Trends in Velocity and Policy Expectations

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Abstract: U.S. velocity of base money exhibits three distinct trends since 1950. After rising steadily for thirty years, it flattens out in the 1980s and falls substantially in the 1990s. This paper explores whether the observed secular movements in velocity can be accounted for exclusively by endogenous responses to changing expectations about monetary and fiscal policy. We use a model with two key features: a substitute for money in transactions and an array of assets that includes money, nominal bonds, and physical capital. The model maps policy expectations into portfolio decisions, making equilibrium velocity a function of expected future money growth, tax rates, and government spending. When expectations are estimated using Bayesian updating, simulated velocity matches the trends in actual velocity surprisingly well.

JEL classification: E50, E63

Key words: velocity, policy expectations, portfolio decisions, transactions services, Bayesian updating

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1. Introduction

Velocity and its determinants lie at the center of long-running debates about the effects of macroeconomic policies.[1] U.S. velocity of base money exhibits three distinct trends since 1950. After rising steadily for 30 years, it flattens out in the 1980s, and falls substantially in the 1990s. Figure 1 depicts these facts.[2] Despite the importance of understanding the determinants of velocity, considerable uncertainty remains about the sources of its observed secular movements. A leading textbook states: "The deep recession that the United States experienced in 1982 is partly attributable to a large, unexpected, and still mostly unexplained decline in velocity" (Mankiw 1997, p. 241).

Several explanations of secular movements in velocity appear in the literature. Many observers in the early 1980s attributed the growth in velocity to financial innovation and technological improvement in the financial sector, as Fisher (1911) predicted. Bordo and Jonung (1987,1990) attribute the long-run behavior of velocity to institutional factors that induce substitutions between different monetary assets.[3] Another perspective comes from empirical

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[1] For example, Friedman (1956,1959), Brunner and Meltzer (1963), Friedman and Meiselman (1963), and Tobin (1961).

[2] In the figure velocity is defined as the ratio of private expenditures (consumption plus investment) to the monetary base deflated by the GDP deflator. The monetary base is adjusted for sweeps. A qualitatively similar pattern emerges when velocity is defined to include government spending or in terms of total GDP. Velocity of M1, after rising steadily until the early 1980s, displays wide swings around a declining trend in the 1980s (see Figure A1 in the appendix).

[3] Increases in velocity are attributed to technological advances in the payments system and the creation of substitutes for money, in both its asset and transaction roles. Decreases in velocity are viewed as due to the spread of commercial banking and improvements in the quality of money.

money demand functions, which have embedded in them a path for velocity implied by estimated income elasticities. The general equilibrium determinants of velocity, however, are rarely emphasized in this branch of work.[4] More closely related to the present paper are the arguments made by Hester (1981), Darby et al. (1987), and Poole (1988) that much of the financial innovation affecting velocity is likely an endogenous response to monetary policy.

This paper explores the extent to which the observed secular movements in velocity can be accounted for exclusively by endogenous responses to changing expectations about monetary and fiscal policy. Velocity in the model considered here is determined by the effect of policy expectations on portfolio choice and by the use of money substitutes to carry out transactions. As a result, both monetary and fiscal policy are potentially important determinants of velocity.

Modern efforts to endogenize velocity in transactions-based models have met with limited empirical success, as Hodrick, Kocherlakota, and Lucas (1991) document.[5] Introducing a real resource that substitutes for money in transactions appears to show some empirical promise (see Bansal and Coleman (1996) and Lacker and Schreft (1996)).[6] To our knowledge, however, these efforts have not attempted to match the time path of velocity.

The idea that monetary policy influences the decision to devote resources to the creation of money substitutes has recently been formalized in a variety of papers.[7] Missing from most of these papers are the asset substitutions emphasized by Gurley and Shaw (1960), Tobin (1961,1980) and Brunner and Meltzer (1972).[8] Incorporating the role of asset substitutability

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[4] For example, Friedman (1959), Meltzer (1963), Goldfeld (1973,1976), McCallum and Goodfriend (1987), and Lucas (1994), to mention a few.

[5] These efforts include Svensson's (1985) modification of the information structure in a cash-in-advance economy and Lucas and Stokey's (1987) distinction between cash and credit goods.

[6] Other methods for modeling money, such as McCallum's (1983) shopping time setup or Marshall's (1992) transactions cost technology, would also endogenize velocity.

[7] A sampling of such papers includes King and Plosser (1984), Prescott (1987), Diaz-Gimenez et al. (1992), Marquis and Reffett (1992), Gillman (1993), Ireland (1994), Aiyagari, Braun, and Eckstein (1996), Bansal and Coleman (1996), and Lacker and Schreft (1996).

[8] Certain aspects of this perspective are reflected in the overlapping generations frameworks employed by Wallace (1981), Bryant and Wallace (1984), and Waldo (1985), among others. In

leads to two distinct influences of policy on velocity: a direct effect from expected money growth and a separate effect from total nominal liability creation. When fiscal financing is posed as a choice among distorting sources of revenues, both the level of government spending and the composition of financing — between real taxation and nominal liability creation — affect agents' portfolio decisions.

We address the secular patterns of velocity in an economic structure with two key features: (1) a substitute for money in transactions; (2) an array of assets that includes money, nominal bonds, and physical capital. The model maps policy expectations into portfolio decisions. The mapping makes equilibrium velocity a function of expected future money growth, tax rates, and government spending. To focus on the role of policy expectations in determining the trend in velocity, we refrain from building in any persistent exogenous disturbances or other sources of trend.

Once expectations of policy are specified, it is possible to simulate time paths of velocity. We ground expectations in realized policy actions and consider three different methods for estimating agents' expectations. The first two use procedures employed in many econometric studies of policy, while the last is a more theoretically motivated exploration of an alternative expectations mechanism. One econometric procedure estimates a Bayesian vector autoregression for policy variables over the entire sample period from 1960:1 to 1997:1. Expectations of policy constructed from this standard technique do not generate any trend in velocity during the 1960-80 period. A second econometric procedure estimates expectations from a Bayesian updating mechanism. In contrast to the first method, updating allows for some adjustment of expectations to changes in the policy process. Bayesian updating of expectations performs surprisingly well, capturing both the secular increase in velocity through the late 1970s and the lack of trend after 1983.

It is interesting to contrast the policy expectations implied by the updating algorithm to those suggested by arguments about how agents adjust to new policy environments. We analyze a simulation that assumes the experience of the 1970s dominated policy expectations entering the

those setups the degree of asset substitutability strongly influences the effects of open market operations and of fiscal financing choices.

1980s. The decade of the 1970s appears to be a policy episode unlike any in the post-World War II period. The simulation assumes agents use the policy rules of the 1970s to form their prior beliefs about policy in the 1980s, and that agents update their expectations as new data arrives. The implied expectations produce a flat path for simulated velocity that lies everywhere above actual velocity from 1981 to 1997. The results are consistent with an inference that expectations shifted in response to the policies initiated by Federal Reserve Chairman Paul Volcker and President Ronald Reagan.

One explanation for the decline in velocity in the 1990s is the sharp increase in flows of U.S. currency overseas. We discuss this explanation and consider its implications for the path of velocity in the 1990s.

Monetary analyses frequently treat velocity as a structural relationship, and it is not uncommon for researchers to draw policy implications based on that treatment. A substantial body of work focuses on the statistical properties of velocity, generally finding velocity follows a random walk (see Nelson and Plosser (1982), Serletis (1995), and references therein). That statistical result has formed the basis for the analysis of optimal policy responses to permanent and transitory "shocks" to velocity (Walsh (1986)).[9] The velocity growth slowdown in the early 1980s has been interpreted as an exogenous event whose macroeconomic effects could have been ameliorated by an expansionary monetary policy response (Mankiw (1997)). A maintained assumption in these analyses is that the time series properties of velocity are invariant to the choice of policy. Our results suggest that assumption is both false and potentially dangerous.

2. The Economic Model

The economy is a standard Ramsey-Cass-Koopmans growth model with a financial transactions sector that provides a costly substitute for the transactions services of money. Private agents have available to them a nominal substitute for money — government bonds — and a real substitute — capital. Those capital goods are interpreted as a composite of physical and

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[9] Walsh studies "base drift," which is the tendency of a policy authority that targets a monetary aggregate to let bygones be bygones and adjust the target growth rates to the realized level of the base. Axilrod (1982) defends this practice by the Federal Reserve, while Poole (1976) and Broaddus and Goodfriend (1984) criticize it.

human capital, with the two types of capital treated symmetrically, as in many endogenous growth model.

Each period the aggregate goods in the economy, $f(k_{t-1})$, are allocated to consumption, c_t, capital, k_t, or government services, g_t. The three types of goods are perfect substitutes in production and there is no cost to converting goods to any of these uses. The resource constraint each period is

$$f(k_{t-1}) \geq c_t + k_t + g_t \tag{1}$$

with $c_t \geq 0$, $k_t \geq 0$, and $g_t \geq 0$. The total supply of goods includes undepreciated capital. We assume the function $f(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable. For analytical tractability, depreciation is introduced as a function of the level of the capital stock via the function $d(k)$ with $d'(k) > 0$ and $d''(k) < 0$. Undepreciated capital is given by

$$d(k_{t-1}) = d \cdot f(k_{t-1}), \quad d \in [0,1], \tag{2}$$

and net investment is defined as

$$x_t = k_t - d(k_{t-1}). \tag{3}$$

The resource constraint, (1), can be rewritten in terms of output net of undepreciated capital, y_t, as

$$c_t + x_t + g_t \leq f(k_{t-1}) - d(k_{t-1}) = y_t. \tag{4}$$

2.1   Firms

Two types of representative firms rent factors of production from households and then sell their output back to households.

Goods producing firms rent k from households at rental rate r and choose k to solve

$$\max \quad D_{Gt} = (1 - t_t) f(k_{t-1}) + t_t d(k_{t-1}) - r_t k_{t-1} \tag{5}$$

taking r and t as given. Taxes are levied on firms' sales of goods, which are gross assets — output — less undepreciated capital.

Firms producing transactions services rent labor, l, from households at wage rate w and sell transactions services, $T(l)$, to households at price P_T. The function $T(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable. Firms choose l to solve

$$\max \; D_{Tt} = P_{Tt} T(l_t) - w_t l_t, \tag{6}$$

taking P_T and w as given.

2.2   Households

Households own the firms and receive factor payments.   They also receive from the government lump-sum transfers whose real value is h_t, so their income is given by

$$I_t = r_t k_{t-1} + D_{Gt} + w_t l_t + D_{Tt} + h_t, \tag{7}$$

where D_G and D_T are dividends received from the goods-producing and transactions-producing firms. Preferences are defined over consumption and leisure. The current period utility function, $U(\cdot)$, is time-separable, strictly increasing in both arguments, strictly concave, and continuously differentiable. Households are endowed with one unit of time each period and choose c, l, T, M, B, and k to solve

$$\max \; E_0 \sum_{t=0}^{\infty} b^t U(c_t, 1 - l_t), \qquad 0 < b < 1 \tag{8}$$

subject to the budget constraint

$$c_t + k_t + \frac{M_t + B_t}{P_t} + P_{Tt} T_t \le I_t + \frac{M_{t-1} + (1 + i_{t-1}) B_{t-1}}{P_t}, \tag{9}$$

the finance constraint

$$\frac{M_{t-1}}{P_t} + T_t(c_t + x_t) \ge c_t + x_t, \tag{10}$$

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and (3), with $0 \le l_t \le 1$, P_t, P_{Tt}, i_t, and t_t taken as given, and $\left(k_{-1}, M_{-1}, (1+i_{-1})B_{-1}\right)$ as initial conditions.[10] Future government policy is the sole source of uncertainty in the model. The operator E in (8) denotes equilibrium expectations of private agents over future policy.

Delivering a given level of consumption or investment goods to the private sector requires the use of transactions services or real money balances, M/P, as described by the finance constraint, (10). T represents the fraction of private transactions executed with transactions services. The finance constraint is specified so that as the size of transactions rises, a given amount of real resources generates a proportional increase in the value of transactions services. In keeping with Friedman's (1956) view of the myriad ways the private sector may endogenously create substitutes for money in transactions, we do not model the precise means by which transactions services substitute for money. Transactions services may be thought of as a clearinghouse, money market mutual funds, or credit cards, although our specification abstracts from any institutional details. The critical point is that the private sector may economize on money holdings by using resources to execute transactions.

The government purchases goods, g, and provides lump-sum transfer payments, h, to private agents. Total government expenditures are financed by printing money, M, and selling nominal bonds, B, which pay a nominal interest rate of i, and levying proportional taxes, t, against net output. The government's decision rules for policy variables satisfy the budget constraint

$$t_t\left[f(k_{t-1}) - d(k_{t-1})\right] + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1+i_{t-1})B_{t-1}}{P_t} = g_t + h_t. \tag{11}$$

To distinguish wealth and substitution effects of government policies, we specify government spending as a share of output.[11] In terms of shares, the government budget constraint may be written as

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[10] Only private transactions are subject to the finance constraint. This assumption is inessential and is designed to prevent government policies from directly affecting the transactions services sector.

[11] If the government makes claims on the economy, it is not clear whether holding constant the private sector's share of output or the level of government spending forms the better basis for comparison. In a growth model where the government makes real claims, fixing the level of

$$\frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1+i_{t-1})B_{t-1}}{P_t} = \left(s_t^g + s_t^h - \boldsymbol{t}_t \right)(1-\boldsymbol{d})\, f(k_{t-1}), \tag{12}$$

where $s_t^g = g_t / y_t$, $s_t^h = h_t / y_t$, and from (2) and (4), $y_t = (1-\boldsymbol{d})\, f(k_{t-1})$.

The model separates the production of goods from the provision of financial services. Goods are produced using capital but not labor, while transactions services are produced using labor but not capital. To focus on the impacts of policy expectations on portfolio decisions, the model keeps to a minimum factor substitutions that can occur within a period. These factor intensity assumptions limit the range of substitutions possible by eliminating direct interactions between resource allocation decisions in the financial (goods) sector and the quantity of goods (financial services) produced.[12]

The elasticity of labor supplied to the transactions sector is a key determinant of velocity. To have any correspondence with reality, the model's employment of labor should be thought of in terms of a variation in intensity of use rather than literally as fluctuations in the supply of labor.

2.3   Functional Forms

We specialize the model by assuming the following functional forms for the production functions and preferences:

$$f(k_{t-1}) = k_{t-1}^{\boldsymbol{s}}, \qquad 0 < \boldsymbol{s} < 1 \tag{13}$$

$$T(l_t) = 1 - (1 - l_t)^{\boldsymbol{a}}, \qquad \boldsymbol{a} > 1 \tag{14}$$

$$U(c_t, 1 - l_t) = \ln(c_t) + \boldsymbol{g}\ln(1 - l_t), \qquad \boldsymbol{g} > 0. \tag{15}$$

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spending implies a monotonically declining role for the government. Policy exercises that hold the government's share of output fixed also fix private wealth, and generate a well-defined substitution effect from the choice between distorting taxes and money creation.

[12] The model is a tractable version of King and Plosser (1984). Our factor intensity assumptions restrict the general perspective that King and Plosser present, and are consistent with the money demand studies by McCallum and Goodfriend (1987) and Lucas (1994). Although similar in spirit, our specification differs from the work of Gillman (1993), Ireland (1994), Aiyagari et al. (1996), and Lacker and Schreft (1996), among others, by limiting factor substitutability.

The first-order conditions for the model are recorded in Appendix A.

3. Mapping Policy Expectations into Portfolio Decisions

The solution of the model requires solving the Euler equation for capital, taking the paths of government spending and taxes as given, and then using the arbitrage between money and transactions services to solve the Euler equation for money (see Appendix A). The result is a valuation of real and nominal assets in the current period as a function of expected policies. Imposing budget constraints and market clearing conditions then determines the current equilibrium.

Define the rate of investment out of private expenditures as

$$s_t = x_t / (c_t + x_t) \tag{16}$$

and the capital stock as a share of total real resources available to the private sector as

$$\widetilde{s}_t = \frac{k_t}{\left[1 - s_t^g(1-d)\right] f(k_{t-1})}. \tag{17}$$

s and $\widetilde{s}$ are related by

$$\frac{1}{1 - s_t} = \frac{1}{1 - \widetilde{s}_t}\left[\frac{(1-d)\left(1 - s_t^g\right)}{1 - s_t^g(1-d)}\right]. \tag{18}$$

The Euler equation for capital implies the difference equation in $\widetilde{s}$

$$\frac{1}{1 - \widetilde{s}_t} = sbE_t\left[\frac{(1-t_{t+1}) + dt_{t+1}}{1 - s_{t+1}^g(1-d)}\right]\frac{1}{1 - \widetilde{s}_{t+1}} + E_t\left[1 - sb\frac{g}{a}\left(\frac{1 - t_{t+1}}{1 - s_{t+1}^g}\right)\right] \tag{19}$$

whose solution is

$$\frac{1}{1 - \widetilde{s}_t} = h_t, \tag{20}$$

where

$$h_{t} = E_{t} \sum_{i=0}^{\infty} (sb)^{i} d_{i}^{h} \left[1 - sb \frac{g}{a} \left(\frac{1 - t_{t+i+1}}{1 - s_{t+i+1}^{g}} \right) \right], \quad d_{i}^{h} = \prod_{j=0}^{i-1} \frac{1 - t_{t+j+1}(1-d)}{1 - s_{t+j+1}^{g}(1-d)}, \quad d_{0}^{h} = 1. \quad (21)$$

The Euler equation for money is solved analogously. Letting $r_{t} = M_{t}/M_{t-1}$ denote the growth rate of the money supply, the Euler equation is

$$(1 - T_{t}) \left[\frac{1}{1 - s_{t}} - \frac{g}{a} \right] = b \frac{1}{r_{t}} E_{t} \left\{ (1 - T_{t+1}) \left[\frac{1}{1 - s_{t+1}} - \frac{g}{a} \right] + \frac{g}{a} \right\}. \quad (22)$$

The solution to this difference equation is

$$(1 - T_{t}) \left[\frac{1}{1 - s_{t}} - \frac{g}{a} \right] = \frac{m_{t}}{r_{t}}, \quad (23)$$

where

$$m_{t} \equiv b \frac{g}{a} E_{t} \sum_{i=0}^{\infty} b^{i} d_{i}^{m}, \qquad d_{i}^{m} \equiv \prod_{j=0}^{i-1} \frac{1}{r_{t+j+1}}, \qquad d_{0}^{m} \equiv 1. \quad (24)$$

The equilibrium at each date can be characterized in terms of the expectations functions (h_{t}, m_{t}), current government claims on goods, s_{t}^{g}, and the current state $\left(k_{t-1}, M_{t-1}, (1+i_{t-1})B_{t-1} \right)$. Equilibrium velocity, the object of interest in this paper, is obtained by combining the finance constraint with expressions (20) and (23):

$$v_{t} = \frac{c_{t} + x_{t}}{M_{t}/P_{t}} = \Delta_{t}, \quad (25)$$

where

$$\Delta_{t} = \frac{1}{m_{t}} \left[\frac{(1-d)(1-s_{t}^{g})}{1 - s_{t}^{g}(1-d)} h_{t} - \frac{g}{a} \right]. \quad (26)$$

There are two distinct influences of policy on velocity. The first works through m, the marginal value of end-of-period real money balances, which is ubiquitous in dynamic monetary models. Changes in m imply changes in the expected rate of return on money and produce the model's direct effects of monetary policy. The higher the expected depreciation in the real value

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of money — the higher is expected money growth — the lower is m and the greater the share of transactions executed with financial services. Substitution out of money raises velocity.

The second influence of policy operates through h. Expression (21) obscures the economic interpretation of h as an index of total future nominal liability creation. That interpretation can be seen most easily by an alternative expression for the terms $(1-t)/(1-s^g)$, which are compounded and discounted in (21). Imposing that the government budget constraint clears in the future and setting transfers to zero imply that in equilibrium:

$$\frac{1-t_t}{1-s_t^g} = 1 + \frac{\left(M_t - M_{t-1} + B_t - (1+i_{t-1})B_{t-1}\right)/P_t}{\left(1-s_t^g\right)(1-d)\,f(k_{t-1})}, \quad \text{for all t.} \tag{27}$$

Thus the terms in $(1-t)/(1-s^g)$ in the expression for h reflect the fraction of private resources absorbed by the acquisition of new nominal liabilities issued by the government.

Equilibrium velocity can now be expressed solely as a function of expected policies and current government spending. When future policies are expected to be constant at (r_F, t_F, s_F^g), velocity is given by

$$\frac{c_t + x_t}{M_t/P_t} = v_t(\overset{(+)}{r_F}, \overset{(-)}{t_F}, \overset{(+)}{s_F^g}, \overset{(-)}{s_t^g}), \tag{28}$$

where the signs over the arguments of $v(\cdot)$ reflect partial derivatives of velocity with respect to the relevant policy variables. Velocity in (28) is defined with respect to end-of-period money holdings and depends on future but not current money growth and taxes. With minor changes in the model specification, current monetary and tax policies could affect velocity, but the dependence on expected future policies is robust to alterations in the specification.[13]

The definition of h_t in (21) and (27) demonstrates clearly that changes in policy affecting private sector after-tax income relative to expenditures, $(1-t)/(1-s^g)$, trigger substitutions that influence portfolio decisions. Changes in fiscal financing that are pure nominal asset swaps affect

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[13] For example, if end-of-period money, M_t/P_t, enters the finance constraint or if total income were taxed, then both r_t and t_t would affect velocity contemporaneously.

velocity only through substitutions between money and transactions services. Those substitutions change the allocation of resources towards the transactions sector, but not the quantity of goods available. The presence of a distorting real tax creates an environment where private agents may respond to the composition of government finance. Holding government's share of output fixed, lower expected real taxes improve the returns on real assets relative to nominal assets. With relatively more of the tax distortion falling on nominal assets, agents substitute into real assets and transactions services. Velocity increases. In contrast, a shift in the expected composition of financing away from nominal liability creation and toward real taxes induces substitutions out of real assets and into nominal assets, including money. Velocity falls.

The opportunity cost of holding money — the equilibrium nominal interest rate — depends only on expected monetary policy through $\boldsymbol{m}$

$$1+\mathrm{i}_t = \frac{\boldsymbol{m}_t}{\boldsymbol{m}_t - \boldsymbol{b}(\boldsymbol{g}/\boldsymbol{a})}. \tag{29}$$

Using (29) to replace $\boldsymbol{m}$ in (26) yields an expression for equilibrium velocity as an increasing function of the nominal rate and the path of expected future fiscal variables. Empirical studies of velocity that fail to account for its dependence on fiscal variables will be misspecified. To the extent that $\boldsymbol{m}_t$ and $\boldsymbol{h}_t$ are correlated, as they will generally be through the government budget constraint, estimates of the interest elasticity of velocity will be biased. The direction and degree of bias depend on the equilibrium policy process.[14]

The implication that equilibrium velocity depends on expected fiscal variables contradicts the results in McCallum and Goodfriend (1987) and Lucas (1994). They derive static portfolio-balance expressions from models in which agents may use real money balances or time to acquire consumption goods. A key difference arises from the broad interpretation we ascribe to transactions services, which leads us to require that both consumption and investment goods be financed. If investment goods must be financed and future taxes affect the return on capital,

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[14] This prediction of the model is closely related to Poole's (1988) argument that estimates of the interest elasticity of money demand have been biased downward by common trends in time series data on velocity and nominal interest rates.

equilibrium real balances will depend on the expected composition of fiscal financing, except under special assumptions about policy.[15]

Expressing the equilibrium in terms of expected time paths of policy variables is consistent with an agnostic view of policy behavior. Specifically, we do not address the underlying process determining policy. That requires specifying the constraints and incentives facing policy makers, and deriving monetary and fiscal policy decision rules as functions of the economic state. We have not done that. The virtue of our more general characterization of policy behavior is that the class of equilibria we consider does not depend on a particular parameterization of policy behavior.

4. Data and Model Parameters

Simulating time paths of equilibrium velocity from the model requires selecting values for parameters — b, d, s, g, a — and obtaining time paths for the policy expectations functions (h_t, m_t). Both of these steps are based on actual U.S. data.

4.1   Data Selection[16]

All data underlying simulations of the model are quarterly U.S. time series from 1960:1 to 1997:1. Velocity is computed as the ratio of private expenditures — consumption plus investment — to real base money. Consumption is defined as real personal consumption expenditures on non-durable goods and services. Investment equals real gross private domestic investment plus real personal consumption expenditures on durable goods. The monetary base is adjusted for changes in reserve requirement changes and for the introduction of sweep accounts beginning in 1994. The chain-weighted GDP deflator is defined as the model's price level and is used to convert all nominal to real variables.

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[15] Specifically, if $t_t = s_t^g$, all t, then h reduces to a constant, and the model exhibits a type of neutrality.

[16] Data sources are listed in Appendix B.

It may seem more natural to equate the model's M to a narrow transactions balances measure like M1. But base money rather than M1 belongs in the government's budget constraint. In any case, base and M1 velocity exhibit similar secular patterns, as noted above.

On the fiscal side one could argue for including all federal and state and local government claims on the economy, along with an analogously broad measure of tax obligations. A thorough analysis would include social security, and possibly build in future Medicare expenditures associated with demographic shifts. Our objective is less ambitious. We include only federal fiscal variables. Government spending is defined as consumption expenditures plus gross investment. Transfer payments include transfers to persons plus subsidies. Grants-in-aid to state and local governments have been netted out. In the model t is both the marginal and the average tax rate on output. No such object exists in data, so t is computed as the ratio of total real federal tax revenues to output, which is closer to an average tax rate than an average of marginal tax rates. The four policy time series are graphed in Figure 2.

4.2   Selecting Parameters

The factor intensity assumptions of the model make it difficult to lift technology parameters values directly from the real business cycle literature. The discount factor, b, is set at .988, as in Cooley and Prescott (1995), to yield an annual rate of time preference of 4.95 percent. Net depreciation is determined by d. According to (3), along a balanced growth path d is related to the investment-output ratio, the capital-output ratio, and the growth rate of output. Equating balanced growth to time averages of U.S. data yields $d = .772$. These settings of b and d, together with $s = .875$, imply a steady state consumption-output ratio of 0.67, which compares to the time average in U.S. data of 0.66.

A setting of $s = .875$ is higher than the value used in real business cycle calibration exercises, but lower than that used in many endogenous growth simulations. Most real business cycle exercises that do not model the transactions sector set s between .33 and .40.[17] We follow the

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[17] See the studies in Cooley (1995).

growth literature more closely by interpreting k as a composite capital good, including physical and human capital. Estimates of s from that literature range from .67 to 1.0.[18]

The parameters (g,a) appear in equilibrium velocity through the expectations functions (m,h) only as the ratio g/a. That ratio is the utility cost of transactions services. The ratio g/a is chosen at .0023 to ensure that the steady state value of resources devoted to producing transactions services as a share of output, $P_T T/y$, is within the range suggested by previous studies.

Diaz-Gimenez et al. (1992) compute the size of the U.S. financial intermediary sector, measured in terms of value added and final product. They estimate that the sector is large, ranging from 3 to 7 percent of GNP from 1950-1989. Aiyagari et al. (1996) adopt a narrower perspective on financial services to calculate the costs to commercial banks of providing demand deposits and credit cards. Those costs varied from 0.5 to 1.1 percent of GNP over the 1975-1993 period. Although the present model's notion of transactions services is broader than simply financial intermediation, the parameter settings imply $P_T T/y = .025$, which seems a reasonable magnitude.

Steady state computations of endogenous variables require steady state values of policy expectations. Time averages of base growth, r, revenues as a share of output, t, and government purchases as a share of output, s^g, were inserted into the expectations functions, (21) and (24), to compute steady state values of m and h. The parameter settings used in all the simulations, along with model's implied steady state values and corresponding U.S. time averages, are summarized in Table 1.

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[18] Mankiw et al. (1992), calculate that the income share of physical plus human capital is about .67. Rebelo (1991) and others assume production is linear in capital, leading to the Ak model. Chari et al. (1995) discuss a variety of monetary growth models, including Ak and one in which the ratio of human-to-physical capital is constant.

Table 1.			Steady State	U.S. Data
β	.988	v	16.90	13.98
σ	.875	c/y	0.67	0.66
δ	.772	s^g	0.111	0.111
g/a	.0023	t	0.216	0.216
		s^h	0.092	0.092
		r	1.017	1.017
Addendum				
		B/M	20.47	5.23
		$P_T T/y$	0.025	

Table 1 also reports the debt-money base ratio in U.S. data and for the model's steady state. These are very much at odds with each other. As discussed below, none of the analysis of how equilibrium velocity varies with policy expectations rests on the model's steady state B/M. In the simulations, debt is treated as a residual that clears the government budget constraint, given policy realizations and market clearing prices.

5. Econometric Modeling of Policy Expectations

The analysis begins by contrasting two econometric methods for estimating policy expectations. In the first case, the Bayesian vector autoregression (BVAR) used to compute policy projections is estimated over the full sample period. The second method employs a Bayesian updating algorithm to reestimate the forecasting model each period. Estimates under the second method converge to those under the first as the algorithm iterates through the sample.

5.1   Estimating Expectations Functions [19]

The expected time paths of policy variables that are used to construct the policy expectations functions $\{m_t, h_t\}$ are computed as linear projections from a BVAR. Coefficients in the BVAR

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[19] Computational details appear in Appendix C.

are taken to be time invariant. The BVAR is fit to actual time series on the policy variables $\{M_t, s_t^g, t_t, s_t^h\}$, using a statistical prior designed to improve out-of-sample forecast performance.[20] Simulations that begin in 1960:1 use data from 1954:4 to 1959:4 in the estimation of the BVAR. Given the functions $\{m_t, h_t\}$ and a path of $\{s_t^g\}$ from U.S. data, equilibrium velocity is simulated using expressions (25) and (26).

The joint policy process is treated as exogenous to private agents. Whatever endogeneity exists in equilibrium policy variables, therefore, is embedded in the coefficients of the estimated BVAR. By computing policy expectations as linear projections, rather than from behavioral relationships for policy, the separate effects of monetary and fiscal policy are not identified. Consequently, we do not address whether expectations of money growth arise from realizations of fiscal policy, or vice versa.

The forecasts as constructed do not use information about future policy contained in the current state of government indebtedness. Doing so requires clearing the government's budget constraint and solving for expectations simultaneously.[21] The simulated equilibria we report imply a process for government debt that clears the budget. We solve for the realized debt path in the simulations to verify that in each case debt satisfies the necessary transversality condition.

5.2   Full Sample Simulation

If policy obeys a stationary process, better estimates of policy expectations are obtained from longer samples of data. It follows that projections of policy variables will be less contaminated by sampling error, producing a more accurate assessment of agents' forecasts of policy. In the first simulation the projections at date t are conditioned on realizations of policy variables at t and earlier, but the coefficients used to compute the projections are estimated using policy realizations over the full sample.

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[20] Although forecasts of transfers as a share of output, s^h, are not needed to compute (m_t, h_t), they are included in the BVAR because they are both exogenous and likely to help predict other policy variables.

The results are at odds with actual velocity. As shown in Figure 3, simulated velocity increases very slightly in first two decades, but the trend is nothing like what actual velocity exhibits. Simulated velocity generally fluctuates around its steady state value. Evidently when policy expectations are estimated using the entire 37-year sample period, the model implies agents economize much more on holding money in the 1960s and 1970s than was observed.

It is possible that this expectations structure accurately characterizes U.S. decision makers' beliefs about future policy. By that interpretation, the secular increase in velocity arose from factors like technological advances in communications and transportation that increased the efficiency of the payments system — factors not directly linked to macroeconomic policies. Before adopting that perspective it is worthwhile to consider an alternative hypothesis about policy expectations.

5.3   Bayesian Updating

Suppose that agents' expectations stem entirely from observable policy actions and that their expectations are permitted to evolve over time as new realizations of policy occur. In this simulation, estimates of the BVAR representing expectations are updated each period, and projections computed at date t embody policy behavior only up to that date. Because the BVAR is time invariant, coefficients describing expectations do not change over the forecast horizon.

Simulated velocity looks surprisingly like actual velocity, as shown in Figure 4. After starting below actual in the early 1960s, model velocity rises sharply and continues to rise until the early 1980s. There is an extended period during which simulated velocity lies below actual before hovering around the actual path from 1983 to 1990. This pattern of results is largely consistent with Blanchard's (1984) finding that by 1982 private beliefs about policy had adjusted to the environment created by Volcker deflation and the Reagan fiscal changes. For most of the 1990s the model overpredicts velocity, an issue we discuss below.

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[21] This is the approach taken in Leeper (1991), Dotsey (1994), and Leeper and Sims (1994). Those papers solve models that have been linearized around their deterministic steady states, substantially easing the computational burden.

When expectations are formed using the simple updating algorithm, the model underpredicts velocity when actual velocity is rising most rapidly. With the exception of a brief spike in 1978, simulated velocity is persistently "too low" from mid-1976 through 1982. In addition, the trend in simulated velocity is somewhat flatter than that of actual velocity starting in the early 1970s. Two possible explanations suggest themselves: Bayesian updating is a poor approximation to expectations formation during the period or it was a time when surprises in policy were quite important. Additional information corroborates the latter explanation. Inflation was rising rapidly in the late 1970s and ex-post real returns on Treasury bills were negative much of this period. These facts suggest that agents held nominal balances whose realized real value was below their expected value. The surprise inflation would raise realized velocity above expected velocity.

6. Did the 1970s Dominate Beliefs About Policy in the 1980s?

The final simulation considers the implications of assuming that agents' expectations adapt very slowly to new policy environments. We depart from the econometrically rooted procedures above and instead consider the possibility that agents' beliefs about policy in the 1980s were strongly influenced by the experience of the 1970s.

We implement this notion by ascribing to agents entering the 1980s expectations about policy that are conditioned entirely by the policy experience of the 1970s. The BVAR is estimated from 1970:1 to 1980:1 and the resulting posterior is used as the prior in the 1980s. Agents then update their expectations just as in the previous simulation.

Figure 5 reports actual and simulated velocity from 1981:1 to 1997:1 for this experiment. Simulated velocity now displays no trend and lies everywhere above actual. The lack of trend and the higher level are consistent, given the model, with agents believing they are in a stable policy environment with chronically high inflation. Those beliefs, however, do not produce a path of simulated velocity consistent with the data. Figures 4 and 5 together imply that agents in the 1980s did not regard policy behavior before 1970 to be old news or they viewed policy in the early 1980s as both different and somewhat believable.

7. Currency Outflows and the Decline of Velocity in the 1990s

It is difficult to interpret the sharp decline in velocity in the early 1990s. Much of the decline has been attributed to exogenous increases in foreign demand for U.S. currency induced first by the Persian Gulf war in 1990-91 and then by political instability in the former Soviet Union in 1993. Porter and Judson (1996) employ a variety of techniques to estimate the percentage of U.S. currency actually within U.S. borders. Precise estimates, especially over long time periods, are impossible to obtain, but many of their techniques imply that the percentage of U.S. currency flowing overseas rose dramatically beginning in 1989.

Using the estimates of domestic currency reported in the Survey of Current Business (1997), velocity displays a far more modest decline in the 1990s. If foreign demand for U.S. currency increased for reasons unrelated to U.S. macroeconomic policies, then domestic velocity is the notion relevant to the present economic structure. During the 1980s the percentage of U.S. currency held abroad was fairly stable at 40 percent. Under the maintained assumption that that percentage continued through the 1990s, domestic velocity as we define it fluctuates between 16.5 in 1990 and 15.1 in 1996. This calculation implies there is some decline of velocity near the end of the sample, but it is much less precipitous than the raw data suggest.

8. Concluding Remarks

We simulated a simple general equilibrium model in which velocity depends on expectations of policy. When policy expectations are estimated by a standard BVAR estimated over the post-Korean War period, model velocity displays no trend. With regard to estimating policy expectations, more information may not be better information. An alternative procedure treats agents as Bayesian updaters whose expectations evolve as new policy realizations become available. Updating does well in matching the observed trend in velocity except during the periods of rapidly increasing inflation in the late 1970s and early 1980s. After a few years into the 1980s, however, the updating algorithm adapts to the policy environment and generates a flat path for velocity through the 1980s and 1990s. The supposition that beliefs about policy in the 1980s were strongly influenced by actual policy in the 1970s, is contradicted by the result that simulated velocity displays no trend and lies everywhere above actual. The result suggests that either agents did not regard the policy experience before 1970s as irrelevant to the formation of expectations in

the 1980s or agents deemed the policy environment in the 1980s to be both different and to some degree credible.

Taken together, the results in Figures 3 and 4 carry implications for at least four areas of monetary analysis:

(1) Empirical velocity studies that fail to include policy expectations other than through the nominal interest rate are likely to be misspecified and the resulting estimates of the interest elasticity will be biased.

(2) Analyses that do not account for endogenous changes in velocity arising in response to changes in expectations of policy will produce misleading predictions of policy effects.

(3) Discussions of desirable monetary policy responses to "velocity shocks" contain a degree of circularity when the time series properties of velocity are themselves determined by policy expectations.

(4) Efforts to identify monetary policy using VARs estimated over long sample periods may confound policy shocks with changes in the policy process.

This paper used a simple economic structure that maps policy expectations into portfolio choices to argue that shifts in the secular behavior of velocity may be explained by endogenous responses to policy expectations. Our explanation does not require exogenous financial innovation or technological improvements in the financial sector to generate trends in velocity. The explanation does require an environment with a sufficiently rich set of asset substitutions and policy expectations based on actual time series data.

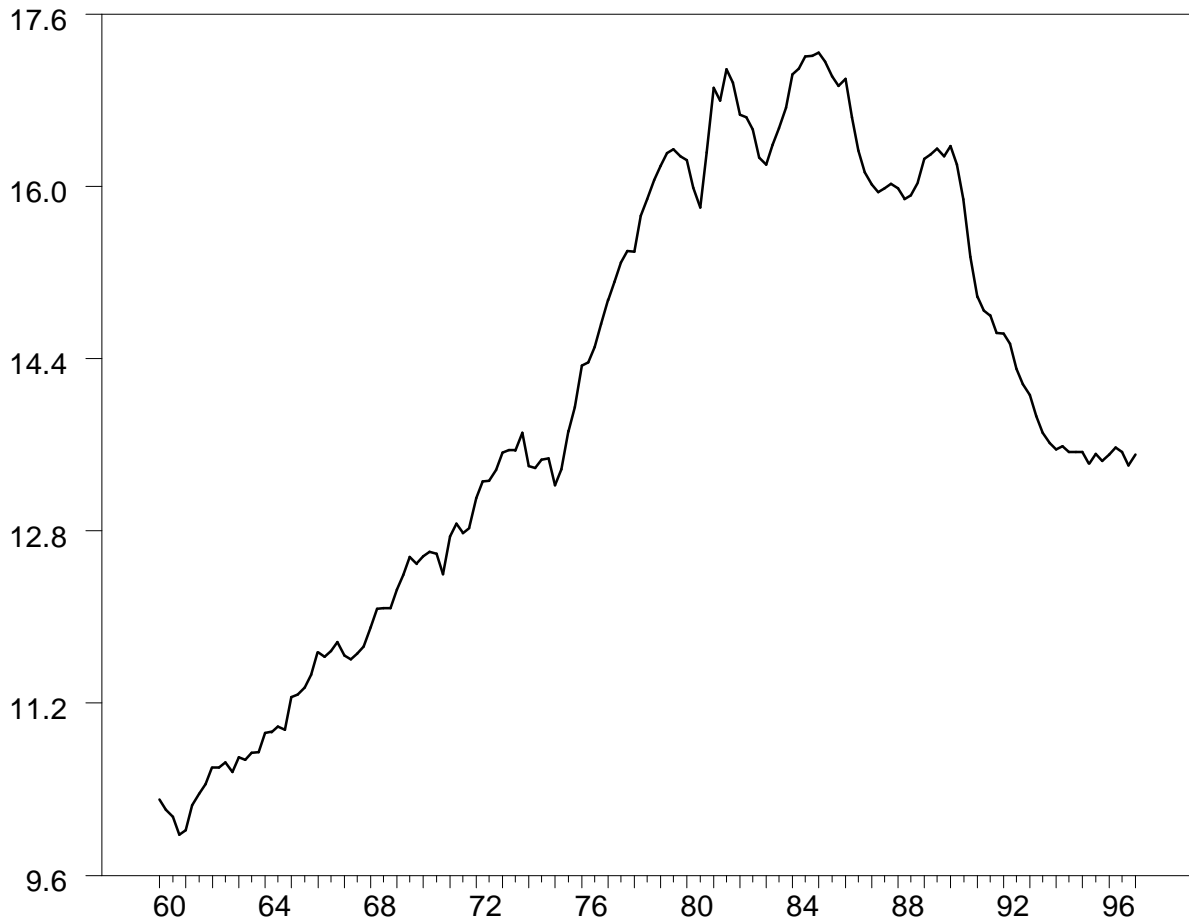# Figure 1. Private Expenditures Velocity of Base Money

Figure 1. Velocity is defined as consumption plus investment divided by the real monetary base. Base money is adjusted for reserve requirement changes and for sweep accounts after 1994.
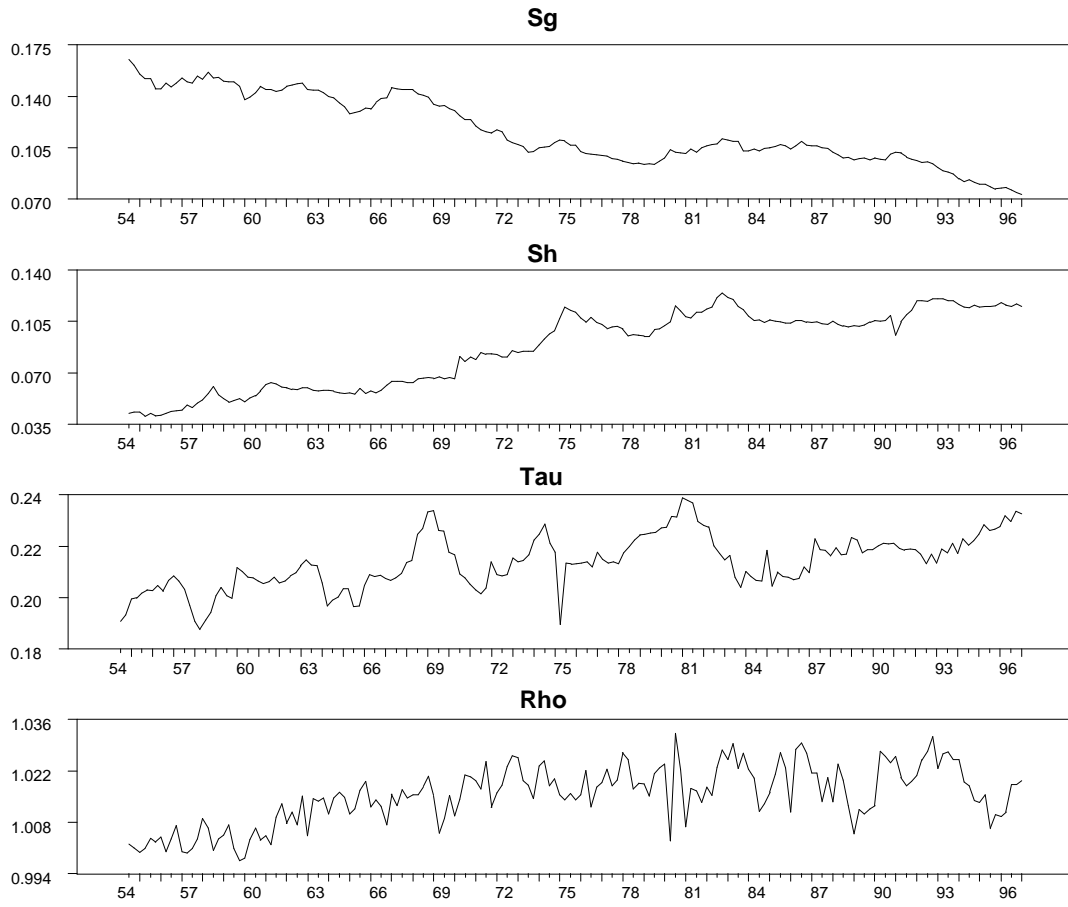
Figure 2. Policy Variables

Sg

Sh

Tau

Rho

Figure 2. $s_t^g = g_t/y_t$, $s_t^h = h_t/y_t$, $\boldsymbol{t}_t = \text{Revenues}_t/y_t$, $\boldsymbol{r}_t = M_t/M_{t-1}$; variables defined in Appendix B.

Figure 3. Velocity: Actual and Full Sample No Updating
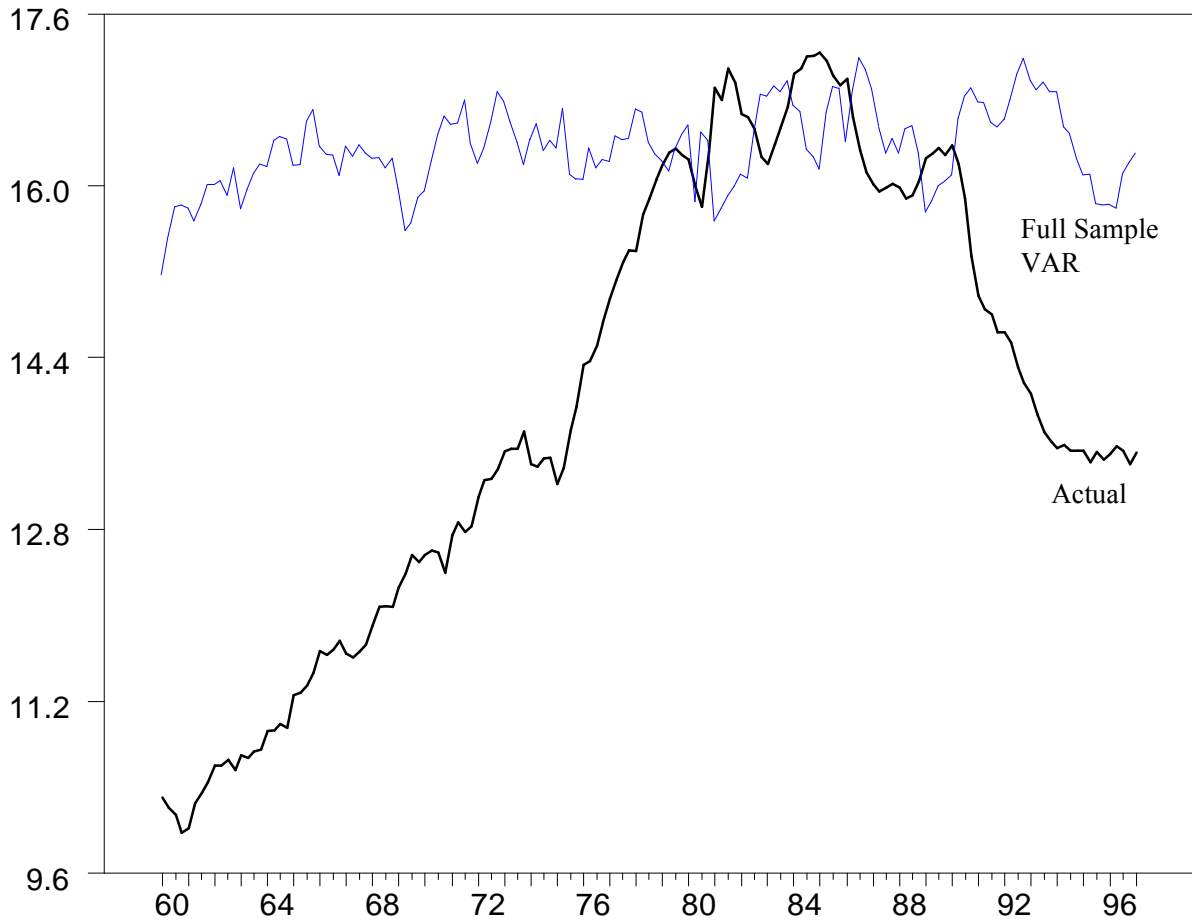

Figure 3. Policy expectations are computed from a BVAR estimated over entire sample.

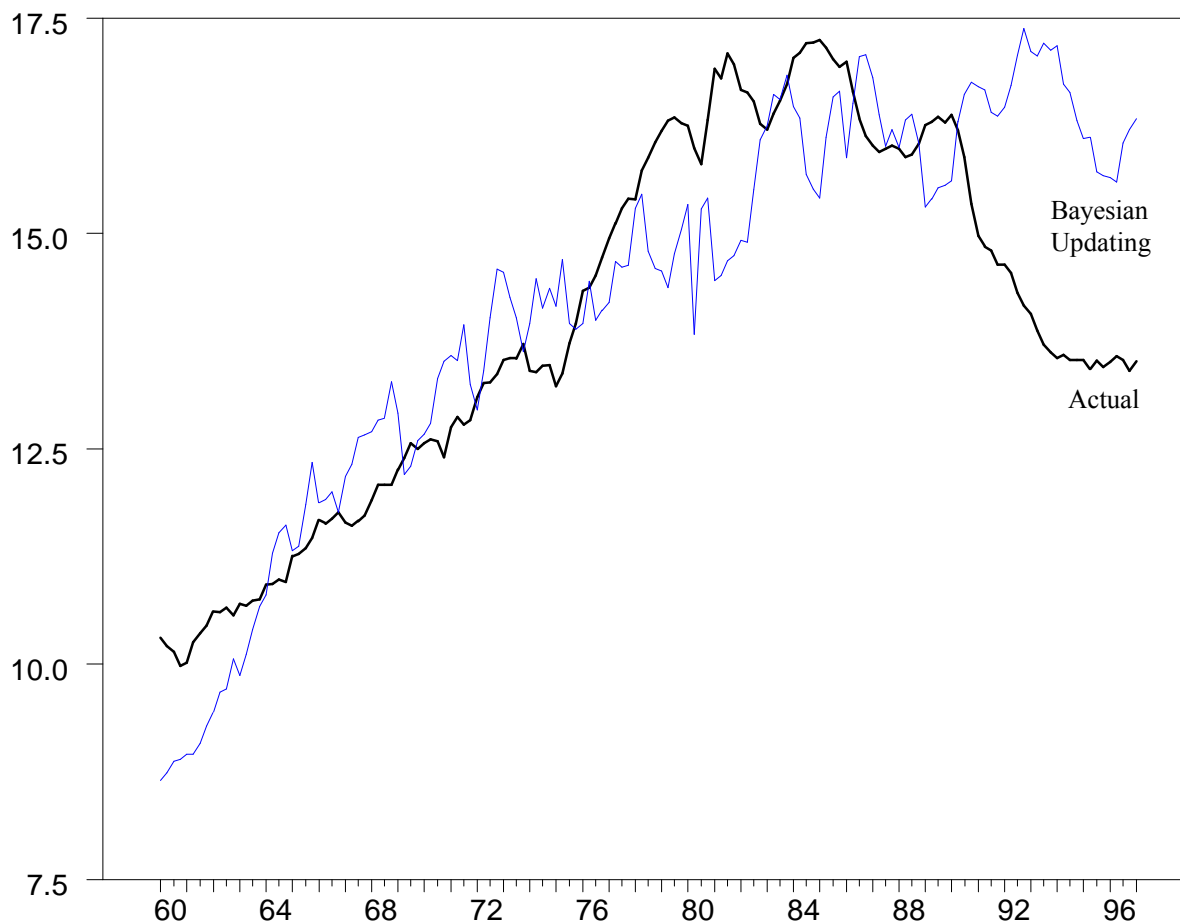**Figure 4. Velocity: Actual and Bayesian Updating**

Figure 4. Policy expectations are estimated by a BVAR that is updated each period as new policy realizations become available. Expectations at date t use data on policy variables at time t and earlier.

Figure 5. Velocity: Actual, Updating, 1970s As Prior

Figure 5. In Bayesian updating policy expectations are estimated by a BVAR that is updated each period as new policy realizations become available. Expectations at date t use data on policy variables at time t and earlier. 1970s as prior uses estimates of the policy process from 1970:1 to 1980:1 as the prior in the 1980s, and updates beliefs through the sample.

Appendix A: Solving the Model

First-Order Conditions

We focus on an interior solution.[22]

Firms':

k_{t-1}:
$$r_t = (1 - t_t) f'(k_{t-1}) + t_t d'(k_{t-1})$$
$$= \left[(1 - t_t) + dt_t \right] f'(k_{t-1}) \tag{A.1}$$

l_t:
$$w_t = P_{Tt} T'(l_t). \tag{A.2}$$

Household's: Let $\boldsymbol{j}$ be the lagrange multiplier for the budget constraint and $\boldsymbol{l}$ be the multiplier for the finance constraint:

c_t:
$$\frac{1}{c_t} = \boldsymbol{j}_t + \boldsymbol{l}_t (1 - T_t) \tag{A.3}$$

l_t:
$$\frac{\boldsymbol{g}}{1 - l_t} = w_t \boldsymbol{j}_t \tag{A.4}$$

T_t:
$$\boldsymbol{j}_t P_{Tt} = \boldsymbol{l}_t (c_t + x_t) \tag{A.5}$$

M_t:
$$\frac{\boldsymbol{j}_t}{P_t} = \boldsymbol{b} \, \mathrm{E}_t \left[\frac{\boldsymbol{j}_{t+1} + \boldsymbol{l}_{t+1}}{P_{t+1}} \right] \tag{A.6}$$

B_t:
$$\frac{\boldsymbol{j}_t}{P_t} = \boldsymbol{b} (1 + i_t) \mathrm{E}_t \left[\frac{\boldsymbol{j}_{t+1}}{P_{t+1}} \right] \tag{A.7}$$

k_t:
$$\boldsymbol{j}_t + \boldsymbol{l}_t (1 - T_t) = \boldsymbol{b} \, \mathrm{E}_t \left[\boldsymbol{j}_{t+1} r_{t+1} + \boldsymbol{l}_{t+1} (1 - T_{t+1}) d'(k_t) \right] \tag{A.8}$$

In addition to (A.3)-(A.8), a solution must satisfy transversality conditions for capital, money, and bonds.

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[22] The finance constraint will be satisfied with equality. If it were not, then excess cash balances would imply that $\boldsymbol{l}_t = 0$. Logarithmic preferences imply there cannot be an equilibrium with $l_t = 1$. If the shadow price of real balances is zero, then the first-order conditions imply we have $P_{Tt} = 0$ and $w_t = 0$. A zero real wage, in turn, implies that $\boldsymbol{g}/(1 - l_t) = 0$, which cannot be satisfied by any feasible values for l_t.

Combine the firms' first-order conditions with the household's first-order conditions for c, l, and T to obtain expressions for the lagrange multipliers

$$l_t = \frac{\dfrac{g}{a}}{(1 - T_t)(c_t + x_t)} \tag{A.9}$$

and

$$j_t = \frac{1}{c_t} - \frac{\dfrac{g}{a}}{c_t + x_t}. \tag{A.10}$$

In equilibrium, the finance constraint will be met with equality, so

$$\frac{1}{P_t} = \frac{1}{M_{t-1}}(1 - T_t)(c_t + x_t). \tag{A.11}$$

First we solve the difference equation in $\tilde{s}$ implied by the Euler equation for capital. Use (A.1) and (A.3) in (A.8) to obtain

$$\frac{1}{c_t} = bE_t \left\{ \left(\frac{1}{c_{t+1}} - \frac{g/a}{c_{t+1} + x_{t+1}} \right) \left[(1 - t_{t+1}) + dt_{t+1} \right] + \frac{(g/a)d}{c_{t+1} + x_{t+1}} \right\} f'(k_t). \tag{A.12}$$

The production function, (13), implies that $f' = sf/k$, so (A.12) can be written as

$$\frac{k_t}{c_t} = sbE_t \left(\frac{\left[(1 - t_{t+1}) + dt_{t+1} \right]}{c_{t+1}} - \frac{g/a}{c_{t+1} + x_{t+1}}(1 - t_{t+1})(1 - d) \right) f(k_t). \tag{A.13}$$

Using (4) and (17), note that $(k_t/c_t) + 1 = 1/(1 - \tilde{s}_t)$. After some minor algebra, (A.13) becomes

$$\frac{1}{1 - \tilde{s}_t} =$$

$$sbE_t \left\{ \frac{f(k_t)}{c_{t+1} + x_{t+1}} \left[(1 - t_{t+1}) + dt_{t+1} \right] \left(\frac{(1 - d)(1 - s_{t+1}^g)}{1 - s_{t+1}^g(1 - d)} \right) \frac{1}{1 - \tilde{s}_{t+1}} - \frac{g}{a}(1 - d)(1 - t_{t+1}) \right] \right\} + 1. \tag{A.14}$$

Use (4) in (A.14) to obtain

$$\frac{1}{1-\widetilde{s}_t} = sbE_t\left[\frac{(1-t_{t+1})+dt_{t+1}}{1-s_{t+1}^g(1-d)}\right]\frac{1}{1-\widetilde{s}_{t+1}} + E_t\left[1-sb\frac{g}{a}\left(\frac{1-t_{t+1}}{1-s_{t+1}^g}\right)\right].$$
(A.15)

The solution to this difference equation is

$$\frac{1}{1-\widetilde{s}_t} = h_t,$$
(A.16)

where

$$h_t = E_t\sum_{i=0}^{\infty}(sb)^i d_i\left[1-sb\frac{g}{a}\left(\frac{1-t_{t+i+1}}{1-s_{t+i+1}^g}\right)\right], \quad d_i = \prod_{j=0}^{i-1}\frac{1-t_{t+j+1}(1-d)}{1-s_{t+j+1}^g(1-d)}, \quad d_0 = 1.$$
(A.17)

Note that convergence of (A.15) requires that

$$\lim_{k\to\infty}(sb)^k E_t\prod_{j=1}^{k}\left[\frac{1-t_{t+j}(1-d)}{1-s_{t+j}^g(1-d)}\right]\frac{1}{1-\widetilde{s}_{t+k}} = 0.$$
(A.18)

To solve the difference equation for money, (A.6), let $r_t = M_t/M_{t-1}$ denote the growth rate of the money supply, and combine (A.9)-(A.11) with the Euler equation for money to obtain

$$(1-T_t)\left[\frac{1}{1-s_t}-\frac{g}{a}\right] = b\frac{1}{r_t}E_t\left\{(1-T_{t+1})\left[\frac{1}{1-s_{t+1}}-\frac{g}{a}\right]+\frac{g}{a}\right\}.$$
(A.19)

The solution to this difference equation is

$$(1-T_t)\left[\frac{1}{1-s_t}-\frac{g}{a}\right] = \frac{m_t}{r_t},$$
(A.20)

where

$$m_t \equiv b\frac{g}{a}E_t\sum_{i=0}^{\infty}b^i d_i, \qquad d_i \equiv \prod_{j=0}^{i-1}\frac{1}{r_{t+j+1}}, \qquad d_0 \equiv 1.$$
(A.21)

Convergence of this difference equation requires that

$$\lim_{k\to\infty}b^k E_t\prod_{j=1}^{k}\frac{1}{r_{t+j-1}}(1-T_{t+k})\left[\frac{1}{1-s_{t+k}}-\frac{g}{a}\right] = 0.$$
(A.22)

Conditions (A.18) and (A.22) are implied by the transversality conditions for capital and money.

Appendix B: Data

The quarterly data are selected to be as close to the definitions in the theoretical model as possible. They cover the period from 1950:1 to 1997:1. The data are seasonally adjusted and from the Bureau of Economic Analysis, the Department of Commerce unless otherwise stated.

c : Real chain-weighted personal consumption expenditures (PCE): non-durable goods plus services.

x : Real chain-weighted gross private domestic investment plus real PCE durable goods.

g : Federal government consumption expenditures and gross investment, deflated by the GDP deflator.

y : Real output, equal to $c + x + g$.

P : GDP deflator, chain-weighted.

t : Ratio of federal government receipts and nominal output (Py).

h : Federal government transfers to persons plus subsidies, deflated by the GDP deflator.

M : Monetary base adjusted for changes in reserve requirements by the Federal Reserve Bank of St. Louis and adjusted for sweep activity in reserves by the Federal Reserve Bank of Atlanta.

k : Real net stock of fixed capital (from the Survey of Current Business) plus real non-farm business inventories.

B: Market value of gross privately held government debt, compiled by the Federal Reserve Bank of Dallas.

Appendix C: Computing Expectations

This appendix describes how we compute agents' expectations of policy variables. We assume that agents do not know the true process generating policy variables, and instead base their expectations on forecasts of policy. The forecasts are constructed from observed data and updated every period using Bayes's rule when new observations become available.

We postulate this updating mechanism in a Bayesian vector autoregression (BVAR) framework. If $y(t)$ is an $(n \times 1)$ vector of time series, we assume it follows a linear, stochastic, and dynamic process:

$$A(L)y(t) = \sum_{s=1}^{p} A_s y(t-s) = e(t), \qquad (C.1)$$

where L is a lag operator and $e(t)$ is an $(n \times 1)$ vector of random disturbances that are uncorrelated with the past data $y(t-s)$ for $s \geq 1$ and have a Gaussian distribution with zero mean and an identity covariance matrix.[23] The dynamic process (C.1) can be expressed in the reduced form:

$$y(t) = B(L)y(t-1) + u(t), \qquad (C.2)$$

where B_0 is an identity matrix and $u(t) = A_0^{-1} e(t)$.

In this paper, the vector $y(t)$ consists of the four policy variables: M_t, s_t^g, t_t, and s_t^h. Calculations use the natural log of M. We transform the remaining variables into the form $\log(z/(1-z))$, where z is s_t^g, t_t, or s_t^h. The transformation ensures that the values of all these variables, when transformed back to their original form, fall within the feasible range of $(0,1)$. Agents use the updating mechanism (C.1) or (C.2) to project policy variables (r, s^g, t, s^h) into

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[23] For expository clarity we omit the constant terms that are included in the actual computation.

the infinite future and, based on the projected values, the expectations functions $\boldsymbol{m}$ and $\boldsymbol{h}$ are computed according to expressions (24) and (21).[24]

The agents' prior on the coefficient matrices $A(L)$ (and thus $B(L)$) has two components. Both components are somewhat complicated. For details, see Sims and Zha (1997). Here, we only outline the main ideas.

The first component is a standard reference prior. Specifically, the matrix A_0 is normalized to be lower triangular and the unrestricted elements have a joint normal distribution. Conditional on A_0, the distribution of A_s is assumed to be also normal and the standard deviations shrink as s increases. This prior is essential for handling a system with a limited number of observations and for producing reasonable forecasts. The prior mean, when transformed to the reduced form $B(L)$, follows a random walk and resembles the original Litterman (1986) prior. The prior variance is chosen so that this prior does not have much influence on the estimates of the coefficients but is essential for eliminating erratic sampling errors.

The second component of the prior is constructed mainly to obtain realistic forecasts over long horizons. As pointed out in Leeper, Sims, and Zha (1996), OLS estimates without any prior tend to overfit the data, producing implausible deterministic trends. This tendency to generate polynomial deterministic trends is elaborated in Sims and Zha (1997). In our four-variable VAR model, for example, it is not uncommon for the out-of-sample forecast of $\boldsymbol{r}$ to approach an arbitrarily large number or for the forecasts of s^g and $\boldsymbol{t}$ to approach 1. To eliminate these absurd forecasts, we specify a prior that M_t follows a random walk and the other three variables are stationary with the root of 0.98. Furthermore, since government spending may influence the formation of monetary policy as well as of fiscal policy (such as a tax policy) in the distant future,

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[24] The convergence of the series in (24) and (21) is relatively fast when checked by simply comparing the successive partial sums. Methods that are more elaborate, such as the Aiken's $\boldsymbol{d}^2$-process documented in Press, et al. (1988), although designed to accelerate the convergence, can perform badly even for a series that is approximately geometric.

our prior also expresses the belief that a long-run relationship among these policy variables is possible.[25]

At each date t, the agents observe the actual policy variables up to time t. They use the Bayesian model (C.1) or (C.2) to obtain the (generalized) maximum likelihood (ML) estimates. The ML estimates are used to project the paths of future policy variables beyond time t; $\boldsymbol{m}_t$ and $\boldsymbol{h}_t$ are then computed. In the period $t+1$, the agents receive new data, update the ML estimates, and form new projections. This Bayes updating process is the updating mechanism used to generate the time path of velocity from the theoretical model in the simulations labeled "Bayesian updating."

The estimated BVARs include a constant term and four lags. Accounting for these plus the dummy initial observations implies the estimation uses data from 1954:4 to 1959:4 in the estimation of the BVAR for the simulations that begin in 1960:1.

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[25] This prior is formulated in the convenient form of dummy observations and its implementation is laid out in Sims and Zha (1997).

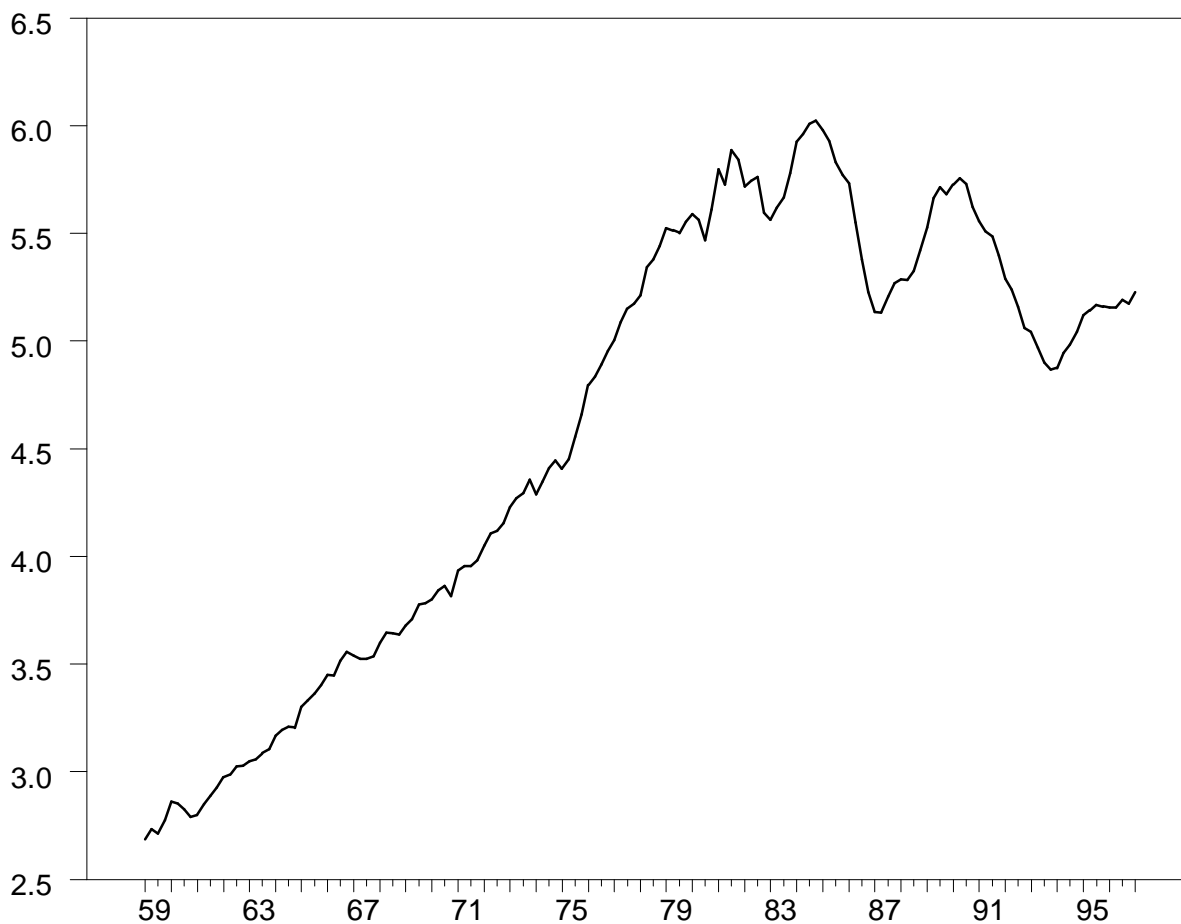# Figure A1. Private Expenditures Velocity of M1

Figure A1. Velocity is defined as consumption plus investment divided by the real M1 money stock. M1 is adjusted for effects of sweep accounts after 1994.

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