow from $0 \leq \widehat{R}_m \leq 1$ (see above), $R > 1$, and equation (25). For $\widehat{\theta}_m \leq 1$ and $\widehat{\theta}_b \leq 1$ we need $\widehat{R}_m \leq \overline{R}_d^p$ and $\widehat{R}_m \leq \overline{R}_d^r$, respectively. Since $\widehat{R}_m = \overline{m}_0/\overline{A}$, these inequalities are equivalent to $\overline{m}_0/\,\overline{A} \leq \overline{R}_d^p$ and $\overline{m}_0/\overline{A} \leq \overline{R}_d^r$, respectively. Thus $\overline{m}_0/\overline{A} \leq \overline{R}_d^p$ is necessary for the existence of a supporting DCR equilibrium.

Equations (20) and (21) imply that the inequalities $\overline{m}_0/\overline{A} < \overline{R}_d^p$ and $\overline{m}_0/\overline{A} < \overline{R}_d^r$ can be rewritten

$$\overline{R}_m \overline{\theta}_m \, \overline{d}^p + \overline{R}_b \overline{\theta}_b \, \overline{d}^r < \overline{R}_d^p \left(\overline{\theta}_m \, \overline{d}^p + \overline{\theta}_b \, \overline{d}^r \right) \tag{.46}$$

and

$$\overline{R}_m \overline{\theta}_m \, \overline{d}^p + \overline{R}_b \overline{\theta}_b \, \overline{d}^r < \overline{R}_d^r \left(\overline{\theta}_m \, \overline{d}^p + \overline{\theta}_b \, \overline{d}^r \right),$$

respectively. Equation (13) ensures $\overline{R}_m < \overline{R}_d^p < \overline{R}_d^r$, and equation (17) implies $\overline{R}_b < \overline{R}_d^r$. Inequality (54) then gives us $\overline{m}_0/\overline{A} < \overline{R}_d^r$, and thus $\widehat{\theta}_b < 1$.

As we have seen, our required choices of $\widehat{R}_m$, $\widehat{\theta}_m$ and $\widehat{\theta}_b$ give us $\widehat{R}_d^p = \overline{R}_d^p$ and $\widehat{R}_d^r = \overline{R}_d^r$, and thus $\widehat{d}^p = \overline{d}^p$ and $\widehat{d}^r = \overline{d}^r$. It follows that

$$\widehat{m}_0 = \widehat{R}_m \left(\widehat{\theta}_m \, \widehat{d}^p + \widehat{\theta}_b \, \widehat{d}^r \right) = \frac{\overline{m}_0}{\overline{A} \left(R - \frac{\overline{m}_0}{\overline{A}} \right)} \left[(R - \overline{R}_d^p) \, \overline{d}^p + (R - \overline{R}_d^r) \, \overline{d}^r \right] \tag{.47}$$

Equations (16) and (17) give us $R - \overline{R}_d^p = \overline{\theta}_m(R - \overline{R}_m)$ and $R - \overline{R}_d^r = \overline{\theta}_b(R - \overline{R}_b)$, respectively. Equation (45) gives us $(R - \overline{R}_d^p)\,\overline{d}^p + (R - \overline{R}_d^r)\,\overline{d}^r = R\,\overline{A} - \overline{m}_0$, so that the right-hand-side of equation (47) simplifies to $\overline{m}_0$. Lemma 2 then gives us $\widehat{G} = \overline{G}$. Thus, $\overline{m}_0/\overline{A} \leq \overline{R}_d^p$ is sufficient for the existence of a supporting DCR equilibrium.

Finally, note that we cannot guarantee $\overline{m}_0/\overline{A} < \overline{R}_d^p$ because it is conceivable for $\overline{R}_b > \overline{R}_d^p$. And since $\overline{R}_d^p > \overline{R}_m$, in the latter case we must have $\overline{R}_b > \overline{R}_m$. Thus, $\overline{R}_b > \overline{R}_m$ is a necessary condition for a public and private allocation to fail to be supportable as a DCR equilibrium.

□

Discussion: Solving equation (45) for $\overline{A}$ produces

$$\overline{A} = \frac{\overline{m}_0 + R\,\overline{d} - \overline{d}^p \overline{R}_d^p - \overline{d}^r \overline{R}_d^r}{R}.$$

Thus, the unsupportability condition $\overline{m}_0/\overline{A} \geq \overline{R}_d^p$ can be rewritten

$$\frac{R\,\overline{m}_0}{\overline{m}_0 + R\,\overline{d} - \overline{d}^p \overline{R}_d^p - \overline{d}^r \overline{R}_d^r} \geq \overline{R}_d^p.$$

Proof of Corollary 1:

Example 2 *Let* $R = 1.02$, $d^p(R_d^p) = 1$, $d^r(R_d^r) = 10 - 4/R_d^r$, $M(0) = 1$ *and* $\overline{G} = 0.07010$*; let the coefficients of the social welfare function be* $a_0 = 0.0142126$, $a_1 = 0.845902$, *and* $a_2 = 0.139886$. *The optimal public and private allocation is* $\overline{m}_0 \doteq 3.13985$, $\overline{R}_d^r = 1.005$, $\overline{R}_d^p = 0.976$. *The multiple reserves policy setting* $(\overline{\theta}_m, \overline{\theta}_b) = (0.2, 0.5)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.8, 0.99)$ *supports this allocation. Notice that* $\overline{R}_b > \overline{R}_m$. *Across alternative multiple reserves policy settings that support this allocation, the lowest gross nominal bond rate is* $R_b^{nom} = 1.00322$, *which occurs when* $R_b = 0.97914$ *and* $R_m = \overline{R}_d^p = 0.976$.

Since $\widehat{R}_m = \overline{m}_0/\overline{A} \doteq 0.978162 > \overline{R}_d^p$, *we know from Proposition 2 that this allocation cannot be supported as a dual currency reserve equilibrium.* □

Example 3 *Let* $R = 1.02$, $d^p(R_d^p) = 1$, $d^r(R_d^r) = 10 - 4/R_d^r$, $M(0) = 1$ *and* $\overline{G} = 0.07010$, *just as in Example 1. However, the coefficients of the social welfare function are* $a_0 = 0.0133428$, $a_1 = 0.84709$, *and* $a_2 = 0.139567$. *The optimal public and private allocation is* $\overline{m}_0 \doteq 2.54103$, $\overline{R}_d^r = 1.00701$, $\overline{R}_d^p = 0.976$. *The multiple reserves policy setting* $(\overline{\theta}_m, \overline{\theta}_b) = (0.2, 0.4)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.8, 0.987516)$ *supports this allocation. Again, notice that* $\overline{R}_b > \overline{R}_m$.

In this case, however, $\widehat{R}_m \doteq 0.973153 < \overline{R}_d^p$, *so Proposition 2 guarantees that this allocation can be supported as a dual currency reserve equilibrium. The supporting DCR policy setting also features* $(\widehat{\theta}_m, \widehat{\theta}_b) \doteq (0.939237, 0.277363)$.

Proof of Proposition 3: Let $(\overline{G}, \overline{R}_d^p, \overline{R}_d^r, \overline{m}_0)$ represent a public and private allocation in a Class B economy, and let $(\overline{R}_m, \overline{R}_b, \overline{\theta}_m, \overline{\theta}_b)$ be a policy setting that supports this allocation as a multiple reserves equilibrium. We wish to identify conditions under which there is a policy setting $(\widehat{R}_m, \widehat{R}_b, \widehat{\theta})$ that supports $(\overline{G}, \overline{R}_d^p, \overline{R}_d^r, \overline{m}_0)$ as a dual reservable liability (DRL) equilibrium.

Equation (21) establishes that in a supporting DRL equilibrium, $\widehat{A} = \overline{A}$. Since $\widehat{A} = \widehat{\theta}\,\overline{d}$, we must have

$$\widehat{\theta} = \frac{\overline{A}}{\overline{d}} = \frac{\overline{\theta}_m\,\overline{d}^p + \overline{\theta}_b\,\overline{d}^r}{\overline{d}^p + \overline{d}^r} \tag{.48}$$

We know $0 < \overline{\theta}_m \leq 1$ and $0 < \overline{\theta}_b \leq 1$; it follows that $0 < \overline{A} \leq \overline{d}$ and thus that $0 < \widehat{\theta} \leq 1$.

A supporting equilibrium also requires $\widehat{R}_d^p = \overline{R}_d^p$ and $\widehat{R}_d^r = \overline{R}_d^r$. Equations (34), (35) and (48) then imply

$$\widehat{R}_m = \frac{\overline{R}_d^p - (1-\widehat{\theta})\,R}{\widehat{\theta}} = R - (R - \overline{R}_d^p)\frac{\overline{d}}{\overline{A}} \qquad (.49)$$

and

$$\widehat{R}_b = \frac{\overline{R}_d^r - (1-\widehat{\theta})\,R}{\widehat{\theta}} = R - (R - \overline{R}_d^r)\frac{\overline{d}}{\overline{A}} \qquad (.50)$$

Together with equation (13), these equations imply $\widehat{R}_m < \widehat{R}_b < R$. However, we must also have $\widehat{R}_m \geq 0$, which is equivalent, given equation (49), to $(R - \overline{R}_d^p)\,\overline{d} < R\,\overline{A}$ or

$$R\left(1 - \frac{\overline{A}}{\overline{d}}\right) < \overline{R}_d^p \qquad (.51)$$

Thus, the latter condition is necessary for the existence of a supporting DRL equilibrium.

When condition (51) holds, our choices of $\widehat{\theta}$, $\widehat{R}_m$ and $\widehat{R}_b$ (recall that $\widehat{R}_b \geq \widehat{R}_m$) are entirely legitimate, and we have $\widehat{R}_d^p = \overline{R}_d^p$, $\widehat{R}_d^r = \overline{R}_d^r$, $\widehat{d}^p = \overline{d}^p$ and $\widehat{d}^r = \overline{d}^r$. Equations (16) and (17) give us $R - \overline{R}_d^p = \overline{\theta}_m(R - \overline{R}_m)$ and $R - \overline{R}_d^r = \overline{\theta}_b(R - \overline{R}_b)$. These equations, along with equations (48), (49) and (50), can be used to verify that

$$\widehat{m}_0 = \widehat{\theta}\left(\widehat{R}_m\,\widehat{d}^p + \widehat{R}_b\,\widehat{d}^r\right) = \overline{R}_m\overline{\theta}_m\,\overline{d}^p + \overline{R}_b\overline{\theta}_b\,\overline{d}^r = \overline{m}_0\,.$$

Lemma 2 then implies $\widehat{G} = \overline{G}$. Thus, inequality (51) is sufficient for the existence of a supporting DRL equilibrium.

Finally, equation (16) and the definition of $\overline{A}$ imply that the condition $(R - \overline{R}_d^p)\,\overline{d} < R\,\overline{A}$ is equivalent to

$$R\,(\overline{\theta}_m - \overline{\theta}_b)\,\overline{d}^r < \overline{\theta}_m\overline{R}_m\,\overline{d}\,.$$

It follows that $\overline{\theta}_b \geq \overline{\theta}_m$ is sufficient for the existence of a supporting DRL equilibrium. □

Discussion: Equation (45) gives us

$$\overline{A} = \frac{\overline{m}_0 + R\,\overline{d} - \overline{d}^p \overline{R}_d^p - \overline{d}^r \overline{R}_d^r}{R}$$

and thus

$$\frac{\overline{A}}{\overline{d}} = 1 + \frac{\overline{m}_0}{R\,\overline{d}} - \frac{\overline{d}^p \overline{R}_d^p + \overline{d}^r \overline{R}_d^r}{R\,\overline{d}}\,.$$

The unsupportability condition (38) can consequently be rewritten

$$\frac{\overline{d}^p \overline{R}_d^p + \overline{d}^r \overline{R}_d^r}{\overline{d}} - \frac{\overline{m}_0}{\overline{d}} \geq \overline{R}_d^p\,.$$

Alternatively, we can solve for $\overline{m}_0$:

$$\overline{m}_0 \leq \overline{d}^p \overline{R}_d^p + \overline{d}^r \overline{R}_d^r - \overline{d}\,\overline{R}_d^p$$

which is

$$\overline{m}_0 \leq \left(\overline{R}_d^r - \overline{R}_d^p\right) \overline{d}^r\,.$$

Proof of Corollary 2:

Example 4 *Let* $R = 1.02$*,* $d^p(R^p_d) = 1$*,* $d^r(R^r_d) = 10$*,* $M(0) = 1$ *and* $\overline{G} = 0.34$*; let the coefficients of the social welfare function be* $a_0 = 0.0117061$*,* $a_1 = 0.907983$*, and* $a_2 = 0.0803112$*. The optimal public and private allocation is* $\overline{m}_0 \doteq 2.06$*,* $\overline{R}^r_d = 1.006$*,* $\overline{R}^p_d = 0.772$*. The multiple reserves policy setting* $(\overline{\theta}_m, \overline{\theta}_b) = (0.4, 0.2)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.4, 0.95)$ *supports this allocation. Notice that* $\overline{\theta}_m > \overline{\theta}_b$ *and* $\overline{R}_b > \overline{R}_m$*. Across alternative multiple reserves policy settings that support this allocation, the lowest gross nominal bond rate is* $R^{nom}_b = 1.19171$*, which occurs when* $R_b = 0.92$ *and* $R_m = \overline{R}^p_d = 0.772$*. The minimum difference between* θ_m *and* θ_b *occurs when* $R_m = 0$*, in which case* $\theta_m = 0.243137$ *and* $\theta_b = 0.215686$*.*

Since $R\,(1 - \overline{A}/\overline{d}) = 0.797455 > \overline{R}^p_d$*, we know from Proposition 3 that this allocation cannot be supported as a dual reservable liability (DRL) equilibrium. A supporting equilibrium would require* $\widehat{\theta}_m = \overline{A}/\overline{d} \doteq 0.218182$*, and this produces* $\widehat{R}_m = -0.1167$*.* □

Proof of Proposition 4: Let $(\overline{G}, \overline{R}^p_d, \overline{R}^r_d, \overline{m}_0)$ represent a public and private allocation in a Class B economy, and let $(\overline{R}_m, \overline{R}_b, \overline{\theta}_m, \overline{\theta}_b)$ be a policy setting that supports this allocation as a multiple reserves equilibrium. We wish to identify conditions under which there is a policy setting $(\widehat{R}_m, \widehat{\theta}_m, \widehat{\theta}_b)$ that supports $(\overline{G}, \overline{R}^p_d, \overline{R}^r_d, \overline{m}_0)$ as a single reserve/deposit tax (SR/DT) equilibrium.

In a supporting equilibrium, we must have $\widehat{R}^r_d = \overline{R}^r_d$. By equation (36), this is

$$\overline{R}^r_d = (1 - \widehat{\theta}_b)R\,,$$

which produces

$$\widehat{\theta}_b = \frac{R - \overline{R}^r_d}{R}\,. \tag{.52}$$

Inequality (11) guarantees $0 \leq \widehat{\theta}_b \leq 1$.

A supporting equilibrium must produce $\widehat{R}^p_d = \overline{R}^p_d$. It follows, using equation (35), that

$$\widehat{\theta}_m = \frac{R - \overline{R}^p_d}{R - \widehat{R}_m}\,. \tag{.53}$$

Given equation (12), we will have $0 \leq \widehat{\theta}_m \leq 1$ if and only if $\widehat{R}_m \leq \overline{R}^p_d$.

A supporting equilibrium must also produce $\widehat{m}_0 = \overline{m}_0$. Equation (21) implies $\widehat{m}_0 = \widehat{\theta}_m \widehat{R}_m \widehat{d}^p$. Since $\widehat{R}^p_d = \overline{R}^p_d$ implies $\widehat{d}^p = \overline{d}^p$, we must have

$$\widehat{\theta}_m \widehat{R}_m = \frac{\overline{m}_0}{\overline{d}^p}. \tag{.54}$$

Equation (35) and the requirement $\widehat{\theta}_m < 1$ imply any supporting SR/DT equilibrium must satisfy $\widehat{\theta}_m \widehat{R}_m < \overline{R}^p_d$. Thus,

$$\frac{\overline{m}_0}{\overline{d}^p} < \overline{R}^p_d \tag{.55}$$

is a necessary condition for the existence of such an equilibrium.

Solving equations (53) and (54) for $\widehat{R}_m$ yields

$$\widehat{R}_m = \frac{R\,\overline{m}_0}{\overline{m}_0 + (R - \overline{R}^p_d)\,\overline{d}^p}. \tag{.56}$$

Condition (11) and the nonnegativity requirement on m_0 imply $\widehat{R}_m \geq 0$. Since a supporting SR/DT equilibrium must also satisfy $\widehat{R}_m < \overline{R}^p_d$, the condition

$$\frac{R\,\overline{m}_0}{\overline{m}_0 + (R - \overline{R}^p_d)\,\overline{d}^p} < \overline{R}^p_d$$

is necessary for the existence of such an equilibrium. A bit of algebra reveals that this condition is equivalent to condition (55).

We have established that when a public and private allocation satisfies condition (55), the values of $\widehat{R}_m$, $\widehat{\theta}_m$, and $\widehat{\theta}_b$ derived above are all entirely legitimate. We have also shown that these values produce $\widehat{R}^p_d = \overline{R}^p_d$, $\widehat{R}^r_d = \overline{R}^r_d$ and $\widehat{m}_0 = \overline{m}_0$. Lemma 2 then implies $\widehat{G} = \overline{G}$. Thus, condition (55) is sufficient for the existence of a supporting SR/DT equilibrium. □

Proof of Corollary 3:

Example 5 *Let* $R = 1.02$, $d^p(R_d^p) = 1$, $d^r(R_d^r) = 5$, *and* $\overline{G} = 0.125$*; let the coefficients of the social welfare function be* $a_0 = 0.0123033$, $a_1 = 0.829757$, *and* $a_2 = 0.15794$*. The optimal public and private allocation is* $\overline{m}_0 \doteq 1.525$, $\overline{R}_d^r = 1.01$, $\overline{R}_d^p = 0.912$*. The multiple reserves policy setting* $(\overline{\theta}_m, \overline{\theta}_b) = (0.4, 0.25)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.75, 0.98)$ *supports this allocation. Note that* $\overline{R}_b > \overline{R}_m$*. Across alternative multiple reserves policy settings that support this allocation, the lowest gross nominal bond rate is* $R_b^{nom} = 1.03408$*, which occurs when* $R_b = 0.943077$ *and* $R_m = \overline{R}_d^p = 0.912$*.*

Since $\overline{m}_0/\overline{d}^p = 1.525 > \overline{R}_d^p$*, we know from Proposition 4 that this allocation cannot be supported as a single reserve/deposit tax (SR/DT) equilibrium. A supporting SR/DT equilibrium would require* $\widehat{R}_m = 0.952541 > \overline{R}_d^p$*.* □

Example 6 *Let* $R = 1.02$, $d^p(R_d^p) = 1$, $d^r(R_d^r) = 5$, *and* $\overline{G} = 0.125$*; let the coefficients of the social welfare function be* $a_0 = 0.0123033$, $a_1 = 0.823217$, *and* $a_2 = 0.16448$*. The optimal public and private allocation is* $\overline{m}_0 \doteq 1.525$, $\overline{R}_d^r = 0.994$, $\overline{R}_d^p = 0.992$*. The multiple reserves policy setting* $(\overline{\theta}_m, \overline{\theta}_b) = (0.4, 0.25)$ *and* $(\overline{R}_m, \overline{R}_b) = (0.95, 0.916)$ *supports this allocation. Note that* $\overline{R}_b < \overline{R}_m$*.*

Since $\overline{m}_0/\overline{d}^p = 1.525 > \overline{R}_d^p$*, we know from Proposition 4 that this allocation cannot be supported as a single reserve/deposit tax equilibrium. A supporting SR/DT equilibrium would require* $\widehat{R}_m = 1.00161 > \overline{R}_d^p$*.*

Proof of Proposition 5: We have already seen that the allocation from Example 4 cannot be supported as a dual reservable liability equilibrium. Proposition 2 establishes that a supporting dual currency reserve equilibrium would require $\widehat{R}_m = \overline{m}_0/\overline{A} \leq \overline{R}_d^p$; since $\overline{m}_0/\overline{A} = 0.858333$ and $\overline{R}_d^p = 0.772$, the allocation cannot be supported as an DCR equilibrium. Finally, since $\overline{m}_0/\overline{d}^p = 2.06 > \overline{R}_d^p$, Proposition 4 establishes that the allocation cannot be supported as a single reserve/deposit tax equilibrium. A supporting SR/DT equilibrium would require $\widehat{R}_m < \overline{R}_d^p$, but $\widehat{R}_m = R\,\overline{m}_0/\left(\overline{m}_0 + (R - \overline{R}_d^p)\,\overline{d}^p\right) = 0.910399$. □