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POLLUTION LINKED TO CONSUMPTION: A STUDY OF POLICY INSTRUMENTS IN AN ENVIRONMENTALLY DIFFERENTIATED OLIGOPOLY

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Abstract

In this paper we evaluate the effectiveness of alternative regulatory policies on reducing aggregate pollution in an environmentally differentiated market. Two firms first choose their environmental quality and then their prices in a market where consumers differ in their valuations of the environmental features of the products. We first show that environmental standards may have an adverse impact on aggregate pollution. Moreover, we find that a uniform ad-valorem tax rate unambiguously increases the level of pollution in the market. When the tax rate is set in favor of the environmentally cleaner product, aggregate pollution decreases. Finally, direct subsidies on the abatement technology always decrease pollution.

Key Words: Aggregate pollution, Vertical Differentiated Oligopoly, Environmental Consciousness, Environmental Policy.

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1 Introduction

The vast majority of goods are nowadays available in a variety of types with substantially different impact on the environment. Environmental friendly products co-exist in the market with more environmentally damaging versions. Examples of *green* products include bio-degradable cleaning products, mercury-free batteries, recycled paper, unleaded fuel, low fuel-consumption vehicles, biologically grown vegetables, products in recyclable containers or recyclable products themselves. In general, green products are sold at higher prices than their competing variants, even when the rest of the product's features are exactly the same. The reason for this successful product coexistence is obviously that consumers, some more than others, are willing to spend more money on green products. Just why consumers behave in this way is a controversial question. It might be argued that consumers believe they can significantly contribute to the reduction of environmental pollution. Although it is collective action what does really change the state of the environment, buying green products is *perse* a gratifying action and it can exert some power to influence firm's attitudes towards the environment.[1] Whatever the reason, consumers' behavior has a potential for positive effects on the environment.[2] Even though green is still a minor feature of markets we can expect more markets developing green varieties or, what really concerns us here, firm's competing at an increasing rate in the environmental features and services associated to the products they offer.

These arguments suggest that vertical product differentiation models provide the appropriate framework to analyze the provision of environmental services in markets and the impact of alternative regulatory policies on this provision and on the resultant aggregate pollution.[3] Recent papers on qual-

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[1] According to the political science literature, individual action is one of the five dimensions of environmental consciousness and it encompases all private environmental behavior including consumption of environmental friendly products, ecolabeled products and waste disposal efforts. The term environmental conciousness comprises elements within five different dimensions: affective, cognitive, connative, individual action and collective action.

[2] Influencing consumption through information and education can be a source of environmental improvement. Indeed, policiy agendas' of industrialized countries include financial and regulatory support to green production (i.e. ecobalances and ecoauditing) as well as social or educational programs.

[3] The effects of environmental policy in homogeneous oligopolistic markets have been widely analyzed in the industrial organization literarure: Buchanan (1969) and Oates and Strassman (1984) in monopoly, Ebert (1991) in a Cournot duopoly and Katsoulakos and Xepapadeas (1992) and Requate (1992) in oligopoly. They all focus on the design of optimal, Pigouvian, emission fees. In general, an optimal emission tax within non-competitive market structures falls short of marginal external damages and has minimum output distortions. Moreover, overinternalization of environmental damages in oligopoly

ity provision in differentiated markets have demonstrated that regulatory policies can undoubtedly be welfare improving. For instance imposing a minimum quality standard improves social welfare as, in the regulated equilibrium, higher quality products are provided and more consumers are active (Ronnen (1991)). When environmental externalities are considered, however, the second fact might be counter-productive as aggregate pollution depends on both total sales and unitary emissions of the products. Our claim is that environmental policy in differentiated industries has to take into account the effects not only derived from quality readjustments but also from consumers reallocations.

To substantiate our claim, we study the effects of technology subsidization, maximum emission standards and ad valorem taxes on the equilibrium aggregate pollution in an environmentally differentiated market. We consider two firms that first choose the environmental quality of their products and then their prices. Consumers differ in their valuation of the environmental features of the products. As a result, the (unregulated) equilibrium is characterized by the coexistence of two varieties of the product identified by their unitary emissions levels.

To the best of our knowledge, three articles have studied the effects of environmental targeting policies in a differentiated market, namely, Motta and Thisse (1993), Cremer and Thisse (1994) and Constantatos and Sartzetakis (1996). As in ours, all these papers take advantage of the analogy between quality and environmental features of a product. Cremer and Thisse (1994) study the provision of environmental quality and the effects of ad-valorem taxation on this provision in a setting in which consumers enjoy a positive externality associated with the average environmental quality. Constantatos and Sartzetakis (1996) evaluates the effects of a commodity tax when the production of a high quality product is associated with a negative environmental externality (i.e. the use of a highly polluting input that cannot be substituted at all). They also measure the environmental externality through the average quality. Using the average environmental quality might sometimes be justified by lacks of information but, in general, it can be misleading because it is a rough approximation. Thus, we have departed from this literature by assuming that product differentiation is related to an observable and measurable variable which captures the unitary emissions level of a specific product variety. Some examples are mercury quantity in batteries, photocopiers' ozone harmful unit emissions, central heating' CO_2 emissions, etc.

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market structures tends to reduce the number of firms, firm's output and, therefore, total pollution levels. This argument has contributed to reduce concern on tax, specifically, and on other environmental policy instruments, in general, in the case of oligopolistic industries.

Our modeling allows us to accurately compute the negative environmental externality and, following our claim, study the impact of regulatory policies on both the emission level per unit of product (associated product quality) and the firms' aggregate pollution level after consumption (industrial environmental externality). Motta and Thisse (1993) study the impact of minimum environmental standards on market structure and welfare. However, they do not investigate the effects of this policy on aggregate pollution.

There is another important difference between our paper and those of Cremer and Thisse (1994) and Constantatos and Sartzetakis (1996). While they study the impact of ad-valorem taxation allowing entry in the industry but limiting to the study of situations in which the market is covered (all consumers are active in equilibrium), we contrastively restrict the analysis to the duopolistic case but allow for the possibility of non-covered markets situations, as in Motta and Thisse (1993). We consider our setting as more accurate for the study of those situations in which *small* taxes (or different policies) are introduced, keeping the market structure unchanged but distorting the allocation of consumers and abatement between firms. However, Cremer and Thisse (1994) and Constantatos and Sartzetakis (1996) seems to be more adequate in situations in which *large* taxes distort market structure.

Our results are as follows: First, we characterize the (unregulated) equilibrium. Two product varieties arise in equilibrium: the *clean* and the *dirty* products. Surprisingly, the clean firm (which is that producing the less polluting product) might actually be the more polluting firm when total pollution after sales is considered. The reason is that in equilibrium the clean firm's market share is higher than the dirty firm's. This happens whenever absolute product differentiation is small, that is, whenever the environmental quality gap between both products is not very large. Second, we show that environmental standards, whether defined as pollution rates or as technology standards, may have adverse effects on industrial pollution levels. While both firms improve their individual environmental quality, the total level of pollution might be higher. Third, we show that a direct subsidy on the abatement technology increases the level of environmental services of both firms and unambiguously decreases aggregate pollution: the reason is that technology subsidization does not affect consumers allocation. Fourth, a uniform ad-valorem tax tends to decrease abatement of both firms in the market and, as a result, total emissions increase. Finally, differential tax treatment through a non uniform ad-valorem tax can achieve the same positive results as a uniform direct subsidy to the firms. Increasing the tax rate on the dirty firm or decreasing the tax rate on the cleaner one, results in aggregate pollution decreasing.

The paper is organized as follows. The next section describes the model and the (unregulated) equilibrium. In Section 3 we explore the effects of a direct subsidy on the costs of the abatement technology. Section 4 analyzes the effects of an environmental standard. Section 5 is devoted to the case where the government introduces an ad-valorem tax, considering both uniform and non-uniform tax rates. Section 6 concludes. Some proofs have been relegated to an appendix.

2 The model

We consider a standard model of vertical product differentiation.[4]

On the demand side of the market there is a continuum of consumers indexed by θ and uniformly distributed on $[0,1]$. The utility function of a consumer with index θ is denoted by $V(\theta, e)$, where θ is the individual's product matching value and e denotes the environmental features of the product (the pollution level derived from consumption, use, production, or disposal of the product).[5] From now on we will refer to this environmental variable as the pollution level associated with the product or the *unitary emissions level of the product*. We assume these emissions per unit of product to be perfectly observable. Further, it is assumed that, for all e, $V_1(\cdot) > 0$, which means that, given any product with pollution level e, a consumer j whose type is θ_j derives higher utility than consumer i whose type is θ_i whenever $\theta_j > \theta_i$. Therefore, θ describes the environmental awareness of the consumer and the higher its value, the greater consumer's willingness to pay to reduce environmental pollution. On the other hand, we assume that, for all θ, $V_2(\cdot) < 0$ which means that all consumers derive higher utility from consumption of the less polluting variety. That is, given a free choice between two products all consumers agree.

For expositional purposes, in what follows, we adopt the following simple indirect utility function specification: consumer θ derives utility $U = \theta(\bar{e} - e) - p$ if she consumes the product whose pollution level is e and pays price p. Under this specification, the individual actually evaluates pollution relative to a generally accepted or known pollution level ($\bar{e}$).[6] This refer-

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[4] See Gabszewicz and Thisse (1979) and Mussa and Rosen (1978) for this framework.

[5] Although many environmental characteristics can exist we assume that all environmental features can be represented by a single unidimensional variable. An example is mercury quantity in batteries, water-use together with chemical contain associated to a unit of cleaning product or the rate of input use within a production process.

[6] Note that $\bar{e}$, the pollution index associated with the product, is of common knowledge for all consumers. It gives the maximum pollution level or the features of the more polluting verision of the product in a (homogeneous) market. Because we focus on the development of green markets, it is plausible to assume that consumers exactly perceive

ence level is normalized to $\bar{e} = 1$ without loss of generality. We assume that no consumption gives zero utility to consumers.[7] In addition, we also assume that consumers only derive utility from the first unit they consume or, alternatively, that each consumer buys one unit of the product only.

On the supply side of the market there are two identical firms producing the good. Each firm offers one product with a unique environmental feature determined by the technology it chooses for the provision of the good. The production technology set is common knowledge and is characterized by the cost function $C(e)$. Provision of the environmentally cleaner products is more costly, that is $C'(e) < 0$, and, we assume that there are decreasing returns to scale, that is, $C''(e) > 0$. In what follows, for computational purposes, we particularly adopt the following quadratic abatement cost function:

$$C(e) = \frac{k(1-e)^2}{2} \tag{1}$$

Unit marginal production costs are assumed to be zero, without loss of generality.

Firms play the following two-stage game: In the first stage, firms simultaneously decide on the abatement technology they will use; that is, e_i is chosen at a cost $C(e_i)$, for $i = 1, 2$. In the second stage, firms simultaneously set prices and consumers choose which variety to buy. This two-stage modelling is motivated by the fact that often firms can rapidly change their prices while a change in the abatement technology takes place in the long run. In this context, it is reasonable to assume that abatement technology decisions are long-run variables while prices are short-run variables. The solution concept used is subgame perfection. We proceed by backward induction, solving first the second stage to find the optimal pricing functions and then, solving the first stage to obtain both products' optimal emission levels.

The firm which chooses a lower abatement technology will be called the *dirty* firm, while the firm which chooses the higher abatement technology will be referred as the *clean* firm. Without loss of generality, we consider firm 1 to be the dirty firm, offering a product with associated pollution level e_1 at price p_1, and firm 2 to be the clean firm, offering a product with pollution level e_2 and price p_2 where, reasonably, $e_1 > e_2$.

Next we derive the demand functions for each product. In the continuum of individuals, there is one consumer indifferent between buying either good

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the improvement of the environmental features associated to a new "green" variety.

[7] Indeed, we can consider that those consumers that do not buy a green variety do buy an homogeneous version of the product in a perfectly competitive market, which price is normalized to zero. An example of this argument can be the Spanish paper market, where two big firms compete in the recycled paper market while there is a large number of firms in the regular paper market.

with preferences determined by the parameter $\theta_h = (p_2 - p_1)/(e_1 - e_2)$. Similarly, there is one consumer indifferent between purchasing the good produced by the dirty firm and not buying at all who is characterized by the parameter $\theta_l = p_1/(1 - e_1)$. Demand for the dirty firm comes from the group of consumers in the lower bound $\theta_l \leq \theta \leq \theta_h$ while demand for the clean firm comes from the upper bound group $\theta_h \leq \theta \leq 1$. Thus, quantity demanded from both firms is given respectively by:

$$q_1 = \frac{p_2 - p_1}{e_1 - e_2} - \frac{p_1}{1 - e_1} \tag{2}$$

$$q_2 = 1 - \frac{p_2 - p_1}{e_1 - e_2} \tag{3}$$

In the second stage of the game, firms simultaneously choose prices to maximize their profits, $\Pi_i = p_i q_i - c(e_i)$, for $i = 1, 2$. From the first order conditions we obtain the following Nash equilibrium prices charged by the dirty and clean firm:

$$p_1^*(e_1, e_2) = \frac{(1 - e_1)(e_1 - e_2)}{3 + e_1 - 4e_2} \tag{4}$$

$$p_2^*(e_1, e_2) = \frac{2(1 - e_2)(e_1 - e_2)}{3 + e_1 - 4e_2} \tag{5}$$

where, as expected, $p_2^*(e_1, e_2) > p_1^*(e_1, e_2)$: the more polluting good is supplied at a lower price.

To solve the first stage of the game, we write profits in terms of their first stage decisions as:

$$\Pi_1(e_1, e_2) = \frac{(1 - e_1)(1 - e_2)(e_1 - e_2)}{(3 + e_1 - 4e_2)^2} - k\frac{(1 - e_1)^2}{2} \tag{6}$$

$$\Pi_2(e_1, e_2) = \frac{4(1 - e_2)^2(e_1 - e_2)}{(3 + e_1 - 4e_2)^2} - k\frac{(1 - e_2)^2}{2} \tag{7}$$

Firms choose their abatement technology $(e_1,\ e_2)$ to maximize profits. First order conditions are:

$$\frac{d\pi_1}{de_1} = (1 - e_2)^2 \frac{(7e_1 - 3 - 4e_2)}{(4e_2 - 3 - e_1)^3} + k(1 - e_1) = 0 \tag{8}$$

$$\frac{d\pi_2}{de_2} = 4(1 - e_2) \frac{(6 - e_1 + 2e_1^2 - 5e_2 - 3e_2 e_1 + 4e_2^2)}{(4e_2 - 3 - e_1)^3} + k(1 - e_2) = 0 \tag{9}$$

We now define the following product differentiation variable,[8] $\lambda = (1 - e_2)/(1 - e_1)$, with $\lambda \in R^+$. Since $e_1 > e_2 > 0$, then $\lambda \geq 1$. Rearranging equations (8) and (9) and solving for λ we derive the unique real solution as $\lambda^* = 5.25123$. By substitution, the following expressions for optimal emission levels and prices e_1, e_2, p_1 and p_2, are obtained in equilibrium:[9]

$$e_1^* = 1 - \frac{0.048238}{k},\ e_2^* = 1 - \frac{0.253311}{k} \tag{10}$$

$$p_1^* = \frac{0.010251}{k},\ p_2^* = \frac{0.10766}{k} \tag{11}$$

which gives the following allocation of consumers or market shares:[10]

$$q_1^* = 0.2625,\ q_2^* = 0.52499 \text{ and } q_o^* = 0.21251 \tag{12}$$

where q_o^* denotes the unserved market portion.

Besides unitary pollution levels, we can measure the total level of pollution associated to each product variety. Thus, we observe that, in equilibrium, the dirty firm sells 0.2625 units whose unitary emission level is given by e_1. The total emission level derived from the dirty product's consumption is $E_1 = e_1 q_1 = 0.262497 - 0.012662/k$. On the other hand, the clean firm enjoys a higher market portion, selling 0.52499 units of the pollutant product characterized by the unitary emission level e_2. The clean firm's total emission level is $E_2 = e_2 q_2 = 0.52499 - 0.132985/k$. Total pollution in the market is $E_T = E_1 + E_2 = 0.78749 - 0.14565/k$.

In equilibrium, the firms differentiate their products and the market is partially divided between them. Those consumers with higher environmental consciousness buy the cleaner variety of the product while those consumers with lower environmental consciousness either purchase the dirty variety or

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[8] This variable is introduced as it facilitates computations (see Motta (1993)). At the same time, it measures the relative degree of product differentiation which is a very useful tool in terms of the interpretation of the model (see Ronnen (1991)). Note, that the higher is λ, the higher is relative product differentiation. In the appendix we report all the relevant variables in terms of the product differentiation variable.

[9] To prove that this is indeed an equilibrium we have to show that none of the firms can improve its profits by leapfrogging the rivals' choice. That is, firm 1 cannot increase its profits by choosing a lower emission level than e_2^* and, analogously that firm 2 does not want to choose $e_2 > e_1^*$. It can be easily shown that there are not such incentives (see appendix).

[10] Note that if we assume nonnegative pollution levels, $e_1 \geq 0$ and $e_2 \geq 0$, there is a lower bound on abatement technology costs: $k \geq \underline{k} = 0.25331$.

do not buy at all. Firms do not deviate from the equilibrium, by approximating their product variety to its rival's in order to attract some consumers, because it would force a price war strong enough to reduce their profits[11].

<insert figures 1 and 2 about here>

At this point, we further characterize the results with respect to the pollution levels generated by each firm. Consider Figure 1, which depicts per unit emissions, and Figure 2, which shows aggregate emissions. Observe first that emissions per unit of product (e_i, $i = 1, 2$) are an increasing and concave function of the cost parameter k, which measures the level of abatement cost. Obviously, as k increases relatively more dirty products will be provided in equilibrium. However, even though both firms' emissions per unit converge to $\overline{e}$ (firms' abatement efforts converge to zero) when the abatement cost goes to infinity, the clean firm's unit emissions increase faster than the dirty firm's unit emissions. In other words, even though relative product differentiation (measured by λ) does not vary with respect to the parameter k, the absolute emissions differential between the firms is reduced with k.

Second, the clean firm's total emissions exceed the dirty firm's total emissions whenever the parameter k is high enough. Recall that total emissions per firm are simply $E_i = e_i q_i$, $i = 1, 2$, which are also increasing and concave functions of the cost parameter. Note that both market shares and unitary emissions levels are endogenously determined in equilibrium. While firms' market shares are constant with respect to k, firms' emissions differential decreases as the cost parameter k increases. Therefore, there exists a $\widetilde{k}$ such that, for all $k > \widetilde{k}$, aggregate pollution associated with the clean firm is higher than the dirty firm aggregate pollution.[12]

The following proposition summarizes these findings:

Proposition 1 *(i) Each firm's emissions per unit of product increase with respect to the abatement cost parameter* k.

(ii) Each firm's total emission level increases with respect to the abatement cost parameter k. *(As a result, the industry total emission level increases too).*

(iii) Even though the clean firm's emissions per unit of product are lower than the dirty firm's ($e_1^* > e_2^*$), *total pollution generated by the clean firm is*

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[11] See Shaked and Sutton (1982).

[12] The fact that equilibrium market shares do not change with k is specific to the model. However, what drives this result is not that feature, but that the environmental gap between both firms narrows as k increases. This is due to the existence of decreasings returns to scale.

higher than the total by the dirty firm ($E_1^* < E_2^*$) *whenever the abatement costs are high enough, that is, as long as* $k > 0.4583855$.

In the following sections we analyze the effects of different command-and-control policies and market based instruments widely used to affect the equilibrium market allocation with the aim of reducing total industrial pollution. We start by analyzing the effects on pollution levels when the government introduces a direct subsidy on technology. In section 4, the effects of imposing maximum emissions standards are analyzed. Finally, the implementation of ad-valorem taxes and their effects are studied.

3 Technology subsidization

Suppose that the government would like to induce the acquisition of cleaner production technologies. Such an environmental target can be reached by offering a subsidy on the overall cost of technology. Indeed, there exist numerous institutional subsidy programs to induce environmental investment, even though international agreements – such as the 'polluter pays principle' of the OECD – are explicitly aimed to avoid industrial subsidization.[13] Suppose then that the government offers the following subsidy to the firms: $S(e) = 0.5s(1-e)^2$, $0 < s < k$. As a result, both firms face the new cost function:

$$C(e) = (k-s)\frac{(1-e)^2}{2} \tag{13}$$

Recomputing the new equilibrium is straightforward and the following values are obtained:

$$e_1 = 1 - \frac{0.048238}{k-s}, \; e_2 = 1 - \frac{0.253311}{k-s} \tag{14}$$

$$p_1 = \frac{0.010251}{k-s}, \; p_2 = \frac{0.10766}{k-s} \tag{15}$$

$$q_1 = 0.2625, \; q_2 = 0.52499 \text{ and } q_0 = 0.21251 \tag{16}$$

Unitary emissions levels decrease for all s. In fact, a technology subsidy reduces abatement technology costs and, as a result, optimal firms' abatement efforts and prices increase. Aggregate emissions levels ($E_1 = 0.262497 - 0.012662/(k-s)$, $E_2 = 0.52499 - 0.132985/(k-s)$) decrease.

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[13] For example, the PITMA program of the spanish Ministry of Industry and Trade have been offering such technological aids since 1991.

The reason is that while the subsidy does not affect firm's market shares because relative product differentiation does not change, unit emission levels decrease.

The following proposition summarizes:

Proposition 2 *Suppose that the government subsidizes abatement technology by the following function:* $S(e) = 0.5s(1-e)^2$, $0 < s < k$. *Then (i) both firms' unitary emissions levels decrease and (ii) both firms' total pollution levels into the market decrease.*

Next, we show how a standard on pollution may have a dramatic impact on the total level of pollution in environmentally differentiated markets.

4 Standard on pollution

It is generally agreed that a maximum emission or a technology standard increases products' environmental quality. Suppose now that the government imposes a maximum emission standard (in the form of emissions' rate or a best or recommended available technology standard (BACT or RACT)) which, unavoidably has to be met by both firms. We analyze the effects of this kind of policy on firm's optimal abatement levels, equilibrium consumers' allocation and total pollution levels.

In the first place, we characterize how the unregulated equilibrium is affected when a maximum emission standard per unit of product is introduced. To avoid duopolistic equilibrium non-existence problems, we consider a maximum emissions standard close enough to the dirty firm's optimal emissions in the unregulated equilibrium.[14] In order to make the problem interesting, the standard, of course, binds for the dirty firm.[15] Previous analysis on product quality standards (see Ronnen (1991) and Crampes and Hollander (1995)) suggest the following intuition on how an environmental standard affects the market equilibrium. As a result of the emission standard policy, the dirty firm will optimally meet the standard (as long as the costs of meeting the standard are not prohibitive). On the other hand, the clean firm optimal reaction consists of increasing its own abatement too, even though the standard is not binding for it, due to strategic behavior.[16]

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[14] A very restrictive maximum emission standard might result in a monopoly. Below, we will come back on this point.

[15] A not binding standard for the dirty firm would not have any effect at all on the market. We would fall into the case previously analyzed.

[16] Note that the clean firm reacts by increasing its own abatement in order to avoid a stronger price competition in the second stage of the game.

Obviously, since the dirty firm's choice set is restricted by the law there is a different market equilibrium. In this new regulated equilibrium abatement efforts are unambiguously higher for both firms. Indeed, Ronnen (1991) and Motta and Thisse (1993) conclude that a standard policy would be a proper policy to reduce pollution levels of both firms. Their analysis however overlooks the effects over total pollution levels in the market as a result of the standard policy. Since the dirty firm's strategy space has been restricted, competition between firms tends to be stronger and market shares of both firms change. Thus, there is some potential for an increase in total pollution levels in the market.

First of all, we emphasize that setting a very restrictive emission standard might result in negative profits for the dirty firm and, therefore, there will not exist an equilibrium in pure strategies. In this case, the market structure will probably change to a monopoly, in which only the clean firm would survive. Lower levels of pollution would result but market surplus would be lower. Since this substantially complicates the analysis by introducing a higher number of cases, we rule out this possibility. Thus, in this paper, we will only consider an emission standard policy that does not affect the market structure. Since an equilibrium necessarily exists when the emission standard is "close" enough to the unregulated equilibrium, we restrict our analysis to its neighborhoods.

The post-regulation equilibrium emission levels are then given by $e_1^{r.e} = e_{\max}$, where $e_{\max}$ is the imposed standard, and $e_2^{r.e} = BR_2(e_{\max})$, where the implicit function $BR_2(\cdot)$ denotes the clean firm's best response (see equation 9). We denote the post-regulation total equilibrium emissions by $E_1^{r.e} = q_1^{r.e}e^{r.e}$ for the dirty firm and $E_2^{r.e} = q_2^{r.e}e_2^{r.e}$ for the clean one. Let $E_T^{r.e}$ denote the total pollution in the market, that is $E_T^{r.e} = E_1^{r.e} + E_2^{r.e}$. For computational convenience, we denote total pollution levels in terms of the product differentiation variable, which is $\lambda^{r.e} = (1-e_2^{r.e})/(1-e_{\max})$. (From now on, we omit the superscript *r.e.* to denote the regulated equilibrium.)

The equilibrium emission levels can be described by the following two equations:

$$1 - e_{\max} = \frac{4(4\lambda^2 - 3\lambda + 2)}{k(4\lambda - 1)^3} \tag{17}$$

$$1 - e_2 = \lambda(1 - e_{\max}) \tag{18}$$

Differentiating, it is easily seen that:

$$\frac{d\lambda}{de_{\max}} = \frac{k(4\lambda - 1)^4}{4(16\lambda^2 - 16\lambda + 21)} > 0 \text{ for all } \lambda > 1 \tag{19}$$

$$\frac{de_2}{de_{\max}} = \frac{4(10\lambda + 1)(4\lambda - 1)^2}{(16\lambda^2 - 16\lambda + 21)} > 0 \text{ for all } \lambda > 1 \tag{20}$$

These two properties are used to support our main results. Equation (19) shows that product differentiation decreases as a result of the maximum standard policy. Note also that this reduction is higher, the higher the cost parameter k. Equation (20) shows that the clean firm's best response to the dirty firm adjustment consists of decreasing its emissions too.

The following proposition states that total pollution can increase as a result of the maximum emissions standard policy. The proof is given in the appendix.

Proposition 3 *After setting an emission standard, close enough to the unregulated equilibrium, that binds for the dirty firm:*

a) pollution derived from consumption of the dirty product increases if $k > 1.040483$

b) pollution derived from consumption of the clean product increases if $k > 56.441126$

c) total pollution in the market increases if $k > 37.9717$.

The intuition goes as follows: Due to regulation, the dirty firm reduces its emissions to meet the standard. In order to reduce price competition, the clean firm's best response consists of reducing its emissions too. However, because of emissions reductions are relatively more expensive for the clean firm, its abatement effort is lower than the effort impelled to the dirty firm. As a result, product differentiation is reduced. The decrease in the product differentiation fosters price competition and therefore, both prices decrease. Both effects, the decrease in prices and the decrease in emissions' levels induce higher sales for both firms, as more consumers are active in the market. If the market effect – which captures this increase in firm's market shares – is strong enough, total pollution in the market will increase, even though both firms reduce their emissions per unit of product. This market effect is stronger, the higher are abatement costs.

The higher are firms' abatement costs the stronger is the market effect. The reason is that the post-regulation product differentiation is lower the higher are abatement costs, as the clean firm has to incur even higher costs to differentiate its product to avoid price competition. Thus, in the second stage of the game, firms are involved in a price war sufficiently strong to decrease prices too much. Quantities sold by each firm increase enough to induce higher total pollution levels in the market.

5 Commodity taxation

Taxation of polluting products has been widely used to reduce their consumption and so, pollution derived from them. Tobacco, fuels and cars are some examples of goods facing this type of taxation. We might also think of the current government agendas to tax electricity energy production, batteries and product containers (i.e. glass, plastic and paper) through recycling charges. In this section we study the effects of commodity taxation policies on both the unitary and the total emissions levels.

Consider an ad-valorem tax t_i – a commodity tax where the per-unit tax is proportional to the price level – imposed on firm i, $i = 1, 2$. Firm i's profit function is then given by:

$$\pi_i = (1 - t_i)p_i q_i - C(e_i),\ i = 1, 2 \tag{21}$$

To simplify the presentation of our results we define $\tau_i = 1/(1 - t_i)$ as the firm i's "tax burden". Note that $\tau_i \geq 1$ defines a positive tax ($0 \leq t_i \leq 1$) while $\tau_i \leq 1$ corresponds to a negative tax or direct subsidy ($t_i \leq 0$). Besides, setting $\tau_1 = \tau_2 = 1$, we obtain the unregulated case analyzed above. Rewriting firm i's profits function in terms of τ_i gives:

$$\pi_i = \frac{1}{\tau_i} p_i q_i - C(e_i),\ i = 1, 2 \tag{22}$$

5.1 Uniform value-added tax (subsidy)

We assume in this section that both firms are charged at the same tax rate, that is $\tau_1 = \tau_2 = \tau$. There are many reasons to justify such a tax setting. In some cases, the government does not have enough information on firms' costs or polluting levels; in other cases, there are legal constraints impeding tax rates differentiation among firms participating in the same market.

We omit here the derivation of the equilibrium values, as it is solved using the same steps we undertook in the previous sections, substituting the profits functions for (22). An interesting implication derived from equation (22) is that both, a) the firm i's optimal strategy facing the commodity tax t_i and the cost function $C(e_i)$, and b) the firm i's optimal strategy facing the cost function $\tau_i C(e_i)$ and not being taxed at all, are exactly the same. When $\tau_1 = \tau_2 = \tau$, this property allows us to assure the existence of a duopolistic equilibrium as long as $k\tau > 0.25331$. We limit the analysis to such cases. Thus, for any $\tau \geq 0$ ($t \leq 1$), the equilibrium is unique and given by the following values:

$$e_1^* = 1 - \frac{0.048238}{k\tau},\ e_2^* = 1 - \frac{0.25331}{k\tau} \tag{23}$$

$$p_1^* = \frac{0.010251}{k\tau}, \; p_2^* = \frac{0.10766}{k\tau} \tag{24}$$

A uniform tax rate does not affect relative product differentiation. As a result, market shares of both firms remain unchanged, that is, as in the unregulated equilibrium ($q_1 = 0.262497$ and $q_2 = 0.52499$). Firms' total emissions levels are then $E_1 = 0.262497 - 0.012662/k\tau$ and $E_2 = 0.52499 - 0.132985/k\tau$, while total emissions level in the market is then $E_T = 0.78749 - 0.14565/k\tau$. Firms' profits are $\pi_1^* = 0.0015274/k\tau^2$, $\pi_2^* = 0.024437/k\tau^2$ and consumers' surplus is $0.0732188/k\tau$.

Optimal unitary emissions are found to be increasing for all $\tau > 0$. This is not surprising because an ad-valorem tax can be seen as a way of increasing firms' costs. Moreover, as market shares are not affected by the tax, introducing taxation in our model unambiguously yields to higher pollution levels.

The following proposition summarizes:

Proposition 4 *Consider the environmentally differentiated duopoly described above. After imposing a uniform ad-valorem tax on both firms:*

a) both firms' optimal unitary emission levels unambiguously increase.

b) both firms' total emission levels unambiguously increase.

5.2 Non-uniform value added tax

In the previous section we have investigated how symmetric tax increases reduce both firms optimal abatement levels which results in higher market pollution. Furthermore, social welfare is reduced. Instead of symmetric taxation policy, the regulator might increase only one firm's tax rate, say for instance the dirtier firm. In this section we focus on the effects of such a policy. We investigate whether the regulator can do better by taxing differently both polluting firms.

We proceed in the following way: starting from a situation in which both firms are taxed at the same tax rate t, we first slightly change the dirty firm tax rate and investigate the effects of this policy on the equilibrium outcome. Secondly we slightly change the clean firm tax rate. Analogously, we emphasize that differentiated enough tax rates might raise duopolistic equilibrium non-existence problems. However, these can be avoided by assuming that the tax differentiation is small enough.[17] For that reason we limit the analysis to situations in which one of the firms' taxes is slightly raised or lowered, that is, assuming $\tau_1 \simeq \tau_2$

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[17] Indeed, it is plausible to assume that it is not feasible to induce high levels of tax differentiation within a market.

Consider then the situation in which the clean firm tax rate remains unchanged that is $t_2 = t$ and the dirty firm tax rate is slightly changed, such that $t_1 \simeq t$ and $\tau_1 \simeq \tau$. It is easily seen (following the same steps as above) that there exists an equilibrium given by the unique solution to the following two equations system:

$$e_1 = 1 - \frac{\lambda^2(4\lambda - 7)}{k\tau_1 (4\lambda - 1)^3} \tag{25}$$

$$e_2 = 1 - \frac{4\lambda(4\lambda^2 - 3\lambda + 2)}{k\tau_2 (4\lambda - 1)^3} \tag{26}$$

Let us define the parameter γ, with $\gamma = \tau_2/\tau_1$, which measures firms' tax differentiation. The following lemma characterizes how the optimal product differentiation variable (λ) varies with respect to the tax differentiation variable (γ).

Lemma 1 *$\lambda(\gamma)$ is a decreasing function.*

Proof: In equilibrium, the product differentiation variable has to satisfy equations (25) and (26). By properly dividing these two equations it is obtained that λ has to satisfy $\lambda = (-12\lambda + 8 + 16\lambda^2)/(\gamma\lambda(4\lambda - 7))$. Proper differentiation gives $\partial\lambda/\partial\gamma = \lambda^2(7-4\lambda)/(12-32\lambda-14\lambda\gamma+12\lambda^2\gamma)$. Evaluating this derivative within a neighborhood of the equilibrium ($\gamma = 1, \lambda = 5.25123$) is obtained $\partial\lambda/\partial\gamma = -3.81053 < 0$.

Q.E.D.

The intuition behind this lemma is the following: either increasing (decreasing) the clean firm tax rate or decreasing (increasing) the dirty firm tax rate, the equilibrium product differentiation decreases (increases). As a result, price competition is fostered (relaxed). Since the reallocation of consumers strongly depends on how intensive is price competition, this observation is determinant when the effects of such policies on total pollution in the market are under consideration.

The change in optimal emission levels of both firms is given by:

$$\frac{de_i^*(\lambda(\gamma), \tau_i)}{d\tau_j} = \frac{\partial e_i}{\partial\lambda}\frac{d\lambda}{d\gamma}\frac{\partial\gamma}{\partial\tau_j} + \frac{\partial e_i}{\partial\tau_j} \text{ for all } i, j = 1, 2 \tag{27}$$

By computing these derivatives is obtained (evaluated in the unregulated equilibrium):

$$\frac{de_1}{d\tau_1} = \frac{\partial e_1}{\partial\lambda}\frac{\partial\lambda}{\partial\gamma}\frac{\partial\gamma}{\partial\tau_1} + \frac{\partial e_1}{\partial\tau_1} = \frac{0.0359918}{k\tau^2} > 0 \tag{28}$$

$$\frac{\partial e_2}{\partial \tau_1} = \frac{\partial e_2}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_1} = \frac{0.00518792}{k\tau^2} > 0 \tag{29}$$

Therefore, after slightly raising (lowering) the dirty firm tax rate, both firms react increasing (decreasing) their optimal emission levels. Total emissions from consumption are given by $E_1 = e_1 q_1$ and $E_2 = e_2 q_2$. Then, we must evaluate the effects of the tax on the reallocation of consumers. It can be easily seen that, if τ_1 is increased (decreased) both firms market shares decrease (increase), that is:

$$\frac{\partial q_1^*}{\partial \tau_1} = \frac{\partial q_1}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_1} = \frac{-0.00952165}{\tau} < 0 \tag{30}$$

$$\frac{\partial q_2^*}{\partial \tau_1} = \frac{\partial q_2}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_1} = \frac{-0.0190433}{\tau} < 0 \tag{31}$$

The following proposition gives the final effects on the firms' total emissions levels. The rest of proof is given in the appendix.

Proposition 5 *Consider the environmentally differentiated duopoly described above in which both firms face symmetric ad-valorem taxation policy. By slightly increasing (decreasing) the nonuniform ad-valorem tax rate to the dirty firm:*

a) total pollution from the dirty firm increases (decreases) if and only if $k\tau < 1.040476$ ($k\tau > 1.040476$).

b) total pollution from the clean firm increases (decreases) if and only if $k\tau < 0.396333$ ($k\tau > 0.396333$).

c) total pollution in the market increases (decreases) if $k\tau < 0.611047$ ($k\tau > 0.611047$).

Obviously, the dirty firm reacts to the tax rate increase (decrease) by raising (lowering) its unitary emissions level. The clean firm best response consists of raising (decreasing) its emissions level as well. More importantly, by slightly raising (lowering) the dirty firm tax rate, the clean firm is given a technological advantage (disadvantage). Thus, the clean firm's emission level increases less (decreases more) than the dirty firm one. As a result, product differentiation increases (decreases) and then price competition lessens (fosters), inducing both firms' sales reductions (increasing). Therefore, when total pollution in the market is under analysis, one has to be careful with this kind of policy because, as explained above, there are two opposite effects: the regulatory effect, which is related to the readjustment of abatement efforts, and the market effect, which is related to the reallocation of consumers between firms. Note that the market effect is larger the larger is the change in product differentiation, which increases with the parameter k. Thus, whenever k is large enough, the market effect exceeds the regulatory effect, and the increase (decrease) of the dirty firm tax rate results in lower (higher) pollution levels. Hence, this policy would be adequate for those situations in which the abatement costs are high.

Next, we investigate the effects of slight changes of the clean firm's tax rate. For instance, an increase of the clean firm's tax rate can be justified since it might be, as shown in Proposition (2.1), the more polluting firm in the market. Analogously to the previous case, both firms' optimal emissions levels change as follows:

$$\frac{de_1}{d\tau_2} = \frac{\partial e_1}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_2} = \frac{0.0122465}{k\tau^2} > 0 \qquad (32)$$

$$\frac{de_2}{d\tau_2} = \frac{\partial e_2}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_2} + \frac{\partial e_2}{\partial \tau_2} = \frac{0.248123}{k\tau^2} > 0 \qquad (33)$$

On the other hand, quantities vary according to:

$$\frac{\partial q_1^*}{\partial \tau_2} = \frac{\partial q_1}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_2} = \frac{0.00952165}{\tau} > 0 \qquad (34)$$

$$\frac{\partial q_2^*}{\partial \tau_2} = \frac{\partial q_2}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_2} = \frac{0.0190433}{\tau} > 0 \qquad (35)$$

From these two latter facts, the following proposition directly follows:

Proposition 6 *Consider the environmentally differentiated duopoly described above in which both firms face symmetric ad-valorem taxation policy. By slightly increasing (decreasing) the nonuniform ad-valorem tax rate to the clean firm:*

a) total pollution from the dirty firm unambiguously increases (decreases)
b) total pollution from the clean firm unambiguously increases (decreases)
c) total pollution in the market unambiguously increases (decreases).

The intuition behind this proposition is similar to the previous case. By slightly raising (lowering) the clean firm tax rate, the dirty firm is given a technological advantage (disadvantage). In this case, the clean firm increases (decreases) its unitary emissions level while the dirty firm's best response is to increase (decrease) its emissions as well, but less (more) than the clean firm one. This results in first, a lower (higher) level of product differentiation, which increases (decreases) price competition, and second, an increase (decrease) of both firm's sales. From both reasons, the level of total emissions of each firm unambiguously increases (decreases). Therefore, if the regulator aims to reduce total pollution in the market, he should not increase the tax rate of the clean firm.

Finally, note that even though it might seem very intuitive that policies consisting of an increase in the dirty firm's tax rate and a decrease in the clean firm's tax rate would yield the same results, it is not the case, as we have seen above. The reason is that there are two kinds of effects to be considered: strategic and direct effects. While an increase in the dirty firm tax rate gives a direct technological disadvantage to this firm, a decrease in the clean firm's tax rate only gives a strategic technological disadvantage to it. These effects drive our results.

6 Conclusions

In this paper we have examined the impact of alternative environmental policy instruments on a vertically differentiated market. We show that partial internalization of environmental damage by the consumers causes firms to supply two different varieties of the product, a cleaner and a dirtier one. In the unregulated equilibrium the *cleaner* firm sells at a higher price and covers a higher portion of the market. The negative externality associated to the commodity is a function of a pollution index per unit of output and market sales of each variety. Interestingly, even though the *cleaner* firm's emissions per unit of output are lower than the *dirtier* one, total pollution derived by the former will be probably higher. This is so since, the lower the level of abatement attained by the firms (the higher abatement costs) the more important becomes market shares in total pollution levels. Hence, what might apparently be the cleaner variety can easily generate higher environmental damages. This result suggests that market coverage and consumers' reallocation between firms play a crucial role in pollution control measures when there is vertical product differentiation.

Indeed, we demonstrate that an environmental standard may increase the level of pollution in the market, instead of decreasing it. The standard induces both firms to improve their abatement technology, reducing product differentiation and fostering price competition. Even though emissions per unit of output are reduced, lower prices increase market shares of both firms. As more consumers are active in the market and both firms end up increasing sales, the standard may have a dramatic impact in total pollution levels. The higher the cost of the abatement technology, the stronger is the net impact of the market effect (higher pollution levels associated to higher consumption levels as products become greener) over the regulatory effect (lower pollution levels induced by regulation). Therefore, instruments such as direct subsidies

– that reduce the cost of the abatement technology – are unambiguously more effective in lowering aggregate pollution levels.

It seems that as firms develop greener varieties, induced by current eco-labeling and other types of regulation, consumption levels of still polluting goods can easily increase. Even though recycled paper, biodegradable products, batteries with lower cadmium content or unleaded fuels, to name just a few examples, may appear as less polluting products to consumers, reallocation of consumers in the market may result in higher pollution levels.[18] Hence, failure to acknowledge the role of consumers preferences for green varieties can lead to higher environmental damages.

Finally, it is shown that *uniform* commodity taxation increases, unambiguously, the level of pollution in a vertically differentiated market. This is so since, any increase in firms' costs induces lower abatement efforts of both firms and it does not affect vertical product differentiation. As a result, consumption levels remain unchanged while per unit emissions increases. However, if the ad-valorem tax rate is set in favor of the environmentally-friendly product it will induce lower pollution levels in the market and, therefore, this policy would potentially be welfare improving. Decreasing the tax rate to the cleaner firm, eventually, seems to be more effective in terms of pollution abatement than increasing the tax rate to the dirtier one.

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[18] Moreover, green markets might shift environmental impacts, reducing some type of pollution while incresing other types.

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7 Appendix

7.1 Unregulated equilibrium

Next, we characterize the equilibrium values in terms of the product differentiation variable. The unregulated equilibrium is given by the unique solution to the following two equations system:

$$e_1 = 1 - \lambda^2 \frac{4\lambda - 7}{k\left(4\lambda - 1\right)^3} \tag{36}$$

$$e_2 = 1 - 4\lambda \frac{4\lambda^2 - 3\lambda + 2}{k\left(4\lambda - 1\right)^3} \tag{37}$$

Equilibrium prices are given by:

$$p_1 = \frac{2\lambda\left(\lambda - 1\right)\left(1 - e_1\right)}{\left(4\lambda - 1\right)} \text{ and } p_2 = \frac{\left(\lambda - 1\right)\left(1 - e_1\right)}{\left(4\lambda - 1\right)} \tag{38}$$

Equilibrium market shares are given by:

$$q_1 = \frac{\lambda}{4\lambda - 1} \text{ and } q_2 = \frac{2\lambda}{4\lambda - 1} \tag{39}$$

Now we prove that firms do not increase profits by leapfrogging the rival's choice. Given the optimal choice of firm 2, the firm 1 maximizes:

$$\Pi(e_1, e_2^*) = \frac{\frac{0.253311}{k}(1 - e_1)(e_1 - 1 + \frac{0.253311}{k})}{(3 + e_1 - 4(1 - \frac{0.253311}{k}))^2} - \frac{k(1 - e_1)^2}{2} \tag{40}$$

subject to $e_1 < e_2^*$. From the first order condition it is obtained that $\widehat{e}_1 = 1 - 1.757448/k$. By substituting $\widehat{e}_1$ into the profits function it yields profits $\widehat{\Pi} = -2.75335/k$ which are clearly negative. Analogously, the clean firm maximizes $\Pi(e_1^*, e_2)$ subject to $e_2 > e_1^*$. It is easily obtained that $\widehat{e}_2 = 1$, which yields zero profits.

7.2 Maximum emission standards

PROOF OF PROPOSITION 4.1.:

a) Dirty firm's total emissions are $E_1 = q_1 e_1$. Taking derivatives with respect to the maximum emission standard we obtain that the change in the dirty firm's pollution level can be divided into two effects: the "market effect" (ME) and the "regulation effect" (RE). In fact, $\frac{\partial S_1}{\partial e_{\max}} = ME + RE = \frac{\partial S_1}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}} = \frac{\partial q_1}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}} e_1 + q_1 \frac{\partial e_1}{\partial e_{\max}}\frac{\partial \lambda}{\partial e_{\max}}$, where ME denotes the market effect and RE denotes the regulation effect. Note that ME measures the increase of the pollution level due to the demand shift caused by setting the maximum standard policy and RE measures the decrease of the pollution level caused by the maximum standard policy. We are interested in determining the sign of these two effects, that is: $Sg(\frac{\partial S_1}{\partial e_{\max}}) = Sg(\frac{\partial q_1}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}} e_1 + q_1 \frac{\partial e_1}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}}) = Sg(EM + ER)$. It is easily seen that $Sg(EM) = Sg(\frac{\partial q_1}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}} e_1) < 0$ as $\frac{\partial q_1}{\partial \lambda} < 0$ and $\frac{\partial \lambda}{\partial e_{\max}} > 0$. On the other hand, $Sg(ER) = Sg(q_1) > 0$. Therefore, the sign of $\frac{\partial S_1}{\partial e_{\max}}$ is indeterminate. If $EM > ER$, it will have positive sign while if $EM < ER$ it will have negative sign. As we show next, this depends on the level of abatement costs.

Substituting appropiately, it can be easily seen that $\frac{\partial S_1}{\partial e_{\max}} = -\frac{(4\lambda-1)^2(k-4.8238\times 10^{-2})}{64\lambda^2-64\lambda+84} + \frac{\lambda}{4\lambda-1}$ which evaluated at the unregulated equilibrium gives $\frac{\partial S_1}{\partial e_{\max}} = -.26455k + .27526$. Therefore, dirty firm's emission level increases after the policy whenever $k > 1.040483$.

b) On the other hand, the clean firm's total emissions are $E_2 = q_2 e_2$. The ME is given by $ME = \frac{\partial q_2}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}} e_2$ which is unambiguously negative. The RE is given by $RE = q_2 \frac{\partial e_2}{\partial \lambda}\frac{\partial \lambda}{\partial e_{\max}}$ which is unambiguously positive. As in the dirty firm case, the total effect will depend on which of these two effects is stronger. It can be easily seen that $\frac{\partial S_2}{\partial e_{\max}} = -\frac{1}{2}\frac{(4\lambda-1)^2(k-.25331)}{16\lambda^2-16\lambda+21} + \frac{2\lambda(10\lambda+1)(4\lambda-1)}{16\lambda^2-16\lambda+21}$ which evaluated at the unregulated equilibrium gives $\frac{\partial S_2}{\partial e_{\max}} = -.5291k + 29.863$. This expression is greater than zero as long as $k > 56.441126$. Thus, total pollution from consumption of the clean product decrease after setting the maximum emission standard whenever the abatement costs satisfy $k < 56.441126$. Otherwise, total pollution derived from consumption of the cleaner product increases.

c)To find out the range of k by which total pollution derived from consumption of both products increase we compute $\frac{\partial S_T}{\partial e_{\max}} = \frac{\partial (S_1+S_2)}{\partial e_{\max}} = -\frac{(4\lambda-1)^2(k-4.8238\times 10^{-2})}{64\lambda^2-64\lambda+84} + \frac{\lambda}{4\lambda-1} - \frac{1}{2}\frac{(4\lambda-1)^2(k-.25331)}{16\lambda^2-16\lambda+21} + \frac{2\lambda(10\lambda+1)(4\lambda-1)}{16\lambda^2-16\lambda+21}$

Again, evaluated at the unregulated equilibrium gives $\frac{\partial S_T}{\partial e_{\max}} = -.79365k + 30.138$. This expression is greater than zero as long as $k > 37.973917$.

Q.E.D.

7.3 Nonuniform ad-valorem tax

PROOF OF PROPOSITION 5.3.:

a) Firm 1 total emissions are $E_1 = e_1 q_1$. Proper differentiation gives $\frac{\partial E_1}{\partial \tau_1} = q_1 \frac{\delta e_1}{\delta \tau_1} + e_1 \frac{\delta q_1}{\delta \tau_1}$. Substituting their corresponding values (in a neighborhood of the unregulated equilibrium) and rearranging terms it is obtained $\frac{\partial E_1}{\partial \tau_1} = \frac{9.90705-9.52165k\tau}{1000k\tau^2}$ which is a positive expression as long as $k\tau < 1.040476$.

b) Firm 2 total emissions are $E_2 = e_2 q_2$. Proper differentiation gives $\frac{\partial E_2}{\partial \tau_1} = q_2 \frac{\delta e_2}{\delta \tau_1} + e_2 \frac{\delta q_2}{\delta \tau_1}$. Substituting their corresponding values (in a neighborhood of the unregulated equilibrium) and rearranging terms it is obtained $\frac{\partial E_2}{\partial \tau_1} = \frac{7.5475-19.0433k\tau}{1000k\tau^2}$ which is positive whenever $k\tau < 0.39634$.

c) To find out the range of $k\tau_1$ for which total pollution in the market increase we analogously compute $\frac{\partial S_T}{\partial \tau_1} = \frac{\partial S_1}{\partial \tau_1} + \frac{\partial S_2}{\partial \tau_1} = \frac{17.4545-28.5649k\tau}{1000k\tau^2} > 0$ for all $k\tau < 0.611047$.

Q.E.D.

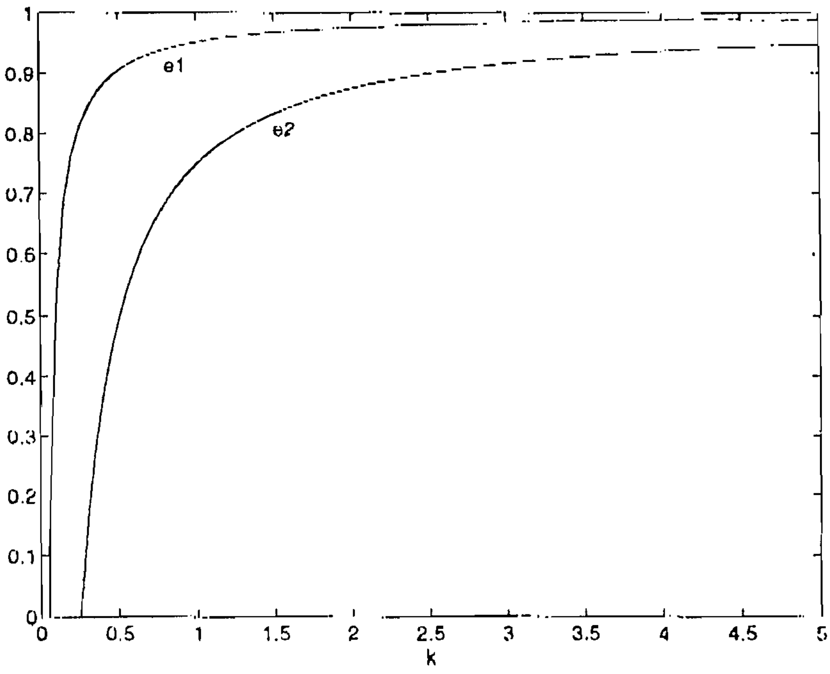

Figure 0.1:

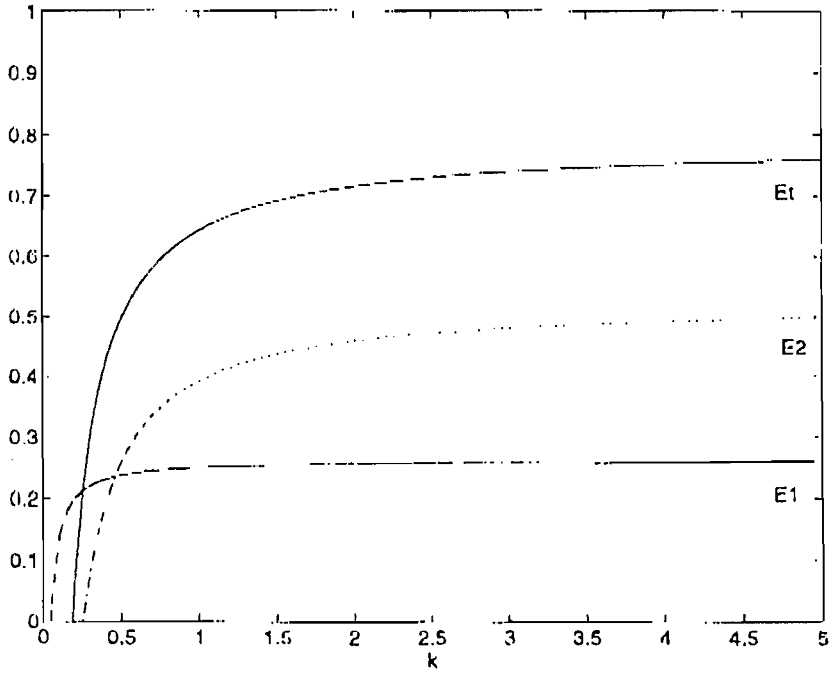

Figure 0.2: