The Effect of Consumption Based Taxes on Agriculture in the United States

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Abstract: Recently several proposals have arisen to replace the current income tax system in the United States with a consumption based or Fair Tax. This study investigates the effect of such a consumption based tax on agricultural investment decisions using stochastic optimal control to model the investment decision at the farm level. The results indicate that a consumption tax rate of 25.9 percent would be equivalent to the income tax rate paid by very large producers in the United States.

Keywords: optimal debt, income tax, consumption tax, stochastic optimal control

1. Introduction

Recently a significant movement has emerged in the United States supporting the replacement of income taxes with consumption-based taxes typically described as a national sales tax. This support includes five prominent agricultural economics that signed an open letter the congress, president, and the American people (Americans for Fair Tax, 2005). Most proponents of this legislation typically extol its possible effect on investment and, hence, productivity. This paper examines whether moving to a consumption-based tax would have this effect using an extension of the optimal debt model proposed by Ramirez, Moss, and Boggess (1997).

2. Modifying the Optimal Debt Model

Ramirez, Moss, and Boggess extend Merton's (1969) lifetime portfolio formulation to derive a model of optimal debt. This formulation extends other formulations of the optimal debt (Collins 1985, Featherstone et al. 1988, Moss, Ford and Boggess 1989) by the derivation of the optimal consumption path. The extension of the optimal debt model to include the consumption investment tradeoff is important in understanding the potential tradeoff between income taxes and consumption based taxes.

Starting from the original stochastic optimal control problem proposed by Ramirez, Moss and Boggess, we introduce income and consumption based taxes. The extended model can be written as:

$$\max_{\delta(t),C(t)} E\left[\int_0^\infty e^{-rt} \frac{\left[C(t)(1-\psi)\right]^b}{b} dt\right]$$

$$s.t.\, dW(t) = \left[W(t)\frac{(\mu_A(t)-K(t)\delta(t))(1-\tau)}{(1-\delta(t))} - C(t)\right]dt + \frac{W(t)\sigma_A(t)(1-\tau)}{(1-\delta(t))}dz(t)$$

(1)

where $E[.]$ is the expectation operator, r is the discount rate, $C(t)$ is the level of consumption, ψ is the consumption tax rate, b is the relative risk aversion coefficient, $W(t)$ is the wealth or equity of the farmer, $\mu_A(t)$ is the mean rate of return to agriculture, $K(t)$ is the cost of debt capital, $\delta(t)$ is the debt-to-asset ratio, τ is the income tax rate, $\sigma_A(t)$ is the standard deviation of the rate of return on assets, and $dz(t)$ is the Wiener motion process.

The optimal consumption and debt-to-asset ratio for this formulation are

$$C^*(t) = W(t) \left\{ \frac{r - K(t)(1-\tau)b}{(1-b)} - \frac{b[K(t) - \mu_A(t)]^2}{2\sigma_A^2(t)(1-b)^2} \right\}$$

$$\delta^*(t) = 1 - \frac{(1-b)(1-\tau)\sigma_A^2(t)}{\mu_A(t) - K(t)} \tag{2}$$

(See the appendix for mathematical derivations). These results provide several significant insights. First, note that a consumption based tax does not affect the optimal level of consumption or debt. However, substituting the optimal consumption path for the general consumption path in Equation 1, we conjecture that a consumption based tax reduces the decision maker's utility through time. Second, if the income tax rate goes to zero this solution is identical to that of Ramirez, Moss, and Boggess which is reassuring.

Taking the derivative of the optimal consumption path in Equation 2 with respect to the income tax rate yields

$$\frac{\partial C^*(t)}{\partial \tau} = W(t) \frac{K(t)b}{(1-b)} \tag{3}$$

Given the result in Equation 3, the sign of the effect of taxation on the optimal path of consumption depends on the magnitude of the risk aversion coefficient. In this study $b < 1$, with a more negative number representing a more risk averse decision maker. If $0 < b < 1$, the results from Equation 3 indicate that the optimal consumption path increases in response to an increase in the tax rate. However, if $b < 0$, the consumption path decreases in response to an increase in the income tax rate.

Next, we solve the consumption based tax that leaves the decision maker indifferent between consumption and income based taxes. Equating the dollars in tax paid from consumption based tax with the dollars of tax paid from an income based tax

$$\psi C^{*}(t) = \tau W(t) \frac{\mu_{A}(t) - K(t)\delta^{*}(t)}{1 - \delta^{*}(t)}$$

$$\Rightarrow \psi = \frac{W(t)}{C^{*}(t)} \tau \frac{\mu_{A}(t) - K(t)\delta^{*}(t)}{1 - \delta^{*}(t)}$$

(4)

Where $C^{*}(t)$ and $\delta^{*}(t)$ are defined using the optimal levels of consumption and debt in Equation 2.

3. Empirical Implementation of the Optimal Debt Model with Taxes

Given the above derivations we derive the consumption based taxes that are equivalent to the current income federal tax regiment in the United States. Specifically, we use data from the Agricultural Resource Management Survey (ARMS) to estimate the expected rate of return on agricultural assets, variance of the rate of return on agricultural assets, cost of debt capital, debt-to-asset ratio, and average federal income tax rate sorted by the typology of U.S. Farms (Hoppe, Perry and Banker 2000). These data are then used to estimate a risk aversion coefficient (b) for each farm typology.

The typology of U.S. Farms divides all farms in the United States into eight groups. Five of these groups are classified as small family farms. These include Limited-Resource farms that have gross sales less than $100,000, less than $150,000 in total farm assets, and total operator household income of less than $20,000. Retirement farms that consist of farmers that are not Limited-Resource farmers and who report that they are retired. Residential/Lifestyle farms whose primary occupation is something other than farming. Farming Occupation/Lower-Sales farms with sales of less than $100,000, but whose operator reports that farming is their primary occupation. And, Farming Occupation/Higher Sales who report between $100,000 and $249,999 sales and that farming is their primary occupation. The three groups of other farms are then Large Family

Farms that report between $250,000 and $499,999 of annual sales, Very Large Family Farms with annual agricultural sales of more than $500,000, and Nonfamily farms that are farms organized as corporations or cooperatives as well as farms with hired managers.

Table 1 presents the wealth, debt, interest paid, consumption, income from farming, and off-farm income by typology group for 2003. The overall variations in these figures agree with our expectations. For the lower total sales typologies income from agriculture is relatively small ($10,156 or less). Consumption for these typologies is funded primarily from off-farm income. Surprisingly, even when farming is the head of the household's primary occupation and agricultural sales are high off-farm income contributes significantly to household consumption.

Table 2 presents the debt-to-asset position, interest rate, and average rate of return on assets based on the values presented in Table 1. In general, the debt-to-asset position increases as the level of sales and investment in agricultural assets increase. Similarly, the rate of return on agricultural assets increases with the level of sales and investment in agricultural assets, while the interest rate paid on debt remains relatively constant. The results do indicate that Residential/Lifestyle farms and whose primary occupation is farming with lower agricultural sales have a lower cost of debt than other typologies. Similarly, non-family farms experience a slightly higher interest rate on debt capital. Finally, Table 3 presents the average income tax rate paid by farm typology (USDA 2005).

Using this data, we can derive the risk aversion coefficient (b) [1]- by rearranging the optimal debt relationship in Equation 2

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[1] As in Ramirez, Moss and Boggess, we consider b as a parameter which incorporates the Arrow-Pratt relative risk aversion coefficient. Otherwise, we can compute the risk aversion coefficient as (1-b).

$$1 - b = \frac{(1 - \delta^*)(\mu_A - K)}{(1 - \tau)\sigma_A^2} \quad or \quad b = 1 - \frac{(1 - \delta^*)(\mu_A - K)}{(1 - \tau)\sigma_A^2} \tag{5}$$

4. Results

The population statistics presented in Tables 2 and 3 raise several difficulties for a direct application of the optimal debt model. First, implicit in the optimal debt relationship presented in Equation 2 is the assumption that the expected rate of return to agricultural assets must be greater than the cost of debt capital. If this condition is not met producers do not invest in agricultural assets, but sell agricultural assets short to lend on the debt market. As depicted in Table 2, only the largest firms have an expected rate of return on agricultural assets greater than the cost of debt capital. Second, the population standard deviation of the operating margin used as a proxy for the variance of the rate of return on agricultural assets understates the relative risk facing the producer. In essence this measures the variance of the means of the rate of return on agricultural assets.

To adjust for these anomalies we focus our attention on the results for the very large and non-family typologies. In addition, we assume that the appropriate farm-level standard deviation is 25 times larger than the population statistic presented in Table 3. Under these assumptions the risk aversion coefficient is 0.355 for very large family farms and 1.029 (e.g., not different from 1.0) for non-family farms. Resolving for the risk aversion coefficient without a tax effect (i.e., consistent with Ramirez, Moss, and Boggess) yields a risk aversion coefficient of 0.421 for very large farms and 1.023 (e.g., not different from 1.0) for non-family farms. These results are interesting from several perspectives. First, very large family farms appear to be risk averse while non-family farms are risk neutral. Second, tax considerations result in a 15.8 percent reduction in the relative risk aversion

for very large family farms. Thus, estimates that fail to incorporate tax considerations understate the relative risk aversion of very large farmers.

Turning to the results on consumption, we compute the fraction of wealth consumed in each time period using the results presented in Equation 2. Assuming a pure discount rate of 6 percent the farmer would consume 4.6 percent of his wealth each year. Given an expected rate of return on equity of 11.6 percent, this implies that farmer wealth would increase at a rate of 6.0 percent. The optimal level of consumption for very large farms would then be $99,893.67 which is significantly below the observed annual consumption of $44.887.08.

Developing the tax consequence, the expected rate of return on equity implies an annual income of $251,208.90 which is close to the observed income level of $246,070.21. This producer would pay on average $25,874.52 in federal income taxes. Collapsing the forgoing derivations into Equation 4 yields an equivalent consumption tax rate of 25.9 percent. This figure is much lower than the 57.6 percent suggested by the data (i.e., dividing the annual income taxes paid by household consumption).

5. Discussion and Implications

Replacing the current income tax system with a consumption based tax has gained support among some political groups. The proponents of the change cite several advantages. Proponents contend that the consumption based tax would replace of federal income and payroll taxes with a simple, progressive, visible and efficient system of taxation. In addition, the rebate provisions of the tax would provide lower income households with a rebate that could be used to purchase necessities. Proponents also contend that replacing the current federal income tax would eliminate various inefficiencies in the economy

including filing costs and government monitoring expenditures. Apart from these contentions, this study examined whether consumption based taxes would improve economic growth and productivity by removing the distortions the federal income tax code imposes on the savings and investment behavior of households. To examine this question we derive investment behavior of farms implicit in the optimal debt model for farm households. Using this formulation we derive the consumption tax rate that would leave investment behavior unaffected by a switch from income tax to a consumption based tax. Our results indicate that a consumption tax of 25.9 percent would be equivalent to the average income tax rate of 10.3 percent paid by very large farms in the United States.

Some debate about the revenue-neutral consumption tax rate exists, commonly cited consumption based taxes vary from 22 to 25 percent. Thus, our results suggest that very large farmers may benefit slightly from a change to a consumption based tax. However, this result must be taken with a healthy skepticism. First, our results hold only for very large farms (those with sales greater than $500,000). The optimal investment behavior could not be derived for the remaining farm typologies because their expected returns on farm assets were lower than their average interest rate on borrowed capital. Given that the average income tax rate for large farms under the U.S. Department of Agriculture typology is 40.8 percent of that for very large farms, we anticipate the break-even consumption tax rate to be around 10.6 percent, which is significantly below the commonly referenced range for the break-even consumption based tax. Thus, the largest proportion of farmers in the United States would be adversely affected by a movement toward consumption based taxes. However, this conjecture does not take into account the sales tax rebate included in most consumption based tax proposals.

References

Americans for Fair Tax. "Letters of Endorsement." Website: http://www.fairtax.org/eco_endorsements.html, Accessed May 14, 2005.

Collins, R.A. "Expected Utility, Debt-Equity Structure, and Risk Balancing." *American Journal of Agricultural Economics* 67(1985): 627–29.

Featherstone, Allen M., Charles B. Moss, Timothy G. Baker, and Paul V. Preckel. "The Theoretical Effects of Farm Policies on Optimal Leverage and the Probability of Equity Loss." *American Journal of Agricultural Economics* 70(1988): 572–79.

Hoppe, R.A., J.E. Perry, and D. Banker. *ERS Farm Typology for a Diverse Agricultural Sector* U.S. Department of Agriculture, Economic Research Service Agricultural Information Bulletin Number 759, September 2000.

Merton, Robert C. "Lifetime Portfolio Selection Under Uncertainty: The Continuous Time Case." *Review of Economics and Statistics* 51(1969): 247–57.

Moss, Charles B., Steven A. Ford, and William G. Boggess. "Capital Gains, Optimal Leverage, and the Probability of Equity Loss: A Theoretical Model." *Agricultural Finance Review* 49(1989): 127–34

Ramirez, Octavio A., Charles B. Moss, and William G. Boggess. "A Stochastic Optimal Control Formulation of the Consumption/Debt Decision." *Agricultural Finance Review* 57(1997): 29–38.

U.S. Department of Agriculture, Economic Research Service. "Federal Taxes: Farm Taxation at a Glance." Website: http://www.ers.usda.gov/Briefing/FederalTaxes/Slide10.htm Accessed: May 14, 2005.

Table 1. Farm Wealth, Debt, Expenditures and Income by USDA Typology for 2003

	Wealth	Debt	Interest Paid	Household Expenditures	Income from Farming	Off-Farm Income
Limited-resources	374,464.01	16,351.19	1,151.64	18,647.64	-883.36	14,460.88
	(25,652.56)[a]	(2,606.80)	(137.78)	(1,051.22)	(1,549.98)	(1,733.45)
Retirement	416,840.34	10,346.33	704.76	30,563.40	5,704.53	49,327.11
	21,869.54	1,343.13	88.86	2,342.11	661.69	2,333.27
Residential/lifestyle	358,008.42	35,321.97	2,345.80	44,097.41	1,121.92	90,366.65
	59,147.07	2,623.65	146.03	2,024.41	1,394.81	3,352.77
Farming occupation/lower-sales	525,655.09	45,092.17	2,769.02	31,376.32	10,154.48	47,476.11
	13,879.73	4,331.25	223.70	1,088.89	2,294.63	2,575.38
Farming occupation/higher-sales	934,274.15	121,179.44	9,063.41	32,758.56	41,485.69	31,194.71
	28,463.20	5,802.33	392.25	797.21	2,692.34	1,776.70
Large	1,272,828.54	224,188.33	16,077.19	38,974.33	84,721.34	40,078.01
	75,886.98	12,007.04	563.01	1,140.75	5,031.60	2,260.56
Very large	2,158,755.80	505,934.54	36,948.16	44,887.08	246,070.21	42,273.66
	95,672.05	33,023.45	2,026.81	1,273.34	18,470.31	2,371.47
Nonfamily	1,643,766.66	249,392.29	19,771.93		98,017.94	
	1,743,547.32	99,280.76	14,233.90		72,911.56	

[a]Numbers in parenthesis denote standard deviations.

Table 2. Computed Debt-to-Asset Ratio, Rates of Return, and Interest Rates for 2003

Typology Group	Debt to Asset	Interest Rate	Expected Return on Assets	Std. Dev. Operating Margins
Limited-resources	0.04367	0.07043	0.00069	0.00826
Retirement	0.02482	0.06812	0.01500	0.00237
Residential/lifestyle	0.09866	0.06641	0.00882	0.00695
Farming occupation/lower-sales	0.08578	0.06141	0.02264	0.00441
Farming occupation/higher-sales	0.12970	0.07479	0.04789	0.00274
Large	0.17613	0.07171	0.06733	0.00333
Very large	0.23436	0.07303	0.10621	0.00838
Nonfamily	0.15172	0.07928	0.06222	0.03107

Table 3. Income Tax Rate by Typology for 2000

Typology Group	Federal Income Tax Rate
Limited-resources	0.1
Retirement	3.5
Residential/lifestyle	3.1
Farming occupation/lower-sales	5.5
Farming occupation/higher-sales	4.2
Large	10.3
Very large	17.2

Source: USDA (2005).

Mathematical Appendix

Following the steps in the appendix Ramirez, Moss, and Boggess (1997) we start with the definition of the value function:

$$J(t_0, W_0) = \max_{\delta(t), C(t)} E\left[\int_{t_0}^{\infty} e^{-rt} \frac{\left[C(t)(1-\psi)\right]^b}{b} dt\right]$$

$$s.t. dW(t) = \left[W(t)\frac{(\mu_A(t) - K(t)\delta(t))(1-\tau)}{(1-\delta(t))} - C(t)\right] dt + \frac{W(t)\sigma_A(t)(1-\tau)}{(1-\delta(t))} dz(t).$$

(A.1)

Using the dynamic programming formulation, this problem can be restated as:

$$J(t, W) \approx \max_{\delta(t), C(t)} E\left[e^{-rt} \frac{(C(t)(1-\psi))^b}{b} \Delta t + J(t + \Delta t, W + \Delta W)\right].$$

(A.2)

Taking the second order Taylor series expansion of the second term in brackets in Equation A.2 yields

$$J(t + \Delta t, W + \Delta W) \approx J(t,W) + J_t(t,W)\Delta t + J_W(t,W)\Delta W + \tfrac{1}{2} J_{WW}(t,W)(\Delta t)^2$$

(A.3)

where $J_t(t,W)$ is the partial derivative of the value function with respect to time, $J_W(t,W)$ is the partial derivative of the value function with respect to wealth, and $J_{WW}(t,W)$ is the second derivative of the value function with respect to wealth. Substituting Equation A.3 and the equation of motion from Equation A.1 into Equation A.2 and applying Ito's lemma yields

$$J(t,W) = \max_{\delta(t),C(t)} E\left[e^{-rt} \frac{(C(t)(1-\psi))^b}{b} \Delta t + J(t,W) + J_t(t,W)\Delta t + \right.$$

$$J_W(t,W)\left(W(t)\frac{(\mu_A(t) - K(t)\delta(t))(1-\tau)}{(1-\delta(t))} - C(t)\right)\Delta t + \qquad (A.4)$$

$$\left. J_W(t,W)\frac{W(t)\sigma_A(t)(1-\tau)}{(1-\delta(t))}\Delta z + \frac{1}{2}J_{WW}(t,W)\frac{W^2(t)\sigma_A^2(t)(1-\tau)^2}{(1-\delta(t))^2}\Delta t \right]$$

Taking the expectation of A.4, using $E(\Delta z) = 0$ and subtracting $J(t,W)$ from both sides

yields

$$0 = \max_{\delta(t),C(t)} \left[e^{-rt} \frac{(C(t)(1-\psi))^b}{b} \Delta t + J_t(t,W)\Delta t + \right.$$

$$J_W(t,W)\left(W(t)\frac{(\mu_A(t) - K(t)\delta(t))(1-\tau)}{(1-\delta(t))} - C(t)\right)\Delta t + \qquad (A.5)$$

$$\left. \tfrac{1}{2}J_{WW}(t,W)\frac{W(t)^2\sigma_A^2(t)(1-\tau)^2}{(1-\delta(t))^2}\Delta t \right].$$

Dividing both sides by Δt and moving $J_t(t,W)$ from both sides yields

$$J_t(t,W) = \max_{\delta(t),C(t)} \left[e^{-rt} \frac{(C(t)(1-\psi))^b}{b} + \right.$$

$$J_W(t,W)\left(W(t)\frac{(\mu_A(t) - K(t)\delta(t))(1-\tau)}{(1-\delta(t))} - C(t)\right) + \qquad (A.6)$$

$$\left. \frac{1}{2}J_{WW}(t,W)\frac{W^2(t)\sigma_A^2(t)(1-\tau)^2}{(1-\delta(t))^2} \right].$$

Rewriting equation (A.6) as a current value function assuming $J(t,W) = e^{-rt}V[W]$

implying that $J_t(t,W) = -re^{-rt}V[W]$, $J_W(t,W) = e^{-rt}V'[W]$, and $J_{WW}(t,W) = e^{-rt}V''[W]$. Next

we divided by e^{-rt} yielding

$$rV\left[W(t)\right] = \max_{\delta(t),C(t)} \left[\frac{\left(C(t)(1-\psi)\right)^{b}}{b} + \right.$$

$$V'\left[W(t)\right]\left(W(t)\frac{\left(\mu_{A}(t)-K(t)\delta(t)\right)(1-\tau)}{\left(1-\delta(t)\right)} - C(t) \right) +$$

$$\left. \frac{1}{2}V''\left[W(t)\right]\frac{W^{2}(t)\sigma_{A}^{2}(t)(1-\tau)^{2}}{\left(1-\delta(t)\right)^{2}} \right]. \qquad (A.7)$$

Substituting $w(t) = \dfrac{1}{\left(1-\delta(t)\right)}$ to simplify the optimization problem yields

$$rV\left[W(t)\right] = \max_{w(t),C(t)} \left[\frac{\left(C(t)(1-\psi)\right)^{b}}{b} + \right.$$

$$V'\left[W(t)\right]\left(W(t)\left(\left(\mu_{A}(t)w(t)+(1-w(t))K(t)\right)(1-\tau)\right) - C(t)\right) \quad (A.8)$$

$$\left. V''\left[W(t)\right]W^{2}(t)\sigma_{A}^{2}(t)w^{2}(t)(1-\tau)^{2} \right]$$

To solve this maximization problem, we begin by letting the expression within the brackets equal to M:

$$M = \frac{\left(C(t)(1-\psi)\right)^{b}}{b} + V'\left[W(t)\right]\left\{W(t)(1-\tau)\left[\left(\mu_{A}(t)-K(t)\right)w(t)+K(t)\right]-C(t)\right\}$$
$$+\frac{1}{2}V''\left[W(t)\right]W^{2}(t)\sigma_{A}^{2}(t)w^{2}(t)(1-\tau)^{2} \qquad (A.9)$$

Taking the derivative with respect to $w(t)$ yields

$$\frac{\partial M}{\partial w(t)} = V'\left[W(t)\right]W(t)(1-\tau)\left[\mu_{A}(t)-K(t)\right]$$

$$+V''\left[W(t)\right]W^{2}(t)\sigma_{A}^{2}(t)w(t)(1-\tau)^{2} = 0 \qquad (A.10)$$

$$\therefore w^{*}(t) = \frac{\left(K(t)-\mu_{A}(t)\right)V'\left[W(t)\right]}{W(t)\sigma_{A}^{2}(t)(1-\tau)V''\left[W(t)\right]}$$

Taking the derivative of A.9 with respect to $C(t)$ yields

$$\frac{\partial M}{\partial C(t)} = C^{b-1}(t)(1-\psi)^b - V'[W(t)] = 0$$

$$\therefore C^*(t) = \left\{ \frac{V'[W(t)]}{(1-\psi)^b} \right\}^{\frac{1}{b-1}} \tag{A.11}$$

Next, we solve for $V[W(t)]$ in Equation A.8 using the optimal solution for $w(t)$ in Equation A.10 and $C(t)$ in Equation A.11

$$rV[W(t)] = \frac{\left[\left\{ \frac{V'[W(t)]}{(1-\psi)^b} \right\}(1-\psi) \right]^b}{b} +$$

$$V'[W(t)]\left\{ W(t)(1-\tau)\left(\left[\mu_A(t) - K(t) \right] \frac{V'[W(t)]\left[K(t) - \mu_A(t) \right]}{V''[W(t)]W(t)(1-\tau)\sigma_A^2(t)} + K(t) \right) \right.$$

$$\left. -\left(\frac{V'[W(t)]}{(1-\psi)^b} \right)^{\frac{1}{b-1}} \right\} +$$

$$\frac{1}{2}V''[W(t)]W^2(t)(1-\tau)^2\sigma_A^2(t)\left\{ \frac{V'[W(t)]\left[K(t) - \mu_A(t) \right]}{V''[W(t)]W(t)(1-\tau)\sigma_A^2(t)} \right\}^2$$

$$rV[W(t)] - [V'[W(t)]]^{\frac{b}{b-1}}(1-\psi)^{\frac{-b}{b-1}}\left(\frac{1}{b} - 1 \right) +$$

$$\frac{(V'[W(t)])^2\left[K(t) - \mu_A(t) \right]^2}{2\sigma_A^2(t)V''[W(t)]} - K(t)W(t)(1-\tau)V'[W(t)] = 0 \tag{A.12}$$

Equation A.12 is a nonlinear second-order differential equation in $V[W(t)]$. To solve this we will use a method of undetermined coefficients hypothesizing a solution of the form $V[W(t)] = BW^b(t)$ where $B > 0$ (as described by Kamien and Schwartz

following Merton). Following this proposed solution $V'\big[W(t)\big]=bBW^{b-1}(t)$,

$V''\big[W(t)\big]=b(b-1)BW^{b-2}(t)$. Substituting this proposed solution into A.12 yields

$$rBW^{b}(t)-\big[BbW^{b-1}(t)\big]^{\frac{b}{b-1}}(1-\psi)^{-\frac{b}{b-1}}\left(\frac{1-b}{b}\right)$$

$$-K(t)W(t)(1-\tau)\big[BbW^{b-1}(t)\big]+\frac{\big(BbW^{b-1}(t)\big)^{2}\big[K(t)-\mu_{A}(t)\big]^{2}}{2\sigma_{A}^{2}(t)Bb(b-1)W^{b-2}(t)}=0$$

$$B=\frac{(1-\psi)^{b}}{b}\left\{\frac{r-K(t)(1-\tau)b}{(1-b)}-\frac{b\big[K(t)-\mu_{A}(t)\big]^{2}}{2\sigma_{A}^{2}(t)(1-b)^{2}}\right\}^{b-1} \tag{A.13}$$

Note letting $\tau,\psi\to0$ yields the same B derived by Ramirez, Moss, and Boggess.

Replacing B in Equation A.13 into Equation A.11 determines the optimal consumption path $\big[C^{*}(t)\big]$.

$$C^{*}(t)=\left[W^{b-1}(t)\left\{\frac{r-K(t)(1-\tau)b}{(1-b)}-\frac{b\big[K(t)-\mu_{A}(t)\big]^{2}}{2\sigma_{A}^{2}(t)(1-b)^{2}}\right\}^{b-1}\right]^{\frac{1}{b-1}}$$

$$=W(t)\left\{\frac{r-K(t)(1-\tau)b}{(1-b)}-\frac{b\big[K(t)-\mu_{A}(t)\big]^{2}}{2\sigma_{A}^{2}(t)(1-b)^{2}}\right\} \tag{A.14}$$

It is interesting that the optimal level of consumption does not depend on the level of consumption taxes. However, the utility generated by the optimal level of consumption is decreasing in consumption taxes. For reasons of consistency with Ramirez, Moss, and Boggess, the fraction of wealth consumed at each increment of time (denoted as D) can be defined from Equation A.14 as

$$D=\frac{r-K(t)(1-\tau)b}{(1-b)}-\frac{b\big[K(t)-\mu_{A}(t)\big]^{2}}{2\sigma_{A}^{2}(t)(1-b)^{2}} \tag{A.15}$$

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Further replacing the solution $V\left[W\left(t\right)\right]=BW^{b}\left(t\right)$ along with B into Equation A.10

yields the optimal debt level at each point in time. First, we define $w^{*}\left(t\right)$

$$
\begin{aligned}
w^{*}\left(t\right) &= \frac{BbW^{b-1}\left(t\right)\left[K\left(t\right)-\mu_{A}\left(t\right)\right]}{Bb\left(b-1\right)W^{b-2}\left(t\right)W\left(t\right)\left(1-\tau\right)\sigma_{A}^{2}\left(t\right)} \\
&= \frac{\mu_{A}\left(t\right)-K\left(t\right)}{\left(1-b\right)\left(1-\tau\right)\sigma_{A}^{2}\left(t\right)}
\end{aligned}
\tag{A.16}
$$

Based on the definition for $w\left(t\right)$

$$
1-\delta\left(t\right)=\frac{1}{w\left(t\right)} \Rightarrow \delta\left(t\right)=1-\frac{1}{w\left(t\right)}.
$$

The optimal debt-to-asset ratio can be defined as

$$
\delta^{*}\left(t\right)=1-\frac{\left(1-b\right)\left(1-\tau\right)\sigma_{A}^{2}\left(t\right)}{\mu_{A}\left(t\right)-K\left(t\right)}.
\tag{A.17}
$$