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ON THE IRREDUCIBILITY OF A COMPETITIVE ECONOMY

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Abstract

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1. Introduction

In the Arrow-Debreu model, the possibility of minimum-wealth situation may lead to non-existence of a Walras equilibrium. Arrow and Debreu (1954) prevent this situation from arising by assuming that every consumer's initial endowment is in the interior of her/his consumption set and that each firm can remain inactive. The strength of this assumption lead several authors to define different concepts of irreducibility, which are milder assumptions than those stated in Arrow and Debreu (1954). Gale (1955) formulated irreducibility in the case of the linear exchange model, obtaining a necessary and sufficient condition for the existence of equilibrium. McKenzie (1959, 1961) and Debreu (1962) adapt this to the standard private ownership model. Under their assumptions, irreducibility is no necessary but sufficient condition for existence of equilibrium. Debreu (1962) introduces an auxilary concept existing without irreducibility, called quasi-equilibrium and where only consumers who are not at minimum-wealth consume a maximal element within their budget set. This allows quite well to focus only on the assumptions under which no minimum-wealth situation may occur. Arrow-Hahn (1971) define resource relatedness or Arrow-Hahn irreducibility, in order to insure existence of a Walras equilibrium in a similar way.

Finally, Bergstrom (1976), gives in the case of an exchange economy a definition of irreducibilty which, as Florenzano (1981, 1982) shows, generalizes McKenzie-Debreu and Arrow-Hahn irreducibilty for exchange economies and is more general than both taken together.

Here, we propose to generalize Bergstrom's irreducibility to economies with production and following Florenzano (1981, 1982) we show that this approach still unifies McKenzie-Debreu and Arrow-Hahn irreducibility. The interest is not only to clearify the links between the different approaches, but given the relative complexity of the definitions, it seems useful to us, to dispose of a unifying definition of irreducibility which is not more complicated to state than any of the other two.

2. The Model and the Main Results

We consider an economy with a positive finite set L of ℓ commodities, a positive finite set I of m consumers and a positive finite set J of n producers. We denote by

X_i the consumption set of the i-th consumer ($i = 1, ..., m$), by $e_i \in R^\ell$ her/his initial endowment vector and by $P_i : X_i \to X_i$ her/his strict preference correspondence. We say that x' in X_i is strictly preferred to x in X_i if $x' \in P_i(x)$.

We let e be the total initial endowment vector, that is, $e = \sum_{i \in I} e_i$. The technological possibilities of the j-th producer ($j = 1, ..., n$) are represented by a subset Y_j of R^ℓ. We denote by Y the total production set of the economy, that is $Y = \sum_{j \in J} Y_j$. For all $(i, j) \in I \times J$, the positive real number θ_{ij} denotes the share of the i-th consumer in the profit of the j-th producer. Of course for every j in J, $\sum_{i \in I} \theta_{ij} = 1$. An allocation is a vector (x_i) in $\prod_{i \in I} X_i$ and it is said feasible if there exists (y_j) in $\prod_{j \in J} Y_j$ such that $\sum_{i \in I} x_i = \sum_{j \in J} y_j + e$. Furthermore, for a convex set C, $T_C(z)$ will denote the tangent cone of C at z and f^k will denote the k-th vector $(0, ..., 0, 1, 0, ...0)$ of the natural base of R^ℓ.

An economy $\mathcal{E}$ is a collection

$$\mathcal{E} = ((X_i, P_i, e_i)_{i \in I}, (Y_j)_{j \in J}, (\theta_{ij})_{(ij) \in I \times J})$$

We can now formally define the notions of quasi-equilibrium and of Walras equilibrium.

Definition 2.1. A quasi-equilibrium of the economy $\mathcal{E}$ is an element $((x_i^*), (y_j^*), p^*)$ of $R^{\ell(m+n+1)}$ such that $p^* \neq 0$ and
(a.1) for all $i \in I$, $x_i^* \in X_i$, $p^* \cdot x_i^* = p^* \cdot (\sum_{j \in J} \theta_{ij} y_j^* + e_i)$;
(a.2) for all $i \in I$, for all $x_i \in P_i(x_i^*)$, $p^* \cdot x_i^* \leq p^* \cdot x_i$;
(b) for all $j \in J$, $y_j^* \in Y_j$ and for all $y_j \in Y_j$, $p^* \cdot y_i \leq p^* \cdot y_i^*$;
(c) $\sum_{i \in I} x_i^* = \sum_{j \in J} y_j^* + e$.

A Walras equilibrium of the economy $\mathcal{E}$ is an element $((x_i^*), (y_j^*), p^*)$ of $R^{\ell(m+n+1)}$ such that $p^* \neq 0$, which satisfies conditions (a.1), (b) and (c) together with
(a.2') for all $i \in I$, for all $x_i \in P_i(x_i^*)$, $p^* \cdot x_i^* < p^* \cdot x_i$.

As we will see later a quasi-equilibrium $((x_i^*), (y_j^*), p^*)$ is a Walras equilibrium if it satisfies the following additional condition:

$$\inf p^* \cdot X_i < p^* \cdot (\sum_{j \in J} \theta_{ij} y_j^* + e_i), \ \forall i \in I.$$

This condition is usually insured by the assumption: for all j in J, $0 \in Y_j$ and for all i in I, e_i is in the relative interior of X_i. Especially, the latter one, being a quite strong assumption, there exist various notions of irreducibility of the economy, insuring existence of a Walras equilibrium under less strong assumtions.

Henceforth, we will make use of the following assumptions.

Assumption CX. For all i in I, X_i is a closed, convex subset of R^ℓ.

Assumption CP. For all i in I, P_i is irreflexive and for all $x \in X_i$, if $z \in P_i(x)$ and $v \in X_i$, then there exists $\lambda, 0 < \lambda \leq 1$ such that $\lambda v + (1-\lambda)z \in P_i(x)$.

Assumption PI. For every j in J, $Y_j \subset R^\ell$ and $0 \in Y_j$.

Assumption PT. Y is closed and convex subset of R^ℓ.

Assumption D. $\sum_{i \in I} X_i - Y$ has a non empty interior in R^ℓ.

Assumption S. The relative interiors of $\{e\} + Y$ and of $\sum_{i \in I} X_i$ have nonempty intersection.

Assumpion D is in fact a free assumption. As McKenzie (1959) points out, if its dimension were $k < \ell$ we can treat the problem in the smallest linear manifold containing $\sum_{i \in I} X_i - Y - \{e\}$. We then obtain the full dimensionality by redefining k goods as bundles of the ℓ goods and solve the problem in R^k. Assumption PI as well is rather for convenience. If we would be faced to an economy where 0 is not in Y_j for some j in J, it is possible to fulfill this condition by a translation argument. For every $j \in J$ let $z_j \in R^\ell$ such that $Y_j - z_j$ contains 0 and let $Y_j' = Y_j - z_j$. For every i in I let her/his initial endowment be $e_i' = e_i + \sum_{j \in J} \theta_{ij} z_j$. Obviously, $((x_i^*), (y_j^*), p^*)$ is a Walras equilibrium (quasi-equilibrium) of the 'original' economy if and only if $((x_i^*), (y_j^* - z_j), p^*)$ is a Walras equilibrium (quasi-equilibrium) of the 'translated' one.

We take from Gale and Mas-Colell (1975) the definition of the augmented preference mappings. This allows us to obtain our results for non-satiated preferences instead of locally non-satiated preferences.

$$\hat{P}_i(x) = \{z \in X_i \mid z = x + \lambda(y - x),\ 0 < \lambda \leq 1, y \in P_i(x)\}$$

We state the different notions of irreducibility in close terms following Florenzano (1981). This allows for an easier comparison between them. Note that all three concepts implicitely imply non-satiation of the consumers preferences.

Definition 2.2 An economy $\mathcal{E}$ is McKenzie-Debreu irreducible if for every partition of I into two non empty subsets (I_1, I_2) and every feasible allocation (x_i), there exists an allocation x', such that

(1) $x'_i \in \overline{\hat{P}_i(x_i)}$, $\forall i \in I_1$ and $\exists i \in I_1$, $x'_i \in \hat{P}_i(x_i)$;

(2) $\sum_{i \in I}(x'_i - e_i) - \sum_{i \in I_2}(e_i - x_i) \in Y$.

In other terms, it is possible to distribute $e + \sum_{i \in I_2}(e_i - x_i) + y'$, $y' \in Y$ amongst the consumers, making group I_1 better of according to (1).

Let D be the largest convex cone with vertex zero contained in $-R^{\ell}_{+}$, such that $Y + D \subset Y$. In the case of free disposal we would have $D = -R^{\ell}_{+}$, without any disposal possibility $D = \{0\}$.

Definition 2.3 An economy $\mathcal{E}$ is Arrow-Hahn irreducible if for every partition of I into two non empty subsets (I_1, I_2) and every feasible allocation (x_i), there exists an allocation x', such that

(1) $x'_i \in \overline{\hat{P}_i(x_i)}$, $\forall i \in I_1$ and $\exists i \in I_1$, $x'_i \in \hat{P}_i(x_i)$;

(2) $\sum_{i \in I} x'_i \in \{e'\} + Y$ with $\forall k = 1, ..., \ell$
$e'^k > \sum_{i \in I} e_i \Rightarrow$ there exists $\lambda^k > 0$ such that $\sum_{i \in I_2} e_i - \lambda^k f^k \in \sum_{i \in I_2} X_i - D$;
$e'^k < \sum_{i \in I} e_i \Rightarrow$ there exists $\lambda^k > 0$ such that $\sum_{i \in I_2} e_i + \lambda^k f^k \in \sum_{i \in I_2} X_i - D$.

An economy $\mathcal{E}$ is Arrow-Hahn irreducible, if for any feasible allocation (x_i) and any proper, non empty subset of consumers I_1, there exists an allocation (x'_i), preferred by everybody and strictly by someone in I_1 in the sense of (1) and such that (x'_i) would be feasible if the total initial endowment were $\{e'\}$.

We have the following conditions on $\{e'\}$:

$\{e'\}$ increases the total initial endowment of the good k, only if the group $I_2 = I \setminus I_1$ is able to give some of their initial endowment in k away and distribute the rest amongst them;

$\{e'\}$ decreases the total initial endowment of the good k, only if the group $I_2 = I \setminus I_1$ is able to take some of the good k and distribute it amongst them, together with their initial endowment.

Note that in the case of free disposal $D = -R^\ell_+$ and if for every i in I $e_i \in X_i$, the the second part of condition (2) is redundant: for any $\lambda^k f^k > 0$, $\lambda^k f^k \in -R^\ell_+$ and

$$\sum_{i\in I_2} e_i + \lambda^k f^k \in \sum_{i\in I_2} X_i - D.$$

Arrow-Hahn stated their irreducibility assumption in slightly different terms. Without the free disposal assumption it can be written as follows.

Definition 2.3' Consumer i' is said to be resource related to consumer i'' in the economy $\mathcal{E}$, if for every feasible allocation (x), there exists an allocation x', such that

(1) $x'_i \in \overline{\hat{P}_i(x_i)}$, $\forall i \in I$ and $x'_{i'} \in \hat{P}_i(x_{i'})$;

(2) $\sum_{i\in I} x'_i \in \{e'\} + Y$ with $\forall k = 1, ..., \ell$
$e'^k > \sum_{i\in I} e_i \Rightarrow$ there exists $\lambda^k > 0$ such that $e_{i''} - \lambda^k f^k \in X_{i''} - D$;
$e'^k < \sum_{i\in I} e_i \Rightarrow$ there exists $\lambda^k > 0$ such that $e_{i''} + \lambda^k f^k \in X_{i''} - D$.

Consumer i' is said to be indirectly resource related to consumer i'' in the economy $\mathcal{E}$ if there is a sequence of consumers, i_ν, $(\nu = 0, ..., k)$, with $i_0 = i', i_k = i''$, and consumer i_ν is resource related to consumer $i_{\nu+1} (\nu = 0, ..., k-1)$.

It is easy to check that an economy, where every couple of consumers is indirectly resource related is Arrow-Hahn-irreducible.

Definition 2.4 An economy $\mathcal{E}$ is Bergstrom-irreducible if for every partition of I into two non empty subsets (I_1, I_2) and every feasible $((x_i),(y_j)) \in \prod_{i\in I} X_i \times \prod_{j\in J} Y_j$, there exists an allocation (x'_i) and a system of m numbers $\lambda_i > 0$, $i = 1, ..., m$, such that

(1) $x'_i \in \overline{\hat{P}_i(x_i)}$, $\forall i \in I_1$ and $\exists i \in I_1$, $x'_i \in \hat{P}_i(x_i)$;

(2) $\sum_{i\in I} \lambda_i (x'_i - e_i - \sum_{j\in J} \theta_{ij} y_j) \in T_Y(y)$.

Theorem 2.1 *Suppose the economy $\mathcal{E}$ satisfies assumptions CP, PT, D, S and is Bergstrom irreducible, then every quasi-equilibrium $((x^*_i),(y^*_j),p^*)$ is a Walras equilibrium.*

Proof: For every i in I, $p^* \cdot x^*_i = p^* \cdot e_i + \sum_{j\in J} \theta_{ij} p^* \cdot y^*_j$ and $x'_i \in P_i(x^*_i) \Rightarrow p^* \cdot x'_i \geq p^* \cdot x^*_i$. Let (I_1, I_2) be a partition of I with $I_1 = \{i \in I \mid \inf p^* \cdot X_i < p^* \cdot e_i + \sum_{j\in J} \theta_{ij} p^* \cdot y^*_j\}$.

Suppose, first that $I_1 = \emptyset$. Hence, $p^* \cdot \sum_{i\in I} x^*_i = \operatorname{Min} p^* \cdot X$ and therefore, since $p^* \cdot y^* = \operatorname{Max} p^* \cdot Y$ and $p^* \cdot (y^* + e) = p^* \cdot \sum_{i\in I} x^*_i$, the hyperplane H with normal

p^*, through $\sum_{i \in I} x_i^*$, seperates $\sum_{i \in I} X_i$ and $\{e\} + Y$. By assumption D, H cannot contain both sets and therefore one set has its relative interior strictly outside of H. From assumption S this cannot happen, since the relatives interiors of $\{e\} + Y$ and $\sum_{i \in I} X_i$ have non empty intersection and therefore $I_1 \neq \emptyset$.

If (I_1, I_2) were a partition of I of non empty subsets, then, since $\mathcal{E}$ is Bergstrom-irreducible, there exists an allocation (x_i') and a system of real numbers $\lambda_i > 0,\ i = 1, ..., n$ satisfying the relations (1) and (2) of definition 2.4 with respect to (x_i) and the partition (I_1, I_2).

Claim: Let $((x_i^*), (y_j^*), p^*)$ be a quasi-equilibrium and x_i^*, $i \in I$ in the relative interior of X_i, then $x \in P_i(x_i^*)$ implies $p^* \cdot x > p^* \cdot x_i^*$ and $x \in \hat{P}_i(x_i^*)$ implies $p^* \cdot x > p^* \cdot x_i^*$.

Proof of the claim. First, we prove that $x \in P_i(x_i^*)$ implies $p^* \cdot x > p^* \cdot x_i^*$. Indeed, otherwise take v, such that $p^* \cdot v < p^* \cdot x_i^*$, then by assumption C, there exists $\lambda, 0 < \lambda \geq 1$ such that $\lambda v + (1 - \lambda)x \in P_i(x_i^*)$ and $p^* \cdot (\lambda v + (1 - \lambda)x) < p^* \cdot x_i^*$. This cannot be, since $((x_i^*), (y_j^*), p^*)$ is a quasi equilibrium. Now, suppose $x \in \hat{P}_i(x_i^*)$, hence there exists z in X_i and $\lambda, 0 < \lambda \leq 1$ such that $z \in P_i(x_i^*)$ and $x = x_i^* + \lambda(z - x_i^*)$ and from the above this implies that $p^* \cdot x > p^* \cdot x_i^*$. ∎

From the claim we deduce that, $p^* \cdot \sum_{i \in I_1} \lambda_i (x_i' - e_i - \sum_{j \in J} \theta_{ij} y_j) > 0$ and as p^* is in $N_Y(y^*)$–the normal cone of Y at y^*, $p^* \cdot \sum_{i \in I_2} \lambda_i (x_i' - e_i - \sum_{j \in J} \theta_{ij} y_j) < 0$. From this, we deduce that there exists $i \in I_2$ such that $p^* \cdot (x_i' - e_i - \sum_{j \in J} \theta_{ij} y_j) < 0$, contradicting the definition of I_2. Therefore, $I_1 = I$ and hence by the claim $((x_i^*), (y_j^*), p^*)$ is a Walras equilibrium of $\mathcal{E}$. ∎

From Florenzano (1981, 1982), we know that Bergstrom irreducibility is in the case of an exchange economy is implied by McKenzie-Debreu or Arrow-Hahn irreducibility. Moreover, from Florenzano (1982), a Bergstrom irreducible economy need not be, neither McKenzie-Debreu nor Arrow-Hahn irreducible. Our aim here is to show that the generalization of Bergstrom irreducibility to economies with production preserves its unifying property. In fact, the proofs of the following two properties are generalizations of Florenzano (1981).

Proposition 2.1 *Every McKenzie-Debreu irreducible economy $\mathcal{E}$ satisfying assumptions CX, PI, PT is Bergstrom irreducible.*

Proof: The relation (2) of definition 2.2 can be written as:

$$\sum_{i\in I_1}(x'_i - e_i - \sum_{j\in J}\theta_{ij}y_j) + 2\sum_{i\in I_2}(\frac{x'_i + x_i}{2} - e_i - \sum_{j\in J}\theta_{ij}y_j) + \sum_{j\in J}\sum_{i\in I_2}\theta_{ij}y_j \in Y - y,$$

therefore for some $y' \in Y$, $\sum_{i\in I_1}(x'_i - e_i - \sum_{j\in J}\theta_{ij}y_j) + 2\sum_{i\in I_2}(\frac{x'_i+x_i}{2} - e_i - \sum_{j\in J}\theta_{ij}y_j) = y' - y - \sum_{j\in J}\sum_{i\in I_2}\theta_{ij}y_j = y' - y + \sum_{j\in J}\sum_{i\in I_1}\theta_{ij}y_j - y$.

By assumption PI, for every $j \in J$, $\sum_{i\in I_1}\theta_{ij}y_j \in \overline{co}Y_j$. Thus, by the convexity of Y, $\sum_{j\in J}\sum_{i\in I_1}\theta_{ij}y_j \in Y$. Therefore, $y' - y + \sum_{j\in J}\sum_{i\in I_1}\theta_{ij}y_j - y \in T_Y(y)$. Hence, the allocation (x''_i) defined by

$x''_i = x'_i$ if $i \in I_1$

$x''_i = \frac{x'_i+x_i}{2}$ if $i \in I_2$

satisfies the conditions of definition 2.4. ∎

Proposition 2.2 *Every Arrow-Hahn-irreducible economy $\mathcal{E}$ satisfying assumptions CX, PI, PT is Bergstrom-irreducible.*

Proof: Let $\mathcal{E}$ be an Arrow-Hahn-irreducible economy and x be a feasible allocation, then there exists an allocation (x'_i) satisfying with respect to (x_i) conditions (1) and (2) of definition 2.3.

$$\sum_{i\in I}(x'_i - e_i) + \sum_{i\in I} e_i - e' \in Y$$

with $\sum_{i\in I} e_i - e' = \sum_{k\in L}\mu^k f^k = \sum_{\mu^k<0}\frac{|\mu^k|}{\lambda^k}(-\lambda^k f^k) + \sum_{\mu^k>0}\frac{\mu^k}{\lambda^k}(\lambda^k f^k)$.

Set $\alpha = \sum_{\mu^k<0}\frac{|\mu^k|}{\lambda^k} + \sum_{\mu^k>0}\frac{\mu^k}{\lambda^k}$;

if $e'^k > \sum_{i\in I} e_i^k$ then $-\lambda^k f^k \in \sum_{i\in I_2} X_i - \{\sum_{i\in I_2} e_i\} - D$;

if $e'^k < \sum_{i\in I} e_i^k$ then $\lambda^k f^k \in \sum_{i\in I_2} X_i - \{\sum_{i\in I_2} e_i\} - D$.

As $\sum_{i\in I_2} X_i - \{\sum_{i\in I_2} e_i\} - D$ is convex, $\frac{1}{\alpha}\sum_{k\in L}\mu^k f^k$ being a convex combination of elements in $\sum_{i\in I_2} X_i - \{\sum_{i\in I_2} e_i\} - D$, there exists $d \in D$ and for every $i \in I_2$, $a_i \in X_i$ such that

$$\sum_{i\in I} e_i - e' = \alpha(\sum_{i\in I_2}(a_i - e_i) - d).$$

Hence, $\sum_{i\in I}(x'_i - e_i) + \alpha\sum_{i\in I_2}(a_i - e_i) \in Y$ or in other terms

$$\sum_{i\in I_1}(x'_i - e_i - \sum_{j\in J}\theta_{ij}y_j) + (1+\alpha)\sum_{i\in I_2}(\frac{x'_i + \alpha a_i}{1+\alpha} - e_i - \sum_{j\in J}\theta_{ij}y_j) + \alpha\sum_{j\in J}\sum_{i\in I_2}\theta_{ij}y_j \in Y - y$$

Thus, for some $y' \in Y$,
$\sum_{i\in I_1}(x'_i - e_i - \sum_{j\in J}\theta_{ij}y_j) + (1+\alpha)\sum_{i\in I_2}(\frac{x'_i+\alpha a_i}{1+\alpha} - e_i - \sum_{j\in J}\theta_{ij}y_j) =$
$y' - y - \alpha\sum_{j\in J}\sum_{i\in I_2}\theta_{ij}y_j = (y'-y) + \alpha(\sum_{j\in J}\sum_{i\in I_1}\theta_{ij}y_j - y)$. From PI and PT, $\sum_{j\in J}\sum_{i\in I_2}\theta_{ij}y_j \in Y$ and therefore

$$\sum_{i\in I_1}(x'_i - e_i - \sum_{j\in J}\theta_{ij}y_j) + (1+\alpha)\sum_{i\in I_2}(\frac{x'_i + \alpha a_i}{1+\alpha} - e_i - \sum_{j\in J}\theta_{ij}y_j) \in T_Y(y).$$

Hence the allocation (x''_i) defined by:

$x''_i = x'_i$ if $i \in I_1$

$x''_i = \frac{x'_i+\alpha a_i}{1+\alpha}$ if $i \in I_2$

satisfies the conditions of definition 2.4. ∎

Note that throughout the paper we did not make any transitivity or completness assumption on the preferences. If one would want the preferences to depend on the price level, the consumption or the production of the firms, the proofs would require no modification whatsoever. Through obvious modifications we would need to complicate the definitions of irreducibility. We refer to Florenzano (1981) for the case of an exchange economy which directly carries over.

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