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Abstract: What happens when liquidity increases in credit markets and more funds are channeled from borrowers to lenders? We examine this question in a general equilibrium model where financial matchmakers help borrowers (firms) and lenders (households) search out and negotiate profitable matches and where the composition of heterogeneous borrowers adjusts to satisfy equilibrium entry conditions. We find that enhanced liquidity causes entry by all borrowers and tends to benefit low-quality borrowers disproportionately. However, liquid credit markets may or may not be associated with higher output and welfare. The result is determined by whether the effect of higher market participation outweighs that of lower average quality. The net effect depends crucially on the source of the liquidity shock (financial matching efficacy, productivity, or entry barriers).

JEL classification: C78, E44, G21

Key words: financial matchmaking, search and entry frictions, credit composition effects

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Please address questions regarding content to Zsolt Becsi, Research Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, N.W., Atlanta, Georgia 30303-2713, 404-521-8785, 404-521-8058 (fax), zsolt.becsi@atl.frb.org; Victor Li, Department of Economics, 001 Fisher Hall, Princeton University, Princeton, New Jersey 08544-1021, 609-720-0330, 609-258-6419 (fax), victor.li@princeton.edu; or Ping Wang, Department of Economics, Vanderbilt University, 214 Calhoun Hall, Nashville, Tennessee 37235, ping.wang@vanderbilt.edu.

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1. Introduction

To paraphrase Fama (1980), financial intermediation is neutral without market frictions. As the subsequent literature has shown, financial intermediation enhances the aggregate efficiency of investment by counteracting informational frictions and overcoming technological non-convexities. However, the literature has neglected the fact that financial intermediaries also help alleviate ***search and entry frictions***. Search frictions arise because seeking out trades and bargaining over contracts takes time and this gives a matchmaking role to financial intermediaries and a mediating role in contract negotiations. Entry frictions arise because market participation for firms is costly and this in turn implies that the pool of credit applicants (or the demand for credit) responds to liquidity in financial markets. Both frictions together mean that financial intermediation may enhance the aggregate efficiency of investment by influencing the quality and size of the credit applicant pool and the frequency with which credit contracts are made.

To explore the role of search and entry frictions in credit markets, we restrict our attention to studying the ***matchmaking function*** of financial intermediaries, developing a model of pairwise meetings where finding a match takes time.[1] For illustrative purposes, let us refer to lenders as households and to borrowers as firms. Households would like to lend a part of their endowment to firms who need to finance their productive activity. Financial matchmakers own a matching technology that facilitates transactions between lenders and borrowers by stochastically bringing them together.[2] Thus, our framework generalizes the "credit search" model of Diamond (1990) and our financial matchmakers

[1]The matching framework is based on ideas developed by Burdett and Mortensen (1981), Pissarides (1984), and Laing, Palivos and Wang (1995).

[2]Our financial intermediaries simply match lenders and borrowers but do not independently demand or supply funds. The latter activity typifies "marketmakers" (cf. Yavas 1992).

are similar to the "middlemen" defined in Rubinstein and Wolinsky (1987), Biglaiser(1993) or Yavas (1994). Financial matchmaking here captures the main features of the "money store"in ancient China and in several developing countries, and the services provided are also similar to those provided by mortgage brokers in modern economies.[3] In particular, financial matchmakers are endowed with a more effective random matching technology than the random matches found in nature, yet their matching technology is imperfect and subject to frictions.

Once a borrower and a lender are brought together, financial matchmakers mediate a loan contract. The contract specifies the loan rate paid by firms and a return to households, and divides the net surplus of the match according to the agents' bargaining power. While firms differ according to their productivity, riskiness, and cost structure, these attributes are common knowledge and hence there does not exist an adverse-selection or a moral hazard problem. Moreover, in order to highlight the role of search and entry frictions in financial matchmaking, we abstract from the risk pooling function of financial intermediaries who offer households a safe return. Equilibrium in this model is reflected by the endogenous matching rates of firms and households that determine market liquidity and the amount of household funds channeled to firms. Equilibrium is also reflected by the number of active and inactive credit relationships whereby credit market tightness is measured by the share of unmatched projects from the credit applicant pool which can be interpreted as a frictional capital unemployment rate.

In his pivotal work, Townsend (1978) compares bilateral and centralized trading and shows that there exists an efficiency-enhancing role for middlemen who actively participate in trades. Yet, the matching process by which trading agents are paired is not modeled, whereas in our paper the process is explicit and serves to overcome search time frictions. There do exist a few models of search in credit

[3]Our matchmakers can therefore be regarded as primitive forms of banks. See Chou (1970) for a discussion of the money store in ancient China, such as kung-hsieh-ben-hong and chien-chuang which were created to balance funds demand and supplies by pairing borrowers and lenders. The rotating saving and credit association considered in Besley, Coate and Loury (1993) is similar to the money store but without pairing.

markets that give a potential role for middlemen such as Diamond (1990) and Den Haan, Ramey, and Watson (1999). Like in our model, financial intermediaries are assumed to play a passive role by enabling more effective matching between borrowers and lenders. However, their papers assume exogenous rates of matching, while ours determines matching probabilities in general equilibrium.[4]

Our paper is also related to the middleman literature which focuses on the emergence and equilibrium pattern of intermediated trade. Rubinstein and Wolinsky (1987) consider random matching between demanders and suppliers or between middlemen and demanders/suppliers under complete information, where middlemen emerge and capture a percentage of the matching surplus. Biglaiser (1993) constructs a bargaining model with asymmetric information about quality and correlated valuations where middlemen become experts in their fields and reduce adverse selection related inefficiencies. Yavas (1994) assumes that matching between demanders/suppliers and middlemen is immediate, whereas with private information and endogenous search intensity middlemen arise when search is too costly and random matching is very ineffective. While the financial matchmakers in our paper are not as active as the middlemen in the above studies, we do not rule out a more active role for our matchmakers that allows them to extract a percentage of the matching surplus (see the discussion in the concluding section). However, we keep our financial activity as simple as possible to focus on how search and entry frictions influence credit market liquidity and the aggregate efficiency of investment.

We emphasize that it is ***not*** the purpose of this paper to establish why financial intermediaries or matchmakers are formed. Also, the model is highly stylized to highlight the main structure of our analysis and the underlying intuition. Notably, there is no closely related study of financial matchmaking or credit search where the equilibrium composition of heterogeneous firms affects the efficiency of investment. However, the conventional financial intermediation literature does suggest that composition

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[4]Also, in a more remotely related study, Huffman (1996) considers search in capital markets with exogenous matching rates.

effects are important for aggregate activity. Many of the traditional arguments of how banks affect the composition of the aggregate investment portfolio boil down to the idea that in one way or another financial intermediaries give agents access to financial economies of scale that they would not have otherwise.[5] By contrast, in our paper financial matchmakers bring households and heterogeneous firms together and mediate contracts that share surpluses between two matching parties depending on relative bargaining positions. At the same time, there is differential entry across different types in response to perceived profit opportunities. Thus, by allowing sufficient flexibility in loan contracts to accommodate changing perceptions, financial matchmakers help allocate funds to investment projects by alleviating search frictions, which is distinct from the existing literature.

We prove the existence and the uniqueness of a nondegenerate steady-state search equilibrium with endogenous entry. Then we analyze the effects of shocks to financial structure and firms' profitability and show that the comparative statics depend crucially on the response of endogenous matching rates and differential entry. Generally, any shock that enhances matching rates causes aggregate liquidity to rise. While an increase in liquidity increases market participation by all firms, we show that low quality firms benefit disproportionately and the average quality of firms falls (unless the shock also raises the relative profitability of low quality firms). Thus, liquid credit markets may or may not be associated with high output and welfare, depending on whether the ***market participation effect*** outweighs the ***composition effect*** on average quality. Welfare may fall when the composition effect is large, so that it is possible to have outcomes in a complete information framework that resemble results found in models with asymmetric information (Biglaiser and Friedman, 1999).

Our analysis demonstrates that the balance of market participation and composition effects depends on the source of increased market liquidity. Specifically, profitability shocks (or shocks to

[5]Specific ways by which financial intermediaries affect the composition of the aggregate investment portfolio include risk pooling, liquidity management and effective monitoring. See Becsi and Wang (1997) among many other surveys of the voluminous financial intermediation literature.

productivity and entry costs) that enhance aggregate liquidity usually have strong market participation effects. By contrast, financial market structure shocks such as enhanced matching efficacy increase market liquidity and participation, but because of a strong composition effect output and welfare may rise or fall. When the financial market structure shock is due to lower contract quit rates, the outcome is enhanced liquidity and strong market participation effects. Thus, because lower contract quit rates tend to lengthen contractual relationships (that are not guaranteed with random contract renegotiation), long-term relationships are positively related to output and loan rates consistent with evidence by Petersen and Rajan (1994). Finally, we also show that ex ante loan rate spreads depend only on differential profitability shocks, but that realized credit spreads also depend on offsetting market participation and composition effects.

2. The Basic Environment

Time is continuous. The economy is populated with a continuum of identical households of unit mass and a continuum of firms of mass I. There are two types of firms indexed by i that are distinguished by their riskiness and productivity and by their mass I^i. Identical financial matchmakers own a matching technology that they use to bring together households and firms, a role that we will refer to as their ***matchmaking function***. While a firm's type is known to all, the number of firms of various types is endogenously determined by unrestricted entry with differential costs. As financial matchmakers devise loan contracts, they influence the equilibrium allocation of investment projects, referred to as their ***allocation function***. Thus, our framework highlights the importance of search and entry frictions in providing a primitive role for financial intermediaries.[6]

Each household is endowed with an apple tree that generates flow income normalized to one

[6]These functions are assigned to the matchmaker without loss of generality. For example, the matchmaking role can be derived from optimizing behavior where intermediaries maximize the measure of matched agents given that each agent pays a fee to use the matching technology.

unit. What they do with their income depends on the financial environment. Since there is a large number of borrowers and lenders, the probability of rematches in a random environment is zero. Thus, in the absence of financial matchmaking, no lender would grant the validity of IOUs from borrowers. Further assume that matches between borrowers and lenders are negligible. Under these circumstances, each household consumes a flow value of one. By contrast, such flow income can earn positive returns in the case of a completely financially intermediated economy. Households do not have access to the production technology and hence are seeking opportunities to gain a gross rate of return of $R > 1$ on their savings, depending on the type of firm they are matched with using the financial matchmaker.

Firms have access to a production technology and are productive only when they have secured an investment loan. Without loss of generality, let type 1 firm have access to a low-risk, low-return investment project whereas the type 2 firm has access to a high-risk, high-return project. Membership in the population set I^i is determined by a completely random lottery process. Upon paying an entry (or start-up) cost v_o^i, type i firms enter the loanable funds market and search for loanable funds to implement the investment project. Let N^i denote the (endogenous) fraction of type i firms entering the loanable funds market, which need not be the same as the ex ante population share I^i. For simplicity, we assume $N^1 + N^2 = 1$. When matched with a household, a type i firm produces flow output A^i with probability p^i and zero with probability $(1-p^i)$. Thus, conditional on receiving a loan, the type i firm's value of output net of resource cost is: $Y^i = A^i$.

Specifically, we assume throughout the paper that the high-type pays a higher entry fee, that is, $v_0^2 > v_0^1$. Moreover, we assume that the high-type firms are more productive both in absolute terms and on average, but face a lower success rate:

(A1) (Productivity) $A^2 > A^1$

(A2) (Success Rate) $p^2 < p^1$

(A3) (Expected Productivity) $p^2 A^2 > p^1 A^1 > 1$.

The output of financial matchmakers is measured as the amount of successful matches between borrowers and lenders. The output is produced according to a matching technology (discussed in more detail below), where the inputs are the total funds available for lending and the amount desired for projects. While the matchmaking mechanism is essentially a Diamond-type anonymous random matching machine, one version of our framework allows financial matchmakers to affect the efficacy of the matching technology after paying a real resource cost. Financial matchmakers write contracts that specify the amount of the loan, equal to the unit endowment of the matched household, and an interest rate, R^i, both corresponding to the type i claimed by the individual firm.

The loanable funds market is spatially separated and funds are channeled from lenders to borrowers every time a match is made between households and firms. Individual households or firms may either be matched or unmatched. Let H be the mass of searching and unmatched households and F be the searching firms, where N^iF represents the population of type i firms searching for funds. The mass of matched households is denoted by S and by construction the mass of matched household equals the mass of matched firms, or S = 1-H. Also, we designate μ as the household's flow probability of finding a match with a firm and η as the firm's probability of matching with a household.

Unmatched households consume their flow endowment or, should lending prove profitable (i.e., R >1), they proceed to search for a match in the loanable funds market. Once matched with a firm, they lend their endowment and consume the returns of their savings, R. In order to isolate the funds pooling function of financial intermediation, we assume that risk pooling is absent and thus a matched household earns returns from an investment project from its contracted firm according to terms set by the financial matchmaker. Once a household and firm are matched, there is also a probability of separation which occurs at rate δ. While this separation rate is treated as exogenous, it's potential impact on the endogenous entry and exit decisions of firms of each type will have implications for the degree of "creative destruction."[7] If households are not matched, they consume at the autarky value and wait for a potential match in the future. Notice that it is the anonymous nature of the matching process prevents individual households and firms from setting up enforceable contracts and long-term relationships with each other; hence providing the contracting role for the matchmaker.

We now formalize the flow value associated with searching and matched households of type i=1,2. Denote J_u as the value associated with an unmatched household and J_m^i as the value associated with a household matched with a type i firm. We then have:

$$rJ_u = 1 + \mu N^1(J_m^1 - J_u) + \mu N^2(J_m^2 - J_u) \tag{1}$$

$$rJ_m^i = p^i R^i + \delta(J_u - J_m^i) \tag{2}$$

Equation (1) says that the flow value associated with an unmatched household is the sum of the flow rate of consumption of the endowment good and net values gained from being matched with a type i firm (J_m^i - J_u) which arrives at rate μ, weighted by their respective proportions N^1 and N^2. Equation (2) says that the flow value of a household matched with a type i firm is the sum of the expected returns to the match generated from the loan contract R^i and the net value of separation from that firm and re-entering the pool of unmatched households, which occurs at rate δ.

Similarly for firms, let Π_u^i and Π_m denote, respectively, the unmatched and matched value associated with a firm of type i. These asset values can be specified as:

$$r\Pi_u^i = \eta(\Pi_m^i - \Pi_u^i) \tag{3}$$

[7]This term is used in the Schumpeterian sense of Aghion and Howitt (1992) that a factor forcing households and firms to separate may generate higher individual welfare.

$$r\Pi_m^i = p^i[A^i - R^i] + \delta(\Pi_u^i - \Pi_m^i) \tag{4}$$

Equation (3) gives the flow value of an unmatched firm of each type as the product of the rate by which they contact searching households, η, and the net value of becoming matched (Π_m^i - Π_u^i). Equation (4) specifies the flow value of a matched firm of each type as the sum of the net expected productivity of the investment project made possible by the loan contract, less the interest costs, and the net gain of separation, Π_m^i - Π_u^i, which occurs at rate δ.

Using (1) through (4), we derive the potential (ex ante) gain that accrues from a successful match:

$$J_m^i - J_u = \frac{(r+\delta)(p^i R^i - 1) + \mu N^j [p^i R^i - p^j R^j]}{(r+\delta)(r+\delta+\mu)} \tag{5}$$

$$\Pi_m^i - \Pi_u^i = \frac{p^i[A^i - R^i]}{r+\delta+\eta} \tag{6}$$

Equation (5) indicates that the relative value of a household being matched to a type i firm is increasing in the discounted flow of endowments and in the expected return spread between lending to a type i firm and lending to a type j firm. A positive expected rate of return ($p^i R^i > 1$) is necessary and sufficient to guarantee $J_m > J_u$ and thus ***active intermediation***. Equation (6) shows that the relative value of a matched type i firm is strictly increasing in the discounted endowment flow and the net expected profits from the investment project.

Since firms are atomistic, each of the type i firms takes Π_u^i as parametric in the loanable funds market.[8] Thus, from (4) we have:

[8]We take Π_u^i parametrically because atomistic, competitive firms have no bargaining power over their unmatched value. See Pissarides (1984) and related work cited in Laing, Palivos and Wang (1995).

$$\Pi_m^i - \Pi_u^i = \frac{p^i[A^i - R^i] - r\Pi_u^i}{r+\delta} \tag{7}$$

Thus, the expected gain for each firm that accrues from a successfully mediated match depends positively on the expected output net of the interest payment ($p^i(A^i - R^i)$) and negatively on the flow market value of being unmatched ($r\Pi_u^i$). In other words, the expected gain for each firm accrued from a successful match depends on the (expected) profitability measure $p^i(A^i - R^i) - r\Pi_u^i$.

3. Bargaining, Matching and Entry

Nash Bargaining

Once households and firms meet through a financial matchmaker's "random matching machine" $\tilde{M}$, they bargain in a cooperative manner based on their respective threat points, i.e., unmatched values J_u. Consider a cooperative Nash bargain which gives a share β of the matched surplus to households and 1-β to firms. Bargaining entails solving $\max_{R^i}\ (J_m^i - J_u)^{\beta}(\Pi_m^i - \Pi_u^i)^{1-\beta}$ for i = 1,2. Thus, the bargaining outcome must satisfy:

$$\frac{\Pi_m^i - \Pi_u^i}{1-\beta} = \frac{J_m^i - J_u}{\beta} \tag{8}$$

for i = 1,2. Regarding Π_u^i as parametric, substitution of (5) and (7) into (8) gives:

$$p^i[A^i - R^i] - r\Pi_u^i = \frac{1-\beta}{\beta}\frac{r+\delta}{r+\delta+\mu}\left\{(p^iR^i - 1) + \frac{\mu N^j}{r+\delta}[(p^iR^i - 1) - (p^jR^j - 1)]\right\} \tag{9}$$

This expression clearly illustrates that the outcomes of both types of firms are interdependent because households' threat points depend on the expected returns of both types .

Total differentiation of (9) produces a preliminary characterization of the "interest offer function" in the following:

Proposition 1: *The interest offer function $R^i(\mu, N^i; A^i, \Pi_u^i, \delta)$ is increasing in the household contact rate, μ, the own-type productivity, A^i, and decreasing in the fraction of type 1 firms entered into the loanable funds market, N^1, the own-type unmatched value of firms, Π_u^i, and the separation rate, δ.*

While most of the results are straightforward and intuitive, the two results related to the endogenous arguments in the interest offer function deserve further comments. First, from equation (1), an increase in the fraction of type 1 firms entering into the loanable funds market lowers the unmatched value of households (J_u) which is their bargaining threat point. As a consequence of the reduced bargaining power of households, the interest offer decreases. Second, an increase in the contact rate of households by contrast raises their threat point, thus leading to higher interest offer.

Steady-State Matching

So far, we have only mentioned the random anonymous matching machine M without detailed specification. Recall that the masses of searching households and firms are H and F, respectively. Searching agents enter the loanable funds market and make use of the random matching technology supplied by the matchmaker. Upon a successful match and after the financial contract has been signed, households realize their returns after production is completed (which may be high or low, revealed ex post). Steady-state matching in the loanable funds market requires that the flow of firms of both types seeking loanable funds equals the flow of households providing loanable funds. Thus, given a household contact rate μ and a firm contact rate η, we have:

$$\mu H = \eta F = m_0 \tilde{M}(H,F) \tag{10}$$

where $\tilde{M}(\ ,\ )$ is a funds matching function that satisfies the following properties: strictly increasing and strictly concave in each argument, homogeneity of degree one, standard Inada conditions and boundary conditions [i.e., $\tilde{M}(0,\ ) = \tilde{M}(\ ,0) = 0$]. The anonymous nature of financial matchmakers' random matching technology prevents matches between a "preferred" household with a high type of firm. This thereby rules out the possibility of coalitions among a subset of actors. Moreover, it may be thought of that in the absence of financial matchmaking, the random matching by nature is captured by $\underline{m}\tilde{M}(H,F)$ where $\underline{m} \in [0, m_0)$. For simplicity, in the benchmark economy we assume throughout that $\underline{m} = 0$.

Dividing through by the second argument in the matching function and substituting for *H/F* yields:

$$\eta = m_0 M\left(\frac{\eta}{\mu}\right) \tag{11}$$

This relationship describes η as a negative function of μ which is a "Beveridge curve" (or BC) for the loanable funds market. Under the above-specified properties, we have:

Lemma 1: (Beveridge Curve) *The Beveridge curve, (11), is downward-sloping in (μ,η)-space, convex to the origin, and asymptotes to both axes. It shifts away from the origin as the matching parameter, m_0, increases.*

In addition to the Beveridge curve relationship, the household population in steady state is fixed at unity. This requirement implies that the inflow of households entering the loanable funds market to search for projects (after having been separated from other projects) must equal the outflow from the market, or

$$\delta S = \mu H \tag{12}$$

Due to free entry, the population of firms is determined endogenously.

Free Entry of Firms

As there is unrestricted entry of firms of both types, subject to entry costs, the unmatched values of firms of each type i will be driven down to their respective entry (start-up) costs[9], ν_0^i:

$$\Pi_u^i = \nu_0^i \tag{13}$$

Substituting (6) into (3) and combining the resulting expressions with the last expression yields two zero-profit (ZP) conditions for i = 1,2:

$$\eta^{ZPi} = \frac{r\nu_0^i(r+\delta)}{p^i[A^i - R^i] - r\nu_0^i} \tag{14}$$

Straightforward differentiation implies:

Lemma 2: *The firm contact rate of type i that satisfies the zero profit condition is increasing in the own-type entry cost, ν_o^i, the interest offer, R^i, and the separation rate, δ; it is decreasing in the own-type expected productivity, p^iA^i.*

The underlying intuition is clear-cut once we keep in mind that zero profit requires a negative relationship between the net gains of firms accrued from a successful match and the firm contact rates. As net gains rise, more firms of a type tend to participate in the credit market (to restore zero profit), which lowers the probability that an individual firm will locate a household.

4. Equilibrium

All conditions that must be met in steady-state equilibrium are summarized by:

[9]This, of course, requires that the ex-ante population of each type of firm, I^i, is sufficiently large.

Definition: A *steady-state equilibrium with full information* is a tuple $\{R^i, \mu, \eta, H, S, F, N^1, \Pi_u^i\}$ for i = 1, 2, satisfying:

(i) Nash bargaining: (9) for i = 1, 2;

(ii) steady-state matching and separation: (10), (11) and (12);

(iii) free entry and zero profit: (13) for i = 1, 2 and (14);

(iv) population identity: S + H = 1.

Note that the free entry conditions immediately pin down Π_u^i, whereas (9) (for i=1,2), (11), and (14) do not depend on any population mass except for N^1. Thus, these five equations can be used jointly to solve for, in addition to N^1, two interest rates and two contact rates.

For notational convenience, let $B \equiv [(1-\beta)/\beta](r+\delta)/(r+\delta+\mu)$, $\bar{a}^i \equiv p^i A^i - r v_0^i$, $a^i \equiv \bar{a}^i + B$, $u^i \equiv N^i u$, and $u \equiv \mu/(r+\delta) = u^1 + u^2$ where $dB/d\mu<0$, $du^1/dN^1>0$, $du^2/dN^1<0$ and $du/d\mu>0$. Then we can rewrite the Nash bargaining conditions (9) to give:

$$a^i = (1+B+Bu^j)p^i R^i - Bu^j p^j R^j \qquad i=1,2, \quad i\neq j$$

This system yields the solution

$$p^i R^i = \frac{\bar{a}^i + B}{1+B} + \frac{(1-\beta)\beta\mu}{r+\delta+\beta\mu}\left(\bar{a}^j - \bar{a}^i\right)N^j \tag{15}$$

To ensure sensible results, we impose:

Condition D: (Net production gain and entry cost differentials) $p^2 A^2 - p^1 A^1 > r(v_0^2 - v_0^1) > 0.$

Moreover, to have active intermediation for low type firms, we need $p^1 R^1 > 1$, which is given by

$\bar{a}^1 > 1$, or,[10]

Condition F: (Active Intermediation) $p^1 A^1 - 1 > r\nu_0^1$.

Obviously, given Condition D, Condition F is stronger than Assumption (A3) and it is sufficient but not necessary for active intermediation. Conditions D and F imply $\bar{a}^2 > \bar{a}^1 > 1$ and both are sufficient to guarantee active intermediation for both high and low type firms (i.e., $J_m^i > J_u$).

Comparative Statics for Expected Interest Rates

From (15) we derive the following comparative statics effects on the (expected) return to investing in each firm type i = 1,2:

Proposition 2: *(Comparative Statics for Expected Interest Rates)*

(i) *Productivity:* $\partial p^i R^i / \partial A^i > 0$ *for i,j = 1,2;*

(ii) *Entry Effect:* $\partial p^1 R^1/\partial N^1 = \partial p^2 R^2/\partial N^1 < 0 < \partial p^1 R^1/\partial N^2 = \partial p^2 R^2/\partial N^2$;

(iii) *Matching Rate:* $\partial p^1 R^1/\partial \mu = \partial p^2 R^2/\partial \mu > 0$;

(iv) *Separation:* $\partial p^1 R^1/\partial \delta = \partial p^2 R^2/\partial \delta < 0$;

(v) *Entry Costs:* $\partial p^i R^i/\partial \nu_0^i < 0$ *for i = 1,2.*

Proof: See Appendix. ■

The intuition behind this Proposition can be given as follows. An increase in A^i increases the net gains to a match for both the household and firm. However, it raises the net gains to the firm by a greater proportion and hence an increase in the expected returns to the household is required to satisfy the bargaining rule. An increase in the productivity of type $j \neq i$ firms increases the threat point of households

[10] This is clear by rewriting (15) as: $p^1 R^1 - 1 = \dfrac{\bar{a}^1 - 1}{1 + B} + \dfrac{(1-\beta)\beta\mu}{r+\delta+\beta\mu}(1-N^1)(\bar{a}^2 - \bar{a}^1)$.

matched to a type i firm and hence raises the expected interest payment required from the match.

A change in the proportion of types of firms in the market, N^i, impacts both the threat point of households and firms. An increase in N^1 lowers the threat point of a matched low type firm and tends to increase the expected returns to the household implied by the bargaining solution. However, it also lowers the threat point of households matched to low type firms by lowering their unmatched value J_u. Because this latter effect dominates, $\partial R^1/\partial N^1 < 0$. An increase in N^1 also lowers J_m but increases the threat point of high type firms, both effects leading to $\partial R^2/\partial N^1 < 0$. Similarly, an increase in the fraction of high type firms, N^2, strengthens the relative bargaining position of households matched with both high and low type firms so that $\partial R^1/\partial N^2 > 0$ and $\partial R^2/\partial N^2 > 0$.

An increase in the household matching rate μ lowers the relative value of being matched to either type firm and hence households require an increase in the expected returns to the match. Conversely, an increase in separation rate δ lowers the threat point of households by a greater proportion than firms and decreases the expected returns to households. Finally, entry costs effects are entirely symmetric and opposite to the effects of changes in productivity.

Interest-Rate Spreads

From (5) we can compute the expected interest spread between high and low type firms and the actual interest rate spread as

$$p^2 R^2 - p^1 R^1 = \beta(\bar{a}^2 - \bar{a}^1) \tag{16}$$

$$R^2 - R^1 = \beta\left(\frac{\bar{a}^2}{p^2} - \frac{\bar{a}^1}{p^1}\right) + \frac{1-\beta}{r+\delta+\beta\mu}\left[\beta\mu(N^1\bar{a}^1 + N^2\bar{a}^2) + r + \delta\right]\left(\frac{1}{p^2} - \frac{1}{p^1}\right) \tag{17}$$

where $\lim_{\mu\to\infty, v_o^i\to 0} R^2 - R^1 = \beta(A^2 - A^1) + (1-\beta)(1/p^2 - 1/p^1)(N^1p^1A^1 + N^2p^2A^2 - 1) \gtrless 0$ depending on the productivity as well as risk differential. We can show:

Proposition 3: (Interest Rate Spreads) *Under Assumption (A1) and Condition D, both the expected* ($p^2R^2 - p^1R^1$) *and the actual* ($R^2 - R^1$) *interest rate spreads are positive. The expected interest rate spread is driven by the expected profitability differential* $(\overline{a}^2 - \overline{a}^1)$. *While the actual interest rate spread in a frictionless economy (with* $\mu \to \infty$ *and* $v_0^i \to 0$*) is determined by productivity as well as risk differentials, such a spread is smaller in an economy with search and entry frictions.*

Proof: See Appendix. ■

Condition D says that a positive expected and actual interest spread between high and low types requires that the productivity differential between high and low types be sufficiently large relative to the flow entry cost differential so that $\overline{a}^2 > \overline{a}^1$. Hence, interest paid type 2 always exceeds that of type 1 in both expectations and realization. In general, both the ex ante and ex post interest rate spreads depend positively on the expected net productivity differential. The difference between the two is that the actual or realized interest rate spread (which may be referred to as the credit spread) also depends on the composition of firms (decreasing in the fraction of low quality firms) and the household contact rate (positively). Both the composition and contact-rate effects on the actual interest rate spread diminishes as firm's bargaining power (1-β) decreases. In a frictionless economy where intermediated matching is instantaneous as in Yavas (1994) and firm entry is costless (i.e., $\mu \to \infty$ and $v_0^i \to 0$), productivity and risk differentials pin down the actual interest rate spread. When intermediated matching is not instantaneous as considered by Rubinstein and Wolinsky (1987) and when there are entry frictions, such a spread becomes smaller due to the composition effect caused by heterogeneity in entry costs and success rates.

Steady-State Equilibrium

Given the properties of the interest rate function considered above, we now consider determination of steady-state equilibrium. From Proposition 2, we can write $R^i = R^i(\mu,\eta,N^1)$ where $R^i_\mu > 0$, $R^i_\delta < 0$, $R^i_{N^1} < 0$. Given the interest functions R^i, steady-state $\{\mu^*,\eta^*,N^{1*}\}$ thus satisfy (11) and (14).

Lemma 3: (Equilibrium Zero-Profit Trace) *Both the ZP^1 and ZP^2 loci described by (14) are downward sloping in (N^1,η)-space with $|d\eta^*/dN^{1*}|_{ZP^1} > |d\eta^*/dN^{1*}|_{ZP^2}$. Furthermore, there exists a unique and upward sloping equilibrium zero-profit trace EZ in (N^1,η)-space, $\eta = \eta^Z(N^1)$ that satisfies (14) for a given μ such that there is a $N^1_{min} > 0$ yielding $\eta^Z(N^1_{min}) = 0$ and $\infty > \eta^{max} \equiv \eta^Z(1) > 0$.*

Proof: See Appendix. ■

The ZP^1 and ZP^2 loci and the EZ trace in (N^1,η)-space are shown in Figure 1. How these curves relate to the Beveridge Curve in (μ,η)-space is also shown. These relationships together pin down the steady-state equilibrium $\{\mu^*,\eta^*,N^{1*}\}$.

Satisfying both ZP conditions in (14) would necessarily imply:

$$D \equiv \frac{p^2[A^2 - R^2]}{r v_0^2} - \frac{p^1[A^1 - R^1]}{r v_0^1} = 0$$

The term D measures the expected net surplus differential between high and low type firms, or, in short, the firm surplus differential. The firm surplus differential is less than the profitability differential because high type firms must pay a larger entry cost to get it.[11] When D is positive, the share of low type firms must rise to drive the firm surplus differential back down to equilibrium. To see this, when N^1 rises for a given μ^*, households are more likely to contact low type firms. This weakens the ability of high

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[11]Note that D is less than the net profitability differential $((1-\beta)(\bar{a}^2 - \bar{a}^1)/(r v_0^2))$ because entry costs differ. This can be seen by using $p^2 R^2 - p^1 R^1 = \beta(\bar{a}^2 - \bar{a}^1)$ to rewrite D as

$$D = \frac{1}{r v_0^2}\left\{(1-\beta)(\bar{a}^2 - \bar{a}^1) - \frac{v_0^2 - v_0^1}{v_0^1}\left(p^1(A^1 - R^1) - r v_0^1\right)\right\}.$$

Also, N^1 affects D (through R^1) only when relative entry costs differ. Finally, note that

$$\frac{dD}{d\delta} = \frac{v_0^2 - v_0^1}{r v_0^1 v_0^2}\frac{dp^i R^i}{d\delta} < 0 < \frac{dD}{d\mu} = \frac{v_0^2 - v_0^1}{r v_0^1 v_0^2}\frac{dp^i R^i}{d\mu}.$$

type firms to extract a higher surplus from households and D falls. Thus, any changes resulting in $D > 0$ would require an increase in N^1 to restore zero profit. For example, an increase in μ^* or a decrease in δ will ultimately increase N^1. These arguments are useful for understanding the comparative static results derived in Section 5.

Given $\{\mu^*, \eta^*\}$, we then use (11) and (12) to solve for the equilibrium masses of firms and households:

$$H^* = \frac{\delta}{\delta + \mu^*} \tag{18}$$

$$S^* = 1 - H^* = \frac{\mu^*}{\delta + \mu^*} \tag{19}$$

$$F^* = \frac{\delta \mu^*}{(\delta + \mu^*)\eta^*} \tag{20}$$

These three equations imply that the mass of searching households is negatively related to the household contact rate. Also, the mass of matched household-firm pairs depends positively on the household contact rate. Finally, the mass of searching firms is increasing in the household contact rate but decreasing in the firm contact rate.

The above arguments lead to the following theorem:

Theorem: (Existence and Uniqueness) *Under Assumptions (A1) and (A2) and Conditions D and F, there exists a unique, non-degenerate steady-state equilibrium with full information if the expected production gains are sufficiently high such that* $\eta^* \in (0, \eta^{max})$.

Proof: The existence and uniqueness will be proved in two steps. First, we claim that the BC and EZ loci uniquely determine steady-state $\{\mu^*, \eta^*, N^{1*}\}$. It is clear from the proof of Lemma 3 and expression (14) that as long as the expected production gains are sufficiently high such that $\eta^* \in (0, \eta^{max})$, $N^1{*}$ is bounded in the interval (0,1]. Then as the determinant of the pre-multiplied matrix of system (11) and (14) is strictly positive, the implicit function theorem implies unique determination of steady-state $\{\mu^*,\eta^*,N^{1*}\}$. Thus, for a given pair $\{\mu,\eta\}$ satisfying (BC), there exists a unique pair $\{\eta,N^1\}$ which satisfies (EZ). Once we obtain the equilibrium matching rates μ^*,η^* and fraction of low to high type firms, N^1*, we can use (18)-(20) to solve for the equilibrium masses {H*, S*, F*} and so $F^1* = N^1F*$ and $F^2* = (1-N^1)F*$. Since (18)-(20) are all well-defined monotone functions, the determination of these masses is also unique. ■

5. Comparative Statics

We are now prepared to characterize the steady-state equilibrium. In addition to examining the determinants of the equilibrium matching rates, the composition and the mass of the matched firms, and the gross rates of interest, we are also interested in the percentage of unmatched projects, the aggregate output of the matched firms, and welfare.

Specifically, from (17) we note that S* the equilibrium number of matches is positively related to μ^*/δ. This term reflects the markets' liquidity because it is also equal to the aggregate share of household funds that is channeled to firms. The size of the credit market is measured by S*+F* which adds market participants that are matched to market participants that are unmatched and still searching. Then define U* = F*/(F*+S*) as the share of unmatched projects in the credit applicant pool which includes all entrants. This term can be regarded as the "capital-unemployment rate," which measures the tightness of the credit market much like the unemployment rate in the labor market. Since $F^*=S^*(\delta/\eta^*)$, we find that $U^* = 1/(1+(\eta^*/\delta))$ and thus, our measure of capital-unemployment depends on search and entry frictions solely through the factor δ/η^*.

Next, we propose the following measure for the aggregate flow output, based only on the steady-state masses of matched firms, S^*N^i (i = 1,2):

$$Y^* = S^* [N^{1*}A^1 + (1-N^{1*})A^2] \qquad \textbf{(21)}$$

The aggregate output measure can be decomposed into two components. One component, S*, reflects aggregate matches and enhanced market liquidity (and enhanced market participation). Another component, the square bracket term, reflects the composition of output and can be interpreted as the average output over all matched firms. Because the two components need not always move in the same direction, we assume that in the event of offsetting movements that movements in Y* reflect movements in S*, or:[12]

Condition P: (Production normality) The market participation effect on Y* dominates the composition effect on Y*.

It may be noted that economic welfare in this simple benchmark model is mainly driven by the net productivity differential. Because ex ante profits of competitive firms reach zero in equilibrium, welfare can be measured purely by the ex ante value of households, which is their unmatched value J_u.[13] From equations (1) , (5) and (15) we have:[14]

$$rJ_u = 1 + \frac{\beta\mu}{r+\delta+\beta\mu}\left[(\bar{a}^2 - 1) - N^1(\bar{a}^2 - \bar{a}^1)\right] \qquad \textbf{(22)}$$

Clearly, the welfare measure is increasing in the net productivity of all firms but decreasing in the net productivity differential and the share of low type firms. Furthermore the welfare measure rises with the household contact rate but falls with the separation rate.

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[12] Equivalently, production normality requires that the direct effect of productivity on output dominates.

[13] Notice from equation (1) that this unmatched value takes the expected values of future matches into account and that consumption is instantaneous.

[14] Also, $(r+\delta)\left(J_m^i - J_u\right) = p^i R^i - 1 - \frac{\beta\mu}{r+\delta+\beta\mu}\left[(\bar{a}^2 - 1) - N^1(\bar{a}^2 - \bar{a}^1)\right]$.

Tedious but straightforward comparative-static analysis yields:

Proposition 4: (Financial Structure Shocks) *Under the circumstances described in the Theorem, the effects of matching efficiency (m_0) and separation rate (δ) on steady-state {η^*, μ^*, N^{l*}, R^{i*}, S^*, U^*, Y^*, J_u^*} are given by:*

(i) *An improvement in bank's matching productivity generates more matches and leads to a greater fraction of low-type firms.*

(ii) *An increase in the contract quit rate will raise the firm contact rate and reduce the capital unemployment rate but lower household contact rates and market liquidity. Also, the share of low type firms will fall. Assuming production normality, interest rates, output and welfare fall, otherwise the effect is uncertain.*

Proof: See Appendix. ■

Table 1 summarizes the comparative statics results for the complete information equilibrium.[15] First, we discuss what happens when bank matching efficiency increases. Intuitively, an increase in bank's matching productivity increases the contact rate for households μ^* and encourages firm's entry. The overall level of matchmaking activity increases (as captured by a rise in S*) because of higher household contact rates. From Proposition 3 we know that a rise in the household contact rate raises the interest offer to each firm by an equal amount. This tends to increase the expected net surplus differential (*D*) so that there is a disproportionate entry by low type firms as indicated by an increase in N^1*.

This composition effect puts downward pressure on loan rates and is sufficiently strong that they return to where they originally were. Thus, there is no net effect on loan rates and on the firm matching

[15]While Figure 1 is useful in illustrating the uniqueness of steady state equilibrium, the comparative statics cannot be easily captured graphically since there are significant feedback effects between the matching rates and the fraction of firm types in the market.

rate η^*. Because the firm matching rate is unchanged, the capital unemployment rate is unaffected. Recall from the discussion of Proposition 1 that N^{1*} and μ^* have opposing effects on the unmatched value of households. Because composition effects can be strong, the net effect of market liquidity and composition effects on household's ex ante welfare, J_u^*, is uncertain. Similarly to the uncertain welfare effect, an increase in matching efficiency creates two offsetting forces on output. More matchmaking means higher output because of greater market participation, but this effect is offset by the fact that there are relatively more low type firms in the economy.

Next we ask what happens following an increase in the contract quit rate δ, which could arguably be interpreted as an exogenous bank run or less dramatically as increased resistance towards long-term financial relationships. An increase in δ reduces the unmatched value of all firms relative to their entry cost, lowers their threat points, and induces firm exit. Thus, firm contact rates of surviving firms rise by the zero-profit condition which reinforces the negative direct effect of contract quits on capital unemployment. From the Beveridge curve relationship, household contact rates fall which causes a reduction in market liquidity and a fall in the overall level of matchmaking, S^*. While fewer productive firms means output falls, output could rise if the average productivity of the remaining firms rises. Average productivity is determined by the firm composition effect. As explained previously, low types enter relative to high types when the firm surplus differential (going to high type) firms is excessive. Contract quits have a negative direct effect on loan rates and a negative indirect effect on loan rates because household contact rates are reduced. Because lower loan rates reduce the surplus differential, N^1 falls. The net effect on output and welfare balances a negative effect from reduced market liquidity with a positive composition effect from an increase in the average productivity of (remaining) firms. Under the assumption that the market participation effect dominates (or production normality), output and welfare fall with a rise in contract quits. Interestingly, our results suggest that long-term relationships (i.e., lower contract quit rates) increase aggregate output and raise loan interest rates to all firms (disproportionately more for high quality firms). While the positive effects of long-term relationships on the quantity and price of credit agree with the findings of Petersen and Rajan (1994) using small business data, the prediction about disproportionate changes is empirically testable.

In order to derive sensible comparative dynamic results for entry cost shocks, we assume [16]

Condition Q: (Financial Matching Efficacy) There exists a m_o such that

$$Q \equiv r\frac{r+\delta}{\eta} + r(1-\beta)(1 - \frac{\mu}{r+\delta+\beta\mu}) \text{ » } 0$$

With this assumption we can show

Proposition 5: (Firm Profitability Shocks) *Under the circumstances described in the Theorem, the effects of productivity (A^i) and entry costs (v_0^i) on steady-state $\{\eta^*, \mu^*, N^{1*}, R^{i*}, S^*, U^*, Y^*, J_u^*\}$ are given by:*

(i) *Productivity and entry cost shocks that raise the profitability of high types increase household contact rates, market liquidity, and the share of low type firms, but lower firm contact rates which raises the capital unemployment rate. Productivity and entry cost shocks that raise the profitability of low types will have opposite effects on these variables.*

(ii) *Assuming production normality, productivity shocks raise loan rates, output, and welfare. Cost shocks tend to have the opposite effect (except for output where the outcome is open), assuming production normality and that financial matching efficacy is not too high.*

Proof: See Appendix. ■

To understand the effects of shocks that increase firm profitability $\bar{a}^i$ (due to either an increase in A^i or a reduction in v_o^i), recall that there are two mechanisms at work. First, when $\bar{a}^1$ (or $\bar{a}^2$) rises,

[16] The restriction on financial matching efficacy is met by imposing a sufficient condition on m_o such that $\mu < (r+\delta)/(1-\beta)$.

$\bar{a}^1 - p^1R^1$ (or $\bar{a}^2 - p^2R^2$) rises less (or more) than proportionately because of differences in net expected productivity.[17] Zero-profit thus requires η to rise (or fall) which causes the capital unemployment rate to fall (rise). The Beveridge Curve translates the change of η into a fall (or rise) of μ so that both market liquidity and market participation fall (rise). Following the discussion of Proposition 1, the direct effect of an increase in $\bar{a}^1$ (or $\bar{a}^2$) raises loan rates, but the indirect effect of lower (or higher) μ causes them to fall (or rise). Loan rates rise when $\bar{a}^1$ or $\bar{a}^2$ rises, where production normality guarantees that indirect effects are not too large when the profitability shock benefits high types. Second, N^1 rises whenever shocks cause the firm surplus differential to be excessive. From Propositions 1 and 3, an increase in $\bar{a}^1$ leads to higher profitability for low type firms (A^1-R^1) and higher loan rate for high types (R^2) - the latter results in lower profitability for high type firms. As a consequence, the firm surplus differential decreases, implying a fall in N^1 so as to restore zero profit. By similar arguments, an increase in $\bar{a}^2$ gives rise to a higher N^1. As before, the effect of profitability shocks on output balances market participation effects and average productivity effects, whereby the latter is a sum of individual productivity effects and the composition effect. Productivity shocks tend to enhance average productivity directly, while their indirect effects tend to be offsetting. Thus, production normality guarantees that output will rise. However, since entry cost shocks do not have a direct effect on output and offsetting indirect effects, the normality argument cannot apply and it is unclear whether output will rise or fall. Notice that the effects of $\bar{a}^1$ and $\bar{a}^2$ on J_u* can be signed unambiguously only when production normality is imposed.

[17] A critical relationship for understanding how the zero profit conditions respond to shocks is given by $\bar{a}^i - p^iR^i = \frac{B}{1+B}(\bar{a}^i - 1) - \frac{(1-\beta)\beta\mu}{r+\delta+\beta\mu}N^j(\bar{a}^j - \bar{a}^i)$, where the second term of the righthand side is negative (or positive) for i = 1 (or 2).

Overall, any shock that enhances matching rates causes aggregate liquidity to rise. While an increase in liquidity increases market participation by all firms, low quality firms enter disproportionately and the average quality of firms falls (unless the shock raises the profitability of low quality firms). Thus, liquid markets may or may not be associated with high output and welfare, depending on whether the composition effect on average quality outweighs the effect on market participation. Profitability shocks (to productivity or entry costs) that benefit high type firms will enhance aggregate liquidity but create a negative composition effect. By assuming production normality, positive productivity shocks are generally associated with higher output and welfare. Positive financial market structure shocks will increase market liquidity and market participation, but because of a strong composition effect output and welfare may rise or fall. Specifically, we find enhanced efficiency of the matching technology may lead to lower welfare, but production normality guarantees that falling contract quit rates tend to raise welfare.

6. Summary and Extensions

In the context of a dynamic general equilibrium model, this paper has studied the role of financial matchmakers in overcoming frictions arising from search and entry. Thus, the analysis identifies new channels through which financial intermediaries affect the size and quality of financial flows. Specifically, our analysis of financial matchmaking suggests that shocks that increase financial market liquidity also lead to increased market participation by firms and a composition effect whereby the participation of low quality firms rises disproportionately. However, more liquid markets only increase output and welfare when the market participation effect dominates the composition effect. This generally is the case when shocks enhance the profitability of firms or when they make contracts less fragile (and financial relationships longer lasting). By contrast, structural shocks that make financial matchmaking more efficient may also have large composition effects. We also demonstrate that the extent of search frictions plays an important role in the determination of market loan rates and loan rate spreads.

There are a number of interesting extensions to pursue. For brevity, we only mention two. First, one could allow financial matchmakers to endogenously choose matchmaking effort by maximizing their output net of a real resource cost (in units of the flow rate of matches). Making intermediaries more active in this fashion provides an alternative argument for the existence of financial intermediaries that can be compared with the coalition arguments of Prescott and Boyd (1986) and the middleman arguments of Rubinstein and Wolinsky (1987). Second, one could introduce asymmetric information about the firm's type. This could be accomplished in two different ways that depend on the timing of firms' actions. When firms make their investment project selection (high or low type) *prior* to bank loan approval, the adverse selection problem may exist as in the middleman theory developed by Biglaiser (1993). Alternatively, when firms select projects *ex post*, the moral hazard problem may arise. In either case, financial contracts must be incentive compatible. Also, credit rationing may be present in equilibria with asymmetric information. This additional source of capital unemployment must be added to the frictional capital unemployment considered here.

Figure 1: Steady-State Search Equilibrium with Differential Firm Entry

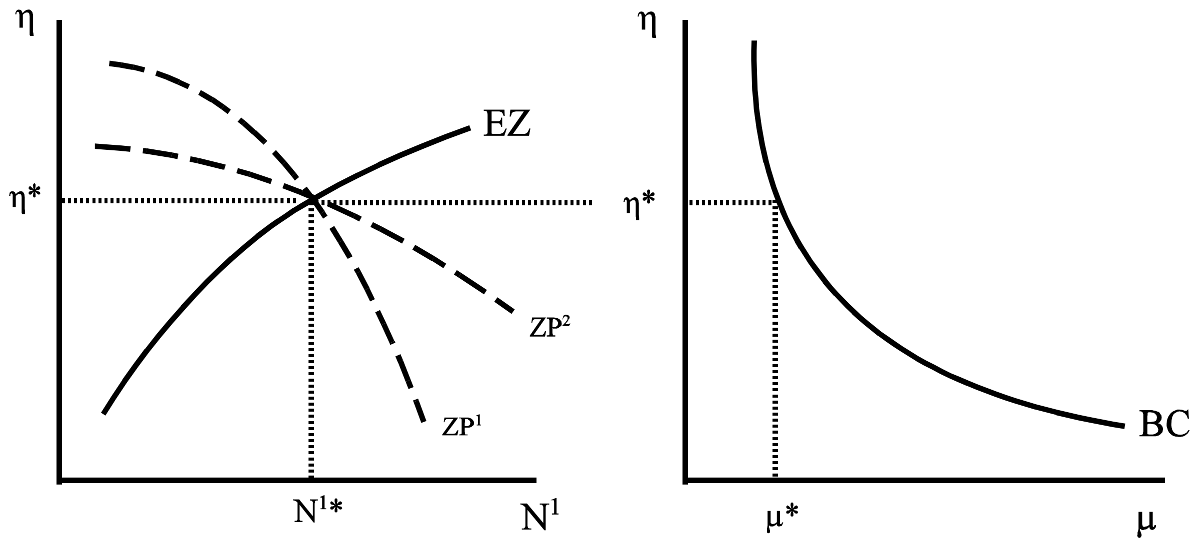

Table 1: Summary of Comparative Statics

Effect of / Effect on	**Financial Market Shocks**		**Firm Profitability Shocks**			
	Bank Matching Productivity m_0	Contract-Quit Rate δ	Type 1 Productivity A^1	Type 2 Productivity A^2	Type 1 Entry Costs[1] ν_0^1	Type 2 Entry Costs[1] ν_0^2
η^*	0	+	+	-	-	+
U^*	0	-	-	+	+	-
μ^*	+	-	-	+	+	-
S^*	+	-	-	+	+	-
N^{1*}	+	-	-	+	+	-
R^{i*}	0	-[2]	+	+[2]	-	-[2]
$R^2 - R^1$	0	-[2]	?	+[2]	?	-[2]
Y^*	?	-[2]	+[2]	+[2]	?	?
J^*_u	?	-[2]	+[2]	+[2]	-[2]	-[2]

Note 1: To sign some of the effects of entry costs we require that m_0 (or μ) is sufficiently small in condition Q.

Note 2: We assume *production normality* or condition P which in essence means the direct effect dominates (and that composition effects are comparatively small).

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Appendix

This Appendix contains proofs of Lemma 3 and Propositions 2-5 in the paper.

Proof of Proposition 2:

(i) From (15) it is immediate that $\partial p^i R^i / \partial A^j > 0$. Differentiating with respect to A^i gives

$$\frac{\partial p^i R^i}{\partial A^i} \propto \frac{1}{1+B}\left\{1 - N^j \frac{(1-\beta)\mu}{r+\delta+\mu}\right\} > 0$$

(ii) Differentiating (15) with respect to N^i gives

$$\frac{\partial (p^i R^i)}{\partial N^i} = -\left[\frac{\beta(1-\beta)\mu}{r+\delta+\beta\mu}\right](\bar{a}^j - \bar{a}^i) = \frac{\partial (p^j R^j)}{\partial N^i}$$

For i = 1 we have $\partial p^1 R^1/\partial N^1 = \partial p^2 R^2/\partial N^1 < 0$ and for i = 2 we have $\partial p^1 R^1/\partial N^2 = \partial p^2 R^2/\partial N^2 > 0$.

(iii) Differentiating (15) with respect to μ gives

$$\frac{\partial (p^i R^i)}{\partial \mu} = \frac{\beta(1-\beta)(r+\delta)}{(r+\delta+\beta\mu)^2}[(1-N^i)(\bar{a}^j - \bar{a}^i) - (1-\bar{a}^i)] = \frac{\partial (p^j R^j)}{\mu} > 0 \ \textit{for all } i,j$$

(iv) Notice that $\partial B/\partial\delta > 0$, and given $\bar{a}^i > 1$, the first term in (15) is strictly decreasing in δ. For i = 1, it is clear that the second term in (15) is also strictly decreasing in δ, implying $\partial p^1 R^1/\partial\delta < 0$. Thus, manipulating (15), we have $p^2 R^2 = p^1 R^1 + \beta(\bar{a}^2 - \bar{a}^1)$, implying $\partial p^2 R^2/\partial\delta = \partial p^1 R^1/\partial\delta$.

(v) Since $\partial p^i R^i/\partial v_0^i \propto - \partial p^i R^i/\partial A^i < 0$, the result is immediate.

Proof of Proposition 3:

From (16), we can derive $R^2 - R^1 = \frac{\beta}{p^2}(\bar{a}^2 - \bar{a}^1) + \frac{p^1 - p^2}{p^1 p^2} p^1 R^1$. Thus both spreads are positive under Condition D that ensures $\bar{a}^2 - \bar{a}^1 > 0$. Utilizing Proposition 2, we can see implies that $R^2 - R^1$ rises with μ and $\bar{a}^2$, falls with N^1 and δ, may rise or fall with $\bar{a}^1$, and is immune to m_0. Moreover, from (17) we obtain:

$$R^2 - R^1 = \lim_{\mu\to\infty, v_o^i \to 0}(R^2 - R^1) - \beta r\left(\frac{v_0^2}{p^2} - \frac{v_0^1}{p^1}\right) - (1-\beta)\left(\frac{1}{p^2} - \frac{1}{p^1}\right)\Theta$$

where $\Theta = \frac{r+\delta}{r+\delta+\beta\mu}(N^1 p^1 A^1 + N^2 p^2 A^2 - 1) + \frac{\beta\mu}{r+\delta+\beta\mu}(N^1 v_0^1 + N^2 v_0^2)$ is a weighted sum of aggregate net outputs and aggregate entry costs, which is positive under (A3) and Condition F. Thus, given (A2), the actual interest rate spread is smaller than that in the absence of search and entry frictions.

Proof of Lemma 3:

From Proposition 3, it is immediate that ZP1 and ZP2 are downward sloping in (η, N^1) space. Differentiating (14) gives

$$\left.\frac{d\eta^*}{dN_1^*}\right|_{zp1} = \frac{d\eta^{zp1}}{dp^1R^1}\frac{dp^1R^1}{dN_1} = -\frac{(\eta^*)^2}{r\nu_0^1(r+\delta)}\frac{\beta(1-\beta)}{r+\delta+\beta\mu}\mu(\overline{a^2}-\overline{a^1}) < 0$$

$$\left.\frac{d\eta^*}{dN_1^*}\right|_{zp2} = \frac{d\eta^{zp2}}{dp^2R^2}\frac{dp^2R^2}{dN_1} = -\frac{(\eta^*)^2}{r\nu_0^2(r+\delta)}\frac{\beta(1-\beta)}{r+\delta+\beta\mu}\mu(\overline{a^2}-\overline{a^1}) < 0$$

since $\nu_0^1 < \nu_0^2$, we have that the pair $\{\eta^*, N^{1*}\}$ satisfying (14) given μ occur where $|d\eta^*/dN^{1*}|_{zp1} > |d\eta^*/dN^{1*}|_{zp2}$. Since both locus' are downward sloping, this pair is unique. To characterize the (EZ) locus, equate ZP1 and ZP2 from (14):

$$\nu_0^2p^1[R^1(N^1,\mu)-1] - \nu_0^1p^2[R^2(N^1,\mu)-1] = \nu_0^2p^1(A^1-1) - \nu_0^1p^2(A^2-1) \qquad \text{(P1)}$$

Notice that from Proposition 3, $\partial p^iR^i/\partial\mu = \partial p^jR^j/\partial\mu > 0$ and $\partial p^iR^i/\partial N^1 = \partial p^jR^j/\partial N^1 < 0$. Consider now that μ increases. Since p^iR^i is higher (for i = 1,2), (14) implies η must be higher. However, this changes the LHS of (P1) away from the RHS: the LHS increases (decreases) iff $\nu_0^1p^2 - \nu_0^2p^1 <$ (>) 0. In either case, N must rise to restore the equality in (P1), implying $d\eta/dN^1|_{EZ} > 0$.

To characterize the limit points of the EZ locus, consider the case where $\mu \to \infty$ which implies $\eta \to 0$ from (11). From (15) $p^iR^i \to [(1-\beta)N^i(\overline{a}^j - \overline{a}^i) + \overline{a}^i] > 0$. The LHS of (P1) can be written as

$$LHS = (\nu_0^1p^2-\nu_0^2p^1)(1-\beta) + \beta\{(1-\beta)(\overline{a^2}-\overline{a^1})[\nu_0^2-N^1(\nu_0^2-\nu_0^1)] + (\nu_0^2\overline{a^1}-\nu_0^1\overline{a^2}) + (\nu_0^1p^2-\nu_0^2p^1)\}$$

Since $RHS = (\nu_0^2\overline{a}^1 - \nu_0^1\overline{a}^2) + (\nu_0^1p^2 - \nu_0^2p^1)$, equating LHS with RHS and solving for N^1 yields

$$N^1 = \frac{\beta(\overline{a^2}-\overline{a^1})\nu_0^2 - (\nu_0^2\overline{a^1}-\nu_0^1\overline{a^2})}{\beta(\overline{a^2}-\overline{a^1})(\nu_0^2-\nu_0^1)}$$

Thus, a condition for $N^1 \in [0,1)$ is given by

$$\beta(\overline{a}^2-\overline{a}^1)\nu_0^1 < (\nu_0^2\overline{a}^1-\nu_0^1\overline{a}^2) \leq \beta(\overline{a}^2-\overline{a}^1)\nu_0^2 \tag{P2}$$

Now consider the limiting case where $\mu \to 0$ which implies $\eta \to \infty$ from (14). From (15) $p^iR^i \to 1 + \beta(\overline{a}^i - 1) > 1$. Thus, there exists an upper bound for η such that $\sup_{N1} \eta(N^1) << \infty$. Furthermore, there exists a finite $\eta^{max} << \infty$ at $N^1 = 1$.

Proof of Propositions 4 and 5:

Totally differentiating (11) and (14) yields

$$C\begin{bmatrix} d\eta \\ d\mu \\ dN^1 \end{bmatrix} = \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix} dm_0 + \begin{bmatrix} 0 \\ r\nu_0^1 + \dfrac{dp^iR^i}{d\delta} \\ r\nu_0^2 + \dfrac{dp^iR^i}{d\delta} \end{bmatrix} d\delta + \begin{bmatrix} 0 \\ -p^1\left(1-\dfrac{dR^1}{dA^1}\right) \\ p^2\dfrac{dR^2}{dA^1} \end{bmatrix} dA^1 + \begin{bmatrix} 0 \\ p^1\dfrac{dR^1}{dA^2} \\ -p^2\left(1-\dfrac{dR^2}{dA^2}\right) \end{bmatrix} dA^2$$

$$+ \begin{bmatrix} 0 \\ \dfrac{r(r+\delta+\eta)}{\eta} + \dfrac{dp^1R^1}{d\nu_0^1} \\ \dfrac{dp^2R^2}{d\nu_0^1} \end{bmatrix} d\nu_0^1 + \begin{bmatrix} 0 \\ \dfrac{dp^1R^1}{d\nu_0^2} \\ \dfrac{r(r+\delta+\eta)}{\eta} + \dfrac{dp^2R^2}{d\nu_0^2} \end{bmatrix} d\nu_0^2$$

where

$$C = \begin{bmatrix} 1-\dfrac{m_0M'}{\mu} & \dfrac{m_0M'\eta}{\mu^2} & 0 \\ \dfrac{r(r+\delta)\nu_0^1}{\eta^2} & -\dfrac{d(p^1R^1)}{d\mu} & -\dfrac{d(p^1R^1)}{dN^1} \\ \dfrac{r(r+\delta)\nu_0^2}{\eta^2} & -\dfrac{d(p^2R^2)}{d\mu} & -\dfrac{d(p^2R^2)}{dN^1} \end{bmatrix} \equiv \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Note that $C_{22} = C_{32} = -\dfrac{dp^iR^i}{d\mu} < 0 < -\dfrac{dp^iR^i}{dN^1} = C_{23} = C_{33}$ and $C_{31} - C_{21} = \dfrac{r(r+\delta)}{\eta^2}(\nu_0^2 - \nu_0^1) > 0$.

Thus, $|C| = -C_{12}(C_{21} - C_{31})C_{33} > 0$. We also compute comparative static effects:

1. Matching Efficiency:

$$\frac{d\eta^*}{dm_0} = \frac{M}{|C|}(C_{22}C_{33} - C_{23}C_{32}) = 0 \quad , \quad \frac{d\mu^*}{dm_0} = \frac{M}{|C|}(C_{31} - C_{21})C_{33} > 0 \quad , \quad \frac{dN^{1*}}{dm_0} = -\frac{M}{|C|}(C_{31} - C_{21})C_{22} > 0$$

$$\frac{dp^iR^i}{dm_0} = \left(\frac{dp^iR^i}{d\mu}\right)\frac{d\mu}{dm_0} + \left(\frac{dp^iR^i}{dN^1}\right)\frac{dN^1}{dm_0} = C_{i2}\frac{d\mu}{dm_0} + C_{i3}\frac{dN^1}{dm_0} = 0$$ after substituting terms from above.

Using this last result and (8) and (9), one sees that $\Pi_m^i - \Pi_u^i$ and $J_m^i - J_u$ also are independent of m_0. Then (2) implies J_m^i and J_u are independent of m_0.

2. Separation Rate:

$$\frac{d\eta^*}{d\delta} = \frac{C_{12}C_{33}}{|C|}r(\nu_0^2 - \nu_0^1) > 0 \quad , \quad \frac{d\mu^*}{d\delta} = -\frac{C_{11}C_{33}}{|C|}r(\nu_0^2 - \nu_0^1) < 0$$

$$\frac{dN^{1*}}{d\delta} = \frac{1}{|C|}\frac{r(\nu_0^2 - \nu_0^1)}{\eta^2}\left((r+\delta)C_{12}\frac{dp^iR^i}{d\delta} - \eta^2 C_{11}\frac{dp^iR^i}{d\mu}\right) < 0$$

3. Productivity:

$$\frac{d\eta^*}{dA^1} = \frac{C_{12}C_{33}}{|C|}\left(p^1(1 - \frac{dR^1}{dA^1}) + p^2\frac{dR^2}{dA^1}\right) > 0 \quad , \quad \frac{d\mu^*}{dA^1} = -\frac{C_{11}C_{33}}{|C|}\left(p^1(1 - \frac{dR^1}{dA^1}) + p^2\frac{dR^2}{dA^1}\right) < 0$$

$$\frac{dN^{1*}}{dA^1} = \frac{C_{11}C_{22}}{|C|}\left(p^1(1 - \frac{dR^1}{dA^1}) + p^2\frac{dR^2}{dA^1}\right) - \frac{C_{12}}{|C|}\left(C_{31}p^1(1 - \frac{dR^1}{dA^1}) + C_{21}p^2\frac{dR^2}{dA^1}\right) < 0$$

Also, the impact of A^2 is inversely related to that of A^1:

$$\frac{d\eta^*}{dA^2} = -\frac{C_{12}C_{33}}{|C|}\left(p^2(1 - \frac{dR^2}{dA^2}) + p^1\frac{dR^1}{dA^2}\right) < 0 \quad , \quad \frac{d\mu^*}{dA^2} = \frac{C_{11}C_{33}}{|C|}\left(p^2(1 - \frac{dR^2}{dA^2}) + p^1\frac{dR^1}{dA^2}\right) > 0$$

$$\frac{dN^{1*}}{dA^2} = -\frac{C_{11}C_{22}}{|C|}\left(p^2(1-\frac{dR^2}{dA^2}) + p^1\frac{dR^1}{dA^2}\right) + \frac{C_{12}}{|C|}\left(C_{21}p^2(1-\frac{dR^2}{dA^2}) + C_{31}p^1\frac{dR^1}{dA^2}\right) > 0$$

4. Entry Costs: The effects are more difficult to sign:

$$\frac{d\eta^*}{d\nu_0^1} = -\frac{C_{12}C_{33}}{|C|}\left(\frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1R^1}{d\nu_0^1} - \frac{dp^2R^2}{d\nu_0^1}\right) \quad , \quad \frac{d\mu^*}{d\nu_0^1} = \frac{C_{11}C_{33}}{|C|}\left(\frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1R^1}{d\nu_0^1} - \frac{dp^2R^2}{d\nu_0^1}\right)$$

$$\frac{dN^{1*}}{d\nu_0^1} = -\frac{C_{11}C_{22}}{|C|}\left(\frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1R^1}{d\nu_0^1} - \frac{dp^2R^2}{d\nu_0^1}\right) + \frac{C_{12}}{|C|}\left(-C_{21}\frac{dp^2R^2}{d\nu_0^1} + C_{31}[\frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1R^1}{d\nu_0^1}]\right)$$

From (15) one obtains $\frac{dp^iR^i}{d\nu_0^i} - \frac{dp^jR^j}{d\nu_0^i} = -r\beta < 0$ and $\frac{dp^jR^j}{d\nu_0^i} = -r\frac{\beta\mu(1-\beta)N^i}{r+\delta+\beta\mu} < 0$ which can be substituted into the above relations. Thus, if a temporary variable is defined by $Q \equiv r\left(\frac{r+\delta}{\eta} + (1-\beta)(1-\frac{\mu}{r+\delta+\beta\mu})\right)$ we can rewrite the above expressions as:

$$\frac{d\eta^*}{d\nu_0^1} = -\frac{C_{12}C_{33}}{|C|}Q \quad , \quad \frac{d\mu^*}{d\nu_0^1} = \frac{C_{11}C_{33}}{|C|}Q \quad , \quad \frac{dN^{1*}}{d\nu_0^1} = \frac{C_{12}C_{31} - C_{11}C_{22}}{|C|}Q + \frac{C_{12}(C_{31}-C_{21})}{|C|}\frac{dp^2R^2}{d\nu_0^1}$$

Thus when $Q > 0$, it follows that $\frac{d\eta^*}{d\nu_0^1} < 0 < \frac{d\mu^*}{d\nu_0^1}$. With a strengthening of this "Q-condition" it is also possible to get $\frac{dN^{1*}}{d\nu_0^1} > 0$. Note that these were the qualitative effects that we had solved for originally.

Using the same approach as above yields:

$$\frac{d\eta^*}{d\nu_0^2} = \frac{C_{12}C_{33}}{|C|}Q \quad , \quad \frac{d\mu^*}{d\nu_0^2} = -\frac{C_{11}C_{33}}{|C|}Q \quad , \quad \frac{dN^{1*}}{d\nu_0^2} = \frac{C_{11}C_{22} - C_{12}C_{21}}{|C|}Q + \frac{C_{12}(C_{31}-C_{21})}{|C|}\frac{dp^1R^1}{d\nu_0^2}$$

The simple Q-condition results in $\frac{d\eta^*}{d\nu_0^2} > 0 > \frac{d\mu^*}{d\nu_0^2}$. Also, because $C_{22} < 0$, $\frac{dN^{1*}}{d\nu_0^2} < 0$ without having to make any further assumptions beyond the simple Q-condition (in contrast to the assumptions needed to get $\frac{dN^{1*}}{d\nu_0^1} > 0$).