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# Stochastic Bio-economic Model of Northern Atlantic and Mediterranean Bluefin Tuna*

Ana Brasão†

Faculdade de Economia, Universidade Nova de Lisboa

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**Abstract**

The purpose of this paper is to study the implications of introducing a stochastic stock in a multi-gear and age structured bio-economic model for the Northern Atlantic Bluefin tuna. In order to account for variations on the recruitment, uncertainty is introduced on a bilinear recruitment function, and it is represented by an exponential random error term leading the stock to be, on average, higher than in the deterministic case.

Both the bionomic equilibrium and the optimal management of this species are examined. In the latter, two alternative instruments are studied: the constant total allowable catch and the constant effort and the purpose is to maximize the expected total net present value. The conclusions enhance that the results do not differ significantly from those in the deterministic model, that is, the fishing effort should be the policy instrument to adopt, both in the East and the West Atlantic.

Despite the previous reasoning, the shock on recruitment should be taken into account, as certain realizations of the random variable may lead to different results. In particular, for low Bluefin tuna stock levels, it is optimal to regulate the caches instead of the fishing effort, to ensure the stock recovery.

**Keywords**: Stochastic Bio-economic model, Bluefin tuna, Optimal management.

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†The author is a Graduate Student at Faculdade de Economia, *Universidade Nova de Lisboa*. Address: Travessa Estevão Pinto, 1070 Lisboa. Telephone: 351-1-3833624. E-mail: PL1778@feunix.

# Contents

## 1. Introduction

An important problem in fisheries economics concerns the optimal management of transboundary and highly migratory fishing stocks. Situations of severe depleted stocks are well known, due mostly to economic incentives and the absence of efficient regulations. The Northern Atlantic Bluefin tuna falls in this group of concerns. Being characterized by two separate stocks, the Eastern and Western Atlantic, it has been harvested at a high rate by several different gears that target different age classes, especially in the East Atlantic[1]. Still, several countries, both coastal and distant water fleets are tempted to enter this fishery because of the high market value placed upon this species, especially by the Japanese market. Owing to this pressure, the Bluefin tuna stock has decreased giving rise to some concern. It, therefore, calls for both national and international regulations, monitoring and enforcement, although the highly migratory nature of the resource has resulted in a difficult management problem.

A bio-economic model was developed in Pintassilgo (1999) in order to study the optimal use of Bluefin tuna in a deterministic context. Two different management policies were considered: the constant total allowable catches and the constant fishing effort. The results enhance that it is optimal to restrict effort, both in the East and the West Atlantic[2].

Nevertheless, it seems obvious, from the observation of the real world that uncertainty should be introduced in the aforementioned model. In fact, for the Bluefin tuna, the recruitment is stochastic and it can result in occasionally huge recruitment that can improve the Bluefin tuna stock, reducing the present concern with this species. In fact, the case of the spring spawning herring that recovered from a severe depletion owing to an occasionally huge recruitment (Bjorndal 1998). Therefore, the present study differs from earlier work in the sense that it introduces uncertainty on the Bluefin tuna recruitment, thus representing an extension to the deterministic model.

In this paper, a stochastic discrete time multi-gear and age structured bio-economic model is developed. In order to account for variations on the recruitment, which result from empirical observation, uncertainty is introduced on the recruitment function, and

[1] In 1982, the International Commission for the Conservation of tunas (commonly known as ICCAT), established a dividing line bewteen the East and West Atlantic separating the stocks based on morphometric differences, in order to facilitate the stock assessement

[2] For more detailed information see Pintassilgo (1999).

according to Kirkwood and Barry (1997), it is represented by an exponential random error term leading the stock to be, on average, higher than in the deterministic case.

The purpose of this paper is to study the implications of uncertainty on the bionomic equilibrium and the optimal management policies. In the latter, the constant total allowable catches (TAC) and the constant fishing effort are examined as economic policies that remain effective over time and across a broad range of objectives and are adaptable in the face of stochastic changes to the resource or its environment.

The open access quickly becomes unprofitable and is seen to involve substantial depletion of the stock. Concerning the optimal management of this species, the regulation should be undertaken through the effort both in the East and the West Atlantic. This result is somehow expected as regulation through effort allows for more flexibility in the adjustment. The results enhance the similarity between the optimal policy choices obtained in both the deterministic and stochastic models. Nevertheless, the shock on the recruitment should be taken into account as for certain realizations of the random variable the results may reveal the constant TAC as the optimal economic instrument to regulate this fishery, due to the low level of the Bluefin tuna stock. Thus, the manager must be aware that the regulation through the fishing effort may turn out to be an inefficient measure.

The paper is organized as follows. In the next section, the stochastic bio-economic model is explained, emphasizing the introduction of the stochastic elements. Section 3 presents the results both for the open access and the optimal management. Section 4 gives some insights on the optimal results obtained after uncertainty is resolved and finally, section 5 concludes the paper.

## 2. Stochastic Bio-economic Model

An age-structured and multi-gear time discrete stochastic bio-economic model is developed to examine the Northern Atlantic Bluefin tuna fishery. Kirkwood and Barry (1997) developed a biological model in order to capture the inter-relations between the different gears, the associated age classes and locations. Both the biological equations and the economic model, including the links between these two models, have been described in detail in Pintassilgo (1999). As the model used in this paper is an extension to a stochastic context, we will not go into detail once again of all the equations. An effort will be made in the explanation of the new elements introduced under uncertainty. Nevertheless, a general description of the model is presented in the Appendix.

## 2.1 Model Description.

In the biological model developed by Kirkwood and Barry (1997), the recruitment is assumed to occur at discrete time intervals. Moreover, recruits will normally join the parent population one-year after spawning. In fact, this kind of approach has been used in several applied studies, namely for the Norwegian Spring Spawning Herring studied in Bjorndal (1988).

Three different recruitment functions were examined: the Shepherd, the Beverton and Holts and the bilinear recruitment functions. The latter recruitment function with a lognormal error provided the best fit to the stock-recruitment data. Figure 1 presents these functions.

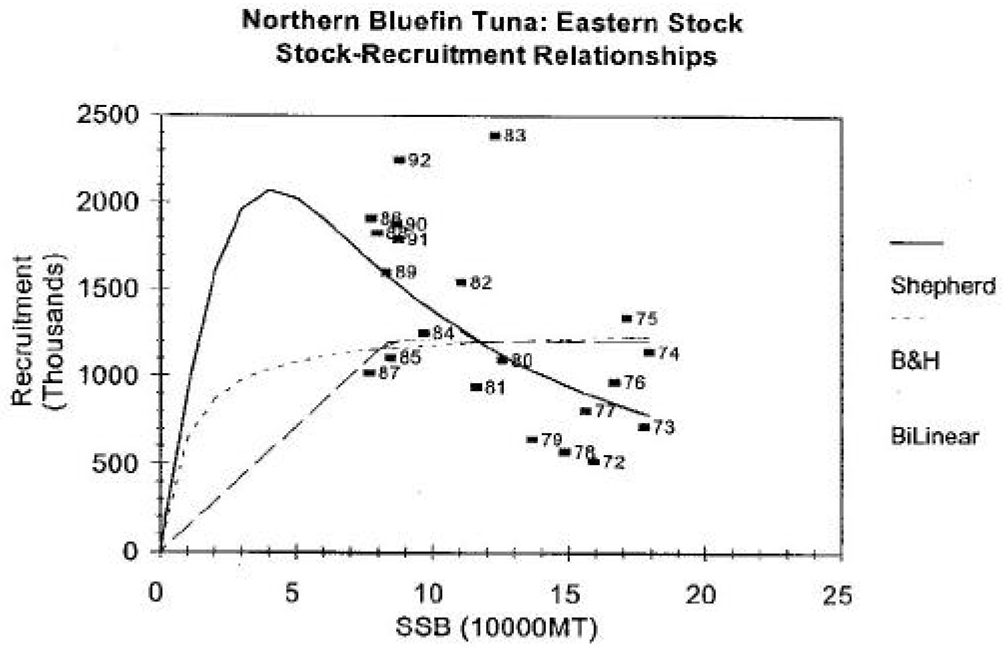


Figure 1

**Source:** Specification of a Biological and Prediction Model for Northern Bluefin Tuna.

In the bilinear relationship, the recruitment is assumed to be on average constant at $R_{max}$, for spawning stock biomasses (SSB) above a minimum level, but on average it declines nearly to zero for SSBs below that minimum.

As uncertainty is introduced, the functional form is given by:

$$R_t = \begin{cases} R_{max} e^{d} \text{ if } S_{t-1} \geq SSB_{min} \\ R_{max} \dfrac{S_{t-1}}{SSB_{min}} e^{d} \text{ if } S_{t-1} < SSB_{min} \end{cases}$$

where $d \sim N(0, s^2)$, and $R_t$ is number of recruits to the juvenile population as a function of the previous year's spawning stock.

Note that an exponential multiplicative error is considered. As the expected value of this random variable is greater than one, the average stochastic biomass will clearly be higher than the deterministic biomass. There is an upward bias that is in accordance with the historical data.

Additionally, a second element of stochastic variation was incorporated: uncertainty around the estimated stock levels at age at the beginning of 1996, reflecting uncertain current stock levels. Therefore, the initial stock levels are described by $N_{j,i,0} = \tilde{N}_{j,i} e^{q}$ for $1 \leq i \leq 10$ , where $\theta$ follows a normal distribution with mean zero and coefficient of variation equal to 0.1. As nothing is said on this matter, a multiplicative lognormal error is assumed to be reasonable. The stock recruitment relationship parameters for the East and the West Atlantic stocks used in this study are presented in table 2 below.

Table 2: Bilinear Stock Recruitment relationship parameters

| | Eastern Stock | Western Stock |
|---|---|---|
| $R_{max}$ | 1572724 | 70581 |
| $SSB_{min}$ (10000 MT) | 8.01 | 0.828 |
| $s^2$ | 0.113 | 0.098 |

## 2.2 Model Implementation

Two stochastic bioeconomic models were developed in Matlab, for the East and West Atlantic respectively, in order to examined the bionomic equilibrium as well as the optimal management of Bluefin tuna. Several Matlab procedures have been created so as to perform simulations and optimizations of the models. The forecasting period is 25 years, as it seems a reasonable horizon for economic analysis and regulation enforcement. Nevertheless, this forecasting horizon was extended to 100 years and the optimal management results were not significantly affected.

An error term for each forecasting period is needed. In order to account for this random variable, two samples of errors were created in Matlab, respecting the distribution, the mean and the variance suggested both for the East and the West Atlantic (these samples are shown in the appendix). These samples were then introduced in the simulation of the open access and in the optimization procedures. Note that the sample is quite small. Nevertheless, a larger sample of errors does not imply changes in the optimal results. Therefore, due to huge complexities introduced in computational terms, we opted for a smaller one.

*Open Access*

The Bluefin tuna stock has been traditionally harvested by several countries, both coastal and distant water fleets. The lack of cooperation between these nations led to a situation of overexploitation of the species. In contrast to the West Atlantic where some regulation was enforced for this fishery in the eighties, the East Atlantic fishery is characterized by an open access scenario, which is leading to the decrease in the Bluefin tuna stock.

In the open access case, the fishing nations are myopic in the sense that they are only concerned with the best decision in each period separately, given the previous year's profit results. Concerning the Matlab procedures, the expected value of each relevant variable is simulated, given on one hand the sample of errors created and on the other hand that each event may happen with equal probability.

As in the deterministic model, the market dynamics in this case is established through the effort in the following way:

$$E_{j,t,s} = \begin{cases} (1-b_{j,s})E_{j,t-1,s} & \text{if} \quad P_{\text{j,t-1,s}} \leq -Pb_{j,s} \\ E_{j,s,t-1} & \text{if} \quad -Pb_{\text{j,s}} \leq P_{\text{j,t-1,s}} \leq Pb_{j,s} \\ (1+b_{j,s})E_{j,s,t-1} & \text{if} \quad P_{\text{j,t-1,s}} \geq Pb_{j,s} \end{cases}$$

Whenever there is a considerable change in profits effort changes accordingly by a certain proportion, otherwise the effort remains unchanged.

*Optimal Management*

In the management of fishery resources several policies may be recommended. Of particular interest are policies that remain effective over time and across a broad range of objectives and are adaptable in the face of stochastic changes to the resource or its environment. In the present work, two alternative policy instruments were examined: the constant total allowable catch and the constant fishing effort.

For the purpose of this analysis, it is assumed that the resource in question is managed by a risk neutral sole owner whose objective is to maximize the discounted expected total net present value over 25 years, in order to choose the best policy instrument to preserve the Bluefin tuna stock, that is:

$$MAX\ E\left\{\sum_{S=1}^{G}\sum_{t=1}^{T}\left[P_{j,s}C_{j,S,t} - TCost_{j,s}\right]\frac{1}{(1+r)^{t}}\right\}$$

where T=25 is the last forecasting year and G is the number of gears. In the East Atlantic G=5 and in the West Atlantic G=4.

For both the East and the West Atlantic, two models of optimal management are presented. The first uses total catches as a control variable while the second assumes that fishing effort can be regulated.

## 3 Results

The results of the two models will be examined, both for the open access and the optimal management.

### 3.1 Open Access

In this model, several simulations were undertaken. It is interesting to begin by examining the average stochastic recruitment evolution. Note that, the recruitment for the 25 years of analysis and for each vector of errors from the sample defined above was calculated and afterwards an average recruitment by year was obtained.

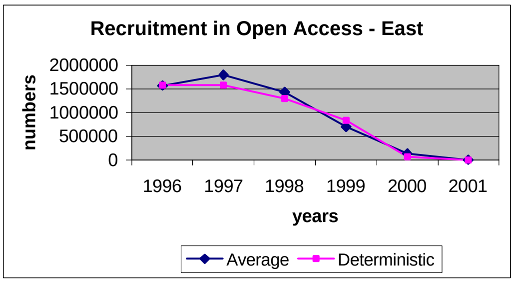


Figure A

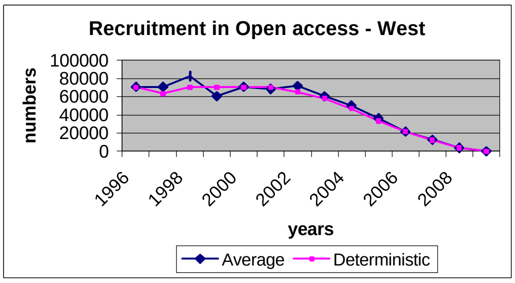


Figure B

The average stochastic recruitment is initially at its maximum but as the spawning stock biomass decreases, it also decreases. It is clear that the trend followed in this case is close to the deterministic model. Note that there are no huge recruitment implied by this sample of errors.

The results for this scenario are displayed in Figures 1 and 2 below. Note that only the evolution of some variables is shown in these figures, both for the East and the West Atlantic. Two arguments are advanced: (i) the evolution of these variables is similar, (ii) these are representative of the overall results, revealing the most important changes that resulted from the uncertainty in recruitment. Nonetheless, all the results are presented in the Appendix.

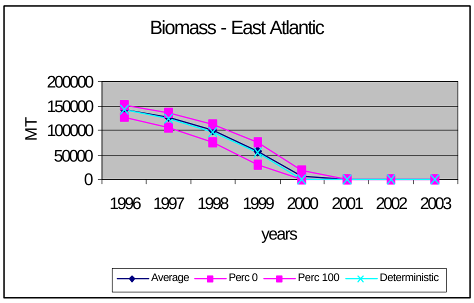


Figure 1a: Biomass evolution

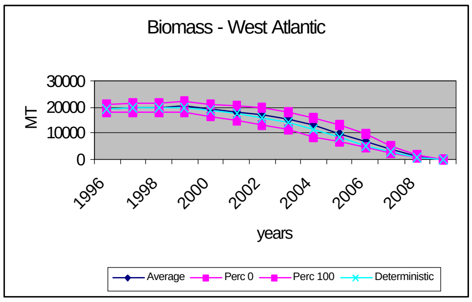


Figure 2a: Biomass evolution

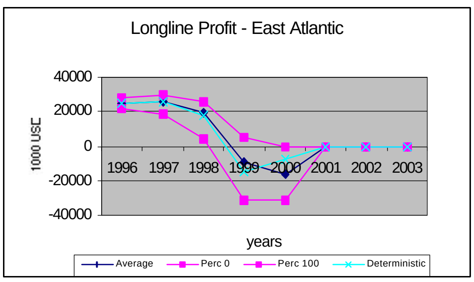


Figure 1b: Longline Profit evolution

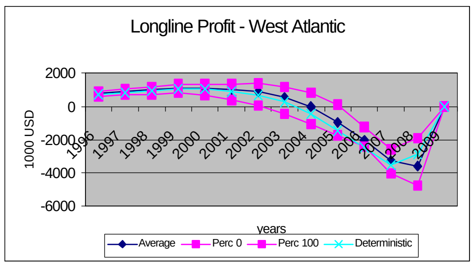


Figure 2b: Longline Profit evolution

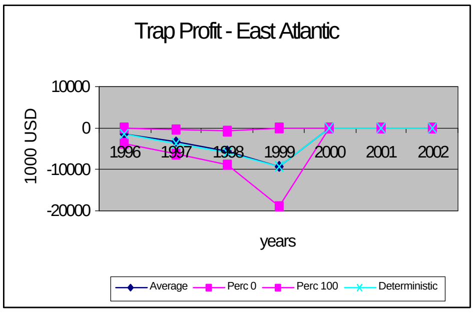


Figure 1c: Trap Profit evolution

Figure 1: Results for the East Atlantic

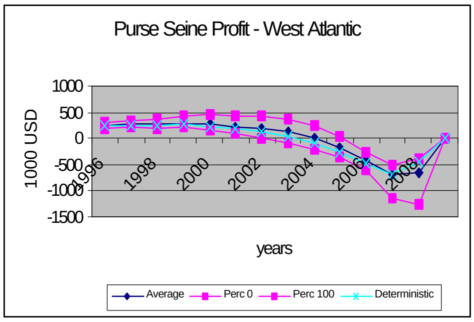


Figure 2c: Purse Seine Profit evolution

Figure 2: Results for the West Atlantic

The situations depicted are not surprising and do not differ significantly from the results obtained in the deterministic model. As expected, the average stochastic stock is above the deterministic stock, given the way the error term was introduced and whose mean is greater than one. Nevertheless, the impact of this uncertainty is not significant on average. As can be seen from the figures above, the deterministic and stochastic trends are very close. This may be explained by the fact that the stock decreases very fast and the error has a smaller effect on small numbers.

Additionally to the average variables, two percentiles were calculated: the percentiles zero and 100, which represent the values below which are zero and one hundred percent of the sample values, respectively. These percentiles give some insight about the variability introduced on each relevant variable. It is important to examine and take into account the limit scenarios that may occur, in order to manage efficiently the resource.

Examining the results, it is clear that some variability exists. Despite the low variability in the biomass, note for instance, the case of the longline and trap profits in the East Atlantic that, in certain situations, may attain very negative levels whereas in others it can remain positive throughout the period before the extinction of the species. In the former situation, the fishing gears experience such huge losses that the expected total net present value is negative, resulting from a smaller Bluefin tuna biomass and consequently lower catches. In the latter scenario, certain fishing gears, the ones shown in the figures above, may not experience any loss while harvesting this species. This is the result of a higher biomass and higher catches. Note however that there is no effect on the moment of total depletion of the stock. This is explained in the following way: in the scenario of greater losses, the biomass is smaller due to low levels of recruitment. Therefore the catches are smaller and the losses are higher. In this case, in the following year, the effort decreases and catches are even lower, and so on. This leads the stock to be less harvested being depleted at the same moment as in the deterministic context. The opposite reasoning can be applied for the scenario of no losses. Both the above results implied by the uncertainty were not possible to attain in the deterministic context.

### 3.2 Optimal Use

As in the deterministic model, the two alternative policies to be enforced are the constant total allowable catches and the constant effort. In the former, a constant catch is determined and in the latter, the effort in 1996 is a percentage of that in 1995.

It is clear that the constant effort is more flexible for the fishing gears, allowing for higher net present values than the former. Nevertheless, when the stocks are very low due to severe depletion, a stricter policy such as the constant total allowable catch may be advisable in order to allow for a better recovery of the stock, as we will see later.

The study focused on two different cases. In the first, it is assumed that all fishing gears will either be subjected to a total allowable catch whose distribution among gears is based on the shares observed in 1995, or will change equally the fishing effort. This scenario will be denoted restricted optimization hereon. In the second, in both the constant allowable catch and the effort change, the gear structure is unrestricted. This will be denoted by unrestricted optimization. The former scenario is examined because, given the existing fishing gears, it seems reasonable to adopt a specific policy that maintains the existing structure of fishing gears and regulates them as a whole. It may be difficult and unpopular to implement a policy that shuts down a specific gear, as it will happen in the second scenario.

Again, it is interesting to start by examining the average stochastic recruitment.

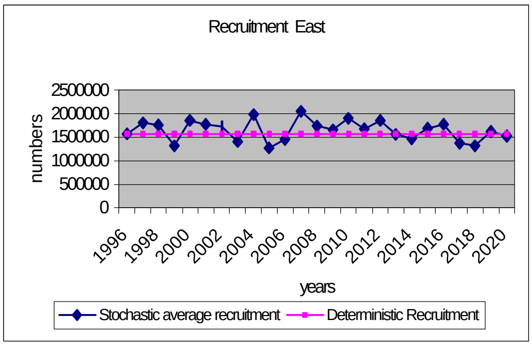


Figure 1

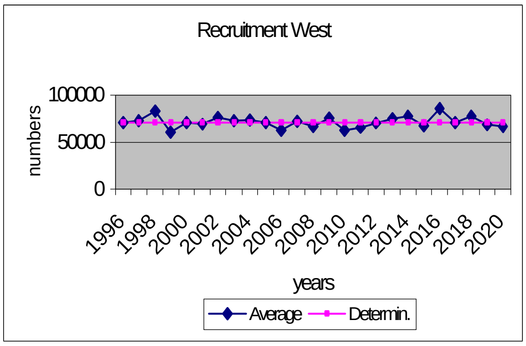


Figure 2

Note that in both the constant TAC and constant effort scenarios, the spawning stock biomass is always above the spawning stock minimum, thus, the recruitment is always at its maximum. The stock evolution is not different from the deterministic one.

The optimal management results are displayed in Tables 3 and 4, beginning with the restricted optimization.

Table 3: Results from the restricted optimization

| | **East Atlantic** | | **West Atlantic** | |
|---|---|---|---|---|
| | Best policy | ETNPV($10^6$) | Best policy | ETNPV($10^6$) |
| Stochastic TAC (MT) | *26 178* | *1 021.0* | *1 987* | *62.071* |
| Deterministic TAC (MT) | 25 706 | 985.5 | 1 980 | 61.389 |
| Stochastic Constant Effort | *0.53* | *1 476.4* | *0.76* | *68.432* |
| Deterministic Constant effort | 0.50 | 1 291.7 | 0.76 | 67.271 |

It is interesting to analyze some of the previous results. Focusing on the expected total net present value (ETNPV), it is interesting to observe the difference in value resulting from the implementation of both a TAC and a constant effort. The explanation may be associated with the more flexibility of the effort instrument, as said earlier. As the Bluefin tuna biomass is larger on average, the economic agents may optimally adjust the recommended effort to the harvest. Therefore, the effort is to be regulated, both in the East and the West Atlantic.

The second situation examined is the unrestricted optimization whose results are given in table 4 below.

Table 4: Results from the unrestricted optimization

| | East Atlantic | | West Atlantic | |
|---|---|---|---|---|
| | [LL,PS,TRAP,BB,Rem] | ETNPV($10^6$) | [LL,PS,TRAP,BB,Rem] | ETNPV($10^6$) |
| Stochastic TAC (MT) | *[13 784,0,10 000,0,5 000]* | *2 424* | *[0, 1 844, 0, 500]* | *93.980* |
| Deterministic TAC (MT) | [13 834,0,10 000,0,5 000] | 2 372 | [0, 1 897, 0, 500] | 93.3 |
| Stochastic Constant Effort | *[1.78, 0,1.5,0,0.34]* | *2 800* | *[0, 7.8, 0, 1]* | *94.672* |
| Deterministic Constant effort | [1.63, 0, 1.5, 0, 0.34] | 2 762 | [0, 7.4, 0, 1] | 94.4 |

The results of the stochastic unrestricted optimization do not differ substantially from the results of the deterministic model, for this sample of errors. Therefore, the fishing effort should be regulated, both in the East and the West Atlantic.

Thus, the optimal management of Northern Atlantic Bluefin tuna can be settled upon a deterministic bio-economic model without any prejudice of obtaining inaccurate economic results, given the parameters considered in the biological model. In fact, owing to this similarity of the results of the two models, the deterministic model should be used as the stochastic one adds a great complexity to the Matlab program.

## 4 Results after uncertainty is resolved

In the previous section, the expected total net present value was optimized given the overall sample of errors. Nevertheless, only one realization of the random error will occur and it is interesting to examine whether the optimal policy decided both in the deterministic context and on the earlier section remains effective for the realizations of all the vectors of errors, given the sample studied. Note that we leave the stochastic context and start working on a sort of deterministic context, given that uncertainty is resolved and a specific error vector is observed. Additionally, this will allow for a better knowledge of the eventual impacts of the different error vectors.

From the results obtained, it can be said that, in the East Atlantic, the constant effort is the optimal policy to be applied either in the stochastic optimization or after the uncertainty is resolved, independently of the realization of the random variable.

Nevertheless, in the West Atlantic the results are not that straight forward. The previous section discussed that a constant fishing effort policy should be implemented instead of a constant TAC. Nonetheless, if the optimization is undertaken for the realization of each vector of error, it can be concluded that, for certain realizations, the optimal policy becomes the constant TAC and not the constant fishing effort. This result occurs whenever the uncertainty leads to lower biomass levels. Comparing the results obtained maximizing the total net present value with respect to both instruments in these specific cases, if the TAC is implemented, the results reveal an initial fishing effort which is higher than that obtained from the implementation of a constant fishing effort but becomes lower in the year 2020. The same reasoning is applied to the catches. Concerning the profits, these follow the same trend as the previous variables. Given this, the total net present value is higher when a TAC is applied. This result is clear. For low levels of the Bluefin tuna stock, it is necessary to implement a more strict policy that control the harvesting rates allowing the stock to recover. In this cases, regulate the fishing effort would not allow for a strict control over the catches and could lead to an excessive catch, endangering the species.

Thus, if the manager chooses to regulate the fishing effort, this decision may turn out to be sub-optimal, depending on the realization of the random variable. The manager should be aware of this possibility.

This analysis is important in that it gives some insight of the possible scenarios that may occur and about what is the best policy and instrument to use for each realization of the error term.

## 5. Conclusions

The Northern Atlantic Bluefin tuna is a species that urges to be regulated in order to prevent its stock from being totally depleted. Owing to its highly migratory nature, both national and international cooperation is necessary to enforce optimal regulations. Furthermore, it is characterized by a stochastic recruitment that is taken into account in the analysis.

Focusing on the bionomic equilibrium, the results show that the open access quickly becomes unprofitable and is seen to involve substantial depletion of the stock, both in the East and the West Atlantic. Additionally, the introduction of uncertainty in the recruitment function does not affect the average results, which are similar to those found in the deterministic model.

Concerning the optimal management results, the conclusions are identical. The similarity between the stochastic and the deterministic results may be explained by the low variance suggested for Bluefin tuna.

Nonetheless, for the purpose of this species and this study, it can be concluded that the optimal policy to be implemented should be decided based on the deterministic model in order to avoid extra and unnecessary complexities to the Matlab model.

In spite of this result similarity, the shocks on the recruitment should be taken into account, to give the manager some insight about the possible scenarios that may occur after uncertainty is resolved. In particular, it is interesting to note that for low levels of recruitment leading to low levels of the biomass, it is optimal to restrict the Bluefin tuna catches so as to allow for the stock recovery. Therefore, in these cases, restrict the fishing effort turns out to be a sub-optimal economic policy.

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## Appendix A

# The Model Equations

## Biological sub-model

Population numbers

$$(1)\quad N_{j,0,a} = \tilde{N}_{j,a} \quad for\ 1 \le a \le A$$

$$(2)\quad N_{j,t,0} = SRR_j\left(SSB_{j,t-2}\right)$$

$$(3)\quad N_{j,t,a} = N_{j,t-1,a-1}e^{-M_{j,a-1}-F_{j,t-1,a-1}} \quad for \quad a = 1,2,...,9;\ t=1,2,...$$

$$(4)\quad N_{j,t,A} = N_{j,t-1,9}e^{-M_{j,9}-F_{j,t-1,9}} + N_{j,t-1,A}e^{-M_{j,A}-F_{j,t-1,A}}$$

$$(5)\quad SSB_{j,t} = \sum_{a=1}^{A} Mat_{j,t,a}N_{j,t,a}W_{j,t,a}$$

$$(6)\quad B_{j,t} = \sum_{a=1}^{A} N_{j,t,a}W_{j,t,a}$$

Catch at age by gear

$$(7)\quad F_{j,t,a,s} = FMax_{j,t,s}.Sel_{j,a,s}$$

$$(8)\quad F_{j,t,a} = \sum_{s=1}^{S} FMax_{j,t,s}.Sel_{j,a,s}$$

$$(9)\quad CN_{j,t,a,s} = \frac{F_{j,t,a,s}.N_{j,t,a}}{\sum_{s=1}^{S}\left(F_{j,t,a,s} + M_{j,a}\right)}\left(1 - e^{-\sum_{s=1}^{S}\left(F_{j,t,a,s}+M_{j,a}\right)}\right)$$

$$(10)\quad CB_{j,t,s} = \sum_{a=1}^{A} CN_{j,t,a,s}.W_{j,a}$$

$$(11)\quad C_{j,t,s} = \sum_{a=1}^{A}\frac{FMax_{j,t,s}.Sel_{j,a,s}.N_{j,t,a}.W_{j,t,a}}{\sum_{s=1}^{S}\left(FMax_{j,t,s}.Sel_{j,a,s} + M_{j,a}\right)}\left(1 - e^{-\sum_{i=1}^{S}\left(FMax_{j,t,s}.Sel_{j,a,si}+M_{j,a}\right)}\right)$$

$$for\ s = 1,...,S$$

## Harvest function

$$(12)\quad C_{j,t,s} = q_{j,s} E_{j,t,s} B_{j,t}^{a_s}$$

$$(13)\quad C_{j,0,s} = sh_{j,0,s} * C_{j,0,s}$$

$$(14)\quad E_{j,0,s} = sh_{j,0,s} * E_{j,0,s}$$

## Economic sub-model

$$(15)\quad Rev_{j,t,s} = \overline{P}_{j,s} * C_{j,t,s}$$

$$(16)\quad Cost_{j,t,s} = wg_{j,s} * E_{j,t,s} + g_{j,s}\left(\overline{P}_{j,s} * C_{j,t,s}\right)$$

$$(17)\quad P_{j,t,s} = Rev_{j,t,s} - Cost_{j,t,s}$$

$$(18)\quad TNPV_j = \sum_{s=1}^{S} \sum_{t=1}^{25} P_{j,t,s} * \left(\frac{1}{1+r}\right)^t$$

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$$(19)\ E_{j,t,s} = \begin{cases} (1-b_{j,s})E_{j,t-1,s} & if \quad P_{,j,t-1,s} \leq -Pb_{j,s} \\ E_{j,s,t-1} & if \quad -Pb_{j,s} \leq P_{,j,t-1,s} \leq Pb_{j,s} \\ (1+b_{j,s})E_{j,s,t-1} & if \quad P_{p,j,t-1,s} \geq Pb_{j,s} \end{cases}$$

Exit condition

$$(20)\ Cost_{j,t-1,s} > \left(1+h_{j,s}\right) * Rev_{j,t-1,s}$$

**Table A1: Glossary of Symbols**

| Variables | | Coefficients | |
|---|---|---|---|
| N | Nº of fish (beginning of year) | M | Instantaneous natural mortality |
| Ñ | Estimated nº fish (beginning of 1995) | Mat | Maturity rate |
| SRR | Stock Recruitment Relation | W | Average weight |
| SSB | Spawning stock Biomass | q | Production function parameter |
| F | Instantaneous fishing mortality | $\alpha$ | Catch-stock elasticity |
| Fmax | Fishing mort. at maximum selectivity | wg | Costs parameter |
| B | Total Biomass | $\gamma$ | Crew share |
| Sel | Selectivity | r | Interest rate |
| CN | Catch numbers | $\beta$ | Effort Adjustment parameter |
| CB | Catch Biomass | $\prod b$ | Profit bound |
| E | Effort | h | Exit condition parameter |
| C | Catch | | |
| Rev | Revenue | | |
| Cost | Cost | | |
| $\overline{P}$ | Average Price | **Indices** | |
| $\Pi$ | Profit | j | Stock (j=East Atl., West Atl.) |
| TNPV | Total Net Present Value | t | Time (t=1,…,T), T=25 (2020) |
| | | a | Age (a=1,…,A), A=10+ |
| | | s | Gear (s=1,2,…,S) |

**Table A2: Economic Parameters of the Model**

| | *East Atlantic* | | | | *West Atlantic* | | | |
|---|---|---|---|---|---|---|---|---|
| *Gears* | Prices (USD/Kg) | $\beta$ | wg | Unit of effort | Prices (USD/Kg) | $\beta$ | wg | Unit of effort |
| *Longline* | 17 | 0.25 | 14,102 | Fishing days | 17 | 0.1 | 15,265 | Fishing days |
| *Purse Seine* | 9 | 0.1 | 45,185* | Fishing days | 18 | 0.1 | 20,092 | Days at sea |
| *Trap* | 25 | 0.2 | 15,738 | Trap days | _ | _ | _ | _ |
| *Bait Boat* | 5 | 0.2 | 4,638 | Days at sea | _ | _ | _ | _ |
| *Rod & Reel* | _ | _ | _ | | 18 | 0.1 | 163 | Fishing hours |
| *Remainder* | 17 | 0.01 | 2,408 | Days at sea | 20 | 0.1 | 22,417 | Fishing days |

*Note that for the PS, in the East Atlantic, one fishing day correspond to more than three days at sea.

**Table A3- Shares of Catches by Gear - in the Base Year (1995)**

| | *East Atlantic* | | | *West Atlantic* | | |
|---|---|---|---|---|---|---|
| *Gears* | EU | OCS | DWFN | USA | CAN | DWFN |
| *Longline* | 0.22 | 0.19 | 0.59 | 0.16 | 0.01 | 0.83 |
| *Purse Seine* | 0.78 | 0.22 | 0 | 1.00 | 0 | 0 |
| *Trap* | 0.57 | 0.43 | 0 | - | - | - |
| *Bait Boat* | 1 | 0 | 0 | - | - | - |
| *Rod & Reel* | - | - | - | 0.81 | 0.19 | 0 |
| *Remainder* | 0.74 | 0.26 | 0 | 0.28 | 0.72 | 0 |

# Appendix B

B1. East Atlantic

Table B1.1: Biomass evolution (in MT)

| | **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** |
|---|---|---|---|---|---|---|---|
| **Average** | 141815 | 125974 | 100154 | 59027 | 6900 | 0 | 0 |
| **Deterministic** | 141180 | 124430 | 97040 | 54950 | 1310 | 0 | 0 |
| **Percentile 0** | 128570 | 106730 | 75600 | 29570 | 0 | 0 | 0 |
| **Percentile 100** | 151310 | 135210 | 112580 | 74820 | 19560 | 0 | 0 |

Table B1.2: Catch evolution by gear (in MT)

| | **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** |
|---|---|---|---|---|---|---|---|
| LONGLINE | | | | | | | |
| **Average** | 15888 | 19393 | 23144 | 24549 | 3047 | 0 | 0 |
| **Deterministic** | 15875 | 19349 | 23013 | 25673 | 541 | 0 | 0 |
| **Percentile 0** | 15581 | 18764 | 21892 | 14189 | 0 | 0 | 0 |
| **Percentile 100** | 16097 | 19673 | 23707 | 27309 | 8525 | 0 | 0 |
| PURSE SEINE | | | | | | | |
| **Average** | 19248 | 20674 | 21712 | 20795 | 2923 | 0 | 0 |
| **Deterministic** | 19232 | 20628 | 21590 | 21196 | 593 | 0 | 0 |
| **Percentile 0** | 18876 | 20004 | 20538 | 12435 | 0 | 0 | 0 |
| **Percentile 100** | 19501 | 20974 | 22241 | 22546 | 8413 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TRAP | | | | | | | |
| **Average** | 1780 | 1349 | 899 | 524 | 0 | 0 | 0 |
| **Deterministic** | 1773 | 1282 | 841 | 427 | 0 | 0 | 0 |
| **Percentile 0** | 1646 | 1134 | 689 | 0 | 0 | 0 | 0 |
| **Percentile 100** | 1875 | 2056 | 1420 | 820 | 0 | 0 | 0 |
| BAITBOAT | | | | | | | |
| **Average** | 3890 | 4558 | 5222 | 4824 | 930 | 0 | 0 |
| **Deterministic** | 3887 | 4548 | 5193 | 3708 | 176 | 0 | 0 |
| **Percentile 0** | 3815 | 4410 | 4940 | 2945 | 0 | 0 | 0 |
| **Percentile 100** | 3941 | 4624 | 5349 | 6438 | 2619 | 0 | 0 |
| REMAINDER | | | | | | | |
| **Average** | 4169 | 3757 | 3094 | 2273 | 0 | 0 | 0 |
| **Deterministic** | 4154 | 3717 | 3016 | 1895 | 0 | 0 | 0 |
| **Percentile 0** | 3855 | 3288 | 2470 | 0 | 0 | 0 | 0 |
| **Percentile 100** | 4391 | 4053 | 3466 | 3592 | 0 | 0 | 0 |

Table B1.3: Effort evolution by gear

| | **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** |
|---|---|---|---|---|---|---|---|
| LONGLINE | | | | | | | |
| **Average** | 11618 | 14522 | 18152 | 21363 | 3729 | 0 | 0 |
| **Deterministic** | 11618 | 14522 | 18152 | 22690 | 1009 | 0 | 0 |
| **Percentile 0** | 11618 | 14522 | 18152 | 14195 | 0 | 0 | 0 |
| **Percentile 100** | 11618 | 14522 | 18152 | 22690 | 9263 | 0 | 0 |
| PURSE SEINE | | | | | | | |
| **Average** | 2325 | 2558 | 2814 | 2994 | 592 | 0 | 0 |
| **Deterministic** | 2325 | 2558 | 2814 | 3095 | 183 | 0 | 0 |
| **Percentile 0** | 2325 | 2558 | 2814 | 2055 | 0 | 0 | 0 |
| **Percentile 100** | 2325 | 2558 | 2814 | 3095 | 1511 | 0 | 0 |
| TRAP | | | | | | | |
| **Average** | 2066 | 1719 | 1375 | 1232 | 0 | 0 | 0 |
| **Deterministic** | 2066 | 1653 | 1322 | 1058 | 0 | 0 | 0 |
| **Percentile 0** | 2066 | 1653 | 1322 | 0 | 0 | 0 | 0 |
| **Percentile 100** | 2066 | 2479 | 1983 | 2086 | 0 | 0 | 0 |
| BAITBOAT | | | | | | | |
| **Average** | 2729 | 3275 | 3930 | 4031 | 1092 | 0 | 0 |
| **Deterministic** | 2729 | 3275 | 3930 | 3144 | 315 | 0 | 0 |
| **Percentile 0** | 2729 | 3275 | 3930 | 2826 | 0 | 0 | 0 |
| **Percentile 100** | 2729 | 3275 | 3930 | 5597 | 2730 | 0 | 0 |
| REMAINDER | | | | | | | |
| **Average** | 21510 | 21312 | 21099 | 24009 | 0 | 0 | 0 |
| **Deterministic** | 21510 | 21295 | 21082 | 20871 | 0 | 0 | 0 |
| **Percentile 0** | 21510 | 21295 | 21082 | 0 | 0 | 0 | 0 |
| **Percentile 100** | 21510 | 21725 | 21508 | 43097 | 0 | 0 | 0 |

Table B1.4: Profit evolution by gear

| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|---|---|---|
| LONGLINE | | | | | | | |
| **Average** | 25242 | 25990 | 19432 | -9114 | -16330 | 0 | 0 |
| **Deterministic** | 25089 | 25471 | 17877 | -14459 | -7795 | 0 | 0 |
| **Percentile 0** | 21589 | 18512 | 4539 | -31322 | -31158 | 0 | 0 |
| **Percentile 100** | 27726 | 29330 | 26135 | 5002 | 0 | 0 | 0 |
| PURSE SEINE | | | | | | | |
| **Average** | 16189 | 14669 | 9651 | -4265 | -8344 | 0 | 0 |
| **Deterministic** | 16091 | 14376 | 8879 | -6320 | -4523 | 0 | 0 |
| **Percentile 0** | 13846 | 10448 | 2255 | -14532 | -16738 | 0 | 0 |
| **Percentile 100** | 17783 | 16554 | 12981 | 2186 | 0 | 0 | 0 |
| TRAP | | | | | | | |
| **Average** | -1324 | -3184 | -5433 | -9162 | 0 | 0 | 0 |
| **Deterministic** | -1481 | -3571 | -6095 | -9179 | 0 | 0 | 0 |
| **Percentile 0** | -3718 | -6164 | -8759 | -18857 | 0 | 0 | 0 |
| **Percentile 100** | 29 | -304 | -636 | 0 | 0 | 0 | 0 |
| BAITBOAT | | | | | | | |
| **Average** | 835 | 617 | -126 | -1997 | -18596 | 0 | 0 |
| **Deterministic** | 824 | 581 | -229 | -1747 | -858 | 0 | 0 |
| **Percentile 0** | 572 | 100 | -1114 | -3680 | -36208 | 0 | 0 |
| **Percentile 100** | 1014 | 848 | 319 | -1327 | 0 | 0 | 0 |
| REMAINDER | | | | | | | |
| **Average** | -2186 | -6613 | -13981 | -30766 | 0 | 0 | 0 |
| **Deterministic** | -2360 | -7039 | -14868 | -27709 | 0 | 0 | 0 |
| **Percentile 0** | -5923 | -12149 | -21366 | -63960 | 0 | 0 | 0 |
| **Percentile 100** | 459 | -4079 | -10547 | 0 | 0 | 0 | 0 |

## B2 . West Atlantic

Table B2.1: Biomass evolution (in MT)

| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Average** | 19383 | 19810 | 19924 | 20382 | 19458 | 18147 | 16978 | 15126 | 12746 | 9813 | 6642 | 3751 | 1219 | 0 |
| **Deterministic** | 19210 | 19593 | 19600 | 19905 | 18873 | 17364 | 15952 | 13943 | 11351 | 8222 | 5367 | 2914 | 697 | 0 |
| **Perc 0** | 17893 | 18145 | 17902 | 18044 | 16585 | 14806 | 13144 | 11085 | 8753 | 6675 | 4369 | 2480 | 382 | 0 |
| **Perc 100** | 20900 | 21291 | 21433 | 22108 | 21196 | 20163 | 19547 | 17924 | 15918 | 13148 | 9419 | 5194 | 1920 | 0 |

Table B2.2: Catch evolution by gear (in MT)

| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONGLINE | | | | | | | | | | | | | | |
| **Average** | 807 | 891 | 982 | 1085 | 1182 | 1282 | 1391 | 1490 | 1545 | 1468 | 1235 | 991 | 509 | 0 |
| **Deterministic** | 805 | 890 | 979 | 1080 | 1175 | 1271 | 1375 | 1472 | 1554 | 1311 | 1084 | 863 | 301 | 0 |
| **Perc 0** | 794 | 876 | 961 | 1059 | 1145 | 1231 | 1323 | 1278 | 1097 | 937 | 786 | 646 | 159 | 0 |
| **Perc 100** | 819 | 904 | 996 | 1103 | 1203 | 1310 | 1432 | 1548 | 1663 | 1761 | 1647 | 1316 | 868 | 0 |
| PURSE SEINE | | | | | | | | | | | | | | |
| **Average** | 251 | 253 | 256 | 262 | 268 | 272 | 276 | 279 | 273 | 259 | 238 | 193 | 87 | 0 |
| **Deterministic** | 250 | 251 | 251 | 252 | 250 | 245 | 241 | 235 | 225 | 211 | 194 | 155 | 44 | 0 |
| **Perc 0** | 247 | 248 | 247 | 247 | 243 | 238 | 232 | 224 | 214 | 203 | 168 | 135 | 30 | 0 |
| **Perc 100** | 255 | 281 | 310 | 342 | 374 | 406 | 443 | 479 | 511 | 486 | 449 | 356 | 177 | 0 |
| ROD & REEL | | | | | | | | | | | | | | |
| **Average** | 1151 | 1272 | 1401 | 1548 | 1687 | 1830 | 1985 | 2133 | 2236 | 2138 | 1798 | 1443 | 622 | 0 |
| **Deterministic** | 1150 | 1270 | 1397 | 1541 | 1677 | 1814 | 1962 | 2101 | 2218 | 1872 | 1547 | 1232 | 352 | 0 |
| **Perc 0** | 1133 | 1250 | 1371 | 1511 | 1634 | 1757 | 1888 | 2007 | 1723 | 1469 | 1214 | 976 | 193 | 0 |
| **Perc 100** | 1169 | 1291 | 1422 | 1574 | 1716 | 1869 | 2043 | 2209 | 2373 | 2513 | 2586 | 2066 | 916 | 0 |
| REMAINDER | | | | | | | | | | | | | | |
| **Average** | 277 | 302 | 330 | 367 | 387 | 398 | 397 | 365 | 305 | 223 | 118 | 6 | 0 | 0 |
| **Deterministic** | 275 | 308 | 338 | 377 | 397 | 409 | 382 | 343 | 262 | 182 | 0 | 0 | 0 | 0 |
| **Perc 0** | 260 | 263 | 260 | 262 | 245 | 224 | 183 | 144 | 107 | 78 | 0 | 0 | 0 | 0 |
| **Perc 100** | 294 | 329 | 364 | 410 | 436 | 461 | 495 | 508 | 462 | 357 | 246 | 137 | 0 | 0 |

Table B2.3: Effort evolution by gear

| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONGLINE | | | | | | | | | | | | | | |
| **Average** | 580 | 638 | 701 | 772 | 849 | 934 | 1027 | 1126 | 1207 | 1206 | 1096 | 987 | 631 | 0 |
| **Deterministic** | 580 | 638 | 701 | 772 | 849 | 934 | 1027 | 1130 | 1243 | 1118 | 1007 | 906 | 421 | 0 |
| **Perc 0** | 580 | 638 | 701 | 772 | 849 | 934 | 1027 | 1027 | 924 | 832 | 749 | 674 | 251 | 0 |
| **Perc 100** | 580 | 638 | 701 | 772 | 849 | 934 | 1027 | 1130 | 1243 | 1367 | 1367 | 1230 | 991 | 0 |
| PURSE SEINE | | | | | | | | | | | | | | |
| **Average** | 145 | 146 | 147 | 150 | 155 | 159 | 164 | 169 | 171 | 171 | 170 | 155 | 87 | 0 |
| **Deterministic** | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 131 | 50 | 0 |
| **Perc 0** | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 145 | 131 | 118 | 38 | 0 |
| **Perc 100** | 145 | 160 | 176 | 193 | 212 | 234 | 257 | 283 | 311 | 311 | 311 | 280 | 173 | 0 |
| ROD & REEL | | | | | | | | | | | | | | |
| **Average** | 81870 | 90060 | 99060 | 108970 | 119870 | 131860 | 145040 | 159550 | 172947 | 173904 | 158054 | 142253 | 76295 | 0 |
| **Deterministic** | 81870 | 90060 | 99060 | 108970 | 119870 | 131860 | 145040 | 159550 | 175500 | 157950 | 142150 | 127940 | 48660 | 0 |
| **Perc 0** | 81870 | 90060 | 99060 | 108970 | 119870 | 131860 | 145040 | 159550 | 143590 | 129230 | 116310 | 104680 | 30080 | 0 |
| **Perc 100** | 81870 | 90060 | 99060 | 108970 | 119870 | 131860 | 145040 | 159550 | 175500 | 193050 | 212350 | 191120 | 103450 | 10 |
| REMAINDER | | | | | | | | | | | | | | |
| **Average** | 156 | 166 | 181 | 197 | 216 | 234 | 245 | 246 | 233 | 210 | 141 | 9 | 0 | 0 |
| **Deterministic** | 156 | 172 | 189 | 208 | 228 | 251 | 251 | 251 | 226 | 204 | 0 | 0 | 0 | 0 |
| **Perc 0** | 156 | 156 | 156 | 156 | 156 | 156 | 140 | 126 | 114 | 102 | 0 | 0 | 0 | 0 |
| **Perc 100** | 156 | 172 | 189 | 208 | 228 | 251 | 276 | 304 | 304 | 274 | 246 | 222 | 0 | 0 |

Table B2.4: Profit evolution by gear (in 1000 USD)

| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONGLINE | | | | | | | | | | | | | | |
| **Average** | 752 | 873 | 974 | 1129 | 1110 | 1004 | 878 | 545 | -42 | -940 | -2041 | -3271 | -3571 | 0 |
| **Deterministic** | 736 | 851 | 937 | 1070 | 1028 | 876 | 684 | 274 | -475 | -1467 | -2469 | -3556 | -2840 | 0 |
| **Perc 0** | 601 | 690 | 728 | 821 | 671 | 402 | 63 | -465 | -1050 | -1722 | -2445 | -4045 | -4788 | -1 |
| **Perc 100** | 899 | 1029 | 1147 | 1343 | 1356 | 1335 | 1363 | 1177 | 819 | 85 | -1267 | -2596 | -1933 | 0 |
| PURSE SEINE | | | | | | | | | | | | | | |
| **Average** | 248 | 263 | 270 | 290 | 269 | 230 | 192 | 119 | 3 | -173 | -414 | -675 | -652 | 0 |
| **Deterministic** | 242 | 255 | 255 | 265 | 231 | 179 | 127 | 46 | -73 | -250 | -468 | -674 | -440 | 0 |
| **Perc 0** | 198 | 206 | 198 | 203 | 151 | 82 | 12 | -86 | -217 | -359 | -593 | -1131 | -1250 | 0 |
| **Perc 100** | 296 | 339 | 378 | 435 | 447 | 427 | 425 | 353 | 245 | 23 | -275 | -520 | -386 | 0 |
| ROD & REEL | | | | | | | | | | | | | | |
| **Average** | 1136 | 1320 | 1471 | 1706 | 1677 | 1517 | 1326 | 821 | -80 | -1463 | -3158 | -5054 | -4623 | 0 |
| **Deterministic** | 1112 | 1286 | 1416 | 1617 | 1553 | 1324 | 1034 | 415 | -717 | -2217 | -3731 | -5374 | -3512 | 0 |
| **Perc 0** | 908 | 1042 | 1100 | 1240 | 1014 | 607 | 94 | -773 | -1745 | -2602 | -3868 | -6113 | -5624 | -1 |
| **Perc 100** | 1358 | 1554 | 1734 | 2029 | 2049 | 2018 | 2059 | 1778 | 1238 | 128 | -2107 | -4316 | -2482 | 0 |
| REMAINDER | | | | | | | | | | | | | | |
| **Average** | 384 | 487 | 555 | 701 | 566 | 305 | 37 | -424 | -996 | -1610 | -1584 | -122 | 0 | 0 |
| **Deterministic** | 357 | 460 | 507 | 623 | 443 | 93 | -283 | -829 | -1402 | -2012 | 0 | 0 | 0 | 0 |
| **Perc 0** | 144 | 185 | 145 | 169 | -71 | -368 | -633 | -1110 | -1615 | -1967 | -2908 | -3045 | 0 | 0 |
| **Perc 100** | 626 | 756 | 859 | 1085 | 985 | 820 | 728 | 290 | -353 | -1082 | 0 | 0 | 0 | 0 |

B3. Sample of errors

Table B3.1: Error Sample - East Atlantic

| **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** | **2003** | **2004** | **2005** |
|---|---|---|---|---|---|---|---|---|---|
| 0,11 | 0,0787 | 0,0072 | -0,3375 | -0,3184 | -0,1259 | -0,3986 | -0,3549 | 0,495 | 0,0187 |
| -0,0267 | 0,516 | -0,2039 | -0,4529 | 0,1578 | -0,3037 | 0,0121 | -0,2109 | 0,18 | 0,1859 |
| 0,0278 | 0,2576 | 0,7519 | 0,1099 | 0,2902 | 0,2284 | 0,1865 | 0,3367 | 0,4233 | 0,0148 |
| -0,1101 | -0,3894 | 0,195 | 0,0806 | -0,118 | 0,2999 | 0,5306 | -0,3725 | -0,0087 | -0,3733 |
| 0,0393 | -0,1987 | -0,2201 | -0,3633 | -0,016 | 0,1275 | -0,1111 | -0,168 | -0,0121 | -0,0587 |
| 0,442 | 0,2237 | -0,0925 | -0,0077 | -0,3052 | -0,3508 | 0,1256 | 0,3031 | 0,4298 | -0,0432 |
| -0,5523 | -0,3553 | -0,0846 | -0,4365 | 0,4146 | 0,5024 | 0,0793 | -0,4721 | 0,2215 | -0,8594 |
| -0,1766 | 0,2413 | 0,3659 | 0,1683 | 0,9318 | -0,0539 | 0,1444 | -0,6612 | -0,1835 | -0,6348 |
| -0,1079 | 0,4157 | -0,2122 | -0,7816 | -0,414 | 0,3549 | -0,0381 | 0,1275 | 0,3174 | -0,7128 |

| **2006** | **2007** | **2008** | **2009** | **2010** | **2011** | **2012** | **2013** | **2014** | **2015** |
|---|---|---|---|---|---|---|---|---|---|
| -0,4092 | -0,0139 | -0,3793 | -0,4536 | -0,0878 | 0,3205 | 0,0432 | 0,2207 | -0,3926 | -0,1548 |
| -0,0685 | -0,6906 | 0,0446 | 0,5355 | 0,3423 | -0,5313 | -0,0264 | -0,2291 | -0,3444 | -0,4149 |
| -0,1056 | 0,0762 | 0,335 | 0,4087 | -0,1824 | 0,3066 | -0,0579 | -0,1129 | 0,182 | 0,3133 |
| 0,2524 | 0,1681 | -0,1739 | -0,188 | -0,2532 | 0,3112 | -0,0835 | -0,0504 | -0,423 | 0,1051 |
| -0,3218 | 0,4345 | 0,1482 | 0,4306 | -0,1673 | -0,3761 | 0,2715 | 0,0138 | -0,2542 | -0,03 |
| 0,206 | 0,6577 | 0,7618 | -0,1257 | 0,7523 | -0,0536 | -0,2364 | 0,1894 | -0,0169 | 0,3911 |
| -0,1798 | 1 | 0,1476 | -0,3865 | 0,2981 | -0,0953 | 0,348 | -0,1227 | 0,4511 | 0,3391 |
| -0,0363 | 0 | -0,2261 | -0,3033 | -0,052 | 0,3184 | 0,5212 | 0,1442 | -0,1885 | 0,0603 |
| -0,2167 | -0,2368 | -0,3423 | -0,0612 | 0,5113 | -0,0129 | 0,4126 | -0,234 | 0,0025 | -0,2632 |

| **2016** | **2017** | **2018** | **2019** | **2020** |
|---|---|---|---|---|
| -0,0882 | -0,4078 | -0,4435 | 0,313 | 0,0038 |
| 0,0971 | -0,1443 | 0,0188 | -0,1237 | -0,1563 |
| -0,1917 | -0,5038 | -0,0169 | 0,1859 | 0,0281 |
| 0,9043 | 0,0974 | -0,4783 | 0,083 | -0,4826 |
| -0,6753 | 0,3644 | -0,3298 | -0,2314 | 0,4503 |
| 0,2215 | -0,5211 | -1 | 0,1817 | -0,3392 |
| 0,0719 | -0,1006 | 0 | -0,0641 | -0,0266 |
| -0,2593 | -0,3171 | 0 | -0,6408 | -0,0219 |
| 0,1973 | -0,0844 | 0,1614 | 0,2246 | -0,0263 |

Table B3.2: Error Sample - West Atlantic

| 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
|---|---|---|---|---|---|---|---|---|---|
| -0,1694 | -0,4175 | 0,3358 | -0,2229 | -0,0035 | -0,0003 | -0,0781 | 0,1241 | -0,0826 | -0,5209 |
| 0,1565 | 0,4001 | -0,1715 | 0,0816 | -0,0041 | -0,1817 | 0,6688 | -0,0806 | -0,4413 | 0,5541 |
| -0,0861 | 0,6926 | 0,4722 | -0,6089 | -0,5261 | -0,1795 | -0,0582 | 0,0028 | 0,262 | -0,2261 |
| 0,0366 | -0,185 | -0,205 | -0,3383 | -0,0149 | 0,1188 | -0,1034 | -0,1565 | -0,0113 | -0,0547 |
| -0,1292 | -0,1585 | 0,5071 | 0,0253 | -0,3384 | -0,352 | 0,5434 | 0,6065 | 0,5119 | -0,3932 |
| 0,4198 | 0,1215 | 0,123 | -0,5345 | 0,0713 | 0,2146 | -0,1993 | -0,3139 | -0,0581 | -0,33 |
| -0,6002 | 0,1471 | 0,3989 | 0,1999 | 0,4323 | 0,4132 | -0,2847 | -0,7218 | 0,56 | 0,1223 |
| 0,204 | -0,1181 | -0,2071 | 0,0779 | -0,1201 | -0,1654 | 0,0173 | 0,3925 | -0,7889 | 0,1831 |
| -0,1035 | 0,2489 | -0,2457 | -0,3954 | 0,2087 | -0,436 | -0,4071 | -0,1894 | -0,466 | 0,1749 |

| 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|---|---|---|
| -0,3221 | 0,0761 | -0,3934 | -0,1087 | -0,2947 | -0,3677 | -0,3197 | -0,1257 | 0,0544 | -0,0364 |
| 0,1019 | -0,3503 | 0,1942 | 0,3975 | -0,2805 | 0,0423 | -0,0435 | -0,3642 | 0,3706 | -0,0048 |
| -0,2259 | -0,063 | -0,0064 | 0,0873 | 0,3313 | 0,1946 | -0,548 | 0,2183 | 0,254 | 0,1992 |
| -0,2997 | 0,4046 | 0,138 | 0,401 | -0,1558 | -0,3502 | 0,2528 | 0,0129 | -0,2367 | -0,0279 |
| -0,0668 | -0,0623 | 0,0963 | -0,1792 | -0,3061 | -0,1399 | 0,3387 | 0,7428 | 0,0718 | -0,0835 |
| -0,0224 | 0,0874 | 0,4299 | 0,0563 | -0,1697 | 0,5116 | 0,2583 | 0,0722 | 0,2103 | -0,1591 |
| 0,0064 | -0,1271 | -0,4805 | 0,0693 | -0,4303 | -0,2627 | -0,0653 | 0,2366 | 0,1176 | -0,4212 |
| -0,3156 | 0,2956 | -0,7588 | -0,0701 | 0,0182 | -0,1329 | -0,0635 | -0,4737 | -0,3526 | -0,2551 |
| -0,0868 | -0,405 | -0,2781 | -0,3088 | -0,0224 | -0,7559 | -0,2174 | -0,4356 | 0,1032 | 0,1874 |

| 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
|---|---|---|---|---|---|
| 0,3331 | -0,0768 | -0,4751 | 0,003 | 0,0223 | 0,0991 |
| 0,1679 | -0,2243 | -0,2052 | 0,0984 | 0,0334 | 0,5786 |
| 0,4101 | 0,1024 | -0,2107 | -0,0467 | -0,7667 | 0,1482 |
| -0,6289 | 0,3393 | -0,3072 | -0,2155 | 0,4193 | -0,2846 |
| 0,2197 | -0,1526 | 0,583 | 0,3465 | -0,3843 | -0,2097 |
| 0,2681 | 0,0841 | 0,1956 | -0,3279 | 0,4807 | 0,136 |
| 0,4639 | 0,0102 | 0,5855 | -0,3785 | -0,245 | -0,2402 |
| 0,1148 | -0,1835 | 0,4813 | 0,0438 | -0,5831 | -0,1422 |
| 0,0461 | -0,0318 | -0,8249 | 0,0088 | -0,2743 | 0,0831 |