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An Extended Accelerator Model of R & D and Physical Investment

Jacques Mairesse and Alan K. Siu

14.1 Introduction

The purpose of the present study is to investigate the determinants of both R & D and physical investment using a panel of firm data. In a standard neoclassical model of investment, the firm is assumed to choose an investment plan to maximize the present discounted value of net cash flow subject to the production technology, cost of adjustment function, initial capital stocks, and other appropriate constraints (or else to minimize the present discounted total cost of production subject to the same constraints and an expected production plan). In full generality, this involves considering nonlinear stochastic control problems, and explicit solutions of the first-order conditions are intractable without very restrictive assumptions. Assumptions such as static expectations about prices, a simple form of the production function, the absence of an explicit cost of adjustment function, and the imposition of a given lag structure are usually made to derive the specification of the investment function.

In view of the complexities of a formal model of investment decisions, and also because of a lack of data on factor prices at the firm level, we

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have to settle for a looser approach in the spirit of data analysis as advocated by Sims (1972a, 1972b, 1977, 1980; Sargent and Sims 1977). A priori, expected demand and expected profitability are important determinants for investment decisions. Both are unobservable. Following Pakes (this volume), we propose to use the stock market one-period holding rate of return, q, as an indicator of changes in expectation about the firm's future profitability. For expected demand, we have used a more traditional distributed lag formulation of the rate of growth of sales, s. These two variables plus the rates of growth of R & D and physical investment, r and i, are embedded in a multivariate autoregressive model. We perform a series of exogeneity tests to investigate the appropriateness of restricted versions of this general model which are of interest. In particular, we vindicate an extended form of the traditional accelerator model: extended both because it applies to R & D as well as to physical investment and because it takes expected profitability, not only demand, as a major explanatory factor. The specification of our model is discussed in section 14.2, while our results are presented in section 14.3. We end with a few remarks in section 14.4.

14.2 Model Specification: Statistical and Economic Considerations

We start from what we call our general model and derive our extended accelerator model, discussing the meaning and specification of each equation in turn.

14.2.1 A General Model

First, let us denote the four variables our study concentrates on by q_{nt} , s_{nt} , r_{nt} , and i_{nt} , where n and t represent firm and year subscripts (n = 1 to N; t = 1 to T), respectively. To simplify matters we shall suppress the firm subscript n in general and, when convenient, we shall also represent by y_{nt} or y_t the column vector of our four variables, that is, $y_t = (q_t, s_t, r_t, i_t)'$.

The variable q_t is the stock market one-period holding rate of return, defined as $q_t = (p_t - p_{t-1} + d_t)/p_t$, where p_t is the price of a share at the end of year t, and d_t is the dividend per share paid during this year. Thus, q_t is equal to the rate of change of the value of a one dollar share over the year plus the corresponding dividend. Variables s_t , r_t , and i_t denote the first difference between year t and year (t-1) of the logarithms of sales, R & D expenditures, and gross investments, respectively, and are thus approximately equal to their rate of change from year to year: $s_t = \log(S_t/S_{t-1})$; $r_t = \log(R_t/R_{t-1})$; $i_t = \log(I_t/I_{t-1})$.

^{1.} In the empirical implementation, q_t is adjusted for stock splits when they occur. Sales are deflated using industry price indexes; $\mathbf{R} & \mathbf{D}$ and investment expenditures are also deflated by an overall price index. There is the possibility of some mismatch in timing between s_t , r_t , and i_t , which are based on the companies' fiscal year, and q_t , which is based on

Given our focus on these four variables, we are interested in investigating thoroughly their mutual dynamic interrelationships. Without pretending too much a priori knowledge about these interrelations, we start by assuming that they can be represented by an autoregressive model:

(1)
$$\mathbf{y}_t = A(L)\mathbf{y}_{t-1} + \mathbf{\lambda}_t + \mathbf{\eta}_t,$$

where A(L) is a matrix of polynomials in the lag operator (L), λ_t is a vector of time-specific effects or year dummies, and η_t is a vector of disturbances assumed to be normally distributed, uncorrelated over time but correlated across equations: η_t serially uncorrelated $N(0, \Sigma)$. The vector η_t is called the vector of "innovations" in the variables. We can write (1) more simply as:

$$(1') y_t = A(L)y_{t-1} + \eta_t,$$

if we take care of the year effects λ_t by measuring our variables relative to their year means, as we shall assume from now on.²

With an adequate number of lags, the autoregressive model is flexible enough to account well for the correlation structure of our variables and simulate their dynamic behavior. From a purely statistical standpoint, equivalent formulations can be obtained by multiplying both sides of (1') by any nonsingular (four by four) matrix B_0 . Among them, recursive formulations may be of practical interest, especially one that corresponds to the causal ordering we are going to hypothesize between our variables; that is, causality running from q to s, and from both q and s to r and s. This particular recursive formulation can be written as:

(1")
$$B_0 y_t = B(L) y_{t-1} + \zeta_t,$$

where $B(L) = B_0 A(L)$, and $\zeta_t = B_0 \eta_t$, B_0 being a triangular matrix with 0 above the diagonal and 1 in the diagonal, such that the transformed disturbances ζ_{jt} are othogonal (i.e., uncorrelated across equations). In fact, B_0 is uniquely determined; its inverse, B^{-1} , has the exact same

the calendar year. From previous work, we know that fiscal and calendar years do not coincide for a large enough proportion of firms; an attempt to correct for this problem had, however, very little impact on our results. We preferred not to make any such correction in the present study.

^{2.} Our adoption of a formulation in terms of the rates of growth of the variables or log differences results from a number of considerations. Using first differences is usually advised in the time-series literature to get more stationary processes (Granger and Newbold 1977). Actually, when we tried to estimate the autoregressive model in the levels of variables, the results suggested a first difference formulation (some of the roots of the characteristic equation associated with the model being close to one in absolute values). Going to first differences is also a simple way to avoid dealing with firm-specific effects, while the formulation in terms of levels raises the well-known difficulties of estimating a dynamic model with such effects (Balestra and Nerlove 1966). First differences have, however, the drawback of magnifying the problems of errors in the variables (augmenting the ratio of error to true variance.)

lower triangular form with 1 on the diagonal and can be obtained from the appropriate Cholewski decomposition of the original variance-covariance matrix σ . This can be written as $\eta_t = B^{-1} \zeta_t$, and amounts, in practice, to successive projections of the original disturbances η_{jt} , which transform them into ζ_{it} 's:

$$\eta_{1t} = \zeta_{1t}; \eta_{2t} = a\zeta_{1t} + \zeta_{2t}; \dots$$

Among the many statistically equivalent formulations, we endeavor to give a specific structural economic meaning to the pure autoregressive form (1), and we therefore refer to it as our general model. All four equations of the general model (q, s, r, and i) can be interpreted and motivated by more or less precise economic considerations, and we can test whether the restrictions suggested by such considerations are compatible with our data.

14.2.2 Interpretation and Motivation

We can justify our *i* equation as an investment demand equation, referring directly to Malinvaud's recent book, *Profitability and Unemployment* (1980; see also Malinvaud 1981). In his book, Malinvaud studies the implications of an investment model in which net investment depends on expected capacity need and expected profitability. While the influence of capacity needs corresponds to the well-known accelerator phenomenon and is supported by the bulk of the vast number of econometric studies of investment, he stresses the importance of profitability as another major determinant. If we assume the investment equation to be log-linear and take first differences, we get:

$$i_t^* = \phi q_{t-1}^e + \gamma s_{t-1}^e$$

where $i_t^* = \log(NI_t/NI_{t-1})$ is the log change in desired net investment between periods (t-1) and t, $s_{t-1}^e = \log(S_{t-1}^t/S_{t-2}^{t-1})$ and $q_{t-1}^e = \log(Q_{t-1}^t/Q_{t-1}^{t-1})$ are the log changes or revisions of capacity need and profitability between these same periods and as expected one period before.

The revision in the expected profitability q_{t-1}^e is presumably because of new information about the future which becomes available between (t-2) and (t-1). Such revisions should have direct bearing on the movements of stock prices during the same period and, hence, will be reflected in the lagged values, q_{t-2} and q_{t-1} , of our stock market holding rate of return variable. Therefore, we will interpret q_{t-1} and q_{t-2} as reasonable indicators of the unobservable q_{t-1}^e in the investment equation.³

3. The usefulness of stock market valuation as an indicator of expectations about future profitability in an investment function can be traced back to Grunfeld (1960), and more recently to the literature on "Tobin's Q" (Tobin 1971). Our q variable will be equal to the

In the absence of any direct information on expectations about capacity, the usual and simple procedure in most econometric studies is to treat them as a function of past levels of output or sales. We can likewise take the revision in the expected capacity need s_{t-1}^e as a distributed lag function of past changes in sales $s_{t-\tau}$, thereby justifying why lagged values $s_{t-\tau}$ should appear in the investment equation. More generally, we can consider s_{t-1}^e as a forecast function depending not only on the past $s_{t-\tau}$, but also on the past values of other relevant variables. Assuming rational expectations, the actual change in sales s_t itself should be an unbiased "forecast" of the expected s_{t-1}^e , conditional on all the information available in period (t-1), and s_{t-1}^e should only differ from s_t by an uncorrelated forecast error. In particular, one would think that q_{t-1} , being a forward-looking variable, has a predictive value for both s_{t-1}^e and s_t , and therefore, will enter significantly in the forecast function even in the presence of lagged $s_{t-\tau}$ terms. Thus, one should find that q_{t-1} influences investment both directly and indirectly via its effect on expected sales.

Finally, the change in the desired net investment variable i_l^* itself is also unobservable, and its relationship with the actual change in gross investment must be specified. The various kinds of delays occurring between the decision and the execution of investment plans, as well as an approximate proportionality of retirements to past investments, suggest reasons why lagged investment terms should also appear in the investment equation.

In sum, starting from Malinvaud's (1980) theoretical equation and taking into account all the necessary transformations for its empirical implementation, we get to an equation that is very close to the investment equation of our general model. Clearly, such a tentative and informal derivation involves many problematic assumptions and issues. Be that as it may, our investment equation consists of two main factors: scale and intensity, as indicated by sales and stock market profitability, respectively, and allows for a quite flexible lag structure. The standard objection one could raise is that more explanatory variables should have been included, mainly the relative cost of labor and capital and the financial liquidity of the firm. It is difficult, though, to get relevant information about factor prices at the firm level; it is also plausible that they tend to move roughly parallel for all firms, and that will be taken care of by the year dummies in the equation. As for financial liquidity of the firm, it

percentage change in Tobin's Q variable, if dobts are proportional to equity and there is no change in the replacement value of the firm. Actually, the correlation between our q variable and the change in Tobin's Q variable, as computed otherwise, is quite high in our sample. Our study is thus related to the studies investigating Tobin's Q as a determinant of investment. See, for example, Engle and Foley (1975), Von Fustenberg (1977), and Summers (1980), among others.

could be gauged by the importance of past profits, and it may be worthwhile to consider this possibility in further research.

The r equation can be justified along the same lines as the i equation and interpreted in terms of an R & D demand equation. One of our basic topics of interest is to assess whether R & D and physical investment behave more or less similarly.

From what we have already said, the s or sales equation can be understood as a forecast function purporting to account for the expectations of firms about their future sales. It seems plausible, however, that these expectations might also depend on other variables besides the ones already included in the equation.

The q or stock market holding rate of return equation has little economic justification. For the sake of symmetry with the s equation, it could also be viewed as a forecast function of expectations on q_t . However, it is usually admitted that q_t cannot be predicted by its own past values or that of any other variable. This property is known as Fama's semistrong test of stock market efficiency (Fama 1970, 1976). Conditional on the information available at the beginning of period t, the expected value of q_t should, by standard arbitrage argument, equal the prevailing market rate of interest. In other words, a trading rule based on public information alone would not allow traders to achieve any excess return on average.

14.2.3 An Extended Accelerator Model

The considerations we have just developed suggest a causal ordering of the variables and specific restrictions on the equations.

We have touched on the issue of stock market efficiency. The hypothesis of stock market efficiency simplifies our general model importantly, the q equation reducing itself to $q_t = \eta_{1t} (= \zeta_{1t})$. In other words, q is exogenous relative to the other variables, or s, r, and i do not cause q in the sense of Granger (Pierce and Haugh 1977, 1979; Granger, 1980). Such a hypothesis has been generally accepted in empirical work, but rather than taking it for granted, it seems better to test it on our data.

Our central interest, however, is in the appropriateness of the traditional formulation of the accelerator model. This formulation postulates that sales or expected sales are exogenous relative to investment, thus ruling out feedback effects from investment to sales. This is a major assumption, since without it not only the usual estimates of the so-called accelerator effect might be biased, but the whole notion itself might not be very meaningful. Within our general model, the accelerator assump-

^{4.} Doubts have recently been expressed about the efficiency of stock markets. Schiller (1981) pointed out that the actual stock prices fluctuate too much to reconcile with the stable and smooth series of the present value of subsequent real dividends. See also Malinvaud (1981) and Summers (1982).

tion is directly testable, requiring i, as well as r by analogy, to not appear in the s equation.

Besides the questions of stock market efficiency and the appropriateness of the accelerator assumption, we have also considered two other issues of lesser significance. The first concerns the interrelations of physical and R & D investment. There seems to be no reason why physical investment should influence R & D investment per se. One might expect, however, that the converse would not be true. A successful R & D program would lead to product or process innovations, which could result in new programs of investment. There is, however, little evidence in our data of such a causal ordering from R & D to investment. While we do not find any significant influence of past i on i, the influence of past i on i is not significant either, and at best appears to be rather weak.

The second issue relates to the existence of contemporaneous reciprocal influences between our variables, or "instantaneous causality". In our general model (1), this amounts to testing the diagonality of the variancecovariance matrix Σ (i.e., no correlation across equations among the disturbances η_{it}), while in the transformed recursive formulation (1"), it becomes the test of the restriction that the contemporaneous value of a variable does not enter as a regressor (i.e., B_0 is an identity matrix, otherwise $\eta_{it} = \zeta_{it}$). A year being a long enough period for interactions between variables to develop, one would expect instantaneous causality to occur and, hence, the diagonality restriction to be strongly rejected. This is indeed what happens. Another explanation of why the disturbances in our model may be correlated across equations is of course the omission of relevant (common or correlated) variables. One would thus expect the disturbances in the investment and R & D equations (η_3 , and η_{4t}) to be correlated with each other and also with the disturbance in the sales equation (η_{2t}) . Indeed, this last disturbance can proxy for variables influencing sales expectations but actually omitted from our forecast equation; as such it should enter in both the investment and the R & D equations, accounting partly for the correlation of their disturbances. The structure of the disturbances and their correlations is clearly revealed by the appropriate Cholewski decompostion, $\eta_t = B^{-1} \zeta_t$, as previously indicated.

We can focus our interest primarily on two restricted versions of the general model: the first one assuming only stock market efficiency; the second one also assuming the appropriateness of the accelerator formulation. We call the latter restricted model the accelerator model or the extended accelerator model since it extends the traditional investment accelerator to research and development expenditures, and because it tries, through the use of the q variable, to incorporate expected prof-

itability as an important determinant of investment and R & D. Since interactions between investment and R & D do not appear to be significant, we generally consider the extended accelerator model without them, but this need not be so in principle.

14.2.4 Moving Average Representation and Multipliers

Changing slightly our notation but still measuring variables relatively to their year means, the extended accelerator model can be written:

(2)
$$q_{t} = \eta_{1t},$$

$$s_{t} = \beta(L) q_{t-1} + \alpha(L) s_{t-1} \eta_{2t},$$

$$r_{t} = \phi(L) q_{t-1} + \gamma(L) s_{t-1} + \theta(L) r_{t-1} + \eta_{3t}$$

$$i_{t} = \psi(L) q_{t-1} + \delta(L) s_{t-1} + \mu(L) i_{t-1} + \eta_{4t},$$

where the η_{jt} are mutually correlated across equations (but uncorrelated over time). The causal structure of the model is simple and can be illustrated by the path diagram in figure 14.1. Changes in q induce variations in s, r, and i, and changes in s move r and i, but there is no feedback from r and i to s or from s to q; there is also no interaction between r and i. As we already stated, in view of this specific structure, there is one appropriate and economically meaningful decomposition of the correlated η_t in terms of uncorrelated ζ_t . Renaming these ϵ_t , u_t , v_t , and w_t (instead of ζ_{it}), we can write

(2')
$$\eta_{1t} = \epsilon_t,
\eta_{2t} = a\epsilon_t + u_t,
\eta_{3t} = b\epsilon_t + cu_t + v_t,
\eta_{4t} = d\epsilon_t + eu_t + fv_t + w_t.$$

In this form the independent errors, ϵ_t , u_t , v_t , and w_t , are intrinsically related to the different equations of the accelerator model. They can be regarded as the exogenous and unobservable (or unobserved) basic factors of our model accounting for the evolution of our observed variables. A change or "shock" in ϵ_t , or an innovation in q_t , can thus be interpreted as a shift in the firm's future profitability as expected by the traders on the stock market. We shall call such a shock an expected profitability shock, or q shock, and the dynamic responses of our variables to it the q effects of q multipliers. Similarly, a change or a "shock" in u_t , or an independent innovation in s_t , can be viewed as a shift in the expectation of the rate of growth of sales, and we shall speak of a demand shock, or s shock, and of the s effects or s multipliers. It is of some interest to separate in the (total) q or s effects the own effects and the additional or cross effects. The own effects are computed in the absence of instantaneous causality (i.e., a = b = c = d = e = f = g = 0 or $\eta_t = \zeta_t$); they result directly from the

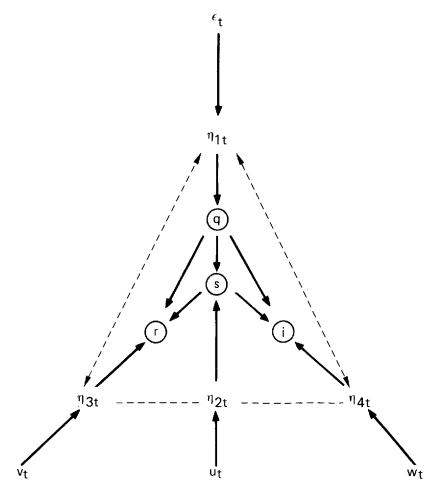


Fig. 14.1 Path diagram of the extended accelerator model.

initial change in q_t or s_t corresponding to a shock in ϵ_t or u_t , as if there was no other immediate impact of such shocks.⁵

To illustrate the q and s multipliers and how a shock in ϵ or u actually affects the movements of our variables, we can consider a simplified version of the accelerator model in which we keep only one lagged variable (i.e., a first-order autoregressive model), ignore the correlations of the disturbances across equations (i.e., Σ is diagonal), and drop the i equation (since i and r behave in the same way). It is enough to consider:

5. The formulations (1') and (1") of the general model can also be written: $y_t = P(L)\eta_t$ and $y_t = T(L)\zeta_t$, P(L) and T(L) being respectively the matrix of the own and total effects with: $P(L) = [I - A(L)L]^{-1} = [I - B^{-1}B(L)L]^{-1}$, and $T(L) = P(L)B_0^{-1}$.

$$q_t = \epsilon_t,$$

 $s_t = \beta q_{t-1} + \alpha s_{t-1} + u_t,$
 $r_t = \varphi q_{t-1} + \gamma s_{t-1} + \theta r_{t-1} + v_t,$

with $|\alpha| < 1, |\theta| < 1$, and ϵ_t , u_t , and v_t mutually uncorrelated. For this simple system, we can write the moving average representation explicitly as:

$$\begin{aligned} q_t &= \epsilon_t, \\ s_t &= \beta \sum_{\tau=1}^{\infty} \alpha^{\tau-1} \epsilon_{t-\tau} + \sum_{\tau=0}^{\infty} \alpha^{\tau} u_{t-\tau}, \\ r_t &= \sum_{t=1}^{\infty} \omega_{\tau} \epsilon_{t-\tau} + \sum_{\tau=0}^{\infty} \rho_{\tau} u_{t-\tau} + \sum_{\tau=0}^{\infty} \theta^{\tau} v_{t-\tau}, \end{aligned}$$

where

$$\rho_{\tau} = \theta \rho_{\tau-1} + \gamma \alpha^{\tau-1}$$
 and $\omega_{\tau} = \phi \theta^{\tau-1} + \beta \rho_{\tau-1}$,

with $\rho_o = 0$, and for $\tau = 1, 2, \ldots$. The response pattern of our variables is described completely by this moving average representation. For example, ω_{τ} is the effect on r after τ years of a one-period, one-unit shock in ϵ . Thus, $\sum_{\tau=1}^k \omega_{\tau}$ is the cumulative effect on r over a period of k years from this shock, that is, the proportional change in the level of R & D after k years from this shock. A shock appears to induce decaying fluctuations in growth rates and to put the levels on higher growth paths. Essentially, the effects on growth rates are transitory, while the changes in levels are permanent.

The long-run effects of a one-period, one-unit shock in ϵ or u on the levels of sales and R & D can be easily computed and are given in table 14.1. A 1 percent increase in u will induce sales and R & D to increase respectively by $\sum_{\tau=1}^{\infty} \alpha^{\tau} = 1/(1-\alpha)$ and $\sum_{\tau=1}^{\infty} \rho_{\tau} = \gamma/(1-\theta)(1-\alpha)$. The ratio of these two effects, $\gamma/(1-\theta)$, is the elasticity of R & D with respect to sales, and thus can be called the long-run accelerator effect or multi-

Table 14.1 Long-Run Multipliers in the First-Order Autoregressive Accelerator Model

Shock or	Perce	ntage Change in Level
Innovation	$\Delta S/S$	$\Delta R/R$
€ or q	$\frac{\beta}{1-\alpha}$	$\frac{\Phi}{1-\theta} + \frac{\beta\gamma}{(1-\theta)(1-\alpha)}$
u or s	$\frac{1}{1-\alpha}$	$\frac{\gamma}{(1-\theta)(1-\alpha)}$

plier. The long-run elasticity of R & D with respect to q is $\sum_{\tau=1}^{\infty} \omega_{\tau} = \phi/(1-\theta) + [\beta\gamma/(1-\theta)(1-\alpha)]$. This expression indicates clearly that q can affect R & D both directly and indirectly through its impact on sales: the direct effect being $\phi/(1-\theta)$; the indirect effect being the product of the impact of q on sales $\beta/(1-\alpha)$ and the long-run accelerator $\gamma/(1-\theta)$.

14.3 Empirical Results

14.3.1 Tests and Estimates

The empirical implementation of our study is based on a sample of ninety-three firms with data from 1962 to 1977. This sample derives from the Griliches and Mairesses (this volume) restricted sample of 103 firms with no major merger problems. We had to discard ten firms because of the lack of all the necessary information to construct the q variable. Although our sample may seem small in terms of number of firms and cannot be taken as representative of the corporate sector in any definite sense, it is, in fact, about the largest size possible for firms doing R & D over a sufficiently long period (at least ten good years for our type of time-series cross-section analysis).

The sample means and standard deviations of our variables over the twelve-year period, 1966–77, as well as the standard deviations of our variables measured relative to their year means, are the following:

$$q = .104, \quad s = .062, \quad r = .025, \quad i = .036$$

(.433) (.120) (.217) (.465)
[.362] [.107] [.211] [.444]

As could be expected, the stock market rate of return is extremely variable. So is physical investment; it is not rare for a firm's physical investment to double (or go down by half) from one year to the next. Note that R & D expenditures are also quite variable, though much less so than physical investment.

We have estimated all our models by Zellner's seemingly unrelated regression least-squares method (based on the variance-covariance matrix Σ estimated once and for all for the general model case). The parameter estimates of the general model, the extended accelerator model, and its simplified first-order autoregressive version are given in tables 14.2 and 14.3, while all the different test results are brought together in table 14.4

The general model uses four lagged values of each of the four variables and is therefore estimated over the twelve-year period, 1966–77, including also twelve-year dummies. We have experimented some with shorter lags, but four lags seemed to be necessary to capture the dynamic behavior of our variables adequately. We have also checked for the possibil-

ity of serial correlation of the disturbances. It is apparently negligible, the first- and second-order autocorrelation coefficients of the residuals $\hat{\eta}_{ji}$ in each equation being rather small uniformly (-.01 and -.06 for the q equation residuals, respectively; -.02 and -.03 for the s equation residuals; -.03 and -.01 for the r equation residuals; and -.01 and -.07 for the t equation residuals).

Table 14.2	Parameter	Estimates	Concept	Modela
rabie 14.2	Parameter	Esumates,	Generai	Monei

	q	s	r	i	_
q_{-1}	005	.044	.067	.172	
	(.027)	(.008)	(.015)	(.030)	
q_{-2}	.002	.008	.034	.171	
	(.025)	(.007)	(.014)	(.028)	
q_{-3}	009	.004	.021	.055	
	(.022)	(.006)	(.012)	(.025)	
q_{-4}	.037	.011	006	.051	
-	(.027)	(.006)	(.012)	(.025)	
S 1	161	.116	.335	.288	
	(.109)	(.031)	(.060)	(.121)	
S-2	043	028	.097	006	
	(.108)	(.031)	(.060)	(.120)	
S-3	076	.089	.102	.097	
	(.109)	(.031)	(.060)	(.121)	
S _ 4	.069	.050	.072	.112	
	(.107)	(.031)	(.059)	(.119)	
r_{-1}	012	.023	243	.140	
	(.055)	(.016)	(.031)	(.061)	
r_{-2}	047	015	132	013	
	(.062)	(.018)	(.034)	(.068)	
r_{-3}	016	.026	.142	103	
	(.065)	(.019)	(.036)	(.072)	
r4	106	016	009	054	
	(.057)	(.016)	(.031)	(.063)	
i_{-1}	064	$001^{'}$.003	344	
•	(.028)	(.008)	(.015)	(.031)	
i_{-2}	062	.012	.003	332	
-	(.029)	(800.)	(.016)	(.032)	
i_{-3}	004	007	023	209	
-	(.029)	(.008)	(.016)	(.032)	
i_{-4}	019	004	.002	143	
•	(.028)	(800.)	(.015)	(.031)	

Weighted residuals sum of squares = 4464 Degrees of freedom = 4346

^aThe parameter estimates of the s, r, and i equations do not differ in the general model without market efficiency nor with market efficiency, while the q equation vanishes in the latter case.

Conversely, the contemporaneous correlations of the residuals $\hat{\eta}_{jt}$ across equations are rather high (.19, .07, and .07 between the q equation and the s, r, and i equation residuals, respectively; .18 and .26 between the s equation and the r and i equation residuals; .18 between the r and i equation residuals). The test of diagonality is indeed strongly rejected. Using the Cholewski decomposition, we can write:

Table 14.3 Parameter Estimates, Extended Accelerator Model

	Extend	ed Accelerator	Model		First-Order Autoregressi Accelerator Model	
	S	r	i	s	r	i
q_{-1}	.043	.068	.174	.041	.063	.194
	(800.)	(.015)	(.030)	(.007)	(.015)	(.029)
q_{-2}		.034	.170			
		(.013)	(.026)			
q_{-3}		.020	.052			
	_	(.012)	(.024)			_
q. 4		012	.038			
		(.012)	(.024)			
s_{-1}	.143	.345	.354	.154	.384	.256
	(.029)	(.058)	(.119)	(.028)	(.057)	(.114)
S-2	000	.108	.047			
	(.028)	(.058)	(.116)	_	_	
s_{-3}	.106	.095	.074			
	(.028)	(.059)	(.117)		_	_
S _ 4	.052	.072	.077			
	(.028)	(.058)	(.115)	_	_	_
r_{-1}	,	258	,		227	
		(.029)	_		(.028)	
r_{-2}		125 [°]			` /	
_	_	(.033)			_	_
r_{-3}		.138				
,	_	(.034)			_	
r ₄		002				
,	_	(.030)			***	
i_{-1}		` ′	337			190
•	_		(.029)	_		(.027)
i_2			346			` /
-	_		(.030)			_
i_{-3}			203			
,		_	(.030)	_	_	
i_{-4}			146			
	_	_	(.029)	_		_

Weighted residuals sum of squares = 4519 Weighted residuals sum of squares = 4777 Degrees of freedom = 4381 Degrees of freedom = 4402

Table 14.4 Test S	Statistics ^a							
	Weighted	:	Test against	Test against the General Model, H_0	odel, H_0	Test the Marko	Test the General Model with Market Efficiency, H ₁	l with
Hypothesis	Residuals Sum of Squares	Degrees of Freedom	Number of Value f of Restrictions F-statistic	Value f of F-statistic	Prob $(F > f)$	Number of Value f of Restrictions F-statistic	Value f of F -statistic	Prob $(F > f)$
H_0 : general model H_0 : general model	4464	4346	l	1	1	l	l	1
with the diagonality	4650	4352	9	30,2	000	I	l	
H ₁ : general model with stock mar- ket efficiency	4490	4362	16	1.6	.060	I	l	I
H_2 : extended accelerator with r and i interactions	4508	4373	27	1.6	.030	11	1.6	.092
H ₃ : extended accelerator without r and i interactions	4519	4381	35	1.5	.030	19	1.5	.075
H_4 : first-order autoregressive accelerator	7774	4402	56	5.4	000.	40	7.0	000

 a The test statistics are the standard F statistics computed from the weighted residuals sum of squares. These are based on the Σ matrix estimated under the alternative hypothesis of the general model.

$$\hat{\eta}_{1t} = \hat{\epsilon}_t,
\hat{\eta}_{2t} = .055 \hat{\epsilon}_t + \hat{u}_t,
\eta_{3t} = .040 \hat{\epsilon}_t + .329 \hat{u}_t + \hat{v}_t,
\hat{\eta}_{4t} = .079 \hat{\epsilon}_t + .974 \hat{u}_t + .284 \hat{v}_t + \hat{w}_t,$$

the standard deviations of the uncorrelated ϵ_t , u_t , v_t , and w_t being .358, .101, .194, and .380, respectively. It appears from these estimates that u, the independent innovation in s, has an immediate and strong impact on i and a more moderate one on r, while the immediate effect of ϵ , the innovation in q, is quite weak. Note also that the independent innovation in r has a sizeable effect on i as well.

Considering the estimated equations of the general model in turn, it is clear that all the implications suggested by the economic interpretation are by and large supported. All the coefficients of the q equation (i.e., the sixteen coefficients of the lagged values of q, s, r, and i except for the time dummies) are insignificant and, even taken together, the hypothesis of their joint nullity cannot be rejected at the 5 percent significance level. This is another confirmation of the unpredictability of q from past information and thus also of the hypothesis of stock market efficiency.

All eight coefficients of the lagged r and i terms are insignificant in the s equation. Assuming stock market efficiency, their joint nullity (together with that of the coefficient of q_{-2} , q_{-3} , and q_{-4} which are also individually insignificant) cannot be rejected at a 5 percent level of significance. We can thus accept the hypothesis that s and q are exogenous relative to r and i, and that the accelerator model is a reasonable specification, even though at first it appeared to be a rather strong simplification.

In the r equation, the four lagged i terms and likewise the four lagged r terms in the i equation are all insignificant, except for the coefficient of r_{-1} on i, which is on the verge of individual significance at the 5 percent level. As a group, they are insignificant at the 5 percent level. We can accept the absence of interactions, other than instantaneous, between r and i and hence we can accept the accelerator model without such interactions. On the other hand, the hypothesis (considered by way of illustration) that the accelerator is first-order autoregressive is strongly rejected.

14.3.2 Dynamic and Long-Run Multipliers

The implications of our results are best described by the dynamic responses of our variables to the different shocks and the q and s effects or multipliers. All long-run multipliers are given in table 14.5, while the q and s dynamic multipliers are represented in figures 14.2 to 14.5. We shall comment on them in turn.

6. The fact that we cannot reject exogeneity tests of both q (stock market efficiency) and s (accelerator model) is all the more meaningful since our sample has a large number of observations (see, for example, Leamer 1978, chap. 4).

Model Shocks on \$\epsilon\$ \text{\$\sigma\$} \$\ell\$	Total Effects	cts v 162 .004 .791 .093 0 0 0 .031	777005	.949 .085 .127 .229	Own] " 414 1.271 .579 .208	Own Effects y 1143	w 690. –
Shocks on e al model	2.528 1.267 .826 .826 .658 0 0 1.357 .936	7162 .004 .791 .093 .093	W [600]	. 949 .085 .127 229 .093	208	v 143 .006	34 690
ancy R 190 ncy R 190 190 190 190 169 169 191 191 191 191	528 1.267 .826 .658 .0 1.357 .936	162 .004 .791 .093 0 0	069 005 015 005		414 1.271 .579 .208	143	
ney R 1.155 11		.102 .004 .791 .093 .031	000.000 000.000 000.000		.208 .208	900.	2000
ncy R .190 1	.826 .658 .0 1.357 .936	.093 .093 .031 .031	015 	1 1003	.279		-900
accel- accel- R	.658 0 1.357 .936 .818	.093 0 .031	0 005	[.229 [1 [093	.208	.795	015
odel 9 169 1 market 8 .169 1 R .208 169 1 I .301 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 1.357 .936 .818	0 .031 .824	0.005	1.093	0	043	.477
ordel 9 1 16	0 1.357 .936 .818	0 .031 .824	.005	1 .093	0		ſ
market S .169 1 accele S .208 I .301 I .301 I .301 I .301 I .141 I .191 I .188	1.357 .936 .818	.031	.005	.093		0	0
accel- S	.936 818	.824	100		1.342	.029	.005
I 301 accel- S 141 1 1 del R 191 I 1.288	818		- 1001	.138	.665	.825	001
g 7 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	070.	.142	.497	.244	.333	.001	.497
9 S141 R191 I288							
S141 R191 I288	0	0	0	1	0	0	0
R .191 I .288	1.431	0	0	.062	1.431	0	0
.288	.978	.805	0	.119	.714	.805	0
	.849	.140	.492	.229	.369	0	.492
ı			l	ı			l
, s	0	0	0	1	0	0	0
R (.114	1.181	0	0	.049	1.181	0	0
I 1.120	.637	.814	0	.067	.369	.814	0
	1.072	.239	.840	.173	.254	0	.840

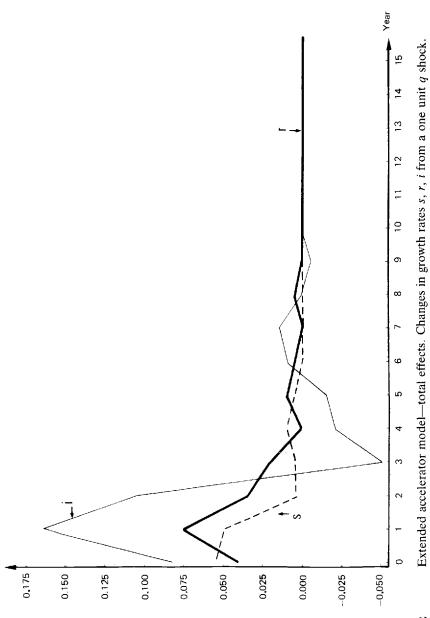
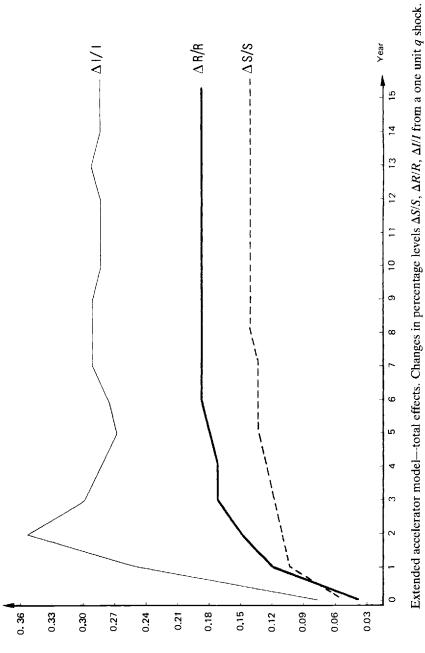


Fig. 14.2



g. 14.3

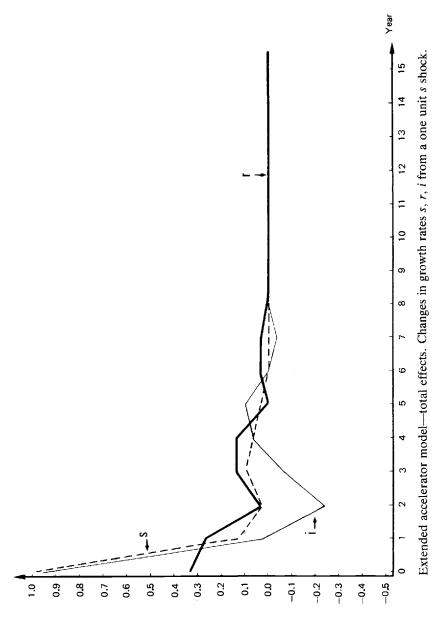
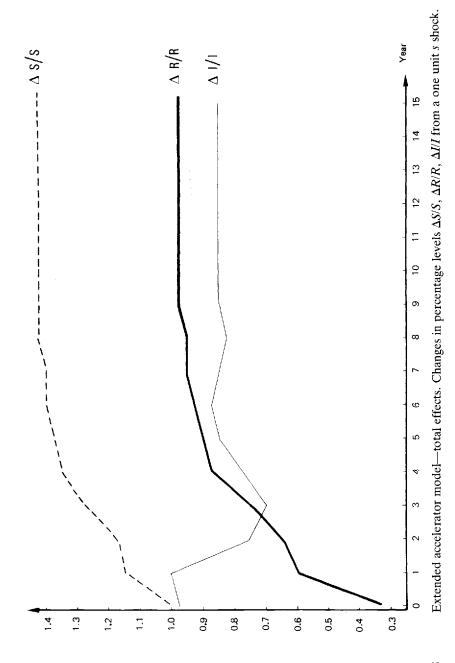


Fig. 14.4



g. 14.5

The eight matrices in table 14.5 consist of the own and total effects estimated for the general model with and without stock market efficiency. the extended accelerator model (without r and i interactions), and the first order autoregressive accelerator model. We have not endeavored to compute the standard deviations of these coefficients.7 However, the comparison of their values for the four different specifications gives us a feeling for their precision. As we have seen, the general model with market efficiency and the extended accelerator are not statistically different at the 5 percent significance level; indeed all the estimated effects for these two models are very close. The general model without market efficiency differs mainly from that with market efficiency by the estimated effect of s (or u) on q; however, this effect should not be statistically significant, corresponding mainly to the large insignificant coefficient of s_{-1} in the q equation (see table 14.2). The largest discrepancies between the extended accelerator model and its first-order autoregressive version occur in the estimated effects of s (or u) on r and i (and also of i [or w] on itself); these discrepancies are probably significant since they correspond to the significant coefficients of s_{-2} and s_{-4} in the r and i equations (and also of i_{-2} , i_{-3} , and i_{-4} in the *i* equation).

The comparison of the own and total effects shows the importance of the contemporaneous influences of q and s on r and i (i.e., the importance of instantaneous causality). This was already clear from the Cholewski decomposition given above, showing the correlation structure of the innovations in our variables. Consider the one very striking case: the long-run impact of a 1 percent s or u shock on the level of physical investment would amount only to .35 percent, instead of about .85, if the contemporaneous dependence between s and i were eliminated.

Figures 14.2 to 14.5 each consist of three graphs, depicting the yearly q or s (total) effects of the three rates of growth: s, r, and i, or on the three percentage changes in levels: $\Delta S/S$, $\Delta R/R$, $\Delta I/I$ (these effects being estimated for the extended accelerator model). The responses of s and r to the q or s shocks are similar enough, damping down rapidly with most of the effects dissipating in three years. The investment growth rate i reacts more strongly and irregularly. In response to a 1 percent q shock, it goes up to about .15 in the first year and down to .10 and - .05 in the second and third years, then cycles down quickly to zero. In response to a 1 percent s shock, after an immediate impact of about 1, it plunges to 0 and - .25 in the first and second years, then cycles back quickly to zero. In coherence with these patterns of response, the levels of sales and R & D expenditures increase steadily toward their new long-run values while

^{7.} The total and own effects are highly nonlinear and complicated expressions of the estimated parameters, making the derivation of their standard deviations a problematic task (see note 5).

investment starts by overshooting its own, all cumulated effects being practically completed in five years.

The long-run (total) effects of a 1 percent q shock on sales, R & D, and investment levels are respectively about .15, .20, and .30. These elasticities appear to be rather small; however, gauged in terms of the standard deviations of the corresponding rates of growth, they are quite sizeable. A one standard deviation q shock induces changes in the levels of sales, R & D, and investment of about .55, .40, and .25 of their respective standard deviations.

The absolute long-run effects of a 1 percent s shock are much larger than those of a 1 percent q shock, moving the levels of sales, R & D, and investment by about 1.4, .95, and .85 repectively. Yet, measured in units of standard deviations, s shocks are not more effective than q shocks in driving R & D and physical investments: the changes induced by the former being about .50 and .20, compared to .40 and .25 by the latter. In this regard it should be noted that only 30 percent of the q effect on R & D and 55 percent of the q effect on investment relies on the direct influence of q, the remaining effect resulting from the impact of q on s. This remark shows that in considering an R & D or investment equation in isolation, one might be led to a serious underestimate of the significance of the q variable.

For comparison with the results of other investment studies, it is interesting to translate the long-run s effects into the usual accelerator elasticities $(\Delta I/I)/(\Delta S/S)$ or $(\Delta R/R)/(\Delta S/S)$: they are about .6 (~.85/1.4) and .7 (\sim .98/1.4) for physical investment and R & D, respectively. The latter estimate of .7 accords well with the elasticity of R & D capital stock reported to be around .5 to .8 by Nadiri and Bitros (1980) in the only other study investigating investment and R & D demand jointly. The former estimate of .6 is, however, lower than their estimated elasticity of around 1 for physical capital stock. A unitary elasticity is implied by the standard Jorgensonian factor demand framework (i.e., the inverse of the returns to scale in the production function, which presumably are not very far from being constant) and is in fact found in many econometric studies (for example, Jorgenson and Stephenson 1967; Jorgenson 1971). Because of the various differences in specification, it is difficult to pinpoint the actual reasons for our relatively low accelerator estimate. It probably arises from our rate of growth formulation. Using a similar formulation, Eisner found an even lower estimate of about .4 (Eisner 1978a, 1978b; see also Oudiz 1978).8 Eisner's explanation, which is similar to Friedman's permanent income hypothesis, may also be applicable

^{8.} To be precise, Eisner's dependent variable is the deviation from the firm mean of the investment-capital ratio, or the rate of growth of the capital stock plus its rate of depreciation.

to our results. In our specification of the accelerator model, the q and s shocks are assumed to be free from errors or contamination by any noise. In reality, the fluctuations in q and s have large transitory components, which will have presumably little impact on i and r. Our estimates of the accelerator elasticity and, more generally, of the q and s effects might be larger if we could disentangle the transitory variations from the permanent changes in q and s.

14.4 Final Remarks

Using a multivariate autoregressive framework, we have found a simple causal structure for the variables of interest, q, s, r, and i, which is consistent with our data. As expected from the stock market efficiency hypothesis, q, the stock market one-period holding rate of return, is exogenous relative to the other three variables (or Granger causes them). As postulated in the traditional accelerator model of investment, the rate of growth of sales, s can also be treated as exogenous to the rates of growth of R & D and physical investment, r and i. Moreover, no strong feedback interaction is detected between r and i.

Within the simple structure of the extended accelerator model, the substantive conclusion is that R & D and physical investment react very similarly to the growth of sales and to movements in q; however, the response of R & D is more stable or less irregular than that of physical investment. Both expected demand and expected profitability thus appear to be important determinants for R & D expenditures and physical investment.

It will be important to check our findings against other data. Also, our study could be improved by incorporating other variables of interest (see Ben-Zion, this volume). In future work, it would be particularly interesting to go further in two directions:

- 1. The multivariate autoregressive setup proved to be useful and convenient for studying the dynamic relationships between variables. However, a more elaborate specification might help to filter out the permanent from the transitory components of the variables. This issue is related to our choice of growth rates formulation, which has many advantages but also tends to magnify the relative importance of transitory components or errors in the variables.
- 2. The fact that past q's, though probably error ridden, are significantly correlated with s, r, and i confirms that movements in stock prices carry valuable expectational information about future profitability. This interpretation of the q variables should be more rigorously substantiated and its relation to "Tobin's Q" clarified. More generally, the extended accelerator model should be grounded more firmly in theory and provided with a more definite behavioral interpretation.

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Comment John J. Beggs

This paper is an extensive attempt at data analysis of the relationships between market value, sales, research and development, and investment at the firm level. The sample is large, being a cross section of 103 firms for a fifteen-year period. The now familiar vector autoregressive formulation of the dynamic process is employed, and unrestricted and restricted formulations of the lagged variable interactions are estimated. The resulting discussion in section 14.3 of the paper provides a thoughtful interpretation of empirical results.

Within the Mairesse-Siu framework at least three major methodological issues must be addressed, though I believe these comments extend to a good number of the papers presented in this volume. The first, and most fundamental, is the complete lack of recognition of the competitive environment in which a firm exist. For instance, it seems quite incon-

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gruous that a firm's R & D should depend on its own R & D four years lagged, yet not be made to depend on that of its major competitor's R & D in the recent year. Both the dictates of fashion and the availability of modeling apparatus have led to this neglect. It should be emphasized that the "fix-up" of adding dummy variables does accommodate this critique, through recentering the data, but the essential interaction among firms remains unaddressed. To illustrate the consequences of the interpretation of so-called multipliers extensively used in the vector autoregressive context (table 14.5; figs. 14.2–14.5), consider the following two simple, extreme cases:

$$(1) Y_i = \alpha + \beta X_i,$$

(2a)
$$Y_i = \alpha + \beta \left(X_i - \frac{\sum X_i}{N} \right)$$

(2b)
$$Y_i = \left[\alpha - \beta \left(\frac{\sum X_i}{N}\right)\right] + \beta X_i, \ i = 1, \dots N.$$

Think of N as the number of firms in the industry, X_i as the R & D of each firm, and y_i as profits. Models (1) and (2b) are observationally equivalent. In the case of model (1) a Δ increase in R & D results in a $\beta\Delta$ increase in profits. In model (2) a Δ increase in R & D only affects profits to the extent that other firms respond by altering their R & D. In the case where all firms respond equally, the multiplier will be exactly zero.

The second issue is that the goal of much recent research effort has been to explain the manner in which R & D effort affects the fortunes of a company, but it remains true that R & D represents only a small proportion of the operating budget of most firms. It is reasonable to question to what extent the R & D tale can wag the VAR. Further, concern about R & D often focuses on the essential uncertainty of the research venture, the implication having been drawn that many unsuccessful attempts must be made before a successful invention is identified. This notion does not fit well in the linear model employed here (equation [2] in Mairesse and Siu) and in other papers in this volume. The model presumes a marginalist-type relationship between the variables, that is, a little more R & D results in a little more sales or investment or profit. Since R & D is a small part of the operating budget and bears such uncertain fruits, it seems that those year-to-year relationships of R & D to other firm-level variables must be swamped by the consequences of wage settlements, cost of raw materials, strikes, advertising, and the response of competitors. Those identified links may more strongly reflect the "continuity" of operations of the firm than a causal-link chain of events.

^{1.} These data are not reported by Mairesse and Siu but seem essential for understanding relative magnitudes in the analysis.

The third comment draws heavily from what has been said above. The vector autoregressive framework for examining links between variables fails to recognize the explicit capacity of a firm to *think*. Why should a firm's investment depend on sales four periods lagged and on R & D four periods lagged? Perhaps there are adjustment costs; perhaps there is some information in this old data. However, how rich is this information in relation to other knowledge available to the firm? Corporate expenditure decisions must reflect, for example, how close the firm is running to full capacity, or what the relative prices of labor and capital and fuel might become in the near future. These factors determine, in a calculated fashion, the levels of R & D and investment and the relative mix in the current year and future years. In the vector autoregressive formulation, this "thinking" is reduced to a sad series of stochastic disturbances in the equation system.²

^{2.} Adjustment costs are a "thin" explanation of lagged R & D's ability to explain current R & D.