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Getulio Vargas

Inflation, Welfare and Public Expenditures

Pedro Cavalcanti Gomes Ferreira

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Pedro Cavalcanti Ferreira*
Graduate School of Economics,
Fundação Getulio Vargas

Abstract

This paper studies welfare effects of monetary policy in an overlapping generations model with capital and no form of taxation other than inflation. Public expenditures have a positive effect on labor productivity. The main result of the paper is that an expansive monetary policy can be welfare improving, at least for "small enough" inflation rates, and that there is an optimal inflation rate. Growth maximization, however, is never optimal. Steady-state capital and output increase with inflation, reproducing the so-called Tobin effect. For large inflation rates, however, the government authorities cannot affect real variables and there are only nominal effects.

1   Introduction

This note presents a simple exercise in inflation, welfare and growth, using a parameterized overlapping generations model with capital in which government expenses positively affect the growth rate of human capital (following Boldrin(1993) and Glomm and Ravikumar(1992), among others) and consequently the growth rate of productivity. In the model it is assumed that money creation is the only source available for the financing of public expenditure, although results would not change qualitatively with the introduction of other distortionary taxes. In this set-up it is shown that inflation does have positive welfare effects by increasing the equilibrium level of private capital and income (a result that reproduces the so-called Tobin effect (e.g., Tobin (1965) and Mundell (1965)) so that price stability is never optimal. Agents end up being better off under inflationary policy because of the impact of public expenditures, financed through seigniorage, on savings and capital accumulation. In a certain sense inflation solves a problem of under-accumulation of capital.

It is also shown that there is an optimal inflation rate (which is always below the rate that maximizes productivity growth). When the rate of money creation is small enough, the positive impact of public expenditures on the economy more than compensates the distortionary effects of inflation. As the rate of inflation increases, seigniorage decreases and so does its impact on growth and capital accumulation. At a certain point the two effects cancel each other out. Hence, there is a non-monotonic relationship between inflation and welfare.

This paper relates to Weiss(1980), Drazen(1981) and other articles if the literature on optimal monetary policy in life-cycle models with production. However, while this literature is mainly theoretical, we present a simulation exercise and the transmission mechanism from inflation to capital and welfare is rather different from these papers, since public expenditure, financed though seigniorage, is a factor of production (more precisely, it enhances labor productivity). The model is presented in the next section, while in Section 3 the simulation results are presented. Some brief concluding remarks are made in Section 4..

2   The Model

Consider an overlapping generations economy with no population growth. Each generation is composed of a large number of individuals who live for two periods, except the first generation, which only lives for one period. In the first period of their lives -"youth" - the individuals are endowed with one unit of labor which they supply inelastically. When young, the individuals work, receive a wage, consume the only good of this economy, and save. In the second and last period of their lives, "old" people do not work, but consume the proceedings of their savings.

There are two different assets in the economy. One is fiat money issued by the government, and the other is a capital asset issued by firms. Money may or may not be valued in equilibrium; and in the latter case individuals will hold only capital in their portfolios. The capital and money levels in the first period (time zero) are given by history.

The budget constraint of a young person born at time t is given by:

$$C_t^y = w_t - S_t$$

and his budget constraint when he is old is:

$$C_{t+1}^0 = S_t R_{t+1} + (\Pi_t - R_{t+1}) M_t / P_t$$

where R_{t+1} is the gross return on capital, Π_t the gross return on money (the inverse of inflation factor), C_t^y and C_{t+1}^0 are the consumption of a young person and a old person born at period t, respectively, S_t stands for savings, M_t is the nominal money demand, P_t the price level and w_t wages. The problem of the

consumer is to maximize his life-time utility by choosing the savings level as well as its distribution between capital and money:

$$Max_{C_t^y, M_t, C_{t+1}^0} (C_t^y)^\gamma (C_{t+1}^o)^{(1-\gamma)} \tag{1}$$

$$s.t.: C_{t+1}^0/R_{t+1} + C_t^y + (\Pi_t/R_{t+1} - 1) M_t/P_t = w_t \tag{2}$$

The first-order conditions for this problem are:

$$\frac{\gamma}{C_t^y} = \frac{(1-\gamma)}{C_{t+1}^0} R_{t+1} \tag{3}$$

$$R_{t+1} > \Pi_t, \quad = \quad if \quad M_t > 0 \tag{4}$$

Equation (3) is the usual Euler equation, while equation (4) is a non-arbitrage condition. Competitive firms maximize profits choosing optimally capital and labor. A Cobb-Douglas production function is assumed, $Y_t = K_t^\alpha L_t^{1-\alpha}$, where K_t is capital stock at time t (which depreciates completely upon use) and L_t the flow of efficiency units of labor of a worker born at time t. Labor force was normalize to one. The first-order conditions of this problem, in efficient units, are given by:

$$w_t = (1-\alpha)k_t^\alpha, \tag{5}$$

$$q_t = \alpha k_t^{\alpha-1}, \tag{6}$$

where q is the rental price of capital. In equilibrium, and under the hypothesis of full depreciation, q_t is equal to R_t.

In this economy, the government budget constraint is $P_t G_t = M_t - M_{t-1}$, where G_t is real government expenditures at time t. Assuming a constant and pre-announced rate of money creation (μ), we obtain $M_t = (1+\mu)M_{t-1}$, which implies

$$g_t = \frac{\mu}{1+\mu} m_t, \tag{7}$$

where g_t is real government expenditures per efficient unit of labor and m_t are real money holdings per efficiency units of labor.

In this economy, public expenditures enhance the productivity of labor, increasing the flow of labor services per unit of time. The idea is that by investing in public education, infrastructure, health services, sanitation, and so on, the government increases the quality of the labor force. In particular, assume that:

$$L_{t+1} = \lambda\left(g_t\right) L_t,\tag{8}$$

where the function $\lambda(g_t)$ is the government expenditure function. It transforms each unit of public investment in infrastructure, by a relative increase of L_{t+1}/L_t in labor productivity. The government expenditure function λ will be of the form $\lambda(g_t) = 2 - exp(-g_t/\phi)$, where ϕ is a real number greater than one.[1] Note that $\lambda(0) = 1$, so that labor productivity does not change when the government investment is zero.

Given the above functional forms, the equilibrium saving function of this economy does not depend on the interest or inflation rate, but only on income, and is given by $(1-\alpha)(1-\gamma)k^\alpha$. The equilibrium in the assets market in efficient units is given by:

$$(1 - \gamma)(1 - \alpha)k_t^\alpha = m_t + \lambda(g_t)k_{t+1}.\tag{9}$$

The dynamic system given by equations (3)-(7) and (9) can be reduced to two equations:

$$\alpha k_{t+1}^{\alpha-1} = \frac{2 - \exp\left(-\frac{\mu}{1+\mu}\frac{m_t}{\phi}\right)}{(1+\mu)},\tag{10}$$

$$(1 - \gamma)(1 - \alpha)k_t^\alpha = m_t + \lambda(g_t)k_{t+1}.\tag{11}$$

At the steady state (actually, the balanced-growth path), equations (10) and (11) become:

$$m = (1 - \alpha)\left(1 - \gamma\right)k^\alpha - \left[2 - \exp\left(-\frac{\mu}{1+\mu}\frac{m}{\phi}\right)\right]k,\tag{12}$$

$$k = \left(\frac{2 - \exp\left(-\frac{\mu}{1+\mu}\frac{m}{\phi}\right)}{(1+\mu)\,\alpha}\right)^{\frac{1}{\alpha-1}}.\tag{13}$$

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[1] In Ferreira(1999) the existence of monetary equilibrium was proved for general functional forms. Results do not depend on this particular form of λ.

In this model, money creation has two opposite effects on the well-being of consumers. The first and usual one is that inflation tax distorts the optimal allocation of the economy. The second effect is that money-creation finances public investment, thus increases the growth rate of output and consumption, and consequently improves consumer utility.

We will study the optimal monetary policy for a government that wants to maximize the utility of its subjects and take their actions as given. The criterion of optimality is the discounted steady-state welfare of the present and all future generations. Hence, the government goal is to choose the rate of money-creation, μ, that maximizes the flow of utility from consumption in the economy. It solves

$$\underset{\mu}{Max} \sum_{t=0}^{\infty} \beta^t (C_t^y)^\gamma (C_t^o)^{(1-\gamma)}.$$

Given that along the balanced growth path all variables grow at a common rate λ, we can rewrite the consumption of the young and the old at time t as $\lambda^t C^y$ and $\lambda^t C^o$. Assuming that the term $(\lambda\beta)^t$ is smaller than one (which will always be the case for all simulations below), the government's problem becomes:

$$\underset{\mu}{Max} \frac{1}{\beta\lambda} (C^y)^\gamma (C^o)^{(1-\gamma)}. \tag{14}$$

Note that C^y and C^o in the above expression stand for the first-period and second-period equilibrium consumption functions, respectively.

3 Results

Simulations were concentrated on steady-state equilibria. Specifically, the behavior of the monetary steady state will be investigated, assuming different combinations of the parameters α, γ and ϕ, when the government changes the monetary policy.

Although the corresponding values are different, the behavior of money and capital vis-a-vis the inflation rate[2] for a wide range of parameters displays a similar pattern. For μ equal or close to zero, the agents hold similar quantities of capital and money on their portfolios. For successively higher rates of inflation, the steady-state level of money decreases until it reaches zero while the capital per efficiency unit of labor increases until the economy reaches the non-monetary steady state. The positive correlation between inflation and capital reproduces the so-called Tobin effect: as inflation increases, agents substitute money for capital in their portfolios until the return on both assets are the same. Overall, savings are higher for higher rates of inflation.

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[2]Note that in this model the inflation rate at the steady state is given by $\{(1+\mu)/\lambda(g) - 1\}$, which is equal to μ only at g = 0. However, it follows μ very closely.

In other worlds, through an inflationary financing scheme the government can stimulate the economy and higher levels of steady-state inflation correspond to higher levels of capital stock. At higher stationary inflation rates, money-demand is very small or null. Thus, there is a bound on the ability of the government to use inflation to boost capital accumulation.

Table I below presents the steady-state level of money holdings for different values of money growth rates, using six combinations of parameters: α equal to one third and one quarter, and γ equal to 0.5, 0.45 and 0.35. For all combinations, ϕ was set to be equal to 10, as changes in this parameter did not significantly affect the results.

Table I						
Steady State Money Stock Levels						
μ	m					
	$\alpha=1/4$			$\alpha=1/3$		
	$\gamma=0.35$	$\gamma=0.45$	$\gamma=0.50$	$\gamma=0.35$	$\gamma=0.45$	$\gamma=0.50$
0.00	0.1496	0.1021	0.0788	0.0611	0.0223	0.0029
0.01	0.1485	0.1010	0.0774	0.0595	0.0205	0.0010
0.02	0.1475	0.1005	0.0761	0.0578	0.0187	1E-06
0.03	0.1463	0.0990	0.0761	0.0562	0.0168	0
0.11	0.1369	0.0887	0.0636	0.0422	0.0013	0
0.12	0.1357	0.0870	0.0622	0.0404	1E-06.	0
0.20	0.1294	0.0751	0.0502	0.0251	0	0
0.32	0.1087	0.0574	0.0311	1E-07	0	0
0.40	0.0968	0.0440	0.0177	0	0	0
0.50	0.0811	0.0276	1E-05	0	0	0
0.64	0.0575	1E-05	0	0	0	0
0.90	0.0097	0	0	0	0	0
0.95	1E-07	0	0	0	0	0

The steady-state value of money holdings falls monotonically with μ for all the combinations of parameters. However, the level at which it will reach zero depends crucially on the values of α and γ. The smaller their values, the higher the inflation rates for which monetary steady state will exist. For the lowest combination (capital share of one quarter and γ equal 0.35) there will be a monetary steady state for rates of money-creation up to 0.95, which corresponds roughly to an inflation rate of 94% per period in the model. On the other hand, for α and γ equal to one third and one half, respectively, there is no monetary steady state for μ larger than 0.02. This is not unexpected because the higher the γ the higher the importance for the individual utility function of consumption in the first period of his life, and consequently the lower the propensity to save. Furthermore, steady-state capital increases with α so that the participation of money in total savings, everything being the same, decreases with this parameter.

The behavior of capital is, in a certain sense, a mirror image of the behavior of money: its level on the steady state is higher for higher values of α and γ, for a given μ, and it increases monotonically with μ and inflation for given capital share and γ. As α and γ fall, there will exist monetary steady states for increasingly higher inflation rates. Consequently, the capital stock level at the non-monetary steady state decreases with α and γ: it is 0.386 for α equal to 0.25 and γ equal to 0.35 and 0.197 for α equal to one third and beta equal to one half.

The welfare effects of money-creation are presented in figure 1 below, where discounted welfare of all generations and growth rates were plotted against inflation rates. The welfare line was obtained from (14) for the different $\mu's$, and $\alpha = 1/4$ and $\gamma = 0.35$ were used in this exercise.
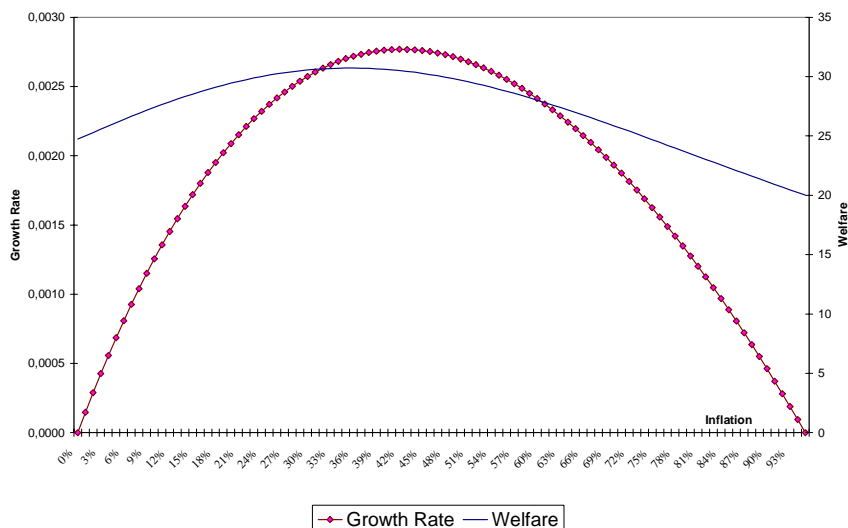

Figure 1: Welfare and growth as functions of inflation

The above result has interesting implications. In a model where government expenditures are not "wasted" or lump-sum transferred, but directly affect the productivity growth of the economy, the best policy always implies some inflation. Positive inflation is always optimal because, up to a certain level, the benefits of money-creation over accumulation are greater than the distortion costs. In these cases, through their impact over productivity, positive inflation rates increase the rate of return of capital and consequently its equilibrium level. This in turn boosts consumption in the first period of life, which dominates the reduction in second-period consumption due to inflation. For higher rates of money-creation, however, the distortionary effect dominates and inflation decreases welfare. The optimal μ is such that these two effects cancel each other out.

The mechanism above can rationalize some inflationary episodes: if the government is unable to increase tax collection (say, for social or political reasons), the importance of public expenses for the economy and their effect over productivity, forces the authorities to resort to money-creation. Even more importantly, it is optimal to do so.

The second implication is that the inflation rate which maximizes the utility of the consumer is lower than that which maximizes growth (35% and 42% respectively). As the rate of money-creation rises, the increase in seigniorage becomes progressively small, and so do the welfare gains from economic growth. At a certain point, the loss due to the distortionary effect of inflation tax overcomes the gains from the growth effect. So if the government wants to maximize the welfare of the present and all future generations, it should consequently operate on the left side of this Laffer curve, below the maximum revenue it can obtain from money creation, and should not maximize economic growth. This result holds for a large number of combinations of γ and α. Barro(1990) obtained a similar result in the context, however, of endogenous-growth models with representative, infinite-lived agents. Moreover, this model has no money but a flat-rate income tax and, in addition to public capital in the production function, there are government consumption services that enter into the household's utility function.

Results would not change qualitatively - but only quantitatively - if any other form of distortionary taxation is introduced in the model. This follows Ramsey's rule, so that given the present functional forms, the optimal tax combination would always imply positive inflation. For instance, with a proportional tax on labor income of 0.25, welfare would still be maximized with positive inflation, but now the optimal rate of money-creation is only 13%, considerably lower than in the case with only inflation tax. However, it remains true that growth maximization is not optimal.

4   Concluding remarks

In this note we have presented a simple model in which optimal monetary policy always implies positive inflation. The basic idea is that if public expenditure is an essential factor of production and seigniorage an important source of revenue, for inflation rates "not large enough", money-creation positively affects steady-state levels of real variables such as capital stock and income, thereby increasing welfare. The result is still valid in the presence of other forms of distortionary taxation. For high inflation rates, however, inflation decreases welfare as its distortionary impact on allocations outweighs the positive effect on capital accumulation.

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