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Time-to-build, Obsolescence and the Technological Paradox

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TIME-TO-BUILD, OBSOLESCENCE AND THE TECHNOLOGICAL PARADOX

FABRIZIO PATRIARCA*

Abstract. The paper focusses on the technological paradox. To analyze the possible temporary negative effect of an innovation, we make use of a flow representation of production. Our aim is to show that such phenomenon can be justified by a simple property of the production process: in real time costs strictly come before proceeds. Moving in the same direction of Amendola (1974), we analyze the obsolescence effect induced by a rise in the interest rate. Furthermore, we analyze the role of capital market stickiness on the timing of the technological paradox and on the distribution of the obsolescence effect among the different stages of a vertical integrated production system.
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1. Introduction

The technological paradox, that is, the temporary worsening of the economic conditions due to the appearing of an innovation, has regained interest in recent growth theory[1]. The paradox is usually considered to occur in a specific period of the innovation process: the time span between the appearance of the innovation, when it comes out of the black box[2], and the time the innovation is completely embodied in the production system.

The first example of such approach can be found in Ricardo[3]. When he analyzes the occupational effect of the introduction of a new and more mechanized technique, he observes that the need of building a more costly productive capacity would bring to a fall in the wage fund leading to technological unemployment. This effect will be reabsorbed later on, when the superiority of the new technique emerges, moving to a new and faster balanced growth. A formal proof of such case of technological paradox, "the machinery effect", was provided by Hicks Hicks (1973) by the mean of its "Neo-austrian" (NA) representation of production, the same we will use in the present work. This representation of the production process as a time flow of primary inputs and outputs provides an approach to the genesis of capital, considering the construction of

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[1]See Yorokoglu (1998) Jovanovic (2000) Campbell (1998).
[2]See Rosenberg (1982).
[3]See Ricardo (London, 1821).

productive capacity[4]. This option is based on the hypothesis that during the first part of the flow no output is produced while positive inputs are needed.

Such an approach was initially limited to the analysis of a technological progress that requires a more indirect production technique as in the case of the industrial revolution. More recently, a similar issue has emerged during the VLSI revolution[5] when the exponential growth of the final product quality has been accompanied by the growth of the fixed capital costs.

A further step towards a generalization of the technological paradox was made by Amendola Amendola (1974). He showed that the phenomenon should occur whatever the kind of technological progress. The proof was based on a typical argument of the Austrian tradition: the variation of the optimal length of the production process. The sudden truncation of old productive capacity causes a sudden temporary fall of output.

This last point allows us to put the Technological Paradox in a different perspective, since it has not a mechanical origin, as in the case of a resource constraint, but instead *it is regarded as depending on the obsolescence effect induced by the rise of the interest rate and hence on a market mechanism.* We deepen the analysis about the way the appearance of a new technique affects the capital value of the processes belonging to the old technique. Differently from the case of the "optimal length" variation[6], we show that the stages of the vertical integrated production system more involved by the osolescence effect are the upstream ones. Furthermore, with the option of truncating processes not already in the utilization phase, time-to-build becomes a sufficient condition for the technological paradox to occur.

The upstream-downstream truncation cases suggest a new perspective about the role of financial markets along a technological traverse: we show that the speed of reaction of the capital market determines which segments of the vertical integrated system are affected by the obsolescence effect. Furthermore, as a result, the time at which the technological paradox takes place depends on the stickiness of the rate of interest. In the case of extra-profits on the investments in the new technology the technological paradox occurs earlier.

In the next section we introduce the framework of NA production. In the third section we analyze the path of the economy after the appearing of the innovation (the traverse). We start considering the mechanical "machinery effect", then move to the "obsolescence effect" involved by the rising of the interest rate and conclude with the role of the speed of reaction of the capital market.

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[4]A further important property of such representation of production is that it is a generalization of the more fashioned vintage linear production models. SeePatriarca (2008).

[5]For further details about the Vertical Large Scale Integrated Systems revolution see Rosenberg (1982).

[6]The variation of the optimal length is shown to be a very particular case. Besides, we argue that such scrapping policy can lack coherence if not better qualified.

2. NA modeling

2.1. **Production Processes.** Within the Neo-austrian approach, a production process is a time flow of primary inputs and final outputs.

From now on, we consider the case of a single output and a single input, in a continuous time context. Such a process π can be graphically represented as in figure 1 where a and b are respectively the input and the output flows, from the activation the initial stage $s = 0$ up to the process' natural length ω[7]; d is the length of the building phase[8].

The core of the neoaustrian approach, the time-to-build, is embodied in the hypothesis that the first part of the output flow is null and the first part of input flow is strictly positive:

$$\pi : a(s),\ b(s) \qquad s \in [0, \omega] \tag{2.1}$$

$$a(s) > 0; b(s) = 0 \qquad \forall\ s \in [0, d)\ . \tag{2.2}$$

For a given output-input ratio w (let's call it the wage rate, referring to labor as the only input), that we will suppose to be fix, the process is characterized by a single sequence of net output q[9]:

$$q(s) = b(s) - wa(s) \qquad s \in [0, \omega] \tag{2.3}$$

$$q(s) < 0 \qquad \forall s \in [0, d)\ . \tag{2.4}$$

If a discount rate ρ is introduced, a capital value k can be associated to each process by considering the integral of the discounted net output flow:

$$k_\rho = \int_0^\omega q(s)e^{-\rho s}ds = \int_0^\omega [b(s) - wa(s)]e^{-\rho s}ds\ . \tag{2.5}$$

The process characterization by means of its net output time profile can be replaced by its capital value profile (figure 2), which expresses the discounted value of the tail of the process at each stage of its life:

$$k_\rho(s) = \int_s^\theta q(u)e^{-\rho(u-s)}du\ . \tag{2.6}$$

[7]The natural length of the process can be finite or not.

[8]We can characterize a Neoaustrian technique either by a sequence of input and output coefficients or by functionals of input and output rates, according to the discrete or continuous representation of time we use. Distinction between discrete and continuous time representation makes sense only in a sequential economic model when, together with the inter-period production sequence, we jointly consider an infraperiod sequence (market interactions, price variations, expectations revisions, and so on). In the latter case we should better use a discrete time version. Since we do not introduce a sequence, each issue concerning the continuous version will have its counterpart in the discrete case. We will use the continuous version where a and b are non-negative functionals of time defined in the interval $[0, \omega]$.

[9]The resulting one-dimensional approach could be exposed to several critiques. Nonetheless, the core of our approach lies on the analysis of the sign of the net output of the different stages of the process. Hence, if one is interested in a multiple goods economy, we could apply a similar analysis defining a building phase (negative net output) as a phase in which no outputs are produced while some inputs are needed. Using the fairly general Georgescu-Roegen's Georgescu Roegen (1971) concept of process boarder, our time-to-build hypothesis corresponds to the case of an economic process in which, at the beginning, the flow factors cross the boarder only in one direction: from outside to inside and not the other way round.

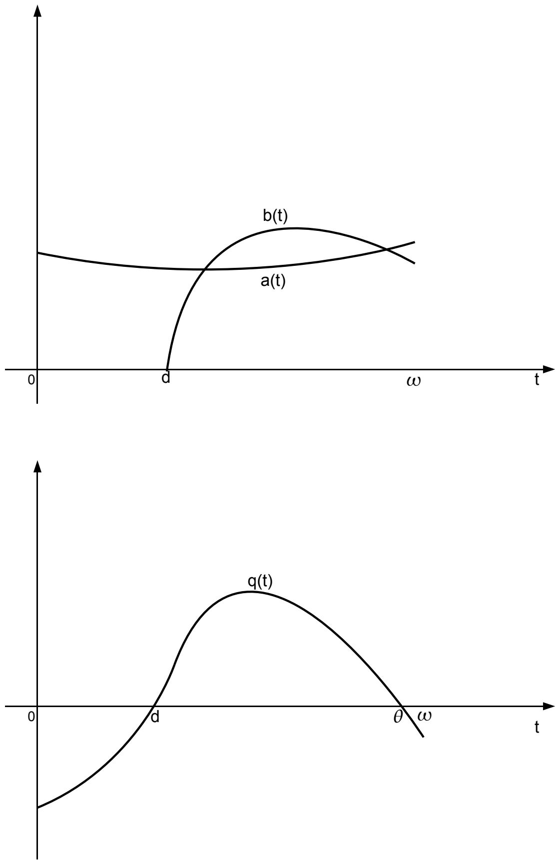

FIGURE 1. Time profiles of a Neoaustrian process

The capital value at the starting point represents the capital value of the process itself:

$$k_\rho(0) = k_\rho \ . \tag{2.7}$$

2.2. **Truncations.** All Neoaustrian analyses are made under the "free truncation" hypothesis: each process can be truncated at any stage of its length without any further cost in terms of input. We are free to decide the length of each process. For a given discount factor, the capital value of the process depends on its length. The optimal length θ is the one associated with the maximum capital value:

$$\theta(\rho) = arg \max_{\theta \leq \omega} [k_\pi(\theta)] = arg \max_{\theta \leq \omega} [\int_0^\theta q(s) e^{-\rho s} ds] \ . \tag{2.8}$$

A trivial example is in figure 1: the optimal length θ is lower than the natural length ω.

The free truncation hypothesis has many important implications.

Trivially, for the process to be activated its capital value at the starting point 0 can not be negative (otherwise it's optimal length would be 0). Moreover, a negative capital

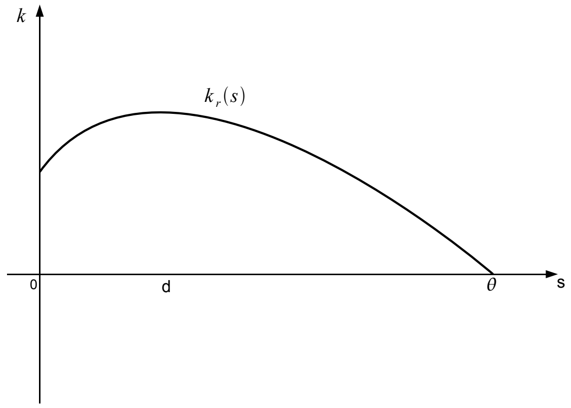

FIGURE 2. Capital value curve of a Neoaustrian process

value at any successive stage would imply a non optimal length choice. As a result, *the capital value at any stage before θ is non-negative.*

Second, *the capital value curve is upward sloping during the whole building phase.* Indeed, in such phase $q(s) < 0$ and with simple calculations we have

$$\frac{dk_\rho(s)}{ds} = \rho k_\rho(s) - q(s) > 0.$$

In other words, moving from the building phase towards the utilization one, the process goes on "facing costs and approaching proceeds".

Another important consequence of free truncation is the so-called "Fundamental Theorem: *the capital value is a decreasing function of the discount rate*[10]

$$\frac{dk_\rho(s)}{d\rho} < 0 \ . \tag{2.9}$$

Thus, there will be a discount rate associated with a zero discounted value of the net output sequence (the capital value at the beginning of the process). This is, by definition, the internal rate of return of the process[11]:

$$r: \qquad k_r = \int_0^{\theta(r)} q(s)e^{-rs}ds = 0 \ . \tag{2.10}$$

[10]In case of a fix and finite terminal date θ we have:

$$\frac{dk(s)}{d\rho} = \int_s^\theta (u-s)q(u)e^{-\rho(u-s)}du = sk(s) - e^{\rho s}\int_s^\theta uq(u)e^{-\rho s}ds =$$

integrating by parts:

$$= e^{\rho s}[e^{-\rho\theta}\theta k(\theta) - \int_s^\theta q(u)e^{-\rho u}du \ ]$$

$k(\theta)$ must be zero and the second term is always positive by the truncation hypothesis. For the case of $\theta(\rho)$ infinite or variable see Hicks Hicks (1973).

[11]The uniqueness of the internal rate of return is a corollary of the Fundamental Theorem.

Finally, as Hicks Hicks (1973) proved, *the optimal length is a non-increasing function of the discount rate* ρ: the higher the discount rate, the shorter (at least equal) will be the optimal length.

2.3. **Production systems.** Having fully defined Neoaustrian technologies, we can now turn our attention to the whole production system. At each period the system is characterized by a population of single production processes each one having a certain age. Let's consider a population of processes X_t belonging to the same technique, with a given optimal length θ:

$$x_t(s) \in \mathcal{R}^+ \qquad s \in [0, \theta] \tag{2.11}$$

the index s is the age of the process, and $x_t(s)$ is the number of processes of age s at time t.

The aggregate results of the production system in terms of total input and total output are:

$$A_t = \int_0^\theta a(s)x_t(s)ds \qquad B_t = \int_0^\theta b(s)x_t(s)ds \tag{2.12}$$

where A_t and B_t are total input (labor)and total output at time t.

At the given wage rate we can also obtain total net output Q_t:

$$Q_t = \int_0^\theta [b(s) - wa(s)]x_t(s)ds = \int_0^\theta q(s)x_t(s)ds \ . \tag{2.13}$$

and the capital value $K_t$of the whole population of productive processes computed at the interest rate r:

$$K_t = \int_0^\theta k(s)x_t(s)ds = \int_0^\theta x_t(s) \int_s^\theta q(u)e^{-(u-s)}(s)du \ ds \ . \tag{2.14}$$

3. The technological paradox

This flow approach to production allows to study the traverse: the behaviour of an economic system facing the appearance of a new and superior technique.

To analyze the technological paradox we will analyze the conditions for a temporary worsening of the economic performance during the initial phase of the traverse.

To this purpose, we will confront the traverse path of input and output, with the path that would result without technical change (the reference path).

3.1. **The simplest cases: the machinery effect.** The first two cases of technological paradox we will take into account come from the original idea of Ricardo Ricardo (London, 1821). In such a perspective the cause of the temporary worsening of the economic conditions is due to the particular kind of progress. In fact, the kind of technical progress that the economy was facing in Ricardo's time was that of a mechanization of the technology, that is, a more indirect production process. We refer to this case of technological paradox as to the "machinery effect".

In the NA context this "forward biased" technological progress can be modeled assuming that the new technique $\tilde{q}$ has a more costly building phase[12]. This higher cost

[12]Obviously, for the new technique to be better, the net output sequence in the utilization phase has to be better.

can be thought in input terms or in time terms.

At time 0 a new technique yielding a higher internal rate appears. In such a context the (only) direct effect of the innovation will be that all new processes activated will belong to this new technique. We will focus on the phase of the traverse along which both techniques coexist, hence up to time ω, when the last process of the old kind ends.

The difference in output terms between the actual path $\tilde{x}(t)$ and the path that would result without the innovation (the reference path) $\bar{x}(t)$ will be:

$$\tilde{B}(t) - B(t) = \int_0^t \tilde{x}(t-s)\tilde{b}(s)ds + \int_t^\omega x(t-s)b(s) - \int_0^\omega \bar{x}(t-s)b(s)$$

before time 0 the reference path $\bar{x}(t)$ corresponds to the actual path $x(t)$ thus:

$$\tilde{B}(t) - B(t) = \int_0^t \tilde{x}(t-s)\tilde{b}(s)ds - \int_0^t \bar{x}(t-s)b(s) \tag{3.1}$$

3.1.1. *Case 1: higher time cost.* We consider first the case of an innovation characterized by a higher lasting of the building phase: $\tilde{d} > d$[13]. During their building phase the processes do not produce any output:

$$b(s) = 0 \ \forall \ s \in [0, d)$$

$$\tilde{b}(s) = 0 \ \forall \ s \in [0, \tilde{d})$$

At any time t between $\tilde{d}$ and d we have from 3.1:

$$\tilde{B}(t) - B(t) = \int_0^t x(t-s)\tilde{b}(s)ds - \int_0^t \bar{x}(t-s)b(s) = -\int_\delta^t \bar{x}(t-s)b(s) < 0$$

In other words, in this interval of time the old kind processes in their utilization phase are not replaced, but the new kind processes are still not in the utilization phases. Thus, the output will fall below the reference path. This is the most simple way in which the "machinery effect" can appear.

3.1.2. *Case 2: higher input cost.* We turn now to the analysis of the second and more general case of forward biased technological progress (when the new technique has a higher building cost in input terms) to shed light on the mechanism that, in Ricardo' thought, is at the origin of the machinery effect: the need of building a new and more costly productive capacity brings to a temporary fall in the wage fund leading to a technological unemployment.

To formalize this issue we focus on the difference between the total employment on the traverse path and the total employment that could have been employed without the technological change (the reference path). In the fix-wage case, total labor $A(t)$ is proportional to the wage fund:

$$A(t) = \frac{1}{w}[B(t) - Q(t)]. \tag{3.2}$$

[13]To simplify the analysis, without loss of generality, from now on we will make the hypothesis that the two techniques have same optimal length: $\omega = \tilde{\omega}$.

As in the NA tradition, we make the hypothesis that the path of aggregate net output $Q(t)$ will follow the same path that would have been followed without the technical change. The difference on the total employment between the two paths becomes:[14]

$$\Delta A(t) = \frac{1}{w}[\tilde{B}(t) - B(t)] \tag{3.3}$$

For the sake of simplicity we suppose the new technique to have the same length parameters θ and d, and also the same output profile[15]:

$$\tilde{b}(t) = b(t) \ \forall \ t \in [0, \theta]. \tag{3.4}$$

The two techniques only differ for their input profiles. Thus, the difference between the output paths can be expressed as a b-weighted average of the difference between the activation rates:

$$\tilde{B}(t) - B(t) = \int_0^t \tilde{b}(s)\tilde{x}(t-s)ds - \int_0^t b(s)\bar{x}(t-s)ds = \int_0^t b(s)[\tilde{x}(t-s) - \bar{x}(t-s)]. \tag{3.5}$$

from 3.3 and 3.5 we have:

$$\Delta A(t) = \frac{1}{w}\int_0^t b(s)[\tilde{x}(t-s) - \bar{x}(t-s)] \tag{3.6}$$

Therefore, to study the sign of the difference between employment paths, we first have to study the sign of $y(t) = \tilde{x}(t) - \bar{x}(t)$.

The hypothesis of an exogenously given net output path implies that, at each point in time, the total net output of processes belonging to the new technique is equal to the total net output that would have been produced by the processes of the old kind which could have been activated from time 0 up to t along the reference path. Thus, for any time $t < \omega$ we have:

$$\int_0^t \tilde{q}(t-s)\tilde{x}(s)ds = \int_0^t q(t-s)\bar{x}(s)ds \tag{3.7}$$

thus

$$\int_0^t \tilde{q}(s)y(t-s)ds = \int_0^t [q(s) - \tilde{q}(s)]\bar{x}(t-s)ds \tag{3.8}$$

where the right hand side is known function and then this expression is a Volterra equation of first kind. As proved in Belloc (Paris, 1980), under quite general hypotheses, the sign of the solution of $y(t)$ depends on the kind of technological change involved. In

[14]We know that $\tilde{Q}(t) = Q(t)$, hence:

$$\Delta A(t) = \tilde{A}(t) - A(t) = \frac{1}{w}[\tilde{B}(t) - \tilde{Q}(t)] - \frac{1}{w}[B(t) - Q(t)] =$$

$$= \frac{1}{w}[\tilde{B}(t) - Q(t)]\frac{1}{w}[B(t) - Q(t)].$$

[15]This sort of normalization is deepened in Belloc (Paris, 1980).

the case considered by Ricardo, the input flow of the new technique is greater than the one of the old technique[16]:

(3.9) $$\tilde{a}(t) > a(t) \qquad \forall\ s < d$$

In this case $y(t)$ could be negative at least up to time d. The effect of the decline of productive capacity will be offset up to the time d when the new processes arrive in their utilization phase. From this point on there will be a period of technological unemployment followed by a rise in occupation over the reference path level. Therefore:

$$\Delta A(t) = 0 \qquad \forall t < d$$

$$\Delta A(t) < 0 \qquad \forall t \in [d, 2d]$$

$$\Delta A(t) > 0 \qquad \forall t \in [d + t_1, d + \theta]$$

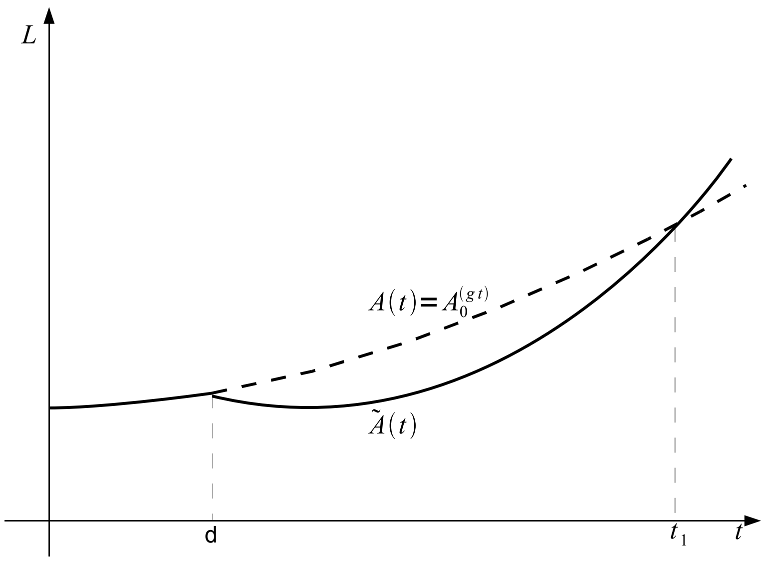

FIGURE 3. The machinery effect

Such employment path is shown in figure 3 (where the reference path is a steady growth path at rate g). The timing of the technological paradox is the same of the previous case: the falling of output begins when the processes of the old kind in utilization phase are not still replaced, but in this case it happens contemporary to the entering in the utilization phase of the first new kind processes.

[16]For the new technique to be better than the older one, its higher labor costs during the building phase must be more than compensated in the utilization phase.

3.2. **Optimal length variation.** In previous section we made use of two restrictive hypotheses: the particular kind of technological progress and the exogenously given path of aggregate net output $Q(t)$. Amendola Amendola (1974) attempted a generalization of the technological paradox removing such hypotheses and considering the option of a variation of the optimal length of the old kind processes. In 2.8 we stated that the optimal length θ, has a negative relationship with the discount rate. Hence, if computed at the (higher) discount rate equal to the new technique's internal rate of return $\tilde{r}$, the optimal length of old kind processes may decrease. For $t \geq 0$:

$$\bar{\theta} = \theta(\tilde{r}) < \theta(r) \tag{3.10}$$

According to the Author, in this case, all the old kind processes older than $\bar{\theta}$ should be suddenly truncated. Thus, the output will immediately fall below the reference path because of the scrapping of a part of the old kind productive capacity. This would happen whatever the kind of technological progress and the demand path we take into account.

The option of different optimal lives of capital goods is a typical issue on capital theory. But the option of a variation of the optimal length of a process *as a result of a rising of the rate of interest* has to be carefully deepened.

In the next section we will show that the variation of the optimal length is a quite particular case, and that the option of anticipating the truncation of *all* already existing processes older than the new optimal length, has to be considered with caution because it could not correspond to the capital value maximization principle.

Before discussing the consequences of the capital value maximization, let's consider an example of how such a truncation policy could not be compatible with this principle. Let's consider the case of a production system that, before the appearing of the innovation, was in a steady growth path at rate g:

$$x(t) = \alpha e^{gt} \qquad \alpha > 0.$$

When the truncation should occur, the aggregate capital value of all the processes older than the new optimal length is:

$$K_T(t) = \int_{\bar{\theta}}^{\theta} k_{\tilde{\rho}}(s) x(t - \bar{\theta} - s) ds = \alpha e^{g(t-\bar{\theta})} \int_{\bar{\theta}}^{\theta} k_{\tilde{\rho}}(s) e^{-gs} ds$$

where $k^{\tilde{\rho}}(s)$ is the capital value of a process of the old kind computed at the interest rate of the new technique.

$$K_T(t) = \alpha e^{g(t-\bar{\theta})} \int_{\bar{\theta}}^{\theta} \int_{s}^{\theta} q(u) e^{\tilde{\rho}(s-u)} e^{-gs} du\ ds$$

We can rearrange the integrand and after simple calculation write:

$$K_T(t) = \alpha e^{g(t-\bar{\theta})} \int_{\bar{\theta}}^{\theta} e^{(\bar{\theta}-u)\tilde{\rho}} \int_{s}^{\theta} q(s) e^{g(u-s)} e^{-gs} ds\ du = \beta \int_{\bar{\theta}}^{\theta} e^{-u\tilde{\rho}} k_g(u) du \tag{3.11}$$

where $k_g(u)$ is the capital value of a process at a stage u computed at a discount rate equal to g, and β is a positive constant.

We already told that, as consequences of the free truncation hypothesis, the capital value computed at the rate r of any stage before θ is positive and that the capital value is decreasing in the discount rate. Thus, at each stage s

(3.12) $$k_\rho(s) > 0 \qquad \forall \rho < r$$

The internal rate of a technique is equal to the maximum rate of steady growth attainable with such a technique[17]. Hence the rate of growth g is lower than the internal rate of return r.

Considering 3.12 we have:

(3.13) $$k_g(u) \geq 0 \ \forall u$$

thus

(3.14) $$K_T(t) > 0$$

As a consequence, contrary to what was implicitly considered in Amendola (1974), truncating all processes older than the new optimal length brings about a capital value loss, not a gain.

Nonetheless, the example of Amendola has many advantages. First, the falling in output and employment during the traverse is possible independently of the kind of technological progress. Second, the technological paradox is a direct consequence of the increase of the market interest rate and it is not due to a shortage of resources. Finally, the timing of the paradox is different in the different cases: in the "machinery" case it occurs when the old kind productive capacity stop to be replaced, and hence after a time span equal to the length of the old technology building phase; instead, *in this truncation case, it occurs suddenly at time* 0.

3.3. **Capital value variation, obsolescence and technological paradox.** In the preceding section a role for capital market was outlined implicitly. However, as we just told, the resulting scrapping policy isn't still fully coherent with the capital value principle. Hicks himself first noticed that a growth of the interest rate could bring below zero the capital value not only of the latest phases of the process. We shall now make a step forward, by analyzing in more depth the consequences on the capital value of old kind processes of an increase of the rate of interest over their internal rate of return.

At the beginning of the traverse, the capital market operates bringing to zero the market value of activating new processes of the new kind by rising the interest rate up to the internal rate of the new technique. Old processes will be truncated if the reallocation of resources to new processes will bring to a capital value gain. Hence, truncation will operate on all processes having a negative capital value.

3.3.1. *The optimal length variation revisited.* Before analyzing the optimal scrapping policy, it is useful to come back to the optimal length variation issue, analyzing the features of the processes that have different optimal lengths according to different discount rates. A variation of the optimal length occurs only if the capital value of the queue of the old kind process, computed at the new and higher discount rate, becomes

[17]This is a consequence of the duality of the prices and quantities frontier. Indeed, along such path the whole output is used to pay inputs and no net output is produced. See Hicks (1973) for more details.

negative. We know that the capital value is a (discounted) sum. A necessary condition for a sum to be negative is that some of its addends is negative. Hence, a necessary (and not sufficient) condition for a variation of the optimal length is that the net output flow of the queue of the process is somewhere negative; it will never occur whenever we make the hypothesis that the net output flow is positive during all the utilization phase as in figure 1. This last is probably the most common case of production process we could consider; instead, the concrete counterpart of the case of optimal length variation could be that of a production process that encompasses a "repairing" phase. To have an example, we consider the profile of net output in figure 4.

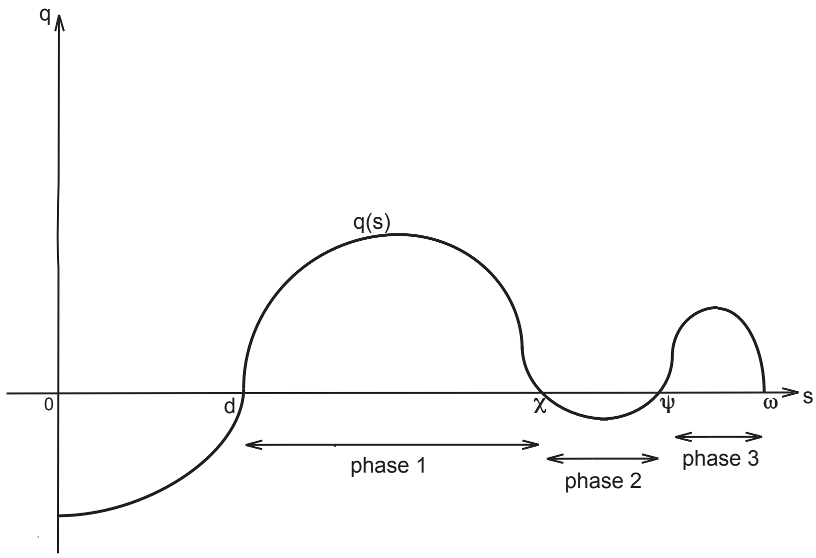

FIGURE 4. general net output profile

In this case to the characterization in the two phases building-utilization depending on the hypotheses in 2.2, we add a non-monotonic net output profile in the utilization phase[18]. The latter phase can be decomposed in three different phases. We can think at phases 1 and 3 as "normal utilization phases" where the net output is positive and decreasing: the process becomes the less and less productive. Phase 2 is exactly what we previously defined as a "repairing" phase: no output is produced, inputs are needed and net output is negative. This phase has to be followed by another utilization phase otherwise we would never face the repairing phase. This is exactly phase 3: the sum of the net output flow of this phase has to be higher than the sum of the negative net output flow yielded during the repairing phase. Indeed, for any phase to be present at some positive interest rate, a necessary condition is that the simple sum of its future net

[18]Obviously, a same net output path can be generated by an infinity of input and output path combination, but allowing for some flexibility, to give an economic meaning to the process' paths we refer to the more intuitive option.

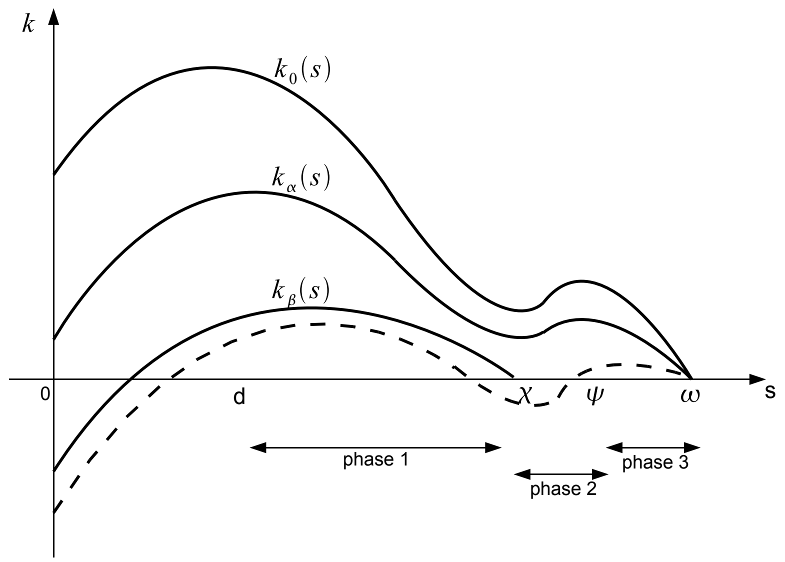

Figure 5. Variations of the capital value curve and of the optimal length induced by an increase of the rate of interest.

output $k_0(s)$) is positive, otherwise their capital value would be negative for any positive interest rate and the optimal length would always be placed before such phase.

In figure 5 we plot the capital value curves of such process for three different interest rates: 0, α and β, with $\alpha < \beta$. As happens in the building phase, the curve is increasing also in the repairing phase. The effect of an increase of the interest rate on the capital value profile is similar to a counterclockwise rotation of the curve around the point ω.

When the interest rate rises, the relative weight of the repairing phase will grow respect to the last utilization phases. When it rises up to β this will bring to a negative capital value of the stage corresponding to the beginning of the repairing phase. If we could not truncate the process, the correspondent capital value curve would be the dotted line. Having the option of truncating, the repairing phase would not be afforded and hence the optimal length changes to χ. The capital value curve, once we consider the new optimal length, is $k_\beta(s)$[19]. For further increases in the rate of interest the capital value curve will still varies in the counterclockwise way, with the new optimal length χ as "pivoting point", but the optimal length will not vary.

As we already told, such repairing phase cases are the only ones that allows for different optimal lengths. The variation of the optimal length is a more general phenomenon when the cause is not the increase of the rate of interest but is instead the increase of the price of primary inputs (wage, in this context) as in the vintage approach in Solow et al.

[19] The dotted line corresponds to the capital value curve of the process when truncation is not allowed. It becomes negative during the repairing phase. The actual curve $k_\beta(s)$ is higher than the dotted one just because the option of truncating at χ allows a capital value gain.

(1966), and in the full employment case in Hicks (1973). In this case the variation of the optimal length is caused by the increase in the price of primary factors that change to negative the sign of the net output flow of the queue of the process: we would not use expensive factors on low productive processes because the result is a negative net output. The most important difference between the case of the interest rate and the resource price induced variations of the optimal length is that in the second case we need a resource constrain that is not needed in our case. We will come back on the difference between resource constrain and market mechanism in the next section.

3.3.2. *Obsolescence as a market effect.* Let's consider now the optimal truncation issue. To this purpose we have to find which stages of the process have negative capital value when the interest rate varies, that is, which segment of a vertical integrated system are involved by the obsolescence effect induced by the appearing of a new production technique.

First, we have to consider that also when a "repairing phase" is present, not all processes older than the new optimal length will be truncated. Actually, the concept itself of optimal length refers to an ex-ante condition: it corresponds to the truncation stage that would have been optimal if the new market conditions (the interest rate) were already that when the process was activated. This optimality condition could not be met ex-post, when the process has already been activated and might have overcome the new optimal lentgh. As an example, considering the case of the dotted line in figure 5, differently from the case in Amendola (1974), the processes that, at the time innovation appears, have already overcome the most of the repairing phase will be surely carried on, at least all those already in phase 3. In this amended version of the optimal length variation, because of the truncated processes are all yielding a negative output flow, the scrapping policy will free more resources and the sudden decrease in total output will not take place.

Summing up, the obsolescence effect can concern only processes in stages yielding negative net output, but these are not necessary the latest. Indeed, a negative net output in the earlier phases is exactly what corresponds to our time-to-build hypothesis. As we noticed before, whatever the profile of the process, the capital value in the building phase is an increasing function starting from the capital value of activating the process. When the new technique appears the capital value of activating a process of the old kind is negative. As shown in figure 6, if the building phase lasts a positive measured time interval (that is, when there is time-to-build), a younger part of the population of processes in the building phase (up to the stage ζ) will have negative capital value, and then they will be truncated[20]. Thus, differently from the optimal length variation case, *when the capital value principle is properly applied, the economic relevance of the obsolescence process will concern the upstream stages of the production system*[21].

All truncated processes are in stages yielding negative net output and so their truncation has an immediate positive effect on total output. As a consequence more resources

[20]This will not happen if the building phase was timeless end only referred to stage 0 as in the point input continuous output case. For a mathematical treatment about the relevance of the non-singularities of the sign change of the net output profile see Patriarca (2008).

[21]A similar scrapping policy ca be found in Amendola and Gaffard (1998).

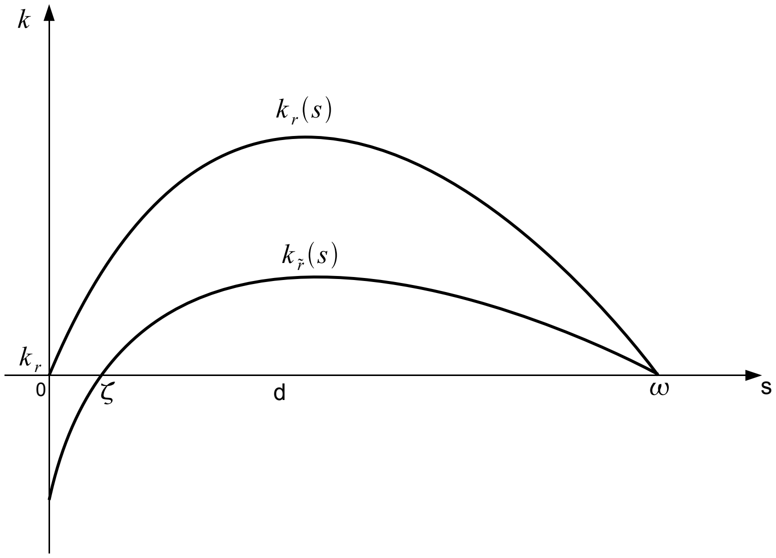

FIGURE 6. Variations of the capital value curve induced by an increase of the interest rate in the case of processes having simple profiles as in figure 1

will be disposable to activate new kind processes and then the diffusion of the new technology, in the building phase, might be faster. Yet, as time goes on, the time at which truncated processes would have arrived in the normal utilization phases comes. Thus the total output will fall (unless the processes of new kind, activated at higher rates had already entered in the utilization phase). We can therefore conclude that *the technological paradox will always take place whatever the kind of technological progress and the hypothesis about savings.*

Here we can find the other side of Ricardo's effect. The machinery effect, thought in a strong forward biased technical progress, is merely a technical effect. Instead, the latter phenomenon is a (capital) market effect of innovation. The appearing of a new technique cancels the capital value of younger old kind processes. *The falling phase of the economy is caused by the truncation of old kind productive capacity in the building phase* and occurs also in the case of a strong backward biased technological change and without shortening of the optimal length.

4. A possible role for capital market stickiness

Up to now we made the hypothesis that when the innovation appears the interest rate suddenly jumps up to the new internal rate of return. Things would radically change if the reaction of the market interest rate would not be so immediate.

If we compute the capital values at the old rate of interest r, the capital value of activating a new kind process $\tilde{k}_r$ is positive. The principle of maximizing the capital value would hence bring to the truncation of all old kind processes with a capital value lower than $\tilde{k}_r$. This could be the case not only for some processes in the downstream sages, but also for some of the processes in the utilization phase. As an example, in figure 7, we can take into account the case in which the rate of interest doesn't change and remain at the old level, corresponding to the internal rate of the old technique. All the processes that are in phases after ψ (and not only those before ζ) have a positive capital value but it is lower than the "extra-profits" $\tilde{k}_r$ allowed by the activation of a new kind process, and hence they should be truncated. In other words, reallocating the resources needed by these processes to the activation of new processes brings to a capital value gain. Hence, a stickiness of the capital market reaction, capable of slowing down the adjustment of the market interest rate to the new technique's internal rate of return, will have two consequences: the investments on new processes belonging to the new technique allows for extra-profits; some old kind processes in utilization phase will be scrapped notwithstanding their positive capital value.

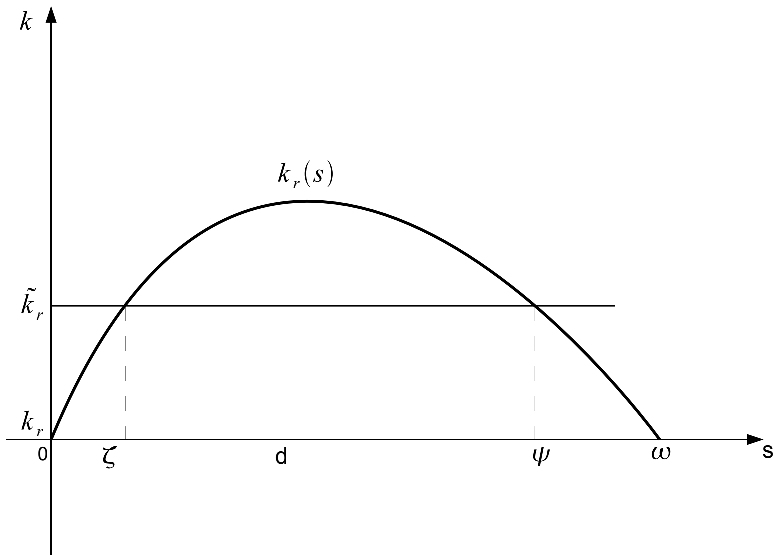

FIGURE 7. Scrapping stages in the case of extra-profits on the innovative process

As we already showed, the scrapping of processes in their utilization phase has the effect of a sudden decrease in total output. Beside, in the previous cases, the technological paradox occurred after a time lag d, when the productive capacity of the old kind that has been not yet produced should have arrived in its utilization phase. This was because the processes interested by the scrapping are only those in the upstream stages: a rise in the interest rate never bring to zero the capital value of a process in its utilization phase; in fact, their capital value were always greater than the null value corresponding to the activation of new kind process[22].

Hence, the timing of the technological paradox is strictly related to the hypothesis we made about the behavior of the interest rate: *a slow adjustment of the market interest rate would anticipate the falling on total output by the mean of a sudden truncation of already productive capacity.*

By the way, the meaningless of the capital value principle in the case of extra-profits needs to be analyzed with caution. Nonetheless, the aim of this last section is to propose an underlying mechanism, and not to present a model in a closed form.

In the other cases such principle brought to the truncation of processes that were in stages yielding negative net output. Hence, applying the capital value principle was only corresponding to the fact that activating a new process brings to higher returns than to keep on building the old kind productive capacity. Instead, when such a principle is applied to truncate processes having positive capital value and being in stages yielding positive net output, it is not so trivial. In such a case a somewhat financial constraint should act[23]. As an example, if at each time the producer has to anticipate the payment of inputs, a financial constraint would force him to rely on the capital value principle and not to pay the inputs for the processes whose capital value is lower, also if positive, as to invest more in the activation of new processes.

5. Conclusions

To analyze the possible temporary negative effect of an innovation, we shed light on the role of the time profile of production and of the time dissociation between costs and proceeds. Moving in the same direction of Amendola (1974), we focussed on the obsolescence effect linked to the option of activating processes belonging to a new technique with a higher internal rate of return. This allowed to generalize the technological paradox and to make it independent on the kind of technological progress and on the hypotheses about consumption.

Contrary to the little relevance (that we have shown) of the cases analyzed in the literature about the "optimal length variation", a deeper analysis of the capital value curve enabled a somewhat different scrapping effect that figures on the timing of the technological paradox. *The mechanism on which this phenomenon is based is not that of a primary resource constraint, instead, the scrapping policy is a result of a capital value principle.* Obsolescence occurs on the upstream stages of the production system just because of activating a new process brings to higher returns than keeping on building the old kind pruductive capacity.

In such a way the paradox can be linked to the schumpeterian concept of creative destruction[24]. The obsolescence process allows for a higher speed of diffusion of the

[22]In the more specific case of processes encompassing a repairing phase, truncations occurred also in the downstream stages, but only the processes in the repairing phase were involved (and not the older). Thus, also in this case, if it occurred, the paradox was lagged.

[23]Otherwise, the rate of activation of new processes would be infinite.

[24]See Schumpeter (1942)

innovation but *the hypothesis of time-to-build implies a lag between the destruction process and its creative effect; it is exactly in such time span that the technological paradox occurs.*

Finally, we sketch out a possible role for a stickiness of the market interest rate or whatever market setting that allows for extra-profits. The analysis suggests that *the speed of reaction of the capital market influences both the timing of the technological paradox and the distribution of truncations among the different stages of the production system.* In case of a sticky (flexible) market interest rate, the technological paradox will be anticipated (lagged). Furthermore, in this case the scrapping effect will involve the downstream stages of the production system, and not only the upstream ones. Thus, the extent of the paradox depends on the level of extra-profits allowed by the new technologies, and hence on the oligopolistic barriers in the innovative sectors, such as the patents protection.

All this, suggests a possible way to interpret the breaking, in the short run, of the virtuous circuit between innovation and growth that has been observed in recent years: the expected extra-profits on new technologies have crowed-out the resources needed to carry on already existing productive capacity, strengthening the "productivity slowdown".

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