I.S.S.N: 1885-6888

ECONOMIC ANALYSIS WORKING PAPER SERIES

Environmental Efficiency Measurement with Translog Distance Functions: A Parametric Approach

♦

Rafael A. Cuesta, C.A.K. Knox Lovell and José L. Zofío

Working Paper 2/2007

UNIVERSIDAD AUTONOMA DE MADRID

DEPARTAMENTO DE ANÁLISIS ECONÓMICO: TEORÍA ECONÓMICA E HISTORIA ECONÓMICA

Environmental Efficiency Measurement with Translog Distance Functions: A Parametric Approach

Rafael A. Cuesta[a] , C.A. Knox Lovell[b] and José L. Zofío[c], [*]

[a] *Caja de Ahorros de Asturias and Departamento de Economía, Universidad de Oviedo, E-33071 Oviedo, Spain.*
[b] *Department of Economics, Terry College of Business, University of Georgia, Athens, GA 30602, USA*
[c] *Departamento de Análisis Económico: Teoría Económica e Historia Económica, Universidad Autónoma de Madrid, E-28049 Cantoblanco, Madrid, Spain.*

December, 2006

Abstract

We use a flexible parametric hyperbolic distance function to estimate environmental efficiency when some outputs are undesirable. Cuesta and Zofio (*J. Prod. Analysis* (2005), 31-48) introduced this distance function specification in conventional input-output space to estimate technical efficiency within a stochastic frontier context. We extend their approach to accommodate undesirable outputs and to estimate environmental efficiency within a stochastic frontier context. This provides a parametric counterpart to Färe *et al.*'*s* popular nonparametric environmental efficiency measures (*Rev. Econ. Stat.* 75 (1989), 90-98). The distance function model is applied to a panel of U.S. electricity generating units that produce marketed electricity and non-marketed SO_2 emissions.

Keywords: Undesirable outputs, parametric distance functions, stochastic frontier analysis, environmental efficiency.
JEL Classification: C32, D25, L95.

[*]Rafael A. Cuesta. Tel: +34 985104879; fax: +34 985104871; e–mail: racuesta@uniovi.es.
C.A. Knox Lovell. E–mail: knox@terry.uga.edu
José L. Zofío. Tel: +34 913482406; fax: +34 913978616; e–mail: jose.zofio@uam.es
We are grateful to William Weber for providing us with the database on U.S. electric utilities that we employ in our empirical application. The earliest draft dates back to September 2003.

1. Introduction

Most firms produce multiple outputs that are difficult or undesirable to aggregate. This necessitates replacing production functions with distance functions in a primal analysis of producer performance. Input- and output-oriented distance functions, introduced by Debreu (1951), Malmquist (1953) and Shephard (1953,1970), are now the cornerstones of primal analysis of producer performance. Nonparametric distance functions (Charnes *et al.* (1978)) dominate empirical analysis, although flexible parametric distance functions have been employed (Lovell *et al.* (1994), Paul *et al.* (2000)).

A particularly significant example of multiple output production involves the simultaneous production of desirable marketed outputs and undesirable, typically non-marketed, byproducts such as emissions and pollutants. Because byproducts are rarely marketed, they are rarely priced, and so environmental performance analysis is frequently based on a primal representation of technology. However conventional distance functions are not well suited for environmental performance analysis because they measure performance radially, in terms of the ability to expand all outputs (or contract all inputs) equiproportionately. They do not discriminate between desirable outputs and their undesirable byproducts. As Zofío and Prieto (2001;67) remark, output distance functions treat the two sets of outputs symmetrically –a business as usual strategy, while what is required is a distance function that treats desirable and undesirable outputs asymmetrically.

Färe *et al.* (1985) introduced such a distance function, a hyperbolic distance function that measures producer performance in terms of the ability to expand outputs and contract inputs equiproportionately. Conventional radial distance functions are oriented toward expanding outputs or contracting inputs, and so are special cases of hyperbolic distance functions. Later Färe *et al.* (1989) (FGLP) adapted a nonparametric hyperbolic distance function to the measurement of environmental performance. This enabled them to treat desirable and undesirable outputs asymmetrically, by measuring environmental performance in terms of the ability to expand desirable outputs and contract undesirable byproducts equiproportionately. A more recent choice when treating outputs and/or inputs asymmetrically can be found in Chambers *et al.* (1996), who introduced an alternative characterization of the production technology by way of the directional distance function. Chung *et al.* (1997) presented the first extension of this distance function for environmental efficiency measurement.

These theoretical breakthroughs have spawned a growing literature in environmental performance analysis. Beyond the seminal contributions, empirical applications based on hyperbolic distance functions include Ball *et al.* (1994, 2004), Hernández-Sancho, Picazo-Tadeo and Reig-Martinez (2000), Zaim and Taskin (2000), Zofío and Prieto (2001), Prieto and Zofío (2004). Among those using a directional distance function we find Weber and Domazlicky (2001), Domazlicky and Weber (2004) and Picazo-Tadeo, Reig-Martinez Hernández-Sancho (2005). All these applications have one common feature. They have been developed within a nonparametric framework that relies on mathematical programming techniques to calculate the hyperbolic distance function. These techniques extend Data Envelopment Analysis (DEA) to identify those producers that are environmentally efficient, and form convex combinations of them to construct a best practice environmental performance frontier for the remaining inefficient producers. However these techniques have two drawbacks: (i) except under constant returns to scale the program is nonlinear, and (ii) the model being deterministic, inference is not possible without bootstrapping (Simar and Wilson, 2004). These drawbacks motivate the use of stochastic frontier techniques to estimate a hyperbolic distance function.

However the use of stochastic frontier techniques to estimate a hyperbolic distance function has been stalled because the existing output- or input-oriented parametric specifications do not allow for an asymmetric treatment of desirable and undesirable outputs.[1] A step in the right direction has been taken by Färe *et al*. (2005), who use mathematical programming techniques to construct a parametric (quadratic) directional distance function to assess the ability of firms to improve their environmental efficiency by simultaneously increasing desirable outputs and reducing undesirable outputs. This model is easy to implement, but it remains vulnerable to the second drawback above.

We extend a recent contribution of Cuesta and Zofio (2005) to develop a hyperbolic distance function model that is both parametric and stochastic. This model is based on a translog specification of production technology introduced by Christensen *et al.* (1971, 1973). It provides a flexible parametric and stochastic counterpart to the influential FGLP (1989) model that is nonparametric and deterministic. It also provides a stochastic and hyperbolic alternative to the Färe *et al.* (2005) model, which is directional and deterministic.

The structural difference between our model and that of Färe *et al.* (2005) is the use of different distance functions. While their model is based on a directional distance function represents the amount by which desirable outputs can be expanded and undesirable outputs and/or inputs can be contracted in an additive manner, our hyperbolic distance function represents the proportion by which desirable outputs can be expanded and undesirable outputs and/or inputs can be contracted in a multiplicative manner. The different properties that these distance functions satisfy have an important influence on their parametric specification. The directional distance

[1] Several authors had tried to overcome the lack of analytical tools in the parametric field proposing SFA alternatives that nevertheless did not treat outputs asymmetrically, *e.g.* Reinhard *et al.* (1999) and Murty and Kumar (2002).

function satisfies a translation property, which can be imposed easily on a quadratic specification. The hyperbolic distance function counterpart of this property is the almost homogeneity property, which can be imposed easily on a translog specification.

In Section 2 we introduce the analytical foundations of the production technology, and we emphasize the properties a hyperbolic distance function oriented toward environmental performance measurement should satisfy. Compared to a conventional output distance function, our hyperbolic specification allows desirable and undesirable outputs to vary in the same proportion, but in opposite directions. We enhance the hyperbolic definitions by allowing for further proportional reduction of inputs. As our intention is to provide the parametric counterparts to the nonparametric distance functions proposed by FGLP (1989), in Section 3 we introduce a pair of translog environmental hyperbolic distance function formulations. In Section 4 we develop the empirical specification and the estimation procedure, which is based on the maximum likelihood panel data model of Pitt and Lee (1981), as extended by Battese and Coelli (1988). In Section 5 we provide an empirical application involving a large database of U.S. electric utilities previously analyzed by Färe *et al.* (2005), in which the desirable output is electricity generated and the undesirable byproduct is SO_2 emissions. Finally, some conclusions are drawn in Section 6.

2. Hyperbolic Distance Functions and Environmental Efficiency

We consider a production technology transforming input vectors $x_i = (x_{1i}, ..., x_{Ki}) \in \Re^K_+$ into output vectors $u_i = (u_{1i}, ..., u_{Vi}) \in \Re^P_+$, consisting of desirable and undesirable output subvectors $v_i = (v_{1i}, ..., v_{Mi}) \in \Re^M_+$ and $w_i = (w_{1i}, ..., w_{Si}) \in \Re^R_+$, and where the subscript $i = (1,2,...,N)$ refers to a set of observed producers.[2] The technology can be represented by the production possibility set

$$\mathrm{T}=\left\{(x, v, w): x \in \Re^K_+, (v, w) \in \Re^P_+, x \text{ can produce } (v, w)\right\}, \tag{1}$$

which is assumed to be a compact set satisfying the axioms found in Färe and Primont (1995). This production structure can be expressed in equivalent terms through the output correspondences, $x \rightarrow \mathrm{P}(x) \subseteq \Re^P_+$, which represents the set of all $u = (v, w)$ output vectors obtainable from x. This output correspondence is inferred from the production possibility set as $\mathrm{P}(x)=\{(v, w): (x, v, w) \in \mathrm{T}\}$, while the graph can be inferred from the output correspondence as $\mathrm{T} = \{(x, v, w): (v, w) \in \mathrm{P}(x), x \in \Re^K_+\}$.

[2] As we introduce the parametric counterpart to Färe *et al.* (1989) we adopt their notation to ease comparability.

Thus, relying on Färe, Grosskopf and Lovell (1985:46), it is verified that $(x, v, w) \in T \Leftrightarrow (v, w) \in P(x)$.

The production technology also can be represented by a hyperbolic distance function, which for a given amount of inputs represents the maximum expansion of the desirable output vector and equiproportionate contraction of the undesirable output vector that places a producer on the boundary of the technology T.

Definition 1: The hyperbolic distance function D_H: $\Re_+^K \times \Re_+^M \times \Re_+^R \rightarrow \Re_+ \cup \{+\infty\}$ is defined by

$$D_H(x, v, w) = \inf\{\theta > 0 : (x, v/\theta, w\theta) \in T\}. \tag{2}$$

The hyperbolic distance function inherits its name from the hyperbolic path that it follows toward the production frontier. It has the virtue of treating desirable and undesirable outputs asymmetrically, thus providing an environmentally friendly characterization of the production technology. The range of the hyperbolic distance function is $0 < D_H(x, v, w) \leq 1$. If the technology satisfies the customary axioms, then the hyperbolic distance function satisfies the following properties: (i) it is almost homogeneous (Aczel (1966, Chs.5,7), Lau (1972)), D_H.1: $D_H(x, \mu w, \mu^{-1}w) = \mu D_H(x, v, w)$, $\mu > 0$, (ii) non-decreasing in desirable outputs, D_H.2: $D_H(x, \lambda v, w) \leq D_H(x, v, w)$, $\lambda \in [0,1]$, (iii) non-increasing in undesirable outputs, D_H.3: $D_H(x, v, \lambda w) \leq D_H(x, v, w)$, $\lambda \geq 1$, and (iv) non-increasing in inputs, D_H.4: $D_H(\lambda x, v, w) \leq D_H(x, v, w)$, $\lambda \geq 1$.

A simpler characterization of the technology is provided by Shephard's (1970) output distance function, which represents the maximum feasible expansion of the desirable output vector required to reach the boundary of the technology set T.

Definition 2: The output distance function D_O: $\Re_+^K \times \Re_+^M \times \Re_+^R \rightarrow \Re_+ \cup \{+\infty\}$ is defined by

$$D_O(x, v, w) = \inf\{\varphi > 0 : (x, v/\varphi, w) \in T\} \tag{3}$$

The output distance function has range $0 < D_O(x, v, w) \leq 1$. It is homogeneous of degree one in outputs, (i) D_O.1: $D_O(x, \mu v, w) = \mu D_O(x, v, w)$, $\mu > 0$, (ii) non-decreasing in outputs, D_O.2: $D_O(x, \lambda v, w) \leq D_O(x, v, w)$, $\lambda \in [0,1]$, while (iii-iv) it is non-increasing in undesirable outputs and inputs, D_O.3: $D_O(x, v, \lambda w) \leq D_O(x, v, w)$, $\lambda \geq 1$, and D_O.4: $D_O(\lambda x, v, w) \leq D_O(x, v, w)$, $\lambda \geq 1$.

Finally, to cover all the alternative efficiency measures defined by FGLP (1989) −except for those that define the technology ignoring undesirable outputs− we can also represent technology with an enhanced hyperbolic distance function, which retains its environmental interpretation, but does not hold inputs constant as its hyperbolic and output counterparts so, calling for further proportional reductions on the inputs side.

Definition 3: The enhanced hyperbolic distance function D_E: $\Re_+^K \times \Re_+^M \times \Re_+^R \to \Re_+ \cup \{+\infty\}$ is defined as

$$D_E(x,v,w) = \inf\{\phi > 0 : (x\phi, v/\phi, w\phi) \in T\}. \qquad (4)$$

As the previous functions, it has range is $0 < D_E(x, v, w) \leq 1$, and besides the last three properties already stated for the hyperbolic distance function $D_H.2$–$D_H.4$, it satisfies a more inclusive degree of almost homogeneity given by $D_E.1$: $D_E(\mu^{-1}x, \mu v, \mu^{-1}w) = \mu D_E(x, v, w)$, $\mu > 0$,

Definition 4: A function $F(x, v, w)$ is almost homogeneous of degrees k_1, k_2, k_3 and k_4 if

$$F(\mu^{k_1}x, \mu^{k_2}v, \mu^{k_3}w) = \mu^{k_4}F(x,v,w), \quad \forall \mu > 0. \qquad (5)$$

The output distance function $D_O(x,v,w)$ is almost homogeneous of degrees 0, 1, 0, 1. The environmental hyperbolic distance function $D_H(x,v,w)$ is almost homogeneous of degrees 0, 1, -1, 1, and the enhanced environmental hyperbolic distance function $D_E(x,v,w)$ is almost homogeneous of degrees –1, 1, -1, 1.[3]

As FGLP (1989) discuss, the hyperbolic and enhanced hyperbolic distance functions (2) and (4) are well suited for defining measures of environmental efficiency. Our hyperbolic distance function (2) corresponds to their *hyperbolic output efficiency measure*, while our enhanced hyperbolic distance function (4) corresponds to their *hyperbolic productive efficiency measure*.[4] Since both distance functions fully characterize the technology assuming weak disposability, $D_H(x,v,w) \leq 1 \Leftrightarrow (x,v,w) \in T$ and $D_E(x,v,w) \leq 1 \Leftrightarrow (x,v,w) \in T$, their magnitudes signaling

---

[3] Cuesta and Zofio (2005;34) prove the almost homogeneity property when the hyperbolic distance function is defined ignoring undesirable outputs: $D_H(x,v) = \inf\{\delta > 0 : (x\delta, v/\delta) \in T\}$, which can be easily extended to $D_H(x,v,w)$ and the remaining distance functions.

[4] Our version of the output distance function (3) differs from the FGLP (1989;93) *conventional hyperbolic output efficiency measure*, which excludes undesirable outputs from the technology set and is defined as $D_O(x,v) = \inf\{\varphi > 0 : (x, v/\varphi) \in T\}$

whether a producer belongs to the isoquant subset of T: Isoq P(x) = {(v, w): (v, w) $\in$ P(x), (v/λ, $w\lambda$) $\notin$ P(x), 0 < λ < 1} or Isoq T = {(x, v, w): (x, v, w) $\in$ T, ($x\lambda$, v/λ, $w\lambda$) $\notin$ T, 0 < λ < 1}. Thus if D_H (x, v, w) = 1 or D_E (x, v, w) = 1, the production occurs respectively on Isoq P(x) or Isoq T, and is said to be weakly efficient. Alternatively, if D_H (x, v, w) < 1 or D_E (x, v, w) < 1, the producer could improve environmental performance by expanding production of marketed outputs and reducing undesirable pollutants and inputs, and is said to be inefficient.

3. Translog Hyperbolic Distance Functions

In this section we develop three specifications of a hyperbolic translog distance function. This popular functional form provides a flexible approximation to the unknown production technology, and it proves to be quite amenable to the imposition of almost homogeneity restrictions.

We depart from Definition 4. Assuming that $F(x, v, w)$ is continuously differentiable, to be almost homogeneous it must satisfy

$$\mathrm{k}_1 \sum_{k=1}^{\mathrm{K}} \frac{\partial F}{\partial x_k} x_k + \mathrm{k}_2 \sum_{m=1}^{\mathrm{M}} \frac{\partial F}{\partial v_m} v_m + \mathrm{k}_3 \sum_{r=1}^{\mathrm{R}} \frac{\partial F}{\partial w_r} w_r = \mathrm{k}_4 F \,. \tag{6}$$

For a translog specification of $F(x,v,w)$,

$$\begin{aligned} \ln F = {} & \alpha_0 + \sum_{k=1}^{\mathrm{K}} \alpha_k \ln x_{ki} + \frac{1}{2} \sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{K}} \alpha_{kl} \ln x_{ki} \ln x_{li} + \sum_{m=1}^{\mathrm{M}} \beta_m \ln v_{mi} + \frac{1}{2} \sum_{m=1}^{\mathrm{M}} \sum_{n=1}^{\mathrm{M}} \beta_{mn} \ln v_{mi} \ln v_{ni} + \\ & \sum_{r=1}^{\mathrm{R}} \chi_r \ln w_{ri} + \frac{1}{2} \sum_{r=1}^{\mathrm{R}} \sum_{s=1}^{\mathrm{R}} \chi_{rs} \ln w_{ri} \ln w_{si} + \sum_{k=1}^{\mathrm{K}} \sum_{m=1}^{\mathrm{M}} \delta_{km} \ln x_{ki} \ln v_{mi} + \sum_{k=1}^{\mathrm{K}} \sum_{r=1}^{\mathrm{R}} \varsigma_{kr} \ln x_{ki} \ln w_{ri} + \\ & \sum_{m=1}^{\mathrm{M}} \sum_{r=1}^{\mathrm{R}} \upsilon_{mr} \ln v_{mi} \ln w_{ri} , \quad (i = 1,2,...,\mathrm{N}), \end{aligned} \tag{7}$$

we can focus on the relevant expressions needed to impose the alternative homogeneity degrees corresponding to the hyperbolic (2), output (3), and enhanced hyperbolic (4) distance functions. Dividing (6) by F, and noting that with a logarithmic specification $\frac{\partial F}{\partial x_k} \frac{x_k}{F} = \frac{\partial \ln F}{\partial \ln x_k}$, $\frac{\partial F}{\partial v_m} \frac{v_m}{F} = \frac{\partial \ln F}{\partial \ln v_m}$, and $\frac{\partial F}{\partial w_r} \frac{w_r}{F} = \frac{\partial \ln F}{\partial \ln w_r}$, the relevant partial derivatives for the translog case (7) are

$$\frac{\partial \ln F}{\partial \ln x_m} = \alpha_k + \sum_{k=1}^{K} \alpha_{kl} \ln x_l + \sum_{m=1}^{M} \delta_{km} \ln v_m + \sum_{r=1}^{R} \varsigma_{kr} \ln w_r \quad (k = 1,2,...,K), \tag{8}$$

$$\frac{\partial \ln F}{\partial \ln v_m} = \beta_m + \sum_{n=1}^{M} \beta_{mn} \ln v_n + \sum_{k=1}^{K} \delta_{km} \ln x_k + \sum_{r=1}^{R} \upsilon_{mr} \ln w_r \quad (m = 1,2,...,M), \text{ and} \tag{9}$$

$$\frac{\partial \ln F}{\partial \ln w_r} = \chi_r + \sum_{s=1}^{R} \chi_{rs} \ln w_s + \sum_{k=1}^{K} \varsigma_{kr} \ln x_k + \sum_{m=1}^{M} \upsilon_{mr} \ln v_m \quad (r = 1,2,...,R). \tag{10}$$

Here we derive the parametric formulation corresponding to the hyperbolic distance function, but we also include at the end of this section expressions corresponding to its output and enhanced hyperbolic distance functions counterparts, which can be easily obtained following the same steps. For the translog hyperbolic distance function (2), almost homogeneity of degrees 0, 1, -1, 1 must be satisfied. Departing from (6), this requires

$$\sum_{m=1}^{M} \frac{\partial \ln F}{\partial \ln v_m} - \sum_{r=1}^{R} \frac{\partial \ln F}{\partial \ln w_r} = 1. \tag{11}$$

For the translog case substituting (9) and (10) into (11) yields

$$\begin{aligned} &\sum_{m=1}^{M} (\beta_m + \sum_{n=1}^{M} \beta_{mn} \ln v_n + \sum_{k=1}^{K} \delta_{km} \ln x_k + \sum_{r=1}^{R} \upsilon_{mr} \ln w_r) - \\ &\quad \sum_{r=1}^{R} (\chi_r + \sum_{s=1}^{R} \chi_{rs} \ln w_s + \sum_{k=1}^{K} \varsigma_{kr} \ln x_k + \sum_{m=1}^{M} \upsilon_{mr} \ln v_m) = 1. \end{aligned} \tag{12}$$

From (12) the necessary (1+M+K+R) restrictions that ensure almost homogeneity of degrees 0, 1, -1, 1 are

$$\sum_{m=1}^{M} \beta_m - \sum_{r=1}^{R} \chi_r = 1, \tag{13}$$

$$\sum_{n=1}^{M} \beta_{mn} - \sum_{m=1}^{M} \upsilon_{mr} = 0, \quad m = 1,2,...,M, \tag{14}$$

$$\sum_{k=1}^{K} \delta_{km} - \sum_{k=1}^{K} \varsigma_{kr} = 0, \quad k = 1,2,...,K, \text{ and} \tag{15}$$

$$\sum_{r=1}^{R} \upsilon_{mr} - \sum_{s=1}^{R} \chi_{rs} = 0, \quad r = 1, 2, ..., R. \tag{16}$$

It is possible to impose this set of restrictions on the translog hyperbolic distance function by modifying the approach introduced by Lovell *et al.* (1994). Using the almost homogeneity condition (5) and choosing the M^{th} desirable output for normalizing purposes, $\mu=1/v_M$, and we obtain

$$D_H\left(x, \frac{v}{v_M}, w v_M\right) = \frac{D_H(x,v,w)}{v_M}, \tag{17}$$

which yields

$$\begin{aligned}\ln(D_{Hi}/v_{Mit}) &= \alpha_0 + \sum_{k=1}^{K}\alpha_k \ln x_{ki} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\alpha_{kl}\ln x_{ki}\ln x_{li} + \sum_{m=1}^{M-1}\beta_m \ln v^*_{mi} + \frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1}\beta_{mn}\ln v^*_{mi}\ln v^*_{ni} + \\ &\sum_{r=1}^{R}\chi_r \ln w^*_{ri} + \frac{1}{2}\sum_{r=1}^{R}\sum_{s=1}^{R}\chi_{rs}\ln w^*_{ri}\ln w^*_{si} + \sum_{k=1}^{K}\sum_{m=1}^{M-1}\delta_{km}\ln x_{ki}\ln v^*_{mi} + \sum_{k=1}^{K}\sum_{r=1}^{R}\varsigma_{kr}\ln x_{ki}\ln w^*_{ri} + \\ &\sum_{m=1}^{M-1}\sum_{r=1}^{R}\upsilon_{mr}\ln v^*_{mi}\ln w^*_{ri}, \quad (i = 1,2,...N),\end{aligned} \tag{18}$$

where $v^*_{mi} = v_{mi}/v_{Mi}$ and $w^*_{ri} = w_{ri}v_{Mi}$. For the normalizing output v_{Mi} the ratio v^*_{mi} is equal to one, and so all terms involving the normalizing output are null. This does not occur for undesirable outputs, which is why the summations involving v^*_{mi} in (18) are over M–1, while summations involving w^*_{ri} are over R. It is straightforward to verify that the translog hyperbolic distance function satisfies properties D_H.1- D_H.4.

As previously anticipated, we can follow the same procedure with regard to the almost homogeneity restrictions and specific conditions that must be satisfied to obtain the translog output (3) and enhanced hyperbolic (4) distance functions. The expression for the translog output distance function is

$$\ln\left(\mathrm{D}_{\mathrm{O}i} / v_{\mathrm{M}it}\right) = \alpha_0 + \sum_{k=1}^{\mathrm{K}} \alpha_k \ln x_{ki} + \frac{1}{2}\sum_{k=1}^{\mathrm{K}}\sum_{l=1}^{\mathrm{K}} \alpha_{kl} \ln x_{ki} \ln x_{li} + \sum_{m=1}^{\mathrm{M-1}} \beta_m \ln v^*_{mi} + \frac{1}{2}\sum_{m=1}^{\mathrm{M-1}}\sum_{n=1}^{\mathrm{M-1}} \beta_{mn} \ln v^*_{mi} \ln v^*_{ni} +$$
$$\sum_{r=1}^{\mathrm{R}} \chi_r \ln w_{ri} + \frac{1}{2}\sum_{r=1}^{\mathrm{R}}\sum_{s=1}^{\mathrm{R}} \chi_{rs} \ln w_{ri} \ln w_{si} + \sum_{k=1}^{\mathrm{K}}\sum_{m=1}^{\mathrm{M-1}} \delta_{km} \ln x_{ki} \ln v^*_{mi} + \sum_{k=1}^{\mathrm{K}}\sum_{r=1}^{\mathrm{R}} \varsigma_{kr} \ln x_{ki} \ln w_{ri} + \qquad (19)$$
$$\sum_{m=1}^{\mathrm{M-1}}\sum_{r=1}^{\mathrm{R}} \upsilon_{mr} \ln v^*_{mi} \ln w_{ri}, \quad (i = 1,2,\ldots\mathrm{N})$$

while the expression for the translog enhanced hyperbolic distance function is

$$\ln\left(\mathrm{D}_{\mathrm{E}i} / v_{\mathrm{M}it}\right) = \alpha_0 + \sum_{k=1}^{\mathrm{K}} \alpha_k \ln x^*_{ki} + \frac{1}{2}\sum_{k=1}^{\mathrm{K}}\sum_{l=1}^{\mathrm{K}} \alpha_{kl} \ln x^*_{ki} \ln x^*_{ki} + \sum_{m=1}^{\mathrm{M-1}} \beta_m \ln v^*_{mi} + \frac{1}{2}\sum_{m=1}^{\mathrm{M-1}}\sum_{n=1}^{\mathrm{M-1}} \beta_{mn} \ln v^*_{mi} \ln v^*_{ni} +$$
$$\sum_{r=1}^{\mathrm{R}} \chi_r \ln w^*_{ri} + \frac{1}{2}\sum_{r=1}^{\mathrm{R}}\sum_{s=1}^{\mathrm{R}} \chi_{rs} \ln w^*_{ri} \ln w^*_{si} + \sum_{k=1}^{\mathrm{K}}\sum_{m=1}^{\mathrm{M-1}} \delta_{km} \ln x^*_{ki} \ln v^*_{mi} + \sum_{k=1}^{\mathrm{K}}\sum_{r=1}^{\mathrm{R}} \varsigma_{kr} \ln x^*_{ki} \ln w^*_{ri} + \qquad (20)$$
$$\sum_{m=1}^{\mathrm{M-1}}\sum_{r=1}^{\mathrm{R}} \upsilon_{mr} \ln v^*_{mi} \ln w^*_{ri}, \quad (i = 1,2,\ldots\mathrm{N})$$

where $x^*_{ki} = x_{ki}v_{\mathrm{M}i}$.

4. Implementing the Translog Hyperbolic Distance Function through SFA

In a stochastic framework one may think of the distance that separates a producer from the production frontier as the combined result of inefficiency and random noise reflecting events beyond producers' control. Enhancing our model to allow for a multi-period framework, the three stochastic translog panel data specifications can be formulated as

$$\ln\left(\mathrm{D}_{\mathrm{H}i} / v_{\mathrm{M}it}\right) = TL(x_{it}, v^*_{it}, w^*_{it}; \alpha, \beta, \chi, \delta, \varsigma, \upsilon) + \omega_{it} \quad (i = 1,2,\ldots,\mathrm{N}), \qquad (21)$$

$$\ln\left(\mathrm{D}_{\mathrm{O}i} / v_{\mathrm{M}it}\right) = TL(x_{it}, v^*_{it}, w_{it}; \alpha, \beta, \chi, \delta, \varsigma, \upsilon) + \omega_{it} \quad (i = 1,2,\ldots,\mathrm{N}), \qquad (22)$$

$$\ln\left(\mathrm{D}_{\mathrm{E}i} / v_{\mathrm{M}it}\right) = TL(x^*_{it}, v^*_{it}, w^*_{it}; \alpha, \beta, \chi, \delta, \varsigma, \upsilon) + \omega_{it} \quad (i = 1,2,\ldots,\mathrm{N}), \qquad (23)$$

for the hyperbolic (18), output (19) and enhanced hyperbolic (20) distance functions. In these formulations deviations from one are accommodated in a composed error $\mathrm{h}(\varepsilon_{it}) = \exp(u_i + \omega_{it})$ (Aigner *et al*. (1977)), comprising the one-sided component u_i that captures time invariant inefficiency, which is assumed to have a half normal distribution $u_i \sim |N(0,\sigma_u^2)|$, and the standard random term symmetrically distributed around zero, ω_{it} ~N(0, σ_v^2). Since – ln $v_{\mathrm{M}it}$ corresponds to the dependent variable and $\ln D_{\mathrm{H}i}$, $\ln D_{\mathrm{O}i}$ and $\ln D_{\mathrm{E}i}$ are the one sided distance components u_i, these expressions can be reformulated to obtain the actual hyperbolic, output, and enhanced hyperbolic distance functions to be estimated:

$$-\ln v_{\mathrm{M}it} = TL\,(x_{it}, v_{it}^*, w_{it}^*; \alpha, \beta, \chi, \delta, \varsigma, \upsilon) + (\omega_{it} - u_i) \quad (i = 1,2,...,\mathrm{N}), \tag{24}$$

$$-\ln v_{\mathrm{M}it} = TL\,(x_{it}, v_{it}^*, w_{it}; \alpha, \beta, \chi, \delta, \varsigma, \upsilon) + (\omega_{it} - u_i) \quad (i = 1,2,...,\mathrm{N}), \text{ and} \tag{25}$$

$$-\ln v_{\mathrm{M}it} = TL\,(x_{it}^*, y_{it}^*, z_{it}^*; \alpha, \beta, \chi, \delta, \varsigma, \upsilon) + (\omega_{it} - u_i) \quad (i = 1,2,...,\mathrm{N}). \tag{26}$$

We estimate these panel data specifications using standard maximum-likelihood techniques introduced by Pitt and Lee (1981) and extended by Battese and Coelli (1988) to obtain the individual conditional distribution of the one sided errors, $E(u_i | \varepsilon_{it})$. Finally, time invariant hyperbolic efficiency estimates can be calculated for each firm substituting these values into the following expressions:

$$\mathrm{TE}_i = \exp[\ln D_{\mathrm{H}i}\,(x_{it}, v_{it}^*, w_{it}^*; \alpha, \beta, \chi, \delta, \varsigma, \upsilon)] = \exp(-u_i), \tag{27}$$

$$\mathrm{TE}_i = \exp[\ln D_{\mathrm{O}i}\,(x_{it}, v_{it}^*, w_{it}; \alpha, \beta, \chi, \delta, \varsigma, \upsilon)] = \exp(-u_i), \text{ and} \tag{28}$$

$$\mathrm{TE}_i = \exp[\ln D_{\mathrm{E}i}\,(x_{it}, v_{it}^*, w_{it}; \alpha, \beta, \chi, \delta, \varsigma, \upsilon)] = \exp(-u_i). \tag{29}$$

5. An application to SO_2 emissions from electric utilities

5.1 Data and model

We illustrate the translog hyperbolic, output and enhanced hyperbolic distance functions by calculating the efficiency scores for a set of U.S. electric utilities. Firm level annual data refers to particular boilers whose technology is represented by one desirable output, megawatt hours of electricity generated, MWh (v), one undesirable output, tons of SO_2 emissions (z), and three inputs: generating capacity in mill. MW (x_1), homogenous fuel measured in million BTU (x_2) and units of labor (x_3). Electricity production, generating capacity, and fuel consumption data come from the *Annual Steam Electric Unit Operation and Design Report*, published within the Department of Energy by the Energy Information Administration, EIA767. SO_2 emissions are available from the Acid Rain Program database compiled by the Environmental Protection Agency. With regard to the last input, labor data is reported by the Federal Energy Regulatory Commission in its *Electric Utility Annual Report*. Further details on the different assumptions that have been made to elaborate these variables and how this database has been assembled, can be found in Färe *et al.* (2005). Table 1 shows the mean firm values over the 1993 and 1997 period and overall descriptive statistics for each variable.

Table 1. Mean firm values and overall descriptive statistics

Variable	Mean	Standard Dev.	Minimum	Maximum
v – MWh	1,774,512	1,401,741	4,352	7,933,261
w – SO_2 Tons	25,419	26,902	3	201,667
x_1 – MWatt	348	249	19	1,300
x_2 – Mill. BTU	17,704,876	13,482,714	47,659	77,800,003
x_3 – Units	278	234	2	1,282

Source: Färe *et al.* (2005).

The particular translog hyperbolic, output and enhanced hyperbolic distance function that have been estimated are the counterparts to (24), (25) and (26), but allowing for a time dummy that captures the presence of neutral technical change from 1993 to 1997, as well as other temporal effects. Specifically,

$$-\ln v_{it} = \Bigg[\alpha_0 + \sum_{k=1}^{3} \alpha_k \ln x_{kit} + \frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3} \alpha_{kl} \ln x_{kit} \ln x_{lit} + \chi_1 \ln w_{it}^{*} + \chi_{11} \ln w_{it}^{*} \ln w_{it}^{*} + \sum_{k=1}^{3} \varsigma_{kr} \ln x_{kit} \ln w_{it}^{*} + \psi_\tau d_\tau \Bigg] + u_i + \omega_{it} \,, \tag{30}$$

$$-\ln v_{it} = \Bigg[\alpha_0 + \sum_{k=1}^{3} \alpha_k \ln x_{kit} + \frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3} \alpha_{kl} \ln x_{kit} \ln x_{lit} + \chi_1 \ln w_{it} + \chi_{11} \ln w_{it} \ln w_{it} + \sum_{k=1}^{3} \varsigma_{kr} \ln x_{kit} \ln w_{it} + \psi_\tau d_\tau \Bigg] + u_i + \omega_{it} \,, \tag{31}$$

and

$$-\ln v_{it}=\left[\alpha_0+\sum_{k=1}^{3}\alpha_k\ln x_{kit}^{*}+\frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3}\alpha_{kl}\ln x_{kit}^{*}\ln x_{lit}^{*}+\right.$$
$$\left.\chi_1\ln w_{it}^{*}+\chi_{11}\ln w_{it}^{*}\ln w_{it}^{*}+\sum_{k=1}^{3}\varsigma_{kr}\ln x_{kit}^{*}\ln w_{it}^{*}+\psi_{\tau}d_{\tau}\right]+u_i+\omega_{it}\;. \tag{32}$$

To avoid convergence problems and ease parameters interpretation, all variables have been corrected prior to estimation, *i.e.* each output –desirable and undesirable– and input variables are divided by their geometric mean. Proceeding this way, first order coefficients can be regarded as distance elasticities evaluated at the sample means. Finally, since for this particular application there is just one desirable output, the almost homogeneity conditions are imposed using electricity production values.

5.2. Results and discussion

Table 2 presents the obtained maximum likelihood estimates of the alternative stochastic models. These MLE parameters for the hyperbolic (30), output (31) and enhanced hyperbolic (32) distance functions' specifications, and their associated standard errors allow us to determine (a) the effect that the undesirable output and the inputs have on the distance functions, and (b) whether the magnitude corresponding to each direct partial elasticity is statistically significant or not. In all three formulations the undesirable output parameters χ_1 present the expected negative sign as any increase in sulfur dioxide emissions would increase the value of the distance functions. A similar reasoning applies to generating capacity, fuel, and labor inputs $-\alpha_k$, as any increment in their amounts would also increase the distance to the frontier. Furthermore, except for the generating capacity elasticity α_1 –an expected result when dealing with utilities whose maximum installed capacity should be able to match peak demand, the *t*-ratios indicate that the remaining estimated parameters are significantly different from zero. These results ensure that the estimated translog hyperbolic, output and enhanced hyperbolic distance functions comply with the aforementioned monotonicity conditions, and reflect that, at the sample mean, they are non-increasing in undesirable outputs and inputs.

In all three specifications the elasticity values of sulfur dioxide emissions χ_1, when compared to the input elasticities, range at the lower end, matching those of generating capacity α_1, and showing its relatively small importance when characterizing the alternative distance functions. Particularly, when compared to fuel α_2, and labor α_3 elasticities, we see that the energy input constitutes the variable essentially responsible for any change in electricity production. The coefficient ψ_{97} corresponding to the dummy intended to capture neutral technical change also presents a negative sign, and is statistically significant in the first specification. Its value reflecting the existence of an upward shift in the environmental frontier –technical progress– by a cumulated aggregate value of 0,95%, in the five years period. The fact that technical progress exists in this specification indicates that the leading firms are able to increase electricity production while making use of more environmentally friendly technologies –for a graphical representation of the industry technological progress see figure 4 in Färe *et al.* (2005;483).

Considering the environmental hyperbolic distance function parameters (30) as the baseline for comparisons with the output and enhanced hyperbolic specifications, we see that the elasticity values for the hyperbolic and output distance functions are about the same except for the undesirable output parameter χ_1, as both of them leave aside inputs reductions, and represent an output enhancing strategy when reaching the production frontier. This is not the case when we take into account the enhanced hyperbolic specification, which also includes an inputs reduction approach. Here we notice that its associated elasticity values are about half the value of those estimated for the hyperbolic distance function. This is consistent with the underlying theory. Had we imposed constant returns to scale in our specifications as in Cuesta and Zofio (2005), we could have recalled an additional property of the hyperbolic distance function, D_H.5: $D_H(x, v, w; \text{CRS}) = D_E(x, v, w; \text{CRS})^2$, which for a translog specifications yields D_H.5: $\ln D_H(x, v, w; \text{CRS}) = 2 \ln D_E(x, v, w; \text{CRS})$, and the enhanced hyperbolic distance function parameters would be one half of those estimated for the hyperbolic distance function.[5] In fact, the same justification applies for the above mentioned difference between the SO_2 parameters of the hyperbolic and output distance functions, being the parameter χ_1 in this latter specification about one half of the parameter of the former specification.

Table 2. Estimated parameters for the alternative distance functions.

Distance Function	$D_H(x, v, w)$, (30)		$D_O(x, v, w)$, (31)		$D_E(x, v, w)$, (32)	
Parameter	Estimated Value	*t*-statistic	Estimated Value	*t*-statistic	Estimated Value	*t*-statistic
α_0	-0.0633	-9.3088	-0.0677	-9.5352	-0.0355	-11.0938
α_1	-0.0133	-0,8210	-0.0143	-0.8773	-0.0122	-1.5062
α_2	-0.9440	-49.1667	-0.9745	-56.0057	-0.4780	-53.7079
α_3	-0.0294	-3.3793	-0.0260	-2.9545	-0.0127	-2.8222
α_{11}	0.1556	3.1820	0.2014	4.1355	0.0937	3.9205
α_{22}	0.0492	1.0446	0.0606	1.6117	0.0462	1.9660
α_{33}	-0.0201	-1.4889	-0.0215	-1.5809	-0.0122	-1.7941
α_{12}	-0.1433	-3.3718	0.0289	3.3605	-0.0787	-3.4367
α_{13}	-0.0284	-1.4416	-0.0261	-3.4800	-0.0092	-0.9388
α_{23}	0.0448	2.3957	0.0022	0.4783	0.0262	2.9111
χ_1	-0.0183	-3.8125	-0.0088	-1.7959	-0.0045	-1.8750
χ_{11}	-0.0082	-1.4386	0.0008	0.1333	0.0006	0.2069
ς_{11}	0.0288	3.3882	0.0289	3.3605	0.0110	2.5581
ς_{21}	-0.0078	-0.6667	-0.0261	-3.4800	-0.0122	-2.9048
ς_{31}	0.0083	1.8864	0.0022	0.4783	0.0012	0.5455
ψ_{97}	-0.0095	-2.1591	-0.0047	-1.0217	-0.0027	-1.2273
σ^2	0.0177	9.461	0.0181	9.5263	0.0044	8.8000
λ	0.0712	6.430	0.0724	6.4071	0.0718	6.5273
Mean L.L.F.	1.1464		1.1323		1.8455	
Mean T.E.	0.9366		0.9373		0.9671	

Source: Own elaboration.
Note: The following parameterization applies: $\sigma^2 = \sigma^2_u + \sigma^2_\omega$, $\lambda = \sigma^2_\omega / \sigma^2_u$

Once the alternative translog distance functions' parameters have been estimated, it is possible to estimate firm specific efficiency scores making use of expressions (27), (28) and (29). With regard to technical efficiency, the significant parameters σ^2 and λ indicate that in all three cases the one sided error is a relevant source when explaining a producer's deviation from the

[5] Cuesta and Zofio (2002) define and provide an example of this equivalence between the translog hyperbolic and output distance functions, which is recalled here for the hyperbolic and enhanced hyperbolic distance functions.

transformation function. For the hyperbolic distance function baseline specification, average environmental technical efficiency is 0.9366, showing how US electric utilities can improve its productive performance by increasing its desirable output by 6.77% (1 / 0.9366 = 1.0677), while simultaneously reducing SO_2 emissions by 6.34% (1 − 0.9366 = 0.0634). This means that on average the industry could increase its electricity production from 1,774,512 MWh to 1,894,646 MWh, while reducing SO_2 from 25,419 tons to 23,807 tons. Applying the additive directional distance function counterpart to our hyperbolic distance function, Färe *et al.* (2005;481) found that on average, production electricity and SO_2 emissions could be respectively increased and reduced by about 20%. This difference between average efficiency values clearly suggests that our translog hyperbolic distance function specification, being stochastic, allocates a significant amount of the one sided error the to the random noise term ω_{it}. An amount that is considered as inefficiency in the deterministic quadratic directional distance function of Färe *et al.* (2005), hence explaining the higher inefficiency values of the latter model –*ceteris paribus* the different specifications.[6]

While similar calculations can be made for the output distance functions, it is worth noting that the mean technical efficiency value of the enhanced hyperbolic distance function is much higher than those corresponding to the hyperbolic distance function. This result can be justified on the grounds that the enhanced hyperbolic distance function represents a more comprehensive path toward the production frontier in so far as firms can adjust both sets of outputs –desirable and undesirable- as well as inputs. Therefore, in this last specification $D_E(x, v, w)$ inefficiency is shared among desirable output increases and undesirable outputs and inputs reductions, while in the output oriented models it hinges on both desirable and undesirable outputs $D_H(x, v, w)$, or just desirable outputs $D_O(x, v, w)$. In fact, our proposal to estimate alternative distance function models matching the efficiency measures proposed by FGLP (1989), yields compatible results to those obtained by these authors, as with regards to the ordering of mean technical efficiency values. No matter

[6] Note that Färe et al. (2005;484) are not able to recover technical efficiency estimates from their stochastic model due to specification and/or estimation problems, and therefore finally decide to apply corrected OLS. Not surprisingly the values they obtain are similar in magnitude to those derived from their deterministic model following Aigner and Chu (1968).

whether non-parametric or parametric techniques are employed, the most comprehensive models including desirable outputs increases and undesirable outputs and inputs reductions yield higher efficiency values than their partially oriented counterparts, *i.e.* do not take into account all outputs and input dimensions. Therefore, it should be comforting to note that all these analytics are entirely consistent with production theory.

6. Conclusions

This paper introduces new definitions and estimation procedures of parametric distance functions intended to be applied in environmental efficiency and productivity studies. Departing from a recent paper by Cuesta and Zofio (2005), we extend their parametric specification of a translog hyperbolic distance function to mirror the theoretical and non-parametric techniques of FGLP (1989), who in their path breaking article treated the outputs vector asymmetrically by allowing equiproportional desirable outputs expansion and undesirable outputs contraction. The paper discusses the relevant properties that characterize the environmental hyperbolic graph distance function, and compares it to its traditional output distance function, as well as an enhanced definition that additionally calls for inputs reductions –all of which can be consistently identified with the alternative efficiency measures introduced by FGLP (1989). It then proceeds to develop the functional conditions necessary to implement them within a translog parametric framework, particularly those restrictions that ensure that these specifications satisfy the almost homogeneity properties discussed by Aczel (1967).

We show that the translog hyperbolic and enhanced hyperbolic distance functions can be easily implemented within an stochastic frontier analysis framework and relying on conventional econometric techniques. The particular specification that has been chosen to illustrate our efficiency analysis based on a translog distance function corresponds to Battese and Coelli's (1988) maximum likelihood panel data methodology. For the empirical application we provide the translog counterpart of the study carried out by Färe *et al.* (2005), who apply a directional distance functions approach using a quadratic specification.

Given the wide non-parametric DEA application of the FGLP (1989) hyperbolic graph efficiency model, we believe that our translog hyperbolic distance function should prove quite useful to econometricians interested in developing the analytical and statistical potential of regression analysis applied to environmental performance.

Bibliography

Aczél, J. (1966), *Lectures on Functional Equations and their Applications*, New York: Academic Press.

Aigner, D., and S. Chu (1968), "On Estimating the Industry Production Function", *American Economic Review* 58(4), 826-839.

Aigner, D., C.A K. Lovell and P. Schmidt (1977), "Formulation and Estimation of Stochastic Frontier Production Functions Models," *International Economic Review* 17, 377-396.

Ball, V.E., C.A.K. Lovell, H. Luu and R. Nehring (2004), "Incorporating Environmental Impacts in the Measurement of Agricultural Productivity Growth," *Journal of Agricultural and Resource Economics* 29, 436-460.

Ball, V.E., C.A.K. Lovell, R. Nehring and A. Somwaru (1994), "Incorporating Undesirable Outputs into Models of Production: An Application to U.S. Agriculture," *Cahiers d'Economie et Sociologie Rurales* 31/2, 60-74.

Battese, G.E., and T.J. Coelli (1988), "Prediction of Firm-level Technical Efficiencies with a Generalized Production Function and Panel Data," *Journal of Econometrics* 38, 387-399.

Chambers, R.G., Y. Chung and R. Färe (1996), "Benefit and Distance Functions," *Journal of Economic Theory* 70, 407-419.

Charnes, A., W.W. Cooper and E. Rhodes (1978), "Measuring the Efficiency of Decision-Making Units," *European Journal of Operational Research* 2, 429-444.

Christensen, L., D. Jorgenson, and L. Lau (1971), "Conjugate Duality and the Transcendental Logarithmic Production Function," *Econometrica* 39, 255-256.

Christensen, L., D. Jorgenson, and L. Lau. (1973). "Trascendental Logarithmic Production Frontiers," *Review of Economics and Statistics* 55(1), 28-45.

Chung, Y.H., R. Färe, and S. Grosskopf (1997), "Productivity and Undesirable Outputs: A Directional Distance Function Approach," *Journal of Environmental Management* 51, 229-240.

Cuesta, R. A., and J.L. Zofío (2002), "A Hyperbolic Distance Function: With Application to Spanish Savings Banks," 3rd Workshop on Efficiency and Productivity, Universidad de Oviedo, Oviedo, Spain.

Cuesta, R. A., and J.L. Zofío (2005), "Hyperbolic Efficiency and Parametric Distance Functions: With Application to Spanish Savings Banks," *Journal of Productivity Analysis* 24, 31-48.

Debreu, G. (1951), "The Coefficient of Resource Utilization," *Econometrica* 19, 273-292.

Domazlicky, B.R., and W.L. Weber (2004), "Does Environmental Protection Lead to Slower Productivity Growth in the Chemical Industry?," *Environmental and Resource Economics* 28(3), 301-324.

Färe, R., S. Grosskopf and C.A.K. Lovell (1985), *The Measurement of Efficiency of Production*. Boston: Kluwer-Nijhoff Publishing.

Färe, R., S. Grosskopf, C. A. K. Lovell and C. Pasurka (1989), "Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Non-parametric Approach," *Review of Economics and Statistics* 75, 90-98.

Färe, R., S. Grosskopf, D.-W. Noh and W. Weber (2005), "Characteristics of a Polluting Technology: Theory and Practice," *Journal of Econometrics* 126, 469-492.

Färe, R., and D. Primont (1995), *Multi-Output Production and Duality: Theory and Applications*. Dordrecht: Kluwer Academic Publishers.

Hernández-Sancho, F., A.J. Picazo-Tadeo and E. Reig-Martinez (2000), "Efficiency and Environmental Regulation," *Environmental and Resource Economics* 15, 365-378.

Lau, L. J. (1972), "Profit Functions of Technologies with Multiple Inputs and Outputs," *Review of Economics and Statistics* 54, 281-289.

Lovell, C.A.K., S. Richardson, P. Travers and L. Wood (1994), "Resources and Functionings: A New View of Inequality in Australia," in W. Eichhorn (ed.), *Models and Measurement of Welfare and Inequality*. Berlin: Springer-Verlag.

Malmquist, S. (1953), "Index Numbers and Indifference Surfaces," *Trabajos de Estadística* 4, 209-242.

Murty, M.N., and S. Kumar (2002), "Measuring the Cost of Environmentally Sustainable Industrial Development in India: A Distance Function Approach," *Environment and Development Economics* 7, 467-486.

Paul, C.J.M., W.E. Johnston and G.A.G. Frengley (2000), "Efficiency in New Zealand Sheep and Beef Farming: The Impacts of Regulatory Reform," *Review of Economics and Statistics* 82, 325-337.

Picazo-Tadeo, A.J., E. Reig-Martinez and F. Hernández-Sancho (2005), "Directional Distance Functions and Environmental Regulation," *Resource and Energy Economics* 27(2), 131-142.

Pitt, M., and L.-F. Lee (1981), "The Measurement and Sources of Technical Inefficiency in the Indonesian Weaving Industry," *Journal of Development Economics* 9, 43-64.

Prieto, A.M., and J.L. Zofio (2004), "Modelización de la gestión ambiental preventiva mediante estándares," *Investigaciones Económicas* 28, 43-66.

Reinhard, S., C.A.K. Lovell and G. Thijssen (1999), "Econometric Estimation of Technical and Environmental Efficiency: An Application to Dutch Dairy Farms," *American Journal of Agricultural Economics* 81, 44-60.

Shephard, R.W. (1953), *Cost and Production Functions.* Princeton: Princeton University Press.

Shephard, R.W. (1970), *Theory of Cost and Production Functions*. Princeton: Princeton University Press.

Simar, L., and P.W. Wilson, (2004), "Estimation and inference in two-stage, semi-parametric models of production processes," *Journal of Econometrics* 137, 31-64.

Weber W.L. and B.R. Domazlicky (2001), "Productivity Growth and Pollution in State Manufacturing," *Review of Economics and Statistics* 83(1), 195-199.

Zaim, O., and F. Taskin (2000), "A Kuznets Curve in Environmental Efficiency: An Application on OECD Countries," *Environmental and Resource Economics* 17, 21-36.

Zofío, J.L., and C.A.K. Lovell. (2001), "Graph Efficiency and Productivity Measures: an Application to US Agriculture," *Applied Economics* 33, 1433-1442.

Zofío, J.L., and A.M. Prieto (2001), "Environmental Efficiency and Regulatory Standards: The Case of CO2 Emissions from OECD Industries," *Resource and Energy Economics* 23, 63-86.