Robustness of stationary tests under long-memory alternatives

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Abstract

This paper investigates the presence of long memory in financial time series using four test statistics: V/S, KPSS, KS and modified R/S. There has been a large amount of study on the long memory behavior in economic and financial time series. However, there is still no consensus. We argue in this paper that spurious short-term memory may be found due to the incorrect use of data-dependent bandwidth to estimating the long-run variance. We propose a partially adaptive lag truncation procedure that is robust against the presence of long memory under the alternative hypothesis and revisit several economic and financial time series using the proposed bandwidth choice. Our results indicate the existence of spurious short memory in real exchange rates when Andrews’ formula is employed, but long memory is detected when the proposed lag truncation procedure is used. Using stock market data, we also found short memory in returns and long memory in volatility.

1 Introduction

In the time domain, a time series exhibits long-range dependence if the absolute values of the autocorrelations are not summable. In practice, the presence of long-memory components in asset returns has important implications for many of the paradigms used in modern financial economics. For example, pricing of derivative securities with martingale methods will no longer be valid, since most of the stochastic calculus employed in that field is inconsistent with long memory. Long memory is also inconsistent with usual statistical inference methods which are largely employed to estimating and conducting hypothesis testing in the capital pricing asset model (CAPM). There has been intense debate on the presence of long memory in financial time series. For instance, Greene and Fielitz (1977) used the the R/S statistic and reported that daily returns of 200
individual stocks on the New York Stock Exchange exhibit long memory. Lo (1991) criticized their results and proposed the modified R/S statistic, in which one accounts for short memory in the null model, and obtained no evidence in favor of long memory of the monthly and daily returns on Center for Research in Security Prices (CRSP). Teverovsky et. al. (1999) show that the modified R/S statistic has a strong preference for accepting the null hypothesis of short memory, even when long memory is present in the data. Based on this finding, Willinger et. al. (1999) conducted further analysis and found some evidence of long memory in the CRSP stock return data. Thus, there seems to be no consensus about the presence of long memory in stock returns.

The same lack of consensus is present in the discussion on the existence of long memory in real exchange rates. By examining the statistical properties of the real exchange rate, past studies have tried to draw some implications on the PPP hypothesis. The PPP hypothesis suggests that national price levels of two countries are equalized when expressed in the common currency unit. Even though this relation is not likely to hold in the short run due to price stickiness, it may hold in the long run. The real exchange rate, by definition, is the relative national price level of two countries expressed in common currency units. If the real exchange rate exhibits short-range dependence, deviations from the PPP are transitory and a mean reversion occurs. In such a case, the relative national price level is equalized in the long run and the PPP hypothesis holds in the long run. On the other hand, if the real exchange rate has long memory, then deviations from the PPP will be highly persistent and mean reversion will take longer to occur. The use of fractional models for exchange rate dynamics is quite common. Cheung and Lai (1993), Diebold et. al. (1991) and Cheung and Lai (2001) find mean-reverting long-memory dynamics in series of real exchange rates. On the other hand, using the KPSS test, Culver and Pappel do not reject the null hypothesis of short-range dependence in real exchange rate at the 5% critical value. Although both long memory and short memory models imply PPP reversion in the long run, they yield different reversion speed toward equilibrium. Taylor (2002), emphasizes that the more important and interesting question is to explain and measure amplitude and persistence of deviations from PPP. For this reason, it is important that statistical tests be able to distinguish correctly short-range from long-range dependence. In this paper,
we re-exam many test statistics used in applications and show that the use of
bandwidth procedures that are not robust against long-range dependence may
lead to spurious short-range dependence in financial time series.

There is a large literature in both theory and applications on testing for
long range dependence (Mandelbrot and Wallis (1968, 1969a, b, c), Mandelbrot
(2003), Robinson, Taqqu, Teverovsky and Willinger (1995), Xiao (1999)).
Among various testing procedures, the rescaled variance V/S statistic recently
proposed by Giraitis et al. (2003), the modified R/S test suggested by Lo
(1991), the KPSS statistic derived by Kwiatkowski et al. (1992), and the KS
statistic proposed by Xiao (2001) are widely used in economics and finance. In
all these test statistics, the scaling factor includes the estimate of the long-run
variance. Compared to the conventional variance estimator in classical tests,
the long-run variance estimator is robust to weak dependence under the null.

In practice, the long-run variance of a time series $x_t$ is usually estimated
by the nonparametric kernel method which entail a choice of bandwidth (lag
truncation) parameter $q$ satisfying the property that $q \to \infty$ and $q/n \to 0$ as
the sample size $n \to \infty$. Just like many other statistical procedures that use
nonparametric estimates, the finite sample performance of these tests depends
largely on the choice of bandwidth, although tests using different bandwidth
values are asymptotically equivalent as long as the bandwidth $q$ satisfies a certain
expansion rate. One of the most popular choices is probably the data-dependent
automatic bandwidth

$$q = \mu_k \delta(f, k) n^{1/(2p+1)}, \tag{1}$$

where $\mu_k$ is a constant associated with the kernel function, $\delta(f, k)$ is a function
of the unknown spectral density and is estimated using a plug-in method, and
$p$ is the characteristic exponent of $k$. This bandwidth choice has been stud-
ied by Andrews (1991) in the estimation of a covariance matrix for stationary
time series and is now widely used in econometrics applications. Such a band-
width has the advantage that it partially adapts the serial correlation in the
underlying time series through the data-dependent component $\hat{\delta}(f, k)$. In many
applications, the following AR(1) plug-in estimator is used.

3
\[ q_1 = \left[ \frac{3n}{2} \right]^{1/3} \cdot \left( \frac{2\hat{\rho}}{1 - \hat{\rho}^2} \right)^{2/3} \]  

(2)

where \( \hat{\rho} \) is an estimate of the first order autoregression coefficient.

Lo (1991) reports that the modified R/S test has good empirical size and power if one employs the automatic truncation lag procedure 2. Giraitis et al (2003) affirm that they did not check how the Andrews’ formula works for the KPSS and V/S tests, but “it can be expected that it will yield fairly good results”.

In a recent paper, Xiao (2002) shows that, the Monte Carlo finding of unusually low power and some empirical findings of short-memory in time series are related to the use of the data-dependent bandwidth choice (1) and (2). In this paper, we conduct extensive monte carlo simulations and show that the power of the modified R/S, V/S and KPSS and KS statistics reduces dramatically when the Andrews’ formula is employed. Indeed, either fixed values or fixed functions of \( n \), have been considered in the existing literature. Fixed bandwidth values are not robust to short-term memory and are generally not recommended. For many fixed functions of \( n \), they generally have better performance than fixed bandwidth values when the short-term memory is strong, but have poor performance if the short-range dependence is weak. For this reason, we propose partially data-dependent bandwidth choices. Monte Carlo simulations are conducted and they indicate that, among the existing choices of the bandwidth parameter, the V/S, KPSS, KS and modified R/S tests have the best sample performance when the partial data-dependent truncation lag procedure is adopted.

We re-visit several economic and financial time series using the proposed approach. In particular, we used the same stock return data as in Lo (1991) and concluded that the null hypothesis of short memory cannot be rejected even when the bandwidth choice takes into account the presence of long memory under the alternative. However, we find that evidence against short memory in absolute and squared values of stock returns is much more strong, indicating the presence of long memory in volatility. Long memory in volatility has been extensively reported in the literature (see, e.g., Huang and Yang, 1995; Lobato and Savin, 1998; Lobato and Robinson, 1998; Ohanissian et al. 2003, among others.).

We also investigated the presence of short memory in real exchange rates. We were unable to reject the null of short memory when one calculates the
bandwidth using the Andrews’ formula. However, the use of the Andrews’ formula may induce the arising of spurious short memory. This problem is resolved when we replace the Andrews’ formula by the partially data dependent one and evidence against short memory is found. This result implies that the PPP holds in the long run and that deviations from PPP are highly persistent.

The paper is organized as follows: section 2 presents the V/S, KPSS, KS and the modified R/S statistics. In section 3, we discuss bandwidth selection in testing I(0) versus I(d). Section 4 motivates and introduces the partially data-dependent bandwidth choice. The Monte Carlo experiment is designed and results are presented in section 5. An empirical application using data on exchange rate and stock returns is carried out in section 6. Section 7 concludes.

2 Test Statistics

This section presents three test statistics largely used to test the null hypothesis of short memory against long memory: the rescaled variance V/S statistic recently proposed by Giraitis et al (2003); the KPSS statistic derived by Kwiatkowski et al. (1992); The KS statistic introduced by Xiao (2001); and the modified R/S statistic proposed by Lo (1991).

We show in the sequel that, unlike what was suggested by Giraitis et al. (2003), these tests may deliver low power when the bandwidth choice is based on Andrews’ formula. We will propose an alternative bandwidth choice that will work well with the above mentioned tests. In particular, when this new bandwidth choice is used, the power of the tests will not only increase as \( d \) moves away from zero, but it will also be very high for distant alternatives besides yielding a reasonable empirical size under the null model.

2.1 V/S statistic

Giraitis et al (2003) proposed the following statistic to test the null hypothesis of short memory against \( I(d) \) alternatives, \( 0 < d < 1/2 \).

\[
V/S = \frac{1}{(\hat{\omega}_n)^2} \left[ \sum_{k=1}^{n} \left( \sum_{j=1}^{k} (X_j - \overline{X}_n) \right)^2 - \frac{1}{n} \left( \sum_{k=1}^{n} \sum_{j=1}^{k} (X_j - \overline{X}_n) \right)^2 \right]
\]
where $\hat{\omega}^2$ is an nonparametric estimator of the long run variance and $\overline{X}_n$ is the sample mean $n^{-1} \sum_{j=1}^{n} X_j$. The $V/S$ statistic can be re-written as

$$V/S = n^{-1} \frac{\hat{Var}(S_1, S_2, ..., S_n)}{\hat{\omega}^2}$$

(3)

where $S_k = \sum_{j=1}^{k} (X_j - \overline{X}_n)$ are the partial sums of the observations and $\hat{Var}(S_1, S_2, ..., S_n)$ is the sample variance of the partial sums.

Under regularity conditions, Giraitis et al (2003) derived the asymptotic distribution of the $V/S$ statistic under the null hypothesis of short memory and the alternative hypothesis of long-memory. Under $H_0$, the $V/S$ statistic has the following behavior

$$V/S \Rightarrow \eta(t)$$

where $\eta(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{(s-t)^2}{2}} ds$ and $W^0(t)$ is the standard Brownian bridge. The distribution function of $\eta(t)$ was derived by Watson (1961) and has the following formula

$$F_{\eta} = 1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{-2k^2 \pi^2 t}.$$  

Giraitis et al (2003) observed that $F_{\eta} = F_K (\pi \sqrt{t})$ where $F_K$ is the asymptotic distribution function of the standard Kolmogorov statistic

$$\sup_{0 \leq t \leq 1} \sqrt{n} \left( \hat{F}_n(t) - t \right).$$

Under the assumption of stationarity with long memory, $V/S \rightarrow \infty$ which yields a consistent test.

2.2 KPSS Statistic

Kwiatkowski, Phillips, Schmidt, and Shin (1992) proposed a test of the null hypothesis of stationarity against unit root. Later, Lee and Schmidt (1996) showed that the KPSS test is consistent against stationary long memory alternatives.

The KPSS statistic is defined as follows

$$KPSS = \frac{1}{(\omega n)^2} \sum_{k=1}^{n} \left( \frac{k}{\sqrt{n}} (X_j - \overline{X}_n) \right)^2$$

(4)

Notice that the KPSS may be seen a rescaled sample uncentered second moment of the partial sums, which differs from the $V/S$ statistic because the
latter corrects for the mean. This imply that the V/S statistic will probably be
more sensitive to shifts in variance and will have higher power than the KPSS
statistic against long memory in squares (Giraitis et al., 2003).

Under the null of short memory, Kwiatkowski et. al. (1992) show that the
KPSS statistic has the following asymptotic behavior

\[ KPSS \Rightarrow \kappa(t) \]

where \( \kappa(t) = \int_0^1 (W^0(t))^2 dt \), which KPSS tabulate. Under the assumption
of stationarity with long memory, Lee and Schmidt (1996) show that \( KPSS \)
diverge in probability to infinity, yielding a consistent test against stationary
long-memory alternatives.

### 2.3 Modified R/S Statistic

Lo (1991) developed a test for long memory that is robust to short memory
dependence. This test is a modification of the rescaled range, or R/S statistic
introduced by Hurst (1951). The modified R/S statistic has the following form

\[
Q_n = \frac{1}{\sigma} \left[ \max_{1 \leq k \leq n} \frac{1}{k} \sum_{j=1}^{k} (X_j - \overline{X}_n) - \min_{1 \leq k \leq n} \frac{1}{k} \sum_{j=1}^{k} (X_j - \overline{X}_n) \right]
\]

Under the null hypothesis of stationarity, Lo (1991) showed that

\[
R/S = \frac{Q_n}{\sqrt{n}} \Rightarrow U_{R/S} = \max_{1 \leq k \leq n} W^0(t) - \min_{1 \leq k \leq n} W^0(t)
\]

The distribution function of the random variable \( U_{R/S} \) was derived by Feller
(1951) and has the following functional form

\[
F_{U_{R/S}}(x) = 1 + 2 \sum_{k=1}^{\infty} \frac{(1 - 4k^2x^2)e^{-2k^2x^2}}{k^2}
\]

This distribution function is used to calculate the critical values of the
modified R/S test. Critical values at various significance level are available
in Table 2 in Lo(1991). Under the alternative hypothesis of stationary long
memory, \( 0 < d < 1/2 \), the R/S statistic diverges in probability to infinity,
yielding a consistent test against stationary long memory alternatives.
2.4 The KS Statistic

Notice that the KPSS statistic uses the Cramér-von Misses measure of the fluctuation in time series \(X_t\). Xiao (2001) proposes testing for stationarity based on the Kolmogoroff-Smirnoff measure of fluctuation, that is:

\[
KS = \max_{1 \leq k \leq n} \frac{1}{\sqrt{n}} \left| \sum_{t=1}^{k} \hat{X}_t - \frac{k}{n} \sum_{t=1}^{n} \hat{X}_t \right|
\]

where \(\hat{X}_t\) is the detrended time series \(X_t\). Under the null hypothesis of short memory and assuming that \(\hat{X}_t\) is the demeaned time series, Xiao (2001) showed that

\[
KS \Rightarrow \sup_{0 \leq r \leq 1} \left| W^0(t) \right|
\]

As any other testing procedures in the short memory context, the assymmetric distribution of \(S\) depends on the limiting function of the deterministic trend. Under the presence of linear deterministic trend

\[
KS \Rightarrow \sup_{0 \leq r \leq 1} \left| W^1(t) \right|
\]

where \(W^1(t)\) corresponds to second-level Brownian bridge (MacNeill, 1978).

The above test was originally proposed in testing stationarity against the alternative of a unit root. However, the test also has power against alternatives of long memory processes. The behavior of the KS test under the alternative of long memory hypothesis is similar to that of the \(R/S\) statistic.

3 Bandwidth Selection

In all tests discussed in Section 2, an estimate of the long-run variance \(\omega\) is needed to standardize the test. The nonparametric kernel estimator of the long-run variance of a time series \(x_t\) has the following form

\[
\hat{\omega}^2 = \sum_{h=-q}^{q} k \left( \frac{h}{q} \right) \hat{\gamma}_{xx}(h), \tag{5}
\]

where \(\hat{\gamma}_{xx}(h)\) is the \(h\)-th order sample autocovariance of \(x_t\), \(k(\cdot)\) is a kernel function, and \(q\) is the bandwidth (lag truncation) parameter satisfying the property
that $q \to \infty$ and $q/n \to 0$ as the sample size $n \to \infty$. For example, when we use the Bartlett kernel, we obtain

$$\hat{a}^2 = \hat{\gamma}_{xx}(0) + 2 \sum_{h=1}^{q} \left(1 - \frac{h}{q+1}\right) \hat{\gamma}_{xx}(h), \quad (6)$$

which is widely used in econometric applications. The finite sample performance of these tests is affected by the choice of bandwidth. In practice, many applications use the data-dependent automatic bandwidth (1) coupled with AR(1) plug-in given by (2).

Simply using the data-dependent bandwidth choice (1) is inappropriate. Notice that the bandwidth choice (1) of Andrews (1991) was derived under a different situation for I(0) time series. In the models that we are considering, the integration order of the time series changes between $H_0$ and $H_1$. The bandwidth selection influences both the size and the power of these tests in finite samples. In choosing the bandwidth, it is necessary that the serial correlation can be captured under the null hypothesis. It is also important that the statistical test be able to discriminate between the null and the alternative hypothesis. To test the I(0) hypothesis against an I(d) alternative, the bandwidth $q$ should be large enough to capture the short-range dependence under the null. On the other hand, it should not be too large that it also captures long-range dependence under the alternatives. Unfortunately, as shown in Xiao (2002), the data-dependent bandwidth (1) not only captures the short-range dependence under the null, but can also (at least partially) capture the long-range dependence under the alternatives. This is reflected on the value of the plug-in component $\hat{\delta}(f,k)$ in (1). As the temporal dependence in $x_t$ increases, the value of the data-dependent $\hat{\delta}(f,k)$ becomes larger and larger. However, there is nothing to prevent $\hat{\delta}(f,k)$ being too large under the alternative!

Under the alternative hypothesis, the divergence rate of the tests discussed in Section 2 and the power of these tests depend on the ratio $n/q$. If the bandwidth value $q$ is too big, the tests will have very low power. Unfortunately, the data-dependent coefficient $\hat{\delta}(f,k)$ adapts to the serial correlation in $X_t$ and often delivers very large bandwidth values under the alternative hypothesis. The tests using such a bandwidth choice generally have very low power even for alternatives that are distant from the null hypothesis. For example, in the case of a unit root, the tests using bandwidth (2) are inconsistent!
A number of other bandwidth choices, including fixed values or fixed functions of $n$, have been considered in the existing literature. Fixed bandwidth values are not robust to short-term memory and are generally not recommended. For many fixed functions of $n$, they generally have better performance than fixed bandwidth values when the short-term memory is strong, but have poor performance if the short-range dependence is weak.

Comparing the existing bandwidth choices, the data-dependent bandwidth (1) generally gives better size properties than other choices, but suffers from low power. Thus, it would be ideal if we could use the data-dependent bandwidth under the null hypothesis (to obtain good size) and a bandwidth that is not too large under the alternative to preserve reasonable power.

Given the above facts, we may consider a partially data-dependent bandwidth choice: The data-dependent plug-in bandwidth $\hat{\mu}_k \delta(f,k)n^{1/(2q+1)}$ coupled with an upper bound. We have two choices: we may either impose the upper bound on the data-dependent coefficient $\mu_k \delta(f,k)$; or impose the upper bound to the bandwidth itself. Our first choice impose the upper bound on the data-dependent coefficient $\mu_k \delta(f,k)$

$$M^*_1 = \min\{\mu_k \delta(f,k), A\} n^{1/(2q+1)},$$

where $A$ is an upper bound. We may choose $A$ to be a fixed number, say $A = 5$; or, corresponding to the kernel function, we may simply choose $A = \delta^*(f,k)$ where $\delta^*(f,k)$ is calculated based on a pre-determined large value of $\rho$.

In our second choice we impose the upper bound to the bandwidth $\mu_k \delta(f,k)n^{1/(2q+1)}$ itself. The upper bound is a fixed increasing function of the sample size $n$ and the suggested bandwidth has the following form:

$$M^*_2 = \min\{\mu_k \delta(f,k)n^{1/(2q+1)}, B(n)\},$$

where $B(n)$ is an upper bound function, say $4(n/100)^{1/3}$ or $8(n/100)^{1/4}$. When the serial correlation is weak function, say $4(n/100)^{1/3}$ or $8(n/100)^{1/4}$ generally has a smaller value than $B(n)$ and $M^*_2$ is determined by the data-dependent formula $\mu_k \delta(f,k)n^{1/(2q+1)}$ and gives better size than fixed bandwidth or fixed functions of $n$. Under the alternative hypothesis, $\delta(f,k)$ is generally very large. In this case the upper bound function $B(n)$ prevents $M^*_2$ from being too big and thus retains reasonable power.
Notice that the above suggested bandwidth choices are not fully data-dependent and still involves choosing an upper bound $A$ or $B(n)$. Again, choosing the upper bound function involves the trade-off between size and power and depends on a specific criterion, and there is no widely accepted criterion for this aspect. However, compared with the existing bandwidth choices, the proposed partially data-dependent bandwidth provides a better trade-off between robustness and efficiency. Monte Carlo evidence indicates that existing formulae like $[4(n/100)^{1/3}]$, $[8(n/100)^{1/4}]$ are reasonable candidates for the upper bound. The partially data-dependent bandwidth gives better size than simple fixed functions of $n$ and retains respectable power, improving upon the data-dependent bandwidth choice (1).

**Remark 1** We may consider lower bound as well, but the lower bound is not as important as the upper bound. The following is a form of the lower bound:

$$M_0^* = \max \left\{ \min \{ \mu_k \delta(f, k), A \}, B \right\} n^{1/(2q+1)},$$

we may choose, say, $A = 5$ and $B = 0.5$.

## 4 Monte Carlo Experiments

We conducted Monte Carlo experiments to investigate finite sample performance of the modified R/S test, the V/S test, KPSS and the KS test under different bandwidth choices. The data were generated by using functions of the Arfima package (Doornik and Ooms, 2001) for Ox programming language. In particular, we generate sequences of ARFIMA(1,d,0) process, $(1-L)^d X_t = \mu_t$, where $(1-\phi L)\mu_t = \epsilon_t$, in which $\epsilon_t \sim iid N(0,1)$, $|\phi| < 1$, and $0 \leq d < 1/2$. The experiment has 5000 replications and considered two sample sizes: $n = 1000$ and $n = 4000$. In order to investigate the power of 5% tests, we considered $\phi = 0.5$, and $d = 0.1, 0.4, 0.49$, and $0.495^1$.

As for the value of the truncation parameter ($\eta$), we considered two choices:

(i) the Andrews’ (1991) formula

$$q_1 = \left( \frac{3n}{2} \right)^{1/3} \cdot \left( \frac{2\hat{\phi}}{1-\hat{\phi}^2} \right)^{2/3}$$

$^1$We also considered $\phi = 0$ and $\phi = 0.9$, but the results were not very different.
where $\hat{\rho} = (\sum X_{t-1}^2)^{-1}(\sum X_{t-1} X_t)$, and $[x]$ is the largest integer less than $x$.

(ii) the partially data-dependent bandwidth choice, in which we impose an upper bound on the bandwidth $\mu_k \delta(f, k)n^{1/(2q+1)}$ itself. The upper bound is a fixed increasing function of the sample size $n$. In other words, we considered the following choice

$$q_2 = \min\{M_1, B(n)\},$$

and $B(n) = [4(n/100)^{1/3}].$ Notice that $q_2$ is numerically equivalent to:

$$q_2^* = \left( \frac{3n}{2} \right)^{1/3} \cdot \left( \frac{2\rho^*}{1 - \rho^*} \right)^{2/3}$$

where $\rho^* = \min(\hat{\rho}, A)$ and $A = 0.3$. Therefore, the above formula imposes an upper bound on the first order autoregressive coefficient estimator $\hat{\rho}$. We only report the results for $q_2$.

### 4.1 Results

We present in this section the results of a simulation study examining the finite sample performance of the V/S, KPSS, KS and modified R/S statistics. The tables below show the percentage of replications in which the rejection of a short memory null hypothesis was observed. We first present the results on the empirical power and then we turn to the empirical size of 5% tests.

#### 4.1.1 Power of Test

Table 1 presents the 5% power of the modified R/S, V/S, KPSS and KS tests. We notice that: (i) the power of each test decreases as $d \to 1/2$ and $q_1$ is used. This happens because $q_1$ captures the long-range dependence under the alternatives; (ii) power of each test increases as $d \to 1/2$ and $q_2$ is used. In other words, our partially data-dependent formula $q_2$ always generates reasonable power because it captures the good performance of $q_1$ when the alternative is near the null model and the good performance of $B(n)$ when $H_1$ is distant from $H_0$; (iii) V/S and modified R/S tests have more power than KPSS and KS tests when $q_2$ is employed.
Table 1: Power of Tests

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<td>0.270</td>
</tr>
<tr>
<td></td>
<td>KSq₂</td>
<td>0.307</td>
<td>0.625</td>
<td>0.950</td>
<td>0.982</td>
</tr>
</tbody>
</table>

Giraitis et al. (2003) conducted Monte Carlo simulations to compare the empirical power of modified R/S, V/S and KPSS tests under different fixed values of q. They concluded that the V/S has always better power than the KPSS statistic, for all fixed values of q considered. Tables 1 shows that the V/S test has more power than the KPSS and KS tests when both q = q₁ and q = q₂ are employed. Another result reported by Giraitis et al. (2003) is that the V/S statistic has more power than the modified R/S statistic for large values of q. Since q₁ → ∞ as d → 1/2 or φ → 1, the finding of Giraitis et al. is also valid if one employs the not recommended Andrews’ formula, q = q₁. On the other hand, if one uses q₂, the V/S statistic is as powerful as the modified R/S. In sum, our results indicate that, among the four test statistics considered, if q₂ is chosen to be the lag truncation procedure, then, in terms of power, the V/S and modified R/S statistic will dominate the KPSS and KS statistics. These result extend the findings of Giraitis et al (2003) for a bandwidth choice that is robust under the alternative hypothesis of long memory.
4.1.2 Size of Test

Table 2 shows the empirical size of 5% tests. We first notice that for all sample sizes and truncation lags considered, the modified R/S statistic is more conservative than the V/S, KPSS and KS statistics for all values of $\phi$. When the short memory dependence gets strong, that is, when $\phi$ gets distant from zero and $q_2$ is used, the empirical size of all three statistics becomes larger than the nominal size, but the difference between empirical and nominal size seems to diminish when the sample size increases. For example, if $\phi = 0.5$ and $q = q_2$, when the sample size increases from 1000 to 4000, the empirical size of the V/S statistic goes from 0.076 to 0.066. In general, the empirical sizes of all four test statistics are not very different from one another when $q = q_2$.

Table 2: Size of Tests

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.0</th>
<th>0.3</th>
<th>0.5</th>
</tr>
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<tr>
<td>Sample size = 1000</td>
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<tr>
<td>R/S$q_1$</td>
<td>0.020</td>
<td>0.042</td>
<td>0.039</td>
</tr>
<tr>
<td>R/S$q_2$</td>
<td>0.020</td>
<td>0.042</td>
<td>0.057</td>
</tr>
<tr>
<td>V/S$q_1$</td>
<td>0.023</td>
<td>0.059</td>
<td>0.061</td>
</tr>
<tr>
<td>V/S$q_2$</td>
<td>0.023</td>
<td>0.059</td>
<td>0.076</td>
</tr>
<tr>
<td>KPSS$q_1$</td>
<td>0.023</td>
<td>0.058</td>
<td>0.060</td>
</tr>
<tr>
<td>KPSS$q_2$</td>
<td>0.023</td>
<td>0.058</td>
<td>0.065</td>
</tr>
<tr>
<td>KS$q_1$</td>
<td>0.021</td>
<td>0.053</td>
<td>0.056</td>
</tr>
<tr>
<td>KS$q_2$</td>
<td>0.021</td>
<td>0.053</td>
<td>0.063</td>
</tr>
<tr>
<td>Sample size = 4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R/S$q_1$</td>
<td>0.021</td>
<td>0.050</td>
<td>0.048</td>
</tr>
<tr>
<td>R/S$q_2$</td>
<td>0.021</td>
<td>0.050</td>
<td>0.060</td>
</tr>
<tr>
<td>V/S$q_1$</td>
<td>0.024</td>
<td>0.055</td>
<td>0.056</td>
</tr>
<tr>
<td>V/S$q_2$</td>
<td>0.024</td>
<td>0.055</td>
<td>0.066</td>
</tr>
<tr>
<td>KPSS$q_1$</td>
<td>0.025</td>
<td>0.057</td>
<td>0.060</td>
</tr>
<tr>
<td>KPSS$q_2$</td>
<td>0.025</td>
<td>0.057</td>
<td>0.065</td>
</tr>
<tr>
<td>KS$q_1$</td>
<td>0.023</td>
<td>0.052</td>
<td>0.055</td>
</tr>
<tr>
<td>KS$q_2$</td>
<td>0.023</td>
<td>0.052</td>
<td>0.060</td>
</tr>
</tbody>
</table>

5 Empirical Results

In this section, we investigate the presence of long-memory in financial time series. We considered data on stock returns from the Center for Research in Security Prices (CRSP) daily files. We included the same period considered by Lo (1991). Specifically, tests are performed for the value-and-equal-weighted CRSP
indexes labelled as VVRETD and EWRETD, respectively. Daily observations range from 3 July 1962 to 31 December 1987, totalizing 6,409 observations. We also considered real exchange rate data. We used monthly data of six bilateral real exchange rates: Japan-USA, United Kingdom-USA, Germany-USA, Germany-United Kingdom, Germany-Japan and Japan-United Kingdom. To construct the real exchange rate, the data on the nominal exchange rate and the price level (Consumer Price Index) are collected from the International Financial Statistics CD-ROM, which is made by the International Monetary Fund (IMF). The sample covers the Post-Bretton Woods period that runs from April 1973 to March 2001.

For each set of tests, we considered three lag truncation procedures: a lag truncation equals to zero; the Andrews’ formula $q_1$; and our partially data dependent choice, $q_2$.

Table 3 displays the results for real exchange rates. We notice that the null hypothesis of short memory is easily rejected when $q = 0$. As mentioned in Lo (1991), the classical tests, in which the long-run variance is replaced by the conventional variance estimator are not robust to the presence of short memory under the null and, consequently, we rejects $H_q$ too often when $q = 0$. Hence, we proceed by adopting the Andrews’ formula, $q_1$, which turns out to be robust against short memory in the null model. As you can see in table 3, the null hypothesis of short memory cannot be rejected at all, excluding the possibility that real exchange rate exhibit long range dependence. Remember that, although robust against short memory, the Andrews’ procedure is not robust under the alternative model. Notice that the first order autoregressive coefficient estimator, $\hat{\rho}$, is too close to unity, suggesting the absence of short memory. Consequently, $q_1$ will be too big causing a dramatic reduction in the power of the tests. The presence of short memory in exchange rate detected when $q_1$ is used may be, in fact, a case of spurious short memory. Thus, the practitioner is led to believe that deviations from PPP are not persistent when $q_1$ is used. If our partially data dependent choice $q_2$ is employed, then the V/S, KPSS, KS and modified R/S statistics become robust against departures from short memory. As a consequence, the null hypothesis is easily rejected when $q = q_2$, supporting the view that deviations from PPP are highly persistent. As an example, if real exchange rate is represented by a fractional white noise

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process, then its impulse response function \( IR_k = \frac{\partial y_k}{\partial u_1} \approx 1/(k^{1-d}) \) for \( 0 < d < 1 \). So, the impact of the innovations vanishes in the long run, but vanishes very slow.

Table 3: Analysis of Real Exchange Rates

<table>
<thead>
<tr>
<th>Variable</th>
<th>JPN-US</th>
<th>GER-US</th>
<th>UK-US</th>
<th>GER-UK</th>
<th>GER-JPN</th>
<th>JPN-UK</th>
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<tbody>
<tr>
<td>( q = q_0 )</td>
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<td></td>
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</tr>
<tr>
<td>R/S</td>
<td>7.76**</td>
<td>5.28**</td>
<td>4.67**</td>
<td>5.85**</td>
<td>7.58**</td>
<td>6.39**</td>
</tr>
<tr>
<td>V/S</td>
<td>4.95**</td>
<td>2.19**</td>
<td>1.78**</td>
<td>1.93**</td>
<td>4.63**</td>
<td>4.11**</td>
</tr>
<tr>
<td>KPSS</td>
<td>19.60**</td>
<td>2.84**</td>
<td>5.31**</td>
<td>9.30**</td>
<td>24.83**</td>
<td>14.38**</td>
</tr>
<tr>
<td>KS</td>
<td>7.68**</td>
<td>3.80**</td>
<td>4.54**</td>
<td>5.81**</td>
<td>7.50**</td>
<td>6.35**</td>
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<td>( q = q_1 )</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R/S</td>
<td>1.01</td>
<td>1.73</td>
<td>1.17</td>
<td>1.16</td>
<td>0.91</td>
<td>0.98</td>
</tr>
<tr>
<td>V/S</td>
<td>0.08</td>
<td>0.23</td>
<td>0.11</td>
<td>0.07</td>
<td>0.07</td>
<td>0.09</td>
</tr>
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<td>0.37</td>
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<td>KS</td>
<td>1.00</td>
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<td>1.14</td>
<td>1.16</td>
<td>0.90</td>
<td>0.95</td>
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<td>( q = q_2 )</td>
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<tr>
<td>R/S</td>
<td>2.99**</td>
<td>2.05**</td>
<td>1.84**</td>
<td>2.27**</td>
<td>2.93**</td>
<td>2.48**</td>
</tr>
<tr>
<td>V/S</td>
<td>0.73**</td>
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<td>0.27**</td>
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<td>0.70**</td>
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<td>1.77**</td>
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</tr>
<tr>
<td>( \hat{\rho} )</td>
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<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
</tbody>
</table>

The Symbol (***) indicates rejection of the null hypothesis at 5% of significance level

Tables 4, 5 and 6 report results for the equal and value weighted returns indexes and its squared and absolute values for the sample periods considered in Lo (1991): the entire sample period (the third row), two equally-partitioned sub-samples (the next two rows), and four equally-partitioned sub-samples (the next four rows). The last column of the tables include estimates of the first order autoregressive coefficient, \( \hat{\rho} \). One can see that the estimates of the first order autoregressive coefficient of the return and its squared and absolute values are small in all sample periods considered. Consequently, we do not expect expressive differences between the value of \( q_2 \) and the value of the bandwidth parameter obtained by the Andrews’ formula. In other words, the tests will probably deliver high power and will have reasonable size no matter we use \( q_2 \) or \( q_1 \).
Table 4: Analysis of Stock Returns

<table>
<thead>
<tr>
<th>Period</th>
<th>Sample size</th>
<th>R/S₀</th>
<th>V/S₀</th>
<th>KPSSS₀</th>
<th>KS₀</th>
<th>R/S₁</th>
<th>V/S₁</th>
<th>KPSSS₁</th>
<th>KS₁</th>
<th>R/S₂</th>
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<th>KPSSS₂</th>
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<td></td>
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<td></td>
</tr>
<tr>
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<td>6409</td>
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<td>0.26**</td>
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<td>1.63</td>
<td>0.088</td>
<td>0.09</td>
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<td>0.09</td>
<td>0.95</td>
<td>0.39</td>
</tr>
<tr>
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<td>1204</td>
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<td>1.61</td>
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<td>0.37</td>
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<td>0.17</td>
<td>0.41</td>
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<tr>
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<td>1.96</td>
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<td>1.19</td>
<td>1.27</td>
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<td>0.35</td>
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<tr>
<td>07/03/62-12/17/68</td>
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<td>0.64**</td>
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<td>1.34</td>
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</tr>
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<td>0.04</td>
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<td>0.11</td>
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<td>0.09</td>
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</table>

Note: The symbol (**) indicates rejections of the null hypothesis at 5% level of significance.
Table 5: Analysis of Squared Returns

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<td>0.11</td>
<td>0.17</td>
<td>0.97</td>
<td>0.46</td>
</tr>
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</tr>
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<td>2.31**</td>
<td>2.51</td>
<td>0.51**</td>
<td>1.95</td>
<td>2.14</td>
<td>2.12**</td>
<td>0.44**</td>
<td>1.57</td>
<td>2.00</td>
<td>0.32</td>
</tr>
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Note: The symbol (***) indicates rejections of the null hypothesis at 5% level of significance
Table 6: Analysis of Absolute Value of Returns

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Note: The symbol (**) indicates rejections of the null hypothesis at 5% level of significance.

Table 4 shows that when one employs the correct bandwidth parameter (\( p_2 \) or \( q_1 \)), the null hypothesis of short memory cannot be rejected in all sample periods. This result validate the findings of Lo (1991) by considering truncation lag procedures that are robust under the alternative hypothesis of long memory. Using other samples and other statistical tests, the presence of short memory in stock returns indexes has also been reported by Hauser, Kunst, and Reschenhofer, 1994; Huang and Yang, 1995, Lobato and Savin (1998), among others.
Tables 5 and 6 show the results for the squared and absolute values of the stock return indexes. The results below suggest the rejection of the null of weak dependence in squared returns for most of the considered sample periods. The rejection is even stronger for absolute returns. These results suggest the presence of long memory in series of volatility, implying that standard techniques of inference can yield misleading results in volatilities models. For instance, the standard errors for the estimates of the coefficients of conventional ARCH or stochastic volatility models will be imprecise and, therefore, wrong confidence intervals for forecasting will be obtained. Using other samples and statistical tests, long memory in volatility has already been reported in the literature by Lobato and Savin (1998), Lobato and Robinson (1998), Ohanissian et al. (2003), among others. In sum, using a truncation lag procedure that is robust under the alternative hypothesis of long memory, we validate standard results already reported in the literature: returns exhibit short memory whereas more evidence of long memory is observed in squared and absolute values of returns, suggesting the existence of long-memory volatility.

6 Conclusion

This paper attempts to contribute to the advance in the literature of long-range dependence by showing that stationarity tests may have low or no power when an inappropriate bandwidth parameter is used to calculate the long-run variance. In particular, we showed that the use of the Andrews’ formula may produce what we called spurious short memory. A new bandwidth procedure is proposed and Monte Carlo experiments indicate that it is robust against long-memory alternatives. Among the four test statistics studied, we found that the V/S statistic proposed by Giraitis et. al. (2003) and the modified R/S statistic proposed by Lo (1991) have the best trade-off between size and power. We recognize that the problem of choosing the optimal $q$ is not solved yet, but providing a bandwidth procedure that is robust against both the null and alternative models may help practitioners in their investigation on the presence of long range dependence in financial time series. We re-visited several economic and financial time series using the proposed approach and found evidence of spurious short memory in real exchange rates, implying that, if the Andrews’ formula is used, then the practitioner is led to believe that deviations from PPP
are not persistent. Short memory in stock returns and long memory in volatility are also found.

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