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A family of autoregressive conditional duration models

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Setembro de 2003

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Abstract: This paper develops a family of autoregressive conditional duration (ACD) models that encompasses most specifications in the literature. The nesting relies on a Box-Cox transformation with shape parameter λ to the conditional duration process and a possibly asymmetric shocks impact curve. We establish conditions for the existence of higher-order moments, strict stationarity, geometric ergodicity and β-mixing property with exponential decay. We next derive moment recursion relations and the autocovariance function of the power λ of the duration process. Finally, we assess the practical usefulness of our family of ACD models using NYSE transactions data, with special attention to IBM price durations. The results warrant the extra flexibility provided either by the Box-Cox transformation or by the asymmetric response to shocks.

1 Introduction

The seminal work of Engle and Russell (1998) hoisted a great interest in the implications of price and trade durations in empirical finance. For instance, the modeling of price duration processes hinges the approaches to option pricing and intraday risk management recently proposed by Prigent, Renault and Scaillet (2001) and Giot (2000), respectively. Although Engle and Russell's (1998) autoregressive conditional duration (ACD) model is the starting point of such analyses, the literature carries several extensions.

Bauwens and Giot (2000) work with a logarithmic version of the ACD model that avoids the nonnegativeness constraints implied by the original specification so as to facilitate the testing of market microstructure hypotheses. Bauwens and Veredas (1999) propose the stochastic conditional duration process, leaning upon a latent stochastic factor to capture the unobserved random flow of information in the market. Ghysels, Gouriéroux and Jasiak (2003) introduce the stochastic volatility duration model to cope with higher order dynamics in the duration process. Zhang, Russell and Tsay (2001) argue for a nonlinear version based on self-exciting threshold autoregressive processes.

This paper develops a family of ACD models encompassing most of the existing models in the literature, such as the nonlinear ACD specifications recently put forward by Dufour and Engle (2000). For that purpose, we exploit the common features shared by the ACD and GARCH processes and follow a similar approach taken by Hentschel (1995) to build a family of asymmetric GARCH models. The nesting relies on a Box and Cox's (1964) transformation with shape parameter $\lambda \geq 0$ to the conditional duration process and on an asymmetric response to shocks. The motivation for the

latter stems from Engle and Russell (1998), who show that standard ACD models applied to financial data tend to overpredict after extreme (very long or very short) durations.

We establish sufficient conditions for the existence of higher order moments, strict stationarity, geometric ergodicity and β-mixing property with exponential decay in this class of augmented ACD models. Although there are no general analytical solutions for the autocorrelation function and moments of the duration process, we show that it is possible to derive the autocovariance function and moment recursion relations for the power λ of the duration process. Alternatively, one must restrict attention to particular subclasses, e.g. $\lambda \to 0$ and $\lambda = 1$, in order to work out expressions for any arbitrary moment and the autocovariance function.

We then demonstrate the practical usefulness of our ACD family modeling IBM price durations and other financial durations from stocks actively trading on NYSE. Our findings clearly reject the restrictions imposed by the existing models in the literature. Further, we show that allowing for a concave shocks impact curve is paramount, because it mitigates the problem of overpredicting short durations. It is thus no wonder that we find some sort of substitutability between the Box-Cox transformation and the asymmetric effects given that both may lead to concavity of the shocks impact curve.

The remainder of the paper is organized as follows. Section 2 outlines the statistical properties of the family of augmented ACD processes. Section 3 collects the findings of the empirical application to NYSE transactions data, focusing on IBM price durations. Section 4 summarizes the main results and offers some concluding remarks.

2 The augmented ACD model

Let $x_i = t_i - t_{i-1}$ denote the time spell between two events occurring at times t_i and t_{i-1}. For example, price durations correspond to the time interval needed to observe a certain cumulative change in the stock price, whereas trade durations stand for the time elapsed between two consecutive transactions. To account for the serial dependence that is common to financial duration data, Engle and Russell (1998) formulate the accelerated time process $x_i = \psi_i \epsilon_i$, where the conditional duration process $\psi_i = E(x_i \,|\, \Omega_{i-1})$ is stochastically independent of the iid sequence formed by ϵ_i and Ω_{i-1} is the set including all information available at time t_{i-1}. As in Hentschel (1995), we generalize the ACD processes by applying a Box-Cox transformation with parameter $\lambda \geq 0$ to the conditional duration process ψ_i, giving way to

$$\frac{\psi_i^\lambda - 1}{\lambda} = \omega_* + \alpha_* \, \psi_{i-1}^\lambda \Big[|\, \epsilon_{i-1} - b \,| - c\,(\epsilon_{i-1} - b) \Big]^v + \beta \, \frac{\psi_{i-1}^\lambda - 1}{\lambda}. \tag{1}$$

The shape parameter λ determines whether the Box-Cox transformation is concave $(\lambda \leq 1)$ or convex $(\lambda \geq 1)$.

The augmented autoregressive conditional duration (AACD) model then ensues by rewriting (1) as

$$\psi_i^\lambda = \omega + \alpha \, \psi_{i-1}^\lambda \Big[|\, \epsilon_{i-1} - b \,| + c(\epsilon_{i-1} - b) \Big]^v + \beta \, \psi_{i-1}^\lambda, \tag{2}$$

where $\omega = \lambda \omega_* - \beta + 1$ and $\alpha = \lambda \alpha_*$. The AACD model provides a flexible functional form that permits the conditional duration process $\{\psi_i\}$ to respond in distinct manners to small and large shocks. The shocks impact curve $g(\epsilon_i) = [|\, \epsilon_i - b \,| + c(\epsilon_i - b)]^v$ incorporates such asymmetric responses through the shift and rotation parameters b and c, respectively.

Because durations are nonnegative, the shift parameter b is key to the identification of the asymmetric response implied by the shocks impact

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curve. In turn, the parameter c determines whether rotation is clockwise ($c < 0$) or counterclockwise ($c > 0$). Interestingly, it is not necessarily the case that shift and rotation reinforce each other. Indeed, the shift parameter affects mostly small shocks, whereas rotation is dominant for large shocks. The shape parameter v plays a similar role to λ, inducing either concavity ($v \leq 1$) or convexity ($v \geq 1$) to the shocks impact curve. Figure 1 illustrates the behavior of the shocks impact curve $g(\cdot)$ according to the values of the shift, rotation and shape parameters.

FIGURE 1

The original ACD model of Engle and Russell (1998) is recovered by imposing $\lambda = v = 1$ and $b = c = 0$, whereas letting $\lambda \to 0$ and $b = c = 0$ renders the Box-Cox ACD specification put forward by Dufour and Engle (2000). Further, (1) reduces to Bauwens and Giot's (2000) logarithmic ACD models either if $\lambda \to 0$, $v = 1$ and $b = c = 0$ (Type I) or if $\lambda, v \to 0$ and $b = c = 0$ (Type II). Following the GARCH literature, one may build other conditional duration models by imposing restrictions on (1). The examples we consider in the sequel include the asymmetric logarithmic ACD ($\lambda \to 0$ and $v = 1$), asymmetric power ACD ($\lambda = v$), asymmetric ACD ($\lambda = v = 1$), and power ACD ($\lambda = v$ and $b = c = 0$). Dufour and Engle (2000) independently propose a version of the asymmetric logarithmic ACD model with $b = 1$ under the name of exponential ACD model. We keep our notation because the linear ACD model with exponential distribution is sometimes referred to as the exponential ACD model. Table 1 summarizes the typology of ACD models under consideration.

TABLE 1

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2.1 Properties

In this section, we build heavily on Carrasco and Chen's (2002) general results to establish sufficient conditions that ensure β-mixing and finite higher order moments for (conditional) duration processes belonging to the augmented ACD family. The first step consists in casting (2) into a generalized polynomial random coefficient autoregressive model

$$X_{i+1} = A(e_i)X_i + B(e_i), \qquad i = 0, 1, 2, \ldots \tag{3}$$

where $\{e_i\}$ forms an iid sequence. Next, we apply Mokkadem's (1990) result for polynomial autoregressive models to derive the mixing properties of $\{\psi_i\}$. For the duration process $\{x_i\}$, we take advantage of Carrasco and Chen's result on the mixing properties of a process $Y_i = X_i + \varepsilon_i$, where X_i is a β-mixing homogeneous Markov process and ε_i is an iid noise with a continuous density. These two results are collected in Propositions 2 and 4 of Carrasco and Chen (2002), respectively.

Proposition 1: *Let $x_i = \psi_i \epsilon_i$, where ψ_i satisfies (2) and ϵ_i is an iid random variable that is stochastically independent of ψ_i. Assume further that the probability distribution of ϵ_i is absolutely continuous with respect to the Lebesgue measure on $(0, \infty)$ and such that the density is positive almost everywhere. Suppose that $|\beta| < 1$ and*

$$E \, | \, \beta + \alpha [| \, \epsilon_i - b \, | + c(\epsilon_i - b)]^v \, |^m < 1, \tag{4}$$

for some integer $m > 1$. Then, $\{\psi_i\}$ is a geometrically ergodic Markov process and, if initialized from its ergodic distribution, is also strictly stationary and β-mixing with exponential decay. Further, $E\left(\psi_i^{\lambda m}\right) < \infty$ and $E\left(x_i^{\lambda m}\right) < \infty$. Condition (4) with $m = 2$ is also necessary to entail geometric ergodicity of $\{\psi_i\}$ and $E\left(\psi_i^{2\lambda}\right) < \infty$. Lastly, if initialized from its ergodic

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distribution, $\{x_i\}$ is strictly stationary and β-mixing with exponential decay.

Proof: The first two results follow immediately from Carrasco and Chen's Proposition 2 with $X_i = \psi_i^\lambda$, $A(e_i) = \beta + \alpha \left[|\, \epsilon_i - b \,| + c(\epsilon_i - b) \right]^\upsilon$, and $B(e_i) = \omega$, where $e_i = (|\epsilon_i - b|, \epsilon_i)'$. The need for condition (4) with $m = 2$ stems from Lemma 2 of Pham (1986). The last result follows from Carrasco and Chen's Proposition 4 with $Y_i = \log x_i$, $X_i = \log \psi_i$, and $\varepsilon_i = \log \epsilon_i$. ■

If the interest were only in deriving sufficient conditions for the duration processes x_i to be nondegenerate and covariance stationary, one could alternatively use the tools provided by Nelson (1990, Theorems 1 to 3) as in Hentschel (1995). Actually, for most of the models in the family spanned by the augmented ACD process, the conditions in the proposition above are both necessary and sufficient. The exceptions are formed by the models that ascertain a positive conditional duration even when at least one of the following restrictions are violated: $\omega > 0$, $\alpha > 0$, $\beta > 0$, and $|\, c \,| \leq 1$ for some odd integer υ. For instance, letting $\lambda \to 0$ ensures nonnegativeness of the duration process without imposing further restrictions.

For the sake of completeness, we establish similar properties for the ACD models belonging to the family of augmented ACD processes. At first glance, it seems that it suffices to consider the parametric restrictions implied by each model in condition (4) to extract the corresponding result. That is not the case, though. To derive (4), one must impose restrictions on $A(\cdot)$ and $B(\cdot)$, which vary according to the specification of the model. More specifically, Carrasco and Chen's (2002) results require that $|A(0)| < 1$ and that, for some integer $m \geq 1$, $E|A(e_i)|^m < 1$ and $E|B(e_i)|^m < \infty$.

The generalized polynomial random coefficient autoregressive representation of the asymmetric logarithmic ACD process ensues from $X_i = \log \psi_i$,

$A(e_i) = \beta$, and $B(e_i) = \omega + \alpha[|\epsilon_i - b| + c(\epsilon_i - b)]$, implying that condition (4) becomes $E(\epsilon_i^m) < \infty$. For the asymmetric power ACD model, $A(e_i) = \beta + \alpha[|\epsilon_i - b| + c(\epsilon_i - b)]^\lambda$ and $B(e_i) = \omega$, so that it suffices to impose that $E\left|\beta + \alpha[|\epsilon_i - b| + c(\epsilon_i - b)]^\lambda\right|^m < 1$. The latter condition also holds for the asymmetric ACD model with $\lambda = 1$ and for the power ACD specification with $b = c = 0$. While the Box-Cox ACD process asks for $E(\epsilon_i^{vm}) < \infty$, the logarithmic ACD models of Bauwens and Giot (2000) require either that $E(\epsilon_i^m)$ exists (Type I) or that $|\alpha + \beta| < 1$ and $E|\log \epsilon_i|^m < \infty$ (Type II). As advanced by Carrasco and Chen (2002), in the linear ACD model, condition (4) reduces to $E|\beta + \alpha\epsilon_i|^m < 1$, which is equivalent to assuming that $\alpha + \beta < [E(\epsilon_i^m)]^{-1/m}$ is finite.

2.2 Higher-order moments and autocovariance function

In general, there is no analytical solution for the moments and autocorrelation function of duration processes belonging to the augmented ACD family. Nonetheless, it is possible to derive moment recursion relations and the autocovariance function of the power λ of the duration process by extension of He and Teräsvirta's (1999) results for the family of GARCH models.

To derive the λm-th moment $\mu_{\lambda m}$ of the duration process, we write (2) in its generalized polynomial random coefficient autoregressive representation

$$\psi_i^\lambda = A_{i-1}\psi_{i-1}^\lambda + B, \tag{5}$$

where $B = \omega$ and $A_i = \beta + \alpha\, g(\epsilon_i)$. Raising both sides to the power $m > 0$ and then applying recursions give

$$\begin{aligned}
\psi_i^{\lambda m} &= \left(A_{i-1}\psi_{i-1}^\lambda + B\right)^m \\
&= A_{i-1}^m \psi_{i-1}^{\lambda m} + \sum_{j=1}^m \binom{m}{j} B^j A_{i-1}^{m-j} \psi_{i-1}^{\lambda(m-j)}
\end{aligned}$$

$$= \psi_{i-n}^{m\lambda} \prod_{k=1}^{n} A_{i-k}^{m} + \sum_{j=1}^{m} \binom{m}{j} \sum_{k=1}^{n} B^{j} A_{i-k}^{m-j} \psi_{i-k}^{\lambda(m-j)} \prod_{\ell=1}^{k-1} A_{i-\ell}^{m}. \qquad (6)$$

We are now ready to state the next proposition that documents moment recursion relations for the augmented ACD class of processes.

Proposition 2: *Let $x_i = \psi_i \epsilon_i$, where ψ_i satisfies (5) with $0 < EA_i^m < 1$ and $\{\epsilon_i\}$ is an iid process stochastically independent of $\{\psi_i\}$. Assume further that the process started at some finite value infinitely many periods ago. It then follows that*

$$\mu_{\lambda m} = \frac{E\epsilon_i^{\lambda m}}{1 - EA_i^m} \sum_{j=1}^{m} \binom{m}{j} \frac{B^j EA_i^{m-j}}{E\epsilon_i^{\lambda(m-j)}} \mu_{\lambda(m-j)} \qquad (7)$$

for some integer $m \geq 1$ and $\mu_0 = 1$.

Proof: Because the process started at some finite value infinitely many periods ago, we can let $n \to \infty$ in (6) and take expectations, resulting in

$$E\psi_i^{\lambda m} = E\psi_i^{m\lambda} (EA_i^m)^n + \sum_{j=1}^{m} \binom{m}{j} B^j EA_i^{m-j} E\psi_i^{\lambda(m-j)} \sum_{k=1}^{n} (EA_i^m)^{k-1}$$

$$= \frac{1}{1 - EA_i^m} \sum_{j=1}^{m} \binom{m}{j} B^j EA_i^{m-j} E\psi_i^{\lambda(m-j)}.$$

To complete the proof, it suffices to observe that ψ_i and ϵ_i are stochastically independent, and hence $\mu_{\lambda m} = E\epsilon_i^{\lambda m} E\psi_i^{\lambda m}$. ∎

Before moving to the autocovariance function of the power λ of the duration process, two remarks are in order. First, assuming that $0 < EA_i^m < 1$ is analogous to imposing condition (4) in Proposition 1. Second, the moment recursion relation in (7) involves moments that are possibly of fractional order. Unfortunately, it is not possible to derive expressions for a moment of an arbitrary order for such a general family of processes without restricting the shape parameter λ of the Box-Cox transformation of the conditional duration process. For instance, imposing linearity ($\lambda = 1$) suffices to extract

a recursion relation involving moments of any integer order. Alternatively, one could also consider the subclass of conditional duration processes determined by the limiting case $\lambda \to 0$. We follow the latter approach in the end of this section in view that the log-transformation of the duration process is quite convenient for avoiding nonnegativeness constraints.

Proposition 3: *Let $x_i = \psi_i \epsilon_i$, where ψ_i satisfies (5) with $0 < EA_i^m < 1$ for some integer $m \geq 2$. Let ϵ_i form an iid sequence stochastically independent of $\{\psi_i\}$ such that $EA_i \epsilon_i^\lambda$ is finite. It then follows that the autocovariance function $\gamma_{\lambda,n} = Ex_i^\lambda x_{i-n}^\lambda - \mu_\lambda^2$ of order $n \geq 1$ reads*

$$\gamma_{\lambda,n} = \frac{B^2 E\epsilon_i^\lambda}{1 - EA_i} \left[\sum_{j=0}^{n-1} (EA_i)^j + \frac{(EA_i)^{n-1} EA_i \epsilon_i^\lambda (1 + EA_i)}{1 - EA_i^2} - \frac{E\epsilon_i^\lambda}{1 - EA_i} \right]. \quad (8)$$

Proof: Multiplying both sides of (5) by ψ_{i-n}^λ yields

$$\psi_i^\lambda \psi_{i-n}^\lambda = \left(A_{i-1} \psi_{i-1}^\lambda + B \right) \psi_{i-n}^\lambda$$

$$= \left[A_{i-1} \left(A_{i-2} \psi_{i-2}^\lambda + B \right) + B \right] \psi_{i-n}^\lambda$$

$$= \left(B + B \sum_{j=2}^{n} \prod_{k=1}^{j-1} A_{i-k} + \prod_{k=1}^{n} A_{i-k} \psi_{i-n}^\lambda \right) \psi_{i-n}^\lambda.$$

Multiplying now both sides by $\epsilon_i \epsilon_{i-n}$ and then taking expectations give

$$Ex_i^\lambda x_{i-n}^\lambda = E\epsilon_i^\lambda B \left[1 + \sum_{j=2}^{n} (EA_i)^{j-1} \right] E\psi_i^\lambda + E\epsilon_i^\lambda (EA_i)^{n-1} EA_i \epsilon_i^\lambda E\psi_i^{2\lambda}$$

$$= E\epsilon_i^\lambda \left[B E\psi_i^\lambda \sum_{j=0}^{n-1} (EA_i)^j + (EA_i)^{n-1} EA_i \epsilon_i^\lambda E\psi_i^{2\lambda} \right].$$

The result then ensues from the fact that equation (6) implies that the first and second moments of ψ_i^λ are respectively $E\psi_i^\lambda = B/(1 - EA_i)$ and $E\psi_i^{2\lambda} = B^2(1 + EA_i)/[(1 - EA_i)(1 - EA_i^2)]$, whereas the moment recursion relation in (7) gives $\mu_\lambda = BE\epsilon_i^\lambda/(1 - EA_i)$. ∎

As an example, consider the linear ACD process with an exponential noise introduced by Engle and Russell (1998), which results from $\lambda = 1$, $A_i = \beta + \alpha\epsilon_i$ and $B = \omega$. Proposition 2 then implies that

$$\mu_m = \frac{\Gamma(m+1)}{1 - E(\beta + \alpha\epsilon_i)^m} \sum_{j=1}^{m} \binom{m}{j} \frac{E(\beta + \alpha\epsilon_i)^{m-j}}{\Gamma(m-j)} \, \omega^j \mu_{m-j}$$

provided that $\alpha + \beta < 1$. Solving for $m = 1$ and $m = 2$ yields the first two moments as derived in Engle and Russell (1998). In turn, it follows from Proposition 3 that the autocovariance function of order n reads

$$\gamma_n = \omega^2 \left\{ \frac{1 - (\alpha+\beta)^n}{1 - (\alpha+\beta)} + \frac{(\alpha+\beta)^{n-1}(1+\alpha+\beta)(2\alpha+\beta)}{[1 - (\alpha+\beta)]\,[1 - (\alpha+\beta)^2 - \alpha^2]} \right\}.$$

This expression provides a sharper result than Bauwens and Giot's (2000) recursive formula for computing the autocovariance function of a linear ACD process with exponential errors.

We now focus on a particular subclass of the augmented ACD family that permits working out expressions for any arbitrary moment as well as the autocorrelation function. This subclass is determined by shrinking the Box-Cox shape parameter to zero ($\lambda \to 0$), yielding

$$\log \psi_i = \omega + \alpha \, g(\epsilon_{i-1}) + \beta \log \psi_{i-1}. \tag{9}$$

This subclass is particularly interesting for ensuring that the duration process is always positive regardless of the sign and magnitude of the parameters. In particular, it nests the asymmetric logarithmic ACD model, Bauwens and Giot's (2000) logarithmic ACD specifications, and the Box-Cox ACD process put forward by Dufour and Engle (2000). He, Teräsvirta and Malmsten (2002) derive analogous results for a class of exponential GARCH models.

To derive the m-th moment $\mu_m = E x_i^m$ of the duration process, it is convenient to write equation (9) in the exponential form. Raising both sides

to the power $m > 0$ and then applying recursions give

$$\psi_i^m = \exp\left[m\,\omega + m\,\alpha\,g(\epsilon_{i-1})\right]\psi_{i-1}^{m\,\beta}$$

$$= \exp\left(m\,\omega\sum_{k=0}^{n-1}\beta^k\right)\prod_{k=1}^{n}\exp\left[m\,\alpha\,\beta^{k-1}g(\epsilon_{i-k})\right]\psi_{i-n}^{m\,\beta^n}.$$

Assuming that $E\left\{\exp\left[\kappa\,g(\epsilon_i)\right]\right\} < \infty$ for $\kappa \in (0,\infty)$ and that $|\beta| < 1$, yields

$$E\psi_i^m = \exp\left(m\,\omega\,\frac{1+\beta^n}{1-\beta}\right)\prod_{k=1}^{n}E\left\{\exp\left[m\,\alpha\beta^{k-1}g(\epsilon_i)\right]\right\}E\psi_i^{m\,\beta^n}. \qquad (10)$$

We are now ready to state the next result that reports the m-th moment of the duration process defined in (9).

Corollary 1: *Let $x_i = \psi_i\epsilon_i$, where ψ_i satisfies (9) with $|\beta| < 1$ and $\{\epsilon_i\}$ is an iid process stochastically independent of $\{\psi_i\}$. Assume that the process started at some finite value infinitely many periods ago. If both $E\epsilon_i^m$ and $E\left\{\exp\left[m\,\alpha\,g(\epsilon_i)\right]\right\}$ are finite for some integer $m \geq 1$, it then follows that*

$$\mu_m = E\epsilon_i^m\exp\left[m\,\omega(1-\beta)^{-1}\right]\prod_{k=1}^{\infty}E\exp\left\{\left[m\,\alpha\beta^{k-1}g(\epsilon_i)\right]\right\}. \qquad (11)$$

Proof: Because the process started at some finite value infinitely many periods ago, we can let $n \to \infty$ in (10), resulting in

$$E\psi_i^m = \exp\left[m\,\omega(1-\beta)^{-1}\right]\prod_{k=1}^{\infty}E\left\{\exp\left[m\,\alpha\beta^{k-1}g(\epsilon_i)\right]\right\}.$$

The result then follows from the fact that ψ_i and ϵ_i are stochastically independent. ∎

Next we move to the autocovariance function of duration processes in the $(\lambda \to 0)$-subclass of augmented ACD models. As before, the exponential form of (9) facilitates the task.

Corollary 2: *Let $x_i = \psi_i\epsilon_i$, where ψ_i satisfies (9) with $|\beta| < 1$ and is stochastically independent of the iid process $\{\epsilon_i\}$. Assume further that both*

$E\left\{\exp\left[\alpha\,g(\epsilon_i)\right]\right\}$ *and* $E\left\{\epsilon_i\exp\left[\alpha\,g(\epsilon_i)\right]\right\}$ *are finite. It then follows that the autocovariance function* $\gamma_n = Ex_ix_{i-n} - \mu_1^2$ *of order* $n \geq 1$ *reads*

$$\gamma_n = E\epsilon_i E\left\{\epsilon_i \exp\left[\alpha\beta^{n-1}g(\epsilon_i)\right]\right\} \prod_{k=1}^{n-1} E\left\{\exp\left[\alpha\beta^{k-1}g(\epsilon_i)\right]\right\}$$

$$\times \prod_{k=1}^{\infty} E\left\{\exp\left[\alpha\left(1+\beta^n\right)\beta^{k-1}g(\epsilon_i)\right]\right\}\exp\left(\frac{2\omega}{1-\beta}\right) - \mu_1^2. \qquad (12)$$

Proof: Consider the exponential form of the conditional duration process (9), then

$$\psi_i\psi_{i-n} = \exp\left(\omega\sum_{k=0}^{n-1}\beta^k\right)\prod_{k=1}^{n}\exp\left[\alpha\beta^{k-1}g(\epsilon_{i-k})\right]\psi_{i-n}^{\beta^n+1},$$

which means that

$$x_ix_{i-n} = \epsilon_i\epsilon_{i-n}\exp\left(\omega\sum_{k=0}^{n-1}\beta^k\right)\prod_{k=1}^{n}\exp\left[\alpha\beta^{k-1}g(\epsilon_{i-k})\right]\psi_{i-n}^{\beta^n+1}.$$

Taking expectations in both sides yields (12). ∎

3   Empirical application

In this section, we estimate different ACD specifications using IBM price durations at the New York Stock Exchange (NYSE) from September to November 1996. Data were kindly provided by Luc Bauwens and Pierre Giot, who have formed a broad data set using the NYSE's Trade and Quote database. We define price duration as the time interval needed to observe a cumulative change in the mid-price of at least \$0.125 as suggested by Giot (2000). Price durations are closely tied to the instantaneous volatility of the mid-quote price process (Engle and Russell, 1997 and 1998); hence it is not surprising that they may have serious implications to option pricing (Prigent et al., 2001) and intra-day risk management (Giot, 2000).

Apart from an opening auction, NYSE trading is continuous from 9:30 to 16:00. Overnight spells, as well as durations between events recorded outside

the regular opening hours of the NYSE, are removed. As documented by Giot (2000), durations feature a strong time-of-day effect. We therefore consider diurnally adjusted durations $x_i = D_i/\varrho(t_i)$, where D_i is the plain duration in seconds and $\varrho(\cdot)$ denotes the diurnal factor determined by first averaging the durations over thirty minutes intervals for each day of the week and then fitting a cubic spline with nodes at each half hour. The resulting (diurnally adjusted) durations serve as input for the remainder of the analysis.

Table 2

Table 2 describes the main statistical properties of the IBM price durations. We compute descriptive statistics for both plain and diurnally adjusted data. It takes on average 4.4 minutes for a cumulative price change of $0.125 to take place, though the median waiting time is much lesser than 2 minutes. Overdispersion is robust to the time-of-day effect, thus it is not an artifact due to data seasonality. Sample autocorrelations reveal that persistence is slightly changed if one accounts for the diurnal factor. Altogether, the combination of overdispersion and autocorrelation in the price durations warrants the estimation of autoregressive conditional duration models.

We then estimate by maximum likelihood the ACD models listed in Table 1 assuming that ϵ_i is iid with Burr density

$$f_B\left(\epsilon_i;\ \theta_B\right) = \frac{\kappa\,\mu_{B,1}^{\kappa}\,\epsilon_i^{\kappa-1}}{\left(1 + \gamma\,\mu_{B,1}^{\kappa}\,\epsilon_i^{\kappa}\right)^{1+1/\gamma}}, \tag{13}$$

where $\kappa > \gamma > 0$ and

$$\mu_{B,m} \equiv \frac{\Gamma(1 + m/\kappa)\,\Gamma(1/\gamma - m/\kappa)}{\gamma^{1+m/\kappa}\,\Gamma(1 + 1/\gamma)}$$

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denotes the m-th moment, which exists for $m < \kappa/\gamma$. The Burr family encompasses both the Weibull ($\gamma \to 0$), exponential ($\gamma \to 0$ and $\kappa = 1$), and log-logistic ($\gamma \to 1$) distributions.

TABLE 3

Tables 3 and 4 report respectively the estimation results for the existing models in the literature and the novel specifications. Asymptotic standard errors are based on the outer-product-of-the-gradient (OPG) estimator of the information matrix since the absolute value function in the shocks impact curve makes Hessian-based estimates tricky to compute due to numerical problems.

TABLE 4

It is interesting to observe that the estimates of the Burr parameters κ and γ are quite robust regardless of the specification of the duration process. They imply that the baseline hazard rate function is nonmonotonic and that there are at most three finite moments in view that $\hat{\kappa}/\hat{\gamma} \in [2.7173, 3.0438]$. The parameter estimates of the linear and logarithmic ACD models are very much in line with the previous results in the literature (see columns ACD, LACD I and LACD II, respectively). Interestingly, the log-likelihood value of the logarithmic ACD Type I model substantially differs from the values of the linear and logarithmic ACD Type II specifications. The asymmetric logarithmic ACD model with $b = 1$ introduced by Dufour and Engle (2000) palpably increases the log-likelihood value (-4,920.5 versus -4,950.5), suggesting that asymmetry may play a role (see column EXACD). The last column BCACD shows however that letting the power υ of ϵ_{i-1} free to vary in the logarithmic ACD processes amplifies even more the log-likelihood

value than introducing asymmetric effects. Indeed, in the Box-Cox ACD model, $\hat{v}$ is significantly different from both zero and one, lending some support against the logarithmic ACD Type I and II models, respectively.

In the power ACD specification, we notice that the shape parameter λ of the Box-Cox transformation is also significantly different from both zero and one (see column PACD). This indicates that the restrictions imposed by the linear and the logarithmic ACD Type I models seem inconsistent with the data, even though the latter is only marginally inferior to the power ACD model in terms of log-likelihood value. Introducing an asymmetric effect to the power ACD specification ameliorates only marginally the fit of the model (see column A-PACD). Despite the fact that b is significantly different from zero, the standard error of c is quite large, showing that the shocks impact curve features no rotation. Although both shift and rotation parameters are significant in the asymmetric ACD specification (see column A-ACD), it violates the constraints usually imposed to ensure the nonnegativeness of the duration process, namely $\alpha > 0$ and $|c| < 1$. The A-LACD column shows that all parameters are significantly different from zero in the asymmetric logarithmic ACD model. In particular, given that $\hat{\alpha}$ is negative, the shift and rotation effects are such that the shocks impact curve is concave.

The figures displayed in the column AACD demonstrate that the double Box-Cox transformation ($\lambda \neq v$) brings about further improvements as indicated by the value of the log-likelihood of the augmented ACD model. The difference between $\hat{v}$ and $\hat{\lambda}$ is striking. Indeed, there is strong evidence supporting that λ converges to zero (i.e. the log transformation), whereas $0.1310 < \hat{v} < 0.5178$ with 99% of confidence. The fact that $\hat{\lambda}$ is close to zero also explains why the estimate of α is not statistically different from zero.

From equations (1) and (2), it happens that $\alpha = \alpha_* \lambda$ only if $\lambda > 0$, while α and α_* are equivalent in the limiting case $\lambda \to 0$. Table 4 reports $\hat{\alpha} = \hat{\alpha_*}\hat{\lambda}$ and the corresponding standard error as computed by the delta method. It is therefore straightforward to retrieve the estimate of α_* from the figures in Table 4: Indeed, $\hat{\alpha_*} = 0.3898$ with standard error equal to 0.1839.

To have a better idea about the fit of the models, we undertake an informal log-likelihood comparison that accounts for overparameterization. We do not pursuit a formal analysis based on log-likelihood ratio tests because, due to the presence of inequality constraints in the parameter space, the limiting distribution of the test statistic is a mixing of chi-square distributions with probability weights depending on the variance of the parameter estimates (Wolak, 1991). Accordingly, it is extremely difficult to obtain empirically implementable asymptotically exact critical values. As an alternative, Wolak suggests applying asymptotic bounds tests. However, bounds are in most instances quite slack, often yielding inconclusive results.

We compute the Akaike information criterion, $\text{AIC} \equiv -2(\log \mathcal{L} - k)/T$, where $\log \mathcal{L}$ denotes the value of the log-likelihood, k the number of parameters and T the number of observations. In terms of AIC values, the horse race winners are augmented ACD, the Box-Cox ACD, the asymmetric power ACD, and the asymmetric logarithmic ACD models.[1] The rewards of the extra flexibility granted by these specifications are in contrast to the poor performance of the linear and logarithmic ACD Type II models. Further, letting λ free to vary and accounting for asymmetric effects seem to operate as substitute sources of flexibility. For instance, it is very rewarding to

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[1] The result does not change much if one considers the Bayesian information criterion so as to penalize more strongly the number of parameters. The only noticeable difference is that the performances of the logarithmic ACD type I and the power ACD models become as good as the above specifications.

introduce asymmetric responses to shocks in specifications with fixed λ.

FIGURE 2

Figure 2 portrays the effective shocks impact curves of each specification by depicting the variation of the conditional duration $\Delta\psi_i \equiv \psi_i - \psi_{i-1}$ in response to a shock ϵ_{i-1} at time t_{i-1}. We fix the conditional duration process ψ_{i-1} at time t_{i-1} to one, while we vary the shock ϵ_{i-1} from zero to five.[2] It is striking that, in all instances, $\Delta\psi_i$ reacts in a very similar fashion to the shock. In particular, it seems that the concavity of the shocks impact curve is the most important feature to account for when modeling IBM price durations, alleviating the problem of overpredicting short durations. In the sequel, we argue that the apparently substitutability between the Box-Cox transformation and the asymmetric effects is chiefly caused by the need to achieve concavity of the shocks impact curve.

The asymmetric linear and logarithmic ACD's shocks impact curves are concave only for certain values of the shift and rotation parameters, namely $b > 0$ and $c < -1$. From this perspective, the parameter estimates reported in the column EXACD in Table 3 and columns A-ACD and A-LACD in Table 4 are not surprising. The estimates of the shift and rotation parameters are significantly different from zero and inferior to minus one, respectively. In contrast, if the shape parameter υ is inferior to one, both the Box-Cox and augmented ACD models produce concave shocks impact curves. In the case of the AACD, this holds regardless of the shift and rotation parameters, hence it comes with no wonder that the corresponding estimates are not jointly significant in the AACD specification. As the power ACD model

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[2] We refrain from plotting the shocks impact curve for larger shocks because it is merely a byproduct of the assumed specification of the duration process, without necessarily representing some meaningful property of the data (Hentschel, 1995).

imposes $\lambda = v$, the estimate of $\hat{\lambda}$ sets in towards the estimates of v in the Box-Cox and augmented ACD models so as to entail a concave shocks impact curve. The same happens with the asymmetric power ACD model, despite the fact that, at first glance, one could also induce concavity through the shift and rotation parameters. It turns out, however, that to ensure a concave shocks impact curve the absolute value of the rotation parameter must exceed one, running counter to the nonnegativeness constraint.[3] Indeed, the augmented ACD model avoids this problem by letting λ converge to zero, thereby mimicking the asymmetric logarithmic ACD specification. All in all, Figure 2 illustrates some of the pitfalls from the specific to general modeling approach: There are various ways to achieve a concave shocks impact curve that the data call for and failing to start from a sufficiently general specification may point to quite misleading directions.

We now infer about the statistical properties of the duration processes by checking whether they satisfy the sufficient conditions for strict stationarity derived in Proposition 1. The aim is to illustrate how to use Proposition 1 for testing purposes. Maximum likelihood requires strict stationary of the duration process to ensure consistency, hence estimates that violate either $|\beta| < 1$ or (4) are not very reliable. In the linear ACD model, this is equivalent to verifying whether $|\alpha + \beta| < \mu_{B,m}^{-1/m} < \infty$ for some integer $m > 1$. The second inequality poses no problem as $\mu_{B,m}$ exists for $m < \hat{\kappa}/\hat{\gamma} = 3.0139$. However, $\hat{\alpha} + \hat{\beta} = 0.9915$, whereas $m = 2$ yields $\hat{\mu}_{B,2}^{-1/2} = 0.4348$. In contrast, all other specifications seem to satisfy the sufficient conditions put forth in Proposition 1. For both versions of the logarithmic ACD model,

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[3] Unlike what occurs in the asymmetric ACD case, the estimation of the asymmetric power ACD model depends heavily on this constraint, since if the shocks impact curve is negative complex numbers would arise disrupting the maximum likelihood algorithm.

condition (4) holds in view that $|\hat{\alpha} + \hat{\beta}| < 1$ in Type I and $|\hat{\beta}| < 1$ in Type II. Further, $|\hat{\beta}| < 1$ guarantees that both restricted (EXACD) and unrestricted (A-LACD) versions of the asymmetric logarithmic ACD model as well as the Box-Cox ACD process are strictly stationary. The power ACD model requires that $E|\beta + \alpha \epsilon_i^{\lambda}|^2 < 1$ for some integer $m > 1$, which reduces to $|\alpha + \beta| < \mu_{B,2\lambda}^{-1/2}$ for $m = 2$. The latter inequality is empirically satisfied as the parameter estimates are such that $0.9738 = \hat{\alpha} + \hat{\beta} < \hat{\mu}_{B,2\hat{\lambda}}^{-1/2} = 1.0058$. Numerical results based on 10,000 Monte Carlo simulations also show that (4) holds for the asymmetric ACD and asymmetric power ACD models. As $\lambda \to 0$ in the augmented ACD model, strict stationarity follows from the fact that $|\hat{\beta}| < 1$.

To check for misspecification, we first inspect whether the standardized durations display any serial correlation by looking at the sample autocorrelation function of n-th order with n varying from 1 to 60. Tables 3 and 4 document that there is no sample autocorrelation greater than 0.05 (in magnitude) irrespective of the specification of the conditional duration process. Moreover, the Ljung-Box statistics also show no evidence of serial correlation in the residuals.[4] We therefore conclude that the conditional duration models are doing a great job of accounting for the serial dependence in the IBM price durations.

Next, we apply Fernandes and Grammig's (2003) D-test to gauge the closeness between the parametric and nonparametric estimates of the density function of the residuals. Under the correct specification of the conditional duration process, both the parametric and kernel density estimates

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[4] To check for nonlinear serial dependence, we also regress the residuals on indicators for the magnitude of the previous duration as in Engle and Russell (1998). As the F-statistic is very close to zero in all instances, we find no evidence supporting any kind of nonlinear dependence.

of the residuals $\hat{\epsilon}_i = \frac{\psi_i}{\hat{\psi}_i} \epsilon_i$ converge to the true Burr density. In contrast, misspecification gives rise to a mixture of Burr distributions since the factor $\frac{\psi_i}{\hat{\psi}_i}$ does not converge to one in probability. The kernel density estimate will then converge to this mixture of Burr densities, whereas the parametric estimate always belongs to the Burr family. The test statistic is thus presumably close to zero under the null, whereas it should be large under the alternative. The motivation to apply the D-test is twofold. First, although it is slightly conservative, the D-test entails excellent power against both fixed and local alternatives. Second, it is nuisance parameter free in that there is no asymptotic cost in replacing errors with estimated residuals.

To avoid boundary effects in the kernel density estimates due to the non-negativeness of standardized durations, we work with log-residuals rather than plain residuals. All nonparametric density estimates use a Gaussian kernel, whereas the bandwidths are chosen according to an adjusted-version of Silverman's (1986) rule of thumb. The adjustment is necessary because the asymptotic theory of the D-test requires a slight degree of undersmoothing so as to avoid additional bias terms (see Fernandes and Grammig, 2003). Despite the fact that the p-values of the D-test seem to decrease with the degree of smoothing, the results are qualitatively robust to minor variations in the bandwidth value.

The D-test results illustrate the rewards of the extra flexibility provided by the AACD family. There is no standard specification that performs well as seen in Table 3. At the 1% level of significance, we soundly reject the linear and logarithmic Type I ACD models, whereas we find a borderline result for the asymmetric logarithmic ACD model with $b = 1$ proposed by Dufour and Engle (2000). At the 5% level, rejection ensues for the Box-Cox

ACD model, while rejecting the logarithmic ACD Type II specification is somewhat arguable given that the p-value is very close to 0.05. The figures in Table 4 are much rosier: There is indeed no clear rejection, though we find a borderline result for the asymmetric ACD model at the 5% significance level. The D-test results also indicate that the asymmetric logarithmic ACD specification is the most successful model, achieving a quite large p-value. Figure 3 illustrates this pattern by plotting the kernel and parametric density estimates of the log-residuals for the two groups of models in the first and second column, respectively. While there are striking discrepancies in the first column, the nonparametric density estimates nicely oscillate around the parametric density estimates of the log-residuals in the second column.

FIGURE 3

To provide further empirical evidence, we consider price duration data, from September to November 1996, relating to four actively traded stocks on NYSE: Boeing, Coca-Cola, Disney, and Exxon. We also examine volume and trade durations referring to the above stocks and IBM. Trade durations measure the time between two successive transactions, whereas volume durations denote the time interval needed to observe a cumulative trading volume of 25,000 shares. We deal with intraday seasonality in the same fashion as before. As expected, all durations keep featuring autocorrelation and overdispersion even after adjusting for the time-of-day effect.

TABLES 5 TO 7

Tables 5 to 7 respectively summarize the results for price, volume and trade durations. We report the Akaike information criterion (AIC) and the p-values for the D-test and Ljung-Box Q statistics of 1st and 12th order.

Restricting attention to the models that are not rejected by the specification tests, the horse race winners (according to the information criterion) belong to the subclass of logarithmic ACD processes given by (9). In particular, we select the LACD Type I model for the Boeing and Disney price durations as well as for the Exxon volume duration data. Furthermore, the asymmetric logarithmic ACD specifications (EXACD and A-LACD) perform well not only for the Coke price and volume durations, but also for the Boeing volume duration. In contrast, we find no suitable ACD specification to model either the IBM volume duration or any trade duration. The rejections are mainly due to the residual serial dependence. We therefore deem that further research must pay more attention to the logarithmic subclass of the AACD family given that it is quite flexible and easy to manipulate in the context of higher order autoregressive structures.

<div align="center">Figure 4</div>

Lastly, the concavity of the shocks impact curve appears to be a quite general feature of financial duration data. Figure 4 displays the shocks impact curves for the models with best overall performance. Although it seems more pronounced for price durations, concavity shows up in all instances.

4   Conclusion

This paper introduces a family of augmented ACD models that encompasses most specifications in the literature. The nesting leans upon a Box-Cox transformation to the conditional duration process and an asymmetric shocks impact curve. The motivation for the latter stems from Engle and Russell's (1998) empirical findings, evincing that the linear ACD model

<div align="center">23</div>

tends to overpredict after either very long or very short durations. We derive sufficient conditions for the existence of higher-order moments, strict stationarity, geometric ergodicity and β-mixing property with exponential decay in this class of ACD models.

Our empirical results on IBM price durations show that the restrictions imposed by the existing models in the literature are incompatible with the data, warranting the extra flexibility granted by the augmented ACD models. Actually, inspecting the parameter estimates of the different specifications reveals that imposing concavity in the shocks impact curve is pivot. The Box-Cox transformation and the asymmetric response to shocks indeed work, to some extent, as substitutes. In particular, the power ACD and asymmetric logarithmic ACD models produce the best fit.

Further empirical investigation reveals that the concavity of the shocks impact curve is not a specific feature of the IBM price durations. Our findings evince the same concave pattern using other financial duration data, though less pronounced for volume and trade durations.

References

Bauwens, L., Giot, P., 2000, The logarithmic ACD model: An application to the bid-ask quote process of three NYSE stocks, Annales d'Economie et de Statistique 60, 117–150.

Bauwens, L., Veredas, D., 1999, The stochastic conditional duration model: A latent factor model for the analysis of financial durations, forthcoming in Journal of Econometrics.

Box, G. E. P., Cox, D. R., 1964, An analysis of transformations, Journal of the Royal Statistical Society B 26, 211–243.

Carrasco, M., Chen, X., 2002, Mixing and moment properties of various GARCH and stochastic volatility models, Econometric Theory 18, 17–39.

Dufour, A., Engle, R. F., 2000, The ACD model: Predictibility of the time between consecutive trades, University of Reading and University of California at San Diego.

Engle, R. F., Russell, J. R., 1997, Forecasting the frequency of changes in quoted foreign exchange prices with the autoregressive conditional duration model, Journal of Empirical Finance 4, 187–212.

Engle, R. F., Russell, J. R., 1998, Autoregressive conditional duration: A new model for irregularly-spaced transaction data, Econometrica 66, 1127–1162.

Fernandes, M., Grammig, J., 2003, Nonparametric specification tests for conditional duration models, Graduate School of Economics, Getulio Vargas Foundation.

Ghysels, E., Gouriéroux, C., Jasiak, J., 2003, Stochastic volatility duration models, forthcoming in Journal of Econometrics.

Giot, P., 2000, Time transformations, intraday data and volatility models, Journal of Computational Finance 4, 31–62.

He, C., Teräsvirta, T., 1999, Properties of moments of a family of GARCH processes, Journal of Econometrics 92, 173–192.

He, C., Teräsvirta, T., Malmsten, H., 2002, Moment structure of a family of first-order exponential GARCH models, Econometric Theory 18, 868–885.

Hentschel, L., 1995, All in the family: Nesting symmetric and asymmetric GARCH models, Journal of Financial Economics 39, 71–104.

Mokkadem, A., 1990, Propriétés de mélange des modèles autoregressifs polynomiaux, Annales de l'Institut Henri Poincaré 26, 219–260.

Nelson, D. B., 1990, Stationary and persistence in the GARCH(1,1) model, Econometric Theory 6, 318–334.

Pham, D. T., 1986, The mixing property of bilinear and generalised random coefficient autoregressive models, Stochastic Processes and Their Applications 23, 291–300.

Prigent, J.-L., Renault, O., Scaillet, O., 2001, An autoregressive conditional binomial option pricing model, in: H. Geman D. Madan S. Pliska T. Vorst (eds), Selected Papers from the First World Congress of the Bachelier Finance Society, Springer Verlag, Heidelberg.

Silverman, B. W., 1986, Density Estimation for Statistics and Data Analysis, Chapman and Hall, London.

Wolak, F. A., 1991, The local nature of hypothesis tests involving inquality constraints in nonlinear models, Econometrica 59, 981–995.

Zhang, M. Y., Russell, J. R., Tsay, R. S., 2001, A nonlinear autoregressive conditional duration model with applications to financial transaction data, Journal of Econometrics 104, 179–207.
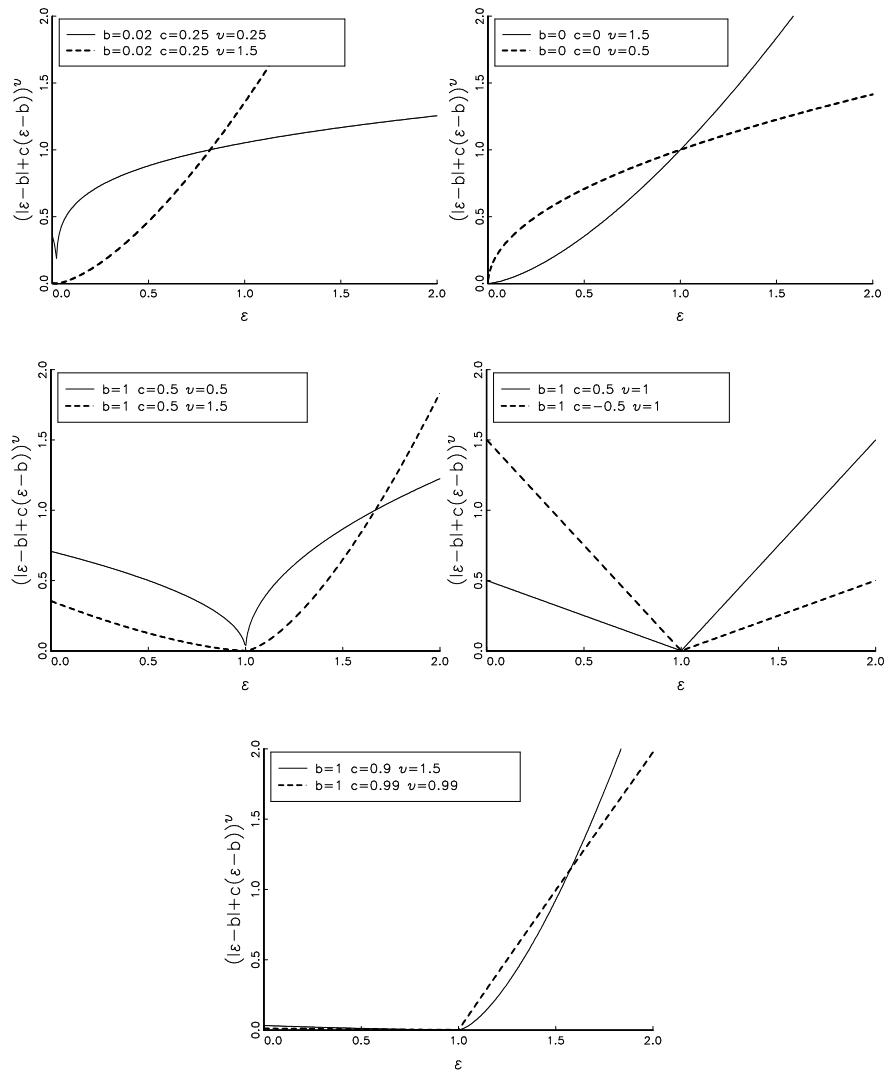
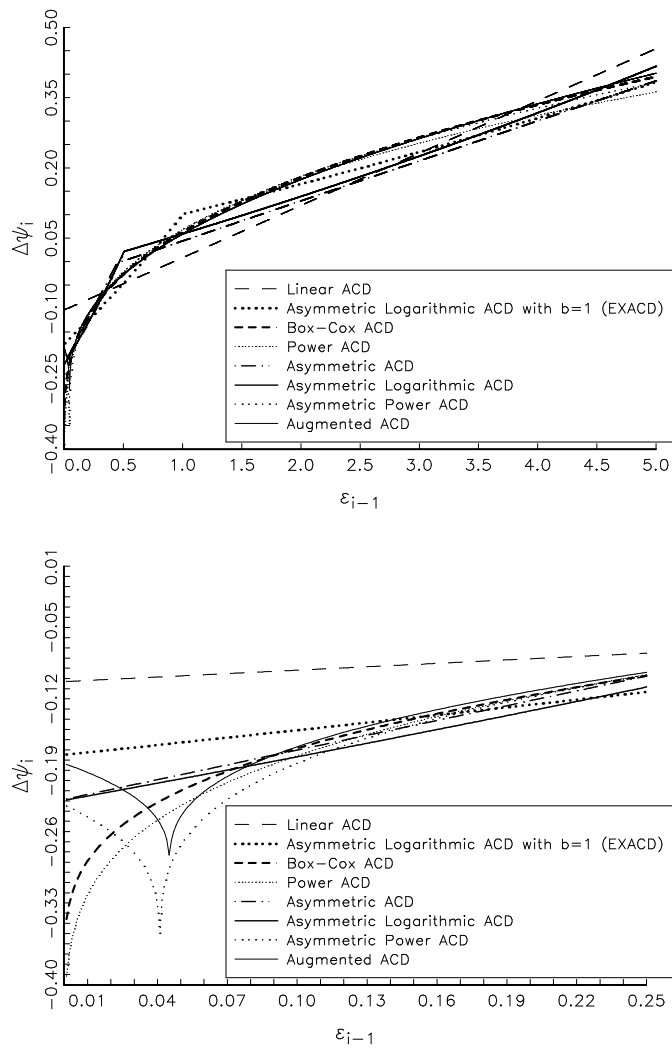Figure 1: Feasible shocks impact curves for the augmented ACD model
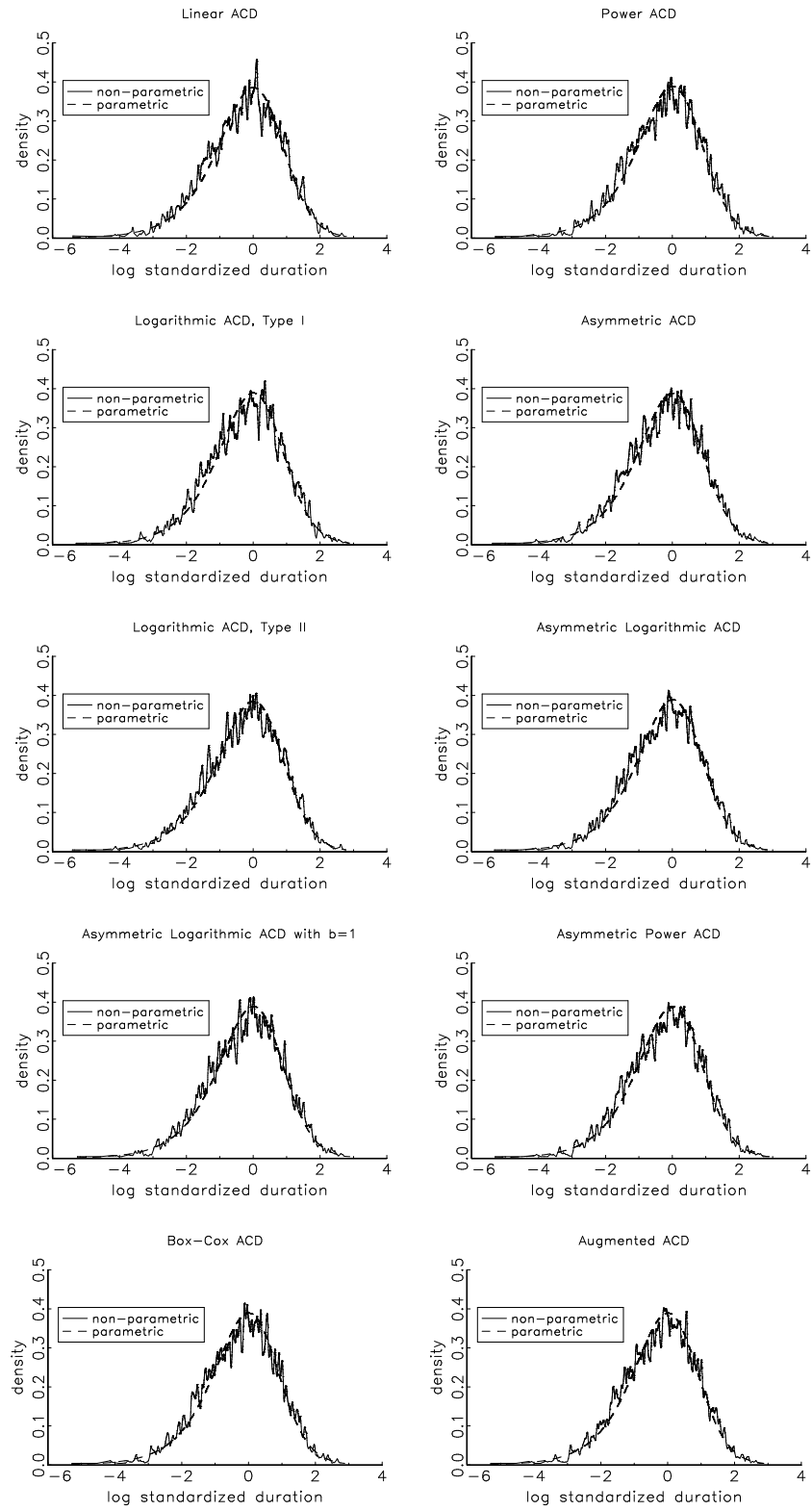
Figure 2: Empirical shocks impact curves
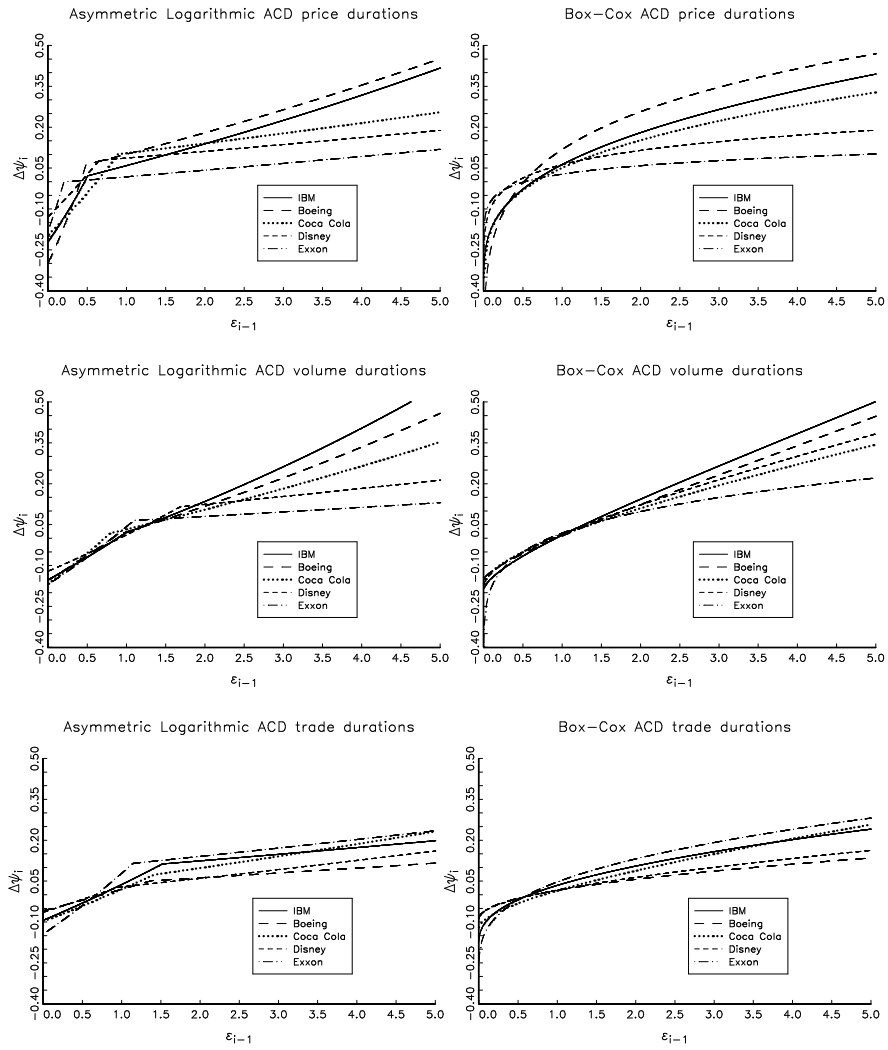
Figure 3: Density estimates for the log-residuals

Figure 4: Shocks impact curves for selected models

Table 1

Typology of ACD models

Augmented ACD

$$\psi_i^\lambda = \omega + \alpha\,\psi_{i-1}^\lambda \Big[\,|\,\epsilon_{i-1} - b\,| + c\,(\epsilon_{i-1} - b)\Big]^\upsilon + \beta\,\psi_{i-1}^\lambda$$

Asymmetric Power ACD $(\lambda = \upsilon)$

$$\psi_i^\lambda = \omega + \alpha\,\psi_{i-1}^\lambda \Big[\,|\,\epsilon_{i-1} - b\,| + c\,(\epsilon_{i-1} - b)\Big]^\lambda + \beta\,\psi_{i-1}^\lambda$$

Asymmetric Logarithmic ACD $(\lambda \to 0 \text{ and } \upsilon = 1)$

$$\log \psi_i = \omega + \alpha\,\Big[\,|\,\epsilon_{i-1} - b\,| + c\,(\epsilon_{i-1} - b)\Big] + \beta\,\log \psi_{i-1}$$

Asymmetric ACD $(\lambda = \upsilon = 1)$

$$\psi_i = \omega + \alpha\,\psi_{i-1}\Big[\,|\,\epsilon_{i-1} - b\,| + c\,(\epsilon_{i-1} - b)\Big] + \beta\,\psi_{i-1}$$

Power ACD $(\lambda = \upsilon \text{ and } b = c = 0)$

$$\psi_i^\lambda = \omega + \alpha\,x_{i-1}^\lambda + \beta\,\psi_{i-1}^\lambda$$

Box-Cox ACD $(\lambda \to 0 \text{ and } b = c = 0)$

$$\log \psi_i = \omega + \alpha\,\epsilon_{i-1}^\upsilon + \beta\,\log \psi_{i-1}$$

Logarithmic ACD Type I $(\lambda, \upsilon \to 0 \text{ and } b = c = 0)$

$$\log \psi_i = \omega + \alpha\,\log x_{i-1} + \beta\,\log \psi_{i-1}$$

Logarithmic ACD Type II $(\lambda \to 0,\ \upsilon = 1 \text{ and } b = c = 0)$

$$\log \psi_i = \omega + \alpha\,\epsilon_{i-1} + \beta\,\log \psi_{i-1}$$

Linear ACD $(\lambda = \upsilon = 1 \text{ and } b = c = 0)$

$$\psi_i = \omega + \alpha\,x_{i-1} + \beta\,\psi_{i-1}$$

Table 2

Descriptive statistics

IBM price durations	plain	adjusted
sample size	4,484	4,484
mean	262.55	1.239
median	128	0.714
maximum	7170	29.121
minimum	1	0.004
overdispersion	1.610	1.330
n-th order sample autocorrelation		
$n = 1$	0.256	0.179
$n = 2$	0.231	0.184
$n = 3$	0.240	0.166
$n = 4$	0.168	0.121
$n = 8$	0.127	0.106
$n = 12$	0.095	0.099
$n = 16$	0.061	0.072
$n = 20$	0.018	0.062
$n = 24$	0.021	0.073
$n = 28$	0.000	0.050
$n = 32$	-0.008	0.047
$n = 36$	0.004	0.054

Table 3

Estimation results for the AACD family of models

IBM price durations ($0.125 mid-price change)

parameter	ACD	LACD I	LACD II	EXACD	BCACD
ω	0.0171	0.0774	-0.0865	-0.0964	-0.5230
	(0.0038)	(0.0069)	(0.0063)	(0.0131)	(0.1708)
α	0.1116	0.1250	0.0912	-0.1157	0.5843
	(0.0088)	(0.0083)	(0.0067)	(0.0152)	(0.1768)
β	0.8799	0.8327	0.9759	0.9614	0.9616
	(0.0089)	(0.0127)	(0.0042)	(0.0064)	(0.0067)
υ					0.2371
					(0.0772)
c				-1.4927	
				(0.1288)	
κ	1.2616	1.3036	1.2592	1.2892	1.2954
	(0.0318)	(0.0331)	(0.0318)	(0.0327)	(0.0330)
γ	0.4186	0.4808	0.4137	0.4519	0.4635
	(0.0471)	(0.0487)	(0.0469)	(0.0478)	(0.0486)
$\log \mathcal{L}$	-4,952.4	-4,924.8	-4,950.5	-4,924.5	-4,920.5
AIC	1.6836	1.6742	1.6830	1.6741	1.6728
D-test	0.0029	0.0025	0.0488	0.0140	0.0266
$Q(4)$	0.1965	0.0845	0.1152	0.5821	0.4990
$Q(8)$	0.0898	0.1433	0.1136	0.2410	0.2327
$Q(16)$	0.0836	0.0534	0.1569	0.1956	0.1891
$Q(24)$	0.0496	0.0897	0.1084	0.2853	0.2698
max ACF	0.0326	0.0403	0.0344	0.0370	0.0378
min ACF	-0.0282	-0.0264	-0.0259	-0.0310	-0.0320

Figures in parentheses correspond to standard errors based on the OPG estimator of the information matrix. $\log \mathcal{L}$ reports the value of the log-likelihood function, whereas AIC denotes the Akaike information criterion. D-test displays the p-values of the nonparametric test proposed by Fernandes and Grammig (2003) applied to the log-residuals. $Q(n)$ correspond to the p-values of Ljung-Box statistic for up to n-th order serial correlation. The last two rows report the maximum and minimum values of the sample autocorrelations from order 1 to 60, respectively.

Table 4
Estimation results for the AACD family of models
IBM price durations ($0.125 mid-price change)

parameter	PACD	A-ACD	A-LACD	A-PACD	AACD
ω	0.0378	0.0208	0.0217	0.0378	0.0361
	(0.0067)	(0.0049)	(0.0147)	(0.0061)	(0.0064)
α	0.1352	-0.1990	-0.2294	0.1270	0.00001
	(0.0110)	(0.0452)	(0.0448)	(0.0268)	(0.1022)
β	0.8386	0.9760	0.9639	0.8468	0.9639
	(0.0123)	(0.0168)	(0.0062)	(0.0116)	(0.0891)
λ	0.1751			0.2573	0.00003
	(0.0848)			(0.0681)	(0.2621)
υ					0.3244
					(0.0747)
b		0.4456	0.5066	0.0411	0.0451
		(0.0741)	(0.0680)	(0.0019)	(0.0015)
c		-1.4294	-1.3172	0.2326	0.1117
		(0.1099)	(0.0724)	(0.9446)	(1.1718)
κ	1.2976	1.2882	1.2926	1.2979	1.2935
	(0.0331)	(0.0328)	(0.0329)	(0.0330)	(0.0329)
γ	0.4688	0.4556	0.4589	0.4685	0.4588
	(0.0487)	(0.0484)	(0.0485)	(0.0482)	(0.0483)
$\log\mathcal{L}$	-4,922.7	-4,930.1	-4,921.1	-4,921.0	-4,918.4
AIC	1.6735	1.6760	1.6730	1.6729	1.6721
D-test	0.0955	0.0494	0.4039	0.1353	0.1370
$Q(4)$	0.3011	0.1794	0.4259	0.3866	0.5031
$Q(8)$	0.2085	0.0600	0.1182	0.2100	0.1840
$Q(16)$	0.1447	0.0719	0.1124	0.1447	0.1726
$Q(24)$	0.2224	0.1085	0.1536	0.2262	0.2594
max ACF	0.0378	0.0351	0.0352	0.0386	0.0381
min ACF	-0.0301	-0.0341	-0.0370	-0.0316	-0.0331

Figures in parentheses correspond to standard errors based on the OPG estimator of the information matrix. $\log\mathcal{L}$ reports the value of the log-likelihood function, whereas AIC denotes the Akaike information criterion. D-test displays the p-values of the nonparametric test proposed by Fernandes and Grammig (2003) applied to the log-residuals. $Q(n)$ correspond to the p-values of the Ljung-Box statistic for up to n-th order serial correlation. The last two rows report the maximum and minimum values of the sample autocorrelations from order 1 to 60, respectively.

Table 5
Estimation results for price durations ($0.125 mid-price change)

		ACD	LACD I	LACD II	EXACD	BCACD	PACD	A-ACD	A-LACD	A-PACD	AACD
Boeing	sample size: 1,746 observations										
	AIC	1.9989	1.9759	2.0030	1.9890	1.9770	1.9771	1.9945	1.9815	1.9789	1.9800
	$Q(1)$	0.551	0.734	0.210	0.539	0.732	0.708	0.754	0.585	0.736	0.679
	Q(12)	0.320	0.219	0.460	0.353	0.220	0.221	0.484	0.417	0.214	0.225
	D-test	0.103	0.173	0.006	0.283	0.157	0.160	0.113	0.140	0.121	0.097
Coke	sample size: 1,072 observations										
	AIC	1.8885	1.8823	1.8909	1.8811	1.8833	1.8833	1.8923	1.8827	1.8863	1.8882
	$Q(1)$	0.138	0.233	0.224	0.232	0.132	0.137	0.139	0.206	0.131	0.135
	Q(12)	0.502	0.482	0.608	0.501	0.436	0.435	0.506	0.494	0.430	0.434
	D-test	0.615	0.059	0.733	0.270	0.250	0.250	0.624	0.184	0.360	0.369
Disney	sample size: 1,439 observations										
	AIC	2.2148	2.2104	2.2146	2.2127	2.2118	2.2118	2.2176	2.2130	2.2145	2.2159
	$Q(1)$	0.926	0.869	0.874	0.877	0.866	0.856	0.889	0.836	0.833	0.848
	Q(12)	0.685	0.500	0.673	0.543	0.524	0.512	0.670	0.430	0.514	0.519
	D-test	0.133	0.271	0.213	0.371	0.295	0.281	0.188	0.087	0.295	0.303
Exxon	sample size: 1,810 observations										
	AIC	1.9556	1.9521	1.9554	1.9549	1.9532	1.9533	1.9536	1.9529	1.9535	1.9545
	$Q(1)$	0.219	0.241	0.267	0.232	0.234	0.267	0.149	0.231	0.304	0.304
	Q(12)	0.228	0.265	0.251	0.224	0.250	0.258	0.208	0.308	0.272	0.272
	D-test	0.099	0.006	0.136	0.043	0.029	0.033	0.091	0.017	0.012	0.012

Table 6
Estimation results for volume durations (cumulative trading volume: 25,000 shares)

		ACD	LACD I	LACD II	EXACD	BCACD	PACD	A-ACD	A-LACD	A-PACD	AACD
Boeing	sample size: 1,050 observations										
	AIC	1.8016	1.8054	1.7990	1.7983	1.8000	1.8021	1.8046	1.7999	1.8044	1.8063
	$Q(1)$	0.964	0.762	0.831	0.772	0.802	0.935	0.898	0.724	0.850	0.849
	$Q(12)$	0.537	0.367	0.586	0.516	0.542	0.510	0.551	0.503	0.515	0.516
	D-test	0.573	0.478	0.767	0.781	0.703	0.567	0.507	0.790	0.610	0.607
Coke	sample size: 2,014 observations										
	AIC	1.8031	1.8060	1.8022	1.7993	1.8009	1.8018	1.8014	1.7994	1.8023	1.8033
	$Q(1)$	0.033	0.029	0.031	0.064	0.057	0.053	0.072	0.091	0.062	0.060
	$Q(12)$	0.096	0.006	0.096	0.059	0.066	0.065	0.054	0.052	0.052	0.053
	D-test	0.390	0.834	0.137	0.499	0.634	0.658	0.696	0.691	0.636	0.631
Disney	sample size: 1,184 observations										
	AIC	1.8913	1.8967	1.8924	1.8915	1.8918	1.8918	1.8941	1.8907	1.8943	1.8960
	$Q(1)$	0.364	0.130	0.284	0.329	0.358	0.364	0.352	0.330	0.368	0.367
	$Q(12)$	0.524	0.297	0.524	0.463	0.493	0.494	0.526	0.517	0.483	0.493
	D-test	0.000	0.010	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
Exxon	sample size: 1,362 observations										
	AIC	1.6471	1.6400	1.6486	1.6404	1.6413	1.6413	1.6435	1.6416	1.6438	1.6453
	$Q(1)$	0.732	0.456	0.947	0.486	0.446	0.446	0.526	0.535	0.417	0.435
	$Q(12)$	0.575	0.572	0.587	0.593	0.573	0.572	0.598	0.598	0.563	0.593
	D-test	0.886	0.989	0.818	0.961	0.964	0.964	0.909	0.987	0.962	0.962
IBM	sample size: 2,869 observations										
	AIC	1.8300	1.8393	1.8282	1.8264	1.8259	1.8286	1.8311	1.8271	1.8274	1.8280
	$Q(1)$	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.001	0.000	0.000
	$Q(12)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	D-test	0.811	0.894	0.941	0.856	0.580	0.866	0.812	0.860	0.664	0.649

Table 7
Estimation results for trade durations

		ACD	LACD I	LACD II	EXACD	BCACD	PACD	A-ACD	A-LACD	A-PACD	AACD
Boeing	sample size: 15,952 observations										
	AIC	2.0765	2.0754	2.0762	2.0730	2.0733	2.0736	2.0738	2.0730	2.0733	2.0735
	$Q(1)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$Q(12)$	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	D-test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Coke	sample size: 26,414 observations										
	AIC	1.8109	1.8142	1.8111	1.8083	1.8086	1.8091	1.8110	1.8082	1.8084	1.8085
	$Q(1)$	0.267	0.000	0.150	0.298	0.289	0.165	0.268	0.270	0.225	0.224
	$Q(12)$	0.007	0.000	0.009	0.034	0.022	0.016	0.007	0.028	0.033	0.033
	D-test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Disney	sample size: 21,880 observations										
	AIC	2.1042	2.1051	2.1042	2.1027	2.1028	2.1029	2.1044	2.1028	2.1029	2.1030
	$Q(1)$	0.011	0.000	0.007	0.023	0.024	0.019	0.011	0.025	0.018	0.019
	$Q(12)$	0.027	0.000	0.025	0.043	0.040	0.037	0.028	0.046	0.034	0.035
	D-test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Exxon	sample size: 18,913 observations										
	AIC	1.9689	1.9641	1.9695	1.9629	1.9633	1.9635	1.9639	1.9629	1.9632	1.9632
	$Q(1)$	0.619	0.094	0.062	0.644	0.779	0.589	0.362	0.641	0.812	0.770
	$Q(12)$	0.031	0.107	0.025	0.115	0.095	0.116	0.133	0.113	0.106	0.095
	D-test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IBM	sample size: 40,302 observations										
	AIC	2.0749	2.0705	2.0745	2.0680	2.0682	2.0687	2.0750	2.0677	2.0680	2.0681
	$Q(1)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$Q(12)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	D-test	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

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