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# Inventory Mistakes and the Great Moderation

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# Inventory Mistakes and the Great Moderation

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**Abstract**

Why did the volatility of U.S. real GDP decline by more than the volatility of final sales with the Great Moderation in the mid-1980s? One possible explanation is that firms shifted their inventory behaviour towards a greater emphasis on production smoothing. In this paper, we investigate the role of inventories in the Great Moderation by estimating an unobserved components model that identifies inventory and sales shocks and their propagation. We find only mixed evidence of increased production smoothing. Instead, it was a reduction in inventory mistakes that accounts for the excess volatility reduction in output relative to sales. The inventory mistakes are informational errors related to production that must be set in advance and their reduction also helps to explain the changed forecasting role of inventories since the mid-1980s. Our findings provide an optimistic prognosis for the continuation of the Great Moderation despite the dramatic movements in output during the recent economic crisis.

*Keywords*: Great Moderation; production smoothing; inventory mistakes; unobserved components model

*JEL codes*: E32; E22; C01

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## 1 Introduction

Lower volatility of the growth rate of the U.S. real GDP since the mid-1980s, first documented by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), has spurred extensive research into its causes. Better inventory management is often put forth as one of the leading explanations for this so-called "Great Moderation".[1] The emphasis on inventories is motivated by a striking but well-known feature of the data—output growth was more volatile than sales growth prior to the mid-1980s, but since then output and sales have shared a similar lower level of volatility. Given the accounting relationship between output, sales, and inventory investment, the excess volatility reduction in output relative to sales directly implies some role for inventories in the Great Moderation.

What is it about inventory behaviour that has changed? One possible answer is that firms shifted their inventory behaviour towards a greater emphasis on production smoothing. Golob (2000) finds that the stylized facts emphasized by Blinder and Maccini (1991) as being so challenging to the relevance of production smoothing theories of inventories have shifted in a more favourable direction in recent years. Kahn, McConnell, and Perez-Quiros (2002) focus on the durable goods sector and find evidence of an improved ability of inventories to forecast future sales, leading them to argue that better information has facilitated improvements in inventory management. By contrast, Herrera and Pesavento (2005) consider industry-level manufacturing and trade data and find little evidence of a change in the relationship between inventories and sales.[2]

In this paper, we estimate an unobserved components model to help disentangle the role of inventories from that of sales in explaining the decline in the volatility of U.S. aggregate output. We find that changes in the

[1]Other explanations are better monetary policy and smaller macroeconomic shocks (a.k.a. "good luck"). See Clarida, Gali, and Gertler (2000), Stock and Watson (2003), and Ahmed, Levin, and Wilson (2004), among many others.

[2]McCarthy and Zakrajsek (2007) consider both aggregate and industry-level data together and conclude that changes in inventory behavior have, along with monetary policy changes, contributed to the volatility decline.

sales process explain about half of the overall decline. However, in terms of the excess decline in output volatility relative to sales, we find that it reflects smaller inventory mistakes rather than a shift towards greater production smoothing, where inventory mistakes reflect informational errors made by firms when their setting production in advance of sales. Moreover, the reduction in inventory mistakes also helps explain the apparent changed forecasting role of inventories with the Great Moderation.

Our findings have important implications for the much-questioned continuation of the Great Moderation. While inventory mistakes will continue to be made, the reduction in their magnitude likely reflects structural changes in the economy such as improved informational flows and/or the rise of "just-in-time" production. Thus, even if the Great Moderation were due to smaller shocks rather than changes in their propagation, as emphasized by Stock and Watson (2003), Ahmed, Levin, and Wilson (2004), and many others, the shocks are not just those that fit under the ephemeral-sounding "good luck" hypothesis. In particular, despite large aggregate shocks during the recent economic crisis, the likely technological and structural reasons for smaller inventory mistakes suggest that we should not expect a return to the ongoing high levels of output volatility experienced during the 1970s and earlier.

The rest of this paper is organized as follows. Section 2 presents some stylized facts in the data that motivate our unobserved components model and presents a simple cost minimization analysis to provides some context for interpreting our empirical results. Section 3 develops the unobserved components model that we use to disentangle the roles of inventory and sales shocks and their propagation in explaining the Great Moderation. Section 4 reports the empirical results for the unobserved components model. Section 5 considers the implications of our findings for the continuation of the Great Moderation and concludes.

## 2 Background

### 2.1 Output volatility and its components

Output, sales, and inventories are related to each other by the following identity:

$$y_t \equiv s_t + \Delta i_t \tag{1}$$

where $y_t$ is the natural logarithm of output, $s_t$ is the natural logarithm of sales, and $\Delta i_t$ is a residual measure of inventory investment.[3] Using quarterly data from the Bureau of Economic Analysis (BEA) on U.S. real GDP and final sales (lines 1 and 2 of NIPA Table 1.2.6), we calculate the volatility of output growth and its components for the respective pre- and post-moderation sample periods of 1960Q1-1984Q1 and 1984Q2-2011Q1.[4] Table 1 reports the basic sample statistics related to the volatility of the variables in equation (1). The most notable stylized fact to emerge from these sample statistics is that real GDP growth stabilized dramatically in recent years, as has been widely reported in the literature. However, the other notable stylized fact is that output was more volatile than sales in the pre-moderation period, but both have a similar lower-level of volatility in the post-moderation period, which has also been discussed previously (see, for example, Kahn, McConnell, and Perez-Quiros (2002) and Golob (2000)).

One possible explanation for these changes in volatility is an increased emphasis on production smoothing by firms. Yet, the sample statistics provide mixed signals about the overall relevance of production smoothing. In

[3]The true accounting identity is between the levels of output, sales, and inventory investment rather than logarithms. However, it will be convenient for us to work with logarithms in terms of specifying our unobserved components model. Meanwhile, sample statistics for the decomposition of output volatility into its components are very similar whether we consider equation (1) or we standardize level changes by the lagged level of output. Put another way, our residual measure of inventory investment $\Delta i_t \equiv y_t - s_t$ is highly correlated with the actual change in inventories expressed as a percentage of the lagged level of output. For the data considered in this paper, the correlation is 0.99996.

[4]Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) both estimate the structural break in the variance of U.S. real GDP growth to have occurred in 1984Q1. In order to keep our analysis focused, we treat this break date as known for the purposes of estimation, although we note there is some degree of uncertainty about its exact timing (see, for example, Stock and Watson (2003) and Eo and Morley (2008)).

TABLE 1. SAMPLE STATISTICS

| | Pre-moderation (1960Q1-1984Q1) | Post-moderation (1984Q2-2011Q1) |
|---|---|---|
| s.d.($\Delta y_t$) | 1.08 | 0.60 |
| s.d.($\Delta s_t$) | 0.84 | 0.58 |
| s.d.($\Delta^2 i_t$) | 0.68 | 0.39 |
| corr($\Delta s_t, \Delta^2 i_t$) | $-0.01$ | $-0.30$ |

Table 1: Sample standard deviation (s.d.) and correlation (corr.) statistics are reported for the first differences of log output, log sales, and a residual measure of inventory investment based on the difference between log output and log sales. All series are multiplied by 100.

the pre-moderation period, both the excess volatility of output relative to sales and the lack of a large negative contemporaneous correlation between sales and inventories directly undermine the idea that firms use inventories to buffer production from fluctuations in sales, as emphasized in the survey article by Blinder and Maccini (1991). By contrast, the shift to more similar levels of volatility and a negative contemporaneous correlation between sales and inventories in the post-moderation period is more consistent with production smoothing, as pointed out by Golob (2000). However, the finding that both sales and inventories also became less volatile in the post-moderation period clearly argues against production smoothing as the sole explanation for the Great Moderation. Meanwhile, the idea that output is still no less volatile than sales in the post-moderation period continues to argue against production smoothing as the primary motive for holding inventories.[5] These mixed signals from the basic sample statistics motivate our development of an unobserved components model in Section 3 to help disentangle the role of increased production smoothing from other factors in explaining the Great Moderation.

[5] Also, as emphasized by Blinder and Maccini (1991), changes in finished goods inventories, which can be most directly related to the production smoothing motive, are neither the largest nor most volatile component of inventory investment.

## 2.2 Inventories and forecasting

In addition to the well-known reduction in volatility, the Great Moderation also corresponded to a change in the forecasting role of inventories (see, for example, Kahn, McConnell, and Perez-Quiros (2002)). Figure 1 motivates why inventories are so useful for forecasting output and sales. The left panel plots log output and log sales based on the BEA data discussed above. Both series are nonstationary, which is easily confirmed by standard unit root and stationarity tests. However, both series appear to share the same stochastic trend. The right panel plots the first-differences of the two series and the difference between the two series, which is our residual measure of inventory investment. All of these series are stationary, which again is confirmed by standard tests. More formally, the idea that our residual measure of inventory investment is stationary corresponds to cointegration between log output and log sales with a cointegrating vector of $[1, -1]$. Cointegration corresponds to the idea that output and sales share the same stochastic trend, which is important because it implies that the cointegrating error term (i.e., inventory investment) *must* forecast future movements in output and/or sales in order for the long-run cointegrating relationship to be restored over time.

We demonstrate the change in the forecasting role of inventories with a simple vector error correction model (VECM), given as follows:

$$\Delta y_t = c_{y,0} + \alpha_y(y_{t-1} - s_{t-1}) + \sum_{j=1}^{p} \gamma_{yy,j}\Delta y_{t-j} + \sum_{j=1}^{p} \gamma_{ys,j}\Delta s_{t-j} + e_{y,t} \quad (2)$$

$$\Delta s_t = c_{s,0} + \alpha_s(y_{t-1} - s_{t-1}) + \sum_{j=1}^{p} \gamma_{ss,j}\Delta s_{t-j} + \sum_{j=1}^{p} \gamma_{sy,j}\Delta y_{t-j} + e_{s,t} \quad (3)$$

where the $\alpha$ parameters are the error-correction coefficients and we determine the lag order $p$ based on the Schwarz Information Criterion (SIC).

Table 2 reports the estimates for the error-correction coefficients for the same sample periods of 1960Q1-1984Q1 and 1984Q2-2011Q1 considered above. In the pre-moderation period, the estimate $\hat{\alpha}_y = -0.70$ suggests that

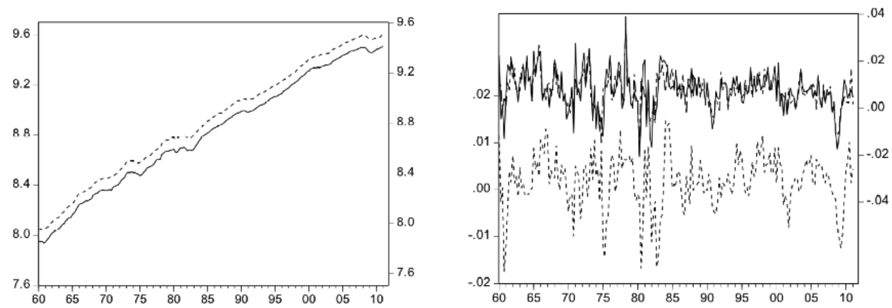


Figure 1: The left panel plots real GDP (solid line, left vertical axis) and final sales (dashed line, right vertical axis), both expressed in natural logarithms. The first differences of the two series (right vertical axis) along with the residual of the change in inventories (thick dashed line, left vertical axis) are plotted in the right panel. The sample period is 1960Q1-2011Q1.

TABLE 2. ERROR CORRECTION COEFFICIENTS

| | Pre-moderation (1960Q1-1984Q1) | Post-moderation (1984Q2-2011Q1) |
|---|---|---|
| $\alpha_y$ | $-0.70$ (0.18) | $-0.26$ (0.15) |
| $\alpha_s$ | $-0.11$ (0.16) | 0.52 (0.15) |

Table 2: OLS estimates are reported, with standard errors in parentheses. SIC selects a lag order of $p = 1$ for the pre-moderation sample and $p = 2$ for the post-moderation sample (and the full sample). The results are qualitatively robust for different numbers of lags and are reported here for $p = 2$, with the lag coefficients suppressed for simplicity.

a positive change in inventories predicts a large decline in future output, all else equal. Meanwhile, inventory investment appears to have no significant predictive impact on future sales. The results for the post-moderation period are strikingly different. First, the estimates suggest that a positive change in inventories still predicts a decline in future output, but there is a much smaller estimated change that is not statistically significant at the 5% level. Second, the estimate $\hat{\alpha}_s = 0.52$ suggests that a positive change in inventories predicts an increase in future sales, all else equal. Put simply, inventories had a strong negative forecasting relationship with future output prior to the Great Moderation, but since then, inventories have had a strong positive forecasting relationship with future sales.

At first glance, the finding that inventories forecast future sales in the post-moderation period might seem supportive of increased production smoothing. For example, Kahn, McConnell, and Perez-Quiros (2002) hypothesize that improvements in information technology have helped firms better anticipate future sales, with inventories being more reflective of intentional production smoothing towards these future sales. However, the forecasting role of inventories might have simply changed due to a different composition of the underlying shocks driving inventory investment. Unfortunately, the role of production smoothing versus a change in the composition of shocks cannot be disentangled from the VECM results alone. Again, as with the stylized facts in Table 1, we are motivated by these competing explanations to develop an unobserved components model in Section 3.[6] Still, the VECM results clearly illustrate that the changed forecasting role of inventories is an important aspect of the Great Moderation that

[6]Also, the finding for the VECM that both output and sales adjust to restore the long-run equilibrium directly implies the presence of a common unobserved stochastic trend rather than one or the other of the variables acting as the *de facto* trend. This result motivates the structure of our unobserved components model in Section 3. Meanwhile, the unobserved components structure also implies a reduced-form dynamics that can only be approximated by a finite-order VECM. Although the reasonableness of the approximation depends on both the data generating process and the number of lags in the VECM, it should be noted that our results in terms of a changed forecasting role for inventories appear to be quite robust to different assumptions about the lag order.

should be compatible with any comprehensive explanation for the reduced volatility.

### 2.3 Cost minimization

In order to be a bit more formal about the motives for holding inventories and to provide some context for understanding our empirical results, we consider a simple linear-quadratic cost minimization problem, similar to Blanchard (1983) and Ramey and West (1999), but modified to reflect both short-run and long-run tradeoffs between production smoothing and stockout avoidance. Specifically, given an exogenous stochastic sales process with the initial level of sales $s_{t-1} = 0$, and exogenous time-varying long-run targets for output and inventories $\tau_t^*$ and $i_t^*$, the representative firm is assumed to solve the following cost minimization problem at date $t$:[7]

$$\lim_{T\to\infty} \min_{\{i_{t+j}\}_{j=0}^{T}} E_t \sum_{j=0}^{T} b^j C_{t+j} \tag{4}$$

where

$$C_t = 0.5a_1(\Delta y_t)^2 + 0.5a_2\left(y_t - \tau_t^*\right)^2 + 0.5a_3(\Delta i_t)^2 + 0.5a_4(i_t - i_t^*)^2, \tag{5}$$

the discount factor $0 < b < 1$, and $a_i > 0$ for $i = 1, 2, 3, 4$.

The cost of changing output is given by the first two terms, $a_1(\Delta y_t)^2$ and $a_2\left(y_t - \tau_t^*\right)^2$. In the short run, the firm finds it costly to alter current output from its lagged level. In the long run, the firm finds it costly to keep output at a level other than its time-varying long-run target level $\tau_t^*$, which we might expect to be linked to the long-run level of sales. Both terms reflect

[7]The cost minimization problem is a version of the Holt, Modigliani, Muth, and Herbert's (1960) partial equilibrium "linear quadratic" framework characterizing inventory decisions at the firm level. Davis and Kahn (2008), Blinder and Maccini (1991), and others have pointed out that the linear-quadratic framework is more applicable for finished goods inventories than for inventories of materials and supplies held by manufactures, which are arguably better captured by an (S,s) model. However, Ramey and West (1999) argue against such a literal interpretation of the cost function for the representative firm. Also, as discussed in Blinder and Maccini (1991), the (S,s) model cannot be easily applied to study aggregate inventory dynamics. See Wen (2005) for general equilibrium analysis of production smoothing and stockout avoidance motives for holding inventories.

the firm's production smoothing motive, with the resulting emphasis on production smoothing increasing with the cost coefficients $a_1$ and $a_2$. Similarly, the short and long run stockout avoidance motives for holding inventories are captured by the terms $a_3(\Delta i_t)^2$ and $a_4(i_t - i_t^*)^2$, respectively.[8] In the short run, the firm finds it costly to alter inventories from their lagged level. In the long run, the firm finds it costly to keep inventories at a different level than the time-varying target level $i_t^*$, which we might expect to be linked to the long-run level of sales and any other exogenous factors that affect the steady-state level of inventories, such as a shift in the nature of production away from goods towards services. The emphasis on stockout avoidance rather than production smoothing is increasing with the cost coefficients $a_3$ and $a_4$.

For simplicity of the theoretical analysis, we abstract from permanent changes in production by assuming a persistent stationary first-order autoregressive (AR(1)) process for sales, $s_t = \phi_s s_{t-1} + \epsilon_{s,t}$ where $\epsilon_s \sim i.i.d.N(0, \sigma_{u_s})$ and $0 < \phi_s < 1$ . The sales process implies a long-run output target $\tau_t^* = 0$. Also, we assume a long-run inventory target $i_t^* = 0$ for all time periods. Then, optimizing with respect to $i_{t+j}$ gives the system of stochastic Euler equations for $j = 0, 1, ..., T-1$:

$$\begin{aligned} E_{t+j}[\{a_1 \Delta y_{t+j} + a_2 y_{t+j} + a_3 \Delta i_{t+j} + a_4 i_{t+j}\} \\ + b\{-2a_1 \Delta y_{t+j+1} - a_2 y_{t+j+1} - a_3 \Delta i_{t+j+1}\} \\ + b^2\{a_1 \Delta y_{t+j+2}\}] = 0. \quad (6) \end{aligned}$$

Simplifying the above equation, we get

$$\begin{aligned} E_{t+j}[a_1\{\Delta y_{t+j} - 2b\Delta y_{t+j+1} + b^2 \Delta y_{t+j+2}\} + a_2\{y_{t+j} - by_{t+j+1}\} \\ + a_3(\Delta i_{t+j} - b\Delta i_{t+j+1}) + a_4 i_{t+j}\}] = 0. \quad (7) \end{aligned}$$

[8] For simplicity, we consider a continuous and symmetric version of the stockout avoidance motive. Instead of just being concerned with a literal "stockout" (i.e., having insufficient inventories to satisfy a large positive sales shock), which would correspond to a discrete and asymmetric specification for the cost, we assume that the representative firm implicitly has a large enough stock of inventories to satisfy any given sales shock, but that it is costly for it to draw down from or add to target levels of inventories, with costs increasing in the deviations from targets.

While both short-run and long-run motives are useful for interpreting some of our results, it is helpful to abstract from short-run motives for the time being by letting $a_1 = a_3 = 0$. Thus, we can rewrite equation (7) as

$$E_{t+j}[a_2\{s_{t+j} + i_{t+j} - i_{t+j-1} - bs_{t+j+1} - bi_{t+j+1} + bi_{t+j}\} + a_4 i_{t+j}\}] = 0. \quad (8)$$

Rearranging the terms we get the following equation

$$bE_{t+j}i_{t+j+1} - \{1 + b + \frac{a_4}{a_2}\}i_{t+j} + i_{t+j-1} = -\{b\phi_s - 1\}s_{t+j}. \quad (9)$$

Following Hansen and Sargent (1980), the optimal level of inventories is determined as

$$i_t = \pi_i i_{t-1} - \pi_i \sum_{j=0}^{\infty} \lambda^j E_t \left[ -\{b\phi_s - 1\}s_{t+j} \right],$$

where $\pi_i = \frac{(1+b+\frac{a_4}{a_2}) - \sqrt{-4b+(1+b+\frac{a_4}{a_2})^2}}{2b}$ is the stable real root of the following polynomial $bx^2 - \{1 + b + \frac{a_4}{a_2}\}x + 1 = 0$ and $\lambda = b\pi_i$. Thus, the inventory process is given by

$$i_t = \pi_i i_{t-1} - \gamma_s \epsilon_{s,t}, \quad (10)$$

where $\gamma_s = \frac{\pi_i(1-b\phi_s)}{1-b\pi_i\phi_s}$.

From equation (10), inventories depend on the relative costs associated with the production smoothing and stockout avoidance motives, as well as with the exogenous sales process. In particular, inventories increase when there is a negative transitory sales shocks—i.e., the contemporaneous correlation between sales and inventories is negative. Also, given the persistent AR(1) structure for sales, the increase in inventories due to a negative sales shock predicts an increase in future sales, as sales return to their long-run level—i.e., inventories have a positive forecasting relationship with future sales. Meanwhile, the persistence of the inventory process, $\pi_i$, is decreasing in $a_4$, the long-run cost that motivates stockout avoidance, and increasing in $a_2$, the long-run cost that motivates production smoothing.[9]

[9]These comparative statics are based on the following partial derivatives $\frac{\partial \pi_i}{\partial a_2} =$

Based on this cost minimization analysis, a change in inventory behaviour could reflect a change in the relative costs motivating production smoothing versus stockout avoidance and/or a change in the sales process. For example, a simple explanation for the excess decline in output volatility presented in Table 1 would be a *relative* reduction in the costs associated with stockout avoidance (i.e., a reduction in costs of accessing inventory stocks compared to costs of changing production plans). A simple explanation for the change in the forecasting role of inventories presented in Table 2 would be a change in the exogenous sales process in such a way that, even given the same relative costs associated with production smoothing and stockout avoidance, inventories adjust more in anticipation of future sales.

Notably, however, this simple cost minimization analysis abstracts from the fact that some production must be set in advance based on noisy signals about sales.[10] As discussed in Blinder and Maccini (1991) Kahn, McConnell, and Perez-Quiros (2002), the nature of informational flows in the production process is such that some changes in inventories will be unintentional and unrelated to actual sales rather than optimal responses to sales shocks. A key question addressed in this paper, then, is how important are these "inventory mistakes" in explaining the Great Moderation relative to changes in the exogenous sales process or to intentional inventory behaviour such as increased production smoothing. Again, to answer this question and to help sort out the competing explanations for the basic

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$\frac{a_4[(a_2+a_2b+a_4)-\sqrt{-4ba_2^2+(a_2+a_2b+a_4)^2}]}{2a_2^2\sqrt{-4ba_2^2+(a_2+a_2b+a_4)^2}}$ and $\frac{\partial \pi_i}{\partial a_2} = \frac{1}{2a_2b}[1 - \frac{(a_2+a_2b+a_4)}{\sqrt{-4ba_2^2+(a_2+a_2b+a_4)^2}}]$. Because $\pi_i$ is a stable real root, $\sqrt{-4ba_2^2 + (a_2 + a_2b + a_4)^2} > 0$ and, given the assumptions on the cost coefficients and the discount factor in equation (5), $\frac{\partial \pi_i}{\partial a_2} > 0$ and $\frac{\partial \pi_i}{\partial a_4} < 0$.

[10]The tradeoff between production smoothing and stockout avoidance can be seen as capturing the idea that it is less costly to set production in advance than at the moment sales are realized. Specifically, the costs associated with accumulating or depleting inventories (i.e., with the stockout avoidance motive) only need to be borne if a firm also finds it costly to change production when a sales shock is realized. Otherwise, the firm will simply adjust production in response to the shock, thus avoiding the costs associated with accessing inventories. Thus, the key abstraction in the cost minimization analysis is in terms of the information flows about sales, rather than setting production in advance.

sample statistics and the VECM results, we develop an unobserved components model in the next section that identifies inventory mistakes, changes in the sales process, and parameters reflecting the intentional responses of inventories to the sales process.

# 3 Model

## 3.1 An unobserved components model

Our unobserved components (UC) model separates each of the observable series for log output, log sales, and a measure of accumulated inventories (derived from the residual measure of inventory investment) into a permanent component and a transitory deviation from the permanent component:

$$y_t = \tau_t^* + (y_t - \tau_t^*), \tag{11}$$

$$s_t = \tau_t^* + (s_t - \tau_t^*), \tag{12}$$

$$i_t = i_t^* + (i_t - i_t^*). \tag{13}$$

The permanent components are specified as follows:

$$i_t^* = \tau_t^* + \kappa_t, \tag{14}$$

$$\tau_t^* = \mu_\tau + \tau_{t-1}^* + \eta_t, \qquad \eta \sim i.i.d.N(0, \sigma_\eta), \tag{15}$$

$$\kappa_t = \mu_\kappa + \kappa_{t-1} + v_t \qquad v \sim i.i.d.N(0, \sigma_v), \tag{16}$$

where $i_t^*$ is long-run target for inventories, $\tau_t^*$ is the common trend for output and sales, and $\kappa_t$ is the trend for the inventory/sales ratio. The trends have deterministic drifts $\mu_\tau$ and $\mu_\kappa$, respectively, and they are driven by $\eta_t$, the permanent sales shock, and $v_t$, the permanent shock to the inventory/sales ratio, respectively. The specification of a common stochastic trend for output and sales corresponds directly to the idea discussed in Section 2.2 that $y_t$ and $s_t$ are cointegrated. The transitory components follow stationary processes:

$$\Psi_y(L)^{-1}(y_t - \tau_t^*) = \lambda_{y\eta}\eta_t + \lambda_{yv}v_t + \lambda_{y\epsilon}\epsilon_t + u_t, \tag{17}$$

$$\Psi_s(L)^{-1}(s_t - \tau_t^*) = \lambda_{s\eta}\eta_t + \epsilon_t, \tag{18}$$

$$\Psi_i(L)^{-1}(i_t - i_t^*) = \lambda_{i\eta}\eta_t + \lambda_{iv}v_t + \lambda_{i\epsilon}\epsilon_t + u_t, \tag{19}$$

where the $\Psi(L)$ lag operators capture invertible Wold coefficients and $\lambda_{y\eta}$, $\lambda_{yv}$, $\lambda_{y\epsilon}$, $\lambda_{s\eta}$, $\lambda_{i\eta}$, $\lambda_{iv}$, and $\lambda_{i\epsilon}$ are the impact coefficients for output, sales, and inventories in response to the shocks. The transitory shocks are $\epsilon \sim i.i.d.N(0, \sigma_\epsilon)$, and $u \sim i.i.d.N(0, \sigma_u)$, where $\epsilon$ is a transitory sales shock and $u$ is a transitory inventory shock, which, as discussed in more detail in Section 3.2, reflects informational errors.

For this UC model, the transitory deviations from trend are driven not only by transitory shocks, but also by adjustments to permanent shocks. By imposing this structure, we are allowing permanent and transitory movements to be correlated, even though the underlying shocks are specified to be mutually uncorrelated. As discussed in Morley, Nelson, and Zivot (2003), a UC model with correlated components is identified given sufficiently rich dynamics. For our application, we estimate the model for sales and inventories, assuming AR(2) dynamics for their transitory components and leaving the process for output implicit. The two-variable model has 14 parameters and corresponds to a reduced-form vector autoregressive moving-average (VARMA) process with 15 independent parameters.[11] As a result, the model is identified, although weak identification is still a potential problem, as discussed in more detail in Section 4.1. A state-space representation of the two-variable UC model is presented in Appendix A.

### 3.2 Interpretation of shocks

The economic interpretation of the various shocks is mostly straightforward. Permanent and transitory sales shocks, $\eta_t$ and $\epsilon_t$, capture technology

[11] There are four AR parameters and two drift terms that are common to both specifications. In addition, the two-variable UC model has four variance parameters and four impact coefficients, while the VARMA model has three variance-covariance parameters and eight MA parameters associated with two-lags of vector MA terms. Note that sales and inventories are not restricted to be cointegrated, making the multivariate UC model here more analogous to the multivariate UC model in Sinclair (2009) than the model in Morley (2007).

and/or demand factors in the aggregate economy. The permanent inventory shocks $v_t$ capture changes in inventory management practices, caused either by shifts in the nature of production (i.e., from goods to services) or changes in the costs of accessing and holding inventories that are not accounted for by changes in the permanent level of sales. The inventory mistakes, $u_t$, capture informational errors that arise due to exogenous noise in the signals firms receive about sales and the fact that some production must be set in advance of sales.[12] The key distinction between the transitory sales shocks and inventory mistakes is that inventory mistakes are assumed to have no direct effect on future sales.[13]

### 3.3 The impact coefficients

Output, sales, and inventory investment are linked together by equation (1). As a result, only a subset of the impact coefficients are, in fact, independent. For the UC model, the following equations describe the relationships between the coefficients implied by equation (1):

$$\lambda_{y\eta} = 1 + \lambda_{i\eta} + \lambda_{s\eta}, \tag{20}$$

$$\lambda_{y\epsilon} = 1 + \lambda_{i\epsilon}, \tag{21}$$

$$\lambda_{yv} = 1 + \lambda_{iv}. \tag{22}$$

Therefore, only four of the seven impact coefficients in the UC model are independently determined.

[12]Kahn, McConnell, and Perez-Quiros (2002) consider similar unintentional inventory shocks and note their magnitude reflects both the flow of information about future sales and the extent to which production needs to be set in advance. For example, a firm may regard an order as a signal of future sales and begin production on this basis, but the order may be subsequently cancelled. To the extent that the firm increased production based on this order, the cancellation was not predicted and the resulting inventory accumulation will be a "mistake". Meanwhile, to the extent that production can be held off closer to the date of the actual sale, fewer mistakes will be made.

[13]Unexpected changes in inventories which do affect aggregate demand will be classified as sales shocks, as will temporary cost shocks that have aggregate effects. Any cost shocks that do not affect aggregate sales will behave much like inventory mistakes and be categorized as such. We further investigate the link between what we identify as "inventory mistakes" and an independent measure of informational errors in Section 4.6.

We impose additional restrictions on the values of the independent impact coefficients based on limits in terms of how output, sales and inventories can respond to exogenous shocks. For example, consider "scenario A" of a positive permanent sales shock to the common stochastic trend $\tau_t^*$. Under this scenario, permanent sales will increase one for one. If actual sales do not change, sales will fall below trend and $\lambda_{s\eta} = -1$. By contrast, if sales increase by the same amount as permanent sales, either due to a ramping up of production and/or due to a running down of existing inventories, then $\lambda_{s\eta} = 0$. Based on these extreme cases, we can bound $\lambda_{s\eta} \in [-1, 0]$. Meanwhile, this scenario implies that permanent inventories rise one for one with permanent sales. If inventories adjust immediately, $\lambda_{i\eta} = 0$. Or, if inventories remain unchanged, then they will be below their long-run target and $\lambda_{i\eta} = -1$. However, it is even possible that inventories temporarily decrease if sales adjust but output does not, in which case $\lambda_{i\eta} = -2$. As a result, we can bound $\lambda_{i\eta} \in [-2, 0]$, which from equation (20) and the bounds on $\lambda_{s\eta}$ implies the bounds $\lambda_{y\eta} \in [-2, 1]$. The lower bound corresponds to the case where sales are accommodated completely by inventories. The upper bound corresponds to the case where output increases one for one and $\lambda_{s\eta} = 0$.[14] In this case, output increases both to prevent a depletion of inventories relative to their long-run target and to accommodate an increase in sales.

The possible values of the impact coefficients for the $\epsilon_t$ and $v_t$ shocks are more straightforward to analyze. A positive temporary sales shock, which we label as "scenario B", leads sales to rise temporarily above their long-run target. If $\lambda_{i\epsilon} = -1$, output remains unchanged and the increase in sales is entirely accommodated by a decline in inventories. However, if output rises and inventories remain unchanged, then $\lambda_{i\epsilon} = 0$. Thus, we can bound $\lambda_{i\epsilon} \in [-1, 0]$, which from equation (21) implies the bounds $\lambda_{y\epsilon} \in [0, 1]$. Meanwhile, a positive permanent shock to the long-run target inventories, which we label as "scenario C", raises $i_t^*$ one for one. If output does not

[14]This case corresponds to the uncorrelated case for the UC structure for sales (the "UC-0" structure in the Morley Nelson and Zivot (2003) terminology).

change then $\lambda_{i\nu} = -1$. However when output does respond, $\lambda_{i\nu} = 0$. Thus, we can bound $\lambda_{i\nu} \in [-1, 0]$, which from equation (22) implies the bounds $\lambda_{y\nu} \in [0, 1]$.

The cost function analysis in Section 2 allows us to relate the different motives for holding inventories to the various impact coefficients. Table 3 reports the implied values of the impact coefficients that are consistent with the production smoothing and stockout avoidance motives under the different scenarios considered above. For the sake of discussion, we focus on the long-run motives, although the table also reports the implied values of the impact coefficients for the short-run motives. As before, consider scenario A of a positive permanent shock to sales. Suppose actual sales increase such that $\lambda_{s\eta} = 0$ (see the left columns in panel (ii)). In this case, if a firm solely wants to smooth production in the long run, it will increase output and slowly adjust it to the new long-run target such that $\lambda_{y\eta} = 0$ and $\lambda_{i\eta} = -1$. But if a firm is solely guided by the stock-out avoidance motive, it will increase output to accommodate the increase in sales and also restore inventories to their long-run target such that $\lambda_{y\eta} = 1$ and $\lambda_{i\eta} = 0$. Meanwhile, consider the case where actual sales remain unchanged after a positive permanent shock to sales and $\lambda_{s\eta} = -1$ (see the right columns in panel (ii)). To smooth production, a firm will increase output to minimize deviations from target with $\lambda_{y\eta} = 0$ and $\lambda_{i\eta} = 0$, while to avoid stock-outs, it will restore inventories to their long-run target, $\lambda_{i\eta} = 0$ and $\lambda_{y\eta} = 0$. The implications under scenario B of a temporary sales shock and scenario C of a permanent inventory shock are once again more straightforward. The impact coefficients will be $\lambda_{i\epsilon} = \lambda_{i\nu} = -1$ when a firm is guided solely by a desire to smooth production and $\lambda_{i_\epsilon} = \lambda_{i\nu} = 0$ when it is guided solely by fear of stockouts. The short-run motives reported in panel (i) are determined in a similar fashion.

### 3.4 Implied forecast errors and forecasting

Because inventory mistakes are informational errors, it might seem like they could be identified as forecast errors for inventories. However, there is

TABLE 3. INVENTORY MOTIVES AND IMPACT COEFFICIENTS

**(i) Short-run motives**

*Scenario A: Permanent shock to sales*

| | $\lambda_{s\eta} = 0$ | | $\lambda_{s\eta} = -1$ | |
|---|---|---|---|---|
| | PS | SA | PS | SA |
| $\lambda_{y\eta}$ | $-1$ | 0 | $-1$ | $-1$ |
| $\lambda_{i\eta}$ | $-2$ | $-1$ | $-1$ | $-1$ |

*Scenario B: Temporary shock to sales*

| | PS | SA |
|---|---|---|
| $\lambda_{y\epsilon}$ | 0 | 1 |
| $\lambda_{i\epsilon}$ | $-1$ | 0 |

*Scenario C: Permanent shock to inventories*

| | PS | SA |
|---|---|---|
| $\lambda_{yv}$ | 0 | 0 |
| $\lambda_{iv}$ | $-1$ | $-1$ |

**(ii) Long-run motives**

*Scenario A: Permanent shock to sales*

| | $\lambda_{s\eta} = 0$ | | $\lambda_{s\eta} = -1$ | |
|---|---|---|---|---|
| | PS | SA | PS | SA |
| $\lambda_{y\eta}$ | 0 | 1 | 0 | 0 |
| $\lambda_{i\eta}$ | $-1$ | 0 | 0 | 0 |

*Scenario B: Temporary shock to sales*

| | PS | SA |
|---|---|---|
| $\lambda_{y\epsilon}$ | 0 | 1 |
| $\lambda_{i\epsilon}$ | $-1$ | 0 |

*Scenario C: Permanent shock to inventories*

| | PS | SA |
|---|---|---|
| $\lambda_{yv}$ | 0 | 1 |
| $\lambda_{iv}$ | $-1$ | 0 |

Table 3: Implied impact coefficients for different shocks are presented for production smoothing (PS) versus stockout avoidance (SA) objectives.

an important distinction between inventory mistakes and the overall forecast error in a given time period. This distinction is key to understanding why the UC model is so helpful in explaining both the role of inventories in the Great Moderation and the changed forecasting role of inventories.

We define an inventory forecast error, or period-to-period "unexpected" inventories as

$$\Delta i_t^u \equiv \Delta i_t - E_{t-1}[\Delta i_t], \tag{23}$$

where $\Delta i_t$ is the actual change in inventories and $E_{t-1}(\Delta i_t)$ is the expected change in inventories. Assuming firms observe the underlying shocks hitting the economy and have rational expectations, the UC model implies the following structure for these forecast errors:

$$\Delta i_t^u = y_t - s_t - E_{t-1}[y_t - s_t] = (\lambda_{y\eta} - \lambda_{s\eta})\eta_t + (\lambda_{y\epsilon} - 1)\epsilon_t + \lambda_{yv} v_t + u_t. \tag{24}$$

Notably, the inventory forecast error depends on date $t$ sales and inventory shocks. Only part of the forecast error is due to informational errors based on noisy signals. For the other shocks, firms implicitly choose how to respond via the impact coefficients, where these coefficients reflect a desire to smooth production versus a fear of stockouts, as discussed in the previous subsection. For instance, again consider scenario A of a positive permanent sales shock. Depending on how much sales immediately adjust to a permanent shock and firms' objectives, there will be accumulation of inventories in the current period by a factor of $(\lambda_{y\eta} - \lambda_{s\eta})$ and this factor is what makes this accumulation intentional.

How does the UC model help in understanding the changed forecasting role of inventories captured by the VECM results in Table 2? One explanation for the results is that inventory changes are more predictable and they provide a better signal of future sales. We consider this possibility by calculating and comparing the variances of the inventory forecast errors and expected inventory investment (i.e., $\Delta i_t^e = \Delta i_t - \Delta i_t^u = E_{t-1}(\Delta i_t)$). Appendix B describes how we calculate these variances for the UC model.

Another explanation for the changed forecasting role is that the compo-

TABLE 4. MARGINAL EFFECTS OF SHOCKS ON FORECASTS

| | Permanent shocks | | Transitory shocks | |
|---|---|---|---|---|
| | $\eta_t$ | $v_t$ | $\epsilon_t$ | $u_t$ |
| $\frac{\partial \Delta y_{t+1}}{\partial \Delta i_t^u}$ | $\frac{\lambda_{s\eta}(\phi_{s,1}-1)+\lambda_{i\eta}(\phi_{i,1}-2)-1}{1+\lambda_{i\eta}}$ | $\frac{\lambda_{iv}(\phi_{i,1}-2)-1}{1+\lambda_{iv}}$ | $\frac{(\phi_{s,1}-1)+\lambda_{i\epsilon}(\phi_{i,1}-2)}{\lambda_{i\epsilon}}$ | $\phi_{i,1}-1$ |
| $\frac{\partial \Delta s_{t+1}}{\partial \Delta i_t^u}$ | $\frac{\lambda_{s\eta}(\phi_{s,1}-1)}{1+\lambda_{i\eta}}$ | 0 | $\frac{(\phi_{s,1}-1)}{\lambda_{i\epsilon}}$ | 0 |

Table 4: Marginal effects of the underlying shocks on forecast errors and forecasts of future output and sales growth are presented.

sition of underlying shocks in an inventory forecast error has changed, with inventory mistakes playing a smaller role and inventory changes no longer leading to offsetting changes in future output. In order to investigate the effects of a change in the composition of shocks and, therefore, relate the UC model to the VECM results, we solve for the partial effects of an inventory forecast error on future output growth and future sales growth: $\frac{\partial \Delta y_{t+1}}{\partial \Delta i_t^u}$ and $\frac{\partial \Delta s_{t+1}}{\partial \Delta i_t^u}$. To do this, we first analytically compute the following marginal effects: (i) impact of each shock on future output and sales growth and (ii) the impact of each shock on an inventory forecast error. Taking the ratio of these marginal effects, we calculate the impact of an inventory forecast error on output growth and sales growth due to a particular shock, holding all else equal. Table 4 presents the implied partial effects of a forecast error, which are clearly different for the various underlying shocks. Thus, a change in the relative importance of these shocks directly implies a change in the reduced-form forecasting implications of inventories.

# 4 Empirical results

## 4.1 Data and methods

As considered in Section 2, the raw data are quarterly U.S. real GDP and final sales from the BEA for the sample periods of 1947Q1-1984Q1 and 1984Q2-2011Q1. We estimate the UC model for sales and inventories, leaving the estimated process for output implicit. Our measure for sales is 100 times the natural logarithms of real sales and our measure for inventories is calculated by i) constructing the change in inventories based on the identity given in equation (1) for 100 times log output and 100 times log sales and ii) accumulating changes given an arbitrary initial level of log inventories. Technically, the inventory series reflects an accumulation of gross inventory investment. However, depreciation is implicitly accounted for in the UC model via the drift and permanent shocks to the inventory/sales ratio given in equation (16).

We estimate our model using Bayesian posterior simulation based on Markov-chain Monte Carlo (MCMC) methods. Specifically, we consider a mutli-block random-walk chain version of the Metropolis-Hastings (MH) algorithm with 100,000 draws after a burnin of 10,000 draws. We check the robustness of our posterior moments to different runs of the chain and for different starting values. The multi-block setup allows us to obtain relatively low correlations between parameter draws, suggesting the sampler is working well. See Chib and Greenberg (1995) for more details on the MH algorithm.

There are two reasons why we consider Bayesian estimation. First, UC models can suffer from weak identification. In particular, UC models are closely related to VARMA models, which are notoriously difficult to estimate due to the problem of near cancellation of AR and MA terms and multiple modes for the likelihood surface. A particularly troublesome estimation difficulty is a so-called "pile-up problem" whereby maximum likelihood estimates tend to hit boundaries even when true parameters are not equal to the boundary values. Preliminary analysis via maximum likelihood estimation (MLE) confirmed multiple modes and some pile-up problems. By contrast, Bayesian estimation with relatively uninformative priors reveals a clear interior mode for the posterior function. Our main inferences about the Great Moderation turn out to be robust to whether we consider the MLE results or the interior mode. However, Bayesian estimation provides a sense of parameter uncertainty that we cannot easily obtain for the

MLE results given that some parameters hit boundaries. The second reason why we consider Bayesian estimation is that it provides posterior moments not only for the model parameters, but also for some particularly interesting, but complicated functions of the model parameters such as counterfactual standard deviations for output growth and implied error-correction parameters.

Our priors are specified as follows: 1) the AR coefficients have standard Normal distributions (i.e., $N(0,1)$), truncated to ensure stationarity (i.e., the roots of the characteristic equations for the AR lag polynomials lie outside the unit circle); 2) the drift for the inventory/sales ratio has a diffuse $N(0,100)$ distribution, while the drift for long-run sales (and output) is concentrated out of the likelihood by recentering the growth rate data; 3) the precisions (inverse variances) have $\Gamma(0.01, 0.01)$ distributions, which correspond to highly diffuse priors for the variances; 4) the impact coefficients have standard Normal distributions with means recentered to be the midpoints of the bounds described in Section 3.3 and truncation to ensure the coefficients lie within or on those bounds; and 5) the initial values for the permanent levels of sales and inventories in the pre-moderation period have diffuse Normal distributions that are centered at initial observations (minus one-period drifts) and have variances of 10000. All of these priors are relatively uninformative in the sense that the posteriors are dominated by the likelihood and our main qualitative inferences are robust to a range of different priors, including the flat/improper priors implicit in the consideration of MLE.

### 4.2 Estimates

Table 5 reports means and standard deviations of the posterior distributions of the parameters for the UC model. From the results, it is clear that many of the parameters governing the process of sales and inventories have changed considerably from the pre-moderation period to the post-moderation period. Overall, the volatility of shocks declined and some of the propagation parameters, captured by the autoregressive coefficients

TABLE 5. PARAMETERS FOR UC MODEL

| | Pre-moderation (1960Q1-1984Q1) | Post-moderation (1984Q2-2011Q1) |
|---|---|---|
| | Sales process | |
| $\sigma_\eta$ | 2.14 (0.71) | 1.15 (0.28) |
| $\sigma_\epsilon$ | 0.58 (0.08) | 0.34 (0.05) |
| $\phi_s^*$ | 0.79 (0.09) | 0.76 (0.07) |
| $\lambda_{s\eta}$ | −0.84 (0.12) | −0.73 (0.10) |
| | Inventory process | |
| $\sigma_\nu$ | 1.05 (0.50) | 0.80 (0.28) |
| $\sigma_u$ | 0.37 (0.08) | 0.15 (0.03) |
| $\phi_i^*$ | 0.88 (0.05) | 0.76 (0.06) |
| $\mu_\kappa$ | −0.72 (0.11) | −0.47 (0.08) |
| $\lambda_{y\eta}$ | −0.91 (0.12) | −0.72 (0.10) |
| $\lambda_{y_\epsilon}$ | 0.78 (0.16) | 0.65 (0.12) |
| $\lambda_{iv}$ | −0.83 (0.14) | −0.88 (0.07) |

Table 5: Posterior means of the parameters for the UC model are reported, with posterior standard deviations in parentheses. The $\phi^*$ parameters refer to sums of autoregressive coefficients for the AR(2) specifications.

and the impact coefficients, have changed.

Because it can be difficult to interpret some of the individual parameters in Table 5, especially the impact coefficients, we calculate implied volatilities, measured by standard deviations, of the underlying variables and key components. Table 6 reports means and standard deviations of the posterior distributions for these implied volatilities. Output growth and sales growth are less volatile in the post-moderation period, consistent with the sample statistics in Table 1. Note that the volatility decline in expected inventory changes is smaller than the change in inventory forecast errors, suggesting an increase in the relative importance of expected inventories in overall inventory investment. At first glance, this change appears consistent with increased production smoothing and potentially explains the changed forecasting role of inventories in the recent sample. We investigate these possibilities in the next few subsections.

TABLE 6. IMPLIED VOLATILITIES

| | Pre-moderation (1960Q1-1984Q1) | Post-moderation (1984Q2-2011Q1) |
|---|---|---|
| s.d.($\Delta y_t$) | 1.16 (0.10) | 0.65 (0.06) |
| s.d.($\Delta s_t$) | 0.94 (0.09) | 0.58 (0.05) |
| s.d.($\Delta i_t$) | 0.75 (0.06) | 0.41 (0.03) |
| s.d.($\Delta i_t^u$) | 0.47 (0.06) | 0.24 (0.03) |
| s.d.($\Delta i_t^e$) | 0.65 (0.11) | 0.50 (0.07) |

Table 6: Posterior means of implied volatilities, measured in terms of standard deviations of variables, are reported, with posterior standard deviations in parentheses.

### 4.3 Increased production smoothing?

Given the decline in output volatility, it is natural to ask whether there is an increase in the use of inventories to smooth production in the post-moderation period. Comparing the impact coefficient estimates in Table 5 with theoretical values in Table 3 in Section 3.3, the only relevant cases that we can consider are the following: the short-run scenario B, and the long-run scenarios B and C. Scenario A is not particularly informative because $\hat{\lambda}_{s\eta}$ is reasonably close to $-1$, at which point the other relevant coefficients are the same for both motives. In the pre-moderation period, based on scenario B for both the long-run and the short-run, the estimated impact coefficient is $\hat{\lambda}_{y\epsilon} = 0.78$, closer to the predicted value of 1 if firms were only concerned about avoiding stockouts. However, the long-run scenario C is more consistent with a focus on production smoothing, given the estimated parameter $\hat{\lambda}_{iv} = -0.83$. Based on these coefficients, the results for the pre-moderation period are ambiguous. In the post-moderation period, both $\hat{\lambda}_{y\epsilon}$ and $\hat{\lambda}_{iv}$ have decreased to 0.65 and $-0.88$, respectively. The decline in $\hat{\lambda}_{y\epsilon}$ suggests that the stockout avoidance has become less important, while a decrease in $\hat{\lambda}_{iv}$ suggests that production smoothing has become more important in the post-moderation period. Broadly, then, these results suggest production smoothing has become more relevant in the recent sample.

As noted in Section 2, the autoregressive coefficient, $\pi_i$, for inventory adjustment in the cost function analysis depends on the cost coefficients $a_2$ and $a_4$. Therefore, we can look at the autoregressive coefficients for transitory inventories in our UC model to infer the relative costs associated with (long-run) production smoothing versus stockout avoidance. The estimate $\hat{\phi}_i^*$ is 0.88 in the pre-moderation period, suggesting that the cost motivating production smoothing was relatively high. However, this relative cost has decreased, as the estimate $\hat{\phi}_i^*$ is 0.76 in the post-moderation period, suggesting somewhat less of a need to emphasize production smoothing in recent years.[15] Thus, this result does not suggest production smoothing has become more relevant and we have mixed results overall based on the model estimates for an increase in production smoothing.

## 4.4 Counterfactuals

We conduct some counterfactual experiments in order to help disentangle the role of inventories from that of sales in explaining the decline in overall output volatility.[16] Our main objective here is to determine whether changes in the inventory process—(i) less volatile shocks and/or (ii) changes

[15]The coefficient $\phi_i^*$ is the sum of the two autoregressive coefficients for an AR(2) specification of transitory inventories. Thus, we are implicitly using the sum of the AR coefficients as our measure of persistence. However, the estimated reduction in persistence is also evident if we consider the largest inverse root of the characteristic equation for the AR lag polynomial or the half-life based on an impulse response function.

[16]See Stock and Watson (2003), Ahmed, Levin, and Wilson (2004), Sims and Zha (2006), and Kim, Morley, and Piger (2008), among many others, for counterfactual experiments with VAR models. Of particular relevance to the analysis here, Kim, Morley, and Piger (2008) discuss the benefits of Bayesian inference for counterfactual quantities. Specifically, Bayesian analysis produces posterior moments for the counterfactual quantities, thus providing a sense of estimation uncertainty that is not available in the classical context. Meanwhile, Benati and Surico (2009) are critical of counterfactual analysis with reduced-form VAR models given an underlying dynamic stochastic general equilibrium (DSGE) structure generating the data. However, unlike with a reduced-form VAR model, our analysis here includes contemporaneous structural transmission within the propagation mechanism and, unlike a finite-order VAR model, our UC model captures VARMA dynamics, as would be implied by a DSGE structure. So, our counterfactual analysis is robust to Benati and Surico's critique of counterfactual analysis based on VAR models, although, of course, it is an open question whether our UC model parameters are "structural" in the Lucas-critique sense that a subset of parameters could have changed without all of the other parameters changing too.

TABLE 7. COUNTERFACTUAL EXPERIMENTS

| | $\Delta(s.d.(\Delta y_t))$ |
|---|---|
| Actual | $-0.51$ (0.12) |
| Sales process alone | $-0.30$ (0.12) |
| Inventory process alone | $-0.11$ (0.14) |
| Inventory shocks alone | $-0.11$ (0.08) |
| $u_t$ shocks alone | $-0.11$ (0.05) |
| Inventory propagation alone | 0.03 (0.15) |

Table 7: Posterior means of implied changes in volatility, measured in terms of the standard deviations of output growth, are reported, with posterior standard deviations reported in parentheses. The counterfactual experiments involve changing a subset of parameters to obtain implied counterfactual changes in volatilities in the post-moderation period.

in the propagation mechanism (autoregressive and impact coefficients)—could have accounted for the Great Moderation. To do this, we hold the parameters of the sales process fixed at their pre-moderation values and let the parameters associated with inventories ($\sigma_v$, $\sigma_u$, $\phi_i^*$, $\lambda_{y\eta}$, $\lambda_{y_\epsilon}$, and $\lambda_{iv}$) change to their post-moderation values. We also try to isolate the role of different inventory shocks ($\sigma_v$ and $\sigma_u$) or the propagation mechanism ($\phi_i^*$, $\lambda_{y\eta}$, $\lambda_{y_\epsilon}$, and $\lambda_{iv}$) by changing only subsets of parameters at a time. For completeness, we also consider an experiment in which the inventory process is fixed and the sales process is allowed to change. Table 7 reports means and standard deviations of the posterior distributions for our counterfactual quantities of interest.

According to the counterfactual results in Table 7, a change in the sales process alone could have generated about half of the overall actual decline in output growth volatility. Given that the autoregressive dynamics for sales are quite similar in the pre- and post-moderation periods, this result is consistent with the "good luck" hypothesis in the sense that a smaller volatility of sales shocks rather than a change in the propagation of the shocks appears to be a key aspect of the Great Moderation. Also, the finding in Table 5 that the autoregressive dynamics did not change much suggests that it is possible to think about changing the values of some parameters of the UC model without other parameters necessarily changing too, thus perhaps mitigating concerns that the Lucas critique is empirically relevant in this setting.

In terms of inventories, the counterfactual results suggest that their primary role in the Great Moderation was in generating an excess reduction in output volatility relative to sales. Furthermore, the counterfactuals in Table 7 suggest very clearly that the excess reduction in output volatility was driven by smaller inventory shocks rather than a change in the propagation of those shocks. Consistent with the mixed findings on the role of production smoothing discussed in the previous subsection, a change in inventory propagation alone would have generated no reduction in output volatility. Instead, the entire excess reduction in output volatility that can be related to inventories appears to be due to a reduction in the magnitude of inventory mistakes. Meanwhile, the sum of the counterfactual reductions in volatility is less than the overall reduction, suggesting there was an important interaction between the changes in the sales and inventory processes in explaining the Great Moderation.

### 4.5 Implied forecasting role of inventories

Even if increased production smoothing does not appear to be responsible for the reduction in output volatility, the question remains as to whether it can explain the changed forecasting role of inventories with the Great Moderation. Based on Table 6, a larger proportion of overall inventory investment is predictable from period to period, consistent with increased production smoothing in advance of future sales. However, the analysis in Section 3.4 suggests that the forecasting role of inventories can also change with the composition of inventory forecast errors, even if the predictability of inventory investment had remained unchanged. Therefore, we consider whether the reduction in inventory mistakes that appears to explain so much of the excess reduction in output volatility relative to sales can also help to explain the changed forecasting role of inventories.

We calculate the implied forecasting role of inventories given a change

TABLE 8. IMPLIED ERROR CORRECTION COEFFICIENTS

| | Pre-moderation (1960Q1-1984Q1) | Post-moderation (1984Q2-2011Q1) |
|---|---|---|
| $\frac{\partial \Delta y_{t+1}}{\partial \Delta i_t^u}$ | $-1.14$ (0.21) | $-0.68$ (0.44) |
| $\frac{\partial \Delta s_{t+1}}{\partial \Delta i_t^u}$ | $-0.13$ (0.17) | 0.28 (0.35) |

Table 8: Posterior means of error correction coefficients implied by the UC model are reported, with posterior standard deviations in parentheses. The marginal impacts of the underlying shocks are weighted by their relative standard deviations.

in the composition of inventory forecast errors by first calculating the marginal effects presented in Table 4 based on our parameter estimates. Then, we weight these marginal effects by the contribution of each underlying shock to the overall forecast error.[17] This calculation provides us with implied error correction coefficients (in the absence of predictable inventory changes). Table 8 reports posterior means and standard deviations for the implied error correction coefficients.

The results in Table 8 are qualitatively in line with the VECM estimates in Table 2. Specifically, there is a diminished negative forecasting relationship between inventories and future output growth and an increased positive forecasting relationship between inventories and future sales growth in the post-moderation period. The estimates are not particularly precise and the quantitative effects are somewhat different than the VECM results in Table 2, but this likely reflects the fact that the predictability of inventory investment has also changed along with the composition of inventory forecast errors. The main point is that the results in Table 8 make it clear that the changing composition of inventory forecast errors, specifically smaller inventory mistakes, can also help to explain the dramatic change in the forecasting role of inventories with the Great Moderation.

[17]The weights are calculated as the ratio of the standard deviation of a shock relative to the standard deviation of the overall inventory forecast error.

## 4.6 Informational Errors?

In terms of the UC model, the inventory mistakes are identified as transitory shocks to inventories that do not affect sales. We interpret these transitory shocks as informational errors. However, they could also reflect deliberate responses to certain cost shocks, such as changes in credit conditions that motivate firms to treat inventories as relatively liquid investments (see Carpenter, Fazzari, and Petersen (1994, 1998)).

How do we justify our interpretation of the inventory mistakes? Beyond the fact that most aggregate cost shocks should have implications for aggregate sales, we also directly consider the link between our estimates of inventory mistakes and an alternative measure of changes in beliefs about actual inventories. In particular, we make use of data revisions for inventory investment in the aggregate data. The data revisions arise for many reasons. However, one reason is that firms sometimes initially report an estimate of their inventory investment in the previous quarter, but subsequently report their actual inventory investment.

We obtain initial release values of inventory investment from the St. Louis Fed's Archival database (ALFRED) and compare them to the values based on the May 26, 2011 vintage of data considered in this paper. The archival database is notable because it contains so many vintages for different series, but it is unavoidably affected by the different data norms that have evolved over time. For our analysis, the main issue is that the vintages for quarterly U.S. real GDP only go back to 1991, when there was a deliberate shift towards emphasizing GDP instead of GNP in the NIPAs. However, the vintages for the real change in private inventories and real final sales go back much further. In particular, we are able to measure the "real-time" real change in private inventories as a fraction of lagged real final sales based on initial release data for the sample period of 1965Q4 to 2011Q1. We also calculate a "revised" version of this measure using the May 26, 2011 vintage. The revised measure has a correlation of 0.99997 with a measure using lagged real GDP as the denominator instead of lagged real final sales.

We then calculate a data revision for the change in inventories by taking the difference between the "revised" measure and the "real-time"' measure. Again, these revisions are affected by many factors, including incomplete sampling with the initial release data and longer-term changes in data collection methodologies (e.g., the shift to chain-weighted measures in the 1990s). However, to the extent that some of revisions reflect new information for firms and not just the data collection agency, we might expect a positive relationship between the data revisions and our estimates of inventory mistakes. Indeed, despite all of the reasons for the data revisions and near-certain measurement error in our model-based estimate of inventory mistakes, we find a positive and significant correlation of 17.6% (with a t-statistic of 2.44) between the data revisions and a filtered estimate of the inventory mistakes based on the posterior mode and the Kalman filter.

Does the positive correlation between data revisions and inventory mistakes really imply that the inventory mistakes reflect informational errors? One reason to question this link would be if data revisions and the overall change in inventories had a positive correlation, perhaps due to an underestimation of inventory changes in the initial release data. However, we find a negative and insignificant correlation of -9.7% (with a t-statistic of -1.31). Thus, if there is a bias in the initial release, it is that it tends to overestimate inventory changes. Therefore, the positive relationship between the data revisions and the estimated inventory mistakes appears to reflect new information that could not be anticipated by the data collection agency and, perhaps, not initially known by the firms reporting their sales and inventory investment.

### 4.7 Robustness

When analyzing inventory behaviour, there is always a question of which data to consider. The Great Moderation is an aggregate phenomenon and any useful explanation for it should show up in the aggregate data. However, inventories are most relevant for the durable goods sector. Many studies of inventory behaviour focus on durable goods data (or sometimes even more specifically on data for retail automobiles).

A reasonable question, then, is whether the findings reported above are robust to consideration of durable goods data instead of the aggregate data. The short answer is yes. Indeed, some key findings are even more pronounced than for the aggregate data. For example, when we considered output and sales data for durable goods (for a slightly shorter sample period of 1960Q1-2009Q2 due to data availability issues for the durable goods data), the residual measure of inventory investment appeared to be responsible for a larger portion of the overall decline in output volatility than for the aggregate data. Consistent with this finding, the counterfactual analysis for the durable goods data suggested that inventories played a larger role than sales in the overall decline in volatility of durable goods output. Unlike with the aggregate data, both inventory shocks and propagation implied a reduction in volatility. However, as in the aggregate case, shocks played the primary role in the excess volatility reduction of output, with smaller inventory mistakes accounting for most of this excess reduction. Meanwhile, the VECM results and forecasting implications from the UC model were remarkably similar to those for the aggregate data. The estimates for the durable goods data are available from the authors upon request.

## 5 Conclusions

In this paper, we have investigated the role of inventories in the Great Moderation. We found only mixed evidence for increased production smoothing in recent years and the estimated changes were not sufficient to explain the excess reduction in U.S. output volatility relative to sales. Instead, we found that smaller inventory mistakes were responsible for the bulk of the excess volatility reduction and help to explain the changed forecasting role of inventories with the Great Moderation.

In contemplating whether or not the Great Moderation is now over, it is important to consider what caused the reduction in inventory mistakes in the first place. The mistakes reflect informational errors about future sales and arise due to the fact that some production must be set in advance. Thus, fewer mistakes could correspond to improved information flows about future sales or to greater flexibility in terms of setting production closer to sales. Distinguishing between these two hypotheses is difficult. However, we might expect improved informational flows to reflect a change in the underlying sales process. Thus, our finding that the dynamics of transitory sales remain unchanged with the Great Moderation does not lend itself to an "improved forecast" hypothesis, although the fact that sales shocks are less volatile is somewhat more supportive.[18] Also, somewhat contrary to improved forecasts, which presumably occur gradually due to learning, is the fact that the volatility reduction appears to have been discrete (see Kim and Nelson (1999) and McConnell and Perez-Quiros (2000)). In addition, the standard deviation of the data revisions for the change in inventories declined by only 25% with the Great Moderation, even given fewer benchmark revisions for the post-moderation data compared to the pre-moderation data. Therefore, the rise of "just-in-time" production (see McConnell, Mosser, and Perez-Quiros (1999)) appears to be the more compelling explanation for smaller mistakes, as it is more plausible that new production processes were implemented somewhat suddenly, especially after the deep recessions of the early 1980s. Also, our finding that the implied costs motivating production smoothing have declined relative to the costs motivating stockout avoidance is consistent with the idea that less production needs to be set in advance.

While inventory mistakes may be smaller for structural and technological reasons, they are not likely to disappear altogether. In particular, the extra volatility in U.S. output relative to sales during the 2007-2009 recession is strongly consistent with the idea that some production must be set

[18]Ramey and Vine (2006) find some evidence of a change in sales dynamics for the U.S. automobile industry, which is the archetypal industry involving production that must be set in advance.

in advance and inventory mistakes will continue to be made.[19] At the same time, given their links to technology and despite some large changes in inventories during the recent recession, a smaller variance for inventory mistakes provides a much more optimistic prognosis for the continuation of the Great Moderation than the "good luck" hypothesis (or, for that matter, the "good policy" hypothesis).

On a related note, it has long been understood that the role of inventories in output fluctuations is asymmetric in terms of business cycle phases, with a much larger role being played in recessions than in expansions (see, for example, Blinder and Maccini (1991) and Golob (2000)). However, the analysis in this paper is based on a linear model and, therefore, does not capture this asymmetry. Thus, given the predominance of expansions in the sample periods covered in this paper, our results likely reflect the past and possibly future behaviour of output, sales, and inventories in expansions more than in recessions (over 80% of the observations in our sample are from NBER-dated expansions). This could, in part, explain some of the differences between our conclusions and those in a recent paper by Maccini and Pagan (2009), which explicitly measures movements in output related to business cycle phases and finds little role for inventories in the changed behaviour of output with the Great Moderation.[20] It also means that we

[19] The dramatic depletion of inventories in late 2008 and early 2009 is also consistent with inventory adjustments in the face of severe cash flow problems for firms in the middle of a deep recession. Carpenter, Fazzari, and Petersen (1994, 1998) highlighted the role of financing constraints in the inventory cycle. In terms of our analysis, it suggests that some of what we have labelled as inventory "mistakes" may, in fact, be deliberate temporary run-downs of inventory stocks during recessions. However, the volatility and forecasting implications of such inventory run-downs should be the same as for inventory mistakes.

[20] Somewhat more consistent with our findings, Maccini and Pagan (2009) find that increased production smoothing does not play a role in the Great Moderation. Instead, they find that an estimated structural model based on pre-moderation data could only have generated the observed reduction in output volatility if the volatilities of the sales process and technology shocks declined by about half. In this sense, their results are strongly supportive of the "good luck" hypothesis. However, their structural model does not incorporate inventory mistakes. As a robustness check, they do consider a modified version of their model in which only past values of sales are observed by firms when setting production. However, this is different from inventory mistakes that arise due to noisy signals about sales.

cannot draw strong conclusions about possible changes in recession and recovery dynamics due to inventories (see Camacho, Perez-Quiros, and Rodriguez-Mendizabal (2009)). Modeling business cycle asymmetries associated with inventories presents its own challenges and opportunities, which we leave for future research.

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# A Appendix

## A.1 State-space representation of UC model

The observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H}\,\boldsymbol{\beta}_t \tag{25}$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} s_t \\ i_t \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} s_t - \tau_t \\ s_{t-1} - \tau_{t-1} \\ i_t - i_t^* \\ i_{t-1} - i_{t-1}^* \\ \tau_t \\ \kappa_t \end{bmatrix} \tag{26}$$

The state equation is

$$\boldsymbol{\beta}_t = \tilde{\boldsymbol{\mu}} + \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \tilde{\boldsymbol{v}}_t \tag{27}$$

where

$$\tilde{\boldsymbol{\mu}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mu_\tau \\ \mu_\kappa \end{bmatrix},\ \mathbf{F} = \begin{bmatrix} \phi_{s,1} & \phi_{s,2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{i,1} & \phi_{i,2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \tilde{\boldsymbol{v}}_t = \begin{bmatrix} \lambda_{s\eta}\eta_t + \epsilon_t \\ 0 \\ \lambda_{i\eta}\eta_t + \lambda_{iv}v_t + \lambda_{i\epsilon}\epsilon_t + u_t \\ 0 \\ \eta_t \\ v_t \end{bmatrix} \tag{28}$$

and the covariance matrix of $\tilde{\boldsymbol{v}}_t$, $\mathbf{Q}$, is given by

$$
\mathbf{Q} = \begin{pmatrix}
\lambda_{s\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2 & 0 & \lambda_{s\eta}\lambda_{i\eta}\sigma_\eta^2 + \lambda_{i\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{s\eta}\sigma_\eta^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_{s\eta}\lambda_{i\eta}\sigma_\eta^2 + \lambda_{i\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{i\eta}^2\sigma_\eta^2 + \lambda_{iv}^2\sigma_v^2 + \lambda_{i\epsilon}^2\sigma_\epsilon^2 + \sigma_u^2 & 0 & \lambda_{i\eta}\sigma_\eta^2 & \lambda_{iv}\sigma_v^2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_{s\eta}\sigma_\eta^2 & 0 & \lambda_{i\eta}\sigma_\eta^2 & 0 & \sigma_\eta^2 & 0 \\
0 & 0 & \lambda_{iv}\sigma_v^2 & 0 & 0 & \sigma_v^2
\end{pmatrix} \tag{29}
$$

# B Appendix

In this appendix, we solve the UC model for inventory investment, sales growth, and output growth. We then show how to calculate the implied variances of inventory investment, unexpected inventory investment, expected inventory investment, sales growth, and output growth for the UC model.

The change in inventories is given by

$$
\Delta i_t = \Delta i_t^* + (1-L)(i_t - i_t^*) = \eta_t + v_t + z_t^i \tag{30}
$$

where $(1-\phi_{i,1}L - \phi_{i,2}L^2)z_t^i = (1-L)x_t^i$ and $x_t^i = \lambda_{i\eta}\eta_t + \lambda_{iv}v_t + \lambda_{i\epsilon}\epsilon_t + u_t$.

The process of sales growth is given by

$$
\Delta s_t = \eta_t + z_t^s \tag{31}
$$

where $(1-\phi_{s,1}L - \phi_{s,2}L^2)z_t^s = (1-L)x_t^s$ and $x_t^s = \lambda_{s\eta}\eta_t + \epsilon_t$.

Using the identity, the change in output can be re-written as

$$
\begin{aligned}
\Delta y_t &= \Delta s_t + (1-L)\Delta i_t \\
&= (\eta_t + z_t^s) + \eta_t + v_t + z_t^i - \eta_{t-1} - v_{t-1} - z_{t-1}^i
\end{aligned} \tag{32}
$$

Note that the state equation for $z_t^s$ and $z_t^i$ is

$$
\mathbf{z}_t = \mathbf{K}\mathbf{z}_{t-1} + \mathbf{w}_t \tag{33}
$$

where

$$\mathbf{z}_t = \begin{bmatrix} z_t^s \\ z_{t-1}^s \\ z_t^i \\ z_{t-1}^i \\ x_t^s \\ x_t^i \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \phi_{s,1} & \phi_{s,2} & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{i,1} & \phi_{i,2} & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{w}_t = \begin{bmatrix} x_t^s \\ 0 \\ x_t^i \\ 0 \\ x_t^s \\ x_t^i \end{bmatrix} \tag{34}$$

Letting $\mathbf{W}$ be the covariance matrix with the following non-zero entries $\mathbf{W}(1,1) = \mathbf{W}(1,5) = \mathbf{W}(5,1) = \mathbf{W}(5,5) = \lambda_{s\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2$, $\mathbf{W}(1,3) = \mathbf{W}(3,1) = \mathbf{W}(1,6) = \mathbf{W}(6,1) = \mathbf{W}(3,5) = \mathbf{W}(5,3) = \mathbf{W}(5,6) = \mathbf{W}(6,5) = \lambda_{s\eta}\lambda_{i\eta}\sigma_\eta^2 + \lambda_{i\epsilon}\sigma_\epsilon^2$, and $\mathbf{W}(3,3) = \mathbf{W}(3,6) = \mathbf{W}(6,3) = \mathbf{W}(6,6) = \lambda_{i\eta}^2\sigma_\eta^2 + \lambda_{iv}^2\sigma_v^2 + \lambda_{i\epsilon}^2\sigma_\epsilon^2 + \sigma_u^2$ The $var(\mathbf{z}_t)$ =reshape$((I - \mathbf{K} \otimes \mathbf{K})^{-1} vec(\mathbf{W}))$.

Then the variance of inventory investment is given by

$$\begin{aligned} var(\Delta i_t) &= var(\eta_t + v_t + z_t^i) \\ &= \sigma_\eta^2 + \sigma_v^2 + var(z_t^i) + 2cov(\eta_t, z_t^i) + 2cov(v_t, z_t^i) \\ &= \sigma_\eta^2 + \sigma_v^2 + var(z_t^i) + 2\lambda_{i\eta}\sigma_\eta^2 + 2\lambda_{iv}\sigma_v^2 \end{aligned}$$

where $var(z_t^i)$ is the $(3,3)$ element of $var(\mathbf{z}_t)$. The variances of the two expectational components of inventory investment are given by

$$var(\Delta i_t^u) = (\lambda_{y\eta} - \lambda_{s\eta})^2\sigma_\eta^2 + (\lambda_{y\epsilon} - 1)^2\sigma_\epsilon^2 + \lambda_{yv}^2\sigma_v^2 + \sigma_u^2.$$

and

$$var(\Delta i_t^e) = var(\Delta i_t) - var(\Delta i_t^u)$$

The variance of sales growth is given by

$$var(\Delta s_t) = var(\eta_t + z_t^s) = \sigma_\eta^2 + var(z_t^s) + 2\lambda_{s\eta}\sigma_\eta^2$$

and $var(z_t^s)$ is the $(1,1)$ element of $var(\mathbf{z}_t)$.

Finally, the variance of output growth is given by

$$\begin{aligned} var(\Delta y_t) &= var(\Delta s_t + \Delta i_t - \Delta i_{t-1}) \\ &= var(\Delta s_t) + 2var(\Delta i_t) + 2cov(\Delta s_t, \Delta i_t) - 2cov(\Delta s_t, \Delta i_{t-1}) - 2cov(\Delta i_t, \Delta i_{t-1}) \end{aligned}$$

where

$$
\begin{aligned}
cov(\Delta s_t, \Delta i_t) &= cov(\eta_t + z_t^s, \eta_t + v_t + z_t^i) \\
&= \sigma_\eta^2 + \lambda_{s\eta}\sigma_\eta^2 + cov(z_t^s, z_t^i) + \lambda_{i\eta}\sigma_\eta^2,
\end{aligned}
$$

$$
\begin{aligned}
cov(\Delta s_t, \Delta i_{t-1}) &= cov(\eta_t + z_t^s, \eta_{t-1} + v_{t-1} + z_{t-1}^i) \\
&= cov(z_t^s, z_{t-1}^i) + cov(z_t^s, \eta_{t-1} + v_{t-1}) \\
&= cov(z_t^s, z_{t-1}^i) + (\phi_{s,1} - 1)\lambda_{s\eta}\sigma_\eta^2
\end{aligned}
$$

and

$$
\begin{aligned}
cov(\Delta i_t, \Delta i_{t-1}) &= cov(\gamma_{i\tau}\eta_t + v_t + z_t^i, \eta_{t-1} + v_{t-1} + z_{t-1}^i) \\
&= cov(z_t^i, \eta_{t-1} + v_{t-1} + z_{t-1}^i) \\
&= (\phi_{i,1} - 1)(\lambda_{i\eta}\sigma_\eta^2 + \lambda_{iv}\sigma_v^2) + cov(z_t^i, z_{t-1}^i)
\end{aligned}
$$

where $cov(z_t^s, z_t^i)$, $cov(z_t^s, z_{t-1}^i)$ and $cov(z_t^i, z_{t-1}^i)$ are the $(1,3)$, $(1,4)$ and $(3,4)$ element of $var(\mathbf{z}_t)$ respectively.