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MONETARY AND FISCAL POLICY INTERACTION: WHAT IS THE ROLE OF THE TRANSACTION COST OF THE TAX SYSTEM IN STABILISATION POLICIES?

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ABSTRACT

In the theory of monetary and fiscal policy interaction, the assumption of Ricardian households isolates the determinants of fiscal policy instrument from the price stabilization policies carried out by the central bank. One of the main implications of the above mentioned Ricardian assumption is that the fiscal policy does not have any distortionary effect for the economy, i.e. it does not affect the behaviour of the households, supporting that way the fiscal policy's neutrality. The argument for this view comes if one assumes that fiscal policy has a distortionary effect on the behaviour of the agents. We relax the above non distortionary assumption by assuming that the imposition of the taxes is consistent with a transaction cost of the tax system that underlies the state - tax payer interaction. In this way we develop a channel through which the stability of prices carried out by the independent central bank is, within optimality, also a function of the fiscal policy determinants (the transaction cost, the tax rates and the debt level). The analysis is carried out in a framework of a monetary union, with two different countries. Within this framework, the effectiveness of a numerical fiscal rule is also examined.

Keywords: Monetary and fiscal policy interactions; Transaction cost of the tax system; Probability of re-election; Stability and growth pact
JEL classification: E52; E58; E62; E63; H60

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1. Introduction

In the theory of monetary and fiscal policy interactions, the assumption of Ricardian households isolates fiscal policy aspects from the determination of equilibrium. The non-distortionary (lump sum) taxation assumption, in the majority of these models, abstracts from the need to analyse the effect of the fiscal instrument, resulting in the neutrality of fiscal policy. Thus, monetary policy –as described by the Taylor rule[1]– plays a deterministic role in equilibrium, increasing the importance of the monetary policy parameters. Hence neither debt dynamics, nor the endogenous effects of the fiscal instruments, play a role in determining equilibrium, implying fiscal policy neutrality with respect to inflation.

By contrast, when one deviates from the assumption of Ricardian equivalence and turns to a distortionary system of taxation, then the inflationary consequences of fiscal policy for inflation have to be examined. Edge and Rudd (2003) show that the coefficient on inflation in the Taylor rule depends positively on the tax rate, indicating non-neutrality of fiscal policy.

Under the assumption of distortionary taxation, fiscal policy affects the behaviour of households, it distorts macroeconomic variables (income -via income and substitution effects-, consumption etc) and affects, in a dynamic way, the effectiveness of this fiscal instrument. Thus, by relaxing the assumption of non-distortionary lump sum taxation, we let fiscal policy have an endogenous effect on fiscal deficits and, consequently, on the debt path, generating consequences for inflation.

The non-neutrality of fiscal policy has been discussed in the literature in an asymmetric way[2] by using either *ad hoc* policy rules, in a stochastic environment, or a proportional tax system, in a general equilibrium framework. As Leeper and Yun (2005) mention, "*although instructive for some purposes, the assumption that all taxes are lump sum prevents the fiscal theory from being understood in the context of broader Public Finance*".

The analysis becomes more interesting when one considers the question of monetary and fiscal policy interactions in a monetary union, like, for example, EMU, with different countries and one central bank. It is then possible for each country's fiscal policy to have spillover effects on price stability. Such effects are validated for the euro area, by

[1] See Taylor (1993).
[2] See, for example, Leeper (1991), Woodford (1994), (2001), (2003), Benhabib et. al (1997), Schmitt-Grohe and Uribe (1997), (2001), Leeper and Yun (2005), Schabert and von Thadden (2006).

simulation results[3]: for example, an increase in Germany's public expenditures by 1% of GDP increases the (common) inflation rate by 0.4%.

Our analysis deviates from the above framework in the following ways:

By deviating from the homogeneity assumption, used in the majority of the literature**,** we analyse monetary and fiscal policy interactions within a Monetary Union, assuming heterogeneous countries. Our main contribution, within this framework, is the introduction of the *operation or transaction cost of the tax system* into economic (monetary and fiscal) policy analysis. In this way, we allow fiscal policy to endogenously affect (via the transaction cost) the price stabilisation policies of the Common Central Bank (CCB), thus resulting to the non-neutrality of fiscal policy.

The notion of this transaction cost (see also Jrbashyan and Harutyunyan (2006)) is similar to the notion of a transaction cost that appears in other market activities. It comprises the notions of tax collection (Barro (1979)), tax evasion (Allingham and Sandmo (1972), Feinstein (1991)) and tax compliance (Clotfelter (1983), Andreoni et. al. (1998)), which are closely related to the behaviour of different types of tax payers that refer to the state–tax-payer interaction.

Our paper focuses on two directions. In the first, we endogenously derive (within optimality) the functional form of the transaction cost. In the second, we investigate the constraints this cost (endogenously) imposes on the efficiency of fiscal policy. In this way, we develop a channel, through which, fiscal policy is related to price stability –the primary objective of the CCB– via the notion of the transaction cost of the tax system. Furthermore, within this framework, we investigate the effectiveness of fiscal rules (like, for example, the European Union Stability and Growth Pact) in building sound fiscal policies and maintaining price stability.

2. The model

We build a model with two types of countries: those with a debt level above their reference value (debtor countries) and those with a debt level below their reference value (non-debtor countries). For simplicity and without loss of generality, our economy consists of two countries, a debtor and a non-debtor one, and the central bank.

[3] See Quarterly Report on the Euro Area, (2007), European Commission, Vol.6(2).

Within each country, we assume that there exists a society, whose decision to re-elect the government is the result of the solution to an optimisation problem. Thus, in our economy, we have three agents: ***society***, ***government*** and the ***central bank***.

2.1 Society

S***ociety*** has preferences over consumption and public goods, which are described by a loss function. Its aim is to minimise this loss function. This minimisation procedure raises the issue of whether society should optimally re-elect the existing government or not, since it is the government's (policy) choices that determine, up to a point, its loss function.

Specifically, the society in either country has preferences over consumption (C) and public expenditure (G). These preferences are described by a loss function of the form

$$L^s = -C^2 - \beta G^2 \tag{1}$$

with $C = W - T$, where W stands for the wage and T for the tax levels.

Assuming $W = 1$, we can write:

$$C = 1 - T \tag{2}$$

2.2 The governments

As far as the ***government*** of each country is concerned, we deviate from the usual assumption of the benevolent dictator and we assume that it cares about the operational cost of its fiscal policy decisions. It produces public goods by issuing debt and by collecting taxes, facing a transaction cost. The government wants to be re-elected; its re-election depends on the probability of no re-election, P, which is discussed in more detail later. Hence the government cares about its probability of (no) re-election[4].

Governments choose their tax policies (tax rates) aiming at minimising their loss functions. In particular, each government's preferences are described by a loss function that depends on the *transaction cost* of the tax system, on *public expenditure* and on the *probability of no re-election*[5] (or, equivalently, the cost of not being re-elected).

More specifically, in our analysis we allow for a monetary union (MU) formed by two countries: the first, country i, has a debt level, expressed as a percentage of its GDP, greater than a reference value[6] $(\bar{b})$ i.e. $b_i > \bar{b}_i$. The second, country j, has joined the MU

[4] It is worth noting at this point, that this self-interest characteristic of the government is consistent with its interest in the frictions created by its policy choices.

[5] See equation (4) in the sequel.

[6] We use the small letters to express the magnitude, expressed in capital letters, divided by the GDP, i.e. for any X, it shall be $x = {}^{X}/_{Y}$.

with a debt level less than or equal to its reference value, that is, $b_j \leq \overline{b}_j$. For simplicity, let the reference debt value be the same for both countries, i.e. let $\overline{b}_i = \overline{b}_j = \overline{b}$.

The *transaction cost* of the tax system corresponds to a micro characteristic of public finance, reflecting the distortionary character of the tax system. Our aim is to enter this into a monetary and fiscal policy interaction model, in order to endogenously investigate, via a micro foundation setting, the way the distortions generated by fiscal policy affect the macroeconomic variables within MU and the conditions under which this takes place, if at all.

The imposition of taxes generates frictions due to state–taxpayer interactions. These frictions relate the behaviour of taxpayers, stipulated by economic, sociological and psychological characteristics, to tax administration aspects, like tax collection and tax compliance, thus resulting in a loss of the tax revenue, with budgetary consequences[7]. Since the effect of the marginal tax rate on the behaviour of tax payers is not controversial in fiscal theory, we consider the transaction cost of the tax system as a function of the tax rate.

Following Barro (1979), we assume[8] the operational cost of the tax system (Z_t) to be equal to net tax collections (T) multiplied by a function (f) of the tax rate (τ), i.e. the transaction cost function is $Z=T \cdot f(\tau)$. However, we deviate from Barro by not pre-assuming the properties of Z and f, but by endogenously deriving their properties from the optimal solution of the model. The only, reasonable, assumption we make is that $f'>0$, which reflects the notion of the operational cost, due to the existence of the tax rate.

As a logical consequence of the above and since the marginal tax rate is the source of frictions between the state and the tax payers, the government's fiscal policy, as implemented through the tax instrument, affects the households' opinion (satisfaction or not) of the government. If one assumes that the government wants to be re-elected by households, then such dissatisfaction entails a cost for the government. In our analysis we assume that this cost enters the governments' preferences; in other words, we assume that each government cares about the effect of its policies on the households' welfare.

More specifically, we assume that the exact effect of society's perception of the government's policy choices is embodied in a function Φ of the form:

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[7] Clotfelter (1983) reports that, in 1976, the unreported income in USA was \$75 billion to \$100 billion, or 7% to 9%, of the reported income.

[8] See Appendix 1.

$$\Phi\left(L^{s}\right)=\begin{cases}1\cdot L^{s} & , \quad \overline{L}<L^{s}\\ P\cdot L^{s} & , \quad \underline{L}<L^{s}<\overline{L}, P\in(0,1)\\ 0\cdot L^{S}=0 & , \quad L^{S}<\underline{L}\end{cases} \tag{3}$$

where P stands for the probability of no re-election, $\underline{L}$ and $\overline{L}$ are the critical values for the society's loss: if the actual loss exceeds $\overline{L}$, then the government is not re-elected (with probability $P=1$); if the loss is below $\underline{L}$, then the government is re-elected for sure (i.e. $P=0$), while it is $P\in(0,1)$, whenever the society's loss is between $\underline{L}$ and $\overline{L}$. Thus, the way society's loss function (L^{S}) affects the objective function of the government is determined by the way changes in L^{S} affect the society's willingness to vote for the government.

We use the *exogenous* probability of no re-election, P, as a measure of this willingness. Specifically, we assume that P corresponds to real numbers, in such a way that a high number is always related to high values of the society's loss function (L^{S}). Intuitively, this implies that the unsatisfied society, i.e. a society with a high loss function, will be less likely to re-elect the government, and hence P will be high. Hence, the loss function of the society affects the government's loss function, to the extent that the probability of no re-election indicates and it should, therefore, be included in the government's loss function.

So, the government's loss function can be written as:

$$L^{G}=Z^{2}+\xi\cdot G^{2}+P\cdot L^{S} \tag{4}$$

where G stands for the public good. The term $P_{i}\cdot L_{i}^{S}$ captures the exact effect the society's loss function has on the government's objective function. With the appropriate rearrangements, the above loss function can be written[9] as:

$$L^{G}=Z^{2}-\left(P\beta-\xi\right)\cdot G^{2}-P\cdot C^{2} \tag{5}$$

thus, in a two country world,

$$L_{i}^{G}=Z_{i}^{2}-\left(P\beta-\xi\right)\cdot G_{i}^{2}-P\cdot C_{i}^{2} \tag{5.i}$$

corresponds to the objective function of the government of country i and

$$L_{j}^{G}=Z_{j}^{2}-\left(P\beta-\xi\right)\cdot G_{j}^{2}-P\cdot C_{j}^{2} \tag{5.j}$$

[9] Notice that if it is the case that $P\beta-\xi>0 \Rightarrow P>\xi/\beta$ then the government's loss reduces with an increase in the public expenditures underlying the characteristics of a myopic government, when it is most likely not to be re-elected (Alesina (1990), Alesina and Tabellini (1987), Cukierman and Meltzer (1989), Tabellini and Alesina (1990), Rogoff (1990)). On the other hand when $P<\xi/\beta$ (i.e when there is, practically, no political uncertainty) an increase in expenditures implies a budgetary cost for the government.

to that of country *j*.

In our analysis, we delve into deriving conclusions about the effectiveness of fiscal rules when the countries are heterogeneous within the MU. The role of the budget constraint is vital for this investigation. Crucial for such a budget constraint is the way the fiscal rules enter and affect the countries of our economy, both the debtor and the non-debtor country.

Following Beetsma and Uhlig (1999) and Debrun (2007) as well as that fiscal rules constitute an inhibitory mechanism for governments to accumulate public debt beyond a certain threshold, $\overline{b}$, we introduce the *cost for breaching this threshold*. One way of approaching this cost is by considering it as a numerical fiscal rule (like, for example, the *Stability and Growth Pact* (SGP)) that corresponds to utility or budgetary losses for the policy makers, when they deviate from the rule[10].

However, in the case of numerical fiscal rules, these losses for the breaching (debtor) country are assumed to be distributed among the other (non-debtor) countries of the MU, implying budgetary gains for them. Additionally, we define $\delta \in (0, 1)$ as the degree of austerity for deviating from the rules. Thus, the term $\delta\left(b_i - \overline{b}\right)$ shows the loss for the debtor country (with $b_i > \overline{b}$), or, equivalently, the gain for the non-debtor country. In other words, $\delta\left(b_i - \overline{b_i}\right)$ stands for the cost to country *i* of breaching the threshold of this reference value.

More specifically, the recourse constraint for the ***debtor country***[11] can be written as:

$$Y_t^i + B_t^i - \delta\left(B_i - \overline{B}_i\right) = G_t^i + B_{t-1}^i\left(1 + \pi^e - \pi\right) \Rightarrow$$

$$G_t^i = Y_t^i - \delta\left(B_i - \overline{B}\right) + B_t^i - B_{t-1}^i\left(1 + \pi^e - \pi\right)$$

Dividing by the country's national income at time *t*, Y_t^i, we get:

$$g_t^i = 1 - \delta\left(b_i - \overline{b}\right) + b_t^i - b_{t-1}^i\left(1 + \pi^e - \pi\right) \tag{6}$$

where π^e and π are the expected and actual levels of inflation, respectively, and b_{t-1}^i is the amount of debt issued by the government and traded in the market in the preceding period.

[10] This can be consistent with the Act (104)c of the Maastricht Treaty, where the cost for deviating from the rules (i.e. for having a debt level greater than 60% of its GDP) is set to be a non negligible amount of GDP.
[11] Recall that country *i*'s debt level is assumed to be greater than its reference value.

Following Beetsma and Uhlig (1999) we assume that the real world market interest rate is zero. In order for risk neutral agents to be willing to hold government bonds the expected rate of return, π^e, includes a mark-up equal to the expected inflation rate. The ex-post real rate of return is given by $\pi^e - \pi$.

For the ***non-debtor country**, j* (whose debt level is below its reference value), the resource constraint captures the idea that following sound fiscal policies results in a gain within the above-mentioned redistribution[12] among the (debtor and the non-debtor) countries. These constraints can be written as

$$Y_t^j + B_t^j + \delta\left(B_i - \overline{B}_i\right) = G_t^j + B_{t-1}^j\left(1 + \pi^e - \pi\right)$$

$$G_t^j = Y_t^j + \delta\left(B_i - \overline{B}\right) + B_t^j - B_{t-1}^j\left(1 + \pi^e - \pi\right)$$

where, dividing by Y_t^j, we get the way the government finances its expenditures.

$$g_t^j = 1 + \delta\left(b_i - \overline{b}\right) + b_t^j - b_{t-1}^j\left(1 + \pi^e - \pi\right) \tag{7}$$

Summarising, the problem the government of each country (*i, j*) faces, can be written as:

$$\min_{b_i, \tau_i} L_i^G = (z_t^i)^2 - \left(P\beta - \xi\right)(g_t^i)^2 - P(c_t^i)^2$$

$$\text{s.t } g_t^i = 1_t^i - \delta\left(b_i - \overline{b}\right) + b_t^i - b_{t-1}^i\left(1 + \pi^e - \pi\right)$$

with $z = {}^{Z}\!/_{Y}$, and

$$\min_{b_j, \tau_j} L_j^G = (z_t^j)^2 - \left(P\beta - \xi\right)(g_t^j)^2 - P(c_t^j)^2$$

$$\text{s.t. } g_t^j = 1 + \delta\left(b_i - \overline{b}\right) + b_t^j - b_{t-1}^j\left(1 + \pi^e - \pi\right)$$

Obviously, $\delta = 0$ corresponds to the case where there exists no numerical fiscal rule.

2.3 The common central bank

The common central banker is appointed in order to carry out price stabilisation policies. He takes into account the total debt level and derives the optimal inflation rule to follow, for any given fiscal policies. This implies that the central banker may also be aware of the constraints that unsustainable public finances impose on the monetary policy rule, which is consistent with the non-Ricardian Regime of our analysis, reflecting that way aggregate demand and aggregate supply, including the effect of fiscal policy, affect price

[12] An alternative way of approaching this is to think of the term $\delta\left(b_i - \overline{b}\right)$ as the level of the penalty, imposed on the debtor country, for deviating from the debt rules and distributed among the other countries.

stability. On the other hand, in our model the rate of inflation works as an interest rate, with its changes affecting the debt level for each country (and hence the total debt for the MU).

The above allow us to write the total debt level for the MU as a function of the rate of inflation π, i.e. $\tilde{b}(\pi)$. So, it is logical to assume that the CCB has preferences about inflation and the fiscal constraints described by the path of the total debt at the MU levels[13]. The CCB's objective is to determine the (common for both countries) inflation level, which is consistent with its stabilisation policies. This objective is described by a loss function of the form:

$$L^{CCB} = \gamma\left(\tilde{b}(\pi)\right)^2 + \theta\pi^2$$

with γ, $\theta \in (0,1)$ and where $\tilde{b}$ stands for the total MU debt, i.e. $\tilde{b} = b_i + b_j$.

The problem the CCB faces can be written as

$$\min_{\pi} L^{CCB} = \gamma\left(\tilde{b}(\pi)\right)^2 + \theta\pi^2$$

3. Solution

We solve the above optimisation problems as a simultaneous equation problem. We look for the optimal policies that each of the countries and the central banker should adopt, in order to minimise their losses.

Within this framework, we also consider the constraints imposed on the efficiency of the fiscal policy through the transaction cost of the tax system, and the way this is related to the political cycle. The existence of a transaction cost captures the fact that, before the time tax payers are called upon to pay taxes, they have already weighed up their gains and losses from this procedure and they decide, accordingly, on the degree to which they will comply with the fiscal rules. This relationship between tax-payers and the state (represented, in each time, by the governing political party) is a direct implication of the tax system that creates frictions. It is these frictions that govern the magnitude of the transaction cost and that, eventually, circumscribe the efficiency of the fiscal policy, putting pressure on the price stabilisation policy of the CCB.

[13] See also Van Aarle et. al (1997), Beetsma and Uhlig (1999), Chari and kehoe (2007) and Nguyen and Kakinaka (2006).

In this way, we describe a mechanism through which the fiscal policy of each country within the MU interacts with the monetary policy of the CCB, where optimality depends on the microeconomic characteristics of the fiscal policy.

The first order conditions for the government of country *i's* maximisation problem, are:

$$\frac{\partial L_i^G}{\partial \tau_i} = 0 \Rightarrow z_i \left(f_i + \tau_i \cdot f_i' \right) = \left(\beta \cdot P_i - \xi \right) g_i - P_i \cdot c_i \tag{8}$$

and

$$\frac{\partial L_i^G}{\partial b_i} = 0 \Rightarrow 1 + \left(1 - \delta\right) b_t^i - c_i + \delta \overline{b} - b_{t-1}^i \left(1 + \pi^e - \pi\right) = 0 \tag{9}$$

For country *j*, the first order conditions, require that

$$\frac{\partial L_j^G}{\partial \tau_j} = 0 \Rightarrow z_j \left(f_j + \tau_t^j \cdot f_j' \right) = \left(\beta \cdot P_j - \xi \right) g_t^j - P_j \cdot c_t^j \tag{10}$$

and

$$\frac{\partial L_j^G}{\partial b_j} = 0 \Rightarrow 1 + b_t^j - c_j + \delta \left(b_t^i - \overline{b} \right) - b_{t-1}^j \left(1 + \pi^e - \pi\right) = 0 \tag{11}$$

From equations (8) and (10), we can easily derive (since $f' > 0$ for both *i* and *j*) the probability of no re-election for any country:

$$P > \frac{\xi \cdot g}{\beta \cdot g - c} \tag{12}$$

This proves that the existence of the tax system is consistent with frictions, reflecting the state – tax-payers' interaction, which is not independent of the households' preferences, as these are expressed by the government's re-election. This is also justified by the existence of the political cycle.

The first order condition for the CCB's optimisation problem, gives the optimal inflation rule:

$$\frac{\partial L^{CCB}}{\partial \pi} = 0 \Rightarrow \frac{\partial \tilde{b}}{\partial \pi} = -\frac{\theta}{\gamma} \cdot \pi \cdot \tilde{b}^{-1}\left(\pi\right) \tag{13}$$

4. Analysis

4.1 The transaction cost of the tax system

In this section we carry out a theoretical investigation of the way the transaction cost of the tax system (expressed as a percentage of GDP) is related to the tax rate and we endogenously derive the properties of the transaction cost function.

From the fist order conditions (8) and (10), it can be shown[14] that in any country, at any time *t*,

$$\left(f_t + \tau f_t'\right) = \frac{1}{z_t}\left[\left(\beta P - \xi\right) g_t - P \cdot c_t\right] \tag{14}$$

The transaction cost is described by the function $z = \tau \cdot f(\tau)$. Hence, differentiating with respect to τ, gives $\frac{\partial z}{\partial \tau} = \left(f + \tau f'\right)$. Thus, $f + \tau f'$ shows the way the transaction cost changes with the tax rate. We treat $f + \tau f'$ as the first order derivative of a function z with $z(\tau)=\tau \cdot f(\tau)$. The second order derivative of this function, $z''(\tau)$, alternates sign and so the transaction cost function $z(t)$ exhibits an inflection point[15] at

$$\left(z^*, \tau^*\right) = \left(P(\beta \cdot g - c) - \xi g, \tau^*\right)$$

As a result, the operation cost of the tax system is related, in a non-linear way, to the tax rate, as can be seen from *Figure 1*[16]. Moreover, from this figure, it is obvious that the transaction cost of the tax system (and hence the frictions as well) increases at a diminishing rate for tax rates less than τ^*, while it increases at an increasing rate, for tax rates greater than τ^*.

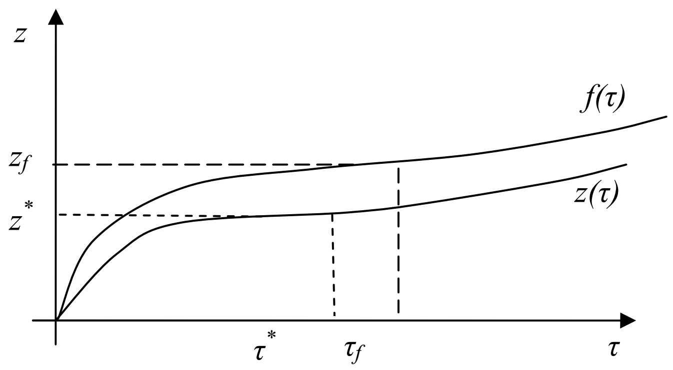

Figure 1: The operation cost of the tax system as a non linear function of the tax rate.

[14] For the moment, we abstract the lower case indices (*i, j)*, which distinguish the countries.
[15] For the proof, see Appendix 1.
[16] See, also, Appendix 3.

Rearranging (14) gives

$$\left(f_t + \tau f_t'\right) = \frac{1}{z_t}\left[P\left(\beta \cdot g_t - c_t\right) - \xi \cdot g_t\right] \tag{14.a}$$

which shows that society's reaction to the government's economic policy choices (as this is reflected in the probability of no re-election *(P)*), affects the rate of change of the transaction cost of the tax system ($f_t + \tau f'_t$). Consequently, the political cycle is related to the government's choices and to the, endogenous, way in which the efficiency of the fiscal policy works, embodying, in this way households' preferences, as these are shaped by the welfare implications of the government's fiscal policy.

In an attempt to qualitatively represent the frictions fiscal policy creates, as a result of the state – tax-payer relation, we describe in the sequel the exact way the political cycle is related to the transaction cost of the tax system:

Abstracting from the mathematical analysis (which is provided in Appendix 4), it can be seen from *Figure 2* that, when the change of the transaction cost, as a result of the government's choice to increase the tax rate, is relatively small, then the probability of no re-election for the government is reduced. The economic, sociological and psychological aspects related to the fiscal policy choices, which reflect the behaviour of the households, are related, via the probability of no re-election, to the political cycle and the way this affects the government's economic choices, if it cares about the political cost (re-election or not) of these choices.

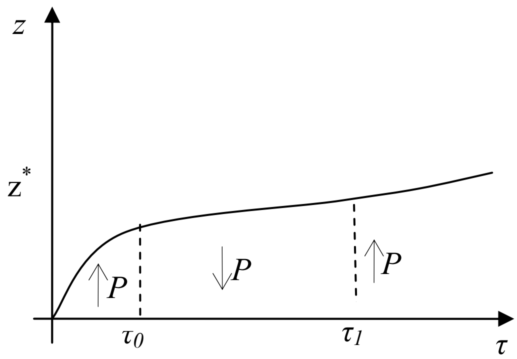

Figure 2: Changes in the probability of no reelection along the transaction cost function.

Thus, when the government chooses to increase the tax rate within the range [τ_0, τ_1], the increase in the transaction cost of the tax system is relatively small and it is related to an <u>efficient</u> fiscal policy, since it is consistent with small frictions. The probability of no re-election decreases within this area and, hence, the government is more likely to be re-elected.

When the government chooses tax rates higher than τ_1, the operation cost of the fiscal choices increases resulting in inefficiencies. The state–tax-payers' interaction is more likely to be characterised by large frictions and it has a political cost, since the probability of no re-election increases.

From *Figure 2*, we can observe that the probability of no re-election (seen as an indicator of the people's dissatisfaction with the government's tax policy) also rises for very small tax rates. This, within optimality and in terms of our analysis, corresponds to an area of tax rates close to zero, where the transaction cost function is very inelastic[17], indicating that small increases in tax rates are consistent with very high rates of growth of the transaction cost.

This is in line with the logic of our model and the notion of the transaction cost. In other words, it describes society's dissatisfaction (an indicator of which is the probability of no re-election, *P*) when, from a world of no taxes (i.e. zero tax rates) the government starts imposing taxes (i.e. positive tax rates).

However, here we do not go into the details of this case since, as real world practice indicates, the tax rates chosen by the fiscal authorities are not negligible. Hence, our analysis focuses on tax rates greater than τ_0.

4.2 Reply functions

In this section we derive the optimal ***reply functions*** for each country, which depict the way the fiscal policy of each country interacts with that of the other, through the constraints imposed by the monetary policy of the independent CCB. This is done by combining the first order conditions of the optimisation problems previously described.

More specifically, these reply functions are derived from the first order conditions (9) and (11) of the countries' problems and the optimal inflation rule, as this is defined by the first order conditions (13) of the CCB's problem.

[17] $\lim_{\tau \to 0^+} \left(f + \tau f' \right) = \lim_{\tau \to 0^+} \frac{P(\beta\gamma - c)}{z} = \frac{-P/Y}{0} = -\infty$

Under the assumption of rational expectations and after the appropriate rearrangements we derive[18] the optimal reply functions for each country. The reaction function $b_j^i(b_i) = b_{jt}^i$, for country *i*, is

$$b_{jt}^i = \frac{1 - c_i + \delta\overline{b} - (1+\pi)b_{t-1}^i}{\dfrac{\gamma}{\theta}\dfrac{\partial\widetilde{b}}{\partial\pi}b_{t-1}^i} + \frac{(1-\delta) - \dfrac{\gamma}{\theta}\dfrac{\partial\widetilde{b}}{\partial\pi}b_{t-1}^i}{\dfrac{\gamma}{\theta}\dfrac{\partial\widetilde{b}}{\partial\pi}b_{t-1}^i}b_t^i \tag{15}$$

Similarly, for country *j*, the reaction function $b_j^j(b_i) = b_{jt}^j$, is

$$b_{jt}^j = -\frac{1 - c_j - \delta\overline{b} - (1+\pi)b_{t-1}^j}{1 - \dfrac{\gamma}{\theta}\dfrac{\partial\widetilde{b}}{\partial\pi}b_{t-1}^j} - \frac{\delta - \dfrac{\gamma}{\theta}\dfrac{\partial\widetilde{b}}{\partial\pi}b_{t-1}^j}{1 - \dfrac{\gamma}{\theta}\dfrac{\partial\widetilde{b}}{\partial\pi}b_{t-1}^j}b_t^i \tag{16}$$

Both optimal reply functions are negatively sloped[19] with

$$\textit{slope of } b_j^i(b_i) < \textit{slope of } b_j^j(b_i)$$

These properties of the loci $b_j^{i,j}(b_i)$ are depicted in *Figure 3*.

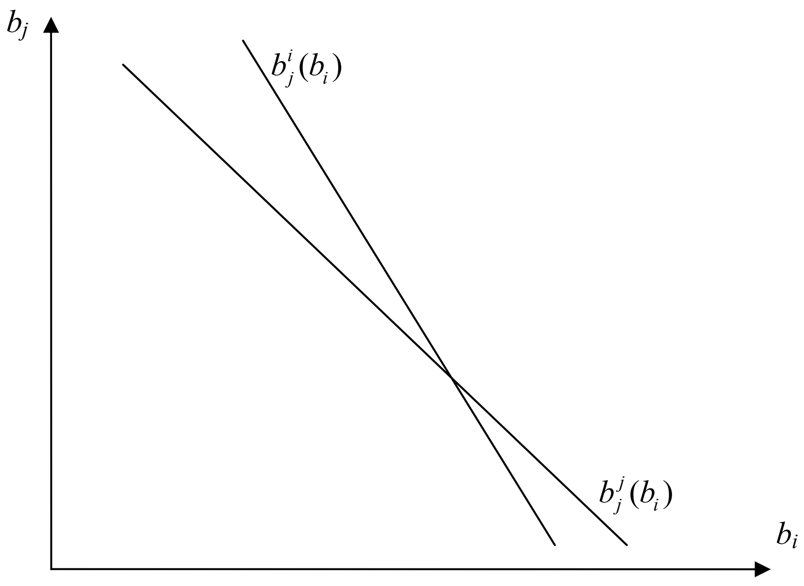

Figure 3: The reaction functions

We will show that these reaction functions embody the way the microeconomic characteristics of the fiscal policy, explained by the frictions of the transaction cost of the

[18] For the proof, see Appendix 5.
[19] See Appendix 6.

tax system, are related to the political cycle and the monetary policy followed by the independent CCB.

The equilibrium is determined by the intersection point of the two reaction functions. This will be derived by simultaneously solving[20] equations (15) and (16), which gives:

$$b_j = \frac{\beta P_j - \xi}{P_j} g_j - \frac{z_j\left(f_j + \tau_j f_j'\right)}{P_j} - \frac{b^j_{t-1}}{b^i_{t-1}}\left[\frac{\beta P_i - \xi}{P_i} g_i - \frac{z_i\left(f_i + \tau_i f_i'\right)}{P_i}\right] + \frac{\tilde{b}_{t-1}}{b^i_{t-1}}\delta\bar{b} + \frac{b^j_{t-1} - b^i_{t-1}}{b^i_{t-1}} - \frac{\delta b^i_{t-1} + (1-\delta) b^j_{t-1}}{b^i_{t-1}} b_i \tag{17}$$

Expression (17) provides a rule for the path of the economy, since it characterises all the intersection points of $b^i_j(b_i)$ and $b^j_j(b_i)$, for any given level of π and τ.

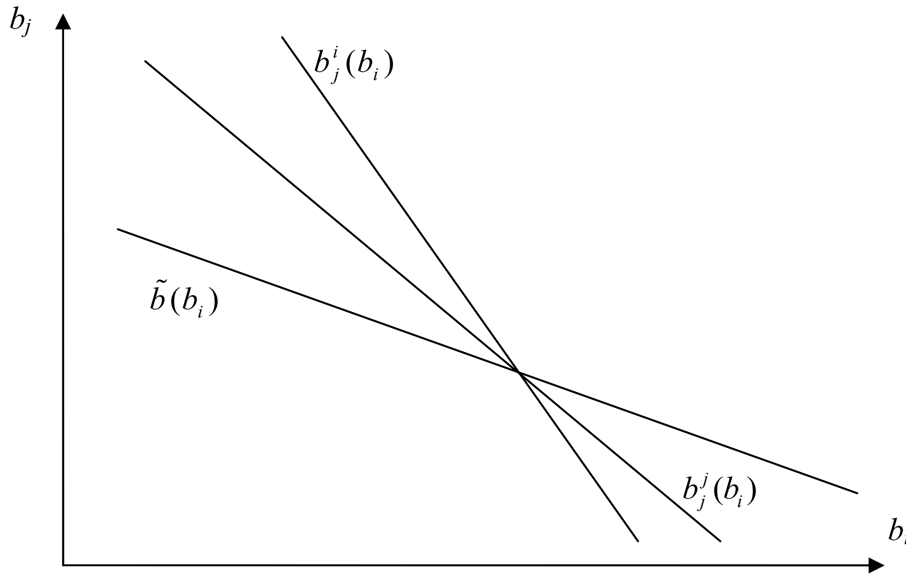

Figure 4: The representation of the economy's total debt, as determined by the intersection points of the countries' reaction functions.

4.3 The CCB response

The Common Central Banker is independent. His only objective is to use his policy instrument, the interest rate, in order to keep the inflation rate below or equal to a reference value[21], under the constraints imposed by the fiscal policies of the country-members. This implies a relationship between the fiscal policies of the two countries (as

[20] See Appendix 7.
[21] In the case of EMU this could be less but close to 2%.

described by the level of total MU debt ($\tilde{b}$)) and price stability (which is the CCB's objective)[22].

As shown before (eq. 13), given optimisation, monetary and fiscal policies are related as follows:

$$\frac{\partial \tilde{b}}{\partial \pi} = -\frac{\theta}{\gamma} \cdot \pi \cdot \tilde{b}^{-1}(\pi)$$

We look at the above relationship, as the first order derivative of a function $\tilde{b}$ *(π)*, which describes the best response of the CCB regarding the changes in the total debt level within the MU. Its second derivative requires that, at the minimum loss[23], $\frac{\partial^2 \tilde{b}}{\partial \pi^2} < 0$. Hence, we have endogenously derived the properties of a function $\tilde{b}$ *(π)*, which shows the optimal response of the CCB to the fiscal outcome and the effect this response has on the total debt level.

The above are described in Figure 5 below.

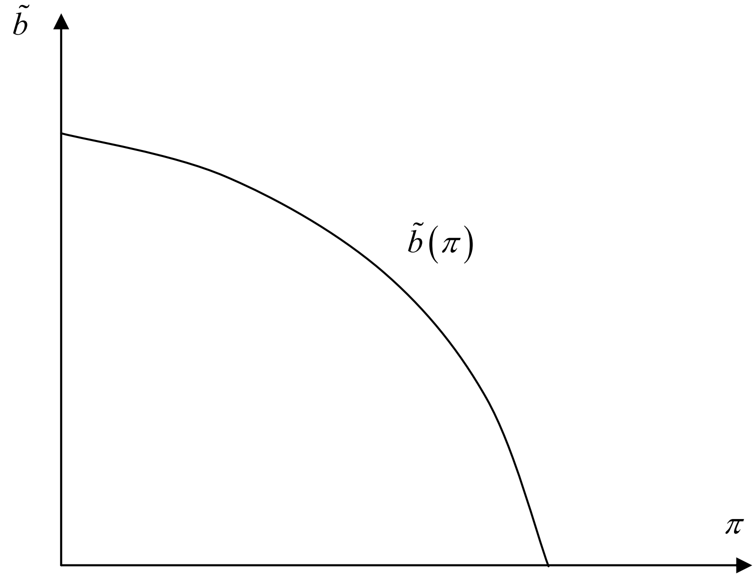

Figure 5: CCB's optimal response to the fiscal outcome ($\tilde{b}$).

Since the first order derivative $\partial \tilde{b} / \partial \pi$, as described by (13), is a function of the inflation level, π, the value of the total debt within the MU ($\tilde{b}$) will depend on the level of inflation. That is, whenever inflation changes, the MU's total debt is affected and this is

[22] See eq. (6), (7), (15) and (16).
[23] See Appendix 8 for the proof.

shown by moving along the line $\widetilde{b}\,(\pi)$. This is because a tight monetary policy by the CCB increases the *ex post* real rate of return, $\pi^e - \pi$ and, hence, the debt level for each country member, via the term $(1+\pi^e-\pi)b_{t-1}$ of their budget constraint. Woodford (2003) notes that "*… monetary policy (the choice of i_t) affects the evolution of the public debt, even in the absence of seignorage revenues, through its consequences for debt service on existing government debt. In most advance economies, this is actually the most important fiscal consequence of monetary policy …*".

Additionally, the shape of the $\widetilde{b}\,(\pi)$ function, as well as the economic interpretation of this shape, is along the lines of King's (1995) analysis. He uses the expression *"some unpleasant fiscal arithmetic"* in order to describe the effects of the central bankers' disinflationary policies on the debt dynamics.

4.4 Economic policy analysis

In this section we aim at making economic policy inferences. For this, we will combine the properties of the above-mentioned transaction cost and the constraints it imposes on the efficiency of fiscal policy, with the derived reaction functions of the country members and the CCB. In this way, we will be able to determine the debt path for each country separately, as well as the path of total debt (for the MU).

This procedure will also enable us to specify the optimal trajectory for our economy, which is not independent of the CCB's optimal policy (as this is formally explained by its reply function), thus implying the existence of second order effects.

Because the degree of the countries' homogeneity is a determining factor of the whole economy's path, in the sequel we study our economy under different scenarios regarding the countries' level of the transaction cost, as this cost is assumed to be the main factor for homogeneity. Hence, we can study the constraints, if any, it imposes on the strategic behaviour of the countries, as well as the way this is related to the political cycle.

We can categorise our scenarios into two groups: The first consists of the cases where the countries face considerably different levels of transaction costs, or equivalently, where they lie within different ranges of the transaction cost function *z*. Within our framework, this is the most "realistic" scenario[24]. The second scenario, includes cases where the countries (although heterogeneous regarding, for example, their initial debt

[24] This also applies to the EMU case of heterogeneous countries (as this is signified by the existence of fiscal rules, like, for example the Stability and Growth Pact).

conditions) face similar transaction costs, that is, they both lie within the same range of the transaction cost function *z*.

We show that when we have a considerable degree of heterogeneity, the transaction cost of the tax system is the factor determining the countries' policy choices and hence, the resulting level of debt and its inflationary consequences. On the other hand, when we have a considerable degree of homogeneity, it is the countries' strategic behaviour that determines the outcome.

Scenario 1: when the countries are asymmetric with respect to their tax system's transaction cost function

When the fiscal authorities choose tax rates that differ considerably from each other, corresponding to different levels of costs on the transaction cost function, then the countries face different properties regarding their tax system's transaction cost and, consequently, they are related to different efficiency levels of their fiscal/tax policies. In this case, the (initial) heterogeneity of the countries results in a different outcome for the economy, depending on which country (the initially better-off or the other one) has set the higher tax rates. We therefore assume that each country increases its tax rate and we look for the economy's outcome within the following cases.

CASE 1.1: the debtor country, *i*, has introduced lower tax rates than the non-debtor one

In this case[25] country *i* has a relatively lower rate of increase of its transaction cost due to each unit increase of the country's tax rates, compared to that of country *j*. This, makes country *i*'s tax instrument more efficient in raising finance in order to serve the debt.

Within optimality[26],

$$\frac{\partial CT_i}{\partial P_i} = -\frac{\xi g_i + z_i\left(f + \tau f'\right)_i}{\frac{\gamma}{\theta}\frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i P_i^2} > 0 \tag{18}$$

where CT_i stands for the constant term of country *i*'s reply function.

This suggests that any increase of P_i (as a result[27] of a government's attempt to increase tax rates in order to finance the country's debt) would shift the locus $b_j^i(b_i)$ backwards from $b_j^i(b_i)_0$ to $b_j^i(b_i)_1$, resulting in a reduction of country *i*'s debt level.

[25] Because of the form of the transaction cost function *z*, as this is described in detail in 1.

[26] See Appendix 5.

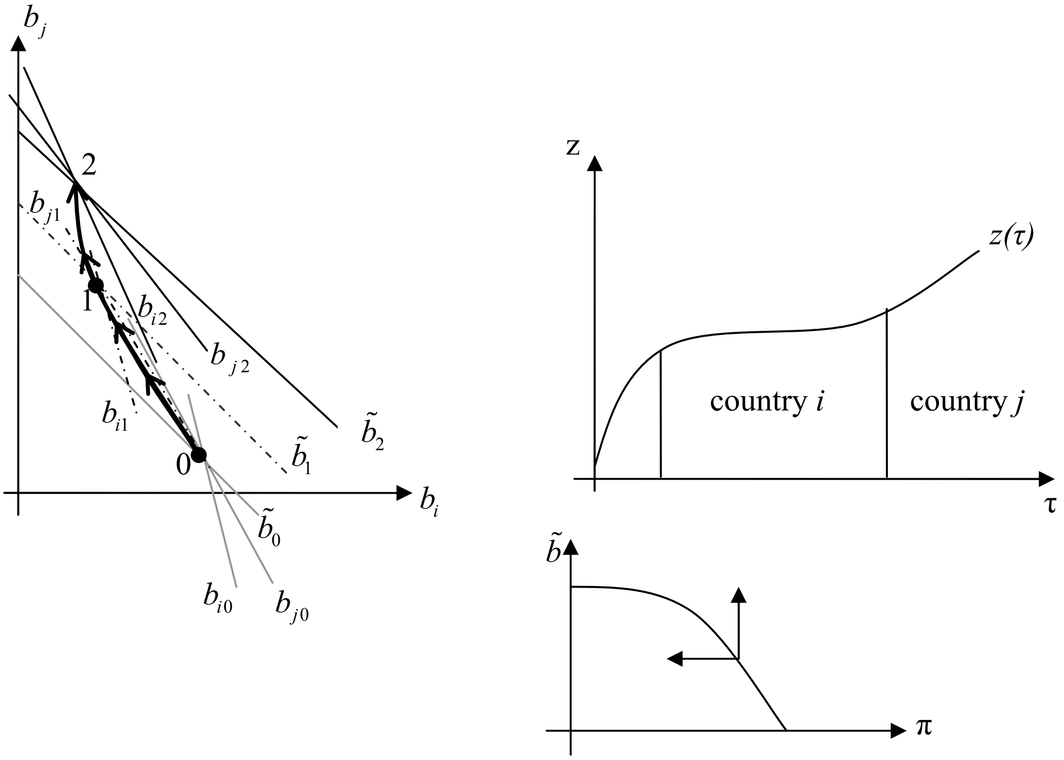

Figure 6: The path of the economy when the debtor country introduces lower taxes than the non debtor one.

Country *j,* chooses high tax rates, being situated at the right end of the transaction cost function, where the rates of change are high, pointing to high frictions. This means that the tax-payers' dissatisfaction, caused by the welfare implications of any increase of the tax rates is high, and hence the probability of no re-election is also high. In this case the government's tax instrument is inefficient in serving country *j*'s debt.

As for country *i*, within optimality[23], it is

$$\frac{\partial CT_j}{\partial P_j} = \frac{\xi g_j + z_j\left(f + \tau f'\right)_j}{\left(1 - \frac{\gamma}{\theta}\frac{\partial \tilde{b}}{\partial \pi} b^j_{t-1}\right) P_j^2} < 0 \tag{19}$$

(where CT_j stands for the constant term of country *j*'s reply function). Any increase in country *j*'s tax rates would moderate society's welfare, driving up the government's probability of no re-election. Consequently, the locus $b^j_j(b_i)$ shifts upwards, from $b^j_j(b_i)_0$ to $b^j_j(b_i)_1$, causing country *j*'s debt level to rise.

[27] Recall that, as indicated by (14a), the rate of change of the transaction cost is monotonically related to the society's dissatisfaction, as this is captured by the probability of no re-election.

In addition to this, if the government in this country, thinking about its re-election, considers increasing public spending, the country's debt would end up even higher, since[28]

$$\frac{\partial P}{\partial g} = \frac{\xi(\beta g - c) - \beta\left[z(f + \tau f') + \xi g\right]}{(\beta g - c)^2} < 0 \tag{20}$$

In this case, more taxes will be required coming with increasing rates of change of the transaction cost:

$$\frac{\partial (f + \tau f')_j}{\partial g_j} = \frac{1}{z_j}\left(\beta \cdot P_j - \xi\right) > 0 \tag{21}$$

As a result, the total MU debt also increases, as shown from the shift from $\tilde{b}_0$ to $\tilde{b}_1$, in Figure 6, triggering off a monetary policy tightening (indicated by (13)), by the CCB. As a second order effect of such a tight policy, the level of inflation falls, while the total MU debt increases, moving the economy from point 1 to point 2 in Figure 6.

CASE 1.2: the debtor country, *i*, has introduced higher tax rates than the non-debtor one

In this case, which is graphically shown in Figure 7, the government of the non debtor country *j* exploits its tax instrument's efficiency[29] which is related to low frictions and hence it enjoys a higher probability of re-election (i.e. a lower P_j).

[28] See Appendix 9.

[29] This efficiency stems from the fact that its tax rates correspond to the "flat" area of the transaction cost function *z*, indicating a *very low rate of change of its tax system transaction cost* and, hence, that an increase in its tax rate would result in relatively higher tax revenues, due to the negligible rate of change of the transaction cost.

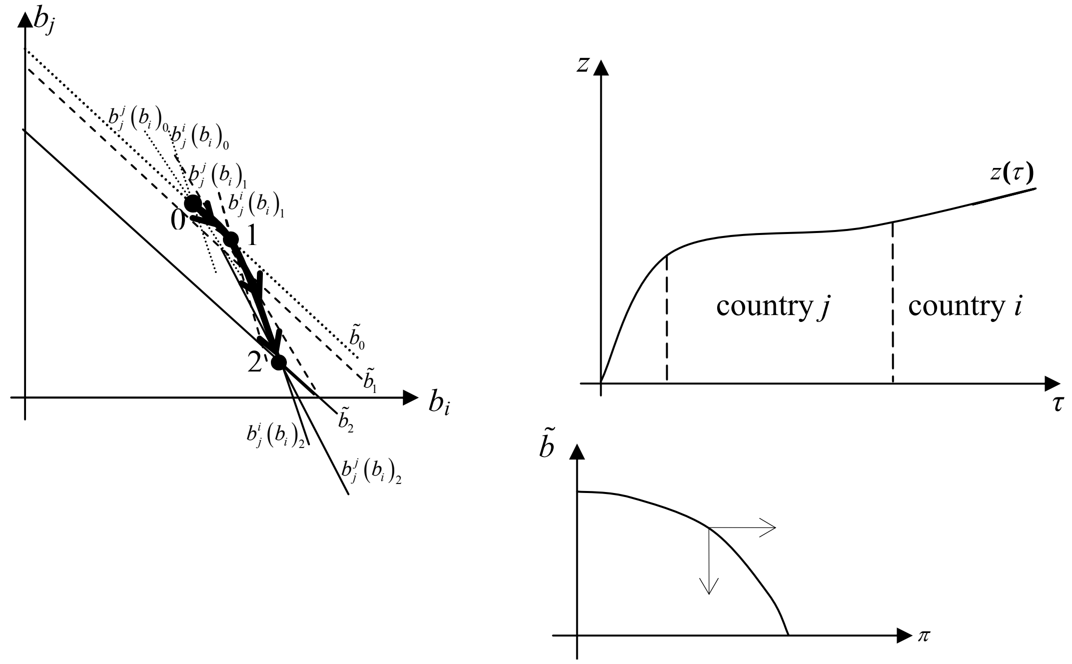

Figure 7: The path of the economy when the debtor country introduces higher taxes than the non debtor one.

Equation (19) suggests that any reduction in the probability P_j would shift the locus $b_j^j(b_i)$ to the right ($\frac{\partial CT_j}{\partial P_j} < 0$), from $b_j^j(b_i)_0$ to $b_j^j(b_i)_1$, resulting in a reduction of country j's debt level.

On the other hand, the high frictions underlying the tax policy of country i are related to a higher level of the transaction cost and, consequently, to an inefficient tax instrument in reducing the country's debt.

Regarding country i, equation (18) holds, implying that any increase in the country's tax rates would cause the government of country i's probability of no re-election to increase ($\frac{\partial CT_i}{\partial P_i} > 0$) and hence, the locus $b_j^i(b_i)$ to shift to the right, from $b_j^i(b_i)_0$ to $b_j^i(b_i)_1$. This will increase the country's debt level. This is depicted in the above graph by a movement from the starting point *0* to point *1*.

This result is justified by the logic of the transaction cost we develop in this paper. The total MU debt is reduced in this case, allowing for a looser monetary policy by the CCB (within the limits set regarding the level of inflation, e.g. π < *2%* for the case of

EMU). A change in the inflation level would affect, in a certain way, the slope of the loci $b_j^j(b_i)$ and $b_j^i(b_i)$, since[30]

$$\frac{\partial \text{slope}_i}{\partial \pi} = -\frac{\theta(1-\gamma)}{\gamma} \frac{\frac{\partial^2 \tilde{b}}{\partial \pi^2}}{b_{t-1}^i \left(\frac{\partial \tilde{b}}{\partial \pi}\right)^2} > 0$$

and

$$\frac{\partial \text{slope}_j}{\partial \pi} = \frac{\frac{\gamma}{\theta}(1-\delta) b_{t-1}^j \frac{\partial^2 \tilde{b}}{\partial \pi^2}}{\left(1 - \frac{\gamma}{\theta}\frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j\right)} < 0$$

The path of the debt level of each country separately, as well as of the total MU debt is shown by the trajectory in the above figure.

<u>Scenario 2:</u> when the countries are <u>symmetric</u> with respect to their tax system's transaction cost function

The existence of symmetric transaction cost functions allows for strategic substitutability[31] among the countries. Specifically, when both countries face similar transaction cost functions, they also face cost functions with the same properties. This means that the countries are also similar regarding the effectiveness of their tax policies and that their governments have similar chances for re-election. Hence there is space for strategic behaviour, which was not the determining factor in the previous scenario, where the outcome was determined mainly by the constraints of the transaction cost, imposed on the effectiveness of fiscal policy by the tax systems.

Technically, the symmetry of this scenario means that both countries lie within the same "area" along the transaction cost function and that they both increase their tax rates within the same range.

We therefore develop our analysis of this scenario, over the following cases:

<u>CASE 2.1:</u> both countries choose tax rates which are related to a transaction cost that exhibits low rates of change

The increase of the tax rates for both countries in this case is consistent with a transaction cost which increases at a low rate as a result of any increase of the tax rates, that is, the fiscal authorities increase the tax rates within the range [τ_0, τ_1] of Figure 8.

[30] As shown in Appendix 5.
[31] See Bulow et al (1985), Beetsma et al (2001).

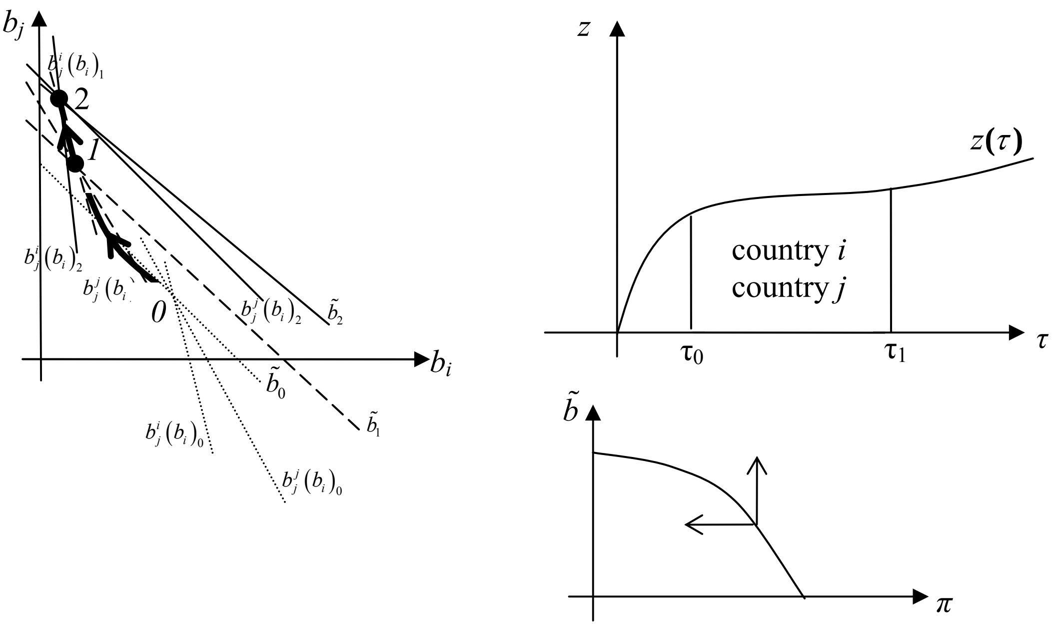

Figure 8: The path of the economy when the transaction cost is symmetric, exhibiting low rates of change in both countries.

Regarding country *i,* it is worth noting that its debt level, b_i , is initially above the reference value ($b_i > \bar{b}$). The fact that the transaction cost of the country's tax system changes at a low rate makes it possible for the government to effectively use its tax instrument in order to reduce its debt level (which falls from b_{i0} to b_{i1}) as a result[32] of a backward shift of the locus $b_j^i(b_i)$ from $b_j^i(b_i)_0$ to $b_j^i(b_i)_1$.

On the other hand, when country *j* is called upon to take a decision, it faces a debt level below the reference value and (the same as country *i*) a low rate of change of its transaction cost. This facilitates the country's public policy, which can result in a lower probability of no re-election, P_j, as a result of an increase in the tax rates.

Since the tax instrument is efficient at reducing the debt level, the incumbent in this country could behave opportunistically[33] and benefit from increasing public spending (since $\frac{\partial P_j}{\partial g_j} < 0$) at the cost of the country's debt level (b_j increases) as well as the total MU level ($\tilde{b}$ also increases). This observation is also consistent and further supported by the

[32] $\frac{\partial CT_i}{\partial P_i} > 0$, $\frac{\partial CT_j}{\partial P_j} < 0$

[33] Beetsma et. al. (2001), Alesina (1990), Alesina and Tabellini (1987)

optimality conditions[34]. Besides, an increase in public spending would –within optimality– be related to a higher rate of change of the tax system's transaction cost ($\frac{\partial\left(f+\tau f'\right)}{\partial g_j}>0$) and, hence, to a higher level of the transaction cost[35] z_j. This gives an additional reason for the increase in the country's debt level, which is depicted in the above graph by a shift in the locus $b_j^j(b_i)$ from $b_j^i(b_i)_0$ to $b_j^i(b_i)_1$.

The total MU debt, $\tilde{b}$, increases, triggering off a tight monetary policy[36] resulting in a lower level of inflation. This creates second order effects on the level of each country's debt as well as on the level of total debt. These correspond to a change in the slopes of the countries' debt loci, moving the economy along the indicated trajectory, to point 2 in the above graph.

At this point it can be mentioned that the Central Banker's behaviour affects the behaviour of the countries, as reflected in the above-mentioned change in the slopes of the corresponding loci. This is also consistent with the findings of Beetsma *et al* (2001) and is indicative of the interaction between the two policies, monetary and public.

CASE 2.2: both countries choose tax rates which are related to a transaction cost that exhibits relatively high rates of change

When the countries choose tax rates within the range $(\tau_1, 1]$, the transaction cost of their tax systems exhibits high rates of change as a result of an increase in their tax rates.

The high debt level of country *i* ($b_i > \bar{b}$), in connection with the high transaction cost (due to its high rate of change) makes its tax instrument inefficient in reducing the country's debt. The country's debt level increases and so does society's dissatisfaction (resulting in a higher probability of no re-election, P_i). This is depicted[37] by a shift in the locus $b_j^i(b_i)$ from $b_j^i(b_i)_0$ to $b_j^i(b_i)_1$ in Figure 9 below.

This increase in country *i*'s debt level could also be explained as the result of strategic behaviour within the scenario of opportunistic behaviour by the party in office:

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[34] As shown in Appendix 10, here, there is no upper limit for g_j. Note that this is not true in the following case 2.1, where an upper limit for the government's public expenditure does exist.

[35] Note that rearranging the optimality conditions generates (see Appendix 10) the result that z_j is greater than a linear function of z_i.

[36] Justified by the optimality conditions, which require that $\frac{\partial\tilde{b}}{\partial\pi}=-\frac{\theta}{\gamma}\frac{\pi}{\tilde{b}}<0$.

[37] Since, as before, optimality indicates that $\frac{\partial CT_i}{\partial P_i}>0$.

having to face low chances of being the elected again (i.e. facing a high probability P_i), the government increases its public expenditure, despite the already high debt, as a means of increasing society's satisfaction and, hence, its probability of re-election ($\frac{\partial P_i}{\partial g_i} < 0$).

However, this has second order effects, since the higher public expenditure requires higher taxes, which need to be collected under the high transaction cost of the tax system (since $\frac{\partial\left(f + \tau f'\right)}{\partial g_i} > 0$). As a result the country's debt increases. On the other hand, from country *j*'s point of view, the high rates of the transaction cost of its tax system, together with the resulting inefficiency of the tax instrument, indicate that it should adopt policies that counteract the financing implications of the high tax rates.

In other words, country *j*'s fiscal policy aims at public expenditure retrenchments[38], which negatively affect the rate of change of the transaction cost ($\frac{\partial\left(f + \tau f'\right)}{\partial g_j} > 0$), smoothing the progress of the contribution of its tax instrument towards decreasing the country's debt level. The economic policy of this country relies on "*switching strategies*"[39], in the sense that the government of the country starts the fiscal consolidation by raising taxes and then moves into more politically sensitive policies, by reducing spending.

With lower public spending, society's welfare will decrease, resulting in a higher probability of no re-election, but at the same time, the debt level will also drop (since $\frac{\partial CT_j}{\partial P_j} < 0$).

The reduction in country *j*'s debt level is enough to drive down the total MU debt, leaving space for the Central Banker to loosen monetary policy[40], which, again, affects the countries' debt levels and, consequently, the total MU debt, which falls further, following the path of the economy shown in the above graph.

[38] In Appendix 10 we prove that, in this case, optimality requires that $z_j < A+Bz_i$, which also indicates that it must be the case that $g_j < \Gamma+\Delta g_i$ (For the A, B, Γ and Δ calculated in the Appendix). That is, in this case, optimality imposes an upper bound in country *j*'s public spending.

[39] See, for example, OECD Economic Outlook (2007), and von Hagen *et al* (2002).

[40] The monetary policy rule $\partial\tilde{b}/\partial\pi < 0$ implies that a monetary loosening is consistent with reductions in the slopes of the loci of the countries' reaction functions.

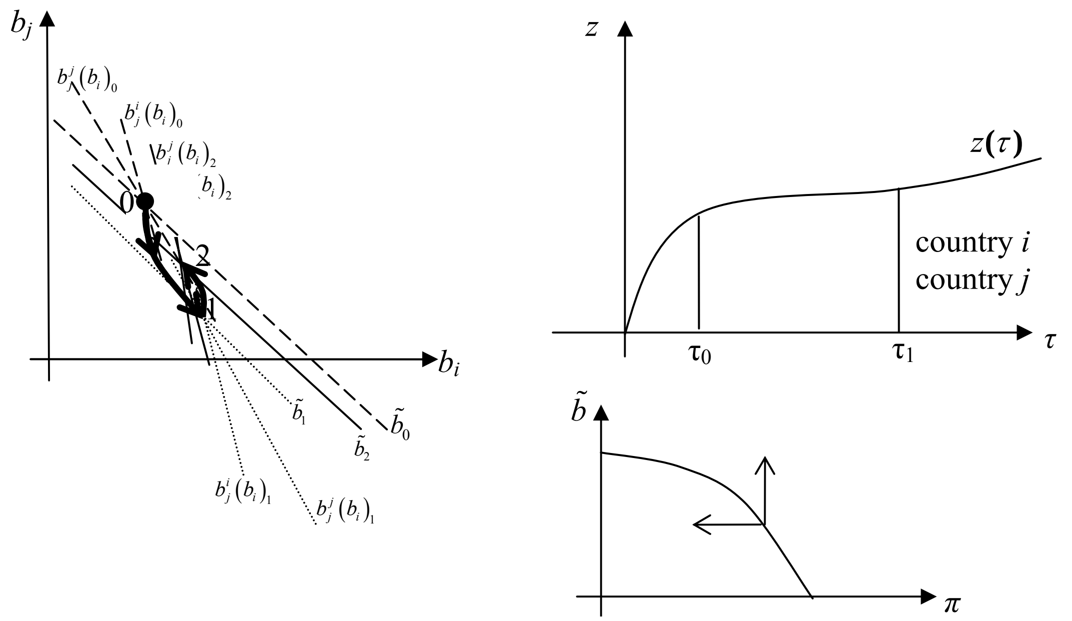

Figure 9: The path of the economy when both countries face transaction costs with high rates of change.

Summing up, from the above analysis it is made clear that there exists a channel, via the transaction cost of the tax system, that affects the stabilisation policies of the CCB. In other words, the optimal solution indicates a mechanism through which the CCB policies, that target price stability, are related to the fiscal policy variables of each MU country.

The transaction cost of the tax systems of the countries we introduce into our analysis, is an endogenous mechanism, which affects the effectiveness of the fiscal policy instruments. Thus, we formally show a way in which the operational cost of the tax system of each MU member has inflationary consequences that depend on the characteristics of each individual country, as a token of fiscal policy non-neutrality.

4.5 Fiscal rules vs. transaction cost

In this section we focus on making inferences regarding the effectiveness of the numerical fiscal rules we have modelled, under the constraint implied by the existence of the transaction cost. It is worth noting that the introduction of this cost into our analysis, affects the efficiency of the fiscal policy's instrument, via an endogenous mechanism, which relates the efficiency of the fiscal policy (which is a quantitative result) with the frictions generated by the state-tax payer interaction and which are related to behavioural issues stipulated by the economic, sociological and psychological characteristics and tax administration aspects.

Keeping this in mind, we are going to use a formal analysis to investigate whether the numerical fiscal rule we introduce into our model (which is also examined by Uhlig and Beetsma (1999) and Debrun (2007)), affects the behaviour of the agents of our model. In terms of our analysis, we are interested in investigating whether the existence (or absence) of fiscal rules alters the slopes of the loci $b_j^j(b_i)$ and $b_j^i(b_i)$. Since the numerical fiscal rules are expressed by the term $\delta\left(b-\bar{b}\right)$, where $\delta \in \left(0,1\right)$, the case of a MU without fiscal rules corresponds to the case where $\delta = 0$. In this latter case, the loci $b_j^j(b_i)$ and $b_j^i(b_i)$ become:

$$b_{jt}^j = \frac{1-c_j-\left(1+\pi\right)b_{t-1}^j}{1-\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^j} + \frac{1-\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^i}{1-\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^j}b_t^i$$

and

$$b_{jt}^i = \frac{1-c_i-\left(1+\pi\right)b_{t-1}^i}{\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^i} + \frac{1-\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^i}{\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^i}b_t^i$$

for country *j* and country *i* respectively.

The above loci retain the sign of their slopes, with

$$\textit{slope of}\ b_j^i(b_i) < \textit{slope of}\ b_j^j(b_i)$$

indicating that the *existence of numerical fiscal rules does not alter the strategic behaviour of the countries*, when the transaction cost of the tax system is explicitly modelled. A reasonable explanation for this is based on the micro-characteristic of the transaction cost: the transaction cost of the tax system relates the aspects of administration to the revenue losses the government incurs due to the behaviour of the tax payers, as a consequence of the existence of the tax system and governments' tax policy. On the other hand, numerical fiscal rules (e.g. the SGP), are consistent with the government's behaviour for deviating from the rules.

But the transaction cost, which is the cost underlying the state–taxpayers interaction, is related to the behaviour of the tax payers, which are not affected by the fiscal rules that underlie the government's decision. This justifies the fact that, in terms of our model, the numerical fiscal rules do not affect the slope of the reaction functions.

This also corresponds to the way we have constructed the government's objective function. Most of the theoretical work regarding economic policy issues is based on the assumption of the benevolent dictator government. This assumption, although consistent with the needs of the models used in these theoretical works, is not consistent with our analysis. This is because the notion of the transaction cost (i.e. the cost that evolves from the state–tax payer interaction) embodies the distinction between the society (that pays the taxes) and the government (that imposes the taxes). This segregation makes it clear that the benevolent dictator assumption is not consistent with the government having identical preferences with the society; an assumption that would imply that there is no transaction cost.

5. Conclusion

We build a model in order to investigate monetary and fiscal policy interactions in the economic environment of a MU, with two heterogeneous countries and a common central banker (the CCB), who is conservative vis-à-vis inflation. In this framework, we are looking for economic policy implications regarding the way the efficiency of the fiscal policy affects the price stability pursued by the CCB.

The fiscal policy non-neutrality implied above is consistent with departure from the Ricardian Equivalence Proposition, whose existence could be considered as a stepping stone toward the view that inflation is only a monetary phenomenon. The theoretical work, consistent with the above-mentioned "Ricardian Regime", is based on the explicit assumption that the tax instrument that does not have any effect on economic activity and, for that reason, a lump sum or a proportional taxation scheme is introduced.

Our analysis does not explicitly assume any specific type of tax instrument (e.g. lump sum or proportional taxation). We cope instead, with a more general consideration of the tax system by introducing the transaction or operation cost of the tax system.

This stems from the fact, that the imposition of the taxes generates frictions which relate the behaviour of the tax payers, stipulated by socioeconomic and psychological characteristics, with tax administration aspects, like tax collection and tax compliance, resulting in the loss of tax revenue or deadweight loss (Feinstein 1998 JEL), with budgetary consequences.

In this way, we have a microfountation of the macroeconomic theory regarding monetary and fiscal policy interactions, where, starting from the operational properties of the tax system, we are looking at the way the efficiency of the fiscal policy is related to the political cycle, the path of the debt level and the level of inflation. Assuming heterogeneous countries enables us to identify possible spillovers in a monetary and fiscal policy interaction framework

We show that when we have a considerable degree of heterogeneity, the transaction cost of the tax system is the factor determining the countries' policy choices and hence the resulting level of debt as well as the inflation path. On the other hand, when we have a considerable degree of homogeneity, it is the countries' strategic behaviour that determines the outcome.

We formally develop a channel, through which the operation of the tax system, related to the fiscal policy objectives is not independent of price stability which is the primary objective of the CCB. Under this framework, we also show that the numerical fiscal rules are not efficient in altering the behavior of the agents as they do not change the path of the economy.

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APPENDIX 1: The transaction cost function

We let Z_t correspond to the real cost incurred during the tax collection process at time *t,* be a function of the total net tax take T_t and the taxable recourses Y_t, i.e. $Z_t = F(T_t, Y_t)$.

Since the transaction cost of the tax system is consistent with the notion that when imposing taxes there exists a cost underlying the state–tax-payer interaction, it is reasonable to assume that this transaction cost depends positively on T_t, and negatively on Y_t, i.e. $\frac{\partial Z_t}{\partial T_t} > 0$, $\frac{\partial Z_t}{\partial Y_t} < 0$.

Assuming that the transaction cost function is *homogeneous of degree one*, we can write

$$Z_t = F\left(T_t, Y_t\right) \Rightarrow \frac{1}{T} Z_t = F\left(T\frac{1}{T}, \frac{Y}{T}\right) = F\left(1, \frac{Y}{T}\right)$$

So, there exists a function[41] *f* of the tax rate $\tau = {}^{T}\!/_{Y}$ such that,

$$\frac{1}{T} Z_t = f\left(\frac{T}{Y}\right) \Rightarrow Z_t = \mathrm{T} \cdot f\left(\frac{T}{Y}\right) \Rightarrow \frac{Z_t}{Y} = \frac{T}{Y} \cdot f\left(\frac{T}{Y}\right) \Rightarrow z_t = \tau \cdot f\left(\tau\right)$$

where, $z = \frac{Z}{Y}$.

APPENDIX 2: The shape of the transaction cost function z

The transaction cost is described by the function $z = \tau \cdot f(\tau)$, with a first order derivative given by

$$\frac{\partial z}{\partial \tau} = \frac{\partial}{\partial \tau}\left(\tau \cdot f(\tau)\right) = f + \tau f'$$

where f' stands for the first order derivative of f with respect to τ.

Recall that rearranging the first order conditions of the government's problem, gives

[41] There exists a function $\phi\left(\frac{Y}{T}\right) = \phi\left(\frac{1}{T/Y}\right) = f\left({}^{T}\!/_{Y}\right)$.

$$f + \tau f' = \frac{1}{z}\left[(\beta P - \xi) g - P \cdot c \right] \tag{14}$$

where, $c = {}^{1}/_{Y} - \tau$.

So, the second order derivative of the transaction cost function, is given by

$$\frac{\partial^2 z}{\partial \tau^2} = z''(\tau) = \frac{\partial}{\partial \tau}(f + \tau f') = \frac{\partial}{\partial \tau}\left(\frac{1}{z}\left[(\beta P - \xi) g - P \cdot c \right] \right) =$$

$$= \frac{z \cdot \frac{\partial}{\partial z}\left[(\beta P - \xi) g - P \cdot c \right] - \left[(\beta P - \xi) g - P \cdot c \right] \frac{\partial z}{\partial \tau}}{z^2} =$$

$$= \frac{z \cdot P - \left[(\beta P - \xi) g - P \cdot c \right](f + \tau f')}{z^2} =$$

$$= \frac{z \cdot P - \left[P(\beta g - c) - \xi g \right](f + \tau f')}{z^2}$$

The sign of this second order derivative depends on the value of the transaction cost z:

$$\frac{\partial^2 z}{\partial \tau^2} > 0 \Rightarrow \frac{z \cdot P - \left[P(\beta g - c) - \xi g \right](f + \tau f')}{z^2} > 0 \Rightarrow$$

$$\Rightarrow z > \frac{P(\beta g - c) - \xi g}{P}(f + \tau f') \ (>0)$$

or, equivalently,

$$\frac{\partial^2 z}{\partial \tau^2} > 0 \Rightarrow z' = f + \tau f' < \frac{Pz}{P(\beta g - c) - \xi g}$$

while

$$\frac{\partial^2 z}{\partial \tau^2} < 0 \Rightarrow (0<) z < \frac{P(\beta g - c) - \xi g}{P}(f + \tau f') = z^*$$

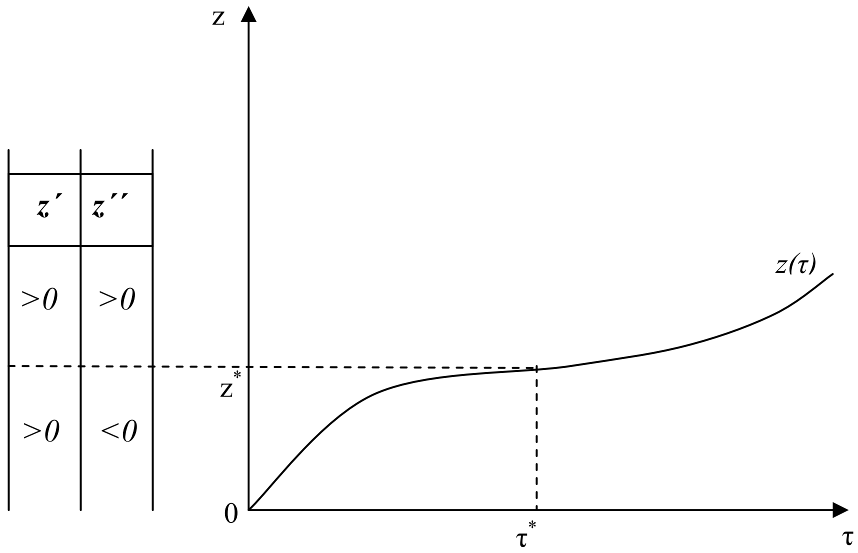
or, equivalently,

$$\frac{\partial^2 z}{\partial \tau^2} < 0 \Rightarrow z' = f + \tau f' > \frac{Pz}{P(\beta g - c) - \xi g}$$

The above indicate that the transaction cost function, z, exhibits an inflection point at

$$(z^*, \tau^*) = \left(P(\beta \cdot g - c) - \xi g, \tau^* \right)$$

of its graph and, hence, that its shape will be as shown in the following.



APPENDIX 3: The relationship between f and z

As shown in Appendix 3, the transaction cost function is $z = \tau \cdot f(\tau)$, with a first order derivative given by

$$\frac{\partial z}{\partial \tau} = \frac{\partial}{\partial \tau}\left(\tau \cdot f(\tau)\right) = f + \tau f' > 0$$

and a second order derivative of the form

$$\frac{\partial^2 z}{\partial \tau^2} = z''(\tau) = \frac{z \cdot P - \left[P\left(\beta g - c\right) - \xi g\right]\left(f + \tau f'\right)}{z^2}$$

where,

$$\frac{\partial^2 z}{\partial \tau^2} < (>) 0 \Rightarrow z < (>) \frac{P\left(\beta g - c\right) - \xi g}{P}\left(f + \tau f'\right) = z^*$$

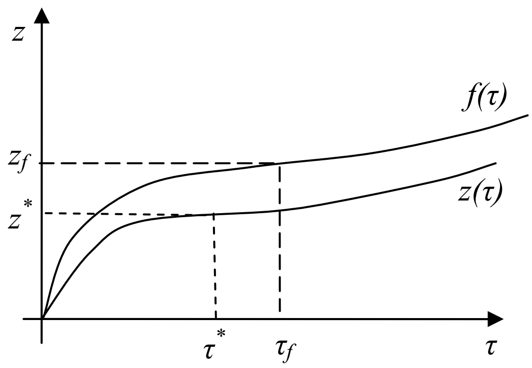
Recall that $f' > 0$. Besides, looking at the second order derivative of z in another way, we have

$$z'' = f' + \left(f' + tf''\right) \Rightarrow z'' = 2f' + tf''$$

and so,

- when $z'' < 0$ (which is the case in the left part of the transaction cost function graph), then $2f' + tf'' < 0 \Rightarrow f'' < -\frac{2f'}{t} < 0$, indicating that, within this range, *f* is a concave function.
- when $z'' > 0$ (which is the case in the right part of the transaction cost function graph), then $f'' > -\frac{2f'}{t} = \tau_f$ i.e. f'' is negative (positive) for relatively low (high) values of the tax rate *τ*, indicating that, within the low *τ* range, *f* is a concave function, while it is convex for higher values of *τ*.

Hence, we can draw the following diagram



APPENDIX 4: The transaction cost function and the political cycle

As shown in Appendix 1, it is $z_t = \tau f(\tau)$ and, so, the first order derivative of the transaction cost function, is

$$z' = \frac{\partial z}{\partial \tau} = \frac{\partial}{\partial \tau}\left[\tau \cdot f(\tau)\right] = f(\tau) + \tau \cdot f'(\tau)$$

Recall, also, that the first order conditions for the governments' optimisation problems give

$$f + \tau f' = \frac{1}{z}\left[P(\beta \cdot g - c) - \xi \cdot g\right] \qquad (14)$$

Hence,

$$z' = f + \tau f' = \frac{P(\beta \cdot g - c) - \xi \cdot g}{z}$$

The second order derivative of the transaction cost function, will then be

$$z'' = \frac{\partial z'}{\partial \tau} = \frac{\partial}{\partial \tau}\left[\frac{P(\beta \cdot g - c) - \xi \cdot g}{z}\right] = \frac{P \cdot z - \left[P \cdot (\beta \cdot g - c) - \xi \cdot g\right] \cdot (f + \tau f')}{z^2}$$

with

$$\frac{\partial z''}{\partial P} = \frac{1}{z^2} \cdot \frac{\partial}{\partial P}\left(P \cdot \left[z - (\beta \cdot g - c) \cdot (f + \tau f')\right] + \xi g \cdot (f + \tau f')\right) = \frac{z - (\beta \cdot g - c) \cdot (f + \tau f')}{z^2}$$

We look for the way the political cycle is related to the shape of the transaction cost function (via the probability of no re-election). This will be shown by the behaviour of the derivative $\frac{\partial z''}{\partial P} = \frac{z - (\beta \cdot g - c) \cdot (f + \tau f')}{z^2}$:

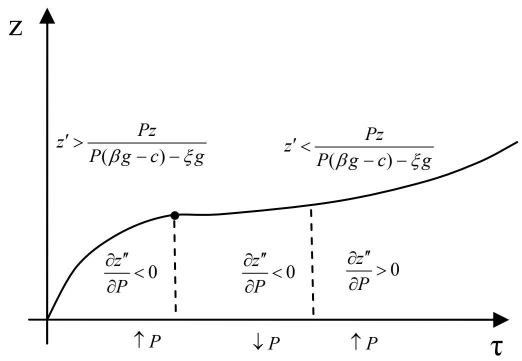
It is,

- $\frac{\partial z''}{\partial P} > 0 \Rightarrow \frac{z}{(\beta \cdot g - c)} > (f + \tau f') = z'$

and

- $\frac{\partial z''}{\partial P} < 0 \Rightarrow \frac{z}{(\beta \cdot g - c)} < (f + \tau f') = z'$

These imply, given the shape of the transaction cost function and the fact that, as shown in Appendix 2, it is $z' > (<) \frac{Pz}{P(\beta g - c) - \xi g}$ for $z'' < (>) 0$, that



✓ when the value of the first order derivative, and hence the slope of the transaction cost function, is high (in particular, when $z' > \frac{Pz}{P(\beta g - c) - \xi g} > \frac{z}{(\beta \cdot g - c)}$, which is the case in the "left part" of the transaction cost function graph), an increase in the probability of no re-election, *P*, is related to a lower z'', that is, to a transaction cost line with a "lower" slope as τ rises. Hence, ***P* is increasing** in this range.

- ✓ when the value of the first order derivative, and hence the slope of the transaction cost function, is low (i.e. when $\frac{z}{(\beta \cdot g - c)} < z' < \frac{Pz}{P(\beta g - c) - \xi g}$, which is the case in the "middle part" of the transaction cost function graph), an increase in the probability of no re-election, *P*, is related to a lower z'', that is to a transaction cost line with a "lower" slope as τ rises. Hence, ***P* is decreasing** in this range.
- ✓ when the value of the first order derivative, and hence the slope of the transaction cost function, is very low (i.e. when $z' < \frac{z}{(\beta \cdot g - c)} < \frac{Pz}{P(\beta g - c) - \xi g}$, which is the case in the "right part" of the transaction cost function graph), an increase in the probability of no re-election, *P*, is related to a higher z'', that is, to a transaction cost line with a "higher" slope as τ rises. Hence, ***P* is increasing** in this range.

APPENDIX 5: Reaction Functions

The first order conditions for country *i*'s optimisation problem give

$$\frac{\partial L_t^G}{\partial b_t^i} = 0 \Rightarrow 1 + (1-\delta) b_t^i - c_t^i + \delta \bar{b} - \gamma\!/\!\theta \cdot \frac{\partial \tilde{b}}{\partial \pi}\left(1 + \pi^e + \gamma\!/\!\theta \cdot \frac{\partial \tilde{b}}{\partial \pi} \cdot \tilde{b}\right) = 0 \Rightarrow^{\tilde{b} = b_i + b_j}$$

$$1 - c_t^i + \delta \bar{b} - (1 + \pi^e) b_{t-1}^i + \left[1 - \delta - \gamma\!/\!\theta \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i\right] b_t^i - \gamma\!/\!\theta \cdot \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i b_t^j = 0$$

where, with the appropriate rearrangements and assuming $\pi = \pi^e$, we get the reaction function for country *i*:

$$b_{jt}^i = \frac{1 - c_t^i + \delta \bar{b} - (1+\pi) b_{t-1}^i}{\gamma\!/\!\theta \cdot \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i} + \frac{1 - \delta - \gamma\!/\!\theta \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i}{\gamma\!/\!\theta \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i} b_t^i = b_j^i(b_i) \tag{15}$$

Similarly, for country *j*, the first order conditions, require that

$$\frac{\partial L_j^G}{\partial b_t^j} = 0 \Rightarrow 1 + b_t^j - c_t^j + \delta\left(b_t^i - \bar{b}\right) - \left(1 + \pi - \pi^e\right) = 0 \Rightarrow^{\tilde{b} = b_i + b_j}$$

$$1 - c_t^j - \delta \bar{b} - (1+\pi^e) b_{t-1}^j + \left(1 - \gamma\!/\!\theta \cdot \tilde{b} \cdot \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j\right) b_t^j + \left(\delta - \gamma\!/\!\theta \cdot \tilde{b} \cdot \frac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j\right) b_t^i = 0$$

Assuming $\pi = \pi^e$ and after the appropriate rearrangements, we get the reaction function for country *j*, given by:

$$b_{jt}^j = -\frac{1 - c_t^j - \delta \bar{b} - (1+\pi) b_{t-1}^j}{1 - \gamma/\theta \cdot \dfrac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j} - \frac{\delta - \gamma/\theta \dfrac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j}{1 - \gamma/\theta \dfrac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j} b_t^i = b_j^j(b_i) \tag{16}$$

From the above, it can be seen that the probabilities P_i, P_j enter the constant terms (CT_i, CT_j) of the corresponding reply functions. In particular, differentiating these constant terms, as given by (15) and (16) gives the exact way the probabilities of no re-election for the governments affect the position of the respective reply functions:

$$\frac{\partial CT_i}{\partial P_i} = -\frac{\xi g_i + z_i (f + \tau f')_i}{\dfrac{\gamma}{\theta} \dfrac{\partial \tilde{b}}{\partial \pi} b_{t-1}^i P_i^2} > 0 \tag{18}$$

and

$$\frac{\partial CT_j}{\partial P_j} = \frac{\xi g_j + z_j (f + \tau f')_j}{\left(1 - \dfrac{\gamma}{\theta} \dfrac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j\right) P_j^2} < 0 \tag{19}$$

The slopes of the reply functions of the countries can be seen as functions of the (common) inflation

$$\frac{\partial \text{slope}_i}{\partial \pi} = -\frac{\theta(1-\gamma)}{\gamma} \frac{\dfrac{\partial^2 \tilde{b}}{\partial \pi^2}}{b_{t-1}^i \left(\dfrac{\partial \tilde{b}}{\partial \pi}\right)^2} > 0 \text{ and } \frac{\partial \text{slope}_j}{\partial \pi} = \frac{\dfrac{\gamma}{\theta}(1-\delta) b_{t-1}^j \dfrac{\partial^2 \tilde{b}}{\partial \pi^2}}{\left(1 - \dfrac{\gamma}{\theta} \dfrac{\partial \tilde{b}}{\partial \pi} b_{t-1}^j\right)} < 0$$

APPENDIX 6: Slopes of the reaction functions

As it can be seen from the analysis in Appendix 5, both reply functions are negative sloping. In order to plot these reply functions we need to know the relationship between their slopes:

slope of b_{jt}^i - slope of b_{jt}^j = $\dfrac{1 - \delta - \gamma/\theta \cdot \dfrac{\partial \bar{b}}{\partial \pi} b_{t-1}^i}{\gamma/\theta \cdot \dfrac{\partial \bar{b}}{\partial \pi} b_{t-1}^i} + \dfrac{\delta - \gamma/\theta \cdot \dfrac{\partial \bar{b}}{\partial \pi} b_{t-1}^j}{1 - \gamma/\theta \cdot \dfrac{\partial \bar{b}}{\partial \pi} b_{t-1}^j} =$

$$= \frac{(1-\delta)-(1-\delta)\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j} - (1-\delta)\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}}{\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}\left(1-\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j}\right)} =$$

$$= \frac{(1-\delta)\left[1-\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j} - \gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}\right]}{\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}\left(1-\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j}\right)} < 0 \Rightarrow$$

slope of b_{jt}^{i} < slope of b_{jt}^{i}

This indicates that country i's reply function is steeper than that of country j.

APPENDIX 7: Equilibrium path $\tilde{b}(b_i)$

The equilibrium is determined by the intersection point of the above derived reply functions. The locus of these points will be derived by simultaneously solving equations (15) and (16), which gives

$b_j^i = b_j^j \Rightarrow$

$$\frac{1-c_t^i+\delta\bar{b}-(1+\pi)b_{t-1}^i}{\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}} + \frac{1-\delta-\gamma/\theta \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}}{\gamma/\theta \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{i}} b_t^i = -\frac{1-c_t^j-\delta\bar{b}-(1+\pi)b_{t-1}^j}{1-\gamma/\theta \cdot \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j}} - \frac{\delta-\gamma/\theta \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j}}{1-\gamma/\theta \frac{\partial\tilde{b}}{\partial\pi} b_{t-1}^{j}} b_t^i \Rightarrow$$

$$b_j = \frac{\beta P_j-\xi}{P_j} g_j - \frac{z_j(f_j+\tau_j f_j')}{P_j} - \frac{b_{t-1}^j}{b_{t-1}^i}\left[\frac{\beta P_i-\xi}{P_i} g_i - \frac{z_i(f_i+\tau_i f_i')}{P_i}\right] + \frac{\bar{b}_{t-1}}{b_{t-1}^i}\delta\bar{b} + \frac{b_{t-1}^j-b_{t-1}^i}{b_{t-1}^i} - \frac{\delta b_{t-1}^i+(1-\delta)b_{t-1}^j}{b_{t-1}^i} b_i$$

$$= \tilde{b}(b_i) \quad (17)$$

Regarding the slope of the line $\tilde{b}(b_i)$, we have

slope of $\tilde{b}(b_i)$ - slope of $b_{jt}^j = -\frac{\delta b_{t-1}^i+(1-\delta)b_{t-1}^j}{b_{t-1}^i} - \left(-\frac{\delta-\gamma/\theta \cdot \frac{\partial\bar{b}}{\partial\pi} b_{t-1}^{j}}{1-\gamma/\theta \cdot \frac{\partial\bar{b}}{\partial\pi} b_{t-1}^{j}}\right) =$

$$= -\frac{(1-\delta)b_{t-1}^j}{b_{t-1}^i\left(1-\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^j\right)} \cdot \left[\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^i + \frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^j - 1\right] =$$

$$= -\frac{(1-\delta)b_{t-1}^{j}}{b_{t-1}^{i}\left(1-\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}b_{t-1}^{j}\right)}\cdot\left[\frac{\gamma}{\theta}\frac{\partial\tilde{b}}{\partial\pi}\left(b_{t-1}^{i}+b_{t-1}^{j}\right)-1\right]\Rightarrow$$

$\Rightarrow$ slope of $\tilde{b}(b_i)$ > slope of b_{jt}^{j} (> slope of b^{i}_{jt})

This indicates that the locus $\tilde{b}(b_i)$ is flatter than the countries' reply functions.

APPENDIX 8: The reaction function of the CCB

The problem the CCB faces can be written as

$$\min_{\pi} L^{ECB} = \gamma\left(\tilde{b}(\pi)\right)^{2} + \theta\pi^{2}$$

The first order condition for this requires that

$$2\gamma\tilde{b}(\pi)\frac{\partial\tilde{b}}{\partial\pi} + 2\theta\pi = 0 \Rightarrow \frac{\partial\tilde{b}}{\partial\pi} = -\frac{\theta\pi}{\gamma\tilde{b}(\pi)} < 0$$

from which,

$$\frac{\partial}{\partial\pi}\left(\frac{\partial\tilde{b}}{\partial\pi}\right) = -\frac{\partial}{\partial\pi}\left(\frac{\theta\pi}{\gamma\tilde{b}(\pi)}\right) \Rightarrow \frac{\partial^{2}\tilde{b}}{\partial\pi^{2}} = -\frac{\theta\gamma\tilde{b}(\pi) - \theta\gamma\pi\frac{\partial\tilde{b}}{\partial\pi}}{\gamma^{2}\cdot\tilde{b}^{2}(\pi)} \Rightarrow$$

$$\Rightarrow \frac{\partial^{2}\tilde{b}}{\partial\pi^{2}} = -\frac{\theta}{\gamma^{2}}\cdot\frac{\gamma\tilde{b}^{2}(\pi) - \theta\pi^{2}}{\tilde{b}^{3}(\pi)} < 0$$

Consequently, the total debt, $\tilde{b}$, is a downward sloping concave function of inflation.

APPENDIX 9: Public spending and the political cycle

From the first order conditions for the governments' optimisation problems, we get

$$f + \tau f' = \frac{1}{z}\left[P(\beta\cdot g - c) - \xi\cdot g\right] \Rightarrow \tag{14}$$

$$\Rightarrow P = \frac{z(f + \tau f') + \xi\cdot g}{\beta\cdot g - c}$$

and so

$$\frac{\partial P}{\partial g}=\frac{\xi(\beta g-c)-\beta\left[z(f+\tau f')+\xi g\right]}{(\beta g-c)^2}<0 \tag{20}$$

APPENDIX 10: Limits for public expenditures

When the total debt level increases, it must be:

$$\Delta\tilde{b}>0\Rightarrow\left|\frac{\partial CT_{\tilde{b}}}{\partial P_j}\right|-\left|\frac{\partial CT_{\tilde{b}}}{\partial P_i}\right|>0\Rightarrow$$

$$\Rightarrow\frac{\xi g_j+z_j(f+\tau f')_j}{P_j^2}-\frac{b_{t-1}^j}{b_{t-1}^i}\cdot\frac{\xi g_i+z_i(f+\tau f')_i}{P_i^2}>0\Rightarrow$$

$$\Rightarrow z_j(f+\tau f')_j>\xi\left[\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}\cdot g_i-g_j\right]+\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}\cdot(f+\tau f')_i\cdot z_i\Rightarrow$$

$$g_j>\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}\left[g_i+\frac{z_i}{\xi}(f+\tau f')_i\right]-\frac{z_j}{\xi}(f+\tau f')_j$$

Similarly, a decreasing total debt would require that:

$$g_j<\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}\left[g_i+\frac{z_i}{\xi}(f+\tau f')_i\right]-\frac{z_j}{\xi}(f+\tau f')_j$$

that is, there exists an upper limit for g_j in this case.

Because $g_j>0$, it must also be

$$\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}\left[g_i+\frac{z_i}{\xi}(f+\tau f')_i\right]-\frac{z_j}{\xi}(f+\tau f')_j>0$$

$$\Rightarrow z_j<\frac{\xi g_i}{(f+\tau f')_j}\cdot\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}+\frac{P_j^2}{P_i^2}\cdot\frac{b_{t-1}^j}{b_{t-1}^i}\frac{(f+\tau f')_i}{(f+\tau f')_j}\cdot z_i$$

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