

This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: Education as an Industry

Volume Author/Editor: Joseph T. Froomkin, Dean T. Jamison and Roy Radner, eds.

Volume Publisher: NBER

Volume ISBN: 0-88410-476-1

Volume URL: <http://www.nber.org/books/jami76-1>

Publication Date: 1976

Chapter Title: Demand for Higher Education in the United States: A Second Progress Report

Chapter Author: Leonard S. Miller, Dean T. Jamison, Roy Radner

Chapter URL: <http://www.nber.org/chapters/c4494>

Chapter pages in book: (p. 291 - 348)

year of schooling
ability to learn in
rowing bananas
ut of bananas in
equent years, so
eyond that year.

put" effects will
Start Programs,
chievement) did
ars later most or
e was no longer
had been part of
roup of students
f these findings;
ding on the date

roduction func-
nt that empirical
old even before
op there. As this
is, I believe, on
estment process
(2) interactions
h be subject to
odels which ac-
goals.

attack for its alleged
lay 24, 1971, p. 52.
Efficiency and the
view 60 (May 1970):

PART THREE

Higher Education

LEONARD
MILL

University of Calif
Ber

This paper is
tion in the Un
Higher Educa
ture of High
estimating the
alternative typ

A student's
of mutually
secondary-sch
the possibilitie
for higher educ
of total enrollm
of institution.
alternative inst
previous studie
equation regre
dependency of

7

LEONARD S.
MILLER

University of California,
Berkeley

Demand for Higher Education in the United States: A Second Progress Report

This paper is the second report on demand estimates for higher education in the United States from our Project on Econometric Models of Higher Education, supported by the Carnegie Commission on the Future of Higher Education.¹ In this report I address the problem of estimating the joint probability distribution of freshman attendance at alternative types of higher education institutions.

A student's post-secondary-school alternatives are, by and large, a set of mutually exclusive possibilities. The choice of a "best" post-secondary-school alternative is dependent upon the characteristics of all the possibilities available to the chooser. Previous studies of the demand for higher education have found relationships describing the percentage of total enrollment and the percentage of enrollment at a particular type of institution. To account for the mutual interdependency between alternative institutional types, the costs of alternative choice types, these previous studies have incorporated as individual explainants in single equation regressions. In our study a more explicit account of the joint dependency of the post-secondary alternatives is made. The exploration

of how the distribution of attending alternative post-secondary-school choices is related to individual and institutional characteristics is reported here.

Stochastic choice theory underlies our demand estimates. A strict model establishes a relationship between utilities and probabilities. A specific form of the utility function, the exponential of a linear sum of variables describing the chooser and his or her choices, is assumed. This function, coupled with the constraints of the choices and the strict model, allows for maximum likelihood estimates of the parameters of the resulting probability distribution function. These estimates are called conditional logit estimates.

The estimated probability functions contain two variables. The first variable represents alternative estimates of the ratio of the cost of a particular post-secondary-school choice, to the student's family income. The second variable is an estimate of the student's SAT (Scholastic Aptitude Test) score multiplied by an estimate of the average freshman SAT score of the particular post-secondary-school choice, divided by 1,000. We call the first the "cost-to-income" variable, and the second the "student-institution achievement interaction" variable.

The original data source was a subsample of SCOPE (School to College: Opportunities for Post Secondary Education). Only a portion of the SCOPE observations had a reliable family-income estimate. The subsample was divided into two groups; one with a reliable income estimate, referred to as Sample II, and one without a reliable estimate, referred to as Sample I (II has more information than I). Sample II was biased; its students had higher achievement scores and higher student-reported family income. As an attempt to understand the biases that would eventually enter our final results we computed demand estimates for both samples.

An aggregation of the observations by state yielded poor results. The "cost-to-income" coefficient was negative and often significant, as anticipated. But the coefficient on the "student-institution achievement interaction variable" fluctuated considerably. Sample I seemed to yield better results than Sample II. Because of our sampling design, it was likely that Sample I was more homogeneous than Sample II. This result suggested that more homogeneous groups might yield better demand estimates.

State observations were pooled and four achievement groups were constructed in each sample. Two significant patterns emerged. The first was that the "cost-to-income" coefficients were significantly negative; and as achievement rose the value of the coefficient became less negative. The low-achievement group had a relatively elastic demand for higher education with respect to cost and income, and the high-

achievement
tion with re

The second
"student-inst
This trend w
achievement
achievement
had low ach
schools, whi
ginally attrac

Variations
additional in
expect, direc
dent's colleg
predicted pr
children, and
about the re
cause some

Section I
Section II pr
specifications
in Section I
which is ava
accompanied
Section V ex
about the ch
with his or h

SECTION I: R OF THE LITE

In 1967, the
higher educa
the first to
demand desc
 R_t (defined a
as resident c
 P_t (defined,
household an
index). Their
the proportio

secondary-school characteristics is re-
nates. A strict probabilities. A linear sum of assumed. This and the strict parameters of the rates are called
bles. The first the cost of a family income. SAT (Scholastic aptitude test) score, divided by the second
e.
(School to College) portion of the state. The subjective income estimate, Sample II was higher student-achievement estimates

or results. The significant, as anticipated, achievement income was yielded to yield
design, it was II. This result better demand

it groups were merged. The first significantly negative; same less negative demand for and the high-

achievement group to a significantly positive coefficient for the high-
tion with respect to cost and income.

The second pattern to emerge was that the coefficients on the
"student-institution achievement interaction variable" also had a trend.
This trend went from a significantly negative coefficient for the low-
achievement group to a significantly positive coefficient for the high-
achievement group. The middle groups were in between. Students who
had low achievement measures were marginally repelled from better
schools, while students who had high achievement measures were marginally
attracted to better schools.

Variations in the conditional logit estimates were introduced with
additional information about the high school seniors. As one would
expect, direct information such as high school curriculum and the stu-
dent's college plans cause the greatest alterations in the demand model's
predicted probabilities. The educational objectives of parents for their
children, and other measures related to the perceptions students have
about the relationship between education and the future labor market
cause some marginal shifts in the demand probability predictions.

Section I presents a review and critique of the previous studies.
Section II presents the model and a description of the estimated demand
specifications. We include a short description of the data and sampling
in Section III. A more complete discussion appears in an appendix
which is available from the author. A presentation of the results is
accompanied by a discussion of the research process in Section IV.
Section V extends the model and considers how additional information
about the chooser (student) alters the probability distribution associated
with his or her choices.

SECTION I: REVIEW AND CRITIQUE OF THE LITERATURE

In 1967, the process of finding empirical estimates for the demand for
higher education in the United States began. Campbell and Siegel were
the first to generate enough information to make the first empirical
demand description.² Their published result consists of a regression of
 R_t (defined as the proportion of people between age 18 and 24 enrolled
as resident degree undergraduates in 4-year institutions), on Y_{Ht} , and
 P_t (defined, respectively, as an estimate of real disposable income per
household and an index of tuition costs deflated by the consumer price
index). Their linear log specification yielded an elasticity of 1.2036 for
the proportion enrollment with respect to household disposable income,

and an elasticity of $-.4404$ for the proportion of enrollment with respect to tuition costs. Both elasticities were significantly different from zero at the .025 level, and 93 per cent of the variance was explained by the regression. Nine observations between 1927 and 1963 served as the estimation data in the Campbell-Siegel work.

One problem with this first study is that an identification problem probably exists in their analysis. Consequently, their demand estimates are probably biased. If equilibrium is assumed in the enrollment market, annual enrollment equals the annual supply of places. R_t , then, is also the probability that an enrollment place in higher education will be supplied to someone between 18 and 24 years old. Like demand, supply R_t could be related to Y_{Ht} and P_t . Due to the likelihood that any increase in the public sector's supply of places would be related to taxable per capita income, Y_{Ht} would be positively related to R_t . P_t could be positively related to R_t for the usual reason that more places will be offered as price rises.

On the other hand, an identification problem might not exist. Many of the most sought after 4-year institutions (those institutions where applicants exceed capacity) are not in a simple price equilibrium. Rather than expand institutional capacity, suppliers apply greater selectivity to applicants. To the extent that the supply schedule reflects this group of institutions, the price parameter may be absent. Here, the demand schedule is identified.

It is my feeling that the results themselves suggest some bias. Consider the income distribution associated with a unit increase of disposable per capita income in relation to the dependent variable in the Campbell-Siegel formulation. One portion of the income increase accrues to families whose children do not presently attend 4-year colleges, and the other portion accrues to families whose children are presently enrolled in 4-year colleges. None of the portion of the income increase accruing to the enrollees can logically increase enrollment. Some of the income may go to increased expenses, but that is not the dependent variable. Any 4-year college enrollment increase attributed to an income increase can only come from students enrolled in less than 4-year colleges, and from nonstudents. We expect, then, that the percentage of 4-year college enrollment with respect to disposable income elasticity will be less than one. The enrollment elasticity computed by Campbell and Siegel, however, was 1.20.

I am acquainted with two other econometric demand studies which attempt to determine income and price responses to enrollment. Both studies were the result of state-level attempts to impose rational planning on a state higher-education system. One study, by Stephen A. Hoernack, was done for the Office of Analytical Studies, at the University

of California.³ the Metropolitan Massachusetts, B

Since each were different school seniors nia (UC). Hoernack on 1965 Calif information of University of represent the campus. A section demand of U residence com campuses at function was e Irvine (UCI). the weighted Hoernack dete separate camp not presented.

In his study enrolled at spe high school, E tions of the co the cost to th junior college, income of fami graduating sen wage rate data equations incl time was valu

Alternative logarithmic, al not logged. Or brackets and o

To give a fl equations as a

$$\log \frac{A_{UCR+UCI}}{E}$$

of California.³ The other study was produced by a team of consultants for the Metropolitan Area Planning Council for the Commonwealth of Massachusetts, Board of Higher Education.⁴

Since each study had different objectives, their equation specifications were different. Hoenack's task was to determine the California high school seniors' demand for freshman places in the University of California (UC). Hoenack estimated two sets of demand specifications with data on 1965 California high school seniors. One set of specifications used information on Los Angeles area eligibles and their attendance at the University of California at Los Angeles (UCLA). This set was used to represent the demand function of all freshmen commuters for any UC campus. A second set of demand functions represented the enrollment demand of UC eligibles for UC campuses outside their high school residence communities. Separate equations were estimated for the UC campuses at Davis, Berkeley, and Santa Barbara, and an aggregate function was estimated for attendance at UCLA, Riverside (UCR), and Irvine (UCI). The aggregate demand for each campus was set equal to the weighted sum of the commuter plus noncommuter demand.⁵ Hoenack determined a UC aggregate demand function by summing the separate campus functions. The actual demand function achieved was not presented, however.

In his study the dependent variable was the percentage of freshmen enrolled at specific UC campuses out of the total UC eligibles from each high school, E . The independent variables were: (1) various combinations of the cost of attending alternative UC campuses, P (campus); (2) the cost to the high school senior of attending the nearest state and junior college, P_{SC} and P_{JC} respectively; (3) estimates of the median income of families in each high school's census area, Y ; (4) the number of graduating seniors from a high school, G ; and (5) unemployment and wage rate data on the high school's region, U and W . The commuter equations included an estimate of the daily commuting cost. Students' time was valued at \$2.40 per hour.

Alternative demand specifications were tried; they were usually linear logarithmic, although sometimes some of the independent variables were not logged. Ordinary least squares were performed on combined income brackets and on various combinations of income quartile disaggregations.

To give a flavor of Hoenack's results I include one of his estimated equations as an example.

$$\log \frac{A_{UCR+UCI+UCLA}}{E} = 19.11 + .002G + 1.992 \log P_{UCB} - 5.740 \log P_{UCLA} + 1.9866 \log P_{UCSB} + 1.5473 \log P_{SC}$$

(9.21)	(1.86)	(3.48)	(-3.44)
	(3.20)	(2.32)	

$$\begin{aligned}
& + .98 \log P_{JC} + .69 \log Y \\
& \quad (1.70) \quad (1.81) \\
& + .05 \log U - 2.45 \log W \\
& \quad (.20) \quad (-3.07) \\
& - .000126 (P_{UCB} + P_{UCSB}) \log Y \\
& \quad (-3.28) \\
& + .000346 P_{UCLA} \log Y \\
& \quad (3.43) \\
& - .00067 P_{SC} \log Y - .00007 P_{JC} \log Y \\
& \quad (-2.27) \quad (-1.77)
\end{aligned}$$

$R^2 = .99$; $DW = 2.10$; all income brackets; 90 observations; t values in parentheses.⁶

Hoенack finds a demand elasticity of $-.85$ for the percentage of enrollment with respect to price for UC campuses. This value varies from -1.12 for the lowest income bracket to $-.71$ for the highest. Hoенack reports that state colleges appear to be a close substitute for UC campuses. If state college prices rise simultaneously with UC price rises, but with increases only two-thirds of the UC price increase, the students' price elasticity for UC diminishes substantially. Average UC price elasticity values fall from $-.85$ to $-.51$; the lowest income group's values decrease from -1.12 to $-.68$, and the highest income group's values decrease from $-.71$ to $-.48$.

The decision to commute or to live away from home was apparently significantly affected by the wage rates prevailing in the hometown area. They were not affected by the unemployment rate.⁷

Income elasticities are not reported. However, the elasticity implied by the sample equation above is approximately $.7$.

Hoенack made a limited attempt at extending his demand formulation to the state and junior colleges in California. No estimates are actually presented, but Hoенack claims to have been unsuccessful at explaining attendance at the state college geographically nearest to each high school, and successful at explaining the proportion of high school seniors attending their nearest junior college. The junior college tuition-enrollment result given is that a \$100 increase in junior college tuition would diminish enrollment by approximately 7 per cent.

The Massachusetts-Metropolitan Area Planning Council study on higher education took as its task the development of a coordinated and comprehensive public policy for higher education in the state. They saw the question of attending versus not attending any higher education institution as their principal problem. The dependent variable in this study was the percentage of 1960 tenth-grade high school students who attended college in 1963.

The following independent variables were used: (1) junior college tuition, P_j ; (2) public 4-year university tuition, P_u ; (3) tuition at teachers' college, P_c ; (4) private 4-year university tuition, P_p ; (5) father's educa-

tion, which was income of parents as measured by employment.

The Massachusetts study shows that the percentage of students attending the highest tier of colleges is .28. Their results are

In the equation only the coefficient of the father's education is significant with respect to public university enrollment. Percentage of students attending colleges as it is

The unemployment rate was .08. Hoенack.

The authors of the study provide information on the father's education. The elasticity is .28.

When the sample is divided into males and females, the elasticity is almost twice as high for males (sex highlights the difference). Both results are consistent with the idea that women receive more education than men. This suggests that women attend college for more years than men.

Computed Elasticities with Respect to

	Junior
Males	
Females	

tion, which was supposed to proxy for family income, E ; (6) average income of production workers, a proxy for opportunity cost, Y ; (7) ability as measured by performance on achievement tests, A ; and (8) unemployment, U .

The Massachusetts study presented regressions for percentage enrollees; percentage enrollees by sex; and because of possible nonlinearities, percentage of enrollees by quartiles of father's education. From lowest to highest these quartiles are denoted by $LSES$, $2SES$, $3SES$, $HSES$. Their results are reported in the table on page 300.⁸

In the equation explaining the aggregate enrollment percentage level only the coefficient on teachers' college tuition is not significantly different from zero. The computed percentage enrollment elasticities with respect to price are: $-.09$ for junior colleges, $-.18$ for 4-year state public universities, and $-.19$ for 4-year private universities in the state. Percentage of enrollment appears to be twice as price inelastic for junior colleges as it is for public universities or for 4-year private institutions.

The unemployment coefficient was significant in this equation and the wage rate was not. This result is just the opposite of that found by Hoenack.

The authors of the Massachusetts study caution against a direct use of the father's education to compute an income elasticity. However, the reader is provided with almost all the figures to do so, and the temptation was simply too great. Assuming the mean family income of the college-age population as ten thousand dollars, the implied income elasticity is $.28$.

When the sample was stratified by sex, we find males showing almost twice the marginal price responsiveness towards public universities as females. Similarly, the marginal responsiveness to ability is almost twice as high for men as for women. Disaggregating by sex highlights the results of institutional sexism (discrimination according to sex). Both results, price responsiveness and ability responsiveness, are consistent with the finding that males have higher rates of return on education than do females. The price result suggests that men attend college for more economic reasons than women, while the ability result suggests that women do not behave as if their abilities will be valued as much as men's abilities will be valued.

Computed Elasticity of Total Percentage Enrollment with Respect to Price

	Junior College	Public University	Private 4-Year College
Males	$-.096$	$-.27$	$-.23$
Females	$-.081$	$-.11$	not significant

Regression Results

Per Cent Enrolled	Constant	P_j	P_u	P_c	P_p	E	Y	A	U	R^2
E_T	14.431	-.0111 (-3.14)	-.0265 (-2.32)	.0081 (1.26)	-.0087 (-2.06)	2.839 (4.21)	-3.622 (-1.03)	.176 (5.72)	.834 (2.04)	.769
E_{TM}	13.993	-.0132 (-3.42)	-.0438 (-2.39)	.0009 (1.04)	-.0116 (2.17)	3.265 (4.87)	-2.066 (-1.81)	.208 (5.22)	1.553 (2.16)	.806
E_{TF}	5.094	-.0093 (-2.96)	-.0153 (-3.60)	-.0231 (-1.37)	.0106 (.87)	2.576 (2.87)	-1.041 (-1.19)	.131 (3.67)	.427 (1.41)	.721
E_{LSES}	1.573	-.0072 (-2.06)	-.0125 (-1.71)	-.0059 (1.04)	.0018 (.57)	1.762 (2.52)	-1.347 (2.29)	.041 (1.56)	.776 (1.34)	.537
E_{2SES}	4.156	-.0093 (-1.49)	-.0137 (-1.81)	-.0122 (.96)	-.0014 (-1.09)	.853 (2.17)	-1.590 (1.08)	.053 (2.02)	.316 (1.22)	.412
E_{3SES}	5.912	-.0051 (-1.19)	-.0097 (-1.91)	-.0017 (-1.36)	-.0045 (-1.74)	.907 (2.08)	-1.003 (.96)	.069 (2.12)	-.315 (-1.58)	.450
E_{HSES}	5.590	.0006 (.57)	-.0126 (1.68)	.0055 (.86)	-.0041 (2.09)	.267 (1.21)	.668 (.54)	.045 (2.23)	-.065 (-1.89)	.491

NOTE: t statistics are in parentheses.

When the M proxy for social significant. For the lowest SES the highest SES coefficients. For quartiles, the group, unemployment affects the up

Uncertainty that the model of this income required special some difficulty 4-year-enrollment cause the speed way.

Many differences like to have key changes in the enrollment in offered by Ho Campbell-Siege tion problems, fixed supply. A higher education tive institution studies reported joint dependence. The reported 3 college prices demand decision

Individual educational possibilities can are perhaps are jointly dependent sections to follow

Before proceeding to market

When the Massachusetts sample was stratified by father's education, a proxy for socioeconomic status (SES), most of the variables stop being significant. For example, only the junior college price is significant for the lowest SES group, and only the private college price is significant for the highest SES group. The two middle groups have no significant price coefficients. Father's education (SES) is significant for the lower three quartiles, the wage rate is negatively significant for the lowest SES group, unemployment is never significant, and ability rather uniformly affects the upper three groups.

Uncertainty about the Campbell-Siegel estimates arises out of the fact that the model used to describe their problem is incomplete. One aspect of this incompleteness is the question of whether the supply function required specification. In addition, if the reader will recall, there was some difficulty interpreting the per capita income coefficient, given the 4-year-enrollment dependent variable. This latter difficulty arose because the specification of the problem was incomplete in yet another way.

Many different types of higher education institutions exist. One would like to have known how additions to per capita income, or how the changes in the cost of attending a specific type of institution affected enrollment in all higher educational types. To some extent the models offered by Hoenack and the Massachusetts study improved on the Campbell-Siegel effort. Neither of the former two studies had identification problems, for both used cross-sectional data and assumed short-run fixed supply. And, both attempted to get at the existence of alternative higher education institution types by incorporating the costs of alternative institutions as independent explainants. However, none of the studies reported address themselves to the problem of explaining the joint dependency of the demand for these different institutional types. The reported 35 per cent drop in UC's own price elasticity when state college prices change dramatically illustrates the interdependency of demand decisions concerning higher education.

Individual choices are made within a framework of jointly dependent post-secondary-school alternatives. These include the alternative institutional possibilities and the choice of not going on to college. These possibilities can be described by attributes; items like cost and quality are perhaps among some of the more important of these attributes. This jointly dependent decision-making framework will be developed in the sections to follow.

Before proceeding, however, one further note is necessary. In addition to market forces, nonmarket forces affect higher education institu-

NOTE: *t* statistics are in parentheses.

tional choice, and some comment on this matter is called for. Galper and Dunn studied the short-run nonmarket effect of the rate of growth of the armed services on the Campbell-Siegel undergraduate enrollment series.⁹ Their contribution to the demand estimation problem is an estimate of the effect the armed forces play on 4-year college enrollment. They estimated the elasticity of enrollment with respect to the annual rate of growth of the armed services at $-.26$, and their enrollment elasticity with respect to discharges was $.13$. One should add that their income elasticity estimate was $.69$, a value consistent with our previous argument, and similar to that found in Hoenack's example.

After estimating the probability distribution of attending alternative types of schools, principally as a function of economic and achievement variables, our attention is addressed to more noneconomic aspects of the higher education choice decision. Specifically, we compute how our estimates of the probability distribution are altered by additional information about the chooser. This analysis appears in the final section.

SECTION II: THE MODEL

Each graduating high school senior faces a supply of higher education institutions to which he or she is eligible. We shall call this institutional collection, along with the alternative of not going on to higher education, the individual's feasible choice set. Individuals have a multiple-choice problem. They choose one and only one of these separate alternatives from their feasible choice set. We are dealing, therefore, with a demand more like that for houses or automobiles than for butter or beer, in the sense that the choice is among a small number of discrete alternatives rather than among different quantities of a divisible good. Estimating the freshman demand for higher education becomes a problem of specifying and estimating this choice process.

Choosers and their choice sets will be described with personal and institutional variables, respectively. We will assume that the objective function will be the same for the entire population; only the variables describing the people or their alternatives will alter any individual's selection of "best." However, our descriptions are rather simplistic. And, it will often be the case that similarly described people choosing from similarly described feasible choice sets differ in their assessment of the best alternative.

The usual theory of choice would infer in this situation that the chooser or choosers are indifferent among the choices; they perceive the same subjective benefit from them. This is so even when nine out of

every ten st
another. An
recourse does
scribed studen
not infer that
tives.

The deman
stochastic cho
accounts for t
secondary-sch
maximization
freshman dem
cludes a brief
and some dis

Stochastic Cho

Let A be a fin
the feasible c
either dealing
are dealing w
the utility der
choice k , wh
chosen nine o
likelihood, w
tween the tw
theory; they c
and the proba
by assuming
elements J of

- (1) $\text{Prob}(j|e_i) =$
(2) $\frac{\sum_{j \in J_i} \text{Prob}(j|J_i)}{J_i}$

The simple
when the utili
to choose j ov
over A such t
weak utility fe

The stochas
imposed more
is imposed by
defined as foll
over the feasi

led for. Galper
 rate of growth
 graduate enroll-
 n problem is an
 college enroll-
 respect to the
 nd their enroll-
 should add that
 istent with our
 ck's example.
 ding alternative
 nd achievement
 ic aspects of the
 npute how our
 dditional infor-
 final section.

higher education
 this institutional
 p higher educa-
 ave a multiple-
 eparate alterna-
 erefore, with a
 : butter or beer,
 ber of discrete
 divisible good.
 becomes a prob-

th personal and
 at the objective
 ly the variables
 any individual's
 ither simplistic.
 eople choosing
 ir assessment of

uation that the
 ey perceive the
 hen nine out of

every ten students choose one alternative and the tenth chooses another. An appeal to data insufficiencies might be made, but such recourse does not solve the underlying logical problem: similarly described students may not choose the same best choice, and we should not infer that students are indifferent between all the chosen alternatives.

The demand functions reported on here are based on a model of *stochastic choice theory*. The logical framework of this theory explicitly accounts for this indifference problem. The selection of the best post-secondary-school alternatives will be viewed as a stochastic utility-maximization problem. In this section the underlying theory for the freshman demand functions is presented. In addition, the section includes a brief account of the specifications, the estimation technique, and some discussion of the expected signs of the parameters.

Stochastic Choice Theory

Let A be a finite set of discrete post-secondary alternatives, and let J_i be the feasible choice set for individual i , $J_i \subset A$. Imagine that we are either dealing with one chooser who makes repeated choices, or that we are dealing with many choosers assumed to behave alike. It is likely that the utility derived from choice j is greater than the utility derived from choice k , when both j and k are in the feasible choice set, and j is chosen nine out of ten times over k . Stochastic choice theory asserts this likelihood, while absolute choice theory establishes indifference between the two choices.¹⁰ There are many models of stochastic choice theory; they differ according to the specificity of the ordering relation and the probabilistic assumptions between the choices. All models begin by assuming that the basic axioms of probability theory apply to the elements J of the feasible choice set for individual i , J_i , where $J_i \subset A$.

$$(1) \quad \text{Prob}(j|j \in J_i) \geq 0$$

$$(2) \quad \sum_{j \in J_i} \text{Prob}(j|J_i) = 1$$

The simplest stochastic theory, a weak stochastic model, asserts that when the utility of j is greater than the utility of k , there is a tendency to choose j over k . Formally, there exists a constant real-valued vector U over A such that: $U(j) \geq U(k)$ IFF $\text{Prob}(j\{j,k\}) \geq 1/2$. U is called a weak utility function.¹¹

The stochastic model actually used to achieve our demand equations imposed more structure on the choice set and the ordering relation than is imposed by the weak model. A *strict utility model* was used, which is defined as follows: There exists a constant positive real-valued vector U over the feasible set J , $J \subset A$, such that for any $j, k \in J$

$$(3) \quad U(j)/U(k) = \text{Prob}(j|J)/\text{Prob}(k|J)$$

Block and Marschak have shown that a strict model implies, but is not implied by, a weak model.¹²

From equation 2 the probability of individual i choosing institutional type j is expressed as

$$(4) \quad \text{Prob}(j|J_i) = \text{Prob}(j|j \in J_i) / \sum_{k \in J_i} \text{Prob}(k)$$

The strict utility model states that utilities are proportionate to probabilities. The probability that individual i chooses choice type j , ϕ_{ij} , can be rewritten as:

$$(5) \quad \phi_{ij} = \text{Prob}(j|J_i) = U_i(j) / \sum_k U_i(k)$$

A specific form for the utility function $U_i(j)$ allows one to relate the descriptions of the student choosers and their alternative choices to utility and relative frequency. Estimation of the parameters of the model are based on a maximum likelihood process.

Let Z_{ij}^h and β^h represent the h th variable and h th constant, respectively, in a linear sum

$$(6) \quad f_{ij} = \sum_h \beta^h Z_{ij}^h$$

It is assumed that the utility or subjective benefit to individual i from institutional type j is given by the explicit equation

$$(7) \quad U_i(j) = F_{ij} = \exp(f_{ij})$$

The probability of i choosing j , equation 5, can be expressed as

$$(8) \quad \phi_{ij} = \frac{F_{ij}}{\sum_{k \in J_i} F_{ik}} = \frac{\exp(\sum_h \beta^h Z_{ij}^h)}{\sum_k \exp(\sum_h \beta^h Z_{ik}^h)}$$

For n individuals, $i = 1, 2, \dots, n$, let

$$X_{ij} = \begin{cases} 1 \\ 0 \end{cases} \text{ according as } i \begin{cases} \text{has} \\ \text{has not} \end{cases} \text{ chosen } j$$

The likelihood function, $L(\beta^h)$ is given by

$$(9) \quad L(\beta^h) = \prod_i \prod_{j \in J_i} \phi_{ij}^{X_{ij}}$$

The maximum likelihood estimates, b^h of β^h , are those values of b^h that maximize $L(\beta^h)$. McFadden has produced a computer program to estimate these parameters and has explored the properties of these estimates.¹³

The availability of data, and the results of experiments with different formulations, led us to concentrate on the following variables whose precise definitions can be found in the appendix available from the author:

A_i = an act
 Y_i = a meas
 S_j = a mea
 j_i and
 C_{ij} = the ou
 alterna

We define variables.

$$Z_{ij}^h = C_{ij}/Y_i$$

and, for the c
 the names "co
 teraction" res

The greater
 alternative. Th
 anticipated as

For any give
 higher is the
 variable. The
 the form of gr
 higher rate of
 a better school
 an increased p
 net subjective
 sign of the co
 to be positive
 exceeding the
 interaction var

Discussion o
 elasticity term
 joint probabili
 individual's fea
 alternative j , is

$$\phi_{ij} = \exp(\beta^h Z_{ij}^h)$$

Differentiat
 ment, A_i , an
 student's famil
 to these variab

$$\frac{\partial \phi_{ij}}{\partial A_i} = \beta^h \frac{\partial}{\partial A_i} \exp(\beta^h Z_{ij}^h)$$

$$\frac{\partial \phi_{ij}}{\partial S_j} = \beta^h \frac{\partial}{\partial S_j} \exp(\beta^h Z_{ij}^h)$$

A_i = an achievement score for student i (an estimate of i 's SAT score);
 Y_i = a measure of family income for student i ;
 S_j = a measure of the "academic selectivity" or "quality" of alternative j ; and
 C_{ij} = the out-of-pocket dollar cost to i of going to j (set equal to zero for the alternative "no school").

We define Z_{ij}^1 and Z_{ij}^2 , in equation 6 above, in terms of these variables.

$$Z_{ij}^1 = C_{ij}/Y_i, \text{ and } Z_{ij}^2 = A_i S_j / 1000$$

and, for the convenience of discussion, we have given these variables the names "cost-to-income" and "student-institutional achievement interaction" respectively.

The greater the "cost-to-income," the less the individual wants the alternative. Thus, the sign of the estimate b^1 for β^1 is unambiguously anticipated as negative.

For any given individual, the better the quality of the institution the higher is the value of the "student-institution achievement interaction" variable. The perceived benefits of a better quality school can occur in the form of greater intellectual and social amenities, or in the form of a higher rate of monetary return upon completion. The perceived costs of a better school can occur in the form of either increased competition, or an increased probability of failure. If an individual interprets his or her net subjective benefits from an alternative as positive, we expect the sign of the coefficient on "student-institution achievement interaction" to be positive. Alternatively, if the individual interprets the costs as exceeding the benefits, we expect the sign of the coefficient on the interaction variable to be negative.

Discussion of the conditional logit estimates focuses on marginal and elasticity terms. What follows is a derivation of these terms from the joint probability distribution function of the demand model. Given an individual's feasible choice set J_i , the probability of individual i choosing alternative j , is given by equation 8.

$$\phi_{ij} = \exp(\beta^1 Z_{ij}^1 + \beta^2 Z_{ij}^2) / \sum_{k \in J_i} \exp(\beta^1 Z_{ik}^1 + \beta^2 Z_{ik}^2)$$

Differentiation of equation 8 with respect to an individual's achievement, A_i , an institution's quality, S_j , an institution's cost, C_{ij} , or the student's family income, Y_i , yields the marginal responses with respect to these variables.

$$\frac{\partial \phi_{ij}}{\partial A_i} = \beta^2 \frac{\phi_{ij}}{1,000} \left[S_j - \sum_{k \in J_i} S_k \phi_{ik} \right]$$

$$\frac{\partial \phi_{ij}}{\partial S_j} = \beta^2 \frac{A_i}{1,000} \phi_{ij} (1 - \phi_{ij})$$

$$\frac{\partial \phi_U}{\partial C_U} = \beta^1 \frac{1}{Y_i} \phi_U (1 - \phi_U)$$

$$\frac{\partial \phi_U}{\partial Y_i} = \frac{-\beta^1}{Y_i^2} \phi_U [C_U - \sum_{k \in J_i} C_{ik} \phi_{ik}]$$

Denoting $\eta_{\phi_U, W}$ as the elasticity of the probability of individual i choosing choice j with respect to the variable W , the elasticities for the four marginals derived above are

$$\eta_{\phi_U, A_i} = \frac{\beta^2 A_i}{1,000} [S_j - \sum_{k \in J_i} S_k \phi_{ik}]$$

$$\eta_{\phi_U, Y_i} = \frac{-\beta^1}{Y_i} [C_U - \sum_{k \in J_i} C_{ik} \phi_{ik}]$$

$$\eta_{\phi_U, C_U} = \beta^1 \frac{C_U}{Y_i} (1 - \phi_U)$$

$$\eta_{\phi_U, S_i} = \beta^2 \frac{A_i S_j}{1,000} (1 - \phi_U)$$

$$\eta_{\phi_{ik}, C_{ij}} = -\beta^1 \frac{C_U}{Y_i} \phi_{ik}$$

$$\eta_{\phi_{ik}, S_j} = -\beta^2 \frac{A_i S_j}{1,000} \phi_U$$

Two cross elasticities are of particular interest. One indicates the percentage change in ϕ_U caused by a 1 per cent change in the cost of choice k , $k \in J_i$. The other indicates the percentage change in ϕ_U caused by a 1 per cent change in the quality of choice k , $k \in J_i$. The appropriate marginals are obtained by differentiating equation 8

$$\frac{\partial \phi_U}{\partial S_k} = -\beta^2 \frac{A_i}{1,000} \phi_U \phi_{ik}$$

$$\frac{\partial \phi_U}{\partial C_{ik}} = -\beta^1 \frac{1}{Y_i} \phi_U \phi_{ik}$$

Their elasticities are given by

$$\eta_{\phi_U, S_k} = -\beta^2 \frac{A_i S_k}{1,000} \phi_{ik}$$

and

$$\eta_{\phi_{ik}, C_{ij}} = \frac{\partial \phi_{ik}}{\partial C_U} \frac{C_U}{\phi_{ik}} = -\beta^1 \frac{C_U}{Y_i} \phi_U$$

Hoenack found that the elasticity of the percentage of University of California enrollees with respect to University of California cost was lower for low-income groups than for high-income groups. Since the formula for the enrollment elasticity with respect to cost contains income

in the denominator
Hoenack's findi

SECTION III: DA

Estimating the
ous demand stu
ble choice sets
actual post-sec
seemed to hav
feasible choice
were Project T
for Post Second
Analysis of P
income were hi
dance. Although
nonreporting b
the residency l
higher education
feasible choice
decided to prod

SCOPE is an
Research and D
nia, Berkeley. I
1966 high schoo
Carolina.

In addition t
alternative chose
was an achiever
income. Further
mail questionna
naires never pr

Of the famil
4,434 in the sa
scores with the
showed that the
achievement. T
by the students

These were
income-reportin
dure. The relat

in the denominator, the conditional logit results will always reproduce Hoenack's finding.

SECTION III: DATA AND SAMPLING

Estimating the proposed model required data unlike any used in previous demand studies. To my knowledge, no information about the feasible choice sets of a population existed. Two data sources contained the actual post-secondary choice selected by high school seniors, and seemed to have sufficient information to estimate the model if the feasible choice sets for observations were constructed. These sources were Project TALENT, and SCOPE (School to College: Opportunities for Post Secondary Education).

Analysis of Project TALENT data had shown that ability and family income were highly correlated, and also correlated with college attendance. Although Project TALENT probably had a better analysis of the nonreporting biases in their follow-up studies, they would not release the residency location of their subjects. Knowing the locally available higher education options was one of the determinants of an individual's feasible choice set. Since SCOPE was willing to release these data, I decided to proceed with the SCOPE information.

SCOPE is an ongoing longitudinal study, conducted at the Center for Research and Development in Higher Education, University of California, Berkeley. I was provided with a subsample, 4,434 observations of 1966 high school seniors in California, Illinois, Massachusetts, and North Carolina.

In addition to high school residency and the post-secondary-school alternative chosen, the relevant information on each SCOPE observation was an achievement measure and the observation's estimate of family income. Further income information had been obtained with a follow-up mail questionnaire to parents in 1967. Unfortunately, mailed questionnaires never produce unbiased samples.

Of the families queried, 31.6 per cent responded (1,402 out of the 4,434 in the sample). A comparison of the students' achievement test scores with their high school answers to the family-income question showed that the sample of youngsters with parent responders had higher achievement. They were also from higher-income families (determined by the students' twelfth-grade self-reporting of family income).

These were significant biases. We could not adequately take the income-reporting bias into account with a corrective sampling procedure. The relationship between parent-reported income and student-

reported income was not strong enough. In addition, other differences between the parent reporters and nonreporters undoubtedly existed. In an attempt to understand the biases which would appear in our final results, we decided to make demand estimates for both the reporter and the nonreporter populations. An income predictor for the nonreporters was required for the demand specification.¹⁴ In addition, we decided to sample from both populations in a way that would reasonably assure coverage of the full range of student achievement and students' family income.¹⁵

Demand samples were drawn in the following manner:

1. The reporter population in each state was allocated into cells determined by quartiles of achievement and six income ranges. The entire achievement-income range was covered by sampling four observations from each cell wherever possible. This sample will be referred to as Sample II.
2. The nonreporter population in each state was allocated into cells determined by quartiles of achievement and six predicted income ranges. Income was then predicted for the Sample II observations and the rather uniformly distributed achievement-income sample was allocated into achievement-predicted-income cells. Sample II's new cell frequency count determined the number of nonreporters that were drawn from the nonreporter achievement-predicted-income distributed population. This sample is referred to as Sample I (I has less information than II).

SECTION IV: RESULTS

Introduction

Results are presented within a twofold context. First, I want to convey what I have learned about higher education demand functions to date. Second, I should like to convey to the reader some of the process Roy Radner and I have gone through to obtain these results. The inclusion of a discussion of the research process may aid readers in interpreting the results, and may also be of value to future investigators.

Our first demand estimates were derived from individuals aggregated on the basis of the state in which they attended high school. We began with this aggregation because: (1) we processed the individual observations on a state basis (and we were anxious to see if this estimation process would work), and, (2) we believed that the demand functions in

the different states was a kind of r
"Demand create
differences betw
the four states,
different higher-

In general, the
The coefficient o
and often signific
ment interaction
and was signific
emerged. The m
ably yielding the
disaggregate on
homogeneous gr
the achievement
distribution in th
tions did not all
were never able
between states.
ranges for each g
mate homogene

Under the
emerged. The co
contained a tren
tivities with resp
low-achievement
students. The s
also had a trend
the low-achieve
high-achievement
the middle-achie
higher-achieving
achieving studen
history of relativ
away from a cou
had.

Alternative Spec

As I have prev
variables. One
income" variabl

Other differences
tedly existed. In
ear in our final
the reporter and
he nonreporters
a, we decided to
reasonably assure
students' family

her:
cated into cells
income ranges.
red by sampling
ble. This sample

located into cells
redicted income
e II observations
t-income sample
ie cells. Sample
number of non-
r achievement-
mple is referred

want to convey
unctions to date.
the process Roy
The inclusion of
interpreting the

duals aggregated
chool. We began
ividual observa-
f this estimation
and functions in

the different states might actually be different. The basis for this belief was a kind of reverse of Say's law. Here, the argument concludes, "Demand creates its own supply." Since there have been historical differences between the extent of the public higher-education effort in the four states, this very well might be evidence for the existence of different higher-education demand functions in the separate states.

In general, these state aggregation specifications were not satisfactory. The coefficient on the cost-to-income ratio was almost always negative and often significant; the coefficient on the "student-institution achievement interaction variable" was sometimes negative, sometimes positive, and was significant about as often as it was not. No definite pattern emerged. The more homogeneous income sample, Sample I, was probably yielding the best results. We decided to pool state observations and disaggregate on the basis of achievement. This produced more homogeneous groups and allowed us to experiment with the idea that the achievement interaction might vary according to the achievement distribution in the student population. The small number of our observations did not allow us to maintain the state level disaggregation, so we were never able to test successfully whether demand functions varied between states. Four averaged SAT groupings were constructed. The ranges for each group were based on an impression acquired of approximate homogeneous decision-making groups.

Under the achievement aggregation, two significant patterns emerged. The cost-to-income coefficients were significantly negative and contained a trend over the achievement groups. The enrollment elasticities with respect to cost and income ranged from elastic values for low-achievement students to inelastic values for the high-achievement students. The student-institution achievement interaction coefficients also had a trend over the achievement groups. The marginal effects for the low-achievement students were negative, the marginal effects for the high-achievement students were positive, and the marginal effects for the middle-achievement groups were in between. It appears that the higher-achieving students are attracted to better schools and the lower-achieving students are repelled from these schools. Apparently their history of relative academic failure is such as to marginally shift them away from a continuation of experiences like those they have already had.

Alternative Specifications

As I have previously stated, the demand specifications contained two variables. One variable, which for convenience we named the "cost-to-income" variable, measured the ratio of institutional cost estimates to

student family-income estimates. The other variable, named the student-institution achievement interaction variable, measured the product of the student's achievement, A_i , and our estimate of the average SAT score of the freshman class at the choice under consideration, S_j , divided by 1,000 (to convert the units back into the SAT range).

For Sample II, we had four income estimates: (1) reported income, Y ; (2) predicted income, YH ; (3) an estimate of disposable reported income, DY ; and (4) an estimate of disposable predicted income, DYH . For Sample I, the reported income estimates, (1) and (3), were not available. A rather complete discussion of these variables, and others introduced below, appears in the appendix available from the author.

Two cost estimates were available for each group; the out-of-pocket costs, C , and the C value with an opportunity cost estimated at \$3,000 added to each choice, except the choice of not going to a higher-education institution. This cost was represented by the symbol CO . Combining the two cost estimates with the four possible income estimates produced eight possible cost-to-income ratios for Sample II. Combining the two possible cost estimates with the two possible income estimates produced four possible cost-to-income ratios for Sample I.

Detailed State Aggregation Results

Table 1 contains the results of estimating the demand function parameters with one variable in the conditional logit estimation function. Each variable previously discussed was estimated separately. A brief glance at Table 1 shows:

1. The C/Y coefficients always yield negative significant results. California and Illinois estimates are about the same, North Carolina's estimates are half as large as California's and Illinois's, and Massachusetts's is two-thirds of North Carolina's.
2. The C/YH coefficients again yield negative significant results (except in the case of the Massachusetts Sample II), but (a) the estimates are about 75 per cent larger than the C/Y estimates for the Sample II groups, and (b) the comparisons of Sample I and Sample II range from fairly similar estimates for California to quite different estimates for North Carolina.
3. The disposable income and predicted income specifications, C/DY and C/DYH , do not seem to contribute much understanding. These estimates are not significant in half of the states.
4. Adding the opportunity cost to the out-of-pocket cost seems to reproduce the results found in the four specifications already commented on. There are, of course, differences in the values of the coefficient estimates.

TABLE 1 Estimates of Coefficients on Individual Variables

Sample	CAL I	CAL II	ILL I	ILL II	MASS I	MASS II	NC I	NC II
--------	-------	--------	-------	--------	--------	---------	------	-------

e, named the
 sured the prod-
 of the average
 nsideration, S_j ,
 T range).

rted income, Y ;
 eported income,
 DYH. For Sam-
 not available. A
 hners introduced
 r.

he out-of-pocket
 mated at \$3,000
 igher-education
 Combining the
 produced eight
 he two possible
 produced four

tion parameters
 tion. Each vari-
 brief glance at

nificant results.
 e, North Caroli-
 d Illinois's, and

ant results (ex-
 II), but (a) the
 Y estimates for
 f Sample I and
 ilifornia to quite

ifications, C/DY
 rstanding. These

cost seems to
 ns already com-
 e values of the

TABLE 1 Estimates of Coefficients on Individual Variables

Sample (Observations)	CAL I		CAL II		ILL I		ILL II		MASS I		MASS II		NC I		NC II	
	96	96	96	96	91	91	90	90	91	91	94	94	91	91	95	95
CY		-4.000 (.934)					-4.584 (.906)					-1.405 (.613)				-2.231 (.657)
CYH	-5.312 (1.126)		-7.501 (1.354)		-11.273 (1.544)		-8.185 (1.267)		-7.600 (1.374)		-8.74 (.775)		-15.418 (2.048)		-3.644 (1.043)	
CIDY			-231 · 10 ⁻¹ (.904 · 10 ⁻²)				-870 · 10 ⁻² (.515 · 10 ⁻²)				-164 · 10 ⁻¹ (.800 · 10 ⁻²)				-143 · 10 ⁻¹ (.567 · 10 ⁻²)	
CIDYH			-201 (.805 · 10 ⁻¹)		-1.166 (1.167)		-688 (.119)		-855 (.149)		+420 · 10 ⁻² (.899 · 10 ⁻²)		-1.559 (.220)		-215 · 10 ⁻¹ (.165 · 10 ⁻¹)	
CON			-402 (.162)				-1.245 (.303)				-605 (.274)				-578 (.199)	
CONH			-905 (.419)		-4.997 (.536)		-2.669 (.455)		-4.306 (.520)		-990 (.443)		-5.488 (.563)		-893 (.464)	
COIDY			-422 · 10 ⁻² (.200 · 10 ⁻²)				-371 · 10 ⁻² (.207 · 10 ⁻²)				-631 · 10 ⁻² (.267 · 10 ⁻²)				-420 · 10 ⁻² (.175 · 10 ⁻²)	
COIDYH			-385 · 10 ⁻² (.463 · 10 ⁻²)		-488 (.571 · 10 ⁻¹)		-181 (.384 · 10 ⁻¹)		-432 (.558 · 10 ⁻¹)		+342 · 10 ⁻³ (.456 · 10 ⁻²)		-518 (.616 · 10 ⁻¹)		-506 · 10 ⁻² (.359 · 10 ⁻²)	
Achievement interaction			-299 · 10 ⁻¹ (.445 · 10 ⁻²)		-175 · 10 ⁻¹ (.402 · 10 ⁻²)		-0.133 · 10 ⁻¹ (.370 · 10 ⁻²)		-252 · 10 ⁻² (.418 · 10 ⁻²)		+397 · 10 ⁻² (.396 · 10 ⁻²)		-309 · 10 ⁻¹ (.527 · 10 ⁻²)		-100 · 10 ⁻¹ (.393 · 10 ⁻²)	

NOTE: Figures in parentheses are standard errors of the coefficient estimates.

5. The student-institution achievement interaction coefficient estimate appears to be negative and significant in all cases except Massachusetts Sample II again. Frankly, we had not anticipated this result.

These five points tentatively suggested that most of our understanding will be derived from the out-of-pocket cost, the reported income, and the predicted income variables. The additional work that went into creating the other variables produced virtually no payoff. Furthermore, we are rather skeptical of single variable specifications.

Results of estimating the specifications with both variables included for Sample II and Sample I, by state, are presented in Table 2. The Sample II results are quite diverse. If one variable is significant in one specification, the other variable, more often than not, is not significant. The variable which is significant in any specification also varies between states. Sample II's estimates had only three specifications in which both variables were significant, the *C/DY* specification in California, the *C/YH* specification in Illinois, and the *C/Y* specification in Massachusetts. In these three cases, the cost-to-income coefficient was always negative, but the achievement interaction coefficient was negative in California and positive in the other two states. To repeat, not much consistency.

The Sample I estimates are less diverse than the Sample II estimates. In California, all the student-institution coefficients were negative and significant, and no cost-to-income coefficient was significant. For North Carolina we have an apparently opposite result: no student-institution coefficient was significant, while all the cost-to-income coefficients were negative and significant. The Illinois and Massachusetts results were a researcher's dream. Both coefficients were significant in every specification. Moreover, their estimates appeared to be reasonably consistent with one another. Both had positive, similar magnitudes on the student-institution variable, and both had negative, similar magnitudes on the cost-to-income variable.

All things considered, it was a very unsatisfactory state of affairs. A comparison of the results of Sample I and Sample II, however, did suggest how to proceed.

Our sampling control for Sample II had assured an achievement and income dispersion. While we could not really know the true income dispersion of Sample I, it was extremely likely that the Sample I groups were more homogeneous in economic status, within each ability group, than were the Sample II groups. Since the demand specifications worked less well in the more diversely designed sample (Sample II), we concluded that it may have been just too simplistic on our part to have expected rather different people to function as if they had similar decision-making functions. In Sample I, where we had less diverse

icient estimate
s except Mas-
nticipated this

understanding
d income, and
that went into
Furthermore,

ables included
Table 2. The
nificant in one
not significant.
varies between
in which both
rnia, the C/YH
ssachusetts. In
ways negative,
e in California
h consistency.
le II estimates.
e negative and
ant. For North
dent-institution
efficients were
results were a
every specifica-
ably consistent
on the student-
nitudes on the

te of affairs. A
however, did

hievment and
e true income
ample I groups
ability group,
l specifications
Sample II), we
ur part to have
ey had similar
d less diverse

TABLE 2-A Demand Estimates by State and by Sample (California Sample II)

Achievement Interaction	C/Y	C/YH	C/DY	C/DYH	CO/Y	CO/YH	CO/DY	CO/DYH
.255 · 10 ⁻²	-4.346*							
(.513 · 10 ⁻²)	(1.18)							
.103 · 10 ⁻¹	-9.638*							
(.527 · 10 ⁻²)	(1.82)							
-.847 · 10 ^{-2*}			-.175 · 10 ^{-1*}					
(.419 · 10 ⁻²)			(.842 · 10 ⁻²)					
-.792 · 10 ⁻²				-.122				
(.454 · 10 ⁻²)				(.766 · 10 ⁻¹)				
-.942 · 10 ^{-2*}					-.234			
(.440 · 10 ⁻²)					(.166)			
-.100 · 10 ^{-1*}						-.309		
(.447 · 10 ⁻²)						(.310)		
-.107 · 10 ^{-1*}							-.235 · 10 ⁻²	
(.416 · 10 ⁻²)							(.203 · 10 ⁻²)	
-.122 · 10 ^{-1*}								-.149 · 10 ⁻²
(.396 · 10 ⁻²)								(.446 · 10 ⁻²)

NOTE: Figures in parentheses are standard errors of the estimates. An asterisk indicates at least .05 level of significance.

TABLE 2-B Demand Estimates by State and by Sample (Illinois Sample II)

Achievement Interaction	C/Y	C/YH	C/DY	C/DYH	CO/Y	CO/YH	CO/DY	CO/DYH
.249 · 10 ⁻²	-4.953*							
(.492 · 10 ⁻²)	(1.178)							
.117 · 10 ^{-1*}		-10.739*						
(.539 · 10 ⁻²)		(1.788)						
-.117 · 10 ^{-1*}			-.729 · 10 ⁻²					
(.384 · 10 ⁻²)			(.552 · 10 ⁻²)					
.568 · 10 ⁻²				-.780*				
(.529 · 10 ⁻²)				(.163)				
-.523 · 10 ⁻²					-.952*			
(.468 · 10 ⁻²)					(.364)			
.106 · 10 ⁻¹						-3.840*		
(.646 · 10 ⁻²)						(.813)		
-.125 · 10 ^{-1*}							-.158 · 10 ⁻²	
(.384 · 10 ⁻²)							(.218 · 10 ⁻²)	
-.342 · 10 ⁻²								-.158*
(.517 · 10 ⁻²)								(.510 · 10 ⁻¹)

NOTE: See note to Table 2-A.

TABLE 2-C Demand Estimates by State and by Sample (Massachusetts Sample II)

Achievement

NOTE: See note to Table 2-A.

TABLE 2-C Demand Estimates by State and by Sample (Massachusetts Sample II)

Achievement Interaction	C/Y	C/YH	C/DY	C/DYH	CO/Y	CO/YH	CO/DY	CO/DYH
.108 · 10 ⁻¹ *	-2.369*							
(.469 · 10 ⁻²)	(.821)							
.694 · 10 ⁻²		-1.674						
(.474 · 10 ⁻²)		(1.066)						
.591 · 10 ⁻²			-200 · 10 ⁻¹ *					
(.416 · 10 ⁻²)			(.901 · 10 ⁻²)					
.242 · 10 ⁻²				.377 · 10 ⁻²				
(.398 · 10 ⁻²)				(.901 · 10 ⁻²)				
.104 · 10 ⁻¹ *					-1.072*			
(.470 · 10 ⁻²)					(.393)			
.123 · 10 ⁻¹ *						-1.988*		
(.493 · 10 ⁻²)						(.653)		
.560 · 10 ⁻²							-1.745 · 10 ⁻² *	
(.410 · 10 ⁻²)							(.296 · 10 ⁻²)	
.246 · 10 ⁻²								.684 · 10 ⁻³
(.398 · 10 ⁻²)								(.486 · 10 ⁻²)

NOTE: See note to Table 2-A.

**TABLE 2-E Demand Estimates by State and by Sample
(California Sample I)**

Achievement Interaction	C/YH	C/DYH	CO/YH	CO/DYH
$-.253 \cdot 10^{-1*}$ (.594 · 10 ⁻²)	-1.271 (1.24)			
$-.296 \cdot 10^{-1*}$ (.451 · 10 ⁻²)		$-.233 \cdot 10^{-2}$ (.486 · 10 ⁻²)		
$-.301 \cdot 10^{-1*}$ (.474 · 10 ⁻²)			$.145 \cdot 10^{-1}$ (.126)	
$-.303 \cdot 10^{-1*}$ (.454 · 10 ⁻²)				$.891 \cdot 10^{-3}$ (.182 · 10 ⁻²)

NOTE: See note to Table 2-A.

groups, the specifications either worked, as in the cases of Illinois and Massachusetts, or for some unknown reason, one variable dominated the explanation, as in the cases of California and North Carolina.

There seemed to be very little discrepancy about the sign of the cost-to-income variable no matter what group or state was considered. But the estimate of the achievement interaction variable differed both within states, by sample, and between states. We thought that we needed more homogeneous achievement groups. We did not have the resources to extend our sample, so we aggregated the state observations and disaggregated the student observations into four achievement groups. The ranges selected for each achievement group were not totally arbitrary; they were based on impressions about homogeneous student decision-making. The lowest-achievement group contained the students

**TABLE 2-F Demand Estimates by State and by Sample
(Illinois Sample I)**

Achievement Interaction	C/YH	C/DYH	CO/YH	CO/DYH
$.198 \cdot 10^{-1*}$ (.584 · 10 ⁻²)	-16.689* (2.412)			
$.187 \cdot 10^{-1*}$ (.584 · 10 ⁻²)		$-1.720*$ (.263)		
$.488 \cdot 10^{-1*}$ (.829 · 10 ⁻²)			$-10.905*$ (1.247)	
$.389 \cdot 10^{-1*}$ (.788 · 10 ⁻²)				$-.960*$ (.121)

NOTE: See note to Table 2-A.

(.346 · 10⁻²)

(.396 · 10⁻²)

NOTE: See note to Table 2-A.

**TABLE 2-G Demand Estimates by State and by Sample
(Massachusetts Sample I)**

Achievement Interaction	C/YH	C/DYH	CO/YH	CO/DYH
.232 · 10 ⁻¹ * (.530 · 10 ⁻²)	-12.527* (1.908)			
.233 · 10 ⁻¹ * (.527 · 10 ⁻²)		-1.373* (.210)		
.468 · 10 ⁻¹ * (.687 · 10 ⁻²)*			-9.476* (1.026)	
.409 · 10 ⁻¹ * (.657 · 10 ⁻²)*				-.887* (.105)

NOTE: See note to Table 2-A.

with average SAT scores of less than 400. The middle-lower group had SAT's average greater than or equal to 400, but less than 475. The upper-middle group covered the range up to 550, and the upper group's average SAT scores were 550 or above.

Achievement Aggregation Results

Table 3, like Table 1 before, contains the results of estimating the demand functions separately for each variable in the utility function. Since we have argued that we are not very interested in these single variable specifications, we shall not dwell on their detailed results. They

**TABLE 2-H Demand Estimates by State and by Sample
(North Carolina Sample I)**

Achievement Interaction	C/YH	C/DYH	CO/YH	CO/DYH
.441 · 10 ⁻² (.611 · 10 ⁻²)	-16.493* (2.585)			
.181 · 10 ⁻² (.611 · 10 ⁻²)		-1.604* (.270)		
.676 · 10 ⁻² (.651 · 10 ⁻²)			-6.011* (.820)	
-.620 · 10 ⁻³ (.604 · 10 ⁻²)				-.512* (.792 · 10 ⁻¹)

NOTE: See note to Table 2-A.

Sample

CO/DYH

- .887*
(.105)

lower group had
than 475. The
upper group's

estimating the
ability function.
In these single
I results. They

Sample

CO/DYH

- .512*
(.792 · 10⁻¹)

TABLE 3 Single Variable Specifications on Achievement Group Samples

Observations	SAT < 400		400 ≤ SAT < 475		475 < SAT < 495		475 ≤ SAT < 550		SAT ≥ 550	
	I	II	I	II	I	II	I	II	I	II
	164	162	92	84	67	72	46	57		
C/N		-7.138* (.962)		-2.700* (.735)		-2.803* (.865)		-2.803* (.865)		-2.803* (.865)
C/NH	-17.247* (1.844)	-4.975* (.837)	-7.610* (1.284)	-6.832* (1.255)	-5.805* (1.506)	-5.780* (1.350)	-2.795* (1.646)	-2.795* (1.646)		1.354 (1.386)
C/DY		-387 · 10 ⁻¹ * (.115 · 10 ⁻¹)		-132 · 10 ⁻¹ * (.520 · 10 ⁻²)		-168 · 10 ⁻¹ * (.835 · 10 ⁻²)		-168 · 10 ⁻¹ * (.835 · 10 ⁻²)		-368 · 10 ⁻² * (.542 · 10 ⁻²)
C/DYH	-6.11* (.939 · 10 ⁻¹)	-725 · 10 ⁻² * (.599 · 10 ⁻²)	-539 · 10 ⁻² * (.863 · 10 ⁻²)	-406* (.101)	-636* (.165)	-165 (.102)	-338* (.160)	-338* (.160)		.133 (.134)
CON		-2.121* (.279)		-0.321* (.149)		-502* (.209)		-502* (.209)		-.907 · 10 ⁻² * (.225)
CONH	-4.003* (.381)	-1.414* (.324)	-2.879* (.471)	-1.872* (.476)	-2.905* (.624)	-1.559* (.540)	-1.823* (.809)	-1.823* (.809)		.914 (.946)
CO/DY		-910 · 10 ⁻² * (.246 · 10 ⁻²)		-282 · 10 ⁻² * (.169 · 10 ⁻²)		-573 · 10 ⁻² * (.247 · 10 ⁻²)		-573 · 10 ⁻² * (.247 · 10 ⁻²)		-.972 · 10 ⁻² * (.295 · 10 ⁻²)
CO/DYH	-730 · 10 ⁻² * (.315 · 10 ⁻²)	-251 · 10 ⁻² * (.216 · 10 ⁻²)	-196 · 10 ⁻² * (.313 · 10 ⁻²)	-452 · 10 ⁻¹ * (.190 · 10 ⁻¹)	-299* (.652 · 10 ⁻¹)	-686 · 10 ⁻² * (.893 · 10 ⁻²)	-182* (.663 · 10 ⁻¹)	-182* (.663 · 10 ⁻¹)		.880* (.941 · 10 ⁻¹)
Ach. int.	-878 · 10 ⁻¹ * (.789 · 10 ⁻²)	-279 · 10 ⁻¹ * (.491 · 10 ⁻²)	-457 · 10 ⁻¹ * (.526 · 10 ⁻²)	-138 · 10 ⁻¹ * (.424 · 10 ⁻²)	-296 · 10 ⁻² * (.402 · 10 ⁻²)	-113 · 10 ⁻¹ * (.377 · 10 ⁻²)	.115 · 10 ⁻¹ * (.417 · 10 ⁻²)	.115 · 10 ⁻¹ * (.417 · 10 ⁻²)		.182 · 10 ⁻¹ * (.395 · 10 ⁻²)

NOTE: Asterisks indicate significance at .05 level. Figures in parentheses are standard errors of estimates.

are presented because they clearly illustrate two of the most interesting aspects of higher-education demand functions that we have found to date, namely, that there are trends in the coefficient estimates as one moves from the low-achievement group to the high-achievement group. These trends exist in almost every cost-to-income specification, and in the student-institution achievement interaction specification, and in both samples.

First, for the cost-to-income coefficients, the trends run from relatively high negative coefficient estimates for low-achievement groups to relatively low negative coefficient estimates for the high-achievement group. The high-achievement group's values are almost always not significantly different from zero. Second, the student-institution achievement interaction coefficients trend from statistically significant negative values for the low-achievement group to statistically significant positive values for the high-achievement group.

The sign reversal in the student-institution achievement interaction coefficients, going from negative for low-achievement students to positive for high-achievement students indicates a complete reversal in the marginal effect of quality. Given the students' eligibility, low-achievement students are repelled from better schools and high-achievement students are attracted to them.

Table 4 presents the achievement-group aggregation estimations with both variables. The general results found for the single variable specifications appear again. For Sample II, the *C/Y* specification has both coefficients significant in the lowest and the highest SAT groups. The cost-to-income coefficients are more negative for the low-achievement group than for the high-achievement group. The achievement variable offers a negative influence on the lower group and a positive influence on the higher group. The two middle achievement groups have their cost-to-income variable coefficients negative and significant, and their magnitude places them in the trend from elastic demand for lower-achieving students to inelastic demand for higher-achieving students with respect to cost and income. The trend in the student-institution achievement coefficient places the middle achievement group coefficients close to zero in magnitude. Their estimates did indeed turn out not significantly different from zero.

In Sample II, none of the derived cost-to-income variables offers as consistent results as the specification that used the reported income directly. The results of Sample II's predicted income specifications present the same pattern as was found in the reported income results. Their estimates are not as consistent, however. As with the reported specification, two of the four *C/YH* specifications are significant on both variables. As achievement rises, there is a downward trend in the *C/YH* coefficient,

TABLE 4 Demand Estimates by Achievement Groups and by Sample

	Achievement Interaction	C/Y	Sample II Achievement Groups	
			C/DY	CO/DY
SAT < 400	-.328 · 10 ^{-1**}	-3.592*		
162 Observations	(.774 · 10 ⁻²)	(.116)		

most interesting
have found to
estimates as one
achievement group.
education, and in
ation, and in

run from rela-
ment groups to
achievement
always not sig-
nificant achieve-
ment negative
ly significant

ent interaction
identifies to posi-
ve reversal in the
ability, low-
s and high-

estimations with
variable spec-
ification has both
achievement
groups. The
achievement
variable
positive influence
groups have their
and, and their
and for lower-
achieving students
achievement-institution
achievement group
did indeed turn

ables offers as
ed income di-
stributions present
results. Their
reported specifica-
both variables.
PH coefficient,

TABLE 4 Demand Estimates by Achievement Groups and by Sample

	Achievement Interaction	C/Y	C/DY	Sample II Achievement Groups C/OY	C/OY
SAT < 400	-.328 · 10 ⁻¹ ** (.774 · 10 ⁻²)	-3.592* (1.116)			
162 Observations	-.494 · 10 ⁻¹ * (.603 · 10 ⁻²)		-0.103 · 10 ⁻¹ (.704 · 10 ⁻²)		
	-.406 · 10 ⁻¹ * (.746 · 10 ⁻²)			-.709* (.316)	
	-.504 · 10 ⁻¹ * (.602 · 10 ⁻²)				-.248 · 10 ⁻² (.200 · 10 ⁻²)
SAT ≥ 400	-.244 · 10 ⁻² (.557 · 10 ⁻²)	-2.450* (.907)			
SAT < 475	-.106 · 10 ⁻¹ * (.451 · 10 ⁻²)		-.921 · 10 ⁻² (.509 · 10 ⁻²)		
84 Observations	-.118 · 10 ⁻¹ * (.468 · 10 ⁻²)			-.146 (.156)	
	-.131 · 10 ⁻¹ * (.452 · 10 ⁻²)				-.734 · 10 ⁻³ (.186 · 10 ⁻²)
SAT ≥ 475	-.325 · 10 ⁻² (.406 · 10 ⁻²)	-2.364* (1.037)			
SAT < 550	-.928 · 10 ⁻² * (.396 · 10 ⁻²)		-.100 · 10 ⁻¹ (.736 · 10 ⁻²)		
72 Observations	-.875 · 10 ⁻² * (.412 · 10 ⁻²)			-.214 (.174)	
	-.946 · 10 ⁻² * (.398 · 10 ⁻²)				-.374 · 10 ⁻² (.256 · 10 ⁻²)

TABLE 4 (continued)

	Achievement Interaction	C/YH	C/DYH	Sample II Achievement Groups CO/YH	CO/DYH
SAT ≥ 550	.211 · 10 ^{-1**}	-1.031*			
57 Observations	(.423 · 10 ⁻²)	(.589)			
	.205 · 10 ^{-1**}		-.132 · 10 ^{-1*}		
	(.415 · 10 ⁻²)		(.684 · 10 ⁻²)		
	.203 · 10 ^{-1*}			-.393	
	(.415 · 10 ⁻²)			(.201)	
	.196 · 10 ^{-1*}				-.476 · 10 ⁻²
	(.413 · 10 ⁻²)				(.310 · 10 ⁻²)
SAT < 400	-.503 · 10 ^{-1*}	-.392			
162 Observations	(.660 · 10 ⁻²)	(.537)			
	-.537 · 10 ^{-1*}		.423 · 10 ⁻²		
	(.593 · 10 ⁻²)		(.500 · 10 ⁻²)		
	-.518 · 10 ^{-1*}			-.571 · 10 ⁻¹	
	(.626 · 10 ⁻²)			(.139)	
	-.538 · 10 ^{-1*}				.214 · 10 ⁻²
	(.593 · 10 ⁻²)				(.255 · 10 ⁻²)
SAT ≥ 400	.130 · 10 ^{-1**}	-9.431*			
SAT < 475	(.673 · 10 ⁻²)	(1.874)			
84 Observations	-.938 · 10 ^{-2*}		-.691 · 10 ⁻¹		
	(.452 · 10 ⁻²)		(.383 · 10 ⁻¹)		
	-.369 · 10 ⁻²			-1.532*	
	(.819 · 10 ⁻²)			(.890)	
	-.142 · 10 ^{-1*}				-.125 · 10 ⁻¹
	(.449 · 10 ⁻²)				(.858 · 10 ⁻²)
SAT ≥ 475	.289 · 10 ⁻²	-6.514*			
SAT < 550	(.515 · 10 ⁻²)	(1.894)			
72 Observations	-.970 · 10 ^{-2*}		-.473 · 10 ⁻¹		
	(.422 · 10 ⁻²)		(.702 · 10 ⁻¹)		
	-.688 · 10 ⁻²			-.817	
	(.552 · 10 ⁻²)			(.764)	

SAT \geq 475
SAT < 550

72 Observations

$-.938 \cdot 10^{-2*}$
(.452 $\cdot 10^{-2}$)
 $-.369 \cdot 10^{-2}$
(.819 $\cdot 10^{-2}$)
 $-.142 \cdot 10^{-1*}$
(.449 $\cdot 10^{-2}$)

$-.691 \cdot 10^{-1}$
(.383 $\cdot 10^{-1}$)

$-1.532*$
(.890)

$-.125 \cdot 10^{-1}$
(.858 $\cdot 10^{-2}$)

$-6.514*$
(1.894)

$.289 \cdot 10^{-2}$
(.515 $\cdot 10^{-2}$)
 $-.970 \cdot 10^{-2*}$
(.422 $\cdot 10^{-2}$)
 $-.688 \cdot 10^{-2}$
(.552 $\cdot 10^{-2}$)
 $-.108 \cdot 10^{-1*}$
(.388 $\cdot 10^{-2}$)

$-.473 \cdot 10^{-1}$
(.702 $\cdot 10^{-1}$)

$-.817$
(.764)

$-.814 \cdot 10^{-2}$
(.965 $\cdot 10^{-2}$)

SAT \geq 550
57 Observations

$.232 \cdot 10^{-1**}$
(.451 $\cdot 10^{-2}$)
 $.223 \cdot 10^{-1**}$
(.446 $\cdot 10^{-2}$)
 $.262 \cdot 10^{-1**}$
(.491 $\cdot 10^{-2}$)
 $.244 \cdot 10^{-1**}$
(.471 $\cdot 10^{-2}$)

$-3.873*$
(1.735)

$-.323*$
(.166)

$-3.721*$
(1.255)

$-.287*$
(.109)

SAT < 400
164 Observations

$-.438 \cdot 10^{-1**}$
(.119 $\cdot 10^{-1}$)
 $-.847 \cdot 10^{-1*}$
(.807 $\cdot 10^{-2}$)
 $-.814 \cdot 10^{-1*}$
(.863 $\cdot 10^{-2}$)
 $-.867 \cdot 10^{-1*}$
(.798 $\cdot 10^{-2}$)

$-9.593*$
(2.460)

$-.265 \cdot 10^{-1}$
(.233 $\cdot 10^{-1}$)

$-.261$
(.211)

$-.158 \cdot 10^{-2}$
(.213 $\cdot 10^{-2}$)

SAT \geq 400
SAT < 475

92 Observations

$.551 \cdot 10^{-1}$
(.536)

$.141 \cdot 10^{-2}$

TABLE 4 (concluded)

Achievement Interaction	C/YH	C/DYH	Sample II Achievement Groups CO/YH	CO/DYH
(.534 · 10 ⁻²)		(.703 · 10 ⁻²)		
-.467 · 10 ^{-1**}			.873 · 10 ⁻¹	
(.591 · 10 ⁻²)			(.225)	
-.467 · 10 ^{-1**}				.337 · 10 ⁻²
(.539 · 10 ⁻²)				(.366 · 10 ⁻²)
.148 · 10 ^{-1**}	-9.704*			
(.514 · 10 ⁻²)	(2.125)			
.138 · 10 ^{-1**}				
(.511 · 10 ⁻²)		-1.017*		
.297 · 10 ^{-1**}		(.231)		
(.657 · 10 ⁻²)			-7.331*	
.228 · 10 ^{-1**}			(1.225)	
(.614 · 10 ⁻²)				-.629*
				(.120)
.224 · 10 ^{-1**}	-8.787*			
(.486 · 10 ⁻²)	(2.319)			
.233 · 10 ^{-1**}				
(.486 · 10 ⁻²)		-.933*		
.330 · 10 ^{-1**}		(.258)		
(.618 · 10 ⁻²)			-7.809*	
.257 · 10 ^{-1**}			(1.511)	
(.540 · 10 ⁻²)				-.527*
				(.126)

NOTE: Figures in parentheses are standard errors of estimates. Single asterisks indicate .05 level of significance. Double asterisks indicate .01 level of significance.

Elasticities

It is our opinion that income variable interaction is significant. Our guess is that the predicted income variable interaction is more significant than the predicted income variable interaction for the lower-achieving group.

Table 5 contains the predicted income variable interaction for the lower-achieving group. There is, in fact, a significant difference in the predicted income variable interaction for the lower-achieving group.

There is, in fact, a significant difference in the predicted income variable interaction for the lower-achieving group.

and an upward trend. The lower-achieving group did not have its achievement variable interaction reported in the lower-achieving group.

Comparing the upper-achieving group with the lower-achieving group, the predicted income variable interaction for the lower-achieving group is significantly different from zero.

The pattern of what the same estimates. The achievement variable interaction for the lower-achieving group is significantly different from zero.

The reason for the lower-achieving group's lower-achieving group is due to the fact that the lower-achieving group's achievement variable interaction is significantly different from zero.

The reason for the lower-achieving group's lower-achieving group is due to the fact that the lower-achieving group's achievement variable interaction is significantly different from zero.

.330 · 10^{-1**}
 (.618 · 10⁻²)
 .257 · 10^{-1**}
 (.540 · 10⁻²)

-7.809*
 (1.511)

-.527*
 (.126)

NOTE: Figures in parentheses are standard errors of estimates. Single asterisks indicate .05 level of significance. Double asterisks indicate .01 level of significance.

and an upward trend on the $A_i S_j / 1,000$ coefficient. But, the lower group did not have its CYH coefficient significant, and the lower-middle group has its achievement interaction coefficient positively significant. Apparently the achievement variable becomes positive faster than it does in the reported income specification, so it is never statistically different from zero.

Comparing the reported and predicted income coefficients for the upper achievement group shows similar magnitudes on the student-institution achievement estimates, and a response to predicted income two and one-quarter times the response to reported income.

The pattern of the Sample I achievement groups estimates was somewhat the same as the pattern of the Sample II achievement groups estimates. There were some minor variations, however, as the lowest achievement group had both coefficients significant in the Sample II CY specification. The lower-middle group had its achievement coefficient significant instead of its CYH coefficient. The trend in the student-institution achievement variable was somewhat augmented by the fact that the lower-middle group's estimate was too low. This is undoubtedly due to the fact that the CYH coefficient was not significant. An important variation is that the trend in the cost-to-income coefficient over the achievement groups was absent.

The reason the coefficient values for Sample I's cost-to-income specifications are all approximately equal to the CYH coefficient in Sample II's lower-middle achievement group is a mystery. Is it possibly related to the sampling design of Sample I? Or is it related to some attribute about the reporting versus the nonreporting families?

Elasticities

It is our opinion that the CY and the CYH specifications of the cost-to-income variable, in conjunction with the student-institution achievement interaction variable, are the best demand specifications we found. Our guess is that, of the two, the reported income is probably better than the predicted specification, but we shall have to do more work to determine which specifications are actually better in each achievement group.

Table 5 contains means and standard deviations of the income and achievement measures by achievement group, and by sample. Also included are the samples' achievement-income correlation coefficients.

There is, in general, a decline in the mean income estimates as the achievement level declines. But the hypothesis that the mean samples significantly differ in mean income is flatly rejected.

TABLE 5 Means, Standard Deviations, and Correlation Coefficients

Achievement Group	Achievement: Mean and Standard Deviation		Income: Mean and Standard Deviation		Sample I Predicted		Sample II Predicted		Correlation Coefficient between Achievement and Income in Achievement Group	
	Sample II	Sample I	Sample II Reported	Sample II Predicted	Sample I Predicted	Sample I Reported	Sample II Reported	Sample II Predicted	Sample I Predicted	Sample II Predicted
High	611.6 (37.7)	602.2 (42.4)	16,912.8 (17,711.9)	16,921.0 (14,825.1)	17,040.8 (15,391.8)	0.116	0.228	0.502		
Medium high	509.8 (22.1)	513.5 (20.7)	13,379.3 (14,170.4)	11,919.3 (5,826.9)	13,685.7 (7,698.6)	-0.059	-0.081	0.070		
Medium low	440.9 (22.0)	440.8 (20.8)	12,656.0 (11,606.8)	11,021.0 (3,838.3)	11,103.4 (3,901.7)	0.066	0.078	-0.023		
Low	317.4 (55.8)	307.5 (63.2)	14,030.0 (16,675.9)	10,318.6 (3,158.9)	10,537.0 (4,131.5)	0.060	0.183	0.131		

NOTE: Standard deviations are in parentheses.

The correlation sign (see Section 6, group, a rather achievement in achievement group that the estimate. The following secondary-school reader in interest.

Descriptive statistics: No higher education; Low cost-low achievement; Low cost-medium achievement; Low cost-high achievement; Medium cost-low achievement; Medium cost-medium achievement; Medium cost-high achievement; High cost-low achievement; High cost-medium achievement; High cost-high achievement.

Table 6 presents types with the reported and predicted achievement groups.

The choosers of the predicted type is probably Section V compared groups. The frequencies in the cost and average choice sets appear in Tables 6, and 7, the predicted relative frequencies, enable comparison respect to achievement the income and types: three by student achievement are based on the

The correlation coefficients indicate the efficacy of the sampling design (see Section III). In all groups except Sample I's high ability group, a rather high degree of independence between the income and achievement measures was obtained. The income heterogeneity in each achievement group coupled with the four achievement samples implies that the estimates do cover the student income-achievement plane.

The following list establishes a correspondence between the post-secondary-school type number and its description. The list will aid the reader in interpreting Tables 6, 7, and 8 which follow.

Description	Type Number
No higher education	1
Low cost-low achievement	2
Low cost-medium achievement	3
Low cost-high achievement	4
Medium cost-low achievement	5
Medium cost-medium achievement	6
Medium cost-high achievement	7
High cost-low achievement	8
High cost-medium achievement	9
High cost-high achievement	10

Table 6 presents the relative frequencies of the chosen institution types with the demand model's predicted relative frequencies for the reported and predicted income specifications in each sample and achievement group.

The choosers' relative frequencies correspond rather well to the model's predicted frequencies, but the level of disaggregation by institution type is probably taxing the limits of the model's ability. Table 10 in Section V compares eight institutional types, and aggregates the ability groups. The deviations between observed and predicted relative frequencies in that table are, of course, much less. Approximate average cost and average institutional quality values for the samples' feasible choice sets appear in Table 7. The information contained in Tables 4, 5, 6, and 7, the conditional logit estimates, the sample means, the predicted relative frequencies, and the average cost and quality, respectively, enable computation of the probability of enrolling elasticities with respect to achievement, quality, cost and income. Tables 8 and 9 display the income and cost elasticities, respectively, for twelve possible student types: three by family income, \$6,000, \$12,000, and \$18,000, and four by student achievement score, 375, 475, 575, and 650. Those elasticities are based on the Sample II estimates.

TABLE 6 Observed and Predicted Relative Frequencies

Type	Sample II		Sample I	
	Observed Relative Frequency	Predicted Relative Frequency with Reported Income	Observed Relative Frequency	Predicted Relative Frequency with Predicted Income
High Achievement				
1	0.0877	0.0294	0.1739	0.0673
2	0.0877	0.0559	0.1304	0.0999
3	0.0877	0.0715	0.0435	0.1453
4	0.0	0.0435	0.0217	0.0842
5	0.0351	0.0584	0.0217	0.0710
6	0.1579	0.1336	0.1739	0.1090
7	0.1404	0.2035	0.1304	0.2095
8	0.0175	0.0006	0.0	0.0
9	0.0526	0.1751	0.0217	0.1014
10	0.3333	0.2285	0.2826	0.1124
Medium-High Achievement				
1	0.1806	0.2398	0.2836	0.1760
2	0.2500	0.1692	0.1194	0.1772
3	0.0556	0.0700	0.0296	0.1535
4	0.0556	0.0201	0.0149	0.0294
5	0.1806	0.1676	0.1045	0.0995
6	0.1667	0.1288	0.1343	0.1527
7	0.0417	0.0519	0.1791	0.1020
8	0.0278	0.0021	0.0	0.0
Medium-Low Achievement				
9	0.0139	0.1314	0.0597	0.0884
10	0.0278	0.0191	0.0746	0.0208
1	0.2024	0.2641	0.5544	0.5175
2	0.3459	0.1831	0.1957	0.1729
				0.2241

1	0.1806	0.2398	0.2432	0.2830	0.1100
2	0.2500	0.1692	0.1905	0.1194	0.1772
3	0.0556	0.0700	0.0945	0.0296	0.1535
4	0.0556	0.0201	0.0311	0.0149	0.0294
5	0.1806	0.1676	0.1492	0.1045	0.0995
6	0.1667	0.1288	0.1216	0.1343	0.1527
7	0.0417	0.0519	0.0643	0.1791	0.1020
8	0.0278	0.0021	0.0013	0.0	0.0

9	0.0139	0.1314	0.0901	0.0597	0.0884
10	0.0278	0.0191	0.0142	0.0746	0.0208

Medium-Low Achievement

1	0.2024	0.2641	0.2705	0.5544	0.5175
2	0.3452	0.1831	0.2241	0.1957	0.1729
3	0.0595	0.0872	0.1542	0.0217	0.0178
4	0.0119	0.0028	0.0056	0.0109	0.0001
5	0.1786	0.1501	0.1109	0.0978	0.1862
6	0.1310	0.1455	0.1379	0.0344	0.0504
7	0.0119	0.0084	0.0121	0.0	0.0001
8	0.0119	0.0027	0.0011	0.0326	0.0034
9	0.0238	0.1499	0.0799	0.0326	0.0510
10	0.0238	0.0063	0.0037	0.0	0.0007

Low Achievement

1	0.5617	0.5134	0.4940	0.7744	0.6828
2	0.1728	0.2304	0.2050	0.1281	0.2046
3	0.0370	0.0160	0.0103	0.0122	0.0120
4	0.0	0.0002	0.0001	0.0	0.0
5	0.1420	0.1549	0.1925	0.0366	0.0729
6	0.0309	0.0384	0.0374	0.0366	0.0192
7	0.0	0.0001	0.0001	0.0	0.0
8	0.0247	0.0028	0.0066	0.0061	0.0003
9	0.0247	0.0433	0.0537	0.0061	0.0083
10	0.0062	0.0005	0.0003	0.0	0.0000

TABLE 7 Average Costs and Quality

Type	Cost (\$)	Quality (SAT)
1	0	374.4
2	402.5	426.5
3	487.4	499.8
4	542.3	561.9
5	1,607.6	445.0
6	1,700.4	496.5
7	1,462.6	562.0
8	2,574.6	426.2
9	2,914.3	517.6
10	3,369.6	573.4

SECTION V: QUALITATIVE INFORMATION

Here I report on the introduction of qualitative information into the demand model. Our intention was to develop variations in the demand model that would incorporate social, psychological, and economic indices about the student choosers. It appears, though, that the questions appearing in the SCOPE high school questionnaire had not been based on well defined indices. And, in order to incorporate qualitative information we had to construct our own indices. Rather than abandon the qualitative project, we proceeded by constructing a few indices of our own. We gingerly refer to this work as our exercise in amateur sociology.

Three models are compared. The first is the conditional logit model, the second is the conditional logit model with a correction to account for the possibility that all the probabilities for a particular choice may be off by a constant. The third model incorporates the qualitative information about the choosers into the second model. Statistical estimation procedures for the choice-type correction and the qualitative-information corrections are developed below. Then, the results are discussed and presented.

Models and Estimation Procedures

Consider three alternative models.

1. The first model is the conditional logit model. For each student i , $\vec{X}_i = (X_{i1}, \dots, X_{iJ})$ has a multinomial distribution with

$$\text{Prob}\{X_{ij} = 1, X_{ik} = 0 \text{ for } k \neq j\} = \phi_{ij}$$

$$\text{and } \sum_{j \in J_i} \phi_{ij} = 1, \text{ and } \phi_{ij} \geq 0 \text{ for all } i.$$

Quality (SAT)

374.4
426.5
499.8
561.9
445.0
496.5
562.0
426.2
517.6
573.4

mation into the
in the demand
economic indi-
at the questions
not been based
qualitative infor-
an abandon the
y indices of our
amateur sociol-

nal logit model,
n to account for
oice may be off
ive information
timation proce-
nformation cor-
discussed and

each student i ,
n with

TABLE 8 Elasticity of the Probability of Enrolling in Type j with Respect to Family Income

Column	-\$6,000			-\$12,000			-\$18,000					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	.160	-.360	-.226	-.313	-.123	-.233	-.120	-.167	-.094	-.167	-.082	-.114
2	-.013	-.245	-.176	-.263	-.036	-.173	-.096	-.142	-.036	-.129	-.066	-.097
3		-.200	-.157	-.244		-.151	-.086	-.133		-.114	-.060	-.091
4			-.133	-.220			-.074	-.121			-.051	-.083
5	-1.160	.522	.152	.065	.536	.210	.068	.022	.345	.127	.044	.012
6		.161	-.003	-.090		.029	-.009	-.056		.006	-.007	-.039
7			.021	-.066			.003	-.043			.000	-.031
8		.924	.324	.237		.410	.154	.108		.260	.101	.070
9		.924	.324	.237		.410	.154	.108		.260	.101	.070
10				.237				.108				.070
Estimate for β^j	-3.592	-2.407	-1.301	-1.031	-3.592	-2.407	-1.031	-1.301	-3.592	-2.407	-1.031	-1.301

TABLE 9 Elasticity of the Probability of Enrolling in Type j with Respect to the Cost of Type j

Y _i	\$6,000			\$12,000			\$18,000			
	375	475	575	375	475	575	375	475	575	
A _i	375	475	575	375	475	575	375	475	575	650
Estimate for β ¹	-3.592	-2.407	-1.031	-1.031	-2.407	-1.031	-1.031	-2.407	-1.031	-1.031
Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	0	0	0	0	0	0	0	0	0	0
2	-.127	-.091	-.047	-.048	-.064	-.047	-.024	-.024	-.043	-.032
3		-.131	-.059	-.062	-.067	-.067	-.030	-.031	-.045	-.020
4			-.071	-.076	-.076	-.036	-.036	-.039	-.024	-.024
5	-1.204	-.794	-.365	-.370	-.559	-.386	-.182	-.185	-.360	-.254
6		-.455	-.196	-.204	-.204	-.226	-.098	-.103	-.150	-.065
7			-.197	-.208	-.208	-.098	-.098	-.105	-.065	-.065
8		-.1.206	-.511	-.523	-.523	-.582	-.253	-.260	-.382	-.168
9		-.1.211	-.487	-.504	-.504	-.585	-.239	-.249	-.384	-.158
10				-.402				-.192		

- The second model. Start for model: \bar{X}
 $\text{Prob}\{X_i$
 and $\sum_{j \in J_i} \phi_j$
- The third model. Each individual answer of this cost. Thus, \bar{X}
 $\text{Prob}\{X_i$
 and $\sum_{j \in J_i} \phi_j$
 We assume one another now to the

Let p_j denote population, and for the students of students in a
 The observed marginal distribution

$$p_j^r = \frac{1}{n^r} \sum_{i \in I^r} X_{ij}$$

and

$$(1) \quad p_j = \frac{1}{n} \sum_i X_{ij}$$

In the constant choice type-ans

$$(2) \quad E(p_j^r) = \frac{1}{n^r} \sum_{i \in I^r} p_j$$

The observed expected value

$$(3) \quad E(p_j) = \frac{1}{n} \sum_i p_j$$

2. The second model will be called the constant augmented demand model. Simply described, the augmented logit model has a constant for each choice type j , k_j , added to the conditional logit model: \bar{X}_i has a multinomial distribution with

$$\text{Prob}\{X_{iu} = 1, X_{ik} = 0 \text{ for } k \neq j\} = \phi_{ij} + k_j$$

and $\sum_{j \in J_i} \phi_{ij} = 1$, and $\phi_{ij} \geq 0$ for all i , and $\sum_j k_j = 0$.

3. The third model is the qualitative information demand model. In the third model indices are considered one at a time. Each index is separated into a set of disjoint answer categories, $r = 1, \dots, R$. Each individual has an index value that allocates him or her to answer category r . Associated with category r is a constant, a_j^r . This constant is added to the augmented conditional logit model. Thus, \bar{X}_i has a multinomial distribution with

$$\text{Prob}\{X_{iu} = 1, X_{ik} = 0 \text{ for } k \neq j\} = \phi_{ij} + k_j + a_j^r$$

and $\sum_{j \in J_i} \phi_{ij} = 1$, and $\phi_{ij} \geq 0$ for all i , and $\sum_j k_j = 0$.

We assume the students make their decisions independently of one another, so $\{\bar{X}_i\}$ are distributed independently. We turn now to the estimation of these models.

Let p_j denote the observed relative frequency of choice j for a student population, and p_j^r denote the observed relative frequency of choice j for the students who have a category answer r . n^r denotes the number of students in category r .

The observed relative frequencies are determined from the multinomial distributions by the expressions

$$p_j^r = \frac{1}{n^r} \sum_{i \in r} X_{ij}$$

and

$$(1) \quad p_j = \frac{1}{n} \sum_r X_{ij} = \sum_r \frac{n^r}{n} p_j^r$$

In the constant augmented model the relative frequency for each choice type-answer response category has an expected value given by

$$(2) \quad E(p_j^r) = \frac{1}{n^r} \sum_{i \in r} (\phi_{ij} + k_j) = \phi_j^r + k_j$$

The observed relative frequency for each category response has an expected value given by

$$(3) \quad E(p_j) = \frac{1}{n} \sum_i (\phi_{ij} + k_j) = \sum_r \left(\frac{n^r}{n} \right) (\phi_j^r + k_j) = \phi_j + k_j$$

In the qualitative information demand model, the expected values of these two relative frequencies are

$$(2') \quad E(p_j^r) = \frac{1}{n^r} \sum_{u^r} (\phi_{uj} + k_j + a_j^r) = \phi_j^r + k_j + a_j^r$$

and

$$(3') \quad E(p_j) = \frac{1}{n} \sum_j (\phi_{uj} + k_j + a_j) = \phi_j + k_j + a_j$$

The expected value of the difference between the observed relative frequency of the choice type-answer responses and the observed relative frequency of the choice type responses, in both the second and third model is given by

$$(4) \quad E(p_j^r - p_j) = \phi_j^r - \phi_j \text{ (for all } k_j)$$

The value of the variance of this difference

$$\begin{aligned} \text{Var}(p_j^r - p_j) &= \text{Var}\left[p_j^r - \sum_s \left(\frac{n^s}{n}\right) p_j^s\right] \\ &= \text{Var}\left[\left(1 - \frac{n^r}{n}\right) p_j^r - \sum_{s \neq r} \left(\frac{n^s}{n}\right) p_j^s\right] \end{aligned}$$

$$(5) \quad = \left(1 - \frac{n^r}{n}\right)^2 \text{Var}(p_j^r) + \sum_{s \neq r} \left(\frac{n^s}{n}\right)^2 \text{Var}(p_j^s)$$

This value can be estimated from the expression

$$(6) \quad \text{Var}(p_j^r) = \left(\frac{1}{n^r}\right)^2 \sum_{u^r} (\phi_{uj} + k_j)(1 - \phi_{uj} - k_j)$$

To obtain estimates for k_j in the second model, and the a_j^r values of the third model, we shall substitute the observed relative frequencies for the expected value of choosing j .

Thus, k_j is estimated from equation 3 by

$$(7) \quad k_j = p_j - \phi_j$$

and a_j^r is estimated from equations 2 and 4 by

$$(8) \quad a_j^r = 3p_j^r - \phi_j^r - k_j = (p_j^r - p_j) - (\phi_j^r - \phi_j)$$

From equations 8 and 4

$$E(a_j^r) = 0$$

and from equation 8

$$\text{Var}(a_j^r) = \text{Var}(p_j^r - p_j)$$

which can be estimated with equations 5 and 6.

In the following list, the 21 indices, their associated response categories, and a meaningful title appears. The actual questions and score

weights which
When an index
separated at the
example, for in
conservative.

Index	Cate
1	1
2	3
3	5
4	7
5	9
6	11
7	13
8	15
9	17
10	19
11	21
12	24
13	27
14	35
15	41
16	43
17	45
18	47
19	49
20	51
21	53

After the ind
the choice type
gated two of th
will help the r

Choice Type

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

ected values of

weights which constitute each index appear in the author's appendix. When an index has two response categories, the observations have been separated at the mean index value into low and high responses. For example, for index 1, category 1 is low conservative, category 2 is high conservative.

Index	Categories	Interpretation
1	1-2	Conservative
2	3-4	Subindex of conservatism
3	5-6	Fate control
4	7-8	Self-motivation for problem solving
5	9-10	Broad academic interests
6	11-12	Parents' concern with education
7	13-14	Student response to parents' concern
8	15-16	Student academic desires
9	17-18	Index 7 + Index 8
10	19-20	Peer group response at the cost of studying
11	21-23	Actual high school program
12	24-26	Will you ever go to college?
13	27-34	Most satisfaction in life
14	35-40	When was post-secondary-school choice made?
15	41-42	Educational implications of preferred jobs
16	43-44	Educational implications of jobs to which the student is uncertain or indifferent
17	45-46	Student attachment to parents
18	47-48	Desire to sacrifice for future payoffs
19	49-50	Parents' higher education desires for student
20	51-52	Index 6 + ½ (Index 19)
21	53-54	Sex of student

observed relative
observed relative
second and third

the a_j^2 values of
ive frequencies

After the indices had been separated into response categories, a few of the choice types consistently had only a few observations. We aggregated two of the choice types with two other types. The following list will help the reader to understand the tables to follow:

Choice Type	Meaning
1	No school
2	Low cost-low achievement
3	Low cost-medium achievement
4	Low and medium cost-high achievement
5	Medium cost-low achievement
6	Medium cost-medium achievement
7	High cost-low and medium achievement
8	High cost-high achievement

response catego-
tions and score

The constants, k_j , appear in the various parts of Table 10 which present choice type frequency statistics for the following samples: Sample II with the reported income conditional logit estimates, Sample II with the predicted income conditional logit estimates, and Sample I with predicted income conditional logit estimates, respectively. The choice type

TABLE 10 Differences between Observed and Predicted Frequencies by Option Type and Model

Type (J)	Observed	Predicted	Observed Relative	Average Predicted Relative	Constant k_j
Sample II: Reported Income Model					
1	131	127.2003	0.3493	0.3392	0.0101
2	80	66.4680	0.2133	0.1772	0.0361
3	20	19.0140	0.0533	0.0507	0.0026
4	17	20.2639	0.0453	0.0540	-0.0087
5	48	51.7918	0.1280	0.1381	-0.0101
6	37	35.3294	0.0987	0.0942	0.0045
7	18	39.9006	0.0480	0.1064	-0.0584
8	24	15.0197	0.0640	0.0401	0.0239
	375	374.9871	1.0000	1.0000	0.0000
Sample II: Predicted Income Model					
1	131	125.1724	0.3493	0.3338	0.0155
2	80	67.9257	0.2133	0.1811	0.0322
3	20	25.8627	0.0533	0.0690	-0.0156
4	17	24.5696	0.0453	0.0655	-0.0202
5	48	52.3310	0.1280	0.1395	-0.0115
6	37	34.3342	0.0987	0.0916	0.0071
7	18	31.1366	0.0480	0.0830	-0.0350
8	24	12.9148	0.0640	0.0344	0.0296
	375	374.2463	1.0000	0.9980	0.0020
Sample I: Predicted Income Model					
1	208	176.4025	0.5637	0.4781	0.0856
2	53	65.1246	0.1436	0.1765	-0.0329
3	8	20.5756	0.0217	0.0558	-0.0341
4	21	22.3369	0.0569	0.0605	-0.0036
5	20	37.9099	0.0542	0.1027	-0.0485
6	28	23.0234	0.0759	0.0624	0.0135
7	13	16.9878	0.0352	0.0460	-0.0108
8	18	6.6308	0.0488	0.0188	0.0308
	369	368.9910	1.0000	1.0000	0.0000

constants are de
observed relativ
frequency. They
limitations only
Sample IIR (re
here.

The most ext
occurs when dir
sion is asked fo
present high sc
possibility of go
probably best to
demand correct
curriculum can
projections.

Parents' obje
educational obje
education. The
up and the est
medium achiev
schools go dow
children produc
the student's ed
logit model. Sin
one-half of Inde

Economists te
to some future
relationship. On
the educational
preferring jobs v
them went to hig
of them did not
tional requireme

If the student
jobs with high
greater nonatten
college attendan
indifference and
tions (Index 16).

Another aspec
with when the s
made, the more
quality alternati

0 which present
 : Sample II with
 ple II with the
 ple I with pre-
 The choice type

**Predicted
 del**

Age
 ted
 ive
 Constant
 k_j

odel	
2	0.0101
72	0.0361
7	0.0026
10	-0.0087
31	-0.0101
2	0.0045
4	-0.0584
1	0.0239
00	0.0000

odel	
8	0.0155
11	0.0322
30	-0.0156
55	-0.0202
5	-0.0115
16	0.0071
30	-0.0350
44	0.0296
30	0.0020

odel	
31	0.0356
65	-0.0329
58	-0.0341
05	-0.0036
27	-0.0485
24	0.0135
60	-0.0108
88	0.0308
00	0.0000

constants are determined by equation 7 as the difference between the observed relative frequency and the average demand predicted relative frequency. They appear in the final column of Table 10. Due to space limitations only the choice type-category response estimates, a_j , for the Sample IIR (reported income conditional logit estimates) are included here.

The most extensive change in the predicted probability distribution occurs when direct information about the higher-education choice decision is asked for: the present desires of the students (Index 8), their present high school curriculum (Index 11), or a direct statement of the possibility of going to college (Index 12). This is not surprising. It is probably best to view these indices as tests of whether the qualitative demand corrections work. Still, it is interesting to note that high school curriculum can readily be incorporated into higher-education demand projections.

Parents' objectives are important (Index 19). If the parent had low educational objectives, the students were allocated away from higher education. The conditional logit estimate for no higher education goes up and the estimates for the local junior college, the medium cost-medium achievement institutions, and high cost-high achievement schools go down. Parents with high educational objectives for their children produced exactly the opposite effect. Parents' concern about the student's education (Index 6) did not seem to alter the conditional logit model. Since Index 6 was not effective, the sum of Index 6 and one-half of Index 17 was also not effective (Index 20).

Economists tend to measure the response to education as a response to some future labor market. A few of the indices are related to this relationship. One factor in this education-labor market relationship is the educational implications of the jobs students prefer. For students preferring jobs with high educational requirements, significantly more of them went to higher quality-higher cost colleges, and significantly fewer of them did not go to college. Students preferring jobs with low educational requirements behave in exactly the opposite manner (Index 15).

If the student demonstrates high indifference and uncertainty toward jobs with high educational implications, he or she shows significantly greater nonattendance at any college, and significantly lower local junior college attendance. The allocation is reversed for students with high indifference and uncertainty toward jobs with low educational implications (Index 16).

Another aspect of the education-labor market relationship has to do with when the student makes a career choice. The earlier the choice is made, the more positively altered were the higher cost and higher quality alternatives. Students who decide (or accept a decision) on a

career before the seventh grade, attend highest-quality-highest-cost institutions in significantly greater numbers, and significantly fewer of them choose not to achieve higher education. If the student has made a career decision by the tenth grade, he or she chooses the no higher education alternative significantly less, but is not allocated to any particular institutional type. Students who have not yet made a career choice decide not to go on to higher education with significantly greater frequency, and they attend medium cost-medium quality schools with significantly less frequency (Index 14).

What may be the most interesting result of this section is that indices which we thought would have been significant were not. Fate control (Index 3), self-motivation at problem solving (Index 4), broad academic interest (Index 5), and time preference or sacrificing ability (Index 18) were examples of indices which did not alter the conditional logit estimates. Are these measures so poor as to have no marginal effect on choice type, or does the achievement variable already incorporate the information contained in the indices?

The Massachusetts study found that disaggregating their total sample by students' sex altered their results. We did not find this result; only the medium cost-low quality institutions seemed to be affected. Males went less frequently and females more frequently. This result was of some surprise to us too.

These results appear in Table 11. Each choice-type-answer response category contains a column of three entries. The upper entry is the equation 8 estimated choice-type-consumer-answer category constant, a_j^i . The middle entry is the ratio of the estimated constant to its variance (as determined by equations 5 and 6). This number is a t -squared, or chi-squared statistic with one degree of freedom. Constants significantly different from zero at the .05 level are denoted by an asterisk; constants different from zero at the .01 level are denoted by two asterisks. The lower entry in the choice-type-answer category classification is the ratio of the estimated constant, a_j^i , to its observed relative frequency, p_j^i .

NOTES AND REFERENCES

1. This project was carried out by Roy Radner at the University of California, Berkeley, and by the author when he was at Berkeley, and then at SUNY, Stony Brook. The work presented here is the result of a joint effort. I accept responsibility for any errors or omissions. I should like to acknowledge that discussions with Daniel McFadden, in the early phase of this study, were extremely important.

For our first report see R. Radner, and L. S. Miller, "Demand and Supply in U.S. *Proceedings* 60 (May 1970): 326-334. For further developments see Roy Radner and

TABLE 11 Response Residuals, Chi-Squared, and Percentage of Observed Relative Frequency

	1	2	3	4	5	6	7	8	9
1	0.0034	-0.0026	-0.0423*	0.0709*	0.0117	-0.0146	0.0509	-0.0222	0.0162
	0.0188	0.0188	5.9174	5.8170	0.2542	0.2544	0.1729	0.1774	0.7000

ity-highest-cost
icantly fewer of
he student has
chooses the no
allocated to any
t made a career
ificantly greater
ity schools with

on is that indices
ot. Fate control
broad academic
bility (Index 18)
tional logit esti-
rginal effect on
incorporate the

their total sample
this result; only
affected. Males
is result was of

answer response
er entry is the
tegy constant,
nt to its variance
a *t*-squared, or
ants significantly
n asterisk; con-
enoted by two
tegy classifica-
bserved relative

California, Berkeley,
, Stony Brook. The
sponsibility for any
ssions with Daniel
nportant.
and Supply in U.S.
see Roy Radner and

TABLE 11 Response Residuals, Chi-Squared, and Percentage of Observed Relative Frequency

	1	2	3	4	5	6	7	8	9
1	0.0034	-0.0026	-0.0423*	0.0709*	0.0117	-0.0146	0.0509	-0.0222	0.0162
	0.0188	0.0188	5.8174	5.8170	0.3543	0.3544	2.1772	2.1774	0.7993
	0.0108	-0.0069	-0.1577	0.1461	0.0321	-0.0443	0.1095	-0.0744	0.0412
2	-0.0162	0.0122	0.0156	-0.0263	-0.0176	0.0220	-0.0265	0.0116	0.0168
	0.5454	0.5452	1.0785	1.0789	1.0642	1.0639	0.8073	0.8070	1.1318
	-0.0869	0.0522	0.0721	-0.1268	-0.0874	0.0966	-0.1312	0.0529	0.0703
3	0.0048	-0.0036	0.0018	-0.0030	0.0114	-0.0142	0.0154	-0.0067	-0.0118
	0.1459	0.1459	0.0543	0.0544	1.4058	1.4059	0.9801	0.9803	1.6672
	0.0705	-0.0861	0.0284	-0.0852	0.1827	-0.3394	0.2505	-0.1349	-0.3268
4	-0.0024	0.0018	0.0073	-0.0122	-0.0079	0.0099	0.0129	-0.0056	-0.0078
	0.0336	0.0335	0.9484	0.9487	0.6738	0.6735	0.9306	0.9308	0.7057
	-0.0383	0.0546	0.1068	-1.7086	-0.2061	0.1831	0.4909	-0.1052	-0.2898
5	-0.0418*	0.0314*	-0.0136	0.0228	-0.0026	0.0032	-0.0421	0.0184	-0.0008
	4.3039	4.3024	0.9229	0.9223	0.0265	0.0264	2.3402	2.3393	0.0034
	-0.6730	0.1771	-0.1332	0.1331	-0.0200	0.0257	-0.4359	0.1296	-0.0063
6	0.0160	-0.0120	0.0146	-0.0245	0.0066	-0.0083	-0.0242	0.0106	-0.0049
	0.8375	0.8383	1.7811	1.7818	0.2408	0.2412	1.2511	1.2505	0.1529
	0.1285	-0.1513	0.1142	-0.4897	0.0692	-0.0814	-0.3946	0.0920	-0.0578
7	0.0378	-0.0285*	0.0053	-0.0088	0.0074	-0.0093	0.0174	-0.0076	-0.0040
	4.1326	4.1349	0.1936	0.1940	0.2638	0.2644	0.5544	0.5550	0.0863
	0.4059	-2.0302	0.0825	-0.4127	0.1547	-0.1936	0.3310	-0.1656	-0.1102
8	-0.0017	0.0013	0.0113	-0.0189	-0.0091	0.0113	-0.0039	0.0017	-0.0036
	0.0222	0.0222	3.3781	3.3781	1.2723	1.2723	0.1233	0.1233	0.2006
	-0.0191	0.0268	0.1263	-0.8843	-0.1573	0.1573	-0.1110	0.0222	-0.0719

TABLE 11 (continued)

	10	11	12	13	14	15	16	17	18
1	-0.0234	0.0296	-0.0298	0.0331	-0.0386	0.1455**	-0.1095**	0.0972**	-0.0731**
	0.7994	1.8211	1.8213	2.6377	2.6380	31.4333	31.4341	14.3660	14.3665
	-0.0815	0.0705	-0.1071	0.0825	-0.1336	0.2415	-0.6889	0.1908	-0.3193
2	-0.0243	0.0067	-0.0068	-0.0273	0.0319	-0.0448*	0.0337*	-0.0253	0.0190
	1.1322	0.1234	0.1235	2.3848	2.3842	4.0143	4.0137	1.2861	1.2857
	-0.1379	0.0287	-0.0351	-0.1415	0.1345	-0.2329	0.1473	-0.1233	0.0866
3	0.0171	0.0018	-0.0018	0.0232*	-0.0271*	-0.0276*	0.0208*	0.0165	-0.0124
	1.6670	0.0281	0.0281	5.4442	5.4446	5.3445	5.3442	1.8667	1.8669
	0.2179	0.0374	-0.0306	0.3121	-0.9362	-2.2209	0.2468	0.2660	-0.2660
4	0.0114	-0.0009	0.0009	-0.0054	0.0064	-0.0049	0.0037	-0.0049	0.0037
	0.7055	0.0063	0.0063	0.2923	0.2922	0.1872	0.1871	0.1740	0.1739
	0.1580	-0.0230	0.0161	-0.1570	0.1099	0.0	0.0460	-0.2638	0.0565
5	0.0012	-0.0297	0.0299	-0.0160	0.0187	-0.0012	0.0009	-0.0419*	0.0315*
	0.0033	2.7848	2.7837	0.9331	0.9324	0.0033	0.0032	4.1839	4.1825
	0.0104	-0.3492	0.1746	-0.1543	0.1199	-0.0090	0.0070	-0.5184	0.1925
6	0.0072	0.0026	-0.0027	-0.0101	0.0118	-0.0669**	0.0503**	-0.0449**	0.0337**
	0.1526	0.0314	0.0316	0.5291	0.5285	15.8874	15.8847	7.0849	7.0829
	0.0610	0.0276	-0.0262	-0.1201	0.1020	-5.3876	0.3078	-1.2039	0.2330
7	0.0057	0.0047	-0.0047	0.0014	-0.0017	0.0117	-0.0088	0.0049	-0.0037
	0.0859	0.0861	0.0864	0.0091	0.0092	0.4268	0.4274	0.0737	0.0740
	0.0880	0.1102	0.0883	0.0319	-0.0320	0.3757	-0.1446	0.1321	-0.0662
8	0.0052	-0.0149	0.0150	0.0012	-0.0014	-0.0118	0.0089	-0.0017	0.0013
	0.2006	2.7215	2.7215	0.0211	0.0211	1.6045	1.6045	0.0289	0.0289
	0.0609	-0.4000	0.1647	0.0189	-0.0224	-0.6323	0.0903	-0.0342	0.0171

TABLE 11 (continued)

	19	20	21	22	23	24	25	26	27
1	-0.0070	0.0115	0.1419**	-0.1209**	0.1773**	-0.0554**	0.2156**	0.1947**	-0.0221
	0.1648	0.1647	10.1793	37.4128	14.3806	19.7618	7.8939	8.9772	0.0315

7	0.0057	0.0047	-0.0047	0.0014	-0.0017	0.0117	-0.0088	0.0049	-0.0037
	0.0859	0.0861	0.0864	0.0091	0.0092	0.4268	0.4274	0.0737	0.0740
	0.0880	0.1102	0.0883	0.0319	-0.0320	0.3757	-0.1446	0.1321	-0.0662
8	0.0052	-0.0149	0.0150	0.0012	-0.0014	-0.0118	0.0089	-0.0017	0.0013
	0.2006	2.7215	2.7215	0.0211	0.0211	1.6045	1.6045	0.0289	0.0289
	0.0609	-0.4000	0.1647	0.0189	-0.0224	-0.6323	0.0903	-0.0342	0.0171

TABLE 11 (continued)

	19	20	21	22	23	24	25	26	27
1	-0.0070	0.0115	0.1419**	-0.1209**	0.1773**	-0.0554**	0.2156**	0.1947**	-0.0221
	0.1648	0.1647	10.1793	37.4128	14.3806	19.7618	7.8939	8.9772	0.0315
	-0.0212	0.0302	0.2249	-0.9197	0.2766	-0.2177	0.2875	0.2954	-0.0608
2	0.0001	-0.0002	-0.0545	0.0478**	-0.0717	0.0475**	-0.1874**	-0.1654**	0.0381
	0.0001	0.0001	2.0350	7.8659	3.4058	20.2618	9.1808	8.4261	0.1076
	0.0005	-0.0009	-0.2695	0.2035	-0.4302	0.1895	-3.3738	-1.8190	0.1396
3	-0.0089	0.0146	-0.0047	0.0068	-0.0135	-0.0052	0.0256	0.0142	0.0315
	1.1142	1.1140	0.0845	0.5812	0.5213	1.3046	1.0172	0.3749	0.2376
	-0.2078	0.2078	-0.1963	0.0902	-0.5253	-0.0967	0.4610	0.3129	0.3468
4	0.0014	-0.0023	0.0073	-0.0020	-0.0026	-0.0033	0.0346**	-0.0061	-0.0087
	0.0313	0.0314	0.2803	0.0528	0.0356	0.8703	7.6775	0.1138	0.0519
	0.0252	-0.0821	0.6153	-0.0260	0.0	-0.0610	1.2456	0.0	0.0
5	-0.0087	0.0142	-0.0501	0.0259	-0.0167	0.0132	-0.0674	-0.0333	-0.0254
	0.3750	0.3746	2.2290	2.7438	0.2269	2.2310	1.5756	0.5643	0.0663
	-0.0778	0.0918	-0.6010	0.1721	-0.1445	0.0905	-1.2133	-0.4891	-0.2797
6	0.0136	-0.0222	-0.0303	0.0316*	-0.0537*	0.0027	-0.0245	0.0020	0.0925
	1.3840	1.3849	1.7746	6.3972	3.9918	0.1483	0.4998	0.0027	1.2872
	0.1170	-0.3159	-1.2721	0.2040	-2.0937	0.0247	-0.8818	0.0224	0.5085
7	0.0023	-0.0038	0.0158	-0.0090	0.0075	-0.0078	0.0310	0.0271	-0.0509
	0.0355	0.0357	0.3777	0.4465	0.0716	1.1947	0.6859	0.4976	0.2978
	0.0416	-0.1084	0.6629	-0.1365	0.2920	-0.1540	1.1163	0.5969	0.0
8	0.0072	-0.0118	-0.0255**	0.0198**	-0.0268**	0.0083**	-0.0275*	-0.0333*	-0.0550
	1.0814	1.0814	7.0962	8.1452	7.0412	7.7016	4.7912	5.2247	1.4123
	0.0988	-0.2398	0.0	0.1761	0.0	0.1023	0.0	0.0	0.0

TABLE 11 (continued)

	28	29	30	31	32	33	34	35	36
1	0.0436	-0.0173	0.0283	-0.1254	-0.1219	0.0099	-0.1456	0.1779**	0.0666
	1.0592	0.0310	1.2905	3.5570	1.1286	0.0227	2.2551	8.8559	3.7256
	0.1034	-0.0494	0.0752	-0.7105	-0.5283	0.0306	-0.7278	0.3216	0.1492
2	-0.0511	0.0000	0.0091	0.1494*	-0.1120	-0.0216	-0.0373	-0.0068	-0.0191
	1.9653	0.0000	0.1769	5.5296	1.4242	0.1352	0.2017	0.0176	0.4065
	-0.3032	0.0001	0.0416	0.3908	-1.4558	-0.1227	-0.2798	-0.0320	-0.0928
3	0.0081	0.0465	-0.0192	-0.0003	0.0409	0.0215	-0.0023	0.0449	-0.0206
	0.1690	1.0457	2.5065	0.0002	0.7467	0.3403	0.0014	3.1880	1.5206
	0.1349	0.4650	-0.5284	-0.0111	0.5318	0.2440	-0.0339	0.5279	-0.5770
4	0.0024	-0.0168	-0.0036	-0.0325	0.0407	0.0225	0.0426	-0.0140	0.0219
	0.0163	0.2714	0.0894	0.7361	0.2343	0.3345	0.3555	0.2833	2.2883
	0.0661	0.0	-0.0991	-1.1049	0.2645	0.2551	0.3196	-0.6560	0.4895
5	-0.0067	0.0340	0.0076	-0.0588	-0.0204	0.0211	0.0298	-0.0859	-0.0104
	0.0375	0.2276	0.1409	0.9599	0.0594	0.1731	0.1400	3.3008	0.1311
	-0.0503	0.2265	0.0544	-0.6664	-0.2655	0.1795	0.2238	-2.0182	-0.0728
6	-0.0261	0.0081	-0.0097	0.0419	0.1497*	-0.0443	0.0479	-0.0931**	-0.0333
	0.9526	0.0175	0.3257	1.0257	4.2624	0.6605	0.4890	5.9325	2.2733
	-0.4328	0.0806	-0.1002	0.3566	0.6487	-0.5015	0.3596	0.0	-0.6211
7	0.0347	-0.0472	-0.0067	0.0171	0.0208	-0.0200	-0.0291	-0.0355	0.0061
	1.4220	0.4957	0.1390	0.0971	0.0569	0.1534	0.1647	0.6596	0.0647
	0.4796	0.0	-0.1590	0.1939	0.2708	-0.6800	0.0	-1.6699	0.1364
8	-0.0049	-0.0073	-0.0057	0.0085	0.0022	0.0108	0.0938	0.0124	-0.0112
	0.1001	0.0502	0.3114	0.0789	0.0016	0.1015	2.3481	0.3832	1.0293
	-0.1022	-0.1460	-0.1036	0.0964	0.0285	0.1223	0.4692	0.1942	-0.4191

TABLE 11 (continued)

	37	38	39	40	41	42	43	44	45
1	-0.0767*	-0.0390	-0.1543*	-0.0589	0.0539*	-0.0461*	-0.0673*	0.0439*	-0.0137
	4.0463	0.6497	5.1214	0.7674	5.0180	5.0184	6.1921	6.1917	0.3939

	0.0347	-0.0472	-0.0067	0.0171	0.0205	-0.0240	0.0231	0.0000	0.0647
	1.4220	0.4957	0.1390	0.0971	0.0569	0.1534	0.1647	0.6596	0.0647
	0.4796	0.0	-0.1590	0.1939	0.2708	-0.6800	0.0	-1.6699	0.1364
8	-0.0049	-0.0073	-0.0057	0.0085	0.0022	0.0108	0.0938	0.0124	-0.0112
	0.1001	0.0502	0.3114	0.0789	0.0016	0.1015	2.3481	0.3832	1.0293
	-0.1022	-0.1460	-0.1036	0.0964	0.0285	0.1223	0.4692	0.1942	-0.4191

TABLE 11 (continued)

	37	38	39	40	41	42	43	44	45
1	-0.0767*	-0.0390	-0.1543*	-0.0589	0.0539*	-0.0461*	-0.0673*	0.0439*	-0.0137
	4.0463	0.6497	5.1214	0.7674	5.0180	5.0184	6.1921	6.1917	0.3939
	-0.2540	-0.1561	-2.0835	-0.2161	0.1123	-0.1941	-0.2553	0.1082	-0.0463
2	0.0336	-0.0023	-0.0295	0.0048	-0.0189	0.0161	0.0554*	-0.0361*	0.0109
	1.0524	0.0029	0.2239	0.0062	0.8186	0.8183	5.5131	5.5139	0.3309
	0.1345	-0.0115	-0.1989	0.0225	-0.0881	0.0758	0.2103	-0.2001	0.0531
3	0.0016	0.0037	-0.0235	0.0138	-0.0088	0.0076	-0.0116	0.0075	0.0079
	0.0083	0.0248	0.3558	0.1122	0.6007	0.6006	0.7330	0.7329	0.5629
	0.0312	0.0732	-0.6357	0.1520	-0.2549	0.1092	-0.2446	0.1317	0.1155
4	-0.0198	0.0050	-0.0237	0.0135	-0.0065	0.0056	0.0204	-0.0133	-0.0093
	1.2669	0.0272	0.1945	0.1427	0.3388	0.3386	2.1469	2.1473	0.7710
	-0.9490	0.0601	-0.3203	0.2229	-0.5658	0.0754	0.2522	-0.6053	-0.1769
5	0.0502	0.0277	-0.0132	-0.0280	0.0076	-0.0065	-0.0006	0.0004	-0.0083
	2.8685	0.4776	0.0573	0.2482	0.1550	0.1552	0.0007	0.0007	0.2239
	0.3010	0.1846	-0.1780	-0.3077	0.0548	-0.0548	-0.0053	0.0026	-0.0793
6	0.0364	0.0019	0.0937	0.0593	-0.0031	0.0026	0.0071	-0.0046	0.0138
	2.0378	0.0027	2.3612	1.6004	0.0373	0.0372	0.1432	0.1435	0.8747
	0.2691	0.0164	0.4218	0.3911	-0.0381	0.0232	0.0620	-0.0528	0.1090
7	-0.0143	0.0182	0.0375	0.0077	0.0056	-0.0048	0.0031	-0.0020	-0.0079
	0.2791	0.2451	0.3543	0.0214	0.1079	0.1082	0.0239	0.0241	0.2486
	-0.4583	0.2733	0.3376	0.1272	0.1619	-0.0811	0.0512	-0.0514	-0.1495
8	-0.0111	-0.0152	0.1130*	-0.0122	-0.0298**	0.0255**	-0.0067	0.0044	0.0066
	0.6233	0.3018	4.6417	0.1446	10.2959	10.2959	0.3445	0.3445	0.5554
	-0.2653	-0.1824	0.4358	-0.2015	-5.1510	0.2240	-0.1104	0.0662	0.0697

TABLE 11 (concluded)

	46	47	48	49	50	51	52	53	54
1	0.0140	-0.0270	0.0231	0.1462**	-0.0996**	0.1297**	-0.0855**	0.0157	-0.0145
	0.3939	1.2922	1.2920	28.7357	28.7364	21.7866	21.7873	0.4750	0.4751
	0.0346	-0.0915	0.0583	0.2497	-0.5291	0.2300	-0.4111	0.0457	-0.0411
2	-0.0112	0.0258	-0.0221	-0.0612*	0.0417*	-0.0429	0.0283	0.0371	-0.0342
	0.3311	1.5553	1.5558	6.8404	6.8395	3.1802	3.1796	3.4574	3.4581
	-0.0505	0.1114	-0.1114	-0.3722	0.1692	-0.2205	0.1254	0.1483	-0.1907
3	-0.0081	0.0045	-0.0039	-0.0045	0.0031	0.0024	-0.0016	0.0022	-0.0020
	0.5630	0.1503	0.1503	0.1281	0.1280	0.0366	0.0366	0.0374	0.0374
	-0.2146	0.0710	-0.0868	-0.1138	0.0487	0.0519	-0.0280	0.0387	-0.0387
4	0.0096	0.0171	-0.0147	-0.0016	0.0011	0.0027	-0.0018	-0.0102	0.0095
	0.7708	2.0887	2.0891	0.0191	0.0191	0.0519	0.0520	0.8283	0.8280
	0.2526	0.2279	-0.7406	-0.1243	0.0166	0.1362	-0.0292	-0.2633	0.1843
5	0.0086	-0.0288	0.0247	-0.0101	0.0069	-0.0295	0.0194	-0.0460*	0.0424*
	0.2236	2.2171	2.2161	0.2210	0.2207	1.8197	1.8188	5.9976	5.9962
	0.0566	-0.2769	0.1661	-0.0902	0.0494	-0.3136	0.1291	-0.3939	0.3063
6	-0.0141	0.0263	-0.0225	-0.0464*	0.0316*	-0.0423*	0.0279*	0.0174	-0.0160
	0.8755	2.6448	2.6462	6.6906	6.6887	5.5925	5.5909	1.2816	1.2823
	-0.2013	0.2070	-0.3036	-1.0066	0.2348	-1.0504	0.2033	0.2085	-0.1422
7	0.0081	-0.0103	0.0088	-0.0011	0.0007	0.0103	-0.0068	-0.0245	0.0226
	0.2480	0.3494	0.3488	0.0034	0.0034	0.2954	0.2959	2.1349	2.1333
	0.1867	-0.2225	0.1778	-0.0553	0.0110	0.3071	-0.1182	-0.6297	0.4006
8	-0.0068	-0.0077	0.0066	-0.0213*	0.0145*	-0.0305**	0.0201**	0.0083	-0.0077
	0.5554	0.6102	0.6102	4.5642	4.5642	9.5548	9.5548	0.7872	0.7872
	-0.2090	-0.1327	0.0948	-1.0806	0.1544	-4.5465	0.1977	0.1153	-0.1362

NOTE: Single asterisks indicate .05 level of significance. Double asterisks indicate .01 level of significance.

1. Leonard S. Carnegie C Radner, De (Berkeley, negie Commi
2. Robert Camp United States
3. Stephen A. H Analytical Sta believe the s
4. Higher Educa Council for th um VI of Th
5. Weights were with their par
6. Hoenack, Pri
7. The wage rat cent increase enrolling awa in the north
8. Higher Educa
9. Harvey Galpe Education in October 1969
10. For an excell Utility, and S Duncan Luce, ical Psycholog only be referr offer an altern and comparis "Random Ord eds., Contribu (Stanford, Cal follows Block
11. This choice n theory j prefer model, j is pr
12. Block and M Choice Behav developed the shown by Blo choosing j from This condition "irrelevance o
13. Daniel McFad Paper dated A ment Bureauc zation of Econ nia, Berkeley the paramete reported in th

0.2480	0.3494	0.3488	0.0034	0.0034	0.2954	0.2959	2.1349	2.1333
0.1867	-0.2225	0.1778	0.0110	0.0553	0.3071	-0.1182	-0.6297	0.4006
8	-0.0068	0.0066	0.0145*	-0.0213*	-0.0305**	0.0201**	0.0083	-0.0077
	0.5554	0.6102	4.5642	4.5642	9.5548	9.5548	0.7872	0.7872
	-0.2090	0.0948	0.1544	-1.0806	-4.5465	0.1977	0.1153	-0.1362

NOTE: Single asterisks indicate .05 level of significance. Double asterisks indicate .01 level of significance.

- Leonard S. Miller, *Demand and Supply in U.S. Higher Education* (Berkeley, Calif.: Carnegie Commission on Higher Education, 1975) and Leonard S. Miller and Roy Radner, *Demand and Supply in U.S. Higher Education: A Technical Supplement* (Berkeley, Calif.: Carnegie Commission on Higher Education, 1975).
- negie Commission on Higher Education, 1975).
 - Robert Campbell and B. W. Siegel, "The Demand for Higher Education in the United States 1919-1964," *American Economic Review* 57 (June 1967): 482-494.
 - Stephen A. Hoenack, *Private Demand for Higher Education in California*, Office of Analytical Studies, University of California. The document bears no date, but I believe the study was completed in 1967.
 - Higher Education In the Boston Metropolitan Area*, Metropolitan Area Planning Council for the Commonwealth of Massachusetts Board of Higher Education, Volume VI of The Board of Higher Education Series, 1969.
 - Weights were set equal to the percentage of freshmen on each campus who lived with their parents and the percentage who did not live with their parents.
 - Hoenack, *Private Demand for Higher Education*, Appendix 4, pp. 3-4.
 - The wage rate effect was stronger in southern than in northern California. A 1 per cent increase in wages in the hometown area decreased the proportion of eligibles enrolling away from home by 3 to 5 per cent in the south, and decreased enrollment in the north by approximately 1.75 per cent.
 - Higher Education in Boston*, pp. 38-39.
 - Harvey Galper and Robert M. Dunn, Jr. "A Short-Run Demand Function for Higher Education in the United States," *Journal of Political Economy* 77 (September/October 1969): 765-777.
 - For an excellent review of the topic see R. D. Luce, and P. Suppes, "Preference, Utility, and Subjective Probability," pp. 249-410 (see Sec. 5). Chapter 19 in R. Duncan Luce, Robert R. Bush, and Eugene Galanter, eds., *Handbook of Mathematical Psychology*. Vol. 3 (New York: John Wiley and Sons, 1965). In this paper we shall only be referring to models with constant utility functions. Random utility functions offer an alternative method of handling stochastic choice processes. For a discussion and comparison see Luce and Suppes cited above or H. D. Block and J. Marschak, "Random Orderings and Stochastic Theories of Responses," in Ingram Olkin et al., eds., *Contributions to Probability and Statistics, Essays in Honor of Harold Hotelling* (Stanford, Calif.: Stanford University Press, 1960). The development of this section follows Block and Marschak's presentation.
 - This choice model is weaker than the nonstochastic theory, for in nonstochastic theory j preferred to k implies j is chosen over k with probability 1. In the weak model, j is preferred to k with a probability only greater than one-half.
 - Block and Marschak, "Random Orderings." They credit R. D. Luce, *Individual Choice Behavior* (New York: John Wiley and Sons, 1959) with having most fully developed the strict constant utility model. Another property of the strict model shown by Block and Marschak is that the strict model implies that the probability of choosing j from a set K , where $K \subset J \subset A$ is the same as choosing j from the set J . This condition is referred to as the "independence of irrelevant choice" or the "irrelevance of added alternatives" condition.
 - Daniel McFadden, "On Measuring Design Criteria for Public Projects." Working Paper dated April 10, 1967 (processed), and "The Revealed Preferences of a Government Bureaucracy," Technical Report No. 17. Project for the Evaluation and Optimization of Economic Growth, Institute of International Studies, University of California, Berkeley (processed). The existence of McFadden's procedures for estimating the parameters was one of the key factors which made the demand estimates reported in this paper feasible. His work and his direct help on the project are

appreciated. McFadden calls his estimation procedure "conditional logit estimation." ϕ_{ij} is the conditional probability that student i chooses alternative j from the set J_i of alternatives open to him. R. D. Luce has shown (in *Individual Choice Behavior*) that the distribution function for the odds of the binary choice between j and k , in a strict model, can yield a "logistic curve."

14. 1962 *SCAT-STEP Supplement*, Educational Testing Service, Princeton, N.J.; Berkeley, California, 1962. The specific linear predictors estimated were
 $SAT_{Verbal} = -1,472.14 + 6.57 SCAT_{Verbal}$: 560 observations
 $SAT_{Math} = -1,651.96 + 6.99 SCAT_{Quantitative}$: 513 observations
15. For a more complete statement of the predicted income models and results see Leonard S. Miller, "Predicting Family Income in the SCOPE Sample," Working Paper No. 7, Stony Brook Working Papers, April 1970 (Stony Brook, N.Y.: State University of New York, 1970) or Leonard S. Miller and Roy Radner, *Demand and Supply in U.S. Higher Education: A Technical Supplement* (Berkeley, Calif.: Carnegie Commission on Higher Education, 1975).

7 | COMMENTS

Stephen A. Hoenack

University of Minnesota

The private demand for higher education is an important topic which has received insufficient attention from economists. Public policy towards higher education could almost certainly be improved if the enrollment effects of a number of controllable variables were taken into account. For example, analysis of the effects of tuition charges on enrollments can form the basis of a policy of using flexibly administered higher-education prices. Such a policy could be used to allocate enrollments among educational programs in a manner which would ensure that student subsidies are no higher or lower than necessary to achieve desired enrollments.

Another example is in decision making regarding where to locate campuses. Estimates of the effects of travel costs on enrollments could be most helpful for this purpose.

Other policy-controllable variables which have enrollment effects include the nature of the high school curriculum and the timing and amount of career counseling provided.

In analyzing some of the complexities of student behavior, Miller's model of demand for higher education does a better job than previous models. The decision of whether to go to college is complicated because it involves a choice among the different types of schools for which the student is qualified

as well as the seen in the follo then decide wh available to the such a questio determination o and to the hyp making. To my takes into acco dance. This is

Another impo and psychologi cluding factors important to es they more fully improve the es ables such as selves, be des

In his resear measures of stu existing empir parental-reporte for students wh income data. M income data so the results.

With regard t family income c the effects of th family income. T costs and incor envisioning the an equation in v not interacted v variable is inde good a priori gr not independent assuming an in come on colleg priori grounds fo equations expre determines the college attenda the estimation o family income at variable. In add ratio form, such

logit estimation." from the set J_i of (vice Behavior) that and k , in a strict

Princeton, N.J.: ed were

is and results see Sample," Working Brook, N.Y.: State lner, Demand and keley, Calif.: Car-

topic which has y towards higher ent effects of a it. For example, form the basis of prices. Such a ational programs are no higher or

to locate cam- is could be most

it effects include amount of career

or, Miller's model ous models. The use it involves a udent is qualified

as well as the choice of whether to go. The importance of this fact can be seen in the following question: Do students first choose to go to college and then decide which college to attend, or do the particular college options available to them affect their decision to go to college? In order to address such a question, econometric analysis of demand must direct itself to the determination of joint probabilities of various types of college attendance and to the hypothesis that college attendance involves sequential decision making. To my knowledge, Miller's model is the first one which explicitly takes into account the determination of joint probabilities of college attendance. This is an important advance over previous work.

Another important aspect of Miller's research is its incorporation of social and psychological determinants of the demand for college attendance, including factors related to the high school curriculum and environment. It is important to estimate the effects of these variables for two reasons. First, they more fully (and accurately) specify demand relationships and, thus, improve the estimates of the effects of costs on attendance. Second, variables such as high school curriculum and career counseling may, themselves, be desirable subjects of policy control.

In his research, Miller was confronted with some difficult problems with measures of students' families' incomes. Miller did what I suspect is the only existing empirical analysis of the differences between student- and parental-reported family incomes. He also developed an income predictor for students whose parents did not respond to a questionnaire requesting income data. Most important, Miller estimated his equations with different income data so that the reader can see the effect of alternative estimates on the results.

With regard to the specification of the effects of educational costs and family income on college attendance, Miller's estimating equations express the effects of these two variables through the ratio of educational costs to family income. This ratio is a special case of possible relationships between costs and income on college attendance choices. The most familiar way of envisioning the influences of costs and income on college attendance is in an equation in which each of these variables is entered individually and is not interacted with the other. Note that in this case the effect of either variable is independent of the level of the other variable. However, these are good a priori grounds for believing that the effects of costs and income are not independent of each other. While there are good a priori reasons for assuming an interactive relationship between the effects of costs and income on college attendance choices, there are not, to my knowledge, a priori grounds for specifying any particular form of this relationship. Miller's equations express a particular relationship in which one coefficient entirely determines the influences of both income and costs on the demand for college attendance. It would be worthwhile to test this hypothesis through the estimation of attendance equations in which educational costs and family income are entered separately, as well as interacted through the ratio variable. In addition, specifications of interaction terms alternative to the ratio form, such as multiplicative relationships, could be attempted. For such

purposes it would be important to have wider ranges of sample cost data for each type of institution.

Much the same type of comment could apply to the student-institution achievement interaction variable. Incidentally, it would be interesting to test the hypothesis that the effect of the institutional component of this variable would differ according to the type of institution.

In summary, I believe that empirical analysis of the private demand for higher education is an important topic and that Miller's work is a valuable contribution.

8

JUNE
O'NEILL

Council of
Economic Advisers

The relation between higher education and universities and university policy. The cost of higher education and services is changing. Since the late 1960s, the demand for higher education funds and private funds for higher education has become a routine interest of the public.

The traditional view of higher education as a public or private good has been challenged by the study along with the private sector. The public sector has generally been characterized by maximization of output or to public goods. The private sector, such as those of nonprofit firms, has been characterized by maximization of output or to public goods. The public sector has generally been characterized by maximization of output or to public goods. The private sector, such as those of nonprofit firms, has been characterized by maximization of output or to public goods. The public sector has generally been characterized by maximization of output or to public goods. The private sector, such as those of nonprofit firms, has been characterized by maximization of output or to public goods.

NOTE: This paper is a contribution to the Higher Education and the Public Good project. The suggestions were received from the Council of Economic Advisers. The views expressed should not be interpreted as those of the Council, or of those who