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Abstract

Recent years have seen the emergence of an extensive literature exploring wage compression and expansion in various countries. In this paper we imbed an N-level human capital hierarchy in a growth model and demonstrate that hierarchical structure is an engine of "endogenous" wage compression and expansion. We also identify "aggregation boundaries" beyond which measures of wage compression and expansion are distorted. Fundamental to these results is the sequential process by which basic human capital is transformed into more advanced varieties. For example, production of a unit of PhD human capital typically requires the prior sequential production of primary, secondary, and tertiary level human capital. These pre-PhD vintages are themselves potentially productive "final" inputs. As this productive human capital migrates through the hierarchy it creates a sequence of stock depletion effects that, in turn, generates a pattern wage compression and expansion.

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1. Introduction

There exists an extensive empirical literature exploring historical patterns of wage compression and expansion in the U.S., and abroad. Katz and Murphy (1992) and Goldin and Margo (1992) (discussed in detail subsequently) are important empirical works in this area. For the most part, this literature focuses on the components of wage compression/expansion patterns explained by exogenous historical episodes (e.g., World War II) or institutional changes in labor market structure. This paper demonstrates that patterns of wage compression and expansion may also have a more fundamental endogenous motivation – the hierarchical structure of human capital. We imbed an N-level human capital hierarchy in a continuous time growth model and show that the optimal program displays a pattern of wage compression and expansion during transition from the initial condition to the endogenous growth steady state.

To convey the intuition behind this endogenous wage compression/expansion engine it is useful to describe the structure of a human capital hierarchy, and its implication for relative wage evolution. A human capital hierarchy is an ordered sequence of qualitatively distinct human capital types that are "built" in sequential, cumulative fashion. For example, one could associate hierarchy levels with the traditional levels of education: primary, secondary, and tertiary. A unit of primary human capital is created by a transformation (at some cost) of primordial human material. To obtain a unit of secondary level human capital the primary unit (which is itself potentially productive) is subject to a further transformation. More generally, in an N-level hierarchy the creation of level i human capital requires level $i-1$ human capital as an input. Of course, it is possible for an individual with a tertiary education to work productively in a capacity that utilizes only their secondary (or primary) level human capital. However, assuming that at a moment in time a person can only be employed in one capacity (e.g., cannot work *simultaneously* as a street sweeper and an engineer) they enter the production function in one capacity or the other. Moreover, since the transformation of human capital from level i to level $i+1$ is costly, the optimal program will never entail utilizing human capital in less than its highest capacity.[1] As we will focus on initial conditions where advanced human capital is relatively scarce, we need not be concerned that the optimal program will require deconstruction of a physicist to a laborer. We will elaborate further on these issues when the formal model is introduced.

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[1]Our model is constructed from a planner's perspective. Since the model contains no externalities, the decentralized

Two fundamental properties of the human capital that occupies our hierarchy are qualitative distinctiveness and intermediate productivity. Qualitative distinctiveness implies that "advanced" human capital cannot be acquired by simply collecting enough "basic" human capital. In a formal sense, this simply means that the different levels of human capital enter the production function as distinct inputs. As an illustration of this qualitative distinctiveness, consider the case where a unit of basic human capital corresponds to a primary school graduate and advanced human capital to a Ph.D. physicist. It is clear that ten (or for that matter a hundred) primary school graduates do not constitute a physicist. However, a physicist *can be created* from a primary school graduate given the requisite additional investment. The primary school graduate is the raw material, which when combined with the advanced investment technology (i.e., a University education), yields a unit of advanced human capital.

The second fundamental property of human capital in our model is intermediate productivity. As the human putty traverses the hierarchy it may cease transformation at any stage and enter the production function. A “half-built” PhD (a secondary school graduate) is productive in a way that a half-built airplane is not.[2] An implication of this structure is a stock dependence between adjacent levels in the hierarchy. Specifically, investment in a particular hierarchy level generates a secondary supply-side effect on the wages of the next lower hierarchy level. As the optimal program entails distinct regimes of focused human capital investment (on different levels of human capital) this supply-side effect migrates through the hierarchy generating a pattern of wage compression and expansion. Interestingly, this supply-side origin of our wage compression/expansion pattern is consistent with the broad finding of much of the empirical work – although again, ours is an endogenous engine of wage compression and expansion.

We now briefly review the wage compression/expansion literature alluded to earlier. Two issues warrant attention in placing this literature (and our model) in their broader context. First, it is important to note that this literature, though conceptually akin, is quite distinct from the extensive inequality and growth literature. One fundamental distinction is that the inequality/growth literature employs highly “aggregated” distribution measures, such as the Gini

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solution will be identical to our centralized solution.

[2] Of course, intermediate physical products are occasionally productive – but these are exceptions rather than rules. An example of a productive intermediate product is logs, which could be used as telephone poles, or transformed into 2x4s. Intermediate human product (e.g., a secondary school graduate), however, is generally productive.

coefficient. The wage compression/expansion (WC/E) literature focuses on the underling relative wages and pays careful attention to the aggregation issues that may be obfuscated by traditional inequality analysis. A second issue is that WC/E is unambiguously a transition, rather than a steady-state, phenomenon. Though the empirical WC/E literature rarely couches their findings in these terms, this is clearly the intended interpretation. One could imagine that a shock (such as the introduction of a new technology – e.g., computers) perturbs the system, and that our analysis tracks the transition towards the new steady state. We believe that such out-of-steady-state analyses provide important insights into the development process since few would argue (and fewer would hope) that contemporary LDC's are in their steady state. Naturally, the initial conditions relative to the steady state dictate the transition path. The initial conditions we analyze are consistent with those found in many developing countries, and in the western industrialized countries at earlier stages of development – relative scarcity of high level human capital.

We begin our literature survey by returning to the seminal works of Katz and Murphy (1992) and Goldin and Margo (1992) mentioned earlier. Goldin and Margo explore the roots of the "great compression" in relative wages that occurred in the 1940s. They trace the dramatic compression of unskilled to educated wages observed during this decade to increased demand for unskilled labor associated with the Second World War, as well as increased supply of educated labor. Katz and Murphy focus on WC/E during the interval from 1963-87. They argue that the significant wage expansion in the 60s and 80s (which surround a decade of wage compression in the 70s) can be largely explained using a simple supply and demand framework. In particular, they find that fluctuations in the growth rate of labor supply in specific wage categories drive much of the observed pattern of wage expansion and compression. As noted, such supply side phenomena are integral to the patterns of wage expansion and contraction generated by the optimal transition program in our model.

Patterns of wage expansion and compression have also been explored in other countries. Blau and Kahn (1996) investigate wage compression patterns in ten OECD countries, contrasting the U.S. pattern with the non-U.S. sample. They argue that much of differences in wage compression patterns were attributable to greater compression at the bottom of the distribution in the non-U.S. countries. Kahn (1998) examines a significant wage compression in Norway in the late 1980s, while Hibbs and Locking (1996) describe wage compression and drift in Sweden.

As noted, one aspect that separates the WC/E literature from the growth/inequality literature is attention to issues of aggregation. Consequently, one encounters various desegregations in the WC/E literature. Margo and Finegan (1997) disaggregate the public and private components of the "Great Compression" and find that about 40 percent of the phenomena is explicable by public sector factors. Another disaggregation explored in the WC/E literature is by race. Margo (1995) argues that systematic wage compression of the 40s was a major factor in cross-racial wage convergence. Maloney (1994) tracks black-white relative wages from the 40s through 60s and provides evidence that the significant wage compression of the 40s ceased in the 50s.

Important exceptions to the empirical orientation of the relative wage literature surveyed above are a series of papers by Acemoglu and Shimer (1996), Acemoglu (1998), Acemoglu and Pischke (1999), and Acemoglu (1999). Acemoglu (1998) develops a theoretical model to shed light on U.S. wage compression and expansion in the 70s and 80s (i.e., the decline and increase in the college premium). In this paper an increase in the supply of skilled workers has an initial depressing effect on the skill premium but induces skill-biased technological change that subsequently reverses the compression. Similarly, the other papers by Acemoglu (et al.) address (broadly) the interaction of technology and efficiency issues in relative wage determination. A cursory examination of these models reveals little similarity in structure with the model of this paper. What they (as well as the empirical literature) do share is the objective of illuminating mechanisms of relative wage determination and evolution. In contrast to the efficiency and skill-matching issues addressed by Acemoglu (et al) the objective of this paper is to identify a heretofore-overlooked engine of wage compression and expansion: the hierarchical nature of human capital. We also show that hierarchical structure implies moving "aggregation boundaries" that should be considered in interpreting time paths of WC/E.

The reminder of the paper is organized as follows. Section 2 develops the general N-level hierarchical human capital growth model and characterizes the steady-state. Section 3 analyzes patterns of wage compression and expansion along the transition path. Section 4 provides interpretation, and concludes.

2. Investment in Hierarchical Human Capital: The General Model

Consider a continuous time setting with an N-level human capital hierarchy.[3] Denote the stock of human capital of hierarchy level i as H_i, where $i = 1, \ldots, N$. Interpret H_1 as the most basic and H_N as pinnacle human capital. Each of these stocks can be interpreted as the population with the corresponding level of education as the highest attained. Let $H(t)$ be the $Nx1$ vector of human capital stocks at time t. We suppress the t argument when the clarity constraint permits. These human capital stocks are used to produce flow output through time, Y, as described by the production function:

$$(1) \qquad Y = f(H), \qquad f_i > 0, \quad f_{ii} \le 0, f_{ij} \ge 0; \quad i,j = 1, \ldots, N.$$

Let x_i denote investment in human capital level i with $x_i \ge 0$, for all i. These investments can be interpreted as education expenditures. The following equations of motion reveal the structure of the hierarchy:

$$(2a) \qquad \dot{H}_i = x_i - x_{i+1} \qquad for \;\; i = 1, \ldots, N\text{-}1$$

$$(2b) \qquad \dot{H}_N = x_N.$$

[3] In adopting a continuous time formulation we abstract from “time to build” or training time.

Note the depletion effect of the hierarchical system. As reflected in the equations of motion, x_{i+1} depletes H_i for $i= 2, . . ., N$. So that focus can be directed to the effect the hierarchical structure, the relative price (opportunity cost) of investment in H_i is normalized to 1 for all i.[4]

Output can be consumed or invested. Consumption, denoted by c yields utility flow $U(c)$ where $U'(c) > 0$ and $U''(c) < 0$. Again, to lay bare the implications of hierarchical structure there is no borrowing, lending, or depreciation. Hence,

$$(3) \qquad c = f - \sum_{i=1}^{N} x_i \geq 0.$$

What distinguishes this problem from the standard investment problem is the relationship between human capital stocks. As noted, any increase in the stock of H_i for $i > 1$ involves an equal decrease in the stock of H_{i-1}. For example, suppose H_i denotes the population with baccalaureate degree as the highest education level attained and H_{i-1} those with high school. Then an increase in H_i is matched by an equal decrease in H_{i-1}, all else equal. Of course, college graduates retain their high school degrees, but at moment in time they work in one capacity or another – as noted, an optimal program will not waste resources transforming human capital unless it will be employed at its highest level. Increments to H_1 can be thought of as coming from an underlying stock of an unproductive resource, e.g., untrained children, sufficiently large that it imposes no binding constraint.

Let r denote the discount rate. The planners' goal is to maximize the present discounted value of utility:

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[4] This assumption can easily be relaxed. However, the uniform price of investment lays bare the implications of the hierarchy without the obfuscating effect of differential investment costs.

(4) $$\max_{x_i} \int_0^\infty U(c)e^{-rt}dt \qquad i = 1, \ldots, N;$$

Subject to: *(1) – (3), initial conditions $H_i(0) > 0$, and $x_i \geq 0$.*

The present-value Hamiltonian for this problem is:

(5) $$\max_{x_i} \boldsymbol{H} = U(c) + \sum_{i=1}^{N-1} \lambda_i [x_i - x_{i+1}] + \lambda_N x_N ,$$

where the $\lambda_i's$ are the costate variables. Noting that $dc/dx_i = -1$ for all i, the necessary conditions for an interior solution are:

(6) $$-U' + \lambda_1 = 0$$

(7) $$-U' - \lambda_i + \lambda_{i+1} = 0, \qquad i = 1, \ldots, N-1$$

(8) $$\dot{\lambda}_i = r\lambda_i - U' f_i \qquad i = 1, \ldots, N$$

Manipulation of these equations yields the following pattern:

(9) $$\lambda_i = i\, U'.$$

Differentiating with respect to time yields:

(10) $$\dot{\lambda}_i = U''\dot{c} \qquad i = 1, \ldots, N.$$

Using *(8) – (10)* provides the following relationship:

$$\textit{(11)} \qquad \frac{U''}{U'}\dot{c} = \left(\frac{1}{i}\right)\left[ir - f_i\right] \qquad i = 1,\ldots,N,$$

which in turn yields:

$$\textit{(12)} \qquad \frac{f_i}{i} = \frac{f_j}{j} \qquad \forall\, i, j = 1,\ldots, N.$$

Equation *(12)* is the steady-state marginal productivity relationship in the hierarchical environment. Instead of equating the value of inputs' marginal products, as in non-hierarchical settings, the optimal program requires equating the *ratios* of marginal products and hierarchy positions. The hierarchy positions in the denominators of *(12)* reflect the cost of the multiple transformations required to traverse the human capital hierarchy. Thus to satisfy *(12)*, *level N* human capital must have a marginal product N times greater than *level 1* human capital in the steady state. To obtain further insight into *(12)* consider any adjacent ratio pair: $f_i / i \;=\; f_{i+1}/(i+1)$ for $i = 1, 2, \ldots, N\text{-}1$. Cross multiplying and subtracting 1 from both sides yields: $\frac{f_{i+1} - f_i}{f_i} = \frac{1}{i}$. So the percentage change in marginal product from traversing each level of the hierarchy is one over the hierarchy level in the steady state. An additional manipulation establishes the following relationship in the steady state: $\frac{f_i}{i} = f_{i+1} - f_i$. Substituting this expression into *(12)* yields:

$$\textit{(13)} \qquad f_2 - f_1 = f_3 - f_2 = \ldots\ldots = f_N - f_{N-1}\,.$$

Equation *(13)* provides a further illustration of the depletion effect in human capital hierarchy. When a unit of human capital is transformed to from level i to level $i+1$ the net change in output is $f_{i+1} - f_i$. The stocks of human capital that satisfy *(12)* or *(13)* depend, of course, on the properties of the production function. We denote a vector of steady state human capital stocks that satisfy *(12)* (and hence *13*) by H^*, with H_i^* the $i^{\underline{th}}$ element of H^*.

The steady state condition *(12)* has a visual representation that will be useful for understanding the evolution of income distribution on the transition path. In particular, one can represent alternative steady state human capital distributions with "pyramids" of various shapes. For example, a traditional pyramid with a wide base and narrow pinnacle represents a large steady-state stock of basic human capital (H_1) and relatively smaller stocks at progressively higher levels in the human capital hierarchy: $H_1^* > H_2^* > \ \ldots > H_N^*$. This would be the shape of the steady state human capital pyramid if *f(H)* were Cobb-Douglas with equal coefficients across hierarchy levels. Alternatively, if productivity increases sufficiently as one ascends the hierarchy, the steady state pyramid may be inverted: $H_1^* < H_2^* < \ \ldots < H_N^*$. Other production functions could yield a steady state hierarchy with large stocks of mid-level human capital (H_i) and smaller stocks of "extreme" human capital types: $H_1^* < \ldots \ < H_{i-1}^* < \ H_i^* > \ H_{i+1}^* > \ \ldots H_N^*$.

Just as the steady-state human capital hierarchy can be represented by a "pyramid," so too can the initial conditions. The initial condition is simply a distribution of human capital (which can be interpreted as population) across the hierarchy. Let Let $\underline{H}(0)$ be the vector of initial stocks, and $H_i(0)$ the initial stock of level *i* human capital. We assume that at $\underline{H}(0)$, human capital stocks are not at their steady state relationships and *(12)* becomes a set of inequalities. The evolution of income distribution along the transition path is dictated by the relationship between initial and steady state pyramids. Figure 1 below illustrates an initial condition with increasing relative scarcity at higher hierarchy levels and several alternative steady state configurations. We will interpret these shapes subsequently in the context alternative development paradigms.

[Figure 1 (attached)]

The remainder of this paper focuses on transition path of relative wage evolution given an initial condition represented in Figure 1. Given our model structure we will demonstrate that equation *(12)* together with an initial condition provide sufficient information to characterize the evolution of relative wages across the human capital hierarchy during transition. The properties of an endogenous growth steady state can also be characterized if we specify explicit functional forms for utility and production. As our interest is with transition income distribution, we relegate this exercise to the Appendix.

Transition Dynamics and Income Inequality

We now turn to the transition dynamics and begin by characterizing classes of initial conditions where *(12)* is not satisfied. For an N level hierarchy there are $N!$ strict inequality orderings of *(12)*:

$$(14) \qquad \begin{array}{ll} (i). & \frac{f_1(\underline{H}(0))}{1} < \frac{f_2(\underline{H}(0))}{2} < \ldots\ldots < \frac{f_{N-1}(\underline{H}(0))}{N-1} < \frac{f_N(\underline{H}(0))}{N} \\ (ii). & \frac{f_2(\underline{H}(0))}{2} < \frac{f_1(\underline{H}(0))}{1} < \frac{f_3(\underline{H}(0))}{3} < \ldots\ldots < \frac{f_N(\underline{H}(0))}{N} \\ & \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ (N!). & \frac{f_N(\underline{H}(0))}{N} < \frac{f_{N-1}(\underline{H}(0))}{N-1} < \ldots\ldots < \frac{f_2(\underline{H}(0))}{2} < \frac{f_1(\underline{H}(0))}{1}. \end{array}$$

Each of these $N!$ orderings can be interpreted as a class of initial conditions. Associated with these inequality orderings are inequality orderings of net gains of human capital transformation in equation *(13)*. We focus on the following:

$$(15) \qquad f \;\; - f_1 < f \;\; - f \;\; < \ldots\ldots < f \quad - f \;\; _{-1} .$$

Initial condition *15* reflects increasing relative scarcity of high-level human capital vis-à-vis low level. We believe this configuration is most consistent with initial conditions in a typical LDC. This initial condition is associated with (the traditional) wide base and narrow pinnacle pyramid illustrated in Figure 1.

For any initial condition that generates inequality ordering (such as *(15)*), the optimal transition investment program follows a similar pattern. Specifically, the optimal transition program in this environment is "bang-bang" with N-1 phases of transition investment.[5] A simple

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[5] The program is bang-bang because of the lack of adjust costs (see Kamien and Schwartz 1981). There are some technical subtleties that may arise on the transition path that are tangential to our focus on transition path income inequality, and we hence ignore. For example, for a sufficiently productive technology investment in multiple hierarchy levels may occur simultaneously. In addition, the relationship between *(14)* and *(15)* is more subtle than that

arbitrage argument provides the intuition for the bang-bang property. That is, if over some interval a feasible program does not invest exclusively in the highest net return human capital, there exists an alternative feasible program with investment reallocated to the high return human capital that arrives at an identical future state with higher utility. Therefore if (*15)* holds, the optimal program begins with investment in level *N* human capital exclusively. During this initial phase of transition (which we denote *Phase 1*), f_N falls and f_{N-1} rises (due to the depletion effect) until the level *N* net return ($f_n - f_{N-1}$) equals the level *N-1* net return ($f_{N-1} - f_{N-2}$). At this point, *Phase 2* of transition begins with investment in *N* and *N-1* to maintain ($f_N - f_{N-1}$) = ($f_{N-1} - f_{N-2}$) or equivalently $f_N / N = f_{N-1}/(N-1)$. Note that during this phase the next net return in the inequality chain, ($f_{N-2} - f_{N-3}$), rises due to the depletion effect on *N-2*. More generally, given initial condition (*15)* the following qualitative expression describes controls in Phase *k* of transition:

$$(16) \qquad x_i > 0 \;\; for \;\; i \geq N+1-k; \qquad x_i = 0 \;\; for \;\; i < N+1-k, \qquad (k \leq N-1)$$

Returning to Phase 1 this control pattern implies the following stock evolution:

$$(17) \qquad \dot{H}_N = -\dot{H}_{N-1}; \qquad \dot{H}_i = 0 \quad for \;\; i = 1, 2, \ldots, N-2.$$

Note that the relationship between the stocks of H_N and H_{N-1} in *(17)* is a pure hierarchical effect -- there is no traditional depreciation in this model so that focus can be directed to the implications of hierarchical structure. Moving again from the specific to the general, stock evolution during Phase k of transition can be described as follows:

$$(18) \qquad \dot{H}_i = 0 \quad for \;\; i < N-1-k; \;\; \dot{H}_{i-1} = -\dot{H}_i < 0 \quad for \;\; i = N-k;$$

$$\dot{H}_i > 0 \;\mid\; \frac{f_i}{i} = \frac{f_{i+1}}{i+1} \qquad for \;\; i > N-k.$$

We can therefore partition the set of human stocks at each moment into three subsets: those with positive, zero, and negative growth rates. We denote the subsets with positive and zero growth

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between *(12)* and *(13)* since *(15)* was derived using adjacent pairs, and the ordering of inequalities in (15) need not be index number adjacent.

rates respectively as $\dot{H}^{+}$, and $\dot{H}^{o}$. Recall that at each moment on the transition path only the hierarchy level adjacent to (below) the lowest indexed element of $\dot{H}^{+}$ has a negative growth rate. This singularity arises because, with the "bang-bang" solution, only one hierarchy level at a time is subject to an *uncompensated depletion effect*. Hierarchy level i is experiencing uncompensated depletion when $\dot{H}_i = -\dot{H}_{i+1} < 0$ and $x_i = 0$. That is, when the stock reduction that accompanies investment in the next highest hierarchy level is not offset by any stock augmenting investment. Therefore if we assign the index number d to the hierarchy level experiencing uncompensated depletion then $\dot{H}_d = -\dot{H}_{d+1}$, where $\dot{H}_{d+1} \in \dot{H}^{+}$, and at each moment there is a single hierarchy level (d) experiencing uncompensated depletion. To avoid confusion we risk redundancy and reemphasize that d is not the index of a unique hierarchy *level*. A particular vintage of human capital has a position in the hierarchy ($1, 2, \ldots, N$) that does not change. Rather, at each moment in time on the transition path there is a single hierarchy level subject to uncompensated depletion and we tag this level with the subscript d. The label d, in essence, migrates through the hierarchy during the course of transition.

Qualitative Characterization of Changes in Relative Wages during Transition

We seek to characterize the evolution of *relative wages* in a human capital hierarchy along the entire transition path. The first step in this process is to derive expressions for the change in relative wages between any two hierarchy levels. To this end suppose each unit of human capital is paid the marginal product of labor in its hierarchy level. Let i and j be two hierarchy levels with $j > i$. During transition, the qualitative change in relative wages (marginal products f_j/f_i) of any two hierarchy levels is either positive, negative, or zero. To identify these classes we first derive a general expression for the change in the ratio of marginal products with respect to time:

$$\dot{\left(\frac{f_j}{f_i}\right)} = \frac{f_i\left(\sum_{n=1}^{N} f_{j,n}\dot{H}_n\right) - f_j\left(\sum_{n=1}^{N} f_{i,n}\dot{H}_n\right)}{(f_i)^2} . \tag{19}$$

In evaluating *(19)* it is useful to distinguish "direct" from "indirect" effects on marginal product.

By a direct effect we mean the reduction in marginal product in a hierarchy level associated with increased own-level stock. This direct effect is captured in the second derivatives f_{ii}, f_{jj}. Indirect marginal product effects are embodied in the cross-partials and reflect the increase in marginal product in one hierarchy level that *may* accompany increases in the stock of another hierarchy level all else equal (recall the weak inequality in *(1)*, $f_{ij} \geq 0$).

Rewriting *(19)* using the fact that along the transition path $\dot{H}_i = 0$ for $i = 1, 2, \ldots, d-1$, and separating the direct from indirect effects we obtain:

$$(20) \qquad \dot{\left(\frac{f_j}{f_i}\right)} = \frac{\dot{H}_j\left(f_i f_{jj} - f_j f_{i,j}\right) + \dot{H}_i\left(f_i f_{j,i} - f_j f_{ii}\right) + \sum_{n=d,i^-,j^-}^{N} \dot{H}_n\left(f_i f_{j,n} - f_j f_{i,n}\right)}{\left(f_i\right)^2},$$

where we use the notation $n = d, i^-, j^-$ to indicate the summation is from d to N excluding i and j (if i and/or j falls between d and N). Note that while the indirect effects ($f_i f_{j,n}$ - $f_j f_{i,n}$.) are of ambiguous sign (recall $n \neq i, j$) the direct effects have unambiguous sign. Specifically, ($f_i f_{jj}$ - $f_j f_{i,j}$) and ($f_i f_{j,i}$ - $f_j f_{ii}$) are respectively negative and positive. The ambiguity of the indirect effects might seem to present a significant problem in characterizing relative wage evolution (equation *(20)*). However, for many common functional forms the ambiguity dissipates. For example, with a Cobb-Douglas production function it is easy to show that the indirect effect ($f_i f_{j,n}$ - $f_j f_{i,n}$; $n \neq i, j$) is zero regardless of returns to scale. We will employ the Cobb-Douglas form as an example at various points in the subsequent analysis. We think it preferable, however, to maintain a high degree of generality and indicate the additional restrictions necessary to generate unequivocal qualitative results as we proceed.

Returning to our characterization of relative wage evolution we begin with the trivial case where $i, j \in \dot{H}^+$. This is the case where both hierarchy levels are in their steady state relationship. Since $f_j / f_i = \ j / i$, (if $i, j \in \dot{H}^+$) the change in relative wages *(19)* is zero for such pairs along the transition path (and in the steady state). Now consider the case where $j \in \dot{H}^+$ and $i = d$. In words, we are evaluating the qualitative evolution of relative wages between a hierarchy level whose stock is growing due to investment ($j \in \dot{H}^+$) and the hierarchy level whose stock is shrinking due to the depletion effect (d). In this case *(20)* becomes:

$$(21) \qquad \left(\frac{\dot{f_j}}{f_d}\right) = \frac{\dot{H}_j\left(f_d f_{jj} - f_j f_{d,j}\right) + \dot{H}_d\left(f_d f_{j,d} - f_j f_{dd}\right) + \sum_{n=d+1,j^-}^{N} \dot{H}_n\left(f_d f_{j,n} - f_j f_{d,n}\right)}{\left(f_d\right)^2}.$$

Intuition would suggest that *(21)* is negative since the stock of level j is rising and the stock of level experiencing depletion (*d)* is falling. Consistent with this intuition the direct effects in the numerator are both negative (recall $\dot{H}_d < 0$). As discussed above, however, at the highest level of generality our analysis identifies indirect effects as a potential countervailing influence to the direct effect.[6] At this level of generality signing *(21)* as negative requires the restriction that direct effects dominate indirect effects. Again, this restriction holds for the Cobb-Douglas: $f = \prod_{i=1}^{N} H_i^{\alpha_i}$, in which case *(21)* becomes:

$$(22) \qquad \left(\frac{\dot{f_j}}{f_d}\right) = \frac{\alpha_j}{\alpha_i}\left[\frac{\dot{H}_d H_j - \dot{H}_j H_i}{H_j^2}\right] < 0 .$$

A negative *(21)* suggests transition path wage compression between hierarchy levels in the accumulation set ($j \in \dot{H}^+$) and the hierarchy level experiencing uncompensated depletion. To assemble the complete picture of transition path relative wage evolution we next consider the case where $j \in \dot{H}^+$, $i \in \dot{H}^o$. Here we are tracking changes in wages in hierarchy level experiencing stock growth relative to a level with constant stock. The direct effect in this case is negative as shown in *(23)*

$$(23) \qquad \left(\frac{\dot{f_j}}{f_i}\right) = \frac{\dot{H}_j\left(f_i f_{jj} - f_j f_{i,j}\right) + \sum_{n=d,j^-}^{N} \dot{H}_n\left(f_i f_{j,n} - f_j f_{i,n}\right)}{\left(f_i\right)^2},$$

---

6 As an example of an indirect effect dominating a direct effect suppose the stock of programmers is rising and systems engineers is falling. The direct effect would have programmers' wage falling vis-à-vis engineers. However it is conceivable (though unlikely) that the reduction in engineers would reduce (through the cross-partial) the marginal product of programmers by more than the own-reduction effect increases marginal product. In this case programmers wage could rise vis-à-vis engineers even though their stock is falling. It is such perverse cases that we de-emphasize with our subsequent focus on direct effects.

and with Cobb-Douglas the relative wage evolution is simply:

(24) $$\left(\frac{\dot{f_j}}{f_i}\right) = \frac{\alpha_j}{\alpha_i}\left[\frac{-\dot{H}_j H_i}{H_j^2}\right] < 0 \,.$$

The final case to be considered is $j = d,\ i \in \dot{H}^o$. In this case the question is how the wage of the hierarchy level experiencing stock depletion changes relative to one not yet receiving positive investment. In this case *(19)* becomes:

(25) $$\left(\frac{\dot{f_d}}{f_i}\right) = \frac{\dot{H}_d\left(f_i f_{dd} - f_d f_{d,i}\right) + \sum_{n=d+1}^{N} \dot{H}_n\left(f_i f_{d,n} - f_d f_{i,n}\right)}{\left(f_i\right)^2} \,,$$

and for the Cobb-Douglas

(26) $$\left(\frac{\dot{f_d}}{f_i}\right) = \frac{\alpha_d}{\alpha_i}\left[\frac{-\dot{H}_d H_i}{H_i^2}\right] > 0 \,.$$

This is again consistent with the anticipation that the falling *d* stock will tend to increase f_d while the constant stock of *i* imparts no direct effect on *i's* marginal productivity.

To summarize the results of this section; for any two hierarchy levels $\{i,\ j\}$ we have associated the possible qualitative states of relative wage evolution (greater, less, or equal to zero) with the membership of either *i* and/or *j* in one of the three transition sets: $\{\dot{H}^+,\ \dot{H}^o,\ d\}$. Specifically, based on direct effects (or a Cobb-Douglas form) we have shown:

(27) (i). $\left(\frac{\dot{f_j}}{f_i}\right) > 0 \;\; if \;\; i \in \dot{H}^o,\ j = d$

(ii). $\left(\frac{\dot{f}_j}{f_i}\right) < 0 \;\; if \;\; j \in \dot{H}^+ , \;\; i \in \dot{H}^o$ or $\;\; i = d, \; j \in \dot{H}^+$

(iii). $\left(\frac{\dot{f}_j}{f_i}\right) = 0 \;\; if \;\; i, j \in \dot{H}^o \;\; or \; i, j \in \dot{H}^+ .$

It is important to recognize that a given hierarchy level (*i* or *j*) may migrate among the sets $\{\dot{H}^+$, $\dot{H}^o$, $d\}$ during transition. It is this property that generates complex patterns of relative wage evolution in the human capital hierarchy. We explore these dynamics in detail in the next section.

Patterns of Relative Wage Evolution In Transition

In the prior section we characterized the instantaneous change in relative wages between any two hierarchy levels in transition. We now seek to track the evolution of relative wages between hierarchy levels pairs *over the full transition path*. We focus in this narrative on the direct effects identified above while recognizing that indirect effects may have an attenuating influence. The transition time path of relative wages for any hierarchy pair depends on their initial membership in one of the three sets: $\dot{H}^o$, $\dot{H}^+$, d. In particular, the initial membership configurations that yield non-trivial inequality transition paths include: (i). $j \in \dot{H}^o$, $i \in \dot{H}^o$; (ii). $j = d$, $i \in \dot{H}^o$; (iii). $j \in \dot{H}^+$, $i \in \dot{H}^o$. It turns out, however, that the transition paths associated with (ii) and (iii) are subsets of (i). We therefore focus on case (i) and then identify the transition segments of (i) that correspond to (ii) and (iii).

Before analysis of the transition path when $j \in \dot{H}^o$, $i \in \dot{H}^o$ at the initial condition it is useful to reemphasize the qualitative nature of our characterization of f_j/f_i evolution and to restate the interpretation of f_j/f_i itself in the context of a human capital hierarchy. In particular it is critical to bear in mind that each human capital type (*1, 2,…, N*) is at a fixed position (level) in the hierarchy. We then select two specific hierarchy levels (call them *i, j*) and track the qualitative evolution of f_j/f_i during transition. We can select the two levels (*i, j*) to be tracked from anywhere in the hierarchy. Our index numbers ($j > i$) reflect the fact that each unit of *j* occupies a more advanced position in the hierarchy than *i*. The specific levels we select for comparison and the initial condition jointly determine their "membership" in the sets { $\dot{H}^o$, $\dot{H}^+$, d } at the beginning of transition. Given the initial condition we analyze in this paper (equation *15*) and that $j > i$, the optimal program would never entail: $i = d, j \in \dot{H}^o$ or $i \in \dot{H}^+, j \in \dot{H}^o$.

Continuing with our interpretation of $j \in \dot{H}^o$, $i \in \dot{H}^o$, it is natural to associate this initial configuration with the selection of two hierarchy levels relatively low in the human capital pyramid. However, as long as the steady-state relationship between N and $N\text{-}1$ is not achieved instantaneously, this configuration holds at the beginning of transition for any i, j selection unless $j = N\text{-}1$ or N. Thus, the initial configuration $j \in \dot{H}^o$, $i \in \dot{H}^o$ imposes only mild restrictions on the choice of $\{i, j\}$. Moreover, as noted, the qualitative properties of an initial $j \in \dot{H}^+$ or $j = d$ (which correspond to $j = N, j = N\text{-}1$) are subsumed in $i, j \in \dot{H}^o$. What will differ as we choose i and j from more dispersed positions in the hierarchy is the length of time of the various "stages" of transition. We return to this issue subsequently.

Given $i, j \in \dot{H}^o$, and that output is not so high that steady-state relationships can be achieved instantaneously, the transition path of f_j/f_i will consist of four qualitative stages. Figure 2 below provides a qualitative illustration of the stages of the stages of transition. During Stage 1 of transition $i, j \in \dot{H}^o$, and investment is concentrated in hierarchy levels greater than $j+1$. The change in relative wages (f_j/f_i) attributable to direct effects is zero so long as i, j retain membership in $\dot{H}^o$. As the optimal transition program progresses, positive investment is initiated at sequentially lower levels, and d migrates downward through the hierarchy. Eventually, investment begins in hierarchy level $j+1$, and level j will be subject to uncompensated depletion ($j = d$). This is the beginning of Stage 2 of transition, and there is a "bang-bang" discontinuity in f_j/f_i at the Stage 1-2 boundary as the influence of uncompensated depletion is introduced. During Stage 2, relative wage evolution is governed by *(25)*, and based on the direct effect $\left(f_j \dot{/} f_i\right) > 0$.

Stage 3 of transition begins when j and $j+1$ attain their steady-state relationship, $f_{j+1}/j+1 = f_j/j$. At this point $d = j\text{-}1$ and j joins the $\dot{H}^+$ set. During Stage 3 of transition relative wage evolution is governed by *(23)* – so long as $i \neq j-1$. Consequently $\left(f_j \dot{/} f_i\right) < 0$, and the trend of relative wages reverse vis-à-vis Stage 2. We thus move from a regime of wage expansion ($\left(f_j \dot{/} f_i\right) > 0$) to one of wage compression ($\left(f_j \dot{/} f_i\right) < 0$) as we transition from Stage 2 to 3. As

Stage 3 continues, positive investment is initiated at $j - 1, j - 2, . . ., i{+}2$, and the inequalities of *(15)* are converted to equalities one by one. When positive investment reaches $i{+}1$ ($i = d$) and we enter Stage 4 – the $j \in \dot{H}^{+}$, $i = d$ regime. Relative wage evolution here obeys *(21)*. Accordingly, f_j/f_i declines at a faster rate than in Stage 3. The accelerated decline of f_j/f_i in Stage 4 is due to the addition of a second negative effect in *(21)* – as compared to the single negative direct effect in *(23)* during Stage 3. During Stage 4 the falling stock of i associated with depletion raises its marginal productivity until $f_{i+1}/i{+}1 = f_i/i$ at which point level i is in the steady-state relationship with all higher indexed human capital levels – including j. At this point Stage 4 ends and the optimal investment program maintains $f_j / f_i = j/i$ for all future periods. Splicing these stages together yields a complex non-monotonic pattern of wage expansion followed by accelerating wage compression.

[Figure 2]

The relative wage transition path illustrated in Figure 2 is a qualitative representation of the sign of f_j/f_i during distinct stages of transition when $j \in \dot{H}^{o}$, $i \in \dot{H}^{o}$ at the initial condition. Now recall the alternative initial relationships discussed at the beginning of this section: (ii). $j = d$, $i \in \dot{H}^{o}$; (iii). $j \in \dot{H}^{+}$, $i \in \dot{H}^{o}$. To see that these initial relationships are subsumed in the full transition time path when $i, j \in \dot{H}^{o}$, as shown in Figure 2, simply note that (ii) and (iii) above correspond precisely with Stage 2 and 3 respectively in Figure 2. Therefore, if the initial condition stocks are such that the optimal program requires $j = d$, $i \in \dot{H}^{o}$ at the beginning of transition, the full transition path corresponds to Stages 2, 3, and 4 in Figure 2. Similarly, if $j \in \dot{H}^{+}$, $i \in \dot{H}^{o}$ at the beginning of transition, the full transition path entails Stages 3 and 4 of Figure 2.

Aggregation Boundaries

Consideration of human capital in the context of growth models typically entails the aggregation of, what are ultimately, non-homogenous units. For example, from the perspective of our paradigm, models with two human capital types have aggregated the N human capital types into two sets. Obviously, any model with a single human (or physical) capital type that purport's to describe economy-wide growth entails aggregation. In this section we demonstrate that the choice of aggregation sets may substantively affect the relative wage evolution if an *aggregation boundary* is crossed. Thus, the partitions of the human capital set used in aggregation, which are often chosen arbitrarily, may be critical. In particular, we will show that aggregation of the hierarchy level experiencing depletion (*d*) with hierarchy levels in the accumulation set ($\dot{H}^+$) may have an obfuscating effect on the evolution of the relative wages.

To illustrate the potentially substantive effect of the choice of an aggregation set, suppose we aggregate various human capital levels into two sets: advanced (H^A) and basic (H^B). Let $\underline{a}$ and $\overline{a}$ be, respectively, the lowest and highest indexed hierarchy levels in H^A. Employing similar notation for H^B we have $\overline{b} < \underline{a}$. The typical aggregation procedure entails computing an average wage within the aggregation set. Given that we associate the hierarchy level stock (H_j) with population, the average wage in the advanced human capital category is:

$$(28) \qquad f_A = \left(\frac{1}{\sum_{j=\underline{a}}^{\overline{a}} H_j} \right) \left(\sum_{j=\underline{a}}^{\overline{a}} f_j H_j \right)$$

Therefore the relative wage of advanced and basic human capital can be written as:

$$(29) \qquad \frac{f_A}{f_B} = \left(\frac{\sum_{i=\underline{b}}^{\overline{b}} H_i \left(\sum_{j=\underline{a}}^{\overline{a}} f_j H_j \right)}{\sum_{j=\underline{a}}^{\overline{a}} H_j \left(\sum_{i=\underline{b}}^{\overline{b}} f_i H_i \right)} \right)$$

For notation convenience we now make the following definitions:

$A = \sum_{j=\underline{a}}^{\bar{a}} H_j$, $B = \sum_{i=\underline{b}}^{\bar{b}} H_i$, $W_A = \sum_{j=\underline{a}}^{\bar{a}} f_j H_j$, $W_B = \sum_{i=\underline{b}}^{\bar{b}} f_i H_i$.

Given this notation we can write relative wages and their evolution during transition *(29)* as:

(30) $$\frac{f_A}{f_B} = \frac{BW_A}{AW_B},$$

(31) $$\dot{\left(\frac{f_A}{f_B}\right)} = \frac{\left(\dot{B}W_A + B\dot{W}_A\right)AW_B - \left(\dot{A}\,W_B + A\,\dot{W}_B\right)B\,W_A}{\left(A\,W_B\right)^2}\ .$$

Now suppose that the depletion level (*d*) is aggregated with the accumulation set ($\dot{H}^+$) over the transition period during which relative wages are tracked. That is: *A = {d, d+1, . . . , N}*, *B = {1, 2, . . . , d-1}.*[7] For this particular partition of the hierarchy $\dot{A} = \dot{B} = 0$. To see this note that $H^B = \dot{H}^o$ so there is no investment or depletion in the basic human capital set. Moreover, since the depletion level is aggregated with levels undergoing augmentation, the total stock of *A* is constant $(\dot{A} = 0)$. Then substituting $\dot{A} = \dot{B} = 0$ into (*31*), its numerator becomes: $AB(\dot{W}_A W_B - \dot{W}_B W_A)$. So the sign of *(31)* is greater than, less than, or equal to zero as the growth rate of the aggregated advanced human capital wage bill is greater than, less than, or equal to the growth rate of the basic human capital wage bill. If, as in the prior analysis, we focus on direct affects only, the numerator becomes $AB\dot{W}_A W_B$. The sign of *(31)* now turns on the sign of $\dot{W}_A$, which may be written after some manipulation as:

(32) $$\dot{W}_A = \sum_{j=d}^{N} \dot{H}_j \left(f_{jj} H_j + f_j\right).$$

It now becomes clear that the choice of aggregation sets is potentially substantive in at least two dimensions. In both cases, the problem arises from the interaction of the wage and stock effects within an aggregation set. The first potential aggregation problem arises when $\dot{H}_j < 0$ for $j = d$,

[7] Note that *A* and *B* need not completely partition the hierarchy.

but $\dot{H}_j > 0$ for $j = d+1, \ldots, N$. Had we chosen instead to define advanced human capital as any subset of $\dot{H}^+$, for example, $H^A = \{d+1, \ldots, N\}$, $\dot{H}_j$ would be positive across this alternative "advanced" human capital set. Measures of *aggregated* relative wage evolution therefore introduce a potential ambiguity vis-à-vis disaggregated relative wages in that $\dot{H}_j$ may switch signs within the aggregation group. This sign switching will occur whenever the aggregation crosses the "*d*" boundary from above. A second fundamental problem arises because of the stock weighting that must accompany aggregation, and is independent of the sign switching of $\dot{H}_j$ per-se. Specifically, to obtain the average wage of a population distributed across a set of hierarchy levels, the within level wage (f_j) must be weighted by its population stock (H_j) prior to averaging. In contrast {to this explicit role of H_j}, since within level wages are uniform no stock weighting is required in computing relative wage evolution.

Population (stock) weighting implies that even though the individual wage (f_j) may be falling, the total within level wage bill (f_jH_j) may be rising if $\dot{H}_j > 0$. While there may be alternative aggregation procedures that may circumvent this effect, the most common procedure is the one described: sum the wages of all individuals in the aggregation set, and divide by the total population of the set. As a concrete example suppose f is Cobb-Douglas and $H^A \subseteq \dot{H}^+$. Then $f_{jj}H_j + f_j > 0$ for all j, $\dot{W}_A > 0$, and $\left(f_A \dot{/} f_B\right)$ is unambiguously positive. Note that this contradicts the result of $\left(f_j \dot{/} f_i\right) < 0$ when $j \neq d$ (see equations *24* and *27*). The intuition for this contradiction is that, within the aggregation set, population is growing and migrating from lower to higher wage hierarchy levels. Thus, even though wages within individual hierarchy levels are falling , the aggregate wage bill is rising. This stock effect is absent when relative wage are restricted to individual hierarchy levels.

4. Summary and Conclusion

In the prior empirical literature, patterns of wage compression and expansion are attributed to exogenous institutional developments or extra-economic shocks (e.g., World War II). Though these exogenous shocks have undoubtedly been major contributors to the most dramatic episodes of wage expansion and contraction, even the most careful empirical attribute only a minority of the observed expansion/contraction to these exogenous factors. Moreover, though less spectacular, cycles of wage expansion and compression occur outside the dramatic episodes that have been the focus of much of the empirical literature.

The model developed in this paper identifies an endogenous engine of wage expansion and compression along the transition path from initial conditions to the steady state. The critical assumptions that drive this engine are the hierarchical structure of human capital, the productivity of the intermediate human product, and an initial condition of relative scarcity of advanced human capital. Given these assumptions, the optimal transition investment program generates a stock depletion effect that migrates through the hierarchy. In the context of the prior empirical literature, this stock-depletion effect is consistent with a supply-side origin for a cycle of expansion and compression.

A second focus of this paper is consideration of implications of the choice of aggregation sets. All analyses of relative wages or income distribution, beyond the individual, involve aggregation. Though most aggregation in the wage compression-expansion literature is based on reasonable criteria, a uniform "objective" principle for choosing the boundaries of the aggregation sets is lacking. Our analysis suggests that aggregating human capital categories experiencing stock depletion (due to hierarchy effects), with those in an accumulation phase, may obfuscate the underlying pattern of wage expansion and compression. Levels of human capital experiencing stock depletion therefore constitute a natural *aggregation boundary*.

As with all growth models, ours is an abstract model designed to facilitate exposition of a particular feature of the growth environment. Clearly, there are many omitted aspects of the wage compression-expansion setting that may attenuate our results – adjustment costs and technological change to mention two. Nevertheless, the hierarchical mechanism we have identified should be operative whenever the basic assumptions stated above are satisfied – though it may be concealed by a myriad of other factors. To move beyond the qualitative identification of this engine to a quantitative assessment of its importance is the natural next step in this line of research.

Appendix

Assume the following standard functional forms for utility and production respectively:

$$(A1) \qquad U(c) = \frac{c^{(1-v)} - 1}{1-v}, \qquad Y = \prod_{i=1}^{N} H_i^{\alpha_i}.$$

If the generalized Cobb-Douglas production function of *(A1)* exhibits constant returns to scale, and there is no depreciation or exogenous population growth, equations *(11), (12)*and *(A1)* can be used to derive the following expression for the (endogenous) steady-state growth rate of consumption for an *N*-level hierarchy:

$$(A2) \qquad \gamma_c = \frac{1}{v}\left[\prod_{i=1}^{N}\left(\frac{\alpha_i}{i}\right)^{\alpha_i} - r\right],$$

where α_i is the Cobb-Douglas productivity weight of human captial level *i*. Again note the relationship between the steady-state consumption growth rate and hierarchy size *(i's)*. One interesting implication of *(A2)* is that the steady-state consumption growth rate is declining in hierarchy size. The functional forms above together with equation *(12)* yield explicit following steady-state human capital ratios:

$$(A3) \qquad \frac{H_i}{H_j} = \frac{j\alpha_i}{i\alpha_j}, \qquad \forall\, i, j = 1, \ldots, N.$$

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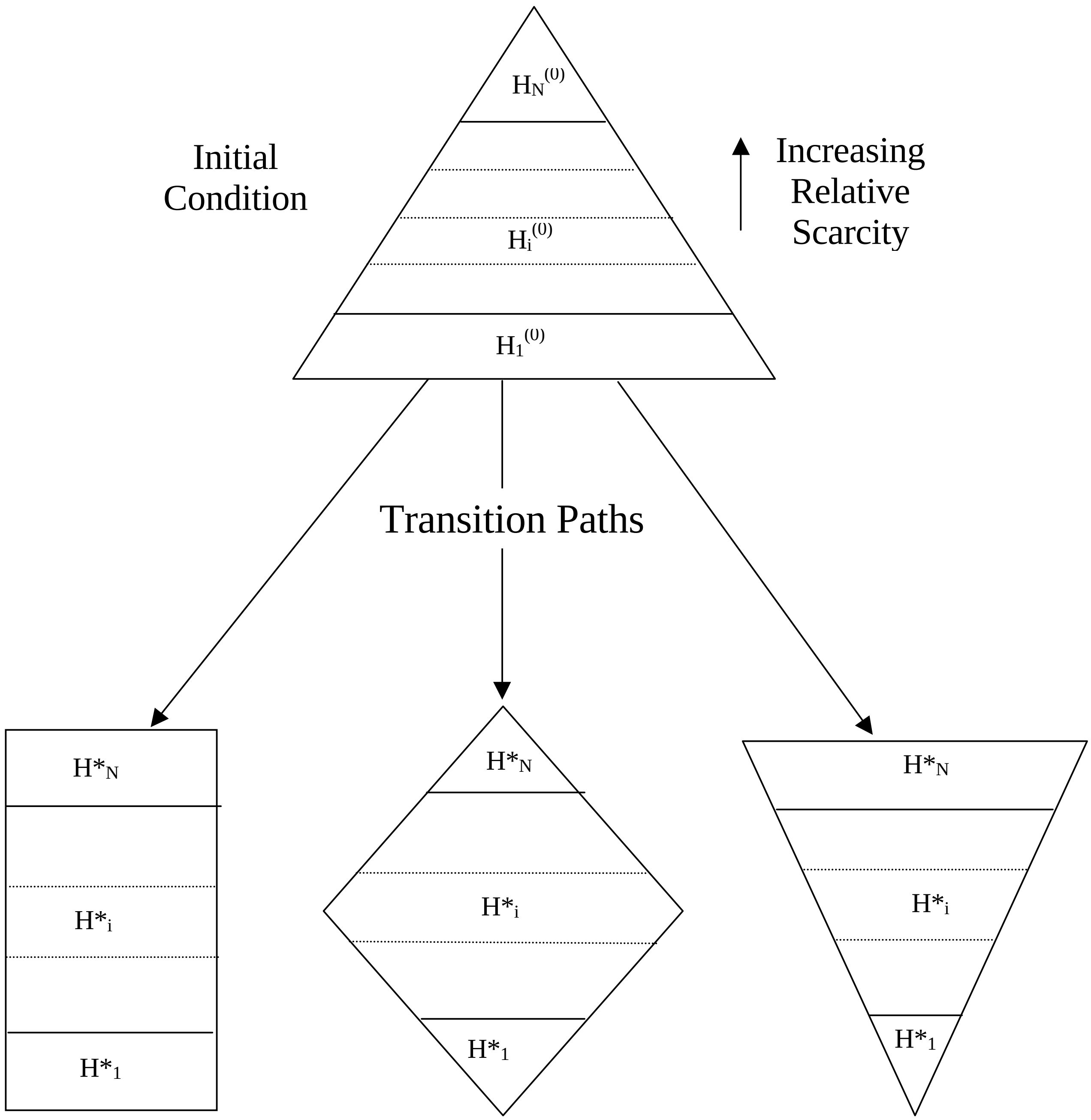
Figure 1



Some Potential Steady State Configurations

Figure 2 –Stages of Transition Path Wage Evolution