

Risk Aversion, Prudence, and the Three-Moment Decision Model for Hedging

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Linear moment preference functions have been widely used in decision analysis as approximations of the Von Neumann-Morgenstern expected utility (EU). The two-moment mean-variance model is the most popular one among them. It was originated by Markowitz (1952) as a portfolio selection tool, extended by Tobin (1958) to include risk-free assets, and applied in equilibrium analysis by Sharpe (1964) and Lintner (1965) to the pricing of capital assets. Comparing to EU models, the two-moment model requires less information from decision makers and from random distributions. However, challenges (Borch, 1969; Feldstein, 1969) to the appropriateness of the approximation have caused the defenders of mean-variance to either modify the conditions or improve the model by adding more moments. Theoretically, the two-moment model can yield a consistent optimal decision with EU if 1) the decision maker's utility function is quadratic, or 2) the stochastic return is normally distributed (Tobin, 1958), or 3) the random variables satisfy the location-scale constraint (Meyer, 1987). However, Arrow and Hicks denounced a quadratic function as absurd because of its limited range of applicability and highly implausible implication of increasing absolute risk-aversion. On the other hand, the assumption of normal distribution of all risky outcomes is not realistic since returns typically are not normally distributed.

Since all of these constraints are very restrictive, many studies have expanded the model to incorporate higher moments. Samuelson (1970) noted that including more than the first two moments can improve the solution for any arbitrarily short, finite time interval. Tsiang (1972) pointed out skewness preference may be prevalent in investors'

behavior because modern financial institutions provide a number of devices for investors to increase the positive skewness of the returns of their investments. The skewness preference has received special attention in the asset pricing and portfolio theory (Kraus and Litzenberger, 1976; Friend and Westerfield, 1980; Sears and Wei, 1988; Lim 1989).

The three-moment model will be important for analytical studies on risk management decision modeling when the distribution is skewed. Poitras and Heaney (1999) compared the optimal demand for put options derived from the two-moment and three-moment models. It is shown theoretically the optimal demand for put options was reduced with positive skewness preference. However, they did not compare their results to the expected utility model, and their derived moment models require a specific utility form. Further development and application of three-moment model in agricultural risk management using derivatives are very limited.

In this paper, we will develop a general three-moment model and compare it and the traditional two-moment model to the expected utility in the setting of an individual producer hedging in the futures market. Specific objectives include (1) to theoretically develop the linear three-moment model analogue to the existing mean-variance model, (2) to apply it in the context of hedging and derive the optimal solution as well as comparative statics, and (3) to numerically compare the optimal hedge ratios (OHR) derived from the two-, three-moment models and the full expected utility model under alternative preference parameters. Only the second and third moments are concerned in this paper because higher moments add little, if any, information about the distribution's physical features (Arditti, 1967).

Model

A decision maker's preference can be represented by a Von Neumann-Morgenstern utility function $U(\pi)$. Using Taylor's expansion,

$$\begin{aligned} EU(\pi) &= EU(\mu + \varepsilon) \\ &\cong EU(\mu) + E[\varepsilon U'(\mu)] + E[\varepsilon^2 u''(\mu)]/2 + E[\varepsilon^3 u'''(\mu)]/3! \end{aligned}$$

where $E()$ is the expected value operator, π is the random profit, μ is the expected profit, and ε is the error term with zero expected value. Because maximizing the utility function of the certainty equivalent is equivalent to maximizing the expected utility function (Robinson and Barry, 1987) and the utility function is monotonically increasing, the three-moment model in terms of mean, variance and skewness are derived as (Appendix):

$$(1) \quad \text{Max } \pi_{CE} = \text{Max } E(\pi) - \frac{\lambda}{2} V(\pi) + \frac{\lambda \eta}{6} S(\pi)$$

where π_{CE} is the certainty equivalent of profit; $E()$, $V()$ and $S()$ are the expectation, variance and skewness¹ operators; λ is Arrow-Pratt's absolute risk aversion coefficient, i.e., $-U''/U'$; η is Kimball (1990)'s absolute prudence level, i.e., $-U'''/U''$ which is isomorphic to Arrow-Pratt's absolute risk aversion. According to Kimball (1990), the absolute prudence measures the sensitivity of the optimal choice of a decision variable to risk. This term suggests the propensity to prepare and forearm oneself in the face of uncertainty while "risk aversion" measures how much one dislikes uncertainty and would turn away from uncertainty if possible. $\pi\eta$ is a measure of relative prudence, just as $\pi\lambda$ is a measure of relative risk aversion.

According to Arrow (1971), the essential properties for an investor's utility function are: (1) positive marginal utility for wealth, i.e., $U' > 0$, (2) decreasing marginal

utility for wealth, i.e., $U'' < 0$, and (3) non-increasing absolute risk aversion, i.e. $U''' \geq 0$.

Thus both λ and η should be positive, i.e., the decision maker is risk averse and prudent.

He/she would always desire positive skewness of return π when the mean and variance of the return remain constant. The two-moment model is equation (1) without the third term, or $\eta = 0$.

Assuming no transaction costs for trading futures contracts, the return π in the futures market for an individual farmer is specified as:

$$(2) \quad \pi = \pi_0 + px - c + (f_0 - f)y$$

where π_0 is the initial wealth; p is the cash price at harvest; x is the nonstochastic production level; c is the cost of producing x ; y is the hedging level in the futures market to be determined; f_0 is the price at planting time; f is the futures price at harvest.

Denoting $\sigma_p^2, \sigma_f^2, \sigma_{pf}, s_p, s_f, \sigma_{p^2f}, \sigma_{pf^2}$ as the variances, covariance, skewnesses and coskewness of the cash and futures prices respectively², the expected value, variance and skewness of the return from hedging become:

$$(2.1) \quad E(\pi) = \pi_0 + xE(p) - c + [f_0 - E(f)]y$$

$$(2.2) \quad V(\pi) = x^2\sigma_p^2 + y^2\sigma_f^2 - 2xy\sigma_{pf}$$

$$(2.3) \quad S(\pi) = x^3s_p - y^3s_f - 3x^2y\sigma_{f^2p} + 3xy^2\sigma_{p^2f}$$

Substituting the specific expected value, variance and skewness of return in equation (1), the first order condition of the model yields:

$$(3) \quad y_{MVS}^* = \frac{f_0 - E(f)}{\lambda\sigma_f^2} + \frac{\sigma_{pf}}{\sigma_f^2}x - \frac{\eta}{2\sigma_f^2} [s_f(y_{MVS}^*)^2 + \sigma_{p^2f}x^2 - 2x\sigma_{f^2p}y_{MVS}^*]$$

where y_{MVS}^* is the optimal hedging levels for the three-moment models. The two-moment optimal hedge level y_{MV}^* equals the first two terms of equation (3). As expected, the optimal hedge levels from the two models are the same, or the three-moment model does not add any information upon the two-moment model, if the decision maker is “prudence neutral”, i.e. $\eta = 0$.

The closed form solution for equation (3) is³:

$$(4) \quad y_{MVS}^* = \frac{x\sigma_{f^2p}}{s_f} + \frac{-\sigma_f^2 + \sqrt{\Delta}}{\eta s_f}$$

where $\Delta = (\sigma_f^2 - \eta x \sigma_{f^2p})^2 - \eta s_f [\eta x^2 \sigma_{f^2p} - 2x\sigma_{pf} - 2f_0/\lambda + 2E(f)/\lambda]$. It suggests that the solutions from the two models can be equal when the decision maker is not prudence neutral only if $\sigma_{fp} = \sigma_f^2$, $\sigma_{f^2p} = s_f$ and in the unbiased futures market ($f_0 = E(f)$). The farmer would fully hedge, namely, he or she will hedge the same amount as the production level in that case. Therefore, we have the following proposition.

Proposition 1: The optimal hedge levels of mean-variance and mean-variance-skewness models are equal if:

- (i) the decision maker is “prudence neutral”, i.e. $\eta = 0$;
- (ii) $\sigma_{fp} = \sigma_f^2$, $\sigma_{f^2p} = s_f$ and the futures market is unbiased; or
- (iii) $\sigma_{f^2p} = 0$, and $s_f = 0$.

We refer $\sigma_{fp} = \sigma_f^2$ as cash and futures prices are perfectly correlated in the two moment, and $\sigma_{f^2p} = s_f$ as perfectly correlated in the third moment, assuming the variance and skewness of the two prices are the same for convenience. Only when the two prices

are perfectly correlated, the mean-variance model yields a full hedge for risk averse farmers in the unbiased market. (ii) says if the two prices are furthermore perfectly correlated in the third moment, the mean-variance-skewness model yields a full hedge for risk averse and prudent farmers. The two cases are implied in a more strong condition when there is no basis risk, ie. $p = f$. Then a decision maker will always make a fully hedge in either model (and in the full expected utility model). (iii) says if the two prices are un-coskewed, and the futures price is not skewed, then the means-variance-skewness model also yields the same optimal hedging levels as the mean-variance model, because the hedging will not affect the skewness of the return and therefore the prudent preference will not affect hedging.

Corollary 1: The risk averse and prudent farmer will make a full hedge in an unbiased market if there is no basis risk.

The following comparative static propositions can be derived by partially differentiating the two optimal hedge levels with respect to each parameter.

Proposition 2: The short position will be increased (or decreased) and the long position will be decreased (or increased) if current futures price goes up (or down) in both mean-variance and mean-variance-skewness models, while holding all other parameters constant.

Proof: Partially differentiate the two optimal hedge levels with the initial futures price:

$$(5) \quad \frac{\partial y_{MV}^*}{\partial f_0} = \frac{1}{\lambda \sigma_f^2} > 0$$

$$(6) \quad \frac{\partial y_{MVS}^*}{\partial f_0} = \frac{1}{\lambda \Delta^{1/2}} > 0$$

The values of the optimal hedge levels are monotonically increasing with the initial futures price. A higher optimal hedge level means “hedge more” for a short position hedger and “hedge less” for a long position hedger because the absolute value is decreased. This is a speculating effect because a higher current futures price means more expected profit gain (loss) for a short (long) hedger. For both models, the response is a smaller for a more risk averse hedger because the speculating position deviates from the optimal risk reducing position, and the more risk averse hedger chooses to deviate less.

Proposition 3: The short position will be increased (or decreased) and the long position will be decreased (or increased) if the covariance between the cash and futures prices increases (or decreases) in both mean-variance and mean-variance-skewness models, while holding all other parameters constant.

Proof: The following are obtained by partially differentiating with the covariance of cash and futures prices.

$$(7) \quad \frac{\partial y_{MV}^*}{\partial \sigma_{pf}} = \frac{x}{\sigma_f^2} > 0$$

$$(8) \quad \frac{\partial y_{MVS}^*}{\partial \sigma_{pf}} = \frac{x}{\Delta^{1/2}} > 0$$

This is a risk reducing effect because a higher covariance means lower basis risk and the risk reducing effect gives more incentive on short hedging but less incentive on long hedging. The responses from both models are proportional to the production level.

Proposition 4: The decision maker hedges more (or less) if the current futures price is lower (or higher) than the expected futures price as the decision maker becomes more

risk averse in both mean-variance and mean-variance-skewness models, while holding all other parameters constant. The risk aversion coefficient will not affect the hedging position when the futures market is unbiased, when other parameters remain constant.

Proof: Differentiate with the risk aversion level and obtain:

$$(9) \quad \frac{\partial y_{MV}^*}{\partial \lambda} = -\frac{f_0 - E(f)}{\lambda^2 \sigma_f^2}$$

$$(10) \quad \frac{\partial y_{MVS}^*}{\partial \lambda} = -\frac{f_0 - E(f)}{\lambda^2 \Delta^{1/2}}$$

According to (9) and (10), the optimal hedge levels from the two models change in the same direction as risk aversion increases. Both equations have a positive sign as $f_0 < E(f)$ and a negative sign as $f_0 > E(f)$. Risk averters will make a full hedge when there is no basis risk in the unbiased futures market. This result will not change with the risk aversion level. The full hedge minimizes risk.

When the current futures price is lower than the expected maturity price, both models advise the decision maker to underhedge, namely, to sell less than their production level. As they become more risk averse they will increase their hedging levels toward the full level, because their risk reducing incentive increases relative to their loss reducing incentive. If the current future price is sufficiently low the decision maker would be likely to take a long position, namely, buy now and sell in the future from the futures market. In that case, the farmers would hedge less as they are more risk averse. On the other hand, when the current futures price is higher than the expected maturity price, both models recommend over hedging, and more risk-averse farmers will over hedge less so as to be closer to the full hedge level.

The comparative statics of the three-moment optimal hedge level on the other parameters are:

$$(11) \quad \frac{\partial y_{MVS}^*}{\partial \eta} = \frac{\sigma_f^2}{\eta^2 s_f} + \frac{[\eta x \sigma_f^2 \sigma_{f^2p} - \eta x s_f \sigma_{pf} - \sigma_f^4 - \eta s_f (f_0 - E(f) / \lambda)] \Delta^{-1/2}}{\eta^2 s_f}$$

$$(12) \quad \frac{\partial y_{MVS}^*}{\partial s_f} = -\frac{x \sigma_{f^2p}}{s_f^2} + \frac{\sigma_f^2}{\eta s_f^2} - \frac{\Delta^{-1/2} \{ (\sigma_f^2 - \eta x \sigma_{f^2p})^2 + \eta s_f [\eta x^2 \sigma_{f^2p} - 2x \sigma_{pf} - 2f_0 / \lambda + 2E(f) / \lambda] / 2 \}}{\eta s_f^2}$$

$$(13) \quad \frac{\partial y_{MVS}^*}{\partial \sigma_{f^2p}} = \frac{x}{s_f} [1 + \Delta^{-1/2} (-\sigma_f^2 + \eta x \sigma_{f^2p} - \frac{1}{2} \eta x s_f)]$$

The signs for equation (11), (12) and (13) are ambiguous. These signs will be examined for the following empirical example.

Simulation and Numerical Results

Numerical analysis of an example examines the level of approximation of the two-moment and three-moment models to the expected utility model by comparing the optimal hedge ratios (OHR). The hedge ratio is the ratio of hedging to the production level. Assume the hedger has the commonly used CRRA (constant relative risk aversion) utility function:

$$(14) \quad U(\pi) = (1 - \theta)^{-1} \pi^{(1-\theta)}$$

where θ is the relative risk aversion coefficient. The optimal hedge ratio for the expected utility model is solved numerically. For two- and three-moment models, the optimal hedge ratios are obtained by (3) ignoring the third term and (4). The values of θ range from 1 to 4 following Dynan (1993). Six levels of relative risk aversion coefficient (θ),

specifically 1.5, 2, 2.5, 3, 3.5, 4, are analyzed. The six levels of the absolute risk aversion coefficient λ and absolute prudence coefficient η are calculated based on the relative risk aversion levels ($\lambda = \theta / \pi$, $\eta = (1 + \theta) / \pi$)⁴.

The analysis assumes a representative farmer who grows wheat in U.S. Pacific Northwest region. The initial wealth determined by average per acre is \$550 per acre. Production cost is \$230 per acre and production level is 68.94 bushels per acre. Bivariate gamma distribution is chosen to simulate the wheat cash and futures prices for the 2002 harvest period because (1) it's positively skewed; (2) gamma random variables (cash and futures prices in this case) are positive; (3) it facilitates including the skewness parameter in the simulation. The approach of Law and Kelton (1982) is used to simulate the correlated bivariate gamma distribution.

The correlation between the wheat cash and futures prices is 0.48. The scale and location parameters for the gamma distribution are calculated from the variances (0.37 and 0.56 for wheat cash and futures prices) and skewnesses (0.12 and 0.29 for wheat cash and futures prices)⁵. The mean values are adjusted to \$3.7 and \$3 per bushel respectively after the simulation. These parameter levels are determined based on the weekly Portland spot market cash price and CBOT futures price data from September 1998 to August 2001. The descriptive statistics for the simulated cash and futures prices are shown on Table 1. The skewness of the simulated cash and futures prices are significant, although they appear small.⁶ The initial future price f_0 is set at three levels (\$3.20, 3.00 and 2.80 per bushel). The futures market is unbiased when f_0 equals \$3.00 per bushel. The hedger is likely to take a short (or long) position if f_0 equals to \$ 3.20 (or 2.80) per bushel.

Table 2 shows the optimal hedge ratios (OHR) from three models under six levels of relative risk aversion and three levels of initial futures prices. The results show that OHRs from the three-moment model are closer to those from expected utility model than two-moment model OHRs in all situations. The evidence from this example strongly favors the three-moment model over the two-moment model.

Comparing the absolute OHR values, the farmer hedges more (or less) in the MVS model than in the MV model when he is in a long (short) position. Based on equation (3), the optimal hedge level of the MVS model has one more term than that of MV model, which is due to skewness of profit. If a farmer with a short (long) position hedges more, the skewness of profit which he desires will be decreased (increased) according to the definition of skewness of profit. Thus compared to MV model, the farmer would hedge less (more) in a short (long) position.

When the initial futures price changes from \$2.8 to \$3 and \$3.2 per bushel, the OHR values of both models increase, consistently with Proposition 2. The OHRs from the MV model increase at a constant rate for each relative risk aversion level. But the values from MVS do not have the same pattern with each level of initial futures price increase, which is also consistent to Proposition 2 as in equations 5 and 6.

The absolute values of OHRs from the MV model do not change in the unbiased futures market while they drop with the relative risk aversion in the biased futures market. This is consistent with Proposition 4 (equation 9). The absolute values of MVS OHRs decrease in biased and unbiased futures as relative risk aversion increases. This seeming inconsistency arises because the particular CRRA utility we choose is not constant in prudence, because the prudence level is related to the risk aversion level. In

order to compare the MVS results to the true utility maximization results, we allow the prudence to vary accordingly. Therefore, the conditions in Proposition 4, i.e., holding all other parameters constant, is violated, and the OHR changes for MVS model in Table 2 is a result of a joint increase in both risk aversion and prudence.

In both models, the farmer hedges more when he is in a short position than in a long one. This is because the minimum risk position is short. When the market goes biased for the same level in both directions, the short hedge is enhanced and the long hedge is only a residual after the short position has been fully reduced.

The comparative statics are also numerically checked so as to illustrate the ambiguous signs of equation (11) and (12). Equation (13) is not checked because the coskewness can not be controlled in the simulation because it changes with the skewness. First, we examine how the MVS OHR changes with the skewness of futures prices. The cash price skewness is fixed because it is not directly related to OHR (Equation 4). Three skewness levels (0.5, 1.0 and 1.5 times of the original skewness) are chosen for the futures prices so that the bivariate gamma distributed cash and futures prices could be simulated.⁷ Two more bivariate gamma distributed cash and futures prices are simulated based on the change of skewness. According to the simulated data, the coskewness, σ_{p^2f} decreases and σ_{pf^2} increases as the skewness of futures goes up and vice versa.

The comparative static results of MVS OHRs on futures price skewness are demonstrated in Table 3. Both unbiased and biased (long and short positions) are considered. The farmer takes a longer position when the futures price is more skewed. The intuition is that the farmer uses hedging to both reduce variance and increase

skewness of the profit, and if futures price is more skewed the long position can amplify the profit skewness more effectively. The increasing skewness motivates the farmer to increase his long hedge position at a cost of decreased variance. The same reasoning can be used to explain the smaller short position when skewness increases in the biased futures market. The opposite behavior occurs under the unbiased market because the short positions are much smaller than in the other two cases. The variance reducing effect dominates the skewness increasing effect comparing the variance equation (2.2) and skewness equation (2.3) of profit. This causes the farmer to take a larger short position. Therefore, the comparative static on futures skewness cannot be simply determined in sign.

The influences of risk aversion and prudence on the OHRs in the MVS model is shown in Figure 1 (a) and (b), respectively. The relative risk aversion and prudence range from 1 to 5. Empirical research on prudence levels is not available. The range is chosen at the same level as risk aversion because the two are often close in commonly used utility functions such as exponential (constant absolute risk aversion preference), log or power functions (constant relative risk aversion preference). The impact on OHR from relative risk aversion has consistent pattern as in proposition 4, when the relative prudence level is fixed at 2.

When relative risk aversion is fixed at 2, the hedging position decreases as the farmer becomes more prudent so that all three lines in Figure 1(b) are downward sloping. The downward slope in the unbiased market is so small that the line looks horizontal. The influence of the prudence on the market is trivial in this case. The decreasing position means hedging less in short and more in long. We have also set risk aversion at

other levels, but the OHRs show the same pattern, i.e., decreasing with prudence. This means the sign of equation (11) is not sensitive to the preference parameters. Compared to Figure 1(a), risk aversion makes a big difference in OHR than prudence in the biased futures market.

Figure 2 demonstrates how relative risk aversion and prudence affect the certainty equivalent in the MVS model in an unbiased futures market. The certainty equivalent is the certain amount of money which leaves the decision maker equally well-off as the specified risky hedging opportunity. The higher certainty equivalent means higher utility achieved with hedging. The results show that changes of certainty equivalent brought by difference prudent levels are small relative to the changes brought by different risk aversion levels.

The certainty equivalent always decreases as the risk aversion increases in both biased and unbiased markets, because the farmer requires higher compensation for risk. For the same reason, one might expect the certainty equivalent to increase as prudence increases. However, it decreases as the prudence increases when in a very large short position (See Figure 2(b)). This occurs because the long position increases very fast (Figure 1(b)) which reduces the profit skewness enough to offset the increased prudence.

Summary and Conclusion

A linear mean-variance-skewness (three-moment) model is developed and applied to the hedging decision in the futures market. The optimal hedge ratio (OHR) and associated comparative statistics are derived and compared theoretically from both three-moment and two-moment models. The term “prudence” introduced by Kimball is

included in the three-moment model. The OHRs from the two models are equal only when: 1) the decision maker is “prudence neutral” or; 2) with unbiased futures markets, assuming perfect correlation of cash and futures prices in both second and third moments. The OHR of the three-moment model changes in the same direction as that of the two-moment model when the initial futures price, covariance of cash and futures prices and risk aversion coefficient change respectively. Otherwise, effects on OHRs are not definite theoretically. The signs on the comparative statics of the three-moment OHR on the other parameters such as the prudence level, skewness of futures prices and coskewness of futures and cash prices are ambiguous.

The two and three moment models are also compared against the expected utility model for a numerical example so as to examine which model provides a closer approximation to expected utility. We assume the hedger is a typical farmer, with the common CRRA utility function, who grows wheat in the Pacific Northwest. The results show the OHRs from the MVS model is closer to those from expected utility model than those from MV model in all situations considered. This strong evidence suggests that the MVS model is superior to the MV model. The farmer hedges more (less) in the MVS model than in the MV model when he/she is in a long (short) position. This results from the additional term skewness of return in the MVS model. The comparative statics of MVS OHRs on futures price skewness indicates the farmer takes a longer position so as to increase the benefit from increased positive profit skewness when the futures price is more skewed. The opposite behavior for the unbiased market is primarily because the short positions are much smaller than in the other two cases, and the variance reducing

effect dominates the skewness increasing effect. There's no clear pattern when the farmer is in a short position.

The influences of risk aversion and prudence on OHRs for the MVS model are also examined. The ranges of relative risk aversion and prudence are extended a little based on the common CRRA utility function. The numerical results show the farmer full-hedges in the unbiased market and hedges less as risk aversion increases in the biased futures market. The hedging position decreases as the farmer becomes more prudent. Risk aversion has a greater influence on OHR than prudence in the biased futures market.

The certainty equivalent consistently decreases as the risk aversion increases in both biased and unbiased market, because the farmer requires his/her certain compensation for the risky hedge. Similarly, the certainty equivalent should be expected to increase with prudence; however, it decreases in a very large short position. This is because the long position increases quickly which reduce the profit skewness thereby offsetting the effect of the increased prudence.

Endnote:

¹ “Skewness” refers to the third moment instead of standardized third moment in this paper.

$$\sigma_{fp^2} = E\{[f - E(f)][p - E(p)]^2\}, \sigma_{f^2p} = E\{[f - E(f)]^2[p - E(p)]\}$$

³ The first order condition equation has two roots and result in two closed forms of y_{MVS}^* actually. The sign operator before the square root could be ‘add’ or ‘subtract’. The ‘add’ operator is chosen in order to achieve the maximum by the second order condition (SOC).

⁴ For the particular CRRA preference, the relative prudence is determined once the relative risk aversion is set at a certain level.

$$\frac{\partial U}{\partial \pi} = \pi^{-\theta}, \frac{\partial^2 U}{\partial \pi^2} = -\theta \pi^{-(1+\theta)}, \frac{\partial^3 U}{\partial \pi^3} = \theta(1+\theta)\pi^{-(2+\theta)}$$

$$\lambda = -U''(\mu)/[U'(\mu)] = \theta/\mu, \eta = -U'''(\mu)/[U''(\mu)] = (1+\theta)/\mu.$$

⁵ Location parameter $\alpha = 4(\sigma^2)^3/S^2$ and scale parameter $\beta = S/2\sigma^2$.

⁶ Formal hypothesis test is conducted. $H_0: S = 0$ vs. $H_1: S \neq 0$ where S is the skewness. Then the statistic z , $z = \hat{S}/\sqrt{6/n}$, where $n = 10,000$ the number of observations, follows the standard normal distribution under the null hypothesis. Here, z is 5.031 and 11.424 for cash and futures prices, respectively, and both are larger than the critical value at 5%. Therefore, the null hypothesis of zero skewness is rejected for both.

⁷ If the skewness is less than 0.5 times, the futures price would be almost normally distributed which is not the interest of this paper. If the skewness is larger than 1.5 times, the bivariate gamma distributed cash and futures prices would not be able to be simulated.

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Table 1: Descriptive Statistics for Simulated Cash and Futures Prices (Units: \$/bushel)

Variable	N	Mean	Skewness	StDev	Minimum	Median	Maximum
cash	10000	3.7	0.123	0.6104	2.052	3.653	6.672
future	10000	3	0.28	0.7418	1.267	2.914	7.048

Table 2: Optimal Hedge Ratios Comparison under Six Relative Risk Aversion Levels and Three Levels of Initial Futures Prices

θ	1.5	2	2.5	3	3.5	4
<u>unbiased futures market ($f_0 = E_f = \\$3/\text{bushel}$)</u>						
Mean-Variance Model	0.392	0.392	0.392	0.392	0.392	0.392
Mean-Variance-Skewness Model	0.385	0.384	0.382	0.381	0.38	0.378
Expected Utility Model	0.385	0.383	0.382	0.38	0.379	0.378
<u>$f_0 = \\$3.2/\text{bushel}$</u>						
Mean-Variance Model	2.325	1.842	1.552	1.358	1.22	1.117
Mean-Variance-Skewness Model	2.089	1.676	1.424	1.253	1.13	1.037
Expected Utility Model	2.046	1.673	1.43	1.261	1.138	1.043
<u>$f_0 = \\$2.8/\text{bushel}$</u>						
Mean-Variance Model	-1.541	-1.058	-0.768	-0.575	-0.437	-0.333
Mean-Variance-Skewness Model	-2.008	-1.361	-0.992	-0.754	-0.588	-0.465
Expected Utility Model	-1.86	-1.295	-0.954	-0.727	-0.566	-0.446

Note: For $f_0 = \$2.8/\text{bushel}$, the negative hedge ratios mean the hedger takes a long position.

Table 3: Impacts of Futures Price Skewness on Three-Moment Optimal Hedge**Ratios**

theta	1.5	2	2.5	3	3.5	4
<u>unbiased futures market ($f_0 = E_f = \\$3/\text{bushel}$)</u>						
0.5*Sf	0.364	0.362	0.359	0.357	0.354	0.351
Sf	0.385	0.383	0.382	0.38	0.379	0.378
1.5*Sf	0.39	0.39	0.39	0.39	0.39	0.39
<u>$f_0 = \\$3.2/\text{bushel}$</u>						
0.5*Sf	2.15	1.71	1.442	1.261	1.131	1.033
Sf	2.089	1.676	1.424	1.253	1.13	1.037
1.5*Sf	1.98	1.599	1.366	1.208	1.094	1.008
<u>$f_0 = \\$2.8/\text{bushel}$</u>						
0.5*Sf	-1.71	-1.18	-0.868	-0.663	-0.518	-0.411
Sf	-2.008	-1.361	-0.992	-0.754	-0.588	-0.465
1.5*Sf	-2.569	-1.619	-1.148	-0.859	-0.663	-0.52

Note: For $f_0 = \$2.8/\text{bushel}$, the negative hedge ratios mean the hedger takes a long position.

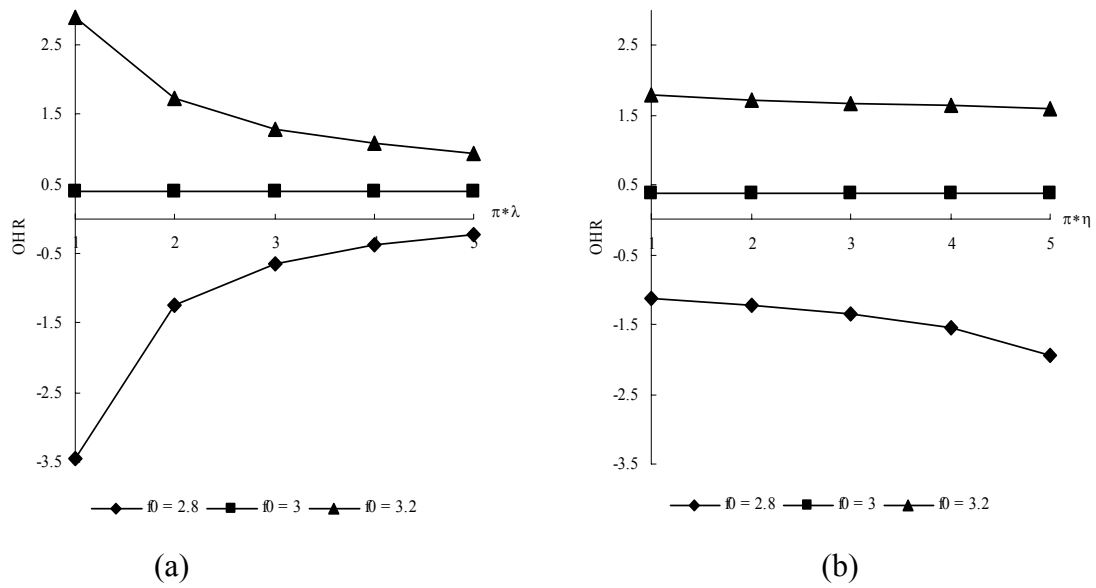
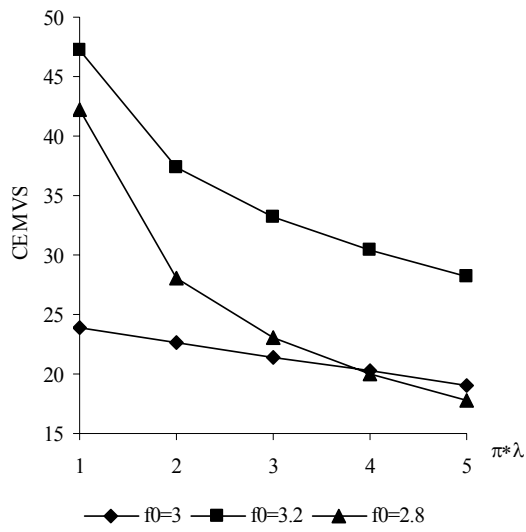


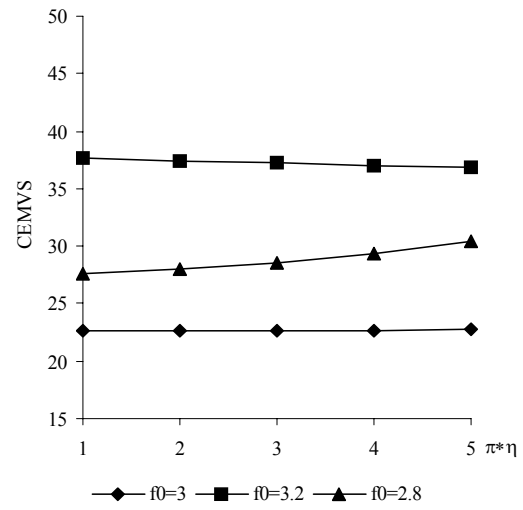
Figure 1: Comparative Statics of Optimal Hedge Ratio (OHR) by Relative Risk Aversion Level and Relative Prudence level.

Note: (1) For $f_0 = \$2.8/\text{bushel}$, the negative hedge ratios mean the hedger takes a long position.

(2) $\pi * \lambda$ is relative risk aversion and $\pi * \eta$ is relative prudence.



(a)



(b)

Figure 2: Certainty Equivalent Changes with Relative Risk Aversion Level and Relative Prudence Level respectively.

Note: $\pi^*\lambda$ is relative risk aversion and $\pi^*\eta$ is relative prudence.

Appendix: Derivation of the three-moment model in terms of mean, variance and skewness.

$$U(\pi_{CE}) = U(\mu - m) = EU(\mu + \varepsilon)$$

where μ is the expected profit return, m is premium and ε is the error term with zero expected value. The profit return π is a random variable which is equal to $\mu + \varepsilon$.

$$U(\mu - m) \cong U(\mu) - mU'(\mu)$$

$$\begin{aligned} EU(\mu + \varepsilon) &\cong EU(\mu) + E[\varepsilon U'(\mu)] + E[\varepsilon^2 U''(\mu)]/2 + E[\varepsilon^3 U'''(\mu)]/3! \\ &= U(\mu) + \sigma^2 U''(\mu)/2 + S_k U'''(\mu)/3! \end{aligned}$$

$$m = \sigma^2 U''(\mu)/[2U'(\mu)] + S_k U'''(\mu)/[3!U'(\mu)]$$

$$\pi_{CE} = \mu - m$$

$$= \mu - \lambda \sigma^2 / 2 + S_k U'''(\mu) / [6U'(\mu)]$$

$$= \mu - \lambda \sigma^2 / 2 + S_k \lambda \eta / 6$$

$$\text{where } \lambda = -U''(\mu)/[U'(\mu)] \text{ , } \eta = -U'''(\mu)/[U''(\mu)]$$