FUNDAÇÃO
GETULIO VARGAS

EPGE
Escola de Pós-Graduação
em Economia

Ensaios Econômicos

Escola de
Pós-Graduação
em Economia
da Fundação
Getulio Vargas

N° 427 ISSN 0104-8910

The Land Assembly Problem Revisited

Flavio Marques Menezes, Rohan Pitchford

Junho de 2001

URL: http://hdl.handle.net/10438/848

Os artigos publicados são de inteira responsabilidade de seus autores. As opiniões neles emitidas não exprimem, necessariamente, o ponto de vista da Fundação Getulio Vargas.

Marques Menezes, Flavio
The Land Assembly Problem Revisited/
Flavio Marques Menezes, Rohan Pitchford - Rio de Janeiro : FGV,EPGE, 2010
(Ensaios Econômicos; 427)

Inclui bibliografia.

CDD-330

The Land Assembly Problem Revisited

EPGE/FGV and
School of Economics
Australian National University
Canberra, ACT, 0200
Australia
Email: Flavio.menezes@anu.edu.au

Rohan Pitchford
NCDS
Australian National University
Canberra, ACT, 0200
Australia
Email: Rohan.pitchford@anu.edu.au

May 23, 2001

Abstract

As in the standard land assembly problem, a developer wants to buy two adjacent blocks of land belonging to two different owners. The value of the two blocks of land to the developer is greater than the sum of the individual values of the blocks for each owner. Unlike the land assembly literature, however, our focus is on the incentive that each lot owner has to delay the start of negotiations, rather than on the public goods nature of the problem. An incentive for delay exists, for example, when owners perceive that being last to sell will allow them to capture a larger share of the joint surplus from the development. We show that competition at point of sale can cause equilibrium delay, and that cooperation at point of sale will eliminate delay.

JEL Classification: C72, C78, R52.
Keywords: land assembly; coordination; inefficient delay.

1 Introduction

Suppose a developer wants to buy two adjacent blocks of land that are currently in the possession of two different owners. The value of the two blocks of land to the developer is greater than the sum of the individual values of the blocks for each owner. Under complete information about individual valuations, the developer would wish to make a take-it-or-leave-it simultaneous offer to both owners equal to their valuations. Such an outcome is efficient, since the owners will accept the offers. The developer will also receive all of the surplus.

Instead, suppose the owners are able to avoid this situation, and approach the developer sequentially. The final division of the surplus will depend on who makes the final offer. This individual will end up with the entire remaining surplus and the efficient allocation will be implemented but at the expense of costly delay. Given the possible advantage that arises from being the last to make an offer, players may strategically delay the start of the sale process. The purpose of our paper is to model this incentive.

The theory of urban land use has been extensively studied since the seminal work of Alonso (1964). The typical assumption is that of a perfectly competitive land market.[1] This assumption is not always reasonable and it has been relaxed. For example, O'Flaherty (1994) studies the problem of a developer who wants to assemble several parcels of land to undertake a

[1] Fujita (19866, 1988) summarizes this literature.

project that generates positive externalities. The acquisition by the developer of some plots of land increase the value of the plots of land that are not acquired. O'Flaherty shows that the existence of this public goods type externality implies that in equilibrium a suboptimal amount of land is acquired by the developer. The reason is the free-rider problem.[2] Previous work has not examined the externality issue but instead either allowed parties to rely on noncredible threats (e.g., Eckart (1985) allows lot owners to make credible and final offers above their values), or examined the problem from a cooperative framework (e.g., Asami (1988)).

The goal of our analysis is to examine incentives for delay that are independent of externality problems. It is our contention that both strategic and externality problems can contribute to inefficiency in land assembly. For example, it is not uncommon to see a development being built close to a property that the developer failed to acquire. This can happen if the remaining property owner enjoys an increase in the value of her land by virtue of being close to the development. Similarly, there are many cases of costly delay in the acquisition of land for development in which all of the necessary land is eventually purchased. Our model is aimed at the latter form of inefficiency.

We model the decision of an owner to sell her land, as a type of intertemporal coordination problem. She may wish to avoid selling her land at the same time as another, in order to reduce the intensity of competition at point of sale. On the other hand, she may wish to sell her land in conjunction with

[2] Grossman and Hart (1980) also rely on this public goods type externality to claim that no land would be acquired when the developer needs all plots of land. O 'Flaherty (1994) weakens this assumption by allowing the developer to buy any subset of the total number of plots of land.

another, if both parties are able to cooperate at point of sale. To focus on the pure strategic effects of our sale-coordination problem, we assume that the sale process is efficient once parties decide to sell to the developer. There is no asymmetric information, or other potential imperfections that might confound the effects that wish to examine. To focus on the problem, rather than public policy solutions, we also abstract from the possibility that a third party (e.g., a public authority) might exercise the power of eminent domain to buy the owners' blocks of land and resell them to the developer – so called 'urban renewal'.[3]

We argue that although the public good nature of the land assembly problem is a very important determinant of inefficient land assembly, that are other forces at work. Even in the absence of public-goods type of externalities, inefficient outcomes may arises from the pure coordination nature of the land assembly problem.

2 The Model and Results

We assume that there is one developer –who wishes to buy land– and two land owners. The developer realizes value v from possession of both blocks of land. Each block is owned by player $i = 1, 2,$ with valuation w_i. The potential gain from trade is therefore $v - w_1 - w_2$. Our analysis allows the buyer to place value $v_i \geq w_i$ on an individual plot of land. Thus, to some extent the plots of land could be seen as substitutes for one-another: If the developer did not purchase both plots, then she would at least make the

[3] O'Flaherty (1994) provides an excellent explanation for the inefficiency of urban renewal policies in some sense formalizing the ideas that have been put forward by other economists such as Arrow (1970).

return $v_i - w_i$ on the block that she does purchase.

We have implicitly assumed that there are no externalities. Specifically, if the developer buys the lot owned by player 1, then the reservation value of player 2's block is unchanged at w_2 (and vice-versa). This contrasts with existing research in the land assembly problem (e.g., O'Flaherty (1994)), that focuses on such externalities. If the developer owns player 1's block of land, then this may increase or decrease w_2. An increase, of course, represents a positive externality. For example, if the developer is building a shopping center, then the convenience of player 2's location nearby may add to player 2's valuation. A decrease is a negative externality. For example, the shopping centre might increase congestion and traffic noise in the neighborhood, and reduce player 2's valuation.

We examine the potential for inefficiency generated purely from the fact that players can delay entry into the sale process. Delay may occur, since a player hopes to extract a higher payment from the buyer by virtue of being last to sell. For simplicity we assume that there are two time periods, now (N) and later (L). We model the possibility of delay by allowing owners $i = 1, 2$ to simultaneously choose probabilities p_1 and p_2 of selling to the developer now. Consequently, $1 - p_i$ denotes the probability of owner i selling later. Let $t_i \in \{0, 1\}$ denote the presence of player i at point of sale now, where 1 indicates presence and 0 indicates absence (and therefore presence later). Thus, t_i is the outcome of $i's$ choice p_i of the probability of participation at date N.

To keep the analysis general, we admit general payoffs at point of sale. The payoff to player i from sale when the outcome is (t_1, t_2) is $s_i : \{0, 1\}^2 \longrightarrow$

$\mathbb{R}$. For example $s_1(1,0)$ is 1's total utility when 1 sells now, and 2 sells later. If both sell now, player 1 receives $s_1(1,1)$, etc. We would expect players to receive no less than their reservation value, i.e. $s_i \geq w_i$. Similarly, the developer should pay no more than v for each block, i.e. $s_1(t_1,t_2)+s_2(t_1,t_2) \leq v$. It turns out that these very basic restrictions do not affect the set of potential equilibria. Therefore, we can ignore the valuations v, w_1 and w_2 and restrict attention instead to the payoffs s_i.

Player i's expected payoff π_i is the probability-weighted sum of payoffs in each state of nature $(t_1,t_2) \in \{(1,1),(1,0),\ (0,1),(0,0)\}$. Player 1's expected payoff is

$$\pi_1 = p_1 p_2 s_1(1,1) + p_1(1-p_2)s_1(1,0) + (1-p_1)p_2 s_1(0,1) + (1-p_1)(1-p_2)s_1(0,0),$$

and player 2's expected payoff is similar, but with subscript 2 replacing 1 in the above.

3 Results

We are interested in finding the Nash equilibria that are generated by the delay game between players 1 and 2. A Nash equilibrium of the game is a vector of probabilities (p_1^*, p_2^*) that satisfies $p_i \in \arg\max \pi_i(p_i, p_{-i}^*)$ for $i = 1,2$. To solve for Nash equilibria, we consider the derivatives of π_1 and π_2 with respect to corresponding probabilities

$$\frac{\partial \pi_1}{\partial p_1} = p_2\left[s_1(1,1) - s_1(0,1)\right] + (1-p_2)\left[s_1(1,0) - s_1(0,0)\right] \tag{1}$$

$$\frac{\partial \pi_2}{\partial p_2} = p_1\left[s_2(1,1) - s_2(1,0)\right] + (1-p_1)\left[s_2(0,1) - s_2(0,0)\right]. \tag{2}$$

Each of the terms in square brackets has a very direct interpretation. For example the first term in square brackets in (1), $s_1(1,1) - s_1(0,1)$, is the gain to player 1 from selling her block now –as opposed to later– given that player 2 sells her block now. The second term in (1) is $s_1(1,0) - s_1(0,0)$, the gain to player 1 from selling now, as opposed to later, given that player 2 sells later. Interpretations are directly analogous for player 2. Define Δ_{iN} as the gain to player i of sale now, given that player $-i$ sells now, and Δ_{iL} as the gain to player i of sale now given that $-i$ sells later. Thus, $\Delta_{1N} = s_1(1,1) - s_1(0,1)$, $\Delta_{2N} = s_2(1,1) - s_1(1,0)$, $\Delta_{1L} = s_1(1,0) - s_1(0,0)$ and $\Delta_{2L} = s_2(0,1) - s_2(0,0)$.

Our method of analysis is to derive Nash equilibria with reference to equations (1) and (2), and the interpretation of the Δ's. This approach gives a direct economic interpretation to each of the different cases that we apply to strategic delay incentives in the land assembly problem.

Suppose first of all, that Δ_{1N}, Δ_{2N}, Δ_{1L} and Δ_{2L} are all positive. From (1) and (2), marginal returns from p_i are positive for both players, so the Nash equilibrium is $(p_1, p_2) = (1,1)$. All of the Δ's positive means that the payoff to both players from immediate sale is higher than the payoff from selling later, independently of what the other player does. In other words, it is a dominant strategy Nash equilibrium for each player to sell now. Such a situation could arise if opportunity for sale is rare: There might be a substantial amount of time between 'now' and 'later', due, for example, to costs to the developer of setting up the sale process. In terms of the Δ's, we could imagine that payoffs to a player from selling later are heavily discounted, and hence are small. The first prediction of the model could be framed as follows:

Proposition 1 *There will be no delay in land assembly if opportunities for sale are rare.*

This conclusion is not altered if at most one of the players would prefer to sell later given that the other sells later. Suppose this is true for player 2, i.e. we have $\Delta_{2L} < 0$ with all other Δ's positive. Here, player 2 wishes to co-ordinate her sale with player 1: $\Delta_{2N} > 0$ means 2 prefers to sell now if 1 sells now, and $\Delta_{2L} < 0$ means 2 prefers to sell later if 1 sells later. The reason for this desire on player 2's behalf, might be that player 1 is a strong bargainer who helps player 2 gain more surplus. Since 1 chooses to sell immediately regardless of player 2's choice, it is also optimal for 1 to sell immediately. In (1), we have $\frac{\partial \pi_1}{\partial p_1} > 0$ so that $p_1 = 1$. Substitution in (2), yields $\frac{\partial \pi_2}{\partial p_2} > 0$, and $p_2 = 1$.

Equilibrium delay occurs if at least one of the players finds it desirable to co-ordinate to be *away* from the other player at point of sale. Suppose all the Δ's are positive, except for $\Delta_{1N} < 0$. This implies that player 1 wishes to sell in a different period to player 2: $\Delta_{1N} < 0$ means 1 wishes to sell later if player 2 sells now, and $\Delta_{1L} > 0$ means 1 prefers to sell now if 2 sells later. Since player 2 always sells now, it is straightforward that player 1 sells later. Thus we have $(p_1, p_2) = (0, 1)$, and potentially inefficient delay occurs in equilibrium. The degree of inefficiency depends on the degree to which the buyer discounts the future value of 1's block.

The crucial feature of this example is that $\Delta_{1N} < 0$ implies $s_1(1,1) < s_1(0,1)$ and $\Delta_{1L} > 0$ implies $s_1(1,0) > s_1(0,0)$. In other words, when both players are present at the same time, player 1 experiences a lower payoff: The presence of player 2 is bad for player 1's sale price. A straightforward interpretation of this situation is that player 2 is a stronger competitor than player 1, so that player 1 avoids player 2 at point of sale.

We can extend the idea of competition to two players, i.e. $\Delta_{1L} > 0$, $\Delta_{1N} < 0$ and $\Delta_{2L} > 0$, $\Delta_{1N} < 0$. Following the same reasoning as above, these conditions are satisfied if

$$\begin{aligned} s_1(1,1) &< s_1(0,1) \\ s_1(1,0) &> s_1(0,0) \\ s_2(1,1) &< s_2(1,0) \\ s_2(0,1) &> s_2(0,0)\,. \end{aligned} \tag{3}$$

Note that in all of these inequalities, the payoff from selling at the same time, falls below the payoff from selling at separate dates. This can be interpreted directly as competition at point of sale: the presence of the other owner reduces each owner's payoff. In this case, both players wish to co-ordinate to be apart at point of sale, to avoid fierce competition. This occurs even though parties might discount the future:

Substitution of the Δ values implied by inequalities (3) into equations (1) and (2) yields two pure-strategy equilibria and one mixed-strategy equilibrium. The latter is found by equating the derivatives to zero, and solving for (p_1, p_2):

$$\begin{aligned} p_2\Delta_{1N} + (1-p_2)\Delta_{1L} &= 0 \\ p_1\Delta_{2N} + (1-p_1)\Delta_{2L} &= 0, \end{aligned}$$

to yield

$$p_1 = \frac{\Delta_{2L}}{\Delta_{2L} - \Delta_{2N}}, \text{ and } p_2 = \frac{\Delta_{1L}}{\Delta_{1L} - \Delta_{1N}}.$$

The two pure strategy equilibria are, of course, $(1, 0)$ and $(0, 1)$. Substitution of $p_1 = 1$ into (2) yields $\frac{\partial \pi_2}{\partial p_2} = \Delta_{2N} < 0$, so $p_2 = 0$ is the best response. Substitution of $p_2 = 0$ in (1) gives $\frac{\partial \pi_1}{\partial p_1} = \Delta_{1L} > 0$, so $p_1 = 1$ is the best response. The argument behind $(0, 1)$ being a Nash equilibrium is directly analogous. We summarize these findings in the following:

Proposition 2 *Delay occurs in equilibrium if there is sufficiently strong competition between players at point of sale*

Now consider the case where players are able to cooperate at point of sale, rather than compete. In other words, assume that the payoffs from being together exceed the payoffs from being separate:

$$\begin{aligned} s_1(1,1) &> s_1(0,1) \\ s_1(1,0) &< s_1(0,0) \\ s_2(1,1) &> s_2(1,0) \\ s_2(0,1) &< s_2(0,0). \end{aligned} \tag{4}$$

Here, we have $\Delta_{1L} < 0$, $\Delta_{1N} > 0$ and $\Delta_{2L} < 0$, $\Delta_{1N} > 0$. The set of Nash equilibria is

$$\left\{(0,0), (1,1), \left(\frac{\Delta_{2L}}{\Delta_{2L} - \Delta_{2N}}, \frac{\Delta_{1L}}{\Delta_{1L} - \Delta_{1N}}\right)\right\}.$$

If players cooperate at point of sale, then they will wish to co-ordinate to sell at the same time, i.e. either now, or later, or a mixed strategy over now and later. We would expect that with discounting, both parties would co-ordinate on selling now. Thus:

Proposition 3 *There is no delay in equilibrium if players cooperate at point of sale, and players discount the future.*

Other equilibria are possible, though these are probably not as applicable to the land assembly problem. For example, both parties might find it advantageous to sell later, independent of the choice of the other (i.e. all Δ's are negative). The complete set of equilibria are summarized in the following tables.

Proposition 4 *The following table summarizes the equilibria, up to symmetry, that obtain for different values of* Δ_{iN} *and* Δ_{iL}*,* $i = 1, 2:$

Rare Sale

Δ_{1L}	Δ_{1N}	Δ_{2L}	Δ_{2N}	(p_1, p_2)
$+$	$+$	$+$	$+$	$(1, 1)$
$+$	$+$	$-$	$+$	$(1, 1)$

Competition at point of sale

Δ_{1L}	Δ_{1N}	Δ_{2L}	Δ_{2N}	(p_1, p_2)
$+$	$-$	$+$	$+$	$(0, 1)$
$+$	$-$	$+$	$-$	$\left(p_1^b, p_2^b\right)$

$$\textit{where } \left(p_1^b, p_2^b\right) \in \{(1,0), (0,1), (\frac{\Delta_{2L}}{\Delta_{2L} - \Delta_{2N}}, \frac{\Delta_{1L}}{\Delta_{1L} - \Delta_{1N}})\}$$

Cooperation at point of sale

Δ_{1L}	Δ_{1N}	Δ_{2L}	Δ_{2N}	(p_1, p_2)
$-$	$-$	$-$	$+$	$(0, 0)$
$-$	$+$	$-$	$+$	(p_1^a, p_2^a)

$$\textit{where } (p_1^a, p_2^a) \in \{(0,0), (1,1), (\frac{\Delta_{2L}}{\Delta_{2L} - \Delta_{2N}}, \frac{\Delta_{1L}}{\Delta_{1L} - \Delta_{1N}})\}$$

Other cases

Δ_{1L}	Δ_{1N}	Δ_{2L}	Δ_{2N}	(p_1, p_2)
$-$	$-$	$-$	$-$	$(0,0)$
$-$	$-$	$+$	$+$	$(1,1)$
$-$	$-$	$+$	$-$	$(0,1)$

4 Conclusion

The purpose of this paper has been to examine how coordination problems might impact on the land assembly problem, independently of consideration of externalities. We do not argue that such strategic coordination motives will always dominate, nor even that they will be important in all situations. However, to the extent that they exist, outcomes from land assembly will be more or less efficient, depending on the nature of the sale process.

We argued that if sale is rare, then the problem of equilibrium delay is mitigated. This suggests the use of commitment devices by developers to make rare sale more credible. When players compete at point of sale, the delay problem becomes worse. Each player prefers to conduct their sale without the other present, in order to mitigate competition. In contrast, cooperation at point of sale eliminates the delay problem, provided sellers discount the future.

References

[1] Alonso, W., 1964, *Location and Land Use.* Harvard University Press, Cambridge, MA.

[2] Arrow, K., 1970, "Criteria, Institutions, and Functions in Urban Development Decisions, in A. Pascal, ed., *Thinking about Cities*. Dickenson, Belmont, CA.

[3] Asami, Y., 1988, "A Game-Theoretic Approach to the Division of Profits from Economic Land Development," *Regional Science and Urban Economics 18*, 233-246.

[4] Fujita, M., 1986, Urban Land Use Theory, in J. J. Gabszewicz, J. F. Thisse, M. Fujita and U. Schweizer, *Location Theory*. Harwood Academic Publishers, 3-149.

[5] Fujita, M., 1988, *Residential Land Use Theory*. Cambridge University Press, Cambridge.

[6] Eckart, W., 1985, "On the Land Assembly Problem," *Journal of Urban Economics 18*, 364-378.

[7] Grossman, S. and O. Hart, 1980, "Takeover Bids, the Free-Rider Problem, and the Theory of the Corporation, " *Bell Journal of Economics 11*, 42-64.

[8] O'Flaherty, B., 1994, "Land Assembly and Urban Renewal," *Regional Science and Urban Economics 24*, 287-300.