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Abstract: Many places, including the island of Oahu in Hawaii, have a number of groundwater aquifers. Consumers located in one aquifer area can be supplied from water extracted and transported from another aquifer if this results in cost savings over local extraction. Incorporating such interdistrict transport is necessary for a fully efficient allocation framework. We derive efficient water management and pricing plans for two of the four aquifer zones in the Central Oahu corridor, taking into account the possibility of inter-district water trade. Efficient management requires not only intertemporal efficiency within zones but also spatial efficiency between zones, where water is transferred from one zone to the next if, without the transfer, the intertemporal efficiency price in the receiving zone is greater than the efficiency price in the source zone plus the cost of transfer.

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Introduction

The central groundwater corridor on the island of Oahu in Hawaii has four major underground aquifers that are the main source of freshwater on the island. These are Honolulu, Pearl Harbor (including Ewa), Schofield, and North (Waialua & Kawailoa). The aquifers on Oahu are interconnected through semi-permeable barriers. Water flows naturally across adjacent aquifers depending, among other things, on the head level gradient between aquifers. Previous studies have shown that efficient intertemporal allocation of groundwater may result in head level drawdown in some aquifers and head level buildup in others. This will result in changes in subsurface flows between aquifers, which have not previously been taken into account in economic modeling. In addition to these natural transfers, consumers located in one aquifer area can be supplied from water extracted and transported from another aquifer if this results in cost savings over local extraction. Incorporating such interdistrict transport is necessary for a fully efficient allocation framework.

We derive efficient water management and pricing plans for two of the four zones in the Central Oahu corridor, namely, Honolulu and Pearl Harbor, taking into account the possibility of inter-district water trade. Efficient management requires not only intertemporal efficiency within zones but also spatial efficiency between zones, where water is transferred from one zone to the next if, without the transfer, the efficiency price in the receiving zone is greater than the efficiency price in the source zone plus the cost of transfer.

The model

To solve for the efficient trajectories of groundwater extraction and shadow values, we envision a social planner maximizing the present value of consumer benefits net of extraction and distribution costs. As groundwater is extracted, the efficiency price (full marginal cost of groundwater, including the marginal user cost) changes over time with the changes in extraction cost and water scarcity. Desalination of seawater is available as a backstop resource. When the efficiency price has risen to the unit cost of desalination, the backstop is used to supplement steady state groundwater recharge.

We set up a regional hydrologic-economic model to optimize groundwater use, along the lines of Krulce et al. (1997), which extends previous models (e.g., Brown and Deacon, 1972; Cummings and Burt, 1969; Moncur and Pollock, 1988, among others) by allowing recharge to continuously vary with the head level. Water is extracted from a coastal groundwater aquifer that is recharged from a watershed and leaks into the ocean from its ocean boundary depending on the aquifer head level, h. Because of the lens shape of the aquifer, the volume of water stored in the aquifer increases at a decreasing rate with the head level. Moreover, as the volume of the stored water increases, leakage to the ocean increases and the net rate of recharge decreases. Thus, we model net recharge, n, (inflow minus leakage) as a positive, decreasing, concave function of head, i.e., $n(h) \geq 0, n'(h) < 0, n''(h) \leq 0$. The aquifer head level, h, changes over time depending on the net aquifer recharge, n, and the quantity extracted, q_t.

The rate of change of head level at time t is given by: $\gamma \cdot \dot{h}_t = n(h_t) - q_t$ where γ is a factor of conversion from volume of water in gallons (on the R.H.S.) to head level in feet. In the remainder of this section, however, we subsume this factor, i.e., h is considered to be in volume, not feet. Thus, we use $\dot{h}_t = n(h_t) - q_t$ as the relevant equation of head motion. If the aquifer is not utilized (i.e., quantity extracted is zero), the head level will rise to the highest level $\bar{h}$, where leakage exactly equal balances inflow, $n(\bar{h}) = 0$ As the head cannot rise above this level, we have $n(h) > 0$ whenever the aquifer is being exploited ($h < \bar{h}$).

The unit cost of extraction is a function of the vertical distance water has to be lifted, $f = e - h$, where e is the elevation of the well location. At lower head levels, it is more expensive to extract water because the water must be lifted further, requiring greater energy. Thus, we model unit extraction cost, c_q as a positive, increasing, convex function of the lift, $c_q(f) \geq 0$, where $c_q'(f) > 0, c_q''(f) \geq 0$. Since the well location is fixed, we can redefine the unit extraction cost as a function of the head level: $c_q(h) \geq 0$, where $c_q'(h) < 0,\ c_q''(h) \geq 0$. The total cost of extracting water from the aquifer at the rate q given head level h is $c_q(h).q$.

The unit cost of distribution from wells to users is c_d. Distribution cost affects welfare estimates as well as optimal extraction paths, and without its inclusion in the model, the welfare estimates will not be realistic. In the presences of fixed costs, average distribution cost might be declining with volume, and allowing such variable distribution costs may provide insight into the effect of the fixed costs on welfare. For simplicity, we assume that the distribution pipes and pumps are already in place and that the distribution costs consist primarily of energy and maintenance costs. These costs are roughly constant per unit of water delivered, at least within the volume range considered.

The unit cost of the backstop (desalination) is represented by c_b and the quantity of the backstop used is b_t. The demand function is $D(p_t, t)$, where p_t is the price at time t, and the second argument, t, allows for any exogenous growth in demand (e.g., due to income or population growth).

A hypothetical social planner chooses the extraction and backstop quantities over time to maximize the present value (with r as the discount rate) of net social surplus.

$$\underset{q_t,\, b_t}{Max} \int_0^{\infty} e^{-rt} \left(\int_0^{q_t + b_t} D^{-1}(x,t)dx - [c_q(h_t) + c_d] \cdot q_t - [c_b + c_d] \cdot b_t \right) dt \qquad \text{..........(1)}$$

Subject to: $\dot{h}_t = n(h_t) - q_t$ ....(2)

The current value Hamiltonian for this optimal control problem is:

$$H = \left(\int_0^{q_t + b_t} D^{-1}(x,t)dx - [c_d + c_q(h_t)] \cdot q_t - [c_d + c_b] \cdot b_t \right) + \lambda_t \cdot \left(n(h_t) - q_t \right) \qquad \text{..........(3)}$$

where λ_t is the shadow price of the stock of water in the aquifer.

Applying Pontryagin's Maximum Principle (see Kamien and Schwartz, 1991, chapter 8) leads to the following condition for optimal groundwater usage:

$$p_t = \underbrace{c_q(h_t) + c_d}_{\text{Extraction and distribution cost}} + \underbrace{\frac{1}{r - n'(h_t)} \left[\dot{p}_t + c_q'(h_t).n(h_t) \right]}_{\text{Marginal User Cost}}, \qquad \text{..........(4)}$$

where $p_t \equiv D^{-1}(q_t + b_t, t)$, is the retail efficiency price of the water delivered to users and, therefore, includes the distribution cost, which would be excluded in computing the wholesale price, i.e. the price before distribution. Equation (4) implies that at the margin, the benefit of extracting water must equal actual physical costs (extraction and distribution) plus marginal user cost (decrease in the present value of the water stock due to the extraction of an additional unit). If water charges omit user costs, as is common in many areas, overuse will occur. Equation (4) also implies that the retail (consumer) price is equal to the distribution cost plus the wholesale price (i.e., the price before distribution).

Re-arranging (4), we get the equation of motion for efficiency prices:

$$\dot{p}_t = [r - n'(h_t)] \cdot [p_t - c_q(h_t) - c_d] + n(h_t) \cdot c_q'(h_t) \qquad \text{……….(5)}$$

The first term on the R.H.S. is positive and the second is negative. Their relative magnitudes determine whether the price is increasing or decreasing at any time. However, if the net recharge is large and the extraction cost is sensitive to the head level, the second term is large and may dominate by the first term, making the price fall.

The solution to the optimal control problem is governed by the system of differential equations (2) and (5). When desalination is used, the price must exactly equal the cost of the desalted water, and we can use $p_t = c_b + c_d \Rightarrow \dot{p}_t = 0$ in (4) to get:

$$c_b = c_q(h_t) - \frac{n(h_t)c_q'(h_t)}{r - n'(h_t)} \qquad \text{……….(6)}$$

Since we have $n(h) \geq 0, n'(h) < 0, n'' \leq 0$, and $c_q'(h) < 0,\ c_q''(h) \geq 0$, the derivative of the R.H.S. with respect to h_t is negative, and the h_t that solves equation (6) is unique. We denote it as h^*. Whenever desalination is being used, the aquifer head is maintained at this optimal level, and therefore, the quantity extracted from the aquifer equals the net inflow to the aquifer, i.e., $q_t = n(h^*)$. In this steady state, any excess of quantity demanded is supplied by desalination.

To obtain the time-paths of prices and the corresponding head levels, a computer algorithm was designed using Mathematica software. The algorithm first solves equation (6) to obtain the steady-state head level and then uses it as a boundary condition to numerically solve equations (2) and (5) simultaneously for the time paths of efficiency price and head level. Welfare is computed as the area under the demand curve minus extraction and distribution cost (objective function (1)).

Application

This section applies the above model to the two adjacent water districts of Pearl Harbor and Honolulu.

Pearl Harbor

The aquifers is a Ghyben-Herzberg lens where the volume of water stored in the aquifer depends on the head level, the aquifer boundaries, lens geometry, and rock porosity (Mink, 1980). Although the freshwater lens is a paraboloid, the upper and lower surfaces of the lens are nearly flat. Thus, the volume of aquifer storage is modeled as linearly related to the head level. Using GIS aquifer dimensions and effective rock porosity of 10%, Pearl Harbor aquifer has 78.149 billion gallons of water stored per foot of head. This value is used to calculate a conversion factor from head level in feet to volume in billion gallons. Extracting 1 billion gallons of water from the aquifer would lower the head approximately by 1/78 or 0.012796 feet, giving us $\gamma = 0.000012796$ ft/million gallons (mg). We follow Krulce et al. (1997) in using Mink's estimates of leakage increasing with head level (Mink, 1980, Table 18) and inflow constant at 281 million gallons per day (mgd) including rainfall and subsurface flows (Mink, 1980, Table 18 and p. 46), yielding a net recharge function (inflow minus leakage): $n(h_t) = 281 - 0.24972h_t^2 - 0.022023h_t$.

The cost is a function of elevation (and, therefore, the head level), specified as:

$c_q(h_t) = c_0\left[(e-h_t)/(e-h_0)\right]^{\theta}$, where c_0 is the initial extraction cost when the head level is at the current level, $h_0 = 15$ feet. There are many wells from which freshwater is extracted and, using a volume-weighted average cost, we have separately estimated the initial average extraction cost in Pearl Harbor at $0.407 per thousand gallons (tg) of water. The average elevation of these wells, e, is estimated at 50 feet, and n is an adjustable parameter that controls the rate of cost growth as head falls. We assume $\theta = 2$. Sensitivity analyses for $\theta = 1$ and $\theta = 3$ did not change the conclusions of this article. Since the head level does not change much over time relative to the elevation, the value of θ does not affect the results appreciably.

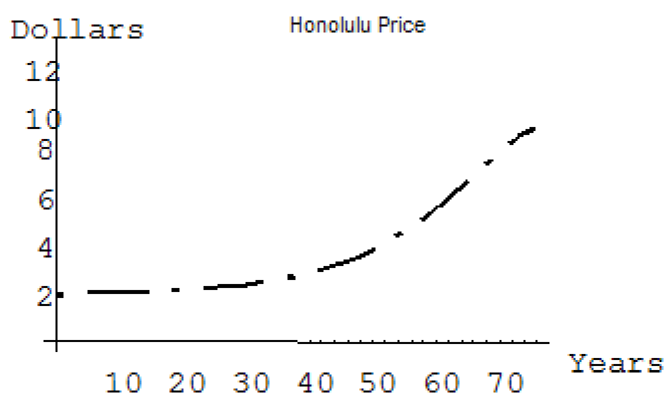
The unit cost (c^b) of desalted water has been separately estimated at $7/tg. This includes a cost of desalting ($6.79/tg) and additional cost of transporting the desalted water from the seaside into the existing freshwater distribution network that we assume to be $0.21/tg.

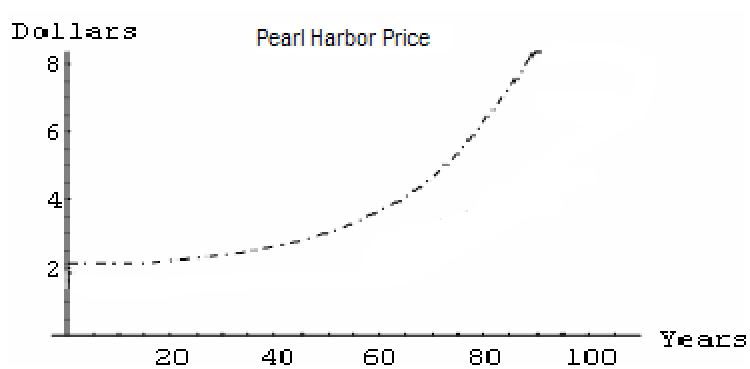
We use a demand function of the form: $D(p_t, t) = A e^{gt} (p_t)^{-\mu}$, where A is a constant, g is the demand growth rate, p_t is the price at time t, and μ is the price elasticity of demand. The demand growth rate, g, is assumed to be 1% (based on projections in DBEDT, 2005). The constant of the demand function, A (=221.35 mgd) is chosen to normalize the demand to actual price and quantity data. Following Krulce et al. (1997), we use $\mu = -0.3$ (also see Moncur, 1987, and Malla, 1996). We calculate the distribution cost, c_d = $1.363/tg, from the water utility data (Honolulu Board of Water Supply, 2002).

Honolulu

Repeating the above calibration methods for the Honolulu aquifer, we find that it has 61 billion gallons of water stored per foot of head. Thus, extracting one billion gallons (or a thousand MG) of water from the aquifer would lower the head by 1/61 or 0.0163934 feet, giving us $\gamma = 0.0000163934$ ft/MG. At the current level, $h_0 = 22$ feet (at Beretannia wells). The initial average extraction cost (c_0) in Honolulu is $0.16/tg, A=83.77 mgd, and c_d = $1.81/tg. The cost of transporting water from Pearl Harbor to Honolulu is about $0.5/tg.

Using these numbers, the model yields the following price paths for the two water districts.





The price in Honolulu rises faster due to smaller aquifer size and larger demand. In year 45, the Honolulu price exceeds by more than the cost of transporting water from Pearl Harbor to Honolulu. Thus, it would be worthwhile to transport water in that direction after that time.

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