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Gustavo M. de Athayde, Renato G. Flôres Jr.

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Gustavo M. de Athayde *and* *Renato G. Flôres Jr.**
Banco Itaú S. A., São Paulo EPGE/Fundação Getulio Vargas, Rio

ABSTRACT

We present explicit formulas for evaluating the difference between Markowitz weights and those from optimal portfolios, with the same given return, considering either asymmetry or kurtosis. We prove that, whenever the higher moment constraint is not binding, the weights are *never* the same. If, due to special features of the first and second moments, the difference might be negligible, in quite many cases it will be very significant. An appealing illustration, when the designer wants to incorporate an asset with quite heavy tails, but wants to moderate this effect, further supports the argument.

Key words: kurtosis, Markowitz solution, portfolio choice, sensitivity analysis, skewness.

JEL classification: C49; C61; C63.

* *Full address (corresponding author):*

EPGE / FGV
Praia de Botafogo 190
Rio de Janeiro 22253-900 RJ Brazil
rflores@fgv.br

1. Introduction.

Is the Markowitz optimal solution very different from the one obtained when considering, say, skewness ? Or kurtosis ?

In this paper we show that, though it might be close to the new optimal solution in some instances, the answer most of the times will be a round *yes*. Indeed, by calling attention to how "wrong" it may be to stick to the Markowitz solution, the results below stress a pledge for due introduction of higher moments in portfolio optimisation. In the next section, after presenting our notation, we develop the analytical results that allow to compare the Markowitz solution with two special higher moments cases, in which variance is minimised given the *same* expected portfolio excess return and either a given skewness or kurtosis. In particular, we prove that apart from a zero-measure set the Markowitz solution is *never* equal to the other two. We then move a little further in section 3, by studying a theoretical example, when only one marginal kurtosis is taken into account. Even in this apparently simple case, the differences can be strikingly.

We believe that the implications of results as those shown here have not been fully exploited yet. Undoubtedly, final testing of the gains brought out by using higher moments relies in extensive practical applications of the idea. If a work like Harvey and Siddique (2000) points to one of the needed directions, the task has however only begun.

2. A general framework.

Portfolio optimisation taking into account moments higher than the second cannot be considered a new theme any more. A mature text like Barone-Adesi (1985), nearly twenty years old, pays witness to the seniority of the problem. However, several issues still contribute to the fact that, though acknowledged by most as an important – or rather crucial – point in *actual* portfolio construction, no systematic approach to globally deal with it, from the practical to the theoretical instances, has been widely accepted yet by the profession.

Since Athayde and Flôres (1997), we have been developing such a systematic way, which has as departure point the treatment of key optimisation problems that are posed to anyone dealing with higher moments in portfolio design. The approach allows several theoretical insights as well as the setting up of software to perform the search of the optimal weights. This encompassing nature is greatly due to a new notation explained in the next sub-section[1].

2.1. A matrix notation for the higher moments arrays.

Given a n-dimensional random vector, the set of its p-th order moments is, in general, a tensor. The second moments tensor is the popular $n \times n$ covariance matrix, while the third moments one is a $n \times n \times n$ cube in three-dimensional space. As the (mathematical) tensor notation, which is so useful in physics, did not appear convenient in the portfolio choice problem, we developed a special notation for the case. Motivation also came from the need to treat the problem in an absolutely general setting – be it either in a utility maximising context or if the optimal portfolio is defined by preference relations -, leaving open the maximum order p of portfolio moments of interest and the possible patterns of their corresponding (higher order) tensors. Beyond providing a synthetic way to treat complicated expressions, it allows performing all the needed operations within the realm of matrix calculus.

We transform the full p-th moments tensor, with n^p elements, into a matrix of order $n \times n^{p-1}$, called M_p ,obtained by slicing all bidimensional $n \times n^{p-2}$ layers defined by fixing one asset and then taking all the moments in which it figures at least once and pasting them, in the same order, sideways. Row *i'* of the matrix layer corresponding to having held the *i-th* asset fixed gives – in a pre-established order – all the moments in which assets *i* and *i'* appear at least once. Of course, assets must be ordered once and for all and this order respected in the sequencing of the layers and in the numbering of the rows of each layer. Accordingly, a conformal ordering must be chosen, and thoroughly used, for the combinations (with repetitions) of the n assets into groups of p-2 elements which will define the columns of each matrix layer.

In the case of kurtosis, for instance, two indices/assets must be held constant in each row of a given layer. Calling σ_{ijkl} a general (co-) kurtosis, when n=2, the final 2 x 8 ($=2^{4-1}$) matrix will result from the juxtaposition of two 2 x 4 ($=2^{4-2}$) layers – one corresponding to the first, and another to the second asset – as shown in Figure 1. Notice that, as pointed out in the figure, the four columns in each layer correspond to ordering the two-by-two combinations, with repetition, of the two assets.

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[1] The basis of the notation has been previously sketched in, for instance, Athayde and Flôres (2004, 2005). We present here a fuller (and hopefully clearer) explanation, making the paper self-contained.

Figure 1: Building up the 2 x 8 matrix M_4 corresponding to the kurtosis tensor, in the case of two assets (or a two-dimensional random vector):

$$
\begin{array}{ll}
\text{“asset 1” row} & \left[\begin{matrix} \sigma_{1111} & \sigma_{1112} & \sigma_{1121} & \sigma_{1122} & \sigma_{1211} & \sigma_{1212} & \sigma_{1221} & \sigma_{1222} \\ \sigma_{2111} & \sigma_{2112} & \sigma_{2121} & \sigma_{2122} & \sigma_{2211} & \sigma_{2212} & \sigma_{2221} & \sigma_{2222} \end{matrix}\right]
\end{array}
$$

<----- “asset 1” layer ----><----- “asset 2” layer ---->

“columns’ meaning” (11) (12) (21) (22) (11) (12) (21) (22)

As happens in a covariance matrix, with the exception of the marginal moments, all other entries in the M_p matrices will share identical values with others in “symmetrical” positions. We shall not pursue this combinatorics here – which can be very important in dealing with special features of the higher moments set -, but only provide a glimpse on its structure, still in the two assets case, in Figure 2.

Figure 2: The general pattern of the 2 x 8 matrix M_4 corresponding to the kurtosis tensor, in the case of two assets (each Roman letter corresponds to one of the (three) possible co-kurtoses):

$$
\begin{bmatrix} \sigma_{1111} & a & a & b & a & b & b & c \\ a & b & b & c & b & c & c & \sigma_{2222} \end{bmatrix} \qquad .
$$

Earlier works generalising portfolio choice to higher moments considered only the *marginal* higher moments of the returns vector, plainly disregarding any co-moment of the same order[2]. Though the full set of co-moments can quickly become too big – even at the third order -, and simplifying assumptions on its pattern will usually be imposed in practice, it is important to have a way to study the general solution to the problem, *irrespective of the simplifying assumptions that might be imposed.* In a second step, due consideration of the shape of the higher moments’ structures – which will give way to special patterns of zeroes in our M_p matrices - is a must for grasping a full knowledge of the market one is dealing with.

[2] This is still the, nowadays unacceptable, hypothesis of most applications

Now suppose that a vector of weights $\alpha \in R^n$ is given, and x, M_2, M_3, ... and M_p stand for the matrices, constructed as above, containing the expected (excess) returns, (co-)variances, skewnesses ... and p-moments of a random vector of n assets. The mean return, variance, skewness ... and p-th moment of the portfolio with these weights will be, respectively:

$$\alpha' x \ , \ \alpha' M_2 \alpha \ , \ \alpha' M_3 (\alpha \otimes \alpha) \ \ldots \text{ and } \ \alpha' M_p (\alpha \otimes \alpha \otimes \alpha \ldots \otimes \alpha) \equiv \alpha' M_p \alpha^{\otimes p-1} \quad ,$$

where '⊗' stands for the Kronecker product and $\alpha^{\otimes p}$ stands for the (Kronecker) product of vector α by itself, p times .

It is immediate to see that, as real functions of α, all expressions above are homogenous functions of the same degree as the order of the corresponding moment. This means that Euler's theorem can be easily used in computing derivatives with respect to α. As an example, the derivative of the portfolio kurtosis with respect to the weights will be:

$$\frac{\partial}{\partial \alpha}[\alpha' M_4 \alpha^{\otimes 3}] = 4 M_4 \alpha^{\otimes 3} \qquad .$$

2.2. Solving the classical portfolio problem controlling for skewness and kurtosis.

With the aid of the above notation we shall derive a general solution to the problem of minimising the portfolio variance given a specified set of (expected excess) return, skewness and kurtosis values, for the portfolio.

Consider a portfolio with n risky equities and a riskless asset with rate of return r_f. Let [1] stand for a nx1 vector of 1's and M_1 be the vector of the equities' expected returns and call $x = M_1 - [1]\, r_f$, the vector of mean excess returns. Minimising the variance, for a given mean return, skewness and kurtosis, amounts to finding the solution to the problem:

$$Min_\alpha L = \alpha' M_2 \alpha + \lambda_1 [(E(r_p) - r_f) - \alpha' x] + \lambda_2 (\sigma_{p^3} - \alpha' M_3 \alpha^{\otimes 2}) + \lambda_3 (\sigma_{p^4} - \alpha' M_4 \alpha^{\otimes 3}) \ , \quad (1)$$

where M_2 , M_3 and M_4 are, resp., the matrices related to the second, third and fourth moments tensors, α is the vector of n portfolio weights – where short sales are allowed, the lambdas are Lagrange multipliers and the three remaining symbols are the α-portfolio given mean return, skewness and kurtosis.

If $R = E(r_p) - r_f$ denotes the given excess portfolio return, the first order conditions (foc) corresponding to (1) are:

$$
\begin{aligned}
&2M_2\alpha = \lambda_1 x + 3\lambda_2 M_3\alpha^{\otimes 2} + 4\lambda_2 M_4\alpha^{\otimes 3} \\
&R = \alpha' x \\
&\sigma_{p^3} = \alpha' M_3\alpha^{\otimes 2} \\
&\sigma_{p^4} = \alpha' M_4\alpha^{\otimes 3} \quad .
\end{aligned}
\tag{2}
$$

Multiplying the first expression by the inverse of M_2 and then successively imposing in it each of the three scalar restrictions leads to the system:

$$
\begin{aligned}
&2R = \lambda_1 A_0 + 3\lambda_2 A_2 + 4\lambda_2 A_3 \\
&2\sigma_{p^3} = \lambda_1 A_2 + 3\lambda_2 A_4 + 4\lambda_3 A_5 \\
&2\sigma_{p^4} = \lambda_1 A_3 + 3\lambda_2 A_5 + 4\lambda_3 A_6 \quad ,
\end{aligned}
\tag{3}
$$

where the new coefficients are:

$$
\begin{aligned}
&A_0 = x' M_2^{-1} x \quad , \\
&A_2 = x' M_2^{-1} M_3\alpha^{\otimes 2} \quad , \\
&A_3 = x' M_2^{-1} M_4\alpha^{\otimes 3} \quad , \\
&A_4 = \alpha^{\otimes 2}{}' M_3{}' M_2^{-1} M_3\alpha^{\otimes 2} \quad , \\
&A_5 = \alpha^{\otimes 2}{}' M_3{}' M_2^{-1} M_4\alpha^{\otimes 3} \quad , \\
&A_6 = \alpha^{\otimes 3}{}' M_4{}' M_2^{-1} M_4\alpha^{\otimes 3} \quad ,
\end{aligned}
\tag{4}
$$

the subscript of the A's corresponding to their degree of homogeneity as real functions of the vector α. Notice that A_0 , A_4 and A_6 are positive because the inverse of the covariance matrix is positive definite.

System (3) can be solved by a straightforward use of Cramer's Rule. Substitution of the solution in the expression below, derived from the first foc in (2):

$$2\alpha = \lambda_1 M_2^{-1} x + 3\lambda_2 M_2^{-1} M_3 \alpha^{\otimes 2} + 4\lambda_2 M_2^{-1} M_4 \alpha^{\otimes 3} \tag{5}$$

yields the nonlinear system that characterises the optimal weights. The algebra, though not difficult, can be formidable, and use of a symbolic calculator (software) is advisable. We show the explicit final expression in two particular cases:

i) when skewness is not taken into account: calling α_K the weights, we have

$$\alpha_K = \frac{A_6 R - A_3 \sigma_{p^4}}{A_0 A_6 - (A_3)^2} M_2^{-1} x + \frac{A_0 \sigma_{p^4} - A_3 R}{A_0 A_6 - (A_3)^2} M_2^{-1} M_4 \alpha_K^{\otimes 3} \quad . \tag{6}$$

ii) when kurtosis is not taken into account: calling α_S the weights, we have

$$\alpha_S = \frac{A_4 R - A_2 \sigma_{p^3}}{A_0 A_4 - (A_2)^2} M_2^{-1} x + \frac{A_0 \sigma_{p^3} - A_2 R}{A_0 A_4 - (A_2)^2} M_2^{-1} M_3 \alpha_S^{\otimes 2} \quad . \tag{7}$$

We shall be interested in making comparisons with the classical Markowitz solution, α_M , which does not take into account both skewness and kurtosis. From (5), and the relevant lines in (2), it is immediately:

$$\alpha_M = \frac{R}{A_0} M_2^{-1} x \quad . \tag{8}$$

Let's define as $\Delta_S = \alpha_S - \alpha_M$ and $\Delta_K = \alpha_K - \alpha_M$, the differences between each higher moment solution and the Markowitz one. Using (6), (7) and (8), it is not very difficult to find that:

$$\Delta_S = \frac{A_0\sigma_{p^3} - A_2 R}{A_0 A_4 - (A_2)^2} M_2^{-1}(I_n - \frac{1}{A_0} xx' M_2^{-1}) M_3 \alpha_S^{\otimes 2} \qquad , \tag{9}$$

$$\Delta_K = \frac{A_0\sigma_{p^4} - A_3 R}{A_0 A_6 - (A_3)^2} M_2^{-1}(I_n - \frac{1}{A_0} xx' M_2^{-1}) M_4 \alpha_K^{\otimes 3} \qquad , \tag{10}$$

two expressions which, thanks to our notation, reveal themselves to be strikingly similar. Both can be used as starting points for sensitivity analyses, of the difference between the respective sets of weights, with respect to either the given value for the (portfolio) higher moment or *a specified (sub)set of the equities' higher moments*.

Nevertheless, such analyses must be carried out with care, as the weights themselves figure in the r.h.s. of the equations, either explicitly or through the "numbers" A_2, A_4 or A_3, A_5 . We shall however prove a more fundamental result:

Proposition. Let a given expected return R be fixed and suppose that a higher moment optimal portfolio (either (6) or (7)) exists *THEN* if the corresponding higher moment constraint is binding, the Markowitz solution *is never equal* to the higher moment solution.

Proof: we prove for the α_K case, the reasoning being identical for α_S . Notice first, from (5) and (6), that if the kurtosis constraint is not biding,

$$A_0\sigma_{p^4} = A_3 R \qquad ,$$

and this is enough to make (6) equal to (8). Now suppose the constraint is binding: this means that the number that multiplies the vector expressed in (10) is non-zero and so the two solutions can only coincide if the vector itself in (10) is zero. As this vector is the image of another one, by a positive definite operator, it ensues that

$$(I_n - \frac{1}{A_0} xx' M_2^{-1}) M_4 \alpha_K^{\otimes 3}$$

must be the null vector. As $M_4\alpha_K^{\otimes 3} \neq 0$, a little algebra shows this implies that, again, *the higher moment constraint is not binding.* As a consequence, the two solutions will *never* coincide.

The above proposition gives a more conclusive finish to results as some in Athayde and Flôres (2004), where a complete solution to the three moments portfolio problem is discussed and the (linear) manifold of "common" Markowitz and α_S solutions is characterised within the geometric structure of the solutions set in moments space. For the practitioner, it says that – apart from the zero (Lebesgue) measure set where the higher moment constraint is not binding – he will be incurring in error by not considering the higher moment. His "Markowitz weights" will certainly be sub-optimal. But, by how much ?

3. A "one non-zero kurtosis" example.

We shall exploit here the case when the fourth moment is considered, but the structure of the kurtosis tensor has only one non-negligible value, related to the marginal kurtosis of the first asset.

Two remarks are due before pursuing. First, as use of the qualifier "non-negligible" calls attention, kurtoses are usually non-zero[3]; what is at stake is which ones to consider as relevant, as signalling heavier (than normal) tails. This means that the analyst will be setting to zero all those values for which, taking *standardised* assets X and Y, for instance, moments like EX^4, EY^4, EXY^3, EX^2Y^2, etc, won't be very far from 3^4. The second is that this very assignment of "zero values" must be made in a consistent way. Considering the same two assets, if one decides not to disregard the two marginal kurtosis, very likely the cross-kurtosis EX^2Y^2 won't be possible to be discarded, and – though not necessarily – the same may apply for the pair EXY^3, EX^3Y. The moral contained in the two remarks is that a much deeper empirical knowledge of the (multivariate) assets distributions is required, for a sensible modelling of the base

[3] Though not very common, some co-kurtoses can be negative.
[4] The reader may consider that all entries in the (*standardised* assets') kurtosis tensor are subtracted from 3.

higher moments structure. Though this means more additional preparatory work, we consider it positive, as obliging a deeper knowledge of the market.

If only one kurtosis is non-zero, the problem in the previous section simplifies greatly, as the kurtosis constraint in (2) directly supplies the weight of the first asset:

$$\alpha_1 = (\sigma_{p^4} / \sigma_{4(1)})^{1/4} \quad , \qquad (12)$$

where the notation used for the relevant kurtosis is self-explanatory. Moreover, the crucial product

$$M_4 \alpha^{\otimes 3} \quad ,$$

becomes a vector of n-1 zeroes but for the first position, whose ordinate is:

$$\alpha_1^3 \sigma_{4(1)} = (\sigma_{p^4}^3 \sigma_{4(1)})^{1/4} \quad ,$$

a weighted geometric average of the two kurtoses at stake.

With these values in hand, one can quickly compute:

$$A_3 = (\sigma_{p^4}^3 \sigma_{4(1)})^{1/4} x' m_{.1} \qquad A_6 = (\sigma_{p^4}^3 \sigma_{4(1)})^{1/2} m_{11} \quad ,$$

where $m_{.1}$ and m_{11} stand, respectively for the first column and entry of matrix

$$M_2^{-1} = [m_{ij}] \quad .$$

It is now a standard matter to go to expressions (6) and (10) and compute the values of the remaining vector of optimal weights and the vector of differences. We shall concentrate on the latter. After not too cumbersome manipulations, one arrives at the following expression for Δ_K :

$$\Delta_K = \frac{\alpha_1 - Rx' m_{.1}}{A_0 m_{11} - (x' m_{.1})^2} [A_0 m_{.1} - (x' m_{.1}) x] \quad . \qquad (13)$$

The above expression conveys two relevant insights:

i) the vector of differences between the Markowitz and the "kurtosis" solution is determined by the structure of the first and second moments arrays; in the case of the latter, particularly the first column of its inverse;

ii) the influence of the kurtosis considered is through the scalar that multiplies all ordinates of the vector mentioned in i).

As, from (12):

$$\frac{\partial \alpha_1}{\partial \sigma_{4(1)}} = -\frac{1}{4\sigma_{4(1)}}(\sigma_{p^4} / \sigma_{4(1)})^{1/4} = -\frac{1}{4\sigma_{4(1)}}\alpha_1 \qquad , \qquad (14)$$

the sensitivity of the difference with respect to the kurtosis is easily found to be:

$$\frac{\partial}{\partial \sigma_{4(1)}}\Delta_K = -\frac{1}{4\sigma_{4(1)}}\frac{\alpha_1}{A_0 m_{11} - (x' m_{.1})^2}[A_0 m_{.1} - (x' m_{.1})x] = \qquad , \qquad (15)$$

$$= -\frac{1}{4}\frac{\sigma_{p^4}^{1/4}}{\sigma_{4(1)}^{5/4}}\frac{1}{A_0 m_{11} - (x' m_{.1})^2}[A_0 m_{.1} - (x' m_{.1})x] \qquad .$$

Changes at the vicinity of a given $\sigma_{4(1)}$ change the signs of the term in (13) which is multiplied by α_1 ; moreover, but for a factor of $1/4\sigma_{4(1)}$, they are of the same intensity as the term itself. They are also, in absolute terms, directly proportional to the set portfolio kurtosis[5], and indirectly, to the non-negligible marginal one.

The last interpretation may be linked to an interesting situation. Suppose the portfolio designer wants to include asset 1 which has a “too” heavy tail. He’s not against heavy tails but wants his portfolio to have a much more moderate one. He’ll then be in the situation of our example, (12) defining a low α_1. If this value can be considered quite small with respect to the relevant elements in (13), he’ll design an optimal portfolio distant from the Markowitz one of:

$$\Delta_K \cong \frac{-Rx' m_{.1}}{A_0 m_{11} - (x' m_{.1})^2}[A_0 m_{.1} - (x' m_{.1})x] \qquad , \qquad (16)$$

a vector invariant to actual values of both kurtoses, provided their ratio allows to discard the corresponding term in (13). Clearly, if a high *R* is aimed at, the differences might be rather significant.

Finally, still under this assumption, the difference between the weights allotted to the riskless asset (in the Markowitz *less* the kurtosis solution) will simply be:

[5] More precisely, its fourth root (or standardised version).

$$\frac{-Rx'm_{.1}}{A_0 m_{11} - (x'm_{.1})^2}[A_0([1]'m_{.1}) - (x'm_{.1})([1]'x)] \quad . \tag{17}$$

Irrespective of this last case, it is worth reminding that, from (8) and (12), it should be expected that at least the first weight will always be significantly different from the Markowitz one. Anyhow, the set of formulas in this section provides a complete toolkit to analyse the effects of the "one non-zero kurtosis" solution.

4. Concluding remarks.

We have presented explicit formulas for evaluating the difference between Markowitz weights and those from optimal portfolios, with the same given return, considering either asymmetry or kurtosis. We proved that, whenever the higher moment constraint is not binding, the weights are *never* the same.

If, even by special features of the first and second moments, the difference, though not the null vector, might be negligible, in quite many cases it will be very significant. This is fully exemplified in a simple and appealing case, when the designer wants to incorporate an asset with quite heavy tails, but wants to moderate this effect.

The results add further support that Markowitz weights are not robust to the introduction of higher moments.

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