Hedging and Production Decisions Under a Linear Mean-Variance Preference Function

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A firm model of production and hedging decisions is developed using a mean-variance preference function. Comparative static analysis of the model generates a number of testable hypotheses. For example, the influence of price risk, production risk and hedging cost on the optimal level of production and hedging is analyzed in this framework.

The Theory of production decisions for a risk averse firm under uncertainty has been examined extensively in the literature [Baron, Sandmo, Batra and Ullah, Pope and Kramer]. The use of futures trading as a price risk management tool for farmers has also been analyzed [Heifner; Ward and Fletcher]. More recently, Danthine, Holthausen and Feder, et al. (hereafter DHF) have attempted to integrate these two approaches. They developed a model that extends the theory of the firm under price uncertainty in a way especially relevant for agricultural firms by considering futures markets. In the DHF model, as input decisions are made, the producer decides to hedge by selling contracts in the futures market for delivery at the end of the production process. Although the cash price is uncertain at the time of the input decision, the futures price is known with certainty, and basis risk is presumed to be insigificant.[1] Also, the DHF model assumes no production uncertainty and no futures trading cost.

Numerous interesting and testable propositions are found from the DHF model. To mention one of particular relevance here: *a ceteris paribus change in the distribution of spot price has no effect on production decisions. That is, production decisions do not depend on expected spot price or its variability*. It should be mentioned that this result in no way hinges on the predictive performance of the futures price for the average spot price but is a function of the micromodel employed. This suggests that Nerlovian models of supply response, specified as a function of expected spot price, may be inappropriate. This is somewhat disturbing given the relatively good record of Nerlovian models in agricultural economics [Askari and Cummings].

This paper presents an alternative formulation of production and hedging decisions which leads to different results which appear more relevant for agricultural production decisions. The analysis is generalized to deal with production uncertainty, and transactions and other hedging costs in the context of a mean-variance utility function.

First, given the nature of agricultural production, production uncertainty should be incorporated in any model of agricultural

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[1]Though basis risk may be significant in some regions and for some commodities, it is likely small in comparison to the other risks investigated in the model.

production decisions under risk. Furthermore, as argued by McKinnon, production uncertainty is expected to influence hedging decisions. Once the farmer faces unpredictable output, going short in some future commodity may increase his income uncertainty. For example, an unexpectedly short crop may be less than the forward sale, forcing the farmer to enter the spot market at harvest time as a buyer to cover his short futures commitment. Since the spot price at that time is not known when the input and hedging decisions are made, output variability exposes the farmer to a price risk that cannot be avoided by hedging. This suggests that the introduction of production uncertainty can have a significant impact on both production and hedging decisions of a competitive firm.

Second, this paper departs from DHF by considering hedging cost. Such costs have already been discussed by Heifner, and Ward and Fletcher in the analysis of optimal hedging decisions. In particular, by taking into consideration commissions and interest on margin deposits, Heifner has shown that, for a risk averse firm, the optimal hedging level is very sensitive to these futures trading costs. This suggests that hedging cost can play an important role in production as well as hedging decisions.

The objective of this paper is to derive testable hypotheses from a theoretical micro model of production and hedging decisions for a competitive firm. The analysis is generally comparative static in nature. The impact of a change in selected parameters on the firm's decisions is derived. For example, among other factors, the influence of price risk, or futures price on the optimum level of production and hedging is discussed. In order to conveniently derive the results, both the primal approach (as in Samuelson) and the primal-dual approach of Silberberg are used. To our knowledge, symbiotic use of the two approaches is not found in the literature. In an attempt to minimize mathematical details, the paper concentrates on presentation and discussion of the results. The details of the mathematical derivations can be found in a companion paper [Chavas and Pope].

To anticipate the results, it is shown that unlike DHF, production responds to the distribution of spot price. Not only does it respond to spot price, but hedging responds in ways often dissimilar to the DHF model. In particular, hedging responses are sometimes ambiguous since hedging may carry substantial risk when production is uncertain. Also expected unhedged production responses are investigated.

The Firm Model

The analysis will focus on the mean-variance preference function of the form

$$L = E(\pi) - \frac{\alpha}{2} V(\pi) \tag{1}$$

where E and V denote mean and variance respectively, π is profit and α is a measure of risk aversion. This model has had extensive use in agricultural economics [Robinson and Barry, Peck, Rolfo, Wiens]. It is generally used in the context of expected utility maximization with constant absolute risk aversion and normality of π. However, Tobin and others have supported mean-variance analysis without necessarily appealing to expected utility axioms. In any case, empirical analysis often focuses on mean and variance as a simple approximation to more complex analyses.[2]

The firm is assumed to make production and hedging decisions at the beginning of the production process. At that time, the firm does not know with certainty the spot price of the output or the production level that will prevail at the end of the production process. Thus, both production and spot price are random. These two random variables are also assumed independent. This seems reason-

[2]Tsiang and others have made such arguments. However, controversy always surrounds simplifications of a general problem to mean-variance analysis.

able for a micro economic analysis in which the competitive firm cannot influence the market price. However, many of the results here are not altered by allowing a non-zero covariance between spot price and production.

Production risk is assumed multiplicative. This choice is justified as follows. First, Turnovsky has argued that multiplicative production risk can be naturally obtained in theoretical models. In this case, it is the slope of the supply function (rather than the intercept as under additive uncertainty) which is stochastic. Second, the assumption of multiplicative uncertainty is frequently used in production analysis [e.g. Hazell and Scandizzo]. This assumption finds further support from empirical research. For example, Just and Pope found that fertilizer has a variance increasing effect on risk. This suggests that a multiplicative production uncertainty would be more appropriate than an additive one. Finally, although Just and Pope suggested flexible functional forms for specifying production uncertainty, a clear trade-off exists between the complexity of the model and its usefulness in terms of being able to derive testable hypotheses. Thus, although the multiplicative production uncertainty is somewhat restrictive, it was chosen over more sophisticated specifications mainly because it is fairly realistic and yet simple to incorporate in a normative model.

At the beginning of the production process, the firm is assumed to hedge by selling contracts for delivery at the date production is realized. The quantity as well as the price of the futures contract are known at the time of the decision-making. Because of arbitrage, and given that they correspond to the same grade and same location, spot price and futures price converge at the expiration of the futures contract. Defining the basis as the difference between futures price and spot price at a particular time, this implies that the basis is zero at the end of the production process. In this case, assuming that the firm does not modify its futures commitments during the production process, it follows that basis risk can be neglected.[3] Also, when production is realized, the firm can either deliver its futures contract, or buy its futures contract back and sell the equivalent amount in the spot market. Since the futures price is assumed equal to the spot price at the end of the production period (zero basis), either action will generate the same revenue.

Thus, the producer makes the production and hedging decisions under uncertain product price and output. These assumptions are particularly relevant for agricultural firms since futures markets exist for all major farm products. Also, production uncertainty characterizes most agricultural production processes because of weather variability or other factors (diseases, mechanical failures, etc.).

The firm's *ex ante* profit function is denoted by

$$(2) \quad \pi = p[y-h]+b\,h-rx-c(h)$$

where

- p = the random spot price of output with mean $\bar{p}$, and variance σ_p,
- y = the random output, obtained from the production of $y = f(x)\varepsilon$, where $E(y) = f(x)$
- ε = a non-negative random disturbance, independent of p, with mean 1 and variance σ_ε
- x = input used in the production process, e.g., energy,
- h = volume of futures contracts sold ($h \geqslant 0$)
- b = price of the futures contract,
- r = unit cost of the input,

[3]Note that the case where spot price and futures price correspond to a different grade or a different location could be accommodated provided that the (now non-zero) basis at the time of the expiration of the futures contract is known at decision time. In such a case, under arbitrage, this basis, reflecting transportation cost or grade premium, would not expose the firm to a basis risk. The following model could easily handle such a situation by defining the futures price net of this basis.

c(h) = total cost of hedging. (commission and margin requirements[4]).

Given this specification, the variance of profit can be written as

$$V(\pi) = V(py) + h^2 V(p) - 2h\, COV(py, p)$$
$$= \{E(p^2) E(y^2) - E(p)^2 E(y)^2\} + h^2 V(p) - 2h \{E(p^2) E(y) - E(p)^2 E(y)\}$$
$$= \{V(p) + E(p)^2\}\{V(y) + E(y)^2\} - E(p)^2 E(y)^2 + h^2 V(p) - 2h \{V(p) + E(p)^2\} E(y) + 2hE(p)^2 E(y)$$
$$= V(y) E(p)^2 + V(p) [h - E(y)]^2 + V(y) V(p)$$

It follows that the objective function to be maximized (1) is

$$(1') \quad L = \bar{p}(f-h) + bh - rx - c - (\alpha/2)[\bar{p}^2 f^2\sigma_\varepsilon + (f-h)^2\sigma_p + f^2\sigma_\varepsilon\sigma_p]$$

The above model states that a farm can sell h units of output on the futures market at a certain price b. Also the output not committed to the futures market (y − h) is sold at the random market price, p. In general, because of production uncertainty, the actual output (y) will be different from the expected output (f). Since the analysis focuses on the *ex-ante* production and hedging decisions, we will consider only the expected output (f) in the rest of the paper. In particular, we will adopt the convention that $(f-h) > 0$ corresponds to an (expected) hedger and $(f-h) < 0$ to an (expected) speculator. It should be clear that these definitions are made *ex-ante*, and that it is possible, for example, for an *ex-ante* (expected) hedger $(f-h > 0)$ to become an *ex-post* speculator $(y-h < 0)$ because of the randomness of production.

The production process is characterized by the expected production function f(x) which satisfies the usual regularity conditions, i.e.

$$f' = \frac{\partial f}{\partial x} > 0$$

and

$$f'' = \frac{\partial^2 f}{\partial x^2} < 0.$$

The hedging cost function c(h) is assumed to exhibit the shape of a typical cost function,

$$c' = \frac{\partial c}{\partial h} > 0$$

and

$$c'' = \frac{\partial^2 c}{\partial h^2} > 0,$$

implying that marginal hedging cost is an increasing function of the hedge placed. This shape is justified on the basis of an increasing opportunity cost of capital (due to higher loan costs and rationing of capital) associated with an increase in futures trading cost. These assumptions make profit in (2) a concave function of x and h. Moreover, they guarantee that the second-order conditions for the maximization of (1′) are satisfied for a risk neutral ($\alpha = 0$) as well as a risk averse firm ($\alpha > 0$). This differs from DHF. Indeed, the DHF model does not allow for the possibility of risk neutral behavior: when the expected cash price differs from the futures price, the optimal hedge then becomes unbounded [Holthausen, p. 989, footnote 2]. In this analysis, the assumption of increasing marginal hedging cost guarantees a finite optimal hedge even for a risk neutral firm.

Maximizing the objective function (1′), the first order conditions that describe the optimal x and h are:

[4]Although the margin calls (funds required to maintain a futures position) are not known at the time of the hedging decision, the risk associated with their randomness has in general been neglected in previous studies [Ward and Fletcher, Heifner]. It is likely to be small compared to the other uncertainties facing the producer. It is also assumed insignificant in this study.

$$(3)\quad \bar{p} - \frac{r}{f'} - \alpha[\bar{p}^2\,\sigma_\varepsilon\,f + (f-h)\,\sigma_p + f\sigma_\varepsilon\,\sigma_p] = 0$$

$$(4)\quad b - \bar{p} - c' + \alpha[(f-h)\,\sigma_p] = 0$$

where $\alpha[\bar{p}^2\,\sigma_\varepsilon\,f + (f-h)\,\sigma_p + f\sigma_\varepsilon\,\sigma_p]$ is the marginal risk premium for production and $-\alpha[(f-h)\,\sigma_p]$ is the marginal risk premium for hedging.

Equation (3) implies that the risk averse firm produces where expected output price equal marginal cost (r/f') plus a marginal risk premium, which is negative for a hedger, $f-h > 0$ (Baron analyzes the marginal risk premium in the no futures market case).

Equation (4) indicates that the marginal risk premium for hedging $-\alpha(f-h)\sigma_p$ equals the net futures price $(b-c')$ minus the expected spot price, or alternatively,

$$(4')\qquad f-h = \frac{-\,[(b-c') - \bar{p}]}{\alpha\,\sigma_p}\,.$$

Equation (4′) implies that when the net futures price $(b-c')$ equals the expected spot price (as argued in some versions of efficient markets and rational expectations), then the firm can make the marginal contribution of hedging to risk (variance of income) zero, by choosing a full expected hedge $(f-h = 0)$.

By inserting equation (4) into (3), we obtain

$$(5)\quad b - \frac{r}{f'} - c' - \alpha\sigma_\varepsilon\,f\,[\bar{p}^2 + \sigma_p] = 0$$

Expression (5) implies that the firm produces optimally where forward price equals marginal cost when c' and σ_ε are zero given risk aversion. Otherwise, the firm restricts production (given h). *Note that the implication of (5) is that the existence of non-constant marginal hedging costs or production uncertainty ($\sigma_\varepsilon > 0$) negates the findings of DHF that production is unresponsive to spot price parameters*.

Before proceeding to the comparative static analysis, a further assumption is discussed. The model in (1′) does not necessarily imply an upward sloping intended supply curve $\partial f^*/\partial\bar{p} > 0$. Indeed, in the absence of production uncertainty, a rise in expected price always increases output as in the classical theory of the firm. However, this result holds under multiplicative production uncertainty only if the risk aversion coefficient is relatively small ($\alpha < (1/2\sigma_p)\,[-c''^2 + \sqrt{c''^2 + 2\sigma_p\,c''/f\bar{p}\sigma_\varepsilon}]$). (See Chavas and Pope). The reason is that increasing output also increases the variance of output under a multiplicative production disturbance; for a highly risk averse firm, this variance effect may be large enough to generate a downward sloping supply curve. An upward sloping intended supply curve is assumed throughout the paper on the ground that it is probably more realistic in empirical situations.

Comparative Static Analysis

The above model provides a basis for investigating the influence of selected factors on the optimum production and hedging decisions. In this section, the impact of prices (input, output and futures prices), uncertainty (both production and price uncertainty) and risk aversion on the decisions of the firm is discussed. The results are obtained using comparative static methodologies. It is extremely helpful in deriving the implications of the model to use the primal-dual function of Silberberg's $U = L^* - L$ where L^* and L are the indirect and direct objective function respectively.[5] Silberberg has shown that the matrix with typical elements

$$(6)\quad \frac{\partial^2 U}{\partial\beta_i\partial\beta_j} = \frac{\partial^2 L}{\partial x\partial\beta_i}\,\frac{\partial x^*}{\partial\beta_j} + \frac{\partial^2 L}{\partial h\partial\beta_i}\,\frac{\partial h^*}{\partial\beta_j}$$

is positive semi-definite and symmetric, where β is some parameter. These deriva-

[5]The asterisk indicates optimality throughout the paper. For example, f^* denotes the expected production corresponding to the optimum input use x^*. Also L^* is simply the objective function L evaluated at the optimum.

tions are calculated in Table 1[6] for selected parameters. The symmetry of Table 1 will be used to yield implications that would be tedious to obtain by conventional methods. However, it is also necessary to derive some results by conventional comparative static analyses (See Chavas and Pope for the mathematical derivations). Though all comparative static results are related, it will be convenient to examine initially results involving production and then focus on the optimal hedging decisions. The results will be discussed mainly for the case where the firm is hedging on average, i.e. $(f-h) > 0$, since this situation is typically more relevant for producing firms.

Production and Input Decisions

Price Risk

As noted earlier, the inclusion of production uncertainty or non-constant marginal hedging cost implies that, contrarily to DHF, production responds to the distribution of spot price. It can be shown that when a firm is a hedger on average $(f-h > 0)$, then an increased variance of price leads to a decline in input use and intended supply. Indeed, as price variance increases, the firm has to adjust its decisions to offset the corresponding increase in the variance of profit. This adjustment involves modifying only the hedging decision when hedging is costless and production is certain, as in DHF. However, the presence of output uncertainty or hedging cost reduces the attractiveness of the forward market for the firm. In this case, facing an increase in price variance, the firm will also adjust its production decision. If the firm is a hedger, then the marginal impact of input use on variability of income is positive. It follows that, as price variance increases, the firm cuts back its input use to help compensate for the corresponding increase in profit uncertainty. This result implies that, although a forward market gives some flexibility for the firm to deal with price risk, it does not eliminate the negative effects of price uncertainty on production. This suggests that, even in the presence of a forward market, price stabilization in the cash market remains a justifiable objective of farm policy.

Production Risk

It can be shown that an increase in the variability of production leads to a fall in input use and intended supply. This is so because, given a multiplicative production disturbance, the only impact of a rise in production risk on the first-order conditions (3) and (4) is to increase the marginal risk premium of production, implying that any risk averse firm (speculator or hedger) would adjust input use and expected output downward.

Risk Aversion

If the firm is a hedger, input use and intended supply fall with increased risk aversion. Indeed, from the first-order condition (3), it is apparent that increasing risk aversion leads to a higher marginal risk premium of production and to a lower input use if the firm hedges on the average. Thus, given the presence of both a cash market and a forward market, the model suggests that price risk, production risk or risk aversion appears detrimental to production since each tends to shift the firm input demand and output supply curves to the left.

Net Futures Price $(b-c')$

The maximization of the objective function (1′) implies that optimal input use generally rises with an increase in the futures price. This marginal effect is zero under risk neutrality or price uncertainty. However, when the firm's decisions are affected by price risk ($\alpha > 0$, $\sigma_p > 0$), then the marginal impact of an increase in the futures price on input use and intended production becomes positive. Note that this result, already derived by

[6]Note that positive semi-definiteness implies that the diagonal elements in Table 1 are non-negative.

TABLE 1. Symmetric Positive Semi-Definite Matrix Derived from Maximization of the Objective Function of a Risk Averse Firm.[a]

	$\bar{p}$	r	σ_ε	σ_p	b	α	γ^b
$\bar{p}$	$\frac{\partial(f^*-h^*)}{\partial \bar{p}} - 2\alpha\bar{p}\sigma_\varepsilon f \frac{\partial f^*}{\partial \bar{p}}$ >0	$\frac{\partial(f^*-h^*)}{\partial r} - 2\alpha\bar{p}\sigma_\varepsilon f \frac{\partial f^*}{\partial r}$	$\frac{\partial(f^*-h^*)}{\partial \sigma_\varepsilon} - 2\alpha\bar{p}\sigma_\varepsilon f \frac{\partial f^*}{\partial \sigma_\varepsilon}$	$\frac{\partial(f^*-h^*)}{\partial \sigma_p} - 2\alpha\bar{p}\sigma_\varepsilon f \frac{\partial f^*}{\partial \sigma_p}$	$\frac{\partial(f^*-h^*)}{\partial b} - 2\alpha\bar{p}\sigma_\varepsilon f \frac{\partial f^*}{\partial b}$	$\frac{\partial(f^*-h^*)}{\partial \alpha} - 2\alpha\bar{p}\sigma_\varepsilon f \frac{\partial f^*}{\partial \alpha}$	$\frac{\partial(f^*-h^*)}{\partial \gamma} - 2\alpha\bar{p}\sigma_\varepsilon \frac{\partial f^*}{\partial \gamma}$
r	$-\frac{\partial x^*}{\partial \bar{p}}$	$-\frac{\partial x^*}{\partial r} \geq 0$	$-\frac{\partial x^*}{\partial \sigma_\varepsilon} \geq 0$	$-\frac{\partial x^*}{\partial \sigma_p}$	$-\frac{\partial x^*}{\partial b}$	$-\frac{\partial x^*}{\partial \alpha}$	$-\frac{\partial x^*}{\partial \gamma}$
σ_ε	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \bar{p}}$	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial r} \geq 0$	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \sigma} \geq 0$	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \sigma_p}$	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial b}$	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \alpha}$	$-\alpha f(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \gamma}$
σ_p	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial \bar{p}}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial \bar{p}}$	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial r}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial r}$	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial \sigma_\varepsilon}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial \sigma_\varepsilon}$	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial \sigma_p}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial \sigma_p} \geq 0$	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial b}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial b}$	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial \alpha}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial \alpha}$	$-\alpha(f-h) \frac{\partial(f^*-h^*)}{\partial \gamma}$ $-\alpha f\sigma_\varepsilon \frac{\partial f^*}{\partial \gamma}$
b	$\frac{\partial h^*}{\partial \bar{p}}$	$\frac{\partial h^*}{\partial r}$	$\frac{\partial h^*}{\partial \sigma_\varepsilon}$	$\frac{\partial h^*}{\partial \sigma_p}$	$\frac{\partial h^*}{\partial b} \geq 0$	$\frac{\partial h^*}{\partial \alpha}$	$\frac{\partial h^*}{\partial \gamma}$
α	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial \bar{p}}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \bar{p}}$	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial r}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial r}$	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial \sigma_\varepsilon}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \sigma_\varepsilon}$	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial \sigma_p}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \sigma_p}$	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial b}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial b}$	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial \alpha}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \alpha} \geq 0$	$-(f-h)\sigma_p \frac{\partial(f^*-h^*)}{\partial \gamma}$ $-f\sigma_\varepsilon(\bar{p}^2+\sigma_p) \frac{\partial f^*}{\partial \gamma}$
γ^b	$-c' \frac{\partial h^*}{\partial \bar{p}}$	$-c' \frac{\partial h^*}{\partial r}$	$-c' \frac{\partial h^*}{\partial \sigma_\varepsilon}$	$-c' \frac{\partial h^*}{\partial \sigma_p}$	$-c' \frac{\partial h^*}{\partial b}$	$-c' \frac{\partial h^*}{\partial \alpha}$	$-c' \frac{\partial h^*}{\partial \gamma} \geq 0$

[a] f^* is the expected production corresponding to the optimum input level x^*.

[b] In order to investigate a change in hedging cost, c is redefined as follows: $c(h) = \gamma C(h)$ where γ is a parameter, $C'(h) > 0$, and $C''(h) > 0$. Then an increase in the parameter γ represents an exogenous increase in hedging cost.

DHF, is not sensitive to the existence of production uncertainty or hedging cost. In view of the previous results, this suggests that *both* the cash market and the futures market play a role in production decisions.

By parametrizing the hedging cost function $c = \gamma C$ and analyzing the increase in the parameter γ, the impact of increased hedging cost can be examined. From the symmetry of table I, $-\partial x^*/\partial\gamma = c'\partial h^*/\partial r$, and $\partial h^*/\partial r = -\partial x^*/\partial b \leq 0$ as noted earlier. Thus $\partial x^*/\partial\gamma$ is zero under risk neutrality or price certainty, but becomes negative when a risk averse firm faces price risk. Therefore, in the latter case, input demand and hence intended supply falls with an increase in hedging cost.

Input Price

From the (2,2) position of Table I, positive definiteness implies that input demand curves are always downward sloping, as in the classical theory of the firm. Compared to DHF, this result remains valid whether or not there is production uncertainty or hedging cost.

Given the above brief discussion, we turn now to the focus of the paper: hedging behavior.

Hedging and Unhedged Expected Output

Expected Output Price

When production is uncertain, contrarily to DHF results, one cannot determine in general the qualitative impact of changes in expected spot price on hedging. However, when production is certain ($\sigma_\varepsilon = 0$), hedging is inversely related to expected price. This follows from (1′), (3) and (4) since, when $\sigma_\varepsilon = 0$, changes in $\bar{p}$ have no direct impact on the variance of income or the marginal risk premiums, but it does directly lower income.

Though little can be said in general regarding the qualitative impact of $\bar{p}$ on h under both production and price uncertainty, more can be said regarding expected unhedged output (f−h). From (4′), it is clear that $\partial(f^* - h^*)/\partial\bar{p} > 0$ when marginal hedging cost is constant. When c′ is non-constant, we expect more ambiguous results. From table I, first row and column, it is noted that an upward sloping supply curve ($\partial f^*/\partial\bar{p} > 0$) implies that $\partial(f^* - h^*)/\partial\bar{p} > 0$. In this case, a rise in expected price always increases expected output by more than the change in hedging. In other words, an increase in $\bar{p}$ increases the expected use (unhedged position) of the cash market.

Price Risk

The marginal impact of increased variability of price on hedging cannot be signed in the general case. This is in contrast with the DHF model. However, when production uncertainty vanishes, it can be shown that $\partial h^*/\partial\sigma_p$ is positive for a hedger. Note that this latter result, while sensitive to the existence of output uncertainty, is not altered by the presence of hedging costs.

Again, more can be inferred regarding unhedged expected output. Given an upward sloping supply curve for a hedger, expected unhedged output is inversely related to the variance of price, i.e. $\partial(f^* - h^*)/\partial\sigma_p < 0$. This is an intuitive result for a risk averse firm: the more risk the cash market, the less the expected use of the cash market. Also, since $\partial f^*/\partial\sigma_p < 0$ and $\partial h^*/\partial\sigma_p$ is ambiguous, this result implies that hedging can never decrease more than expected output.

Production Risk

In the case of production risk, qualitatively unambiguous results can be derived for hedging: hedging falls as the variance of production rises. As is apparent in (3), an increase in σ_ε increases the marginal risk premium for production leading to decreased input use. Since the marginal risk premium for hedging in (4) does not depend on production risk, it follows that production uncertainty will affect the optimal hedge only indirectly through the induced change in optimum expected production. Since, from (4), $\partial^2 L/\partial x\, \partial h = \alpha f'\sigma_p > 0$, a reduction in x increases the marginal risk premium for hedging and thus decreases the optimal hedging level. Therefore an increase in σ_ε leads to a decrease in both input use and hedging. This result suggests that production uncertainty inhibits the use of forward markets.

Also, as production risk rises, expected unhedged output falls if the supply curve is upward sloping. In this case, both hedging and expected production fall but expected output always falls more than hedging. Thus, production uncertainty affects negatively the (expected) use of both the cash market (f−h) and the forward market (h) by a risk averse firm. Moreover, since production risk tends to reduce expected output by more than hedging, it follows that a rise in σ_ε increases the proportion of expected production that is hedged.

Risk Aversion

It can be shown that, when production is certain, then increases in risk aversion lead to increased hedging, as in DHF (i.e. $\partial h^*/\partial\alpha > 0$ when $f-h > 0$). However, this result does not hold under production risk. This is so because, from (3) and (4) increasing risk aversion decreases the marginal risk premium for hedging but increases the marginal risk premium for production, thus leading to ambiguous results.

However, more precise results can be obtained concerning unhedged expected output: expected unhedged output falls with a rise in risk aversion for a hedger ($f-h > 0$) when the supply curve is upward sloping. In such a case, though hedging may decrase, it can never fall more than expected production falls.

Net Futures Price $(b-c')$

From the fifth row and column of table I, it follows that the optimal hedge is an increasing function of the futures price for a risk averse or risk neutral firm. This result, already derived in DHF, is thus not sensitive to the addition of production risk and hedging costs. This is so because the marginal risk premiums in (3) and (4) are unaffected by changes in b. Thus, as expected, the use of the futures market by a producing firm increases with the futures price.

The maximization of the objective function (1′) also implies that unhedged expected output falls as the futures price increases for a risk neutral firm or when production is certain. However, in general one cannot determine unambiguously the impact of b on f−h.

Considering now hedging cost, let $c = \gamma C$. From the last row and column of table I, it follows that an increase in hedging cost (γ) reduces hedging. Thus, as expected, a large (small) cost of access to the futures market would limit (facilitate) its use by producing firms.

Input Price

It can be shown that, as in DHF, an increase in an input price leads to a decrease in hedging under risk aversion and price uncertainty, and no impact on hedging when price is certain or the firm is risk neutral. These results are not sensitive to the existence of production risk or hedging cost. Thus, as an input price rises, input demand and intended supply fall. This reduces the variability of income and the incentive to reduce risk through hedging. Further, if the intended supply curve is upward sloping, then, an increase in input price leads to a fall in unhedged output. Thus, a rise in production cost implies a fall in the (expected) use of both the futures market (h) and the cash market (f−h) by the firm. It follows that expected output has to fall more than hedging, i.e. that the proportions of expected production that is hedged increases with input cost.

Summary and Concluding Remarks

A conditional normative model has been developed to investigate the behavior of a risk averse competitive firm that uses both a cash market and a futures market to sell its product. Optimal production and hedging decisions have been discussed in this framework. As in the case of a typical agricultural producer, both price uncertainty and production uncertainty are incorporated in the analysis, which is comparative static in nature. The impact of selected parameters on expected production, hedging and expected unhedged output is summarized in table 2.

Table 2 illustrates the complexity of the production and hedging decisions under production uncertainty. For example, some ambiguity is found in the model for hedging response to a change in parameters such as expected price, price variance or risk aversion: such response can be either positive or negative depending upon the particular characteristics of the firm. This is in marked contrast to the DHF model.

Table 2 also provides testable hypotheses for empirical research. Some of these hypotheses appear to be particularly relevant for the investigation of agricultural supply response. For example, production uncertainty implies that both the futures price as well as the parameters of the distribution of spot price enter the supply function. This differs from the DHF model or the standard Nerlovian supply or acreage response models. Also, considering the question asked by Gardner: "Which of the futures price or expected spot price should be used in supply analysis?", our results suggest that the answer is "both prices".

Further testable hypotheses concern optimal hedging. We already mentioned that hedging response to a change in expected price, price variance or risk aversion is ambiguous (either positive or negative). However, in table 2, the influence of such parameters on expected unhedged output has been signed: it is positive for expected price and negative for price variance and risk aversion. This indicates that testing of the model in positive economic analysis should perhaps focus on both hedging and expected unhedged output.

Table 2 also illustrates the negative relationship that exists between hedging and production uncertainty. It implies that risky production, which characterizes most agricultural production processes, tends to restrict the use of the futures market by farmers. However, it was shown that, since production risk has even a stronger negative influence on expected production, the proportion of expected output that is hedged by the firm in fact increases with output uncertainty. This should provide hypotheses for cross-sectional analysis of hedging decisions

TABLE 2. Impact of Selected Parameters on the Production and Hedging Decisions for a Risk Averse Firm.[a]

Decision variables	x^*		h^*		$(f^* - h^*)$	
Parameters	$\sigma_\varepsilon=0$	$\sigma_\varepsilon>0$	$\sigma_\varepsilon=0$	$\sigma_\varepsilon>0$	$\sigma_\varepsilon=0$	$\sigma_\varepsilon>0$
$\bar{p}$	+	?	–	?	+	+[b]
σ_p	–[c]	–[c]	–[c]	?	–[bc]	–[bc]
σ_ε	–	–	–	–	–	–[b]
α	–[c]	–[c]	+[c]	?	–[bc]	–[bc]
b	+	+	+	+	–	?
γ	–	–	–	–	+	?
r	–	–	–	–	–[b]	–[b]

[a]The following symbols are used: + indicates a positive impact; – indicates a negative impact; ? indicates an ambiguous impact.

[b]Provided that

$$\frac{\partial f^*}{\partial \bar{p}} \geqslant 0.$$

[c]If $(f-h)>0$.

as production risk varies across regions and commodities.

The above results have policy and management implications. For example, when considering hedging cost, price risk as well as production risk tend to restrict optimum production of a risk averse firm. This implies that although the use of a futures market can provide a means of dealing with price uncertainty, it cannot eliminate the negative effect of this uncertainty on production. This is in marked contrast to the DHF model, where production is not affected by price risk. This suggests that, since the farmer cannot totally avoid price uncertainty, a public policy that intends to reduce price risk would improve the farmers' welfare and stimulate production. Similar arguments can be made about production uncertainty: any policy or management program that reduces output risk would make the farmers better-off and increase supply. Moreover, the identifed influences of price and production uncertainties on optimal choices can be useful in the design of extension efforts. For example, the targeting of farmers who are likely to use futures markets could be based on both their yield variance and price variance.

Finally, although the above mean-variance model could be extended, for example by including basis risk or by allowing for a nonzero covariance between price and output, it appears a reasonably realistic yet simple representation of a production firm participating in a futures market. As such, it has significance in the classroom when discussing potentials for farmers to reduce risk through market and self insurance mechanisms.

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