Social Status, the Spirit of Capitalism, and the Term Structure of Interest Rates in Stochastic Production Economies[1]

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Abstract

This paper studies capital accumulation and equilibrium interest rates in stochastic production economies with the concern of social status. Given a specific utility function and production function, explicit solutions for capital accumulation and equilibrium interest rates have been derived. With the aid of steady-state distributions for capital stock, the effects of fiscal policies, social-status concern, and stochastic shocks on capital accumulation and equilibrium interest rates have been investigated. A significant finding of this paper is the demonstration of multiple stationary distributions for capital stocks and interest rates with the concern of social status.

Key Words: Stochastic growth; Social status; Fiscal policies; Interest rates.

JEL Classification: E0, G1, H0, O0.

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Abstract

This paper studies capital accumulation and equilibrium interest rates in stochastic production economies with the concern of social status. Given a specific utility function and production function, explicit solutions for capital accumulation and equilibrium interest rates have been derived. With the aid of steady-state distributions for capital stock, the effects of fiscal policies, social-status concern, and stochastic shocks on capital accumulation and equilibrium interest rates have been investigated. A significant finding of this paper is the demonstration of multiple stationary distributions for capital stocks and interest rates with the concern of social status.

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1. Introduction

Capital accumulation, interest rates, fiscal policies, and asset pricing under uncertainty have been studied extensively since the 1960s, e.g., Phelps (1962), Levhari and Srinivisan (1969), Brock and Mirman (1972), Mirrlees (1965). Merton (1975) studied the asymptotic theory of growth under uncertainty, and Foldes (1978) explored optimal saving with risk in continuous time. As for the term structure of interest rates, Cox, Ingersoll, and Ross (1981, 1985) considered the equilibrium theory of the term structure of interest rates, and presented the general theory for interest rates in a production economy. Sunderason (1983) provided a plausible equilibrium model, in which the assumption of a constant interest rate is valid. Bhattacharya (1981), Constantinides (1980), and Stapleton and Subrahmanyam (1978) also studied these topics and presented the conditions for a constant interest rate. Constantinides (1980) showed that the term structure of interest rate evolves deterministically over time under the assumptions of perfect capital markets, homogeneous expectations, and the state independent utility. Sunderason (1984) also derived the conclusion of a constant interest rate under Constantinides (1980)'s assumptions on capital markets, expectations, and utility. For the effects of fiscal policies on capital accumulation, interest rates, and asset pricing in stochastic economies, Eaton (1981), Turnovsky (1993, 1995), Grinols and Turnovsky (1993, 1994), and Obstfeld (1994) introduced taxations and government expenditure into the stochastic continuous-time growth and asset-pricing models. Under a linear production technology and other specified assumptions on preferences and stochastic shocks, they have derived explicit solutions of growth rates of consumption, savings, and equilibrium returns on assets.

In all these neoclassical models of capital accumulation, interest rates and asset pricing models, wealth accumulation is often taken to be solely driven by one's desire to increase consumption rewards. The representative agent chooses his consumption path to maximize his discounted utility, which is defined only on consumption. This motive is important for wealth accumulation. It is, however, not the only motive. Because man is a social animal, he also accumulates wealth to gain prestige, social status, and power in the society; see Frank (1985), Cole, Mailath and Postlewaite (1992, 1995), Fershtman and

Weiss (1993), Zou (1994, 1995), Bakshi and Chen (1996), and Fershtman, Murphy and Weiss (1996). In these wealth-is-status models, the representative agent accumulates wealth not only for consumption but also for wealth-induced status. Mathematically, in light of the new perspective, the utility function can be defined on both consumption, c, and wealth, w: $u(c_t,\ w_t)$. Another interpretation of these models is in line of the spirit of capitalism in the sense of Weber (1958): capitalists accumulate wealth for the sake of wealth[3].

With the wealth-is-status and the-spirit-of-capitalism models, many authors mentioned above have tried to explore diverse implications for growth, savings, interest rates, and asset pricing. Cole, Mailath, and Postlewaite (1992) have demonstrated how the presence of social-status concern leads to multiple equilibria in long-run growth. Zou (1994, 1995) has studied the spirit of capitalism and long-run growth and showed that a strong capitalistic spirit can lead to unbounded growth of consumption and capital even under the neoclassical assumption of production technology, Gong and Zou (2002) have studied fiscal policies, asset pricing, and capital accumulation in a stochastic model with the spirit of capitalism. Bakshi and Chen (1996) have explored empirically the relationship between the spirit of capitalism and stock market pricing and offered an attempt towards the resolution of the equity premium puzzle in Mehra and Prescott (1985). Smith (2001) has studied the effects of the spirit of capitalism on asset pricing and has shown that when investors care about status they will be more conservative in risk taking and more frugal in consumption spending. Furthermore, stock prices tend to be more volatile with the presence of the spirit of capitalism.

This paper explores capital accumulation and equilibrium interest rates in a stochastic model with the spirit of capitalism and with diminishing return to scale technology. Under a CES utility function defined on both consumption and wealth accumulation and a Cobb-Douglas production function, explicit solutions for capital accumulation and equilibrium interest rates have been derived. These multiple optimal paths and stationary distributions of capital stock and interest rates are quite significantly different from many existing neoclassical models. With the aid of the steady-state

[3]See Cole, Mailath, and Postlewaite (1992), Zou (1994); and Bakshi and Chen (1996) for details.

distributions for capital stock, the effects of fiscal policies on the long-run economy and the equilibrium interest rates have been investigated. In particular, the equilibrium interest rates are constant when the technology is linear and when the utility function is extended to include the wealth-is-status concern. Moreover, the equilibrium interest rates are a mean reserve process with these special assumptions.

This paper is organized as follows: in section 2, we set up a stochastic growth model in a production economy with the social-status concern. Allowing some special utility function and production function with selected parameters, explicit solutions for the optimal paths and stationary distributions of consumption, capital accumulation and interest rates have been derived in section 3. With the aid of the steady-state distribution of the endogenous variables, the effects of fiscal policies, production shocks, and the spirit of capitalism on the long-run economy have been examined in section 4. In section 5, we present the equilibrium interest rates under both a nonlinear technology and a linear technology and analyze the dynamic behavior of equilibrium interest rates and discuss the effects of fiscal policies and stochastic shocks on the interest rates. In section 6, we present some examples to show the existence of multiple stationary distributions of optimal capital accumulation and equilibrium interest rates. We conclude our paper in section 7.

2. The model

Following Eaton (1981) and Smith (2001), we assume that output y is given by

$$dy = f(k)dt + \varepsilon k dz, \tag{1}$$

where z is the standard Brown motion, ε is the stochastic shocks of production.

Equation (1) asserts that the accumulated flow of output over the period $(t,\ t+dt)$, given by the right-hand side of this equation, consists of two components. The deterministic component is described as the first term on the right-hand side, which is the firm's production technology and has been specified as a neoclassical production function, $f(k)$. The second part is the stochastic component, $\varepsilon k dz$, which can be viewed as the shocks to the production and assumed to be temporally independent, normally distributed.

Suppose the government levy an income tax and a consumption tax. Then, the agent's budget constraint can be written as[4]

$$dk = ((1-\tau)f(k) - (1+\tau_c)c)dt + (1-\tau')\varepsilon k dz \qquad (2)$$

where τ and τ' are the tax rates on the deterministic component of capital income and stochastic capital income, respectively, and τ_c is the consumption tax rate.

With the social status concern, the utility function can be written as $u(c,\ k)$. Suppose the marginal utilities of consumption and capital stock are positive, but diminishing, i.e.

$$u_1(c,\ k) > 0,\ u_2(c,\ k) > 0,\ u_{11}(c,\ k) < 0,\ u_{22}(c,\ k) < 0 \qquad (3)$$

The representative agent is to choose his consumption level and capital accumulation path to maximize his expected discounted utility, namely,

$$\max E_0 \int_0^\infty u(c,\ k)e^{-\rho t}dt$$

subject to a given initial capital stock $k(0)$ and the budget constraint (2). Where $0 < \rho < 1$ is the discount rate.

Associated with the above optimization problem, the value function $J(k,t)$ is defined as

$$J(k,\ t) = \max E_t \int_t^\infty u(c,\ k)e^{-\rho t}dt$$

subject to the given initial capital stock $k(t)$ and the budget constraint (2). Define the

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[4] Merton (1975) assumed that output is produced by a strictly concave production function, $Y = AF(K,L)$, where $K(t)$ denotes capital stock, $L(t)$ the labor force, and $A(t)$ is technology progress. Production is

$$Y = AF(K,L)$$

and the labor force follows

$$dL = aLdt + \varepsilon L dz,$$

where z is the standard Brown motion.

Defining the capital-labor ratio $k = K/L$, from the Itô's Lemma, we can derive the capital accumulation equation similar to equation (2). Or we assume the technology progress follows

$$dA/A = adt + \varepsilon dz$$

Defining the efficiency capital $k = K/(AL)$, we can also derive the capital accumulation similar to equation (2).

current value function $X(k)$ as

$$X(k) = J(k,\ t)e^{\rho t} \tag{4}$$

The recursive equation associated with the above optimization problem is

$$\max_c\{u(c,\ k) - \rho X(k) + X'(k)((1-\tau)f(k) - (1+\tau_c)c) + \frac{1}{2}X''(k)(1-\tau')^2\varepsilon^2 k^2\} = 0$$

Therefore, we get the first-order condition

$$u_c(c,\ k) = (1+\tau_c)X'(k) \tag{5}$$

and the Bellman equation

$$u(c,\ k) - \rho X(k) + X'(k)((1-\tau)f(k) - (1+\tau_c)c) + (1-\tau')^2\frac{1}{2}X''(k)\varepsilon^2 k^2 = 0 \tag{6}$$

Equation (5) states that the marginal utility of consumption equals the after-tax marginal utility of capital stock. Equation (6) determines the value function $X(k)$. In the next section, we will specify the utility function and the production function to present an explicit solution for the value function.

3. An explicit solution

3.1 The explicit solution under the separable utility function

In order to derive an explicit solution, we specify the utility function as[5]

$$u(c,\ k) = \frac{c^{1-\sigma}}{1-\sigma} + \xi\frac{k^{1-\sigma}}{1-\sigma}, \tag{7}$$

where $\sigma > 0$ is the constant relative risk aversion, and it also represents the elasticity of intertemporal substitution. $\xi \geq 0$ measures the investor's concern with his social status or measures his spirit of capitalism. The larger the parameter ξ, the stronger the agent's spirit of capitalism or concern for social status.

The production function is specified as

$$f(k) = Ak^{\alpha} \tag{8}$$

where $A > 0$ and $0 < \alpha < 1$ are positive constants.

For the special utility function and production function in equations (7) and (8), we

[5]In Appendix B, we present a similar analysis when the utility function is non-separable.

conjecture that the value function takes the following form

$$X(k) = a + \frac{b^{-\sigma}k^{1-\sigma}}{1-\sigma} \tag{9}$$

where a and b are constants, and they are to be determined.

Under the specified value function in equation (9), we rewrite the first-order condition (5) as

$$c^{-\sigma} = (1+\tau_c)X_k = (1+\tau_c)b^{-\sigma}k^{-\sigma}$$

namely,

$$c = (1+\tau_c)^{-\frac{1}{\sigma}}bk \tag{10}$$

Upon the relationship (10), the Bellman equation (6) is reduced to

$$\begin{aligned}&\frac{(1+\tau_c)^{1-\frac{1}{\sigma}}b^{1-\sigma}k^{1-\sigma}}{1-\sigma} + \xi\frac{k^{1-\sigma}}{1-\sigma} - \rho(a + \frac{b^{-\sigma}k^{1-\sigma}}{1-\sigma}) + b^{-\sigma}k^{-\sigma}((1-\tau)Ak^{\alpha} - (1+\tau_c)^{1-\frac{1}{\sigma}}bk)\\&+\frac{1}{2}X_{kk}\varepsilon^2(1-\tau')^2k^2 = 0\end{aligned} \tag{11}$$

If $\alpha = 1$, from equation (11), we have $a = 0$ and b is determined by the following equation

$$0 = \sigma(1+\tau_c)^{1-\frac{1}{\sigma}}b + \xi b^{\sigma} - (\rho - (1-\tau)A(1-\sigma) + \frac{1}{2}(1-\sigma)\sigma(1-\tau')^2\varepsilon^2)$$

In general, for the case of $\alpha \neq 1$, we cannot determine the constants a and b. Following Xie (1994), we specified the parameters as $a = \sigma$, then from equation (11), we have

$$a = \frac{(1-\tau)A}{\rho b^{\sigma}}$$

and b is determined by

$$0 = (1+\tau_c)^{1-\frac{1}{\sigma}}\sigma b + \xi b^{\sigma} - (\rho + \frac{1}{2}(1-\sigma)\sigma(1-\tau')^2\varepsilon^2).$$

Summarizing the discussions above, we have

Proposition 1. *Under the special utility function and production function in equations* (7) *and* (8)*, if* $\alpha = 1$*, then the explicit solutions for the economy system are*

$$\frac{c}{k} = (1+\tau_c)^{-\frac{1}{\sigma}}b \tag{12}$$

$$dk = ((1-\tau)Ak - (1+\tau_c)^{1-\frac{1}{\sigma}} bk)dt + (1-\tau')\varepsilon k dz \tag{13}$$

and the TVC

$$\lim_{t\to\infty} E(X'(k)e^{-\rho t}) = 0 \tag{14}$$

where b *is determined by*

$$0 = (1+\tau_c)^{1-\frac{1}{\sigma}}\sigma b + \xi b^{\sigma} - (\rho - (1-\tau)A(1-\sigma) + \frac{1}{2}(1-\sigma)\sigma(1-\tau')^2\varepsilon^2). \tag{15}$$

If $\alpha \neq 1$ *and* $\alpha = \sigma$*, then the explicit solutions for the economy are*

$$\frac{c}{k} = (1+\tau_c)^{-\frac{1}{\sigma}} b, \tag{12'}$$

$$dk = ((1-\tau)Ak^{\alpha} - (1+\tau_c)^{1-\frac{1}{\sigma}} bk)dt + (1-\tau')\varepsilon k dz \tag{13'}$$

and the TVC (14) *holds, whereas* b *is determined by*

$$0 = (1+\tau_c)^{1-\frac{1}{\sigma}}\sigma b + \xi b^{\sigma} - (\rho + \frac{1}{2}(1-\sigma)\sigma(1-\tau')^2\varepsilon^2). \tag{15'}$$

When $\alpha = 1$, the capital stock follows the stochastic growth path (13), and we get the mean growth rate for the capital stock

$$E(\frac{dk}{k}) = ((1-\tau)A - (1+\tau_c)^{1-\frac{1}{\sigma}} b)dt$$

It is easy to show from equation (12') and the production function that the mean growth rates for consumption level, output, and capital stock are equal. Let us denote the common mean growth rate as ϕ, which is given by

$$\phi = (1-\tau)A - (1+\tau_c)^{1-\frac{1}{\sigma}} b .$$

From the expression above, it is clear that capital income taxation, consumption taxation, stochastic shocks, and various preference and production parameters jointly determine the growth rate of the economy. Please also note that when the parameters satisfy the condition $\alpha = \sigma$, the deterministic income tax rate has no effects on the equilibrium consumption-capital stock ratio.

3.2 Steady-state distributions for endogenous variables

Similar to the certainty model, we will examine the existence and the properties of the steady state economy. As in Merton (1975), we are seeking the conditions under which there is a unique stationary distribution for the capital stock k, which is time and initial condition independent.

From equation (13'), the capital stock follows the following stochastic process,

$$\begin{aligned} dk &= ((1-\tau)Ak^{\alpha} - (1+\tau_c)^{1-\frac{1}{\sigma}} bk)dt + (1-\tau')\varepsilon k dz \\ &\equiv b(k)dt + (a(k))^{1/2} dz, \end{aligned} \tag{16}$$

where we denote $a(k) = (1-\tau')^2 \varepsilon^2 k^2$ and $b(k) = (1-\tau)Ak^{\alpha} - (1+\tau_c)^{1-\frac{1}{\sigma}} bk$.

Let $\pi_k(k)$ be the steady-state density function for the capital stock. As in Merton (1975), $\pi_k(k)$ exists and it can be shown to be

$$\pi_k(k) = \frac{m}{a(k)} \exp \int_0^k \frac{2b(x)}{a(x)} dx,$$

where m is a constant chosen so that $\int_0^{\infty} \pi_k(x)dx = 1$.

Substituting the expressions for $a(k)$ and $b(k)$, we have

$$\begin{aligned} \pi_k(k) &= \frac{m}{(1-\tau')^2 \varepsilon^2 k^2} \exp \int_0^k \frac{2((1-\tau)Ak^{\alpha} - (1+\tau_c)^{1-\frac{1}{\sigma}} bk)}{(1-\tau')^2 \varepsilon^2 k^2} dx \\ &= mk^{-2\frac{(1+\tau_c)^{1-\frac{1}{\sigma}} b + (1-\tau')^2 \varepsilon^2}{(1-\tau')^2 \varepsilon^2}} \exp(-\frac{2A(1-\tau)}{(1-\alpha)(1-\tau')^2 \varepsilon^2} k^{\alpha-1}) \end{aligned}$$

Defining variable $R = k^{\alpha-1}$, we have

$$\pi_R(R) = \pi_k(k) / |\frac{dR}{dk}| = \frac{m}{1-\alpha} R^{\gamma-1} \exp(-\beta R),$$

where $\gamma = 2\frac{(1+\tau_c)^{1-\frac{1}{\sigma}} b + (1-\tau')^2 \varepsilon^2}{(1-\alpha)(1-\tau')^2 \varepsilon^2} > 0$, $\beta = \frac{2A(1-\tau)}{(1-\alpha)(1-\tau')^2 \varepsilon^2} > 0$, and b is determined by equation (12').

Therefore, we have

$$m = \frac{(1-\alpha)\beta^{\gamma}}{\Gamma(\gamma)},$$

where $\Gamma(.)$ is the gamma function[6].

Thus, the steady-state distribution for the capital stock is

[6]The Gamma function $\Gamma(\alpha)$ is defined as

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx.$$

$$\pi(k) = \begin{cases} 0, & k \le 0 \\ \frac{(1-\alpha)\beta^{\gamma}}{\Gamma(\gamma)} k^{-\frac{2((1+\tau_c)^{1-\frac{1}{\sigma}} b + (1-\tau')^2 \varepsilon^2)}{(1-\tau')^2 \varepsilon^2}} \exp(-\beta k^{-(1-\alpha)}), & k > 0 \end{cases} \tag{17}$$

and the moment-generating function

$$\Phi_k(\theta) = E\{k^{\theta}\} = \frac{\Gamma(\gamma - \theta/(1-\alpha))}{\Gamma(\gamma)} \beta^{\theta/(1-\alpha)} \tag{18}$$

The steady-state distribution for $R = k^{\alpha-1}$ is

$$\pi(R) = \begin{cases} 0, & R \le 0 \\ \frac{\beta^{\gamma}}{\Gamma(\gamma)} R^{\gamma-1} \exp(-\beta R), & R > 0 \end{cases} \tag{19}$$

and the moment-generating function

$$\Phi_R(\theta) = E\{R^{\theta}\} = \frac{\Gamma(\gamma+\theta)}{\Gamma(\gamma)} \beta^{-\theta} \tag{20}$$

The steady-state distribution for $y (= Ak^{\alpha})$ is

$$\pi(y) = \begin{cases} 0, & y \le 0 \\ \frac{\eta\beta^{\gamma}}{A\Gamma(\gamma)} (\frac{y}{A})^{-(\eta\gamma+1)} \exp(-\beta(\frac{y}{A})^{-\eta}), & y > 0 \end{cases} \tag{21}$$

and the moment-generating function

$$\Phi_y(\theta) = E\{y^{\theta}\} = \frac{A^{\theta}\Gamma(\gamma - \theta/\eta)}{\Gamma(\gamma)} \beta^{\theta/\eta} \tag{22}$$

With the aid of these steady-state distributions and moment-generating functions, we can derive quite a few long-run properties of our endogenous variables in the following section of comparative static analysis.

4. Comparative static analysis

With the given steady-state distributions of the capital stock, output, and the interest rate, we can derive the long-run expected values of capital stock, consumption level, and output

$$E(k) = \int_0^{\infty} k\pi(k)dk = \frac{\Gamma[\gamma - 1/(1-\alpha)]}{\Gamma(\gamma)} \beta^{1/(1-\alpha)}, \tag{23a}$$

$$E(c) = (1+\tau_c)^{-\frac{1}{\sigma}} bE(k), \tag{23b}$$

$$E(y) = \frac{A\Gamma(\gamma - 1/\eta)}{\Gamma(\gamma)} \beta^{1/\eta}, \tag{23c}$$

where b, β, η, and γ are presented in section 3 above, and $\Gamma(.)$ is the Gamma function.

4.1 Effects of uncertainty on expected capital, output, and consumption

As in Zou (1994), the modified golden rule for the long-run capital stock in a deterministic model with the spirit of capitalism can be derived as

$$(1-\tau) f'(k) = \rho - (1+\tau_c) \frac{u_k(c,\ k)}{u_c(c,\ k)}$$

With our special utility function and production function and with the parameter condition of $\alpha = \sigma$, we have

$$(1-\tau) A\alpha k^{\alpha-1} + (1+\tau_c)\xi\left(\frac{1-\tau}{1+\tau_c} A k^{\alpha-1}\right)^{\sigma} = \rho$$

and the associated consumption level and output are

$$c^* = \frac{1-\tau}{1+\tau_c} A(k^*)^{\alpha}, \quad y^* = A(k^*)^{\alpha}$$

Comparing with the uncertainty case, we have

$$E(k) < k^*, \ \ E(c) < c^*, \ \ E(y) < y^*$$

Thus, the long-run expected capital stock, expected consumption level, and expected output are smaller than the deterministic steady-state ones, respectively. This is because that the output is a strictly concave function of the capital stock, and Jensen's inequality implies that an increase in capital risk must reduce the expected capital stock and expected output. The fall in the expected output results in a fall in the expected consumption.

4.2 Effects of the spirit of capitalism

In our special utility function, we know that the parameter ξ measures the representative agent's concern with his social status or his spirit of capitalism. Because we have specified the parameters as $\alpha = \sigma$, we have $\sigma \in [0,\ 1]$.

(**Please insert figure 1 about here**)

Figure 1 presents the effects of the spirit of capitalism on the economy under the cases of $\alpha = 1$ (the solid line) and $\alpha \neq 1$ and $\alpha = \sigma$ (the star line). It is easy to see that with a stronger spirit of capitalism, the long-run expected capital stock, consumption level, and output will be higher.

4.3 Effects of production shocks

(**Please insert figure 2 about here**)

Figure 2 shows that with increasing production shocks, the long-run expected capital stock, output, and consumption will be decreasing. Therefore, uncertainty in production reduces investment, output and consumption. This result is rather clear-cut because other related studies have indicated an ambiguous result of production shocks on investment and output, see Turnovsky (1993, 2000), Obstfeld (1994), and Gong and Zou (2002).

4.4 Effects of fiscal policies

(**Please insert figure 3 about here**)

The solid line in figure 3 shows the effects of income tax rate on the long-run economy. From which, we find the with a rise in the deterministic income tax rate, the long-run capital stock, output, and consumption will be decreasing (solid lines in figure 3). The effects of stochastic income tax rate (starred lines in figure 3) on the economy are just opposite to the effects of deterministic income tax rate: A rising stochastic income tax rate raises expected capital stock, output and consumption.

As for the effects of consumption tax rate on the economy, from the circled line in figure 3, we find that with an increasing consumption tax rate, the long-run expected capital stock and output will be rising, whereas the long-run expected consumption will be decreasing. This is true because a rising consumption tax raises the cost of consumption, which leads to a reduction in consumption and an increase in investment, capital stock and output. Please note that this positive effect of a consumption tax rate on capital accumulation and output is a significant feature of stochastic growth model. In the traditional, deterministic literature such as Rebelo (1990), a consumption tax has no effect on the long-run capital accumulation.

5. Equilibrium interest rates

From Cox, Ingersoll, and Ross (1985), we know that the equilibrium interest rates can be written as

$$r = \rho - \frac{L(X_k)}{X_k},$$

where $L(.)$ is the differential operator.

Thus we have[7]

Proposition 2. *With the utility function and technology in equations* (7) *and* (8), *the equilibrium interest rate is given by*

$$r = \rho + \sigma((1-\tau)Ak^{\alpha-1} - (1+\tau_c)^{1-\frac{1}{\sigma}}b) - \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1) \qquad (24)$$

when $\alpha \neq 1$ *and* $\alpha = \sigma$, *where* b *is determined by equation* (15'). *Furthermore, the equilibrium interest rate is given by*

$$r = \rho + \sigma((1-\tau)A - (1+\tau_c)^{1-\frac{1}{\sigma}}b) - \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1) \qquad (25)$$

when $\alpha = 1$, *where* b *is determined by equation* (15).

From proposition 2, the equilibrium interest rate is a constant when the technology is linear. Among many existing studies, Sundareson (1983, 1984) has also presented a constant interest rate for a constant absolute risk aversion utility function in an infinite horizon dynamic portfolio and consumption choice problem. Our model obtains the same result while allowing the utility function to be dependent on both consumption and

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[7]With the utility function and technology in equations (B1) and (8), the equilibrium interest rate is given by

$$r = \rho + (\sigma+\lambda)((1-\tau)Ak^{\alpha-1} - (1+\tau_c)^{1-\frac{1}{\sigma}}b) + \frac{1}{2}(1-\tau')^2\varepsilon^2(\sigma+\lambda)(-\sigma-\lambda-1)$$

when $\alpha \neq 1$ and $\alpha = \sigma + \lambda$, where b is determined by

$$b = \frac{\rho\frac{1-\sigma}{1-\sigma-\lambda} + \frac{1}{2}(1-\sigma)(\sigma+\lambda)(1-\tau')^2\varepsilon^2}{(1+\tau_c)^{1-\frac{1}{\sigma}}\sigma}.$$

On the other hand, the equilibrium interest rate is given by

$$r = \rho + (\sigma+\lambda)((1-\tau)A - (1+\tau_c)^{1-\frac{1}{\sigma}}b) + \frac{1}{2}(1-\tau')^2\varepsilon^2(\sigma+\lambda)(-\sigma-\lambda-1)$$

when $\alpha = 1$, where b is determined by

$$b = \frac{\rho\frac{1-\sigma}{1-\sigma-\lambda} - (1-\tau)A + \frac{1}{2}(1-\sigma)(\sigma+\lambda)(1-\tau')^2\varepsilon^2}{(1+\tau_c)^{1-\frac{1}{\sigma}}\sigma}.$$

wealth.

Comparative static analysis shows that, with a rise of technology shocks, the equilibrium interest rate will be decreasing; with a rise in the deterministic income tax rate, the equilibrium interest rate will be increasing; but the equilibrium interest rate will be decreasing with a rise of the stochastic income tax rate. Also, we find that with the increase of the consumption tax rate, the equilibrium interest rate will be increasing; please see figure 4.

When $\alpha \neq 1$, the equilibrium interest rate is stochastic, not a constant anymore. Using the expression for the equilibrium interest rate, the dynamics for the capital stock can be rewritten as

$$\frac{dk}{k} = \frac{1}{\sigma}(r - \rho + \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1))dt + \varepsilon dz \tag{26}$$

Thus, the dynamics of the interest rate is

$$\begin{aligned} dr = [&\frac{1}{\sigma}(r - \rho + \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1)) + (1+\tau_c)^{1-\frac{1}{\sigma}}b] \\ &\times\{(r - \rho + \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+\alpha-1))dt + (1-\tau')\varepsilon dz\} \end{aligned} \tag{27}$$

If $\varepsilon = 0$ and $\xi = 0$, we have

$$\frac{dk}{k} = \frac{1}{\sigma}(r-\rho)dt, \quad \frac{dr}{r} = \frac{1}{\sigma}(r-\rho)dt$$

These are dynamic accumulation paths for the capital stock and the interest rate without production shocks and the spirit of capitalism. It is obviously that the equilibrium interest rate will convergent to ρ.

Equations (26) and (27) can be used to study the behavior of the interest rate in this economy. For example, when the initial interest rate is very high, say it is larger than $\rho - \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1)$, then the capital stock will be growing. And from the expression for the interest rate, it will go down. If the initial interest rate is lower enough, say lower than $\rho - \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1)$, then the capital stock will be increasing, thus the interest rate will be go up. Thus, the equilibrium interest rate will fluctuate around a value depending on ρ, ε^2, and σ.

Similarly, we can find the stationary distribution for the interest rate. For simplicity, we define $\overline{r} = \alpha(1-\tau)Ak^{\alpha-1}$, the steady-state distribution and the moment-generating function for variable $\hat{r}$ can be found as

$$\pi(\overline{r}) = \begin{cases} 0, & \overline{r} \le 0 \\ \frac{(\frac{\beta}{\alpha(1-\tau)A})^{\gamma}}{\Gamma(\gamma)} \overline{r}^{\gamma-1} \exp(-\frac{\beta}{\alpha(1-\tau)A}\overline{r}), & \overline{r} > 0 \end{cases} \tag{28}$$

$$\Phi_{\hat{r}}(\theta) = E\{\hat{r}^{-\theta}\} = \frac{\Gamma(\gamma+\theta)}{\Gamma(\gamma)}(\frac{\beta}{\alpha(1-\tau)A})^{-\theta} \tag{29}$$

Thus, we get the long-run behavior of the equilibrium interest rate

$$E(r) = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma)}(\frac{\beta}{\alpha(1-\tau)A})^{-1} + \rho - \sigma(1+\tau_c)^{1-\frac{1}{\sigma}}b - \frac{1}{2}\varepsilon^2(1-\tau')^2\sigma(\sigma+1), \tag{30}$$

where b is determined by equation (15).

(Please insert figure 4 about here)

From figure 4, we know that the long-run expected interest rate will be decreasing with a rise in technology shocks and the deterministic income tax rate (the star line in figure 4c). At the same time, the stochastic income tax rate, the consumption tax rate, and the spirit of capitalism all have positive effects on the long-run expected interest rate.

6. Multiple optimal paths and stationary distributions

From equation (15'), we cannot determine the unique solution for variable b. In this section, we examine the existence of multiple solutions for the consumption-capital ratio for a few selected parameters. Because there exists a unique path for the capital accumulation associated with the consumption-capital ratio, there will exist a unique steady-state distribution associated with each path[8]. Below, we will present examples to show the existence of multiple optimal paths or stationary distributions and their associated long-run expected capital stocks, consumption levels, equilibrium interest rates, and output.

If we select the parameters as $A = 0.5$, $\alpha = \sigma = 0.6$, $\tau = 0.3$, $\tau' = 0.3$, $\xi = 0.1$, $\rho = 0.1$, $\tau_c = 0$, and let ε^2 vary from 0.5, 1, and 1.1, and we get the following results

[8]For the non-separable utility function in (B1) in Appendix B, we can determine the unique steady state.

Table 1: Multiple optimal paths when $\alpha = \sigma = 0.6$

	$\varepsilon^2 = 0.5$			$\varepsilon^2 = 1$			$\varepsilon^2 = 1.1$		
	path 1	path 2	path 3	path 1	path 2	path 3	path 1	path 2	path 3
c/k	0.0210	0.3519	0.1601	0.4217	0.2010	0.0430	0.2093	0.4355	0.0476
$E(k)$	150.1	0.8	4.4	0.4	2.0	25.8	1.7	0.4	20.1
$E(r)$	0.0559	0.0559	0.0559	0.0118	0.0118	0.0118	0.003	0.003	0.003
$E(c)$	3.1568	0.2732	0.6984	0.1810	0.3967	1.1067	0.3619	0.1688	0.9566
$E(y)$	4.5098	0.3902	0.9977	0.2585	0.5667	1.5811	0.5171	0.2411	1.3665

For the case of a linear technology， i.e.,$\alpha = 1$, we have derived the mean growth rate of the economy and the equilibrium interest rate as follows

$$\phi = (1-\tau)A - (1+\tau_c)^{1-\frac{1}{\sigma}} b$$

$$r = \rho + \sigma((1-\tau)A - (1+\tau_c)^{1-\frac{1}{\sigma}} b) - \frac{1}{2}\varepsilon^2 (1-\tau')^2 \sigma(\sigma+1)$$

where b is determined by equation (15).

In this case, we select the parameters as: $\alpha = 1$, $A = 0.43$, $\sigma = 0.6$, $\tau = 0.3$, $\tau' = 0.3$, $\rho = 0.21$, and $\tau_c = 0$. When $\xi = 0$, we have a unique path or stationary distribution for consumption-capital ratio, the growth rate, and the equilibrium interest rate. When $\xi = 0.025$, we have three stationary distributions for these variables. See Table 2 for details.

Table 2: Multiple optimal paths when $\alpha = 1$

	$\xi = 0$	$\xi = 0.025$		
	path 1	path 1	path 2	path3
c/k	0.2473	0.2245	0.2578	0.2301
mean growth rate	0.0537	0.0765	0.0432	0.0709
interest rate	0.007	0.0207	0.0007	0.0174

Finally, we select the parameters as: $\alpha = 1$, $A = 0.46$, $\sigma = 0.6$, $\tau = 0.3$, $\tau' = 0.3$, $\rho = 0.25$, and $\tau_c = 0$. That is to say, we only change the values of A and the discount rate of ρ slightly. Again, we have multiple expected values or multiple stationary distributions in the economy. See details in Table 3:

Table 3: Multiple optimal paths when $\alpha = 1$

	$\xi = 0$	$\xi = 0.025$		
	path 1	path 1	path 2	path 3
c/*k*	0.3000	0.2744	0.3119	0.2806
mean growth rate	0.0220	0.0476	0.0101	0.0414
interest rate	0.0280	0.0434	0.0209	0.0397

This existence of multiple stationary distributions for asset accumulation and interest rates is significant different from the unique stationary distribution in Brock and Mirman (1972), Merton (1973), Lucas (1978), Brock (1982), Cox, Ingersoll and Ross (1985), and many other classical models on stochastic capital theory and the term structure of interest rates. In fact, multiple stationary distributions in asset markets and returns may provide a more realistic picture of the real world because it admits the rationality and plausibility of different expectations and heterogeneity, though our model is still in line with the representative agent framework.

7. Conclusion

This paper has studied capital accumulation and the equilibrium interest rates in stochastic production economies with the spirit of capitalism. Under the specified utility function, production function, and selected parameters, we have presented the explicit solutions for consumption and capital accumulation. With the aid of steady-state distributions for the capital stock, we presented the effects of fiscal policies, the spirit of capitalism, and stochastic shocks on the long-run economy.

We find that the long-run capital stock, output, and consumption level in the uncertainty case are less than those ones in the deterministic case; the long-run interest rate in the uncertainty case is larger than the deterministic case. These conclusions are different from the one presented by Merton (1975), similar to the one in Smith (1998).

As for the effects of the spirit of capitalism on the long-run economy, we find that with the increase of the spirit of capitalism, the long-run interest rate, consumption level, and output will be decreasing. The effect of the spirit of capitalism on the long-run capital stock will be negative when the production shocks are small; its effect will be positive when the production shocks are larger. These findings are different from the ones in Gong and Zou (2001, 2002), and Zou (1994, 1995).

The effects of the income tax rate on the long-run economy have also been investigated in this paper, and we have found the similar effects of a deterministic income tax and a stochastic income tax rate presented by Gong and Zou (2002), Turnovsky (1993, 2000). With the rise in the deterministic income tax rate, the long-run capital stock, output, and consumption level will be decreasing, but the interest rate will be increasing. The effects of the stochastic income tax rate on the long-run economy are just opposite to the effects of the deterministic income tax rate on the long-run economy. We have also shown that the consumption tax rate will affect the long-run expected capital stock, output, and consumption, which are different from the ones in the traditional, deterministic models.

The equilibrium interest rate has been shown under a linear technology and a nonlinear technology, respectively. When the production technology is linear, we can still obtain a constant interest rate for this stochastic model with the spirit of capitalism and social status. This result is similar to the one in Sunderason (1983), who presented the conclusion of constant interest rate under the assumption of CES utility function and a linear technology. Of course, his utility function is independent of the state variable of capital stock. But, with a nonlinear technology, we find that the interest rate follows the mean reserve process and fluctuates around a value depending on the parameters of ρ and σ.

Finally, the existence of multiple stochastic optimal paths or multiple stationary distributions for capital accumulation is presented in this paper. This is a main feature of a model with the spirit of capitalism or social-status concern. Associated with the multiple stationary distributions for capital accumulation, there exist multiple expected interest rates. This line of investigation enriches our understanding of the complexity of asset markets and the term structure of interest rates.

This paper considers an economy with one consumption good and production technology. A first extension of this paper is to follow Sunderason (1983) to study the equilibrium interest rate in an economy with many consumption goods and production technologies. Secondly, this paper has not considered monetary policy, and we should follow Grinols and Turnovsky (1998) and extend this model to a monetary one with the spirit of capitalism. Thirdly, we can extend this model to consider habit formation, catching up with the Joneses, and the non-expected utility.

Appendix A: The steady-state distribution for a diffusion process

We follow Merton (1975) and consider the steady-state distribution for a diffusion process. Let $X(t)$ be the solution to the Itô equation

$$dx = b(x)dt + (a(x))^{1/2} dz ,$$

where $a(.)$ and $b(.)$ are twice-differentiable function on $[0,\infty)$ and independent of t with $a(x) > 0$ and $a(0) = b(0) = 0$.

The steady-state distribution will always exist, and it can be expressed as

$$\pi(x) = \frac{m}{a(x)} \exp \int_0^x \frac{2b(y)}{a(y)} dy ,$$

where m is chosen such that $\int_0^\infty \pi(x)dx = 1$.

Appendix B: The case of non-separable utility function

If we specified the utility function as in Bakshi and Chen (1996), Gong and Zou (2002)

$$u(c,\ k)=\frac{c^{1-\sigma}k^{-\lambda}}{1-\sigma}, \tag{B1}$$

where σ is the constant absolute risk aversion, and it is assumed $\sigma>0$, and $\lambda\ge 0$ when $\sigma\ge 1$, and $\lambda<0$ otherwise; $|\lambda|$ measures the investor's concern with his social status or measures his spirit of capitalism. The larger the parameter $|\lambda|$, the stronger the agent's concern for social status.

For the specified utility function and production function, we conjecture that the value function takes the following form

$$X(k)=a+\frac{b^{-\sigma}k^{1-\sigma-\lambda}}{1-\sigma-\lambda},$$

where a and b are constant, and they are to be determined as follows.

From the first-order condition, we have

$$c=(1+\tau_c)^{-\frac{1}{\sigma}}bk$$

and the Bellman equation (6) is reduced to

$$\frac{(1+\tau_c)^{1-\frac{1}{\sigma}}b^{1-\sigma}k^{1-\sigma-\lambda}}{1-\sigma}-\rho(a+\frac{b^{-\sigma}k^{1-\sigma-\lambda}}{1-\sigma-\lambda})+b^{-\sigma}k^{-\sigma-\lambda}((1-\tau)Ak^{\alpha}-(1+\tau_c)^{1-\frac{1}{\sigma}}bk)$$
$$+(1-\tau')^2\frac{1}{2}X_{kk}\varepsilon^2k^2=0.$$

If $\alpha=1$, from the above equation, we have

$$a=0,$$
$$b=\frac{\rho\frac{1-\sigma}{1-\sigma-\lambda}-(1-\tau)A+\frac{1}{2}(1-\sigma)(\sigma+\lambda)(1-\tau')^2\varepsilon^2}{(1+\tau_c)^{1-\frac{1}{\sigma}}\sigma}$$

In generally, for the case of $\alpha\neq 1$, we cannot determine the constants a and b. Following Xie (1994), we specified the parameters as $\alpha=\sigma+\lambda$, then, we have

$$a=\frac{(1-\tau)A}{\rho b^{\sigma}},$$
$$b=\frac{\rho\frac{1-\sigma}{1-\sigma-\lambda}+\frac{1}{2}(1-\sigma)(\sigma+\lambda)(1-\tau')^2\varepsilon^2}{(1+\tau_c)^{1-\frac{1}{\sigma}}\sigma}.$$

The remaining discussions are similar to ones in the main text.

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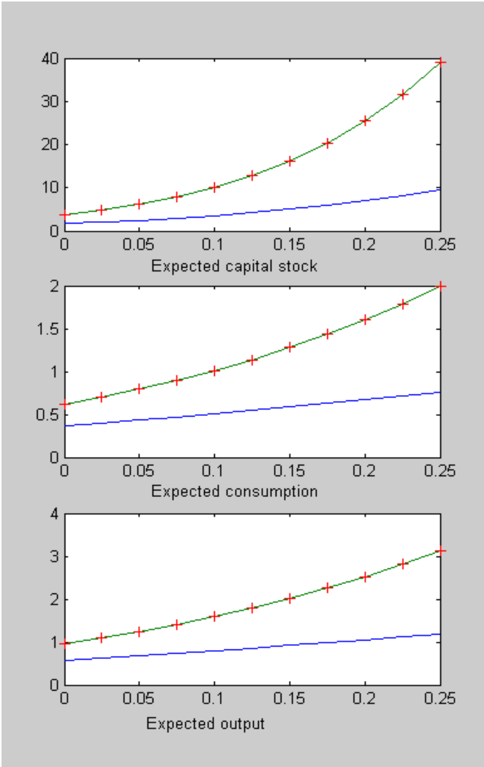

Figure 1: The effects of the spirit of capitalism on the long-run economy.

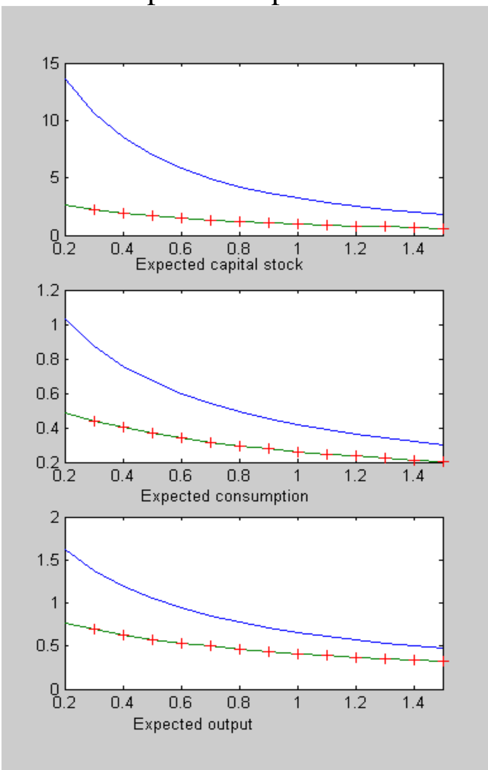

Figure 2: The effects of production shocks on the long-run economy.

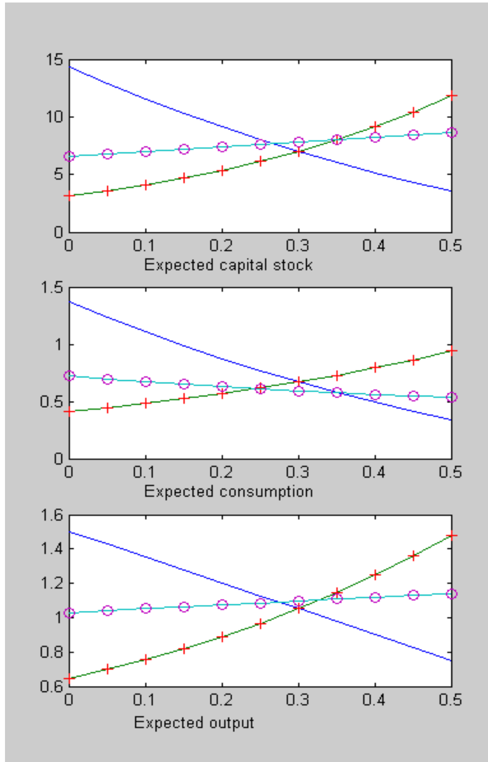

Figure 3: The effects of the deterministic income tax rate, the stochastic income tax rate, and the consumption tax rate on the long-run economy.

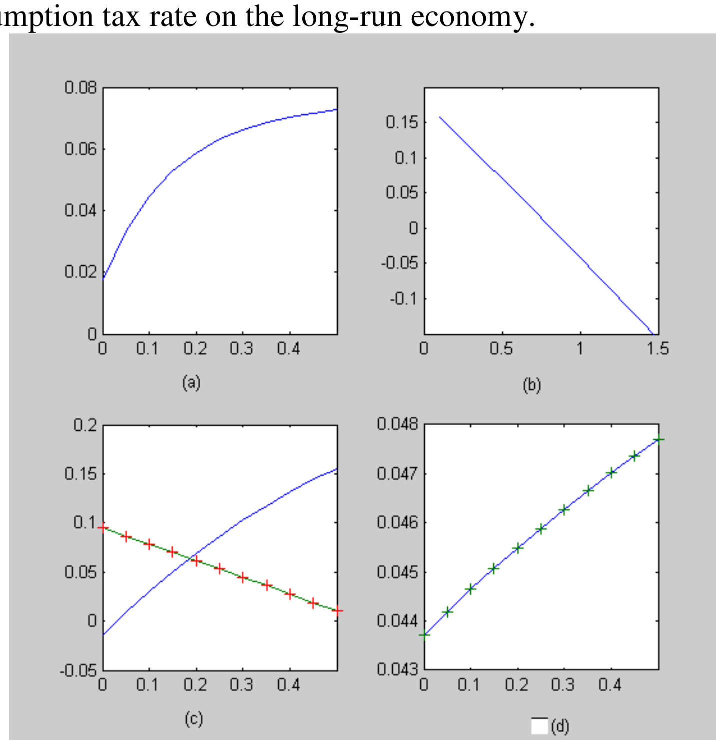

Figure 4: (a) Effects of the spirit of capitalism on the interest rate; (b) Effects of production shocks on the interest rate; (c) Effects of the deterministic income tax rate and the stochastic income tax rate on the interest rate; (d) Effects of the consumption tax rate on the interest rate.